MAGNETIC EFFECT OF CURRENT - I

- **1. Magnetic Effect of Current Oersted's Experiment**
- 2. Ampere's Swimming Rule
- 3. Maxwell's Cork Screw Rule
- 4. Right Hand Thumb Rule
- 5. Biot Savart's Law
- 6. Magnetic Field due to Infinitely Long Straight Current carrying Conductor
- 7. Magnetic Field due to a Circular Loop carrying current
- 8. Magnetic Field due to a Solenoid

Magnetic Effect of Current:

An electric current (i.e. flow of electric charge) produces magnetic effect in the space around the conductor called strength of Magnetic field or simply Magnetic field.

Oersted's Experiment:

When current was allowed to flow through a wire placed parallel to the axis of a magnetic needle kept directly below the wire, the needle was found to deflect from its normal position.

When current was reversed through the wire, the needle was found to deflect in the opposite direction to the earlier case.





Rules to determine the direction of magnetic field:

Ampere's Swimming Rule:

Imagining a man who swims in the direction of current from south to north facing a magnetic needle kept under him such that current enters his feet then the North pole of the needle will deflect towards his left hand, i.e. towards West.

Maxwell's Cork Screw Rule or Right Hand Screw Rule:

If the forward motion of an imaginary right handed screw is in the direction of the current through a linear conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.



Right Hand Thumb Rule or Curl Rule:

If a current carrying conductor is imagined to be held in the right hand such that the thumb points in the direction of the current, then the tips of the fingers encircling the conductor will give the direction of the magnetic lines of force.

Biot – Savart's Law:

The strength of magnetic field dB due to a small current element dI carrying a current I at a point P distant r from the element is directly proportional to I, dI, sin θ and inversely proportional to the square of the distance (r²) where θ is the angle between dI and r.

i) dB α I
ii) dB α dI
iii) dB α sin θ

iv) dB α 1 / r^2

$$dB \alpha \frac{I dI \sin \theta}{r^2}$$
$$dB = \frac{\mu_0 I dI \sin \theta}{4\pi r^2}$$





Biot – Savart's Law in vector form:

$$\vec{dB} = \frac{\mu_0 \ I \ dI \ x \ r}{4\pi \ r^2}$$
$$\vec{dB} = \frac{\mu_0 \ I \ dI \ x \ r}{4\pi \ r^2}$$
$$\vec{dB} = \frac{\mu_0 \ I \ dI \ x \ r}{4\pi \ r^3}$$

Value of $\mu_0 = 4\pi \times 10^{-7} \text{ Tm A}^{-1}$ or Wb m⁻¹ A⁻¹

Direction of \overrightarrow{dB} is same as that of direction of $\overrightarrow{dI} \times \overrightarrow{r}$ which can be determined by Right Hand Screw Rule.

It is emerging • at P' and entering (at P into the plane of the diagram.

Current element is a vector quantity whose magnitude is the vector product of current and length of small element having the direction of the flow of current. (I dl)

Magnetic Field due to a Straight Wire carrying current:



Magnetic field due to whole conductor is obtained by integrating with limits - Φ_1 to Φ_2 . (Φ_1 is taken negative since it is anticlockwise)

$$\mathbf{B} = \int \mathbf{dB} = \int_{-\Phi_1}^{\Phi_2} \frac{\mu_0 \, I \cos \Phi \, \mathrm{d}\Phi}{4\pi \, a}$$

$$\mathbf{B} = \frac{\mu_0 \, \mathbf{I} \, (\sin \Phi_1 + \sin \Phi_2)}{4\pi a}$$



Magnetic Field due to a Circular Loop carrying current:

1) At a point on the axial line:



The plane of the coil is considered perpendicular to the plane of the diagram such that the direction of magnetic field can be visualized on the plane of the diagram.

