ELECTROSTATICS - I
– Electrostatic Force

1. Frictional Electricity
2. Properties of Electric Charges
3. Coulomb's Law
4. Coulomb's Law in Vector Form
5. Units of Charge
6. Relative Permittivity or Dielectric Constant
7. Continuous Charge Distribution
   i) Linear Charge Density
   ii) Surface Charge Density
   iii) Volume Charge Density

Frictional Electricity:

Frictional electricity is the electricity produced by rubbing two suitable bodies and transfer of electrons from one body to other.

Electrons in glass are loosely bound in it than the electrons in silk. So, when glass and silk are rubbed together, the comparatively loosely bound electrons from glass get transferred to silk.
As a result, glass becomes positively charged and silk becomes negatively charged.

Electrons in fur are loosely bound in it than the electrons in ebonite. So, when ebonite and fur are rubbed together, the comparatively loosely bound electrons from fur get transferred to ebonite.
As a result, ebonite becomes negatively charged and fur becomes positively charged.

It is very important to note that the electrification of the body (whether positive or negative) is due to transfer of electrons from one body to another.

i.e. If the electrons are transferred from a body, then the deficiency of electrons makes the body positive.

If the electrons are gained by a body, then the excess of electrons makes the body negative.

If the two bodies from the following list are rubbed, then the body appearing early in the list is positively charges whereas the latter is negatively charged.

Fur, Glass, Silk, Human body, Cotton, Wood, Sealing wax, Amber, Resin, Sulphur, Rubber, Ebonite.

Column I (+ve Charge)	Column II (-ve Charge)
Glass	Silk
Wool, Flannel	Amber, Ebonite, Rubber, Plastic
Ebonite	Polythene
Dry hair	Comb

Properties of Charges:

1. There exists only two types of charges, namely positive and negative.
2. Like charges repel and unlike charges attract each other.
3. Charge is a scalar quantity.
4. Charge is additive in nature. eg. $+2\text{ C} + 5\text{ C} - 3\text{ C} = +4\text{ C}$
5. Charge is quantized.

   i.e. Electric charge exists in discrete packets rather than in continuous amount.

   It can be expressed in integral multiples fundamental electronic charge ($e = 1.6 \times 10^{-19}$ C)

   $$q = \pm ne \qquad \text{where } n = 1, 2, 3, \ldots\ldots\ldots$$

6. Charge is conserved.

   i.e. The algebraic sum of positive and negative charges in an isolated system remains constant.

   eg. When a glass rod is rubbed with silk, negative charge appears on the silk and an equal amount of positive charge appear on the glass rod. The net charge on the glass-silk system remains zero before and after rubbing.

   It does not change with velocity also.

Note: Recently, the existence of quarks of charge ⅓ e and ⅔ e has been postulated. If the quarks are detected in any experiment with concrete practical evidence, then the minimum value of 'quantum of charge' will be either ⅓ e or ⅔ e. However, the law of quantization will hold good.

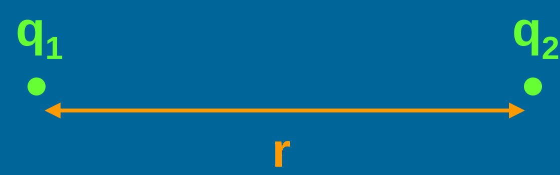
Coulomb's Law – Force between two point electric charges:

The electrostatic force of interaction (attraction or repulsion) between two point electric charges is directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the line joining the two charges.

Strictly speaking, Coulomb's law applies to stationary point charges.

$F \alpha q_1 q_2$

$F \alpha 1 / r^2$



or $F \alpha \dfrac{q_1 q_2}{r^2}$ or $F = k \dfrac{q_1 q_2}{r^2}$

where k is a positive constant of proportionality called electrostatic force constant or Coulomb constant.

In vacuum, $k = \dfrac{1}{4\pi\varepsilon_0}$ where ε_0 is the permittivity of free space

In medium, $k = \dfrac{1}{4\pi\varepsilon}$ where ε is the absolute electric permittivity of the dielectric medium

The dielectric constant or relative permittivity or specific inductive capacity or dielectric coefficient is given by

$$\boxed{K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}}$$

∴ In vacuum, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2}$

In medium, $F = \dfrac{1}{4\pi\varepsilon_0\varepsilon_r}\dfrac{q_1 q_2}{r^2}$

$$\varepsilon_0 = 8.8542 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$$

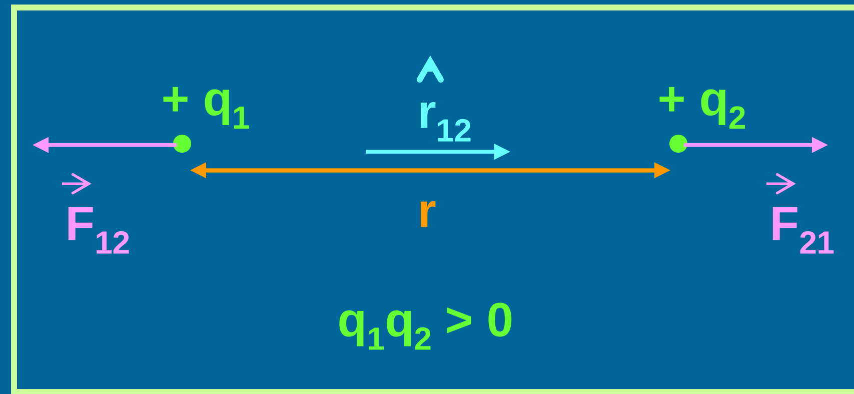
$$\frac{1}{4\pi\varepsilon_0} = 8.9875 \times 10^{9}\ N\ m^2\ C^{-2} \quad \text{or} \quad \frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}\ N\ m^2\ C^{-2}$$

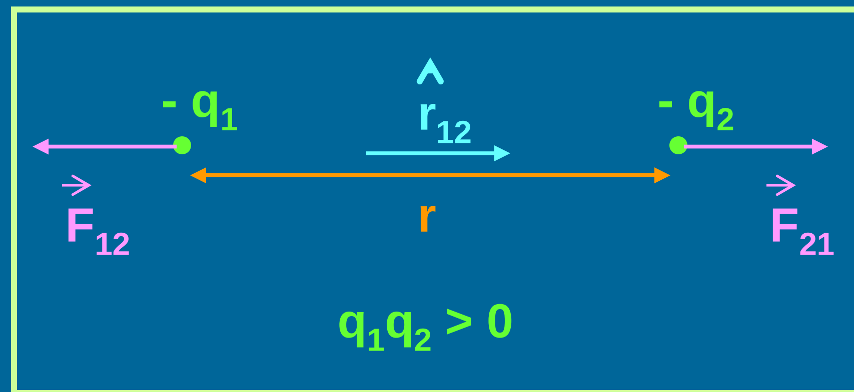
Coulomb's Law in Vector Form:

In vacuum, for $q_1\, q_2 > 0$,

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1\, q_2}{r^2}\,\hat{r}_{21}$$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1\, q_2}{r^2}\,\hat{r}_{12}$$

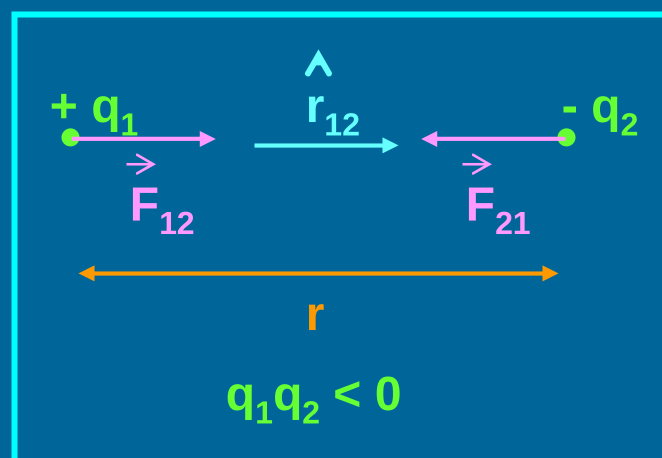




In vacuum, for $q_1\, q_2 < 0$,

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1\, q_2}{r^2}\,\hat{r}_{12} \quad \& \quad \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1\, q_2}{r^2}\,\hat{r}_{21}$$

$\therefore$ $\boxed{\vec{F}_{12} = -\,\vec{F}_{21}}$ (in all the cases)



$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{12} \quad \& \quad \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{21}$$

Note: The cube term of the distance is simply because of vector form.

Otherwise the law is 'Inverse Square Law' only.

Units of Charge:

In SI system, the unit of charge is coulomb (C).

One coulomb of charge is that charge which when placed at rest in vacuum at a distance of one metre from an equal and similar stationary charge repels it and is repelled by it with a force of 9×10^9 newton.

In cgs electrostatic system, the unit of charge is 'statcoulomb' or 'esu of charge'.

In cgs electrostatic system, $k = 1 / K$ where K is 'dielectric constant'.

For vacuum, $K = 1$.

$$\therefore \quad F = \frac{q_1 q_2}{r^2}$$

If $q_1 = q_2 = q$ (say), $r = 1$ cm and $F = 1$ dyne, then $q = \pm 1$ statcoulomb.

In cgs electromagnetic system, the unit of charge is 'abcoulomb' or 'emu of charge'.

1 emu of charge = c esu of charge

1 emu of charge = 3×10^{10} esu of charge

1 coulomb of charge = 3×10^{9} statcoulomb

1 abcoulomb = 10 coulomb

Relative Permittivity or Dielectric Constant or Specific Inductive Capacity or Dielectric Coefficient:

The dielectric constant or relative permittivity or specific inductive capacity or dielectric coefficient is given by the ratio of the absolute permittivity of the medium to the permittivity of free space.

$$K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

The dielectric constant or relative permittivity or specific inductive capacity or dielectric coefficient can also be defined as the ratio of the electrostatic force between two charges separated by a certain distance in vacuum to the electrostatic force between the same two charges separated by the same distance in that medium.

$$K = \varepsilon_r = \frac{F_v}{F_m}$$

Dielectric constant has no unit.

Continuous Charge Distribution:

Any charge which covers a space with dimensions much less than its distance away from an observation point can be considered a point charge.

A system of closely spaced charges is said to form a continuous charge distribution.

It is useful to consider the density of a charge distribution as we do for density of solid, liquid, gas, etc.

(i) Line or Linear Charge Density (λ):

If the charge is distributed over a straight line or over the circumference of a circle or over the edge of a cuboid, etc, then the distribution is called 'linear charge distribution'.

Linear charge density is the charge per unit length. Its SI unit is C / m.

$$\lambda = \frac{q}{l} \quad \text{or} \quad \lambda = \frac{dq}{dl}$$

dq

$$+ + + + + \boxed{+ +} + + + + +$$

dl

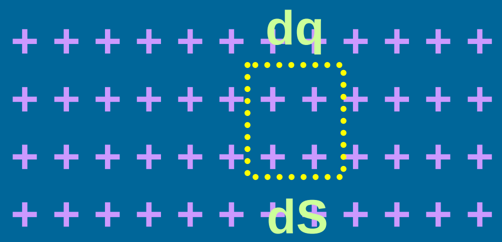
Total charge on line l, $$q = \int_{l} \lambda \, dl$$

(ii) Surface Charge Density (σ):

If the charge is distributed over a surface area, then the distribution is called 'surface charge distribution'.

Surface charge density is the charge per unit area. Its SI unit is C / m².

$$\sigma = \frac{q}{S} \quad \text{or} \quad \sigma = \frac{dq}{dS}$$



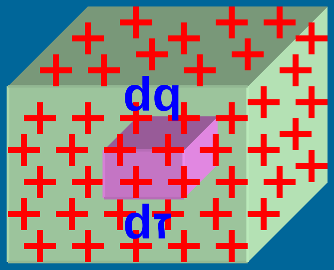
Total charge on surface S, $q = \int_S \sigma \, dS$

(iii) Volume Charge Density (ρ):

If the charge is distributed over a volume, then the distribution is called 'volume charge distribution'.

Volume charge density is the charge per unit volume. Its SI unit is C / m³.

$$\rho = \frac{q}{\tau} \quad \text{or} \quad \rho = \frac{dq}{d\tau}$$



Total charge on volume τ, $q = \int_\tau \rho \, d\tau$

ELECTROSTATICS - II : Electric Field

1. Electric Field
2. Electric Field Intensity or Electric Field Strength
3. Electric Field Intensity due to a Point Charge
4. Superposition Principle
5. Electric Lines of Force
   i) Due to a Point Charge
   ii) Due to a Dipole
   iii) Due to a Equal and Like Charges
   iv) Due to a Uniform Field
6. Properties of Electric Lines of Force
7. Electric Dipole
8. Electric Field Intensity due to an Electric Dipole
9. Torque on an Electric Dipole
10. Work Done on an Electric Dipole

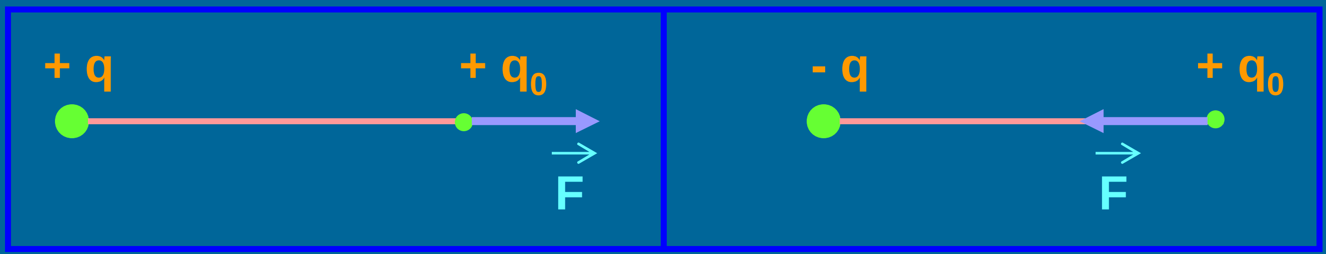
Electric Field:

Electric field is a region of space around a charge or a system of charges within which other charged particles experience electrostatic forces.

Theoretically, electric field extends upto infinity but practically it is limited to a certain distance.

