RAY OPTICS - I

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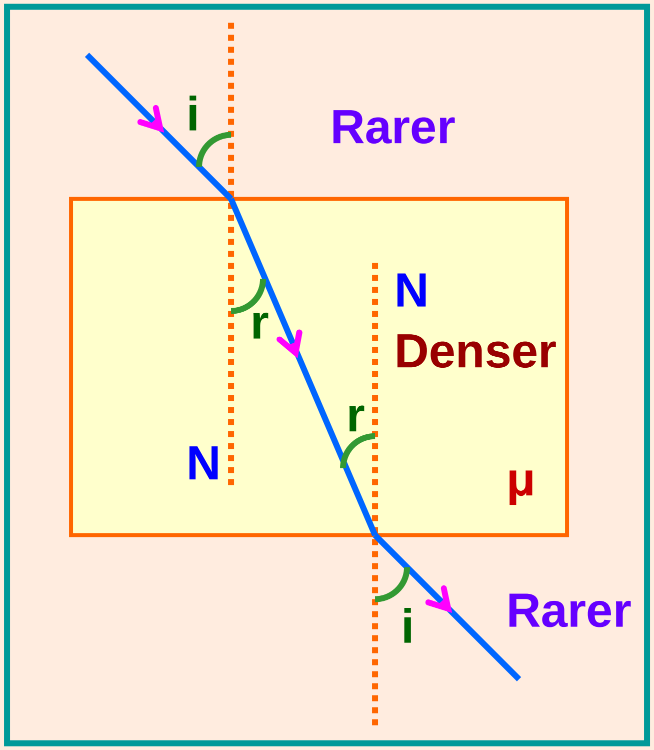
Refraction of Light:

Refraction is the phenomenon of change in the path of light as it travels from one medium to another (when the ray of light is incident obliquely).

It can also be defined as the phenomenon of change in speed of light from one medium to another.



Laws of Refraction:

I Law: The incident ray, the normal to the refracting surface at the point of incidence and the refracted ray all lie in the same plane.

II Law: For a given pair of media and for light of a given wavelength, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant. (Snell's Law)

$$\mu = \frac{\sin i}{\sin r}$$

(The constant μ is called refractive index of the medium, i is the angle of incidence and r is the angle of refraction.)

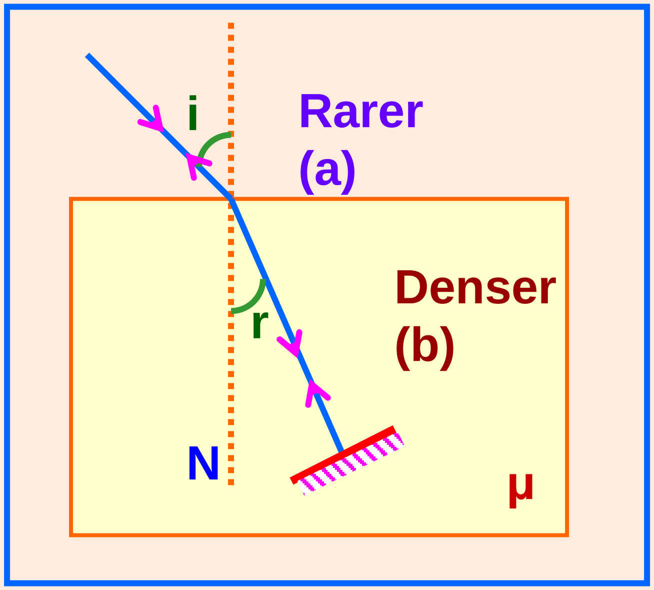
TIPS:

1. μ of optically rarer medium is lower and that of a denser medium is higher.
2. μ of denser medium w.r.t. rarer medium is more than 1 and that of rarer medium w.r.t. denser medium is less than 1. ($\mu_{air} = \mu_{vacuum} = 1$)
3. In refraction, the velocity and wavelength of light change.
4. In refraction, the frequency and phase of light do not change.
5. ${}_a\mu_m = c_a / c_m$ and ${}_a\mu_m = \lambda_a / \lambda_m$