At C and D current elements XY and X'Y' are considered such that current at C emerges out and at D enters into the plane of the diagram.

$$dB = \frac{\mu_0 \ I \ dI \ \sin \theta}{4\pi \ r^2} \quad or \quad dB = \frac{\mu_0 \ I \ dI}{4\pi \ r^2}$$

The angle θ between dI and r is 90° because the radius of the loop is very small and since sin 90° = 1

The semi-vertical angle made by r to the loop is Φ and the angle between r and dB is 90°. Therefore, the angle between vertical axis and dB is also Φ . dB is resolved into components dB cos Φ and dB sin Φ .

Due to diametrically opposite current elements, cos components are always opposite to each other and hence they cancel out each other.

SinΦ components due to all current elements dl get added up along the same direction (in the direction away from the loop).

 $B = \int dB \sin \Phi = \int \frac{\mu_0 \ I \ dI \ \sin \Phi}{4\pi \ r^2} \quad \text{or} \quad B = \frac{\mu_0 \ I \ (2\pi a) \ a}{4\pi \ (a^2 + x^2) \ (a^2 + x^2)^{\frac{1}{2}}}$

 $\mathbf{B} = \frac{\mu_0 \, \mathbf{I} \, \mathbf{a}^2}{2(\mathbf{a}^2 + \mathbf{x}^2)^{3/2}}$

(μ_0 , I, a, sinΦ are constants, $\int dI = 2\pi a$ and r & sinΦ are replaced with measurable and constant values.)

Special Cases:

i) At the centre O,
$$x = 0$$
.

$$\mathbf{B} = \frac{\mu_0 \mathbf{I}}{2a}$$

•

ii) If the observation point is far away from the coil, then a << x. So, a^2 can be neglected in comparison with x^2 .

• **B** =
$$\frac{\mu_0 \ I \ a^2}{2 \ x^3}$$



Different views of direction of current and magnetic field due to circular loop of a coil:



2) B at the centre of the loop:

The plane of the coil is lying on the plane of the diagram and the direction of current is clockwise such that the direction of magnetic field is perpendicular and into the plane.

$$dB = \frac{\mu_0 | d| \sin \theta}{4\pi a^2} \qquad dB = \frac{\mu_0 | d|}{4\pi a^2}$$
$$B = \int dB = \int \frac{\mu_0 | d|}{4\pi a^2}$$
$$B = \frac{\mu_0 | d|}{4\pi a^2}$$

 $(\mu_0, I, a \text{ are constants and } \int dI = 2\pi a)$



The angle θ between dI and a is 90° because the radius of the loop is very small and since sin 90° = 1





When we look at any end of the coil carrying current, if the current is in anti-clockwise direction then that end of coil behaves like North Pole and if the current is in clockwise direction then that end of the coil behaves like South Pole.

MAGNETIC EFFECT OF CURRENT - II

- **1. Lorentz Magnetic Force**
- 2. Fleming's Left Hand Rule
- 3. Force on a moving charge in uniform Electric and Magnetic fields
- 4. Force on a current carrying conductor in a uniform Magnetic Field
- 5. Force between two infinitely long parallel current-carrying conductors
- 6. Definition of ampere
- 7. Representation of fields due to parallel currents
- 8. Torque experienced by a current-carrying coil in a uniform Magnetic Field
- 9. Moving Coil Galvanometer
- **10. Conversion of Galvanometer into Ammeter and Voltmeter**
- **11. Differences between Ammeter and Voltmeter**

Lorentz Magnetic Force:

A current carrying conductor placed in a magnetic field experiences a force which means that a moving charge in a magnetic field experiences force.

 $\vec{F}_m = q (\vec{v} \times \vec{B})$

or

 $\vec{F}_{m} = (q v B sin \theta) \hat{n}$

where θ is the angle between v and B



- i) If the charge is at rest, i.e. v = 0, then $F_m = 0$. So, a stationary charge in a magnetic field does not experience any force.
- ii) If $\theta = 0^{\circ}$ or 180° i.e. if the charge moves parallel or anti-parallel to the direction of the magnetic field, then $F_m = 0$.
- iii) If $\theta = 90^{\circ}$ i.e. if the charge moves perpendicular to the magnetic field, then the force is maximum. $F_{m (max)} = q \vee B$





Fleming's Left Hand Rule:

If the central finger, fore finger and thumb of left hand are stretched mutually perpendicular to each other and the central finger points to current, fore finger points to magnetic field, then thumb points in the direction of motion (force) on the current carrying conductor.