Electric Field Strength or Electric Field Intensity or Electric Field:

Electric field strength at a point in an electric field is the electrostatic force per unit positive charge acting on a vanishingly small positive test charge placed at that point.



q – Source charge, q_0 – Test charge, F – Force & E - Field

$$\vec{E} = \lim_{\Delta q \to 0} \frac{\vec{F}}{\Delta q} \quad \text{or} \quad \vec{E} = \frac{\vec{F}}{q_0} \quad \text{or} \quad \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

The test charge is considered to be vanishingly small because its presence should not alter the configuration of the charge(s) and thus the electric field which is intended to be measured.

Note:

1. Since q_0 is taken positive, the direction of electric field ($\vec{E}$) is along the direction of electrostatic force ($\vec{F}$).

2. Electrostatic force on a negatively charged particle will be opposite to the direction of electric field.

3. Electric field is a vector quantity whose magnitude and direction are uniquely determined at every point in the field.

4. SI unit of electric field is newton / coulomb ($N\ C^{-1}$).

Electric Field due to a Point Charge:

Force exerted on q_0 by q is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q\ q_0}{r^2} \hat{r}$$

or $$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q\ q_0}{r^3} \vec{r}$$

Electric field strength is $$\vec{E} = \frac{\vec{F}}{q_0}$$

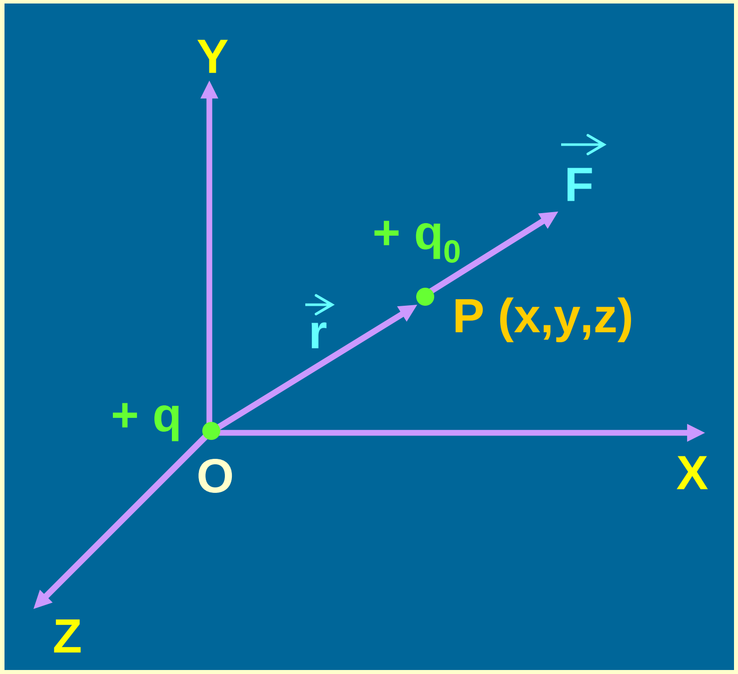
$$\therefore \quad \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^3} \vec{r}$$

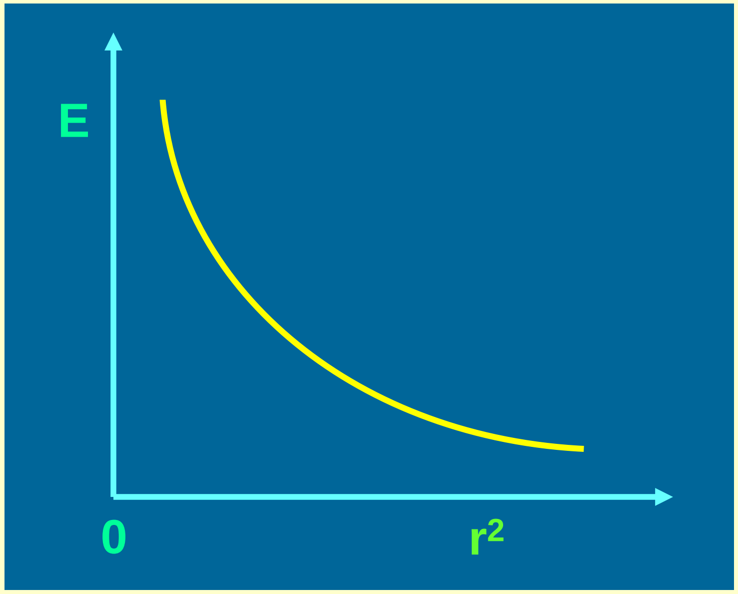
or $$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

The electric field due to a point charge has spherical symmetry.

If q > 0, then the field is radially outwards.

If q < 0, then the field is radially inwards.



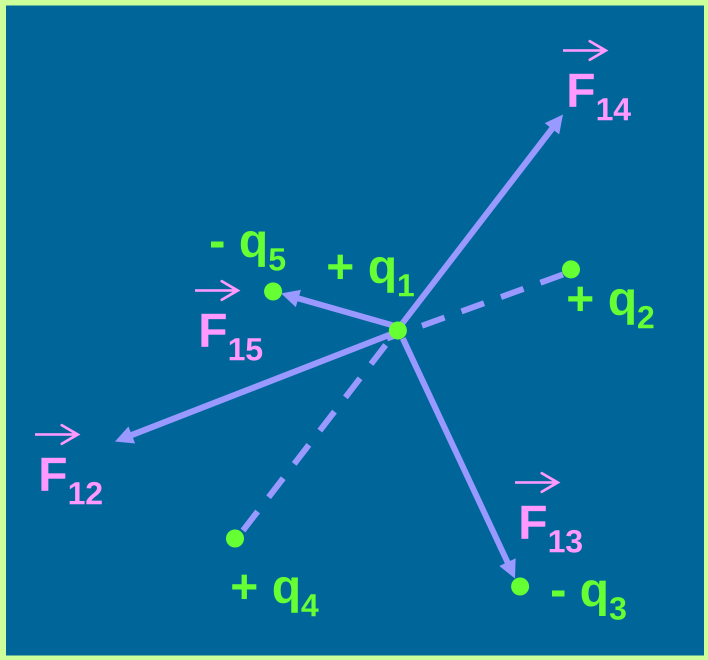


Electric field in terms of co-ordinates is given by

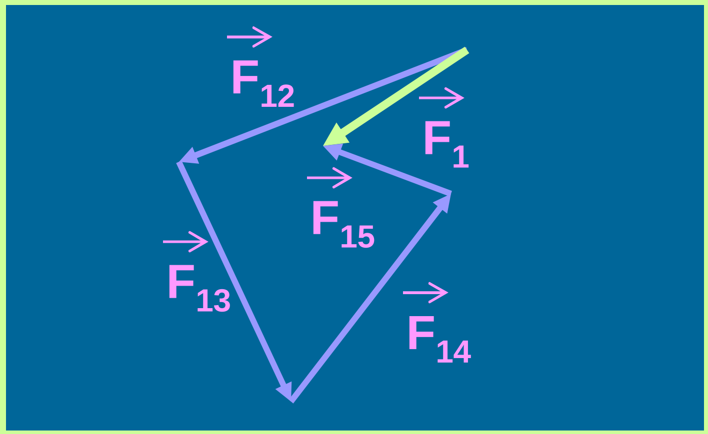
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x^2 + y^2 + z^2)^{3/2}} (x\hat{i} + y\hat{j} + z\hat{k})$$

Superposition Principle:

The electrostatic force experienced by a charge due to other charges is the vector sum of electrostatic forces due to these other charges as if they are existing individually.



$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15}$$



$$\vec{F}_a(\vec{r}_a) = \frac{1}{4\pi\varepsilon_0} \sum_{\substack{b=1 \\ b\neq a}}^{N} q_a q_b \frac{\vec{r}_a - \vec{r}_b}{|\vec{r}_a - \vec{r}_b|^3}$$

In the present example, a = 1 and b = 2 to 5.

If the force is to be found on 2nd charge, then a = 2 and b = 1 and 3 to 5.

Note:

The interactions must be on the charge which is to be studied due to other charges.

The charge on which the influence due to other charges is to be found is assumed to be floating charge and others are rigidly fixed.

For eg. 1st charge (floating) is repelled away by q_2 and q_4 and attracted towards q_3 and q_5.

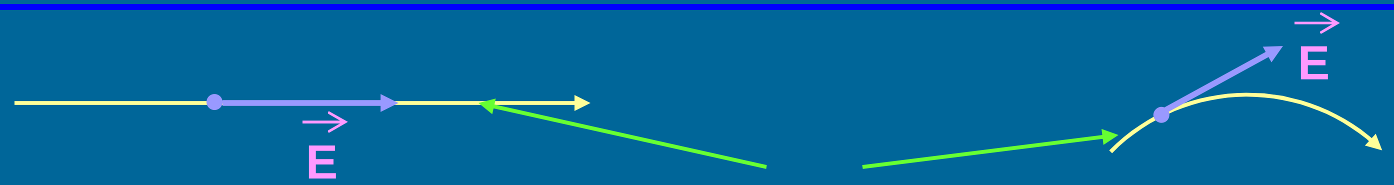
The interactions between the other charges (among themselves) must be ignored. i.e. F_{23}, F_{24}, F_{25}, F_{34}, F_{35} and F_{45} are ignored.

Superposition principle holds good for electric field also.

Electric Lines of Force:

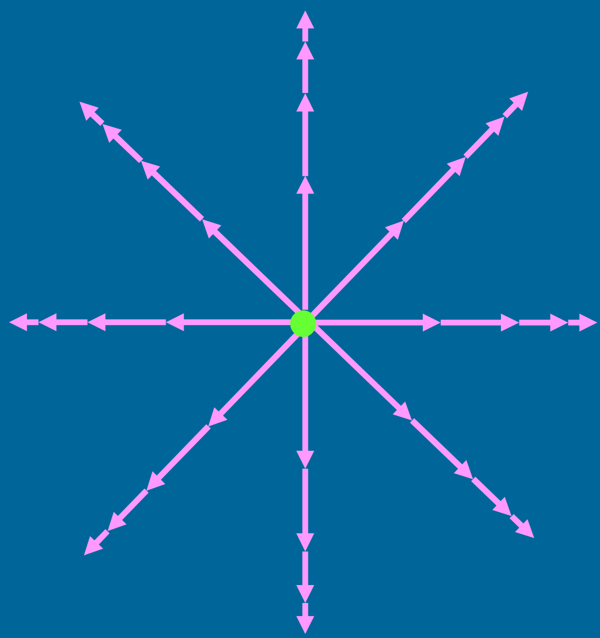
An electric line of force is an imaginary straight or curved path along which a unit positive charge is supposed to move when free to do so in an electric field.

Electric lines of force do not physically exist but they represent real situations.

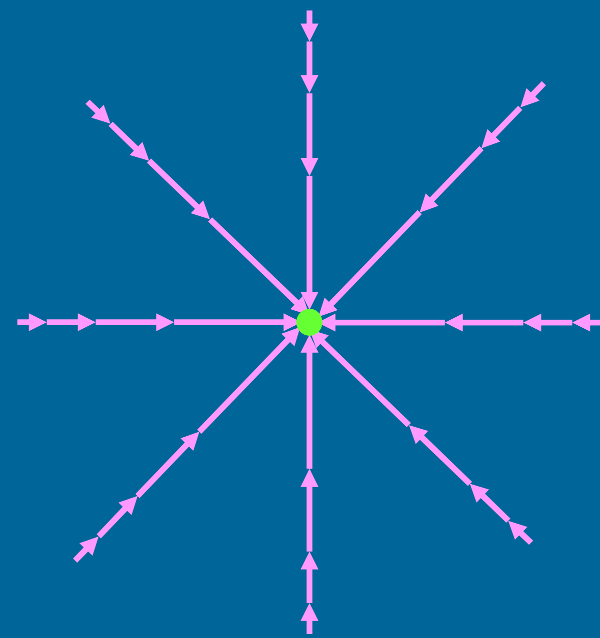


Electric Lines of Force

1. Electric Lines of Force due to a Point Charge:



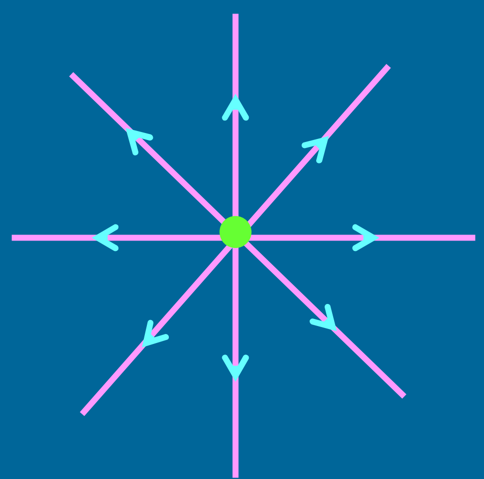
q > 0

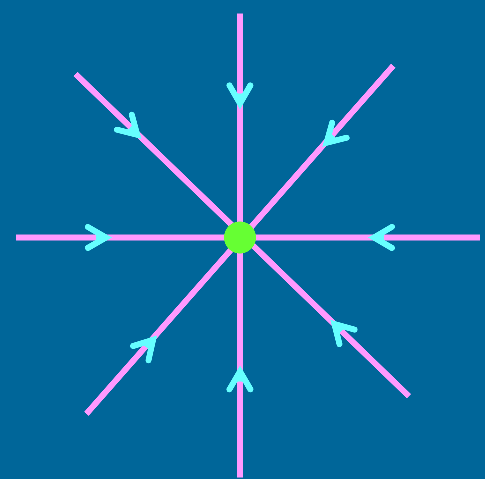


q < 0

a) Representation of electric field in terms of field vectors:

The size of the arrow represents the strength of electric field.