Principle of Reversibility of Light:

$${}_a\mu_b = \frac{\sin i}{\sin r} \qquad {}_b\mu_a = \frac{\sin r}{\sin i}$$

$\boxed{{}_a\mu_b \times {}_b\mu_a = 1}$ or $\boxed{{}_a\mu_b = 1 / {}_b\mu_a}$

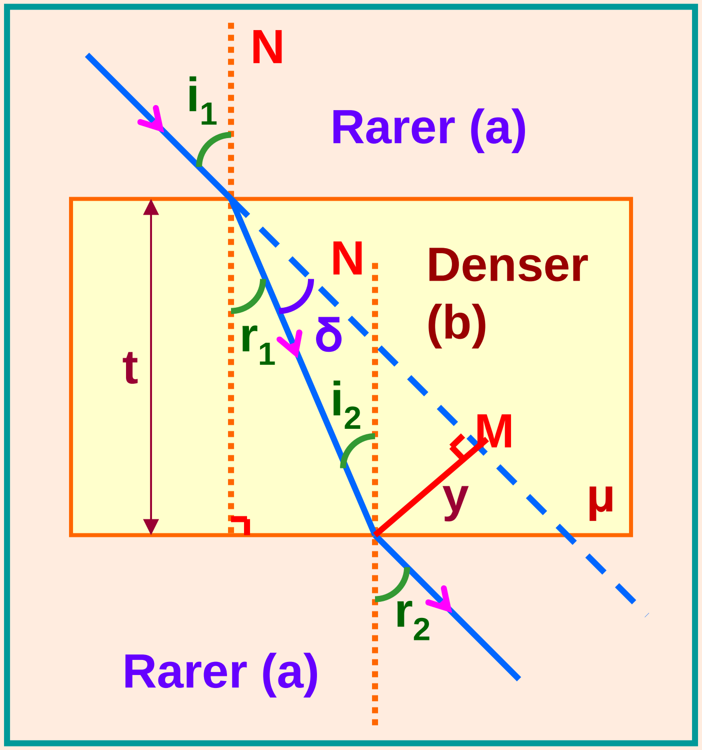


If a ray of light, after suffering any number of reflections and/or refractions has its path reversed at any stage, it travels back to the source along the same path in the opposite direction.

A natural consequence of the principle of reversibility is that the image and object positions can be interchanged. These positions are called conjugate positions.

Refraction through a Parallel Slab:

$${}_a\mu_b = \frac{\sin i_1}{\sin r_1} \qquad {}_b\mu_a = \frac{\sin i_2}{\sin r_2}$$



But ${}_a\mu_b \times {}_b\mu_a = 1$

$$\therefore \frac{\sin i_1}{\sin r_1} \times \frac{\sin i_2}{\sin r_2} = 1$$

It implies that $i_1 = r_2$ and $i_2 = r_1$
since $i_1 \neq r_1$ and $i_2 \neq r_2$.

Lateral Shift:

$$\boxed{y = \frac{t \sin \delta}{\cos r_1}} \quad \text{or} \quad \boxed{y = \frac{t \sin(i_1 - r_1)}{\cos r_1}}$$

Special Case:

If i_1 is very small, then r_1 is also very small.
i.e. $\sin(i_1 - r_1) = i_1 - r_1$ and $\cos r_1 = 1$

$$\therefore \boxed{y = t\,(i_1 - r_1)} \quad \text{or} \quad \boxed{y = t\, i_1(1 - 1/{}_a\mu_b)}$$

Refraction through a Compound Slab:

$$_a\mu_b = \frac{\sin i_1}{\sin r_1}$$

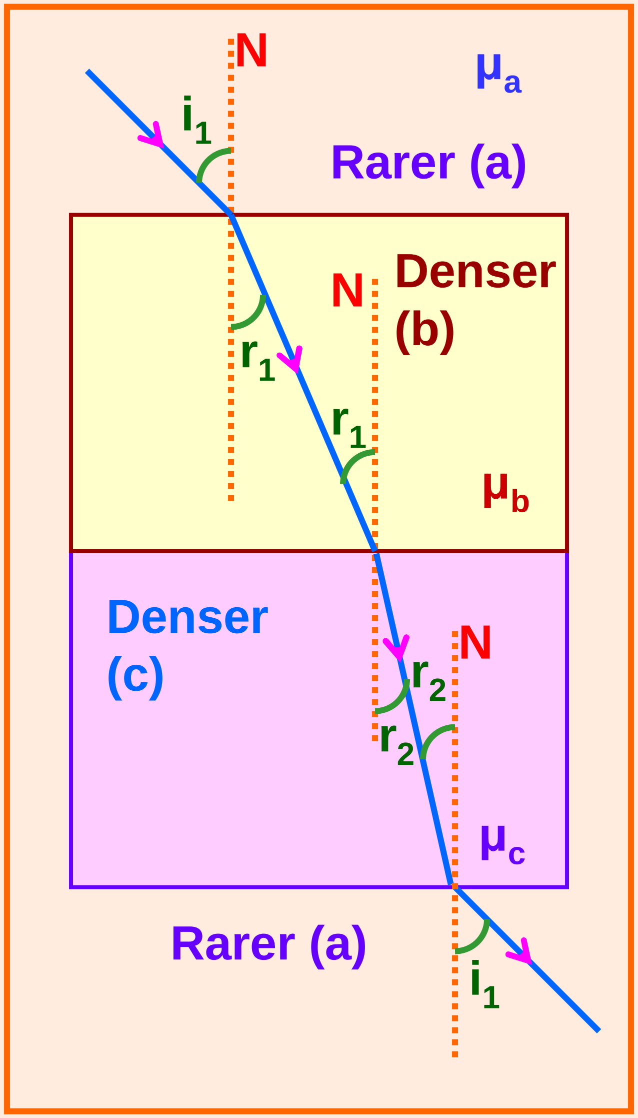
$$_b\mu_c = \frac{\sin r_1}{\sin r_2}$$

$$_c\mu_a = \frac{\sin r_2}{\sin i_1}$$

$$\boxed{_a\mu_b \times {}_b\mu_c \times {}_c\mu_a = 1}$$

or $$\boxed{_a\mu_b \times {}_b\mu_c = {}_a\mu_c}$$

or $$\boxed{_b\mu_c = {}_a\mu_c \,/\, {}_a\mu_b}$$

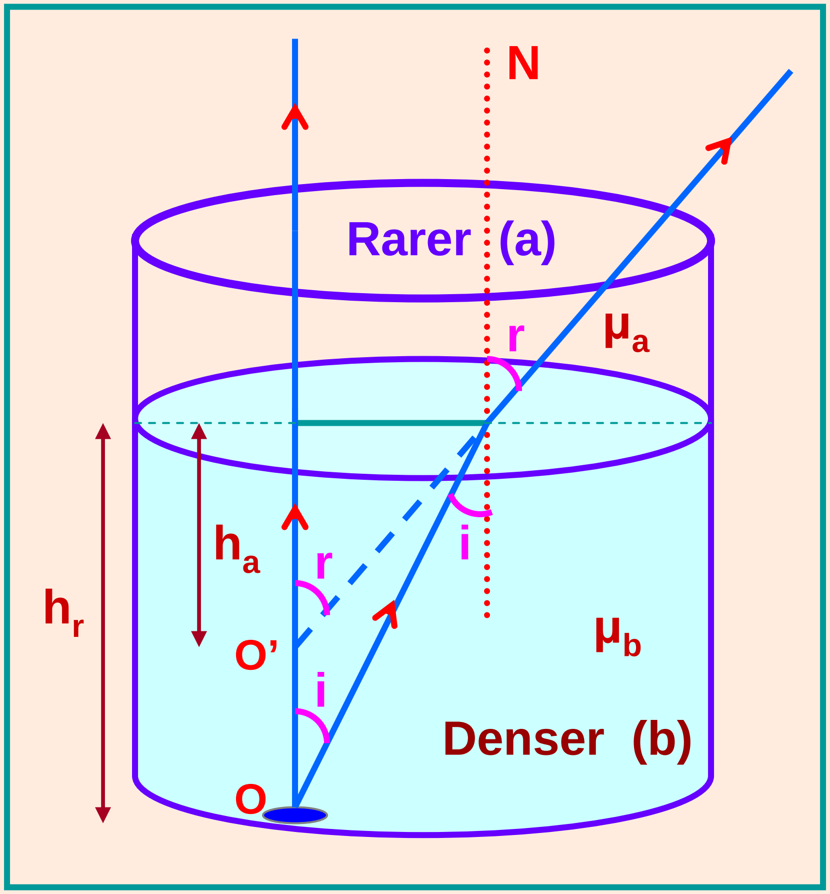


$\mu_c > \mu_b$

Apparent Depth of a Liquid:

$${}_b\mu_a = \frac{\sin i}{\sin r} \quad \text{or} \quad {}_a\mu_b = \frac{\sin r}{\sin i}$$

$${}_a\mu_b = \frac{h_r}{h_a} = \frac{\text{Real depth}}{\text{Apparent depth}}$$



Apparent Depth of a Number of Immiscible Liquids:

$$h_a = \sum_{i=1}^{n} h_i / \mu_i$$

Apparent Shift:

Apparent shift = $h_r - h_a = h_r - (h_r / \mu)$

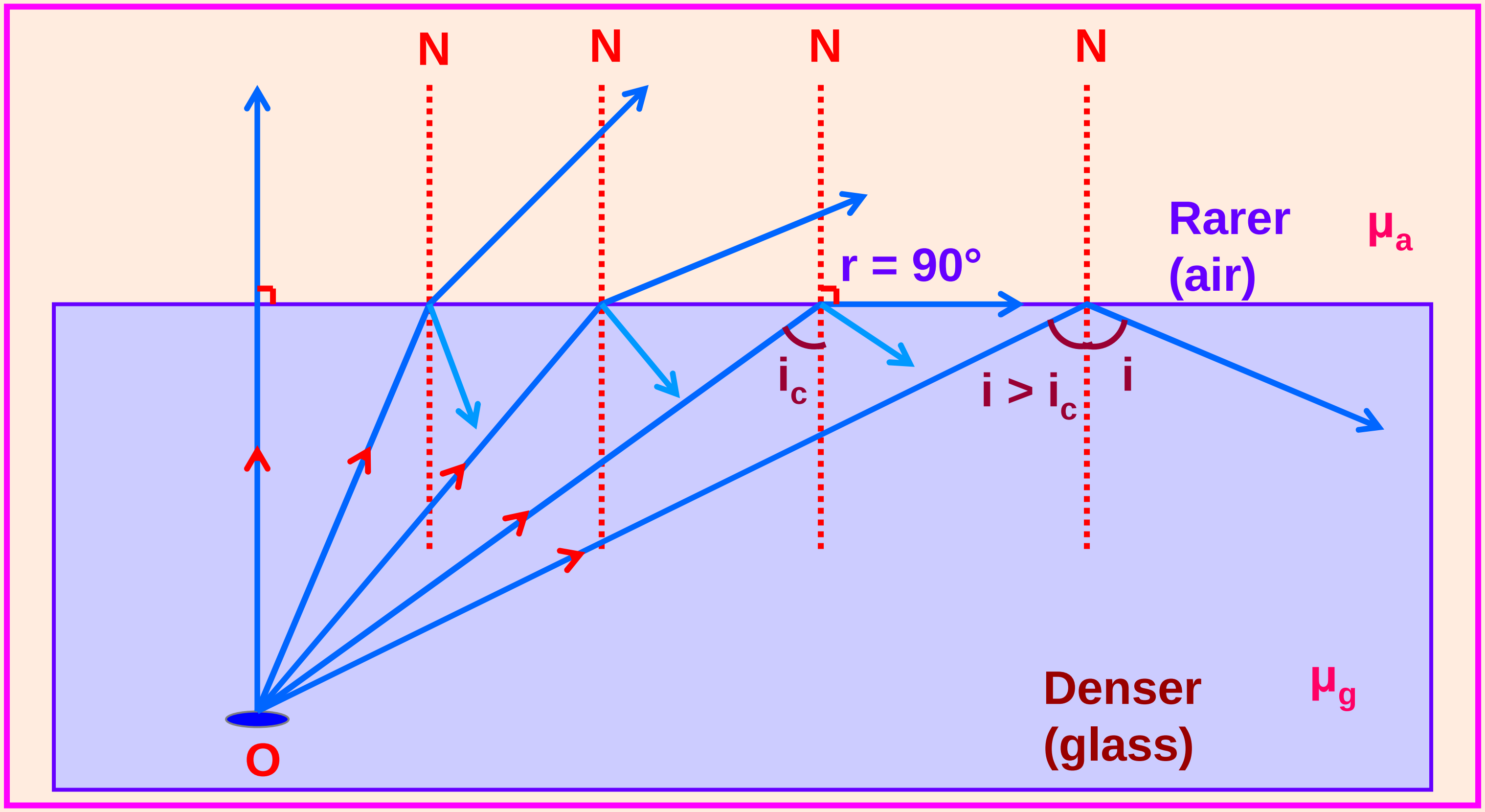
$= h_r [1 - 1/\mu]$

TIPS:

1. If the observer is in rarer medium and the object is in denser medium then $h_a < h_r$. (To a bird, the fish appears to be nearer than actual depth.)
2. If the observer is in denser medium and the object is in rarer medium then $h_a > h_r$. (To a fish, the bird appears to be farther than actual height.)

Total Internal Reflection:

Total Internal Reflection (TIR) is the phenomenon of complete reflection of light back into the same medium for angles of incidence greater than the critical angle of that medium.



Conditions for TIR:

1. **The incident ray must be in optically denser medium.**