TIP:

Remember the phrase 'e m f' to represent electric current, magnetic field and force in anticlockwise direction of the fingers of left hand.

Force on a moving charge in uniform Electric and Magnetic Fields:

When a charge q moves with velocity \vec{v} in region in which both electric field \vec{E} and magnetic field \vec{B} exist, then the Lorentz force is $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$ or $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Force on a current-carrying conductor in a uniform Magnetic Field:

Force experienced by each electron in the conductor is $\vec{f} = -e(\vec{v}_d \times \vec{B})$

If n be the number density of electrons, A be the area of cross section of the conductor, then no. of electrons in the element dl is n A dl.



Force experienced by the electrons in dI is $d\vec{F} = n \ A \ dI \left[- e \left(\vec{v}_d \times \vec{B} \right) \right] = - n \ e \ A \ v_d \ (dI \ X \ B)$ $= I \ (dI \ x \ B)$ $\vec{F} = \int d\vec{F} = \int I \ (dI \ x \ B)$ where $I = neAv_d$ and -ve sign represents that the direction of dI is opposite to that of v_d)

$$\vec{F} = I (\vec{I} \times \vec{B})$$
 or $F = I I B \sin \theta$

Forces between two parallel infinitely long current-carrying conductors:

Magnetic Field on RS due to current in PQ is

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

(in magnitude)

Force acting on RS due to current I₂ through it is

$$F_{21} = \frac{\mu_0 I_1}{2\pi r} I_2 I \sin 90^\circ \text{ or } F_{21} = \frac{\mu_0 I_1 I_2 I}{2\pi r}$$

B₁ acts perpendicular and into the plane of the diagram by Right Hand Thumb Rule. So, the angle between I and B_1 is 90°. I is length of the conductor.

Magnetic Field on PQ due to current in RS is

 $B_2 = \frac{\mu_0 I_2}{2\pi r}$

Force acting on PQ due to current I₁ through it is

$$F_{12} = \frac{\mu_0 l_2}{2\pi r} l_1 l \sin 90^\circ \text{ or } F_{12} = \frac{\mu_0 l_1 l_2 l}{2\pi r} \qquad \begin{array}{c} \text{(The angle between I and} \\ B_2 is 90^\circ \text{ and } B_2 ls \\ emerging out) \end{array}$$

$$F_{12} = F_{21} = F = \frac{\mu_0 l_1 l_2 l}{2\pi r}$$
Force per unit length of the conductor is
$$F/l = \frac{\mu_0 l_1 l_2}{2\pi r} N/m$$



ls





By Fleming's Left Hand Rule, the conductors experience force towards each other and hence attract each other. By Fleming's Left Hand Rule, the conductors experience force away from each other and hence repel each other.

Definition of Ampere:

Force per unit length of the $F/I = \frac{\mu_0 I_1 I_2}{2\pi r}$ N/m conductor is

When $I_1 = I_2 = 1$ Ampere and r = 1 m, then $F = 2 \times 10^{-7}$ N/m.

One ampere is that current which, if passed in each of two parallel conductors of infinite length and placed 1 m apart in vacuum causes each conductor to experience a force of 2×10^{-7} Newton per metre of length of the conductor.

Representation of Field due to Parallel Currents:



Torque experienced by a Current Loop (Rectangular) in a uniform Magnetic Field:

Let θ be the angle between the plane of the loop and the direction of the magnetic field. The axis of the coil is perpendicular to the magnetic field.

 $\vec{F}_{SP} = I (b \times B)$ $|F_{SP}| = I b B \sin \theta$ $\vec{F}_{OR} = I (b \times B)$

 $|\mathbf{F}_{QR}| = \mathbf{I} \mathbf{b} \mathbf{B} \sin \theta$

Forces F_{SP} and F_{QR} are equal in magnitude but opposite in direction and they cancel out each other. Moreover they act along the same line of action (axis) and hence do not produce torque.