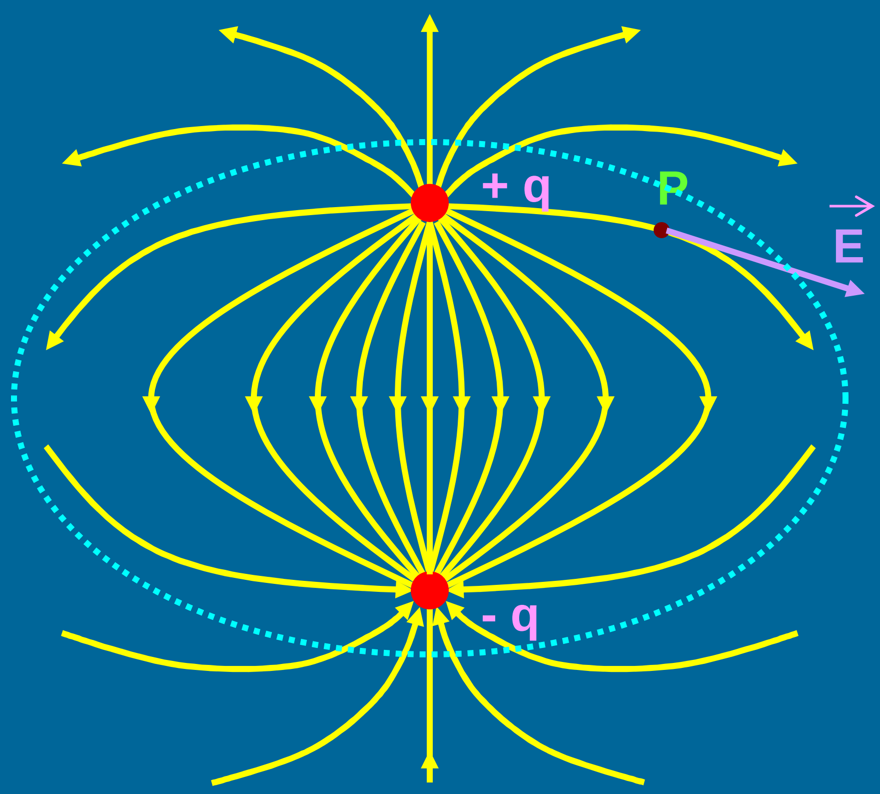




b) Representation of electric field in terms of field lines

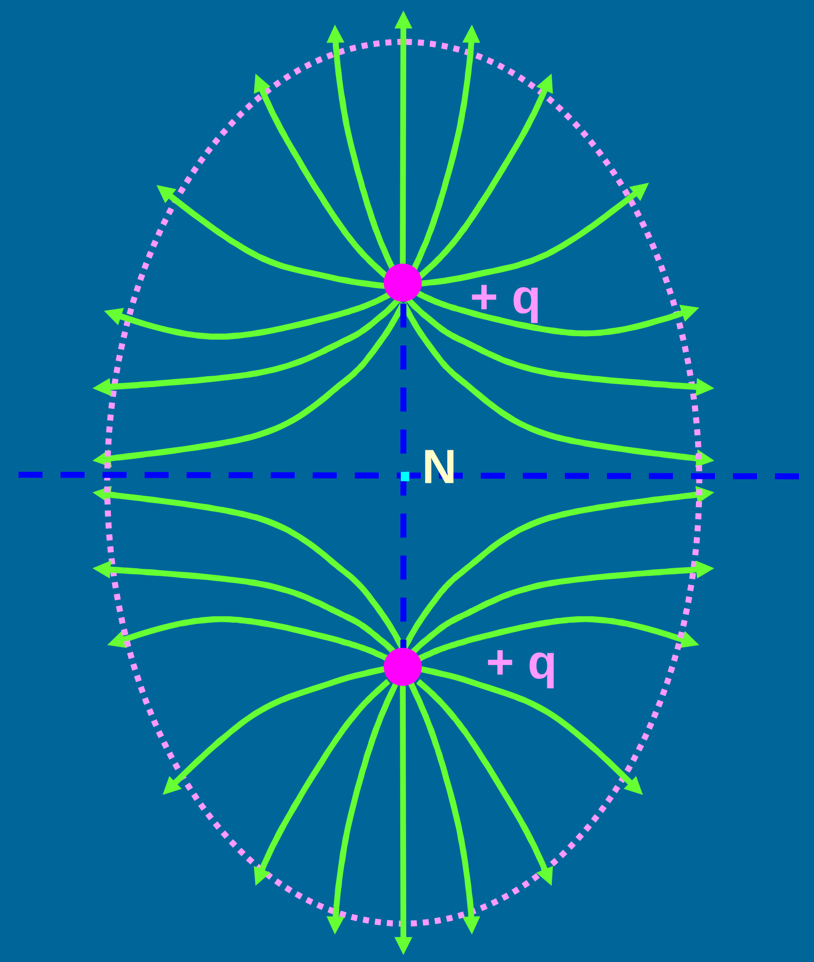
(Easy way of drawing)

2. Electric Lines of Force due to a pair of Equal and Unlike Charges: (Dipole)



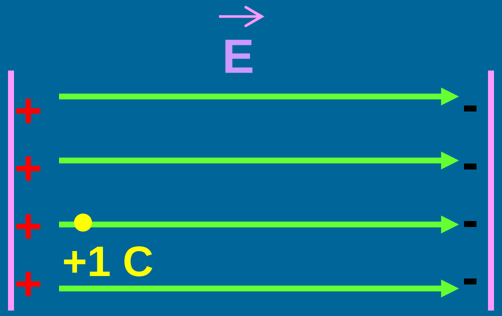
Electric lines of force contract lengthwise to represent attraction between two unlike charges.

3. Electric Lines of Force due to a pair of Equal and Like Charges:



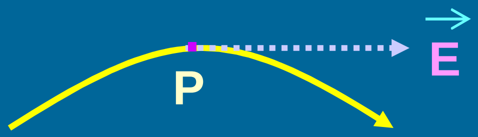
Electric lines of force exert lateral (sideways) pressure to represent repulsion between two like charges.

4. Electric Lines of Force due to a Uniform Field:



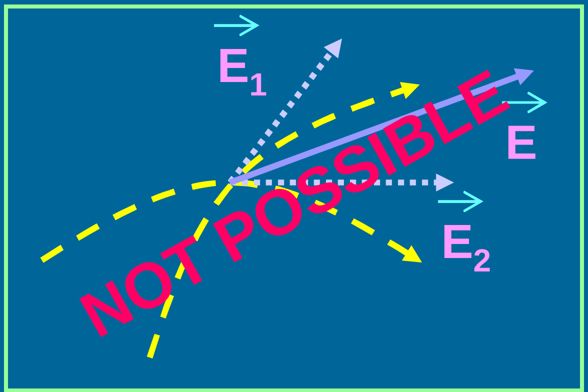
Properties of Electric Lines of Force or Field Lines:

1. The electric lines of force are imaginary lines.
2. A unit positive charge placed in the electric field tends to follow a path along the field line if it is free to do so.
3. The electric lines of force emanate from a positive charge and terminate on a negative charge.
4. The tangent to an electric field line at any point gives the direction of the electric field at that point.

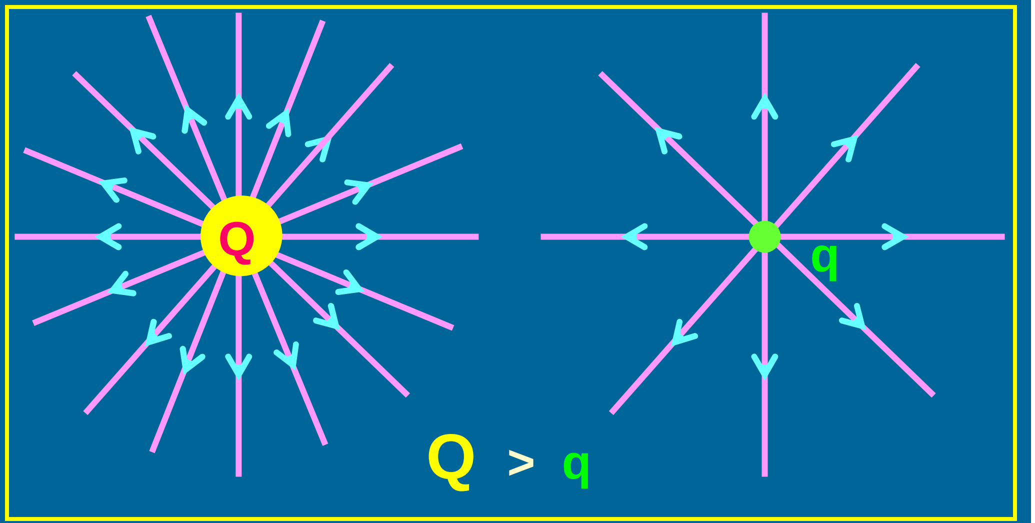


5. Two electric lines of force can never cross each other. If they do, then at the point of intersection, there will be two tangents. It means there are two values of the electric field at that point, which is not possible.

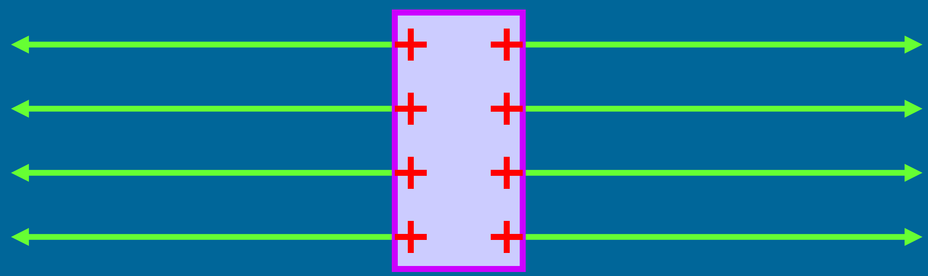
   Further, electric field being a vector quantity, there can be only one resultant field at the given point, represented by one tangent at the given point for the given line of force.

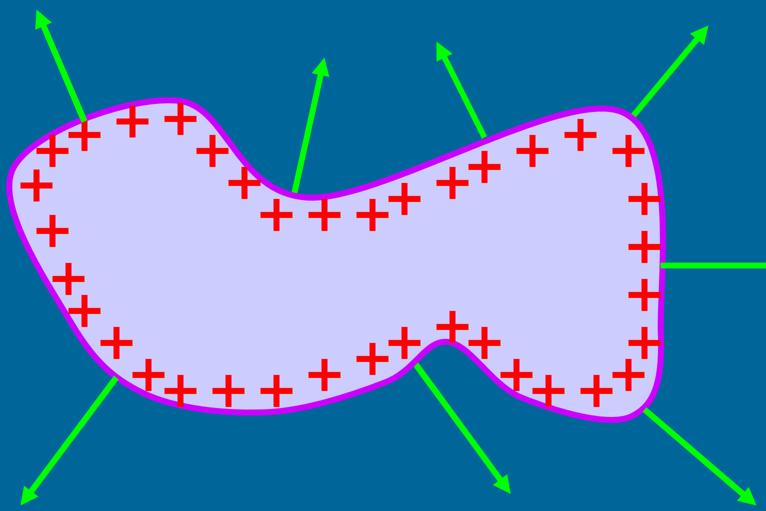


6. Electric lines of force are closer (crowded) where the electric field is stronger and the lines spread out where the electric field is weaker.



7. Electric lines of force are perpendicular to the surface of a positively or negatively charged body.





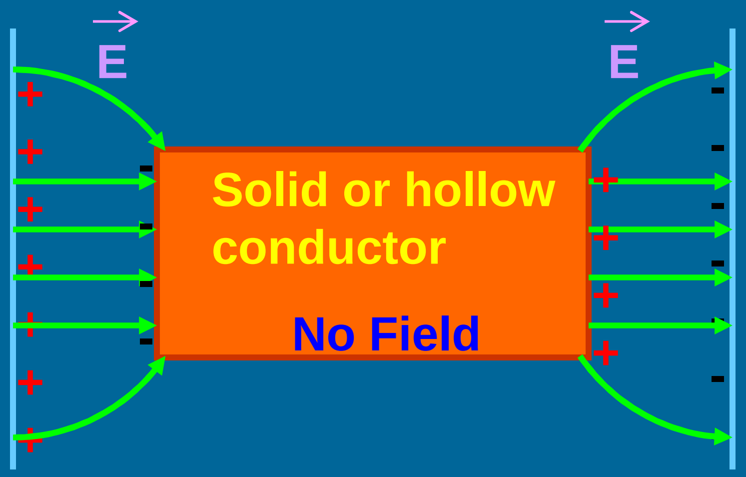
8. Electric lines of force contract lengthwise to represent attraction between two unlike charges.

9. Electric lines of force exert lateral (sideways) pressure to represent repulsion between two like charges.

10. The number of lines per unit cross – sectional area perpendicular to the field lines (i.e. density of lines of force) is directly proportional to the magnitude of the intensity of electric field in that region.

$$\frac{\Delta N}{\Delta A} \propto E$$

11. Electric lines of force do not pass through a conductor. Hence, the interior of the conductor is free from the influence of the electric field.



(Electrostatic Shielding)

12. Electric lines of force can pass through an insulator.

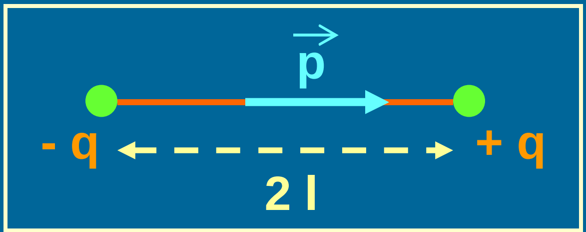
Electric Dipole:

Electric dipole is a pair of equal and opposite charges separated by a very small distance.

The electric field produced by a dipole is known as dipole field.

Electric dipole moment is a vector quantity used to measure the strength of an electric dipole.

$$\vec{p} = (q \times 2l)\ \hat{l}$$



The magnitude of electric dipole moment is the product of magnitude of either charge and the distance between the two charges.

The direction is from negative to positive charge.

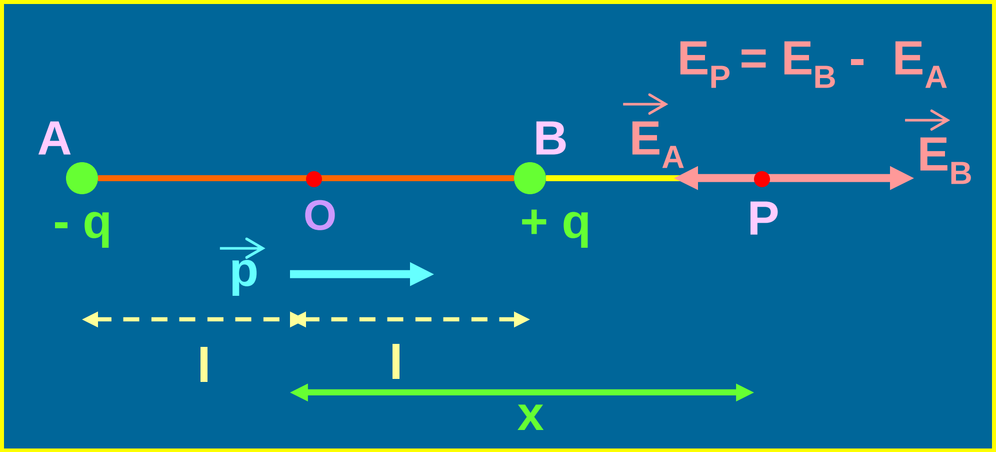
The SI unit of ‘p’ is ‘coulomb metre (C m)’.