2. **The angle of incidence in the denser medium must be greater than the critical angle for the pair of media in contact.**

Relation between Critical Angle and Refractive Index:

Critical angle is the angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90°.

$$ {}_g\mu_a = \frac{\sin i}{\sin r} = \frac{\sin i_c}{\sin 90°} = \sin i_c $$

or $${}_a\mu_g = \frac{1}{{}_g\mu_a} \quad \therefore \quad \boxed{{}_a\mu_g = \frac{1}{\sin i_c}} \quad \text{or} \quad \boxed{\sin i_c = \frac{1}{{}_a\mu_g}} \quad \text{Also} \quad \boxed{\sin i_c = \frac{\lambda_g}{\lambda_a}}$$

Red colour has maximum value of critical angle and Violet colour has minimum value of critical angle since,

$$\boxed{\sin i_c = \frac{1}{{}_a\mu_g} = \frac{1}{a + (b/\lambda^2)}}$$

Applications of T I R:

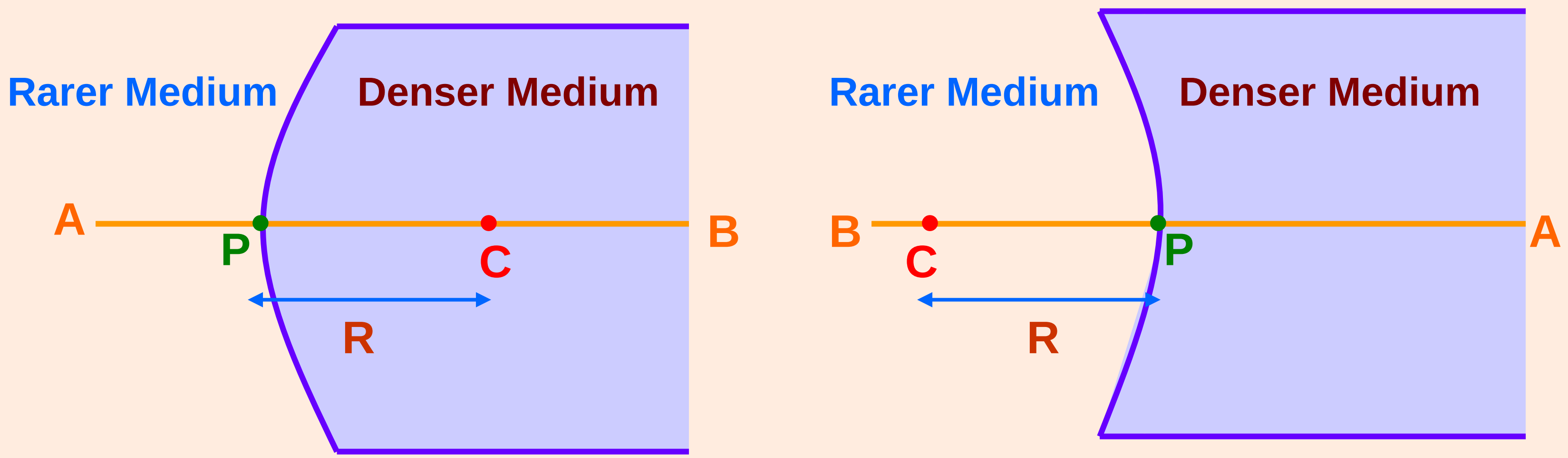
1. **Mirage formation**
2. **Looming**
3. **Totally reflecting Prisms**
4. **Optical Fibres**
5. **Sparkling of Diamonds**