 $F_{PQ} = I(I \times B)$

 $|F_{PQ}| = ||B| \sin 90^{\circ} = ||B|$ $\overrightarrow{F}_{RS} = |(| \times B)$ $|F_{RS}| = ||B| \sin 90^{\circ} = ||B|$

Forces F_{PQ} and F_{RS} being equal in magnitude but opposite in direction cancel out each other and do not produce any translational motion. But they act along different lines of action and hence produce torque about the axis of the coil.





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\tau = N I A B \sin \Phi
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NOTE:

One must be very careful in using the formula in terms of cos or sin since it depends on the angle taken whether with the plane of the coil or the normal of the coil.

Torque in Vector form:

$\tau = N I A B \sin \Phi$

 $\vec{\tau} = (N | A | B | sin \Phi) \hat{n}$ (where \hat{n} is unit vector normal to the plane of the loop)

$$\vec{\tau} = N I (\vec{A} \times \vec{B})$$
 or $\vec{\tau} = N (\vec{M} \times \vec{B})$

(since M = I A is the Magnetic Dipole Moment)

Note:

- 1) The coil will rotate in the anticlockwise direction (from the top view, according to the figure) about the axis of the coil shown by the dotted line.
- 2) The torque acts in the upward direction along the dotted line (according to Maxwell's Screw Rule).
- 3) If $\Phi = 0^\circ$, then $\tau = 0$.
- 4) If $\Phi = 90^{\circ}$, then τ is maximum. i.e. $\tau_{max} = N I A B$
- 5) Units: B in Tesla, I in Ampere, A in m² and T in Nm.
- 6) The above formulae for torque can be used for any loop irrespective of its shape.

Moving Coil or Suspended Coil or D' Arsonval Type Galvanometer:



T – Torsion Head, TS – Terminal screw, M – Mirror, N,S – Poles pieces of a magnet, LS – Levelling Screws, PQRS – Rectangular coil, PBW – Phosphor Bronze Wire

Radial Magnetic Field:

The (top view PS of) plane of the coil PQRS lies along the magnetic lines of force in whichever position the coil comes to rest in equilibrium.

So, the angle between the plane of the coil and the magnetic field is 0°.

or the angle between the normal to the plane of the coil and the magnetic field is 90°.

i.e.
$$\sin \Phi = \sin 90^\circ = 1$$

$$\cdot I = \frac{k}{NAB} \alpha \text{ or } I = G \alpha \text{ where } G = \frac{k}{NAB} \text{ Scale}$$

Current Sensitivity of Galvanometer:

It is the defection of galvanometer per unit current.

 $\frac{\alpha}{l} = \frac{NAB}{k}$

S

Mirror

Ν

Lamp

Voltage Sensitivity of Galvanometer:

It is the defection of galvanometer per unit voltage.



Conversion of Galvanometer to Ammeter:

Galvanometer can be converted into ammeter by shunting it with a very small resistance.

Potential difference across the galvanometer and shunt resistance are equal.

••
$$(I - I_g) S = I_g G$$
 or $S = \frac{I_g G}{I - I_g}$



Conversion of Galvanometer to Voltmeter:

Galvanometer can be converted into voltmeter by connecting it with a very high resistance.

Potential difference across the given load resistance is the sum of p.d across galvanometer and p.d. across the high resistance.

••
$$V = I_g (G + R)$$
 or $R = \frac{V}{I_g} - G$



Difference between Ammeter and Voltmeter:

S.No.	Ammeter	Voltmeter
1	It is a low resistance instrument.	It is a high resistance instrument.
2	Resistance is GS / (G + S)	Resistance is G + R
3	Shunt Resistance is $(GI_g) / (I - I_g)$ and is very small.	Series Resistance is (V / I _g) - G and is very high.
4	It is always connected in series.	It is always connected in parallel.
5	Resistance of an ideal ammeter is zero.	Resistance of an ideal voltmeter is infinity.
6	Its resistance is less than that of the galvanometer.	Its resistance is greater than that of the voltmeter.
7	It is not possible to decrease the range of the given ammeter.	It is possible to decrease the range of the given voltmeter.