Note:

An ideal dipole is the dipole in which the charge becomes larger and larger and the separation becomes smaller and smaller.

Electric Field Intensity due to an Electric Dipole:

i) At a point on the axial line:



Resultant electric field intensity at the point P is

$$\vec{E}_P = \vec{E}_A + \vec{E}_B$$

The vectors $\vec{E}_A$ and $\vec{E}_B$ are collinear and opposite.

$$\therefore \quad |\vec{E}_P| = |\vec{E}_B| - |\vec{E}_A|$$

$$\vec{E}_A = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x+l)^2} \hat{i}$$

$$\vec{E}_B = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x-l)^2} \hat{i}$$

$$|\vec{E}_P| = \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{(x-l)^2} - \frac{q}{(x+l)^2} \right]$$

$$|\vec{E}_P| = \frac{1}{4\pi\varepsilon_0} \frac{2\,(q\,.\,2l)\,x}{(x^2 - l^2)^2}$$

$$\boxed{|\vec{E}_P| = \frac{1}{4\pi\varepsilon_0} \frac{2\,p\,x}{(x^2 - l^2)^2}}$$

$$\boxed{\vec{E}_P = \frac{1}{4\pi\varepsilon_0} \frac{2\,p\,x}{(x^2 - l^2)^2} \hat{i}}$$

If l << x, then

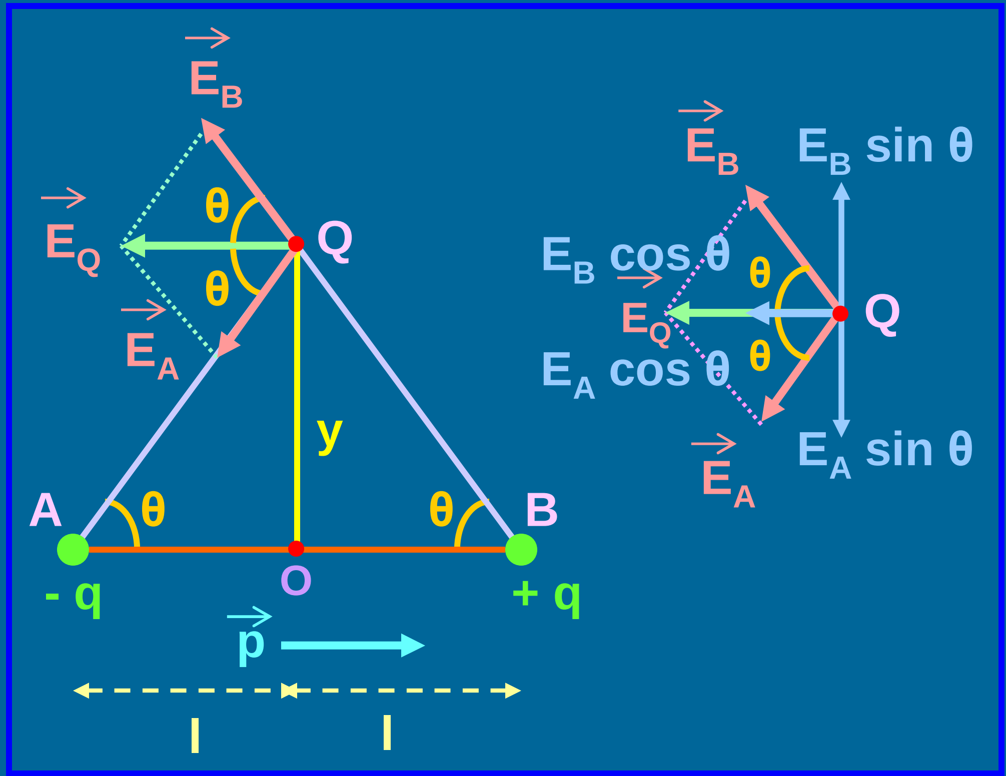
$$\boxed{E_P \approx \frac{2\,p}{4\pi\varepsilon_0\, x^3}}$$

The direction of electric field intensity at a point on the axial line due to a dipole is always along the direction of the dipole moment.

ii) At a point on the equatorial line:

Resultant electric field intensity at the point Q is

$$\vec{E}_Q = \vec{E}_A + \vec{E}_B$$



The vectors $\vec{E}_A$ and $\vec{E}_B$ are acting at an angle 2θ.

$$\vec{E}_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x^2 + l^2)}\,\hat{i}$$

$$\vec{E}_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x^2 + l^2)}\,\hat{i}$$

The vectors $E_A \sin\theta$ and $E_B \sin\theta$ are opposite to each other and hence cancel out.

The vectors $E_A \cos\theta$ and $E_B \cos\theta$ are acting along the same direction and hence add up.

$$\therefore \quad E_Q = E_A \cos\theta + E_B \cos\theta$$

$$E_Q = \frac{2}{4\pi\varepsilon_0}\frac{q}{(x^2 + l^2)}\frac{l}{(x^2 + l^2)^{1/2}}$$

$$E_Q = \frac{1}{4\pi\varepsilon_0}\frac{q \cdot 2l}{(x^2 + l^2)^{3/2}}$$

$$\boxed{E_Q = \frac{1}{4\pi\varepsilon_0}\frac{p}{(x^2 + l^2)^{3/2}}}$$

$$\vec{E}_Q = \frac{1}{4\pi\varepsilon_0} \frac{p}{(x^2 + l^2)^{3/2}} (-\hat{i})$$

If l << y, then

$$E_Q \approx \frac{p}{4\pi\varepsilon_0 \; y^3}$$

The direction of electric field intensity at a point on the equatorial line due to a dipole is parallel and opposite to the direction of the dipole moment.

If the observation point is far away or when the dipole is very short, then the electric field intensity at a point on the axial line is double the electric field intensity at a point on the equatorial line.

i.e. If l << x and l << y, then $E_P = 2\, E_Q$

Torque on an Electric Dipole in a Uniform Electric Field:

The forces of magnitude pE act opposite to each other and hence net force acting on the dipole due to external uniform electric field is zero. So, there is no translational motion of the dipole.

However the forces are along different lines of action and constitute a couple. Hence the dipole will rotate and experience torque.

Torque = Electric Force x ⊥ distance

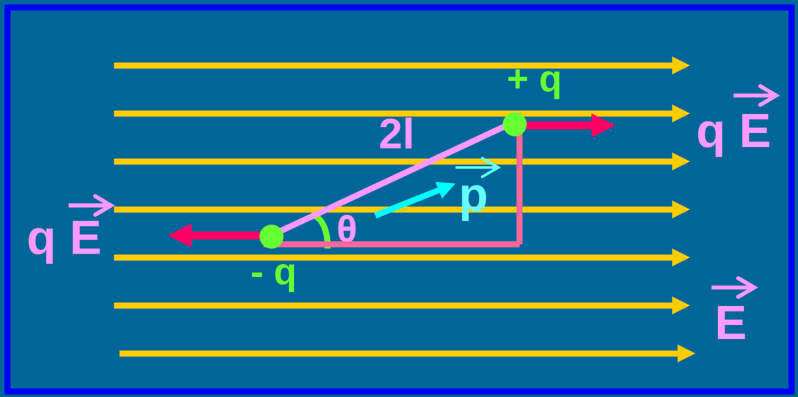
$$t = q E (2l \sin \theta)$$

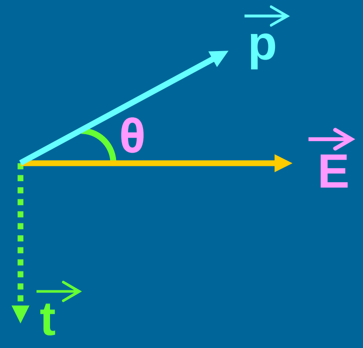
$$= p E \sin \theta$$

$$\boxed{\vec{t} = \vec{p} \times \vec{E}}$$

Direction of Torque is perpendicular and into the plane containing $\vec{p}$ and $\vec{E}$.

SI unit of torque is newton metre (Nm).





Case i: If $\theta = 0°$, then $t = 0$.

Case ii: If $\theta = 90°$, then $t = pE$ (maximum value).

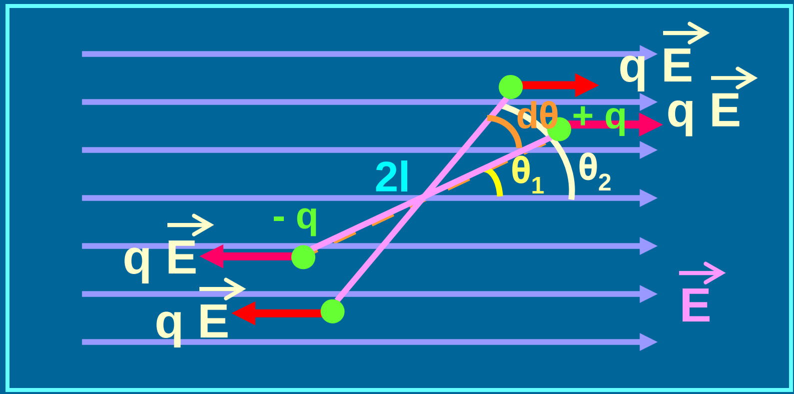
Case iii: If $\theta = 180°$, then $t = 0$.

Work done on an Electric Dipole in Uniform Electric Field:

When an electric dipole is placed in a uniform electric field, it experiences torque and tends to allign in such a way to attain stable equilibrium.

$dW = \tau d\theta$

$= p E \sin\theta \, d\theta$



$$W = \int_{\theta_1}^{\theta_2} p E \sin\theta \, d\theta$$

$$W = p E (\cos\theta_1 - \cos\theta_2)$$

If Potential Energy is arbitrarily taken zero when the dipole is at 90°, then P.E in rotating the dipole and inclining it at an angle θ is

Potential Energy $U = - p E \cos\theta$

Note: Potential Energy can be taken zero arbitrarily at any position of the dipole.

Case i: If $\theta = 0°$, then $U = -pE$ (Stable Equilibrium)

Case ii: If $\theta = 90°$, then $U = 0$

Case iii: If $\theta = 180°$, then $U = pE$ (Unstable Equilibrium)

ELECTROSTATICS - III

- Electrostatic Potential and Gauss's Theorem

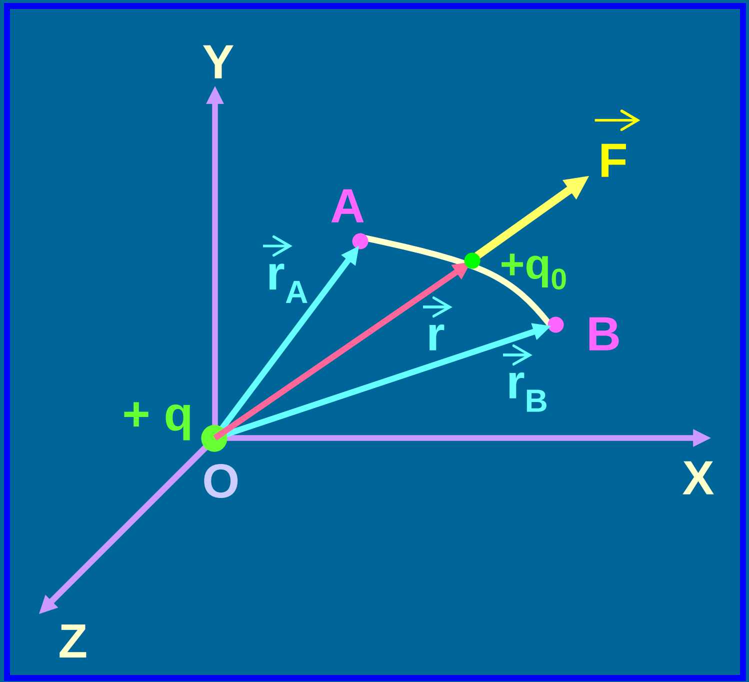
1. Line Integral of Electric Field
2. Electric Potential and Potential Difference
3. Electric Potential due to a Single Point Charge
4. Electric Potential due to a Group of Charges
5. Electric Potential due to an Electric Dipole
6. Equipotential Surfaces and their Properties
7. Electrostatic Potential Energy
8. Area Vector, Solid Angle, Electric Flux
9. Gauss's Theorem and its Proof
10. Coulomb's Law from Gauss's Theorem
11. Applications of Gauss's Theorem:

    Electric Field Intensity due to Line Charge, Plane Sheet of Charge and Spherical Shell

Line Integral of Electric Field (Work Done by Electric Field):

Negative Line Integral of Electric Field represents the work done by the electric field on a unit positive charge in moving it from one point to another in the electric field.

$$W_{AB} = \int dW = -\int_A^B \vec{E} \cdot \vec{dl}$$



Let q_0 be the test charge in place of the unit positive charge.

The force $\vec{F} = +q_0\vec{E}$ acts on the test charge due to the source charge +q.

It is radially outward and tends to accelerate the test charge. To prevent this acceleration, equal and opposite force $-q_0\vec{E}$ has to be applied on the test charge.

Total work done by the electric field on the test charge in moving it from A to B in the electric field is

$$W_{AB} = \int dW = -\int_A^B \vec{E} \cdot \vec{dl} = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$W_{AB} = \int dW = -\int_A^B \vec{E} \cdot \vec{dl} = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

1. The equation shows that the work done in moving a test charge q_0 from point A to another point B along any path AB in an electric field due to +q charge depends only on the positions of these points and is independent of the actual path followed between A and B.
2. That is, the line integral of electric field is path independent.
3. Therefore, electric field is 'conservative field'.
4. Line integral of electric field over a closed path is zero. This is another condition satisfied by conservative field.

$$\oint_A^B \vec{E} \cdot \vec{dl} = 0$$

Note:

Line integral of only static electric field is independent of the path followed. However, line integral of the field due to a moving charge is not independent of the path because the field varies with time.

Electric Potential:

Electric potential is a physical quantity which determines the flow of charges from one body to another.

It is a physical quantity that determines the degree of electrification of a body.

Electric Potential at a point in the electric field is defined as the work done in moving (without any acceleration) a unit positive charge from infinity to that point against the electrostatic force irrespective of the path followed.

$$W_{AB} = -\int_A^B \vec{E} \cdot \vec{dl} = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \quad \text{or} \quad \frac{W_{AB}}{q_0} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

According to definition, $r_A = \infty$ and $r_B = r$

(where r is the distance from the source charge and the point of consideration)

$$\therefore \frac{W_{\infty B}}{q_0} = \frac{q}{4\pi\varepsilon_0 r} = V \qquad \therefore \boxed{V = \frac{W_{\infty B}}{q_0}}$$

SI unit of electric potential is volt (V) or J C^{-1} or Nm C^{-1}.

Electric potential at a point is one volt if one joule of work is done in moving one coulomb charge from infinity to that point in the electric field.

Electric Potential Difference:

Electric Potential Difference between any two points in the electric field is defined as the work done in moving (without any acceleration) a unit positive charge from one point to the other against the electrostatic force irrespective of the path followed.

$$W_{AB} = -\int_A^B \vec{E} \cdot \vec{dl} = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \quad \text{or} \quad \frac{W_{AB}}{q_0} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$\frac{W_{AB}}{q_0} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r_B} - \frac{q}{4\pi\varepsilon_0}\frac{1}{r_A} = V_B - V_A$$

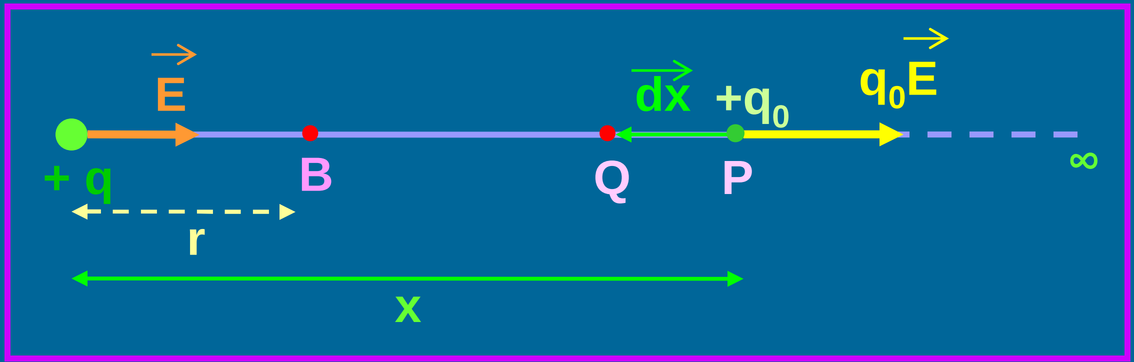
$$\therefore \quad \boxed{V_B - V_A = \Delta V = \frac{W_{AB}}{q_0}}$$

1. Electric potential and potential difference are scalar quantities.
2. Electric potential at infinity is zero.
3. Electric potential near an isolated positive charge ($q > 0$) is positive and that near an isolated negative charge ($q < 0$) is negative.
4. cgs unit of electric potential is stat volt. 1 stat volt = 1 erg / stat coulomb

Electric Potential due to a Single Point Charge:

Let $+q_0$ be the test charge placed at P at a distance x from the source charge +q.

The force $F = +q_0E$ is radially outward and tends to accelerate the test charge.



To prevent this acceleration, equal and opposite force $-q_0E$ has to be applied on the test charge.

Work done to move q_0 from P to Q through 'dx' against q_0E is

$dW = \vec{F} \cdot \vec{dx} = q_0\vec{E} \cdot \vec{dx}$ or $dW = q_0E\,dx\cos 180° = -q_0E\,dx$

$$dW = -\frac{q\,q_0}{4\pi\varepsilon_0\,x^2}\,dx \quad \because \quad E = \frac{q}{4\pi\varepsilon_0\,x^2}$$

Total work done to move q_0 from A to B (from ∞ to r) is

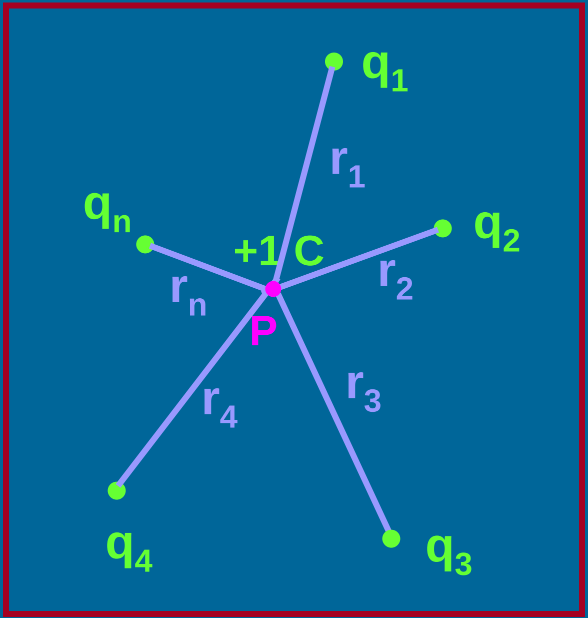
$$W_{\infty B} = \int_{\infty}^{B} dW = -\int_{\infty}^{r} \frac{q\,q_0}{4\pi\varepsilon_0\,x^2}\,dx = -\frac{q\,q_0}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{1}{x^2}\,dx$$

$$\frac{W_{\infty B}}{q_0} = \frac{q}{4\pi\varepsilon_0\,r}$$

$$\boxed{V = \frac{q}{4\pi\varepsilon_0\,r}}$$

Electric Potential due to a Group of Point Charges:

The net electrostatic potential at a point in the electric field due to a group of charges is the algebraic sum of their individual potentials at that point.



$$V_P = V_1 + V_2 + V_3 + V_4 + \ldots\ldots\ldots\ldots + V_n$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}}$$

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\vec{r} - \vec{r}_i|}$$

(in terms of position vector)

1. Electric potential at a point due to a charge is not affected by the presence of other charges.
2. Potential, $V \propto 1 / r$ whereas Coulomb's force $F \propto 1 / r^2$.
3. Potential is a scalar whereas Force is a vector.
4. Although V is called the potential at a point, it is actually equal to the potential difference between the points r and ∞.

Electric Potential due to an Electric Dipole:

i) At a point on the axial line:

$$V_{P_{q+}} = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x - l)}$$

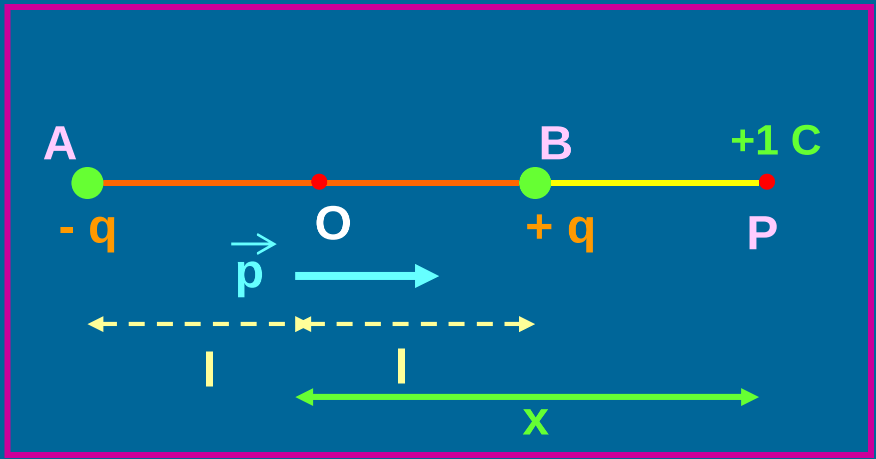
$$V_{P_{q-}} = \frac{1}{4\pi\varepsilon_0} \frac{-q}{(x + l)}$$

$$V_P = V_{P_{q+}} + V_{P_{q-}}$$

$$V_P = \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{(x - l)} - \frac{1}{(x + l)} \right]$$

$$V_P = \frac{1}{4\pi\varepsilon_0} \frac{q \cdot 2l}{(x^2 - l^2)}$$

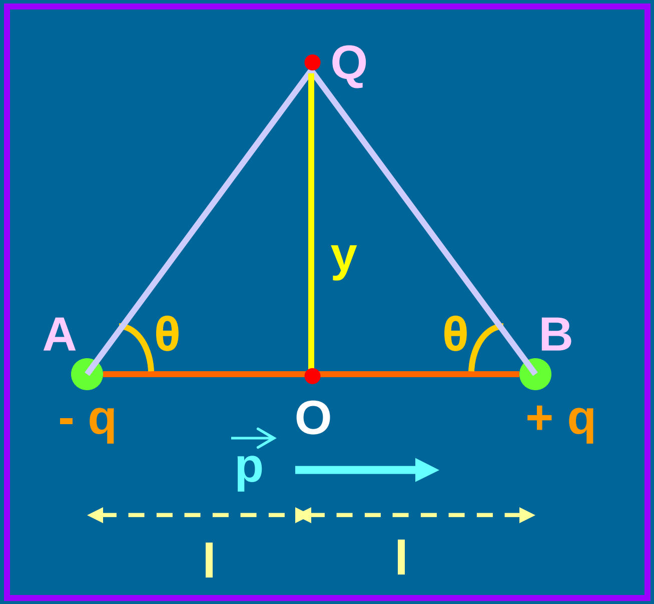
$$\boxed{V_P = \frac{1}{4\pi\varepsilon_0} \frac{p}{(x^2 - l^2)}}$$



ii) At a point on the equatorial line:

$$V_{Q\ q+} = \frac{1}{4\pi\varepsilon_0}\frac{q}{BQ}$$

$$V_{Q\ q-} = \frac{1}{4\pi\varepsilon_0}\frac{-q}{AQ}$$



$$V_Q = V_{P\ q+} + V_{P\ q-}$$

$$V_Q = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{BQ} - \frac{1}{AQ}\right]$$

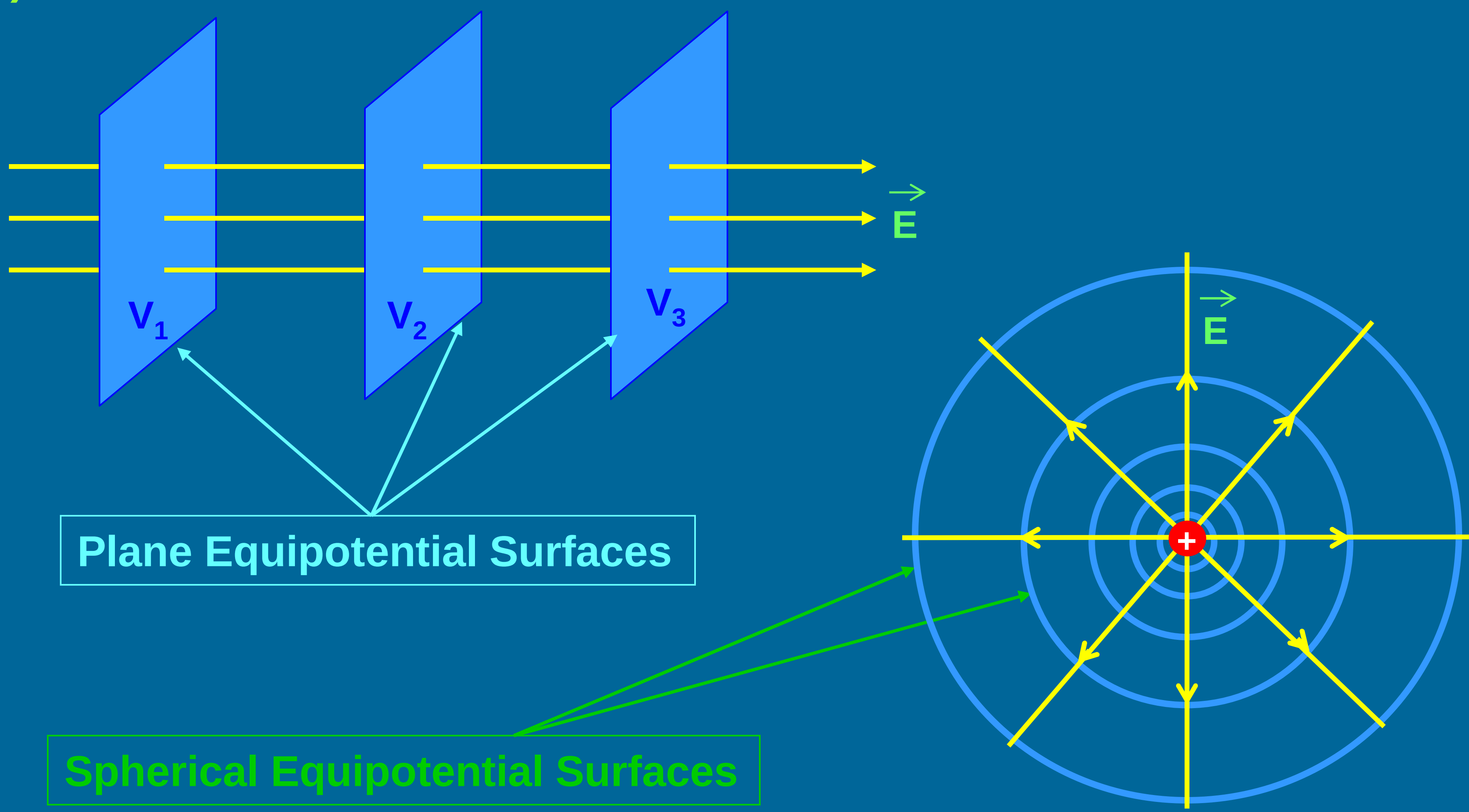
$$\boxed{V_Q = 0} \qquad \because \quad BQ = AQ$$

The net electrostatic potential at a point in the electric field due to an electric dipole at any point on the equatorial line is zero.