MAGNETIC EFFECT OF CURRENT - III

- 1. Cyclotron
- 2. Ampere's Circuital Law
- 3. Magnetic Field due to a Straight Solenoid
- 4. Magnetic Field due to a Toroidal Solenoid



Working: Imagining D_1 is positive and D_2 is negative, the + vely charged particle kept at the centre and in the gap between the dees get accelerated towards D_2 . Due to perpendicular magnetic field and according to Fleming's Left Hand Rule the charge gets deflected and describes semi-circular path.

When it is about to leave D_2 , D_2 becomes + ve and D_1 becomes – ve. Therefore the particle is again accelerated into D_1 where it continues to describe the semi-circular path. The process continues till the charge traverses through the whole space in the dees and finally it comes out with very high speed through the window.

Theory:

The magnetic force experienced by the charge provides centripetal force required to describe circular path.

•
$$mv^2/r = qvB \sin 90^\circ$$

 $v = \frac{B q r}{m}$

(where m – mass of the charged particle, q – charge, v – velocity on the path of radius – r, B is magnetic field and 90° is the angle b/n v and B)

If t is the time taken by the charge to describe the semi-circular path inside the dee, then

$$t = \frac{\pi r}{v}$$
 or $t = \frac{\pi m}{Bq}$

Time taken inside the dee depends only on the magnetic field and m/q ratio and not on the speed of the charge or the radius of the path.

If **T** is the time period of the high frequency oscillator, then for resonance,

$$T = 2 t$$
 or $T = \frac{2\pi m}{B q}$

If f is the frequency of the high frequency oscillator (Cyclotron Frequency), then

$$f = \frac{B q}{2\pi m}$$

Maximum Energy of the Particle:

Kinetic Energy of the charged particle is

K.E. =
$$\frac{1}{2}$$
 m v² = $\frac{1}{2}$ m ($\frac{Bqr}{m}$)² = $\frac{1}{2}$ $\frac{B^2q^2r^2}{m}$

Maximum Kinetic Energy of the charged particle is when r = R (radius of the D's).

K.E.
$$_{max} = \frac{1}{2} \frac{B^2 q^2 R^2}{m}$$

The expressions for Time period and Cyclotron frequency only when m remains constant. (Other quantities are already constant.)

But **m** varies with **v** according to Einstein's Relativistic Principle as per

$$m = \frac{m_0}{[1 - (v^2 / c^2)]^{\frac{1}{2}}}$$

If frequency is varied in synchronisation with the variation of mass of the charged particle (by maintaining B as constant) to have resonance, then the cyclotron is called synchro – cyclotron.

If magnetic field is varied in synchronisation with the variation of mass of the charged particle (by maintaining f as constant) to have resonance, then the cyclotron is called isochronous – cyclotron.

NOTE: Cyclotron can not be used for accelerating neutral particles. Electrons can not be accelerated because they gain speed very quickly due to their lighter mass and go out of phase with alternating e.m.f. and get lost within the dees.

Ampere's Circuital Law:

The line integral $\oint \vec{B}$. dl for a closed curve is equal to μ_0 times the net current I threading through the area bounded by the curve.



Magnetic Field at the centre of a Straight Solenoid:



Magnetic Field due to Toroidal Solenoid (Toroid):

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_0$$

$$\oint \vec{B} \cdot d\vec{l} = \oint \vec{B} \cdot d\vec{l} \cos 0^\circ$$

$$= \vec{B} \oint d\vec{l} = \vec{B} (2\pi r)$$

And
$$\mu_0 I_0 = \mu_0 n (2\pi r)$$

$$\therefore B = \mu_0 n I$$

NOTE:

The magnetic field exists only in the tubular area bound by the coil and it does not exist in the area inside and outside the toroid.

i.e. B is zero at O and Q and non-zero at P.