Equipotential Surfaces:

A surface at every point of which the potential due to charge distribution is the same is called equipotential surface.

i) For a uniform electric field:



ii) For an isolated charge:

Properties of Equipotential Surfaces:

1. No work is done in moving a test charge from one point to another on an equipotential surface.

$$\boxed{V_B - V_A = \Delta V = \frac{W_{AB}}{q_0}}$$

If A and B are two points on the equipotential surface, then $V_B = V_A$.

$$\therefore \quad \frac{W_{AB}}{q_0} = 0 \qquad \text{or} \qquad W_{AB} = 0$$

2. The electric field is always perpendicular to the element dl of the equipotential surface.

Since no work is done on equipotential surface,

$$W_{AB} = -\int_A^B \vec{E} \cdot \vec{dl} = 0 \qquad \text{i.e.} \qquad E \;\; dl \cos\theta = 0$$

As $E \neq 0$ and $dl \neq 0$, $\cos\theta = 0$ or $\theta = 90°$

3. Equipotential surfaces indicate regions of strong or weak electric fields.

Electric field is defined as the negative potential gradient.

$$\therefore \quad E = -\frac{dV}{dr} \quad \text{or} \quad dr = -\frac{dV}{E}$$

Since dV is constant on equipotential surface, so

$$dr \propto \frac{1}{E}$$

If E is strong (large), dr will be small, i.e. the separation of equipotential surfaces will be smaller (i.e. equipotential surfaces are crowded) and vice versa.

4. Two equipotential surfaces can not intersect.

If two equipotential surfaces intersect, then at the points of intersection, there will be two values of the electric potential which is not possible.

(Refer to properties of electric lines of force)

Note:

Electric potential is a scalar quantity whereas potential gradient is a vector quantity.

The negative sign of potential gradient shows that the rate of change of potential with distance is always against the electric field intensity.

Electrostatic Potential Energy:

The work done in moving a charge q from infinity to a point in the field against the electric force is called electrostatic potential energy.

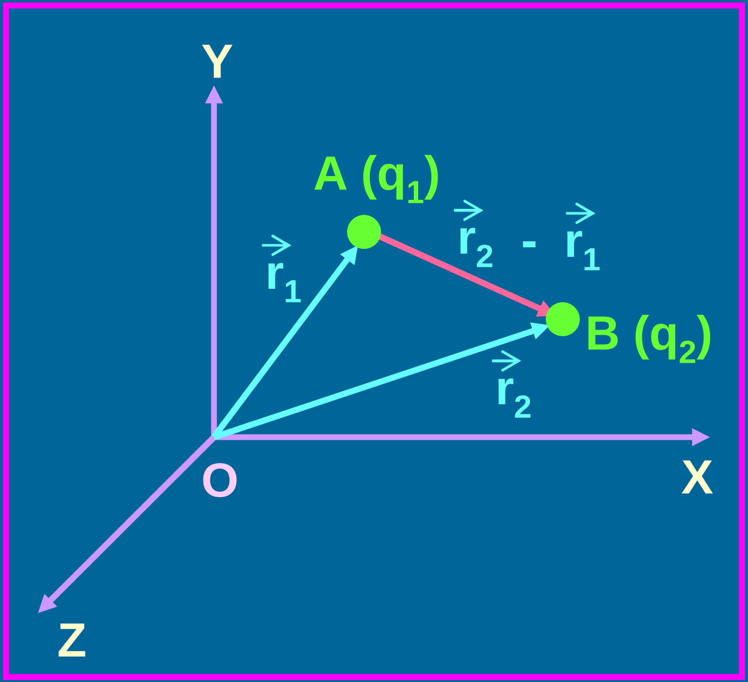
$$\boxed{W = q\,V}$$

i) Electrostatic Potential Energy of a Two Charges System:

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{\left|\vec{r}_2 - \vec{r}_1\right|}}$$

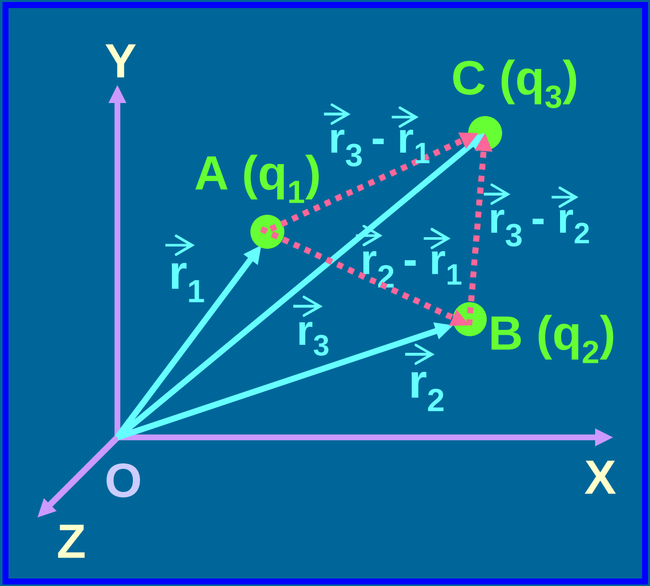
or

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r_{12}}}$$



ii) Electrostatic Potential Energy of a Three Charges System:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{|\vec{r}_3 - \vec{r}_1|} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{|\vec{r}_3 - \vec{r}_2|}$$



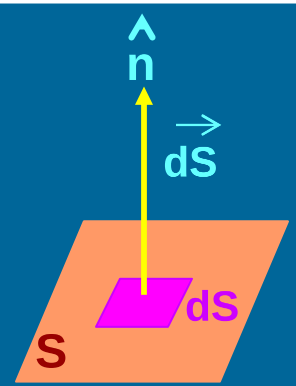
or

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{31}} + \frac{q_2 q_3}{r_{32}}\right]}$$

iii) Electrostatic Potential Energy of an n - Charges System:

$$\boxed{U = \frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{\substack{j=1 \\ i\neq j}}^{n}\frac{q_i\, q_j}{|\vec{r}_j - \vec{r}_i|}\right]}$$

Area Vector:

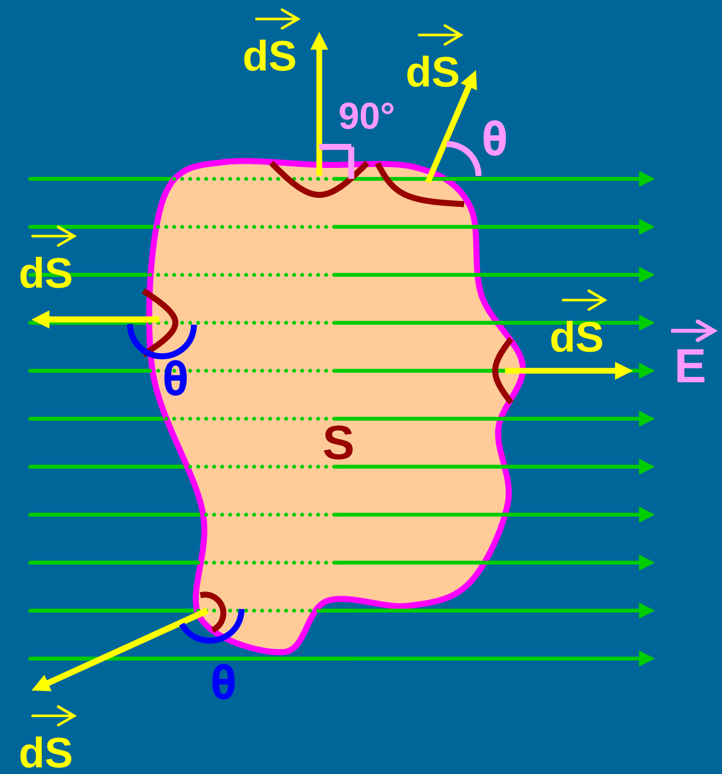


Small area of a surface can be represented by a vector.

$$\vec{dS} = dS\,\hat{n}$$

Electric Flux:

Electric flux linked with any surface is defined as the total number of electric lines of force that normally pass through that surface.



Electric flux dΦ through a small area element dS due to an electric field E at an angle θ with dS is

$$d\Phi = \vec{E} \cdot \vec{dS} = E\ dS \cos\theta$$

Total electric flux Φ over the whole surface S due to an electric field E is

$$\Phi = \int_S \vec{E} \cdot \vec{dS} = E\ S \cos\theta = \vec{E} \cdot \vec{S}$$

Electric flux is a scalar quantity. But it is a property of vector field.

SI unit of electric flux is N m^2 C^{-1} or J m C^{-1}.

Special Cases:

1. For $0° < \theta < 90°$, Φ is positive.
2. For $\theta = 90°$, Φ is zero.
3. For $90° < \theta < 180°$, Φ is negative.

Solid Angle:

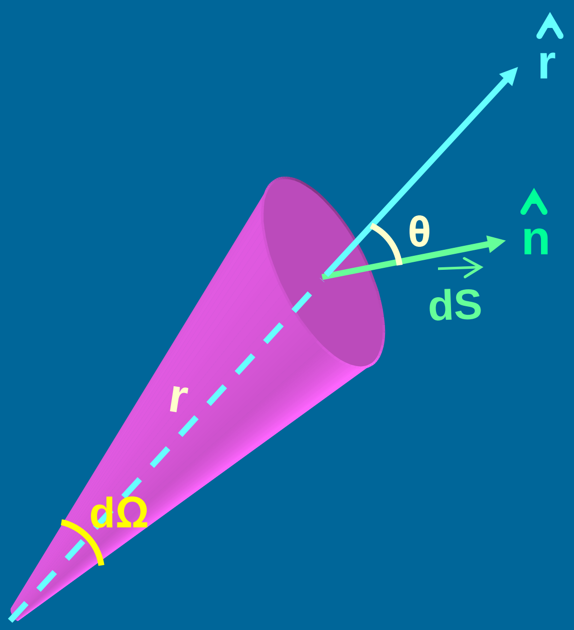
Solid angle is the three-dimensional equivalent of an ordinary two-dimensional plane angle.

SI unit of solid angle is steradian.

Solid angle subtended by area element dS at the centre O of a sphere of radius r is

$$d\Omega = \frac{dS \cos\theta}{r^2}$$

$$\Omega = \int_S d\Omega = \int_S \frac{dS \cos\theta}{r^2} = 4\pi \text{ steradian}$$



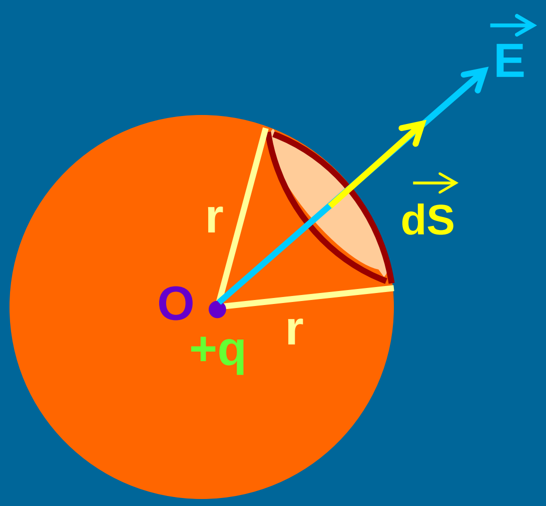
Gauss's Theorem:

The surface integral of the electric field intensity over any closed hypothetical surface (called Gaussian surface) in free space is equal to 1 / ε_0 times the net charge enclosed within the surface.

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i$$

Proof of Gauss's Theorem for Spherically Symmetric Surfaces:

$$d\Phi = \vec{E} \cdot \vec{dS} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} \cdot dS \, \hat{n}$$



$$d\Phi = \frac{1}{4\pi\varepsilon_0} \frac{q\,dS}{r^2} \hat{r} \cdot \hat{n}$$

Here, $\hat{r} \cdot \hat{n} = 1 \times 1 \ \cos 0° = 1$

$$\therefore \quad d\Phi = \frac{1}{4\pi\varepsilon_0} \frac{q\,dS}{r^2}$$

$$\Phi_E = \oint_S d\Phi = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \oint_S dS = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} 4\pi r^2 = \frac{q}{\varepsilon_0}$$

Proof of Gauss's Theorem for a Closed Surface of any Shape:

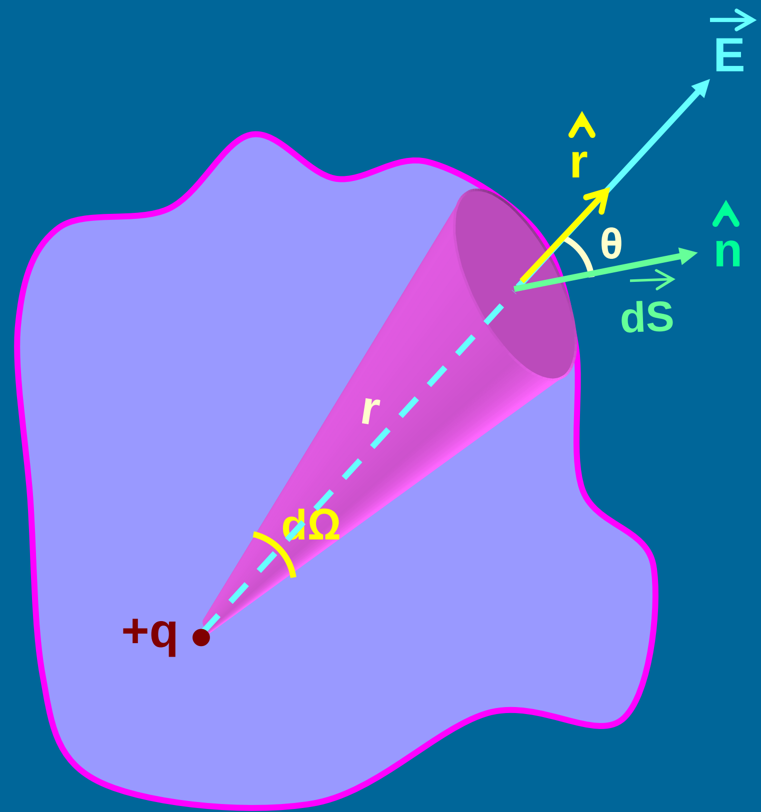
$$d\Phi = \vec{E}\,.\,\vec{dS} = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2}\,\hat{r}\,.\,dS\,\hat{n}$$

$$d\Phi = \frac{1}{4\pi\varepsilon_0}\,\frac{q\,dS}{r^2}\,\hat{r}\,.\,\hat{n}$$

Here, $\hat{r}\,.\,\hat{n} = 1 \times 1\ \cos\theta = \cos\theta$

$$\therefore\quad d\Phi = \frac{q}{4\pi\varepsilon_0}\,\frac{dS\cos\theta}{r^2}$$

$$\Phi_E = \oint_S d\Phi = \frac{q}{4\pi\varepsilon_0}\oint_S d\Omega = \frac{q}{4\pi\varepsilon_0}\,4\pi = \frac{q}{\varepsilon_0}$$

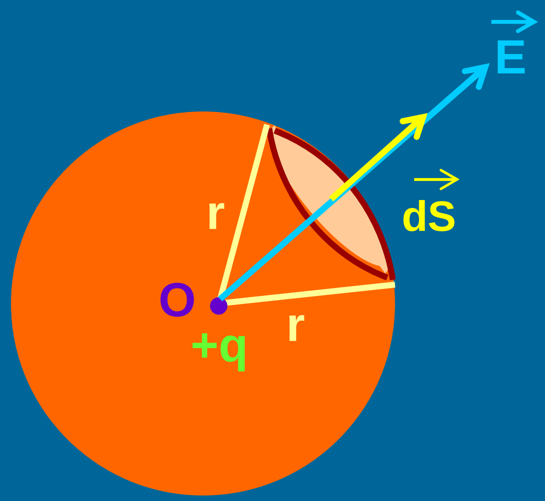


Deduction of Coulomb's Law from Gauss's Theorem:

From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \, . \, \vec{dS} = \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ and $\vec{dS}$ are in the same direction,



$$\therefore \quad \Phi_E = \oint_S E \;\; dS = \frac{q}{\varepsilon_0}$$

$$\text{or} \quad \Phi_E = E \oint_S dS = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi \, r^2 = \frac{q}{\varepsilon_0} \qquad \text{or} \qquad E = \frac{q}{4\pi\varepsilon_0 \; r^2}$$

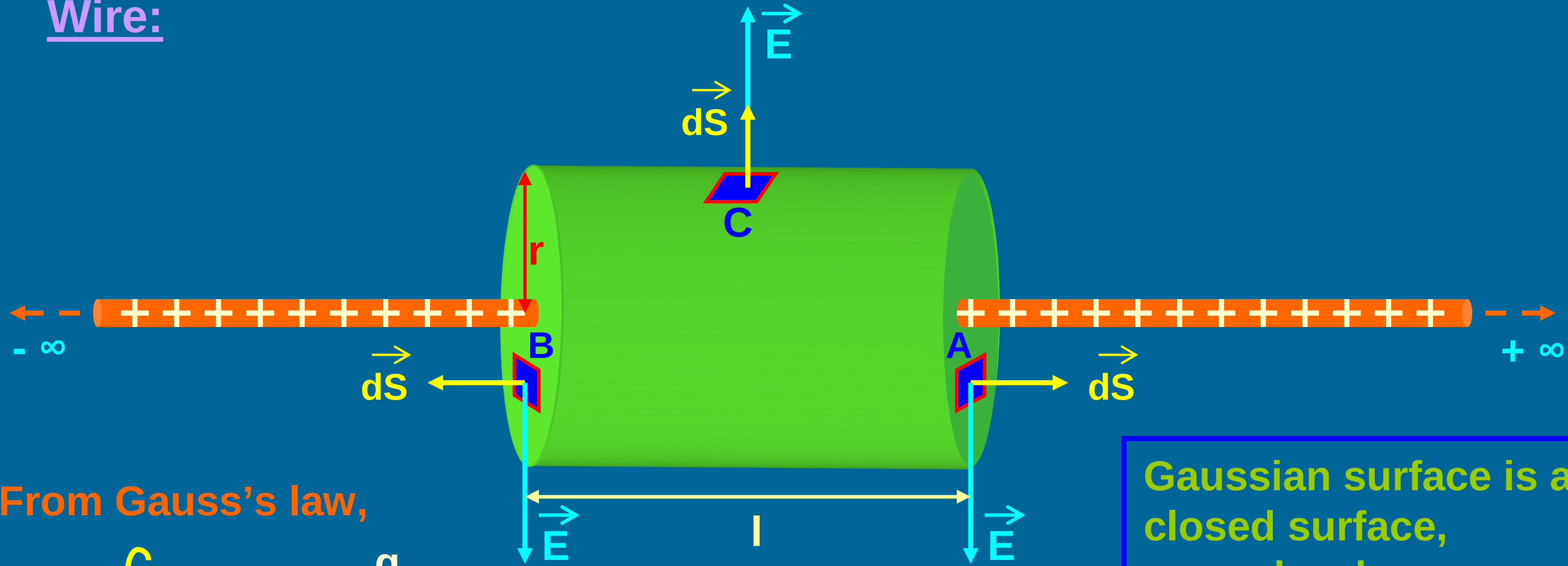
If a charge q_0 is placed at a point where E is calculated, then

$$\boxed{F = \frac{qq_0}{4\pi\varepsilon_0 \; r^2}}$$

which is Coulomb's Law.

Applications of Gauss's Theorem:

1. Electric Field Intensity due to an Infinitely Long Straight Charged Wire:



From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

$$\oint_S \vec{E} \cdot \vec{dS} = \int_A \vec{E} \cdot \vec{dS} + \int_B \vec{E} \cdot \vec{dS} + \int_C \vec{E} \cdot \vec{dS}$$

$$\oint_S \vec{E} \cdot \vec{dS} = \int_A E\, dS \cos 90° + \int_B E\, dS \cos 90° + \int_C E\, dS \cos 0° = E \int_C dS = E \times 2\pi r l$$

Gaussian surface is a closed surface, around a charge distribution, such that the electric field intensity has a single fixed value at every point on the surface.

$$\frac{q}{\varepsilon_0} = \frac{\lambda\, l}{\varepsilon_0}$$ (where λ is the liner charge density)

$$\therefore \quad E \times 2\pi r l = \frac{\lambda\, l}{\varepsilon_0}$$

or $$E = \frac{1}{2\pi\varepsilon_0}\,\frac{\lambda}{r}$$

or $$\boxed{E = \frac{1}{4\pi\varepsilon_0}\,\frac{2\lambda}{r}}$$

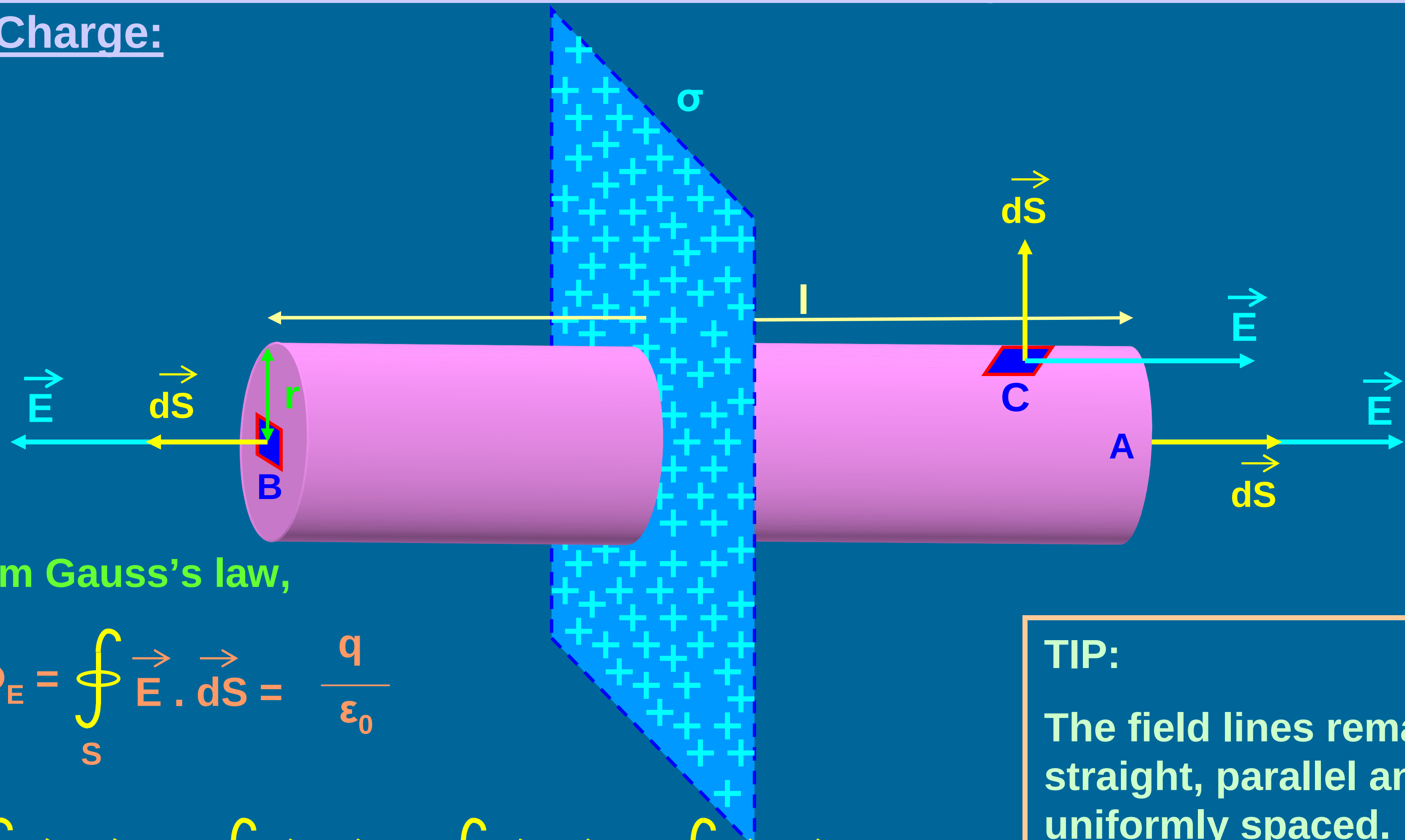
In vector form, $$\boxed{\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\,\frac{2\lambda}{r}\,\hat{r}}$$

The direction of the electric field intensity is radially outward from the positive line charge. For negative line charge, it will be radially inward.

Note:

The electric field intensity is independent of the size of the Gaussian surface constructed. It depends only on the distance of point of consideration. i.e. the Gaussian surface should contain the point of consideration.

2. Electric Field Intensity due to an Infinitely Long, Thin Plane Sheet of Charge:



From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

$$\oint_S \vec{E} \cdot \vec{dS} = \int_A \vec{E} \cdot \vec{dS} + \int_B \vec{E} \cdot \vec{dS} + \int_C \vec{E} \cdot \vec{dS}$$

$$\oint_S \vec{E} \cdot \vec{dS} = \int_A E\, dS \cos 0° + \int_B E\, dS \cos 0° + \int_C E\, dS \cos 90° = 2E \int dS = 2E \times \pi r^2$$

TIP:

The field lines remain straight, parallel and uniformly spaced.

$$\frac{q}{\varepsilon_0} = \frac{\sigma \pi r^2}{\varepsilon_0}$$ (where σ is the surface charge density)

$$\therefore \quad 2 E \times \pi r^2 = \frac{\sigma \pi r^2}{\varepsilon_0}$$

or $$\boxed{E = \frac{\sigma}{2 \varepsilon_0}}$$

In vector form, $$\boxed{\vec{E}(\vec{l}) = \frac{\sigma}{2 \varepsilon_0} \hat{l}}$$

The direction of the electric field intensity is normal to the plane and away from the positive charge distribution. For negative charge distribution, it will be towards the plane.

Note:

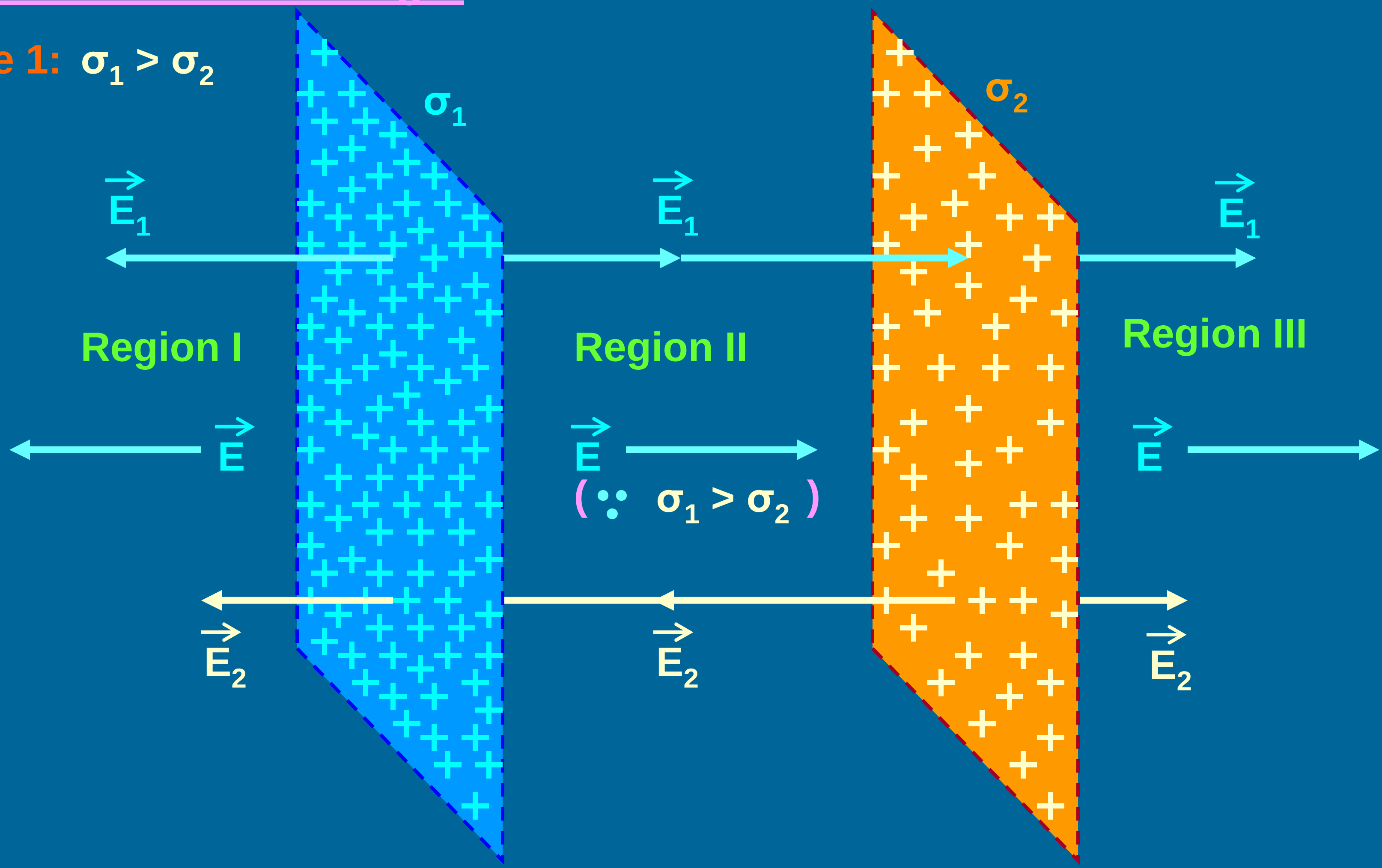
The electric field intensity is independent of the size of the Gaussian surface constructed. It neither depends on the distance of point of consideration nor the radius of the cylindrical surface.

If the plane sheet is thick, then the charge distribution will be available on both the sides. So, the charge enclosed within the Gaussian surface will be twice as before. Therefore, the field will be twice.

$$\therefore \quad \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

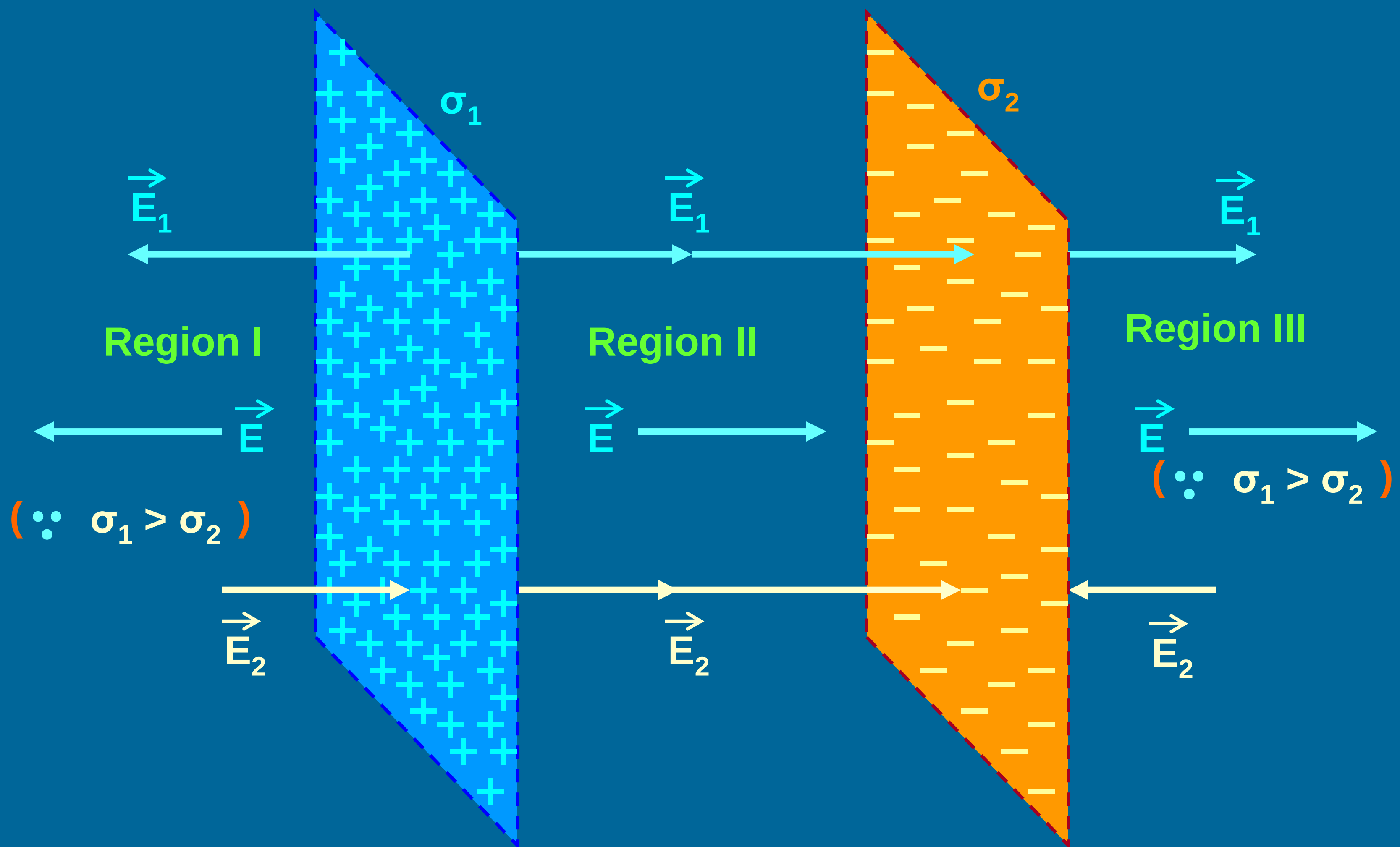
3. Electric Field Intensity due to Two Parallel, Infinitely Long, Thin Plane Sheet of Charge:

Case 1: $\sigma_1 > \sigma_2$



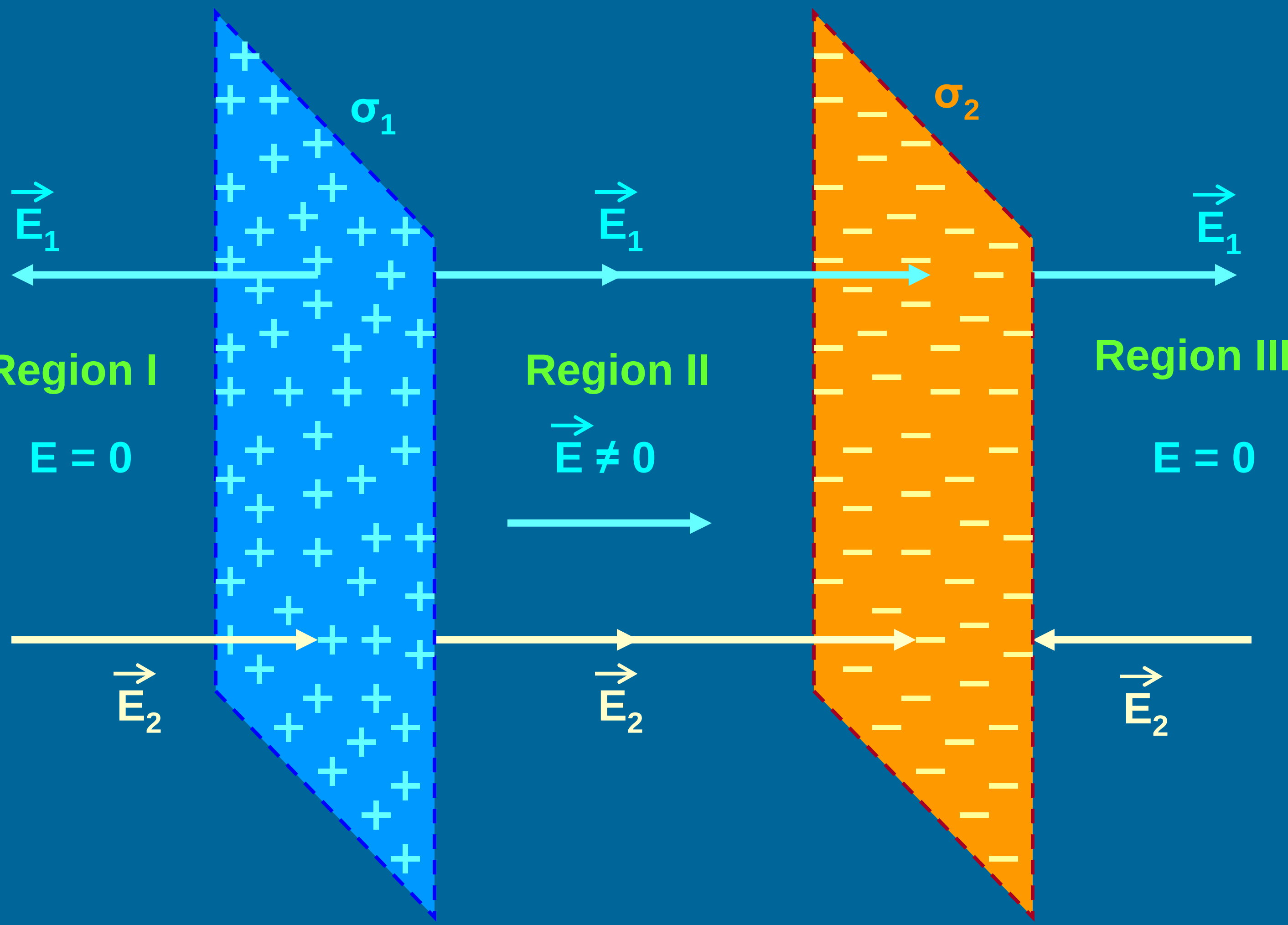
$E = E_1 + E_2$ $E = \dfrac{\sigma_1 + \sigma_2}{2\,\varepsilon_0}$	$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\,\varepsilon_0}$	$E = E_1 + E_2$ $E = \dfrac{\sigma_1 + \sigma_2}{2\,\varepsilon_0}$

Case 2: $+\sigma_1$ & $-\sigma_2$



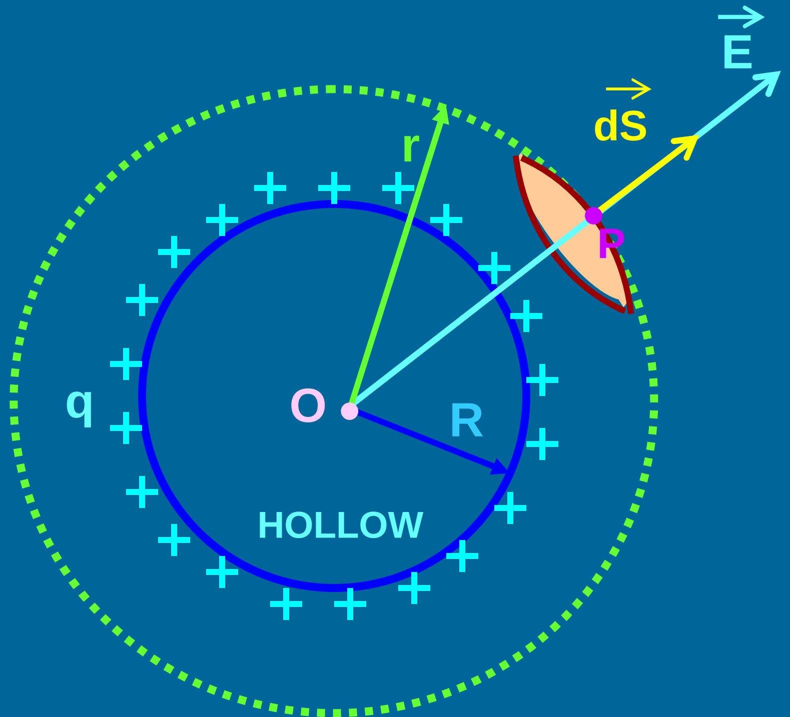
$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\,\varepsilon_0}$	$E = E_1 + E_2$ $E = \dfrac{\sigma_1 + \sigma_2}{2\,\varepsilon_0}$	$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\,\varepsilon_0}$

Case 3: + σ & - σ



$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\,\varepsilon_0} = 0$	$E = E_1 + E_2$ $E = \dfrac{\sigma_1 + \sigma_2}{2\,\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0}$	$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\,\varepsilon_0} = 0$

4. Electric Field Intensity due to a Uniformed Charged This Spherical Shell:



.... Gaussian Surface

i) At a point P outside the shell:

From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ and $\vec{dS}$ are in the same direction,

$$\therefore \quad \Phi_E = \oint_S E \; dS = \frac{q}{\varepsilon_0}$$

or $$\Phi_E = E \oint_S dS = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

or $$\boxed{E = \frac{q}{4\pi\varepsilon_0 \; r^2}}$$

Since $q = \sigma \times 4\pi R^2$,

$$\therefore \quad \boxed{E = \frac{\sigma \; R^2}{\varepsilon_0 \; r^2}}$$

Electric field due to a uniformly charged thin spherical shell at a point outside the shell is such as if the whole charge were concentrated at the centre of the shell.

ii) At a point A on the surface of the shell:

From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ and $\vec{dS}$ are in the same direction,

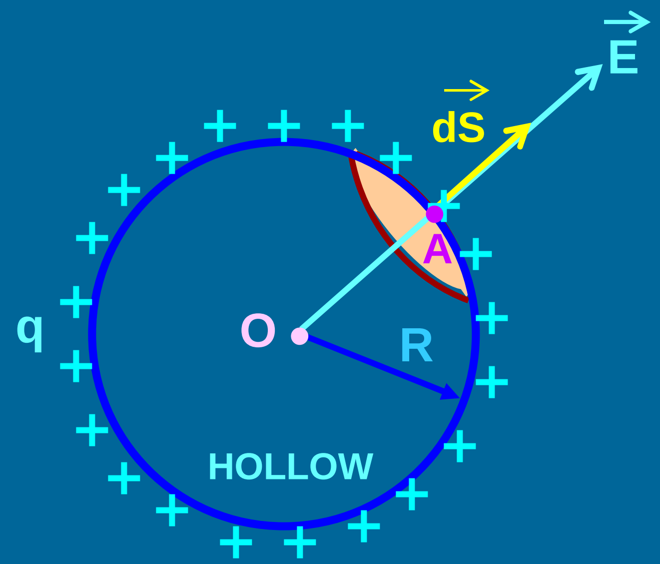
$$\therefore \quad \Phi_E = \oint_S E \; dS = \frac{q}{\varepsilon_0}$$

$$\text{or} \quad \Phi_E = E \oint_S dS = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi R^2 = \frac{q}{\varepsilon_0} \quad \text{or} \quad \boxed{E = \frac{q}{4\pi\varepsilon_0 R^2}}$$

Since $q = \sigma \times 4\pi R^2$,

$$\therefore \quad \boxed{E = \frac{\sigma}{\varepsilon_0}}$$



Electric field due to a uniformly charged thin spherical shell at a point on the surface of the shell is maximum.

iii) At a point B inside the shell:

From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ and $\vec{dS}$ are in the same direction,

$$\therefore \quad \Phi_E = \oint_S E \; dS = \frac{q}{\varepsilon_0}$$

$$\text{or} \quad \Phi_E = E \oint_S dS = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi r'^2 = \frac{q}{\varepsilon_0} \quad \text{or} \quad \boxed{E = \frac{0}{4\pi\varepsilon_0 r'^2}}$$

(since q = 0 inside the Gaussian surface)

$$\therefore \quad \boxed{E = 0}$$

This property E = 0 inside a cavity is used for electrostatic shielding.