Spherical Refracting Surfaces:

A spherical refracting surface is a part of a sphere of refracting material.

A refracting surface which is convex towards the rarer medium is called convex refracting surface.

A refracting surface which is concave towards the rarer medium is called concave refracting surface.



APCB – Principal Axis
C – Centre of Curvature
P – Pole
R – Radius of Curvature

Assumptions:

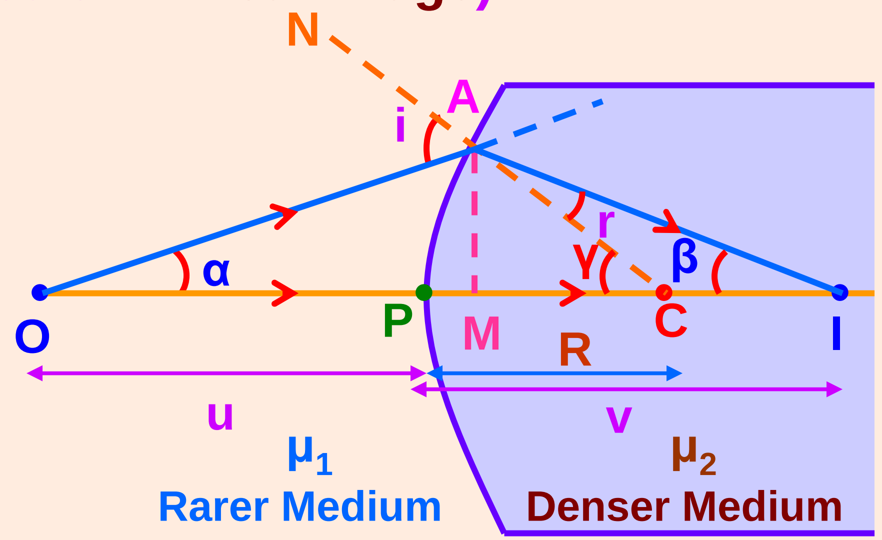
1. Object is the point object lying on the principal axis.
2. The incident and the refracted rays make small angles with the principal axis.
3. The aperture (diameter of the curved surface) is small.

New Cartesian Sign Conventions:

1. The incident ray is taken from left to right.
2. All the distances are measured from the pole of the refracting surface.
3. The distances measured along the direction of the incident ray are taken positive and against the incident ray are taken negative.
4. The vertical distances measured from principal axis in the upward direction are taken positive and in the downward direction are taken negative.

Refraction at Convex Surface:

(From Rarer Medium to Denser Medium - Real Image)



$i = \alpha + \gamma$

$\gamma = r + \beta$ or $r = \gamma - \beta$

$\tan\alpha = \dfrac{MA}{MO}$ or $\alpha = \dfrac{MA}{MO}$

$\tan\beta = \dfrac{MA}{MI}$ or $\beta = \dfrac{MA}{MI}$

$\tan\gamma = \dfrac{MA}{MC}$ or $\gamma = \dfrac{MA}{MC}$

According to Snell's law,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \text{or} \quad \frac{i}{r} = \frac{\mu_2}{\mu_1} \quad \text{or} \quad \boxed{\mu_1 i = \mu_2 r}$$

Substituting for i, r, α, β and γ, replacing M by P and rearranging,

$$\frac{\mu_1}{PO} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Applying sign conventions with values,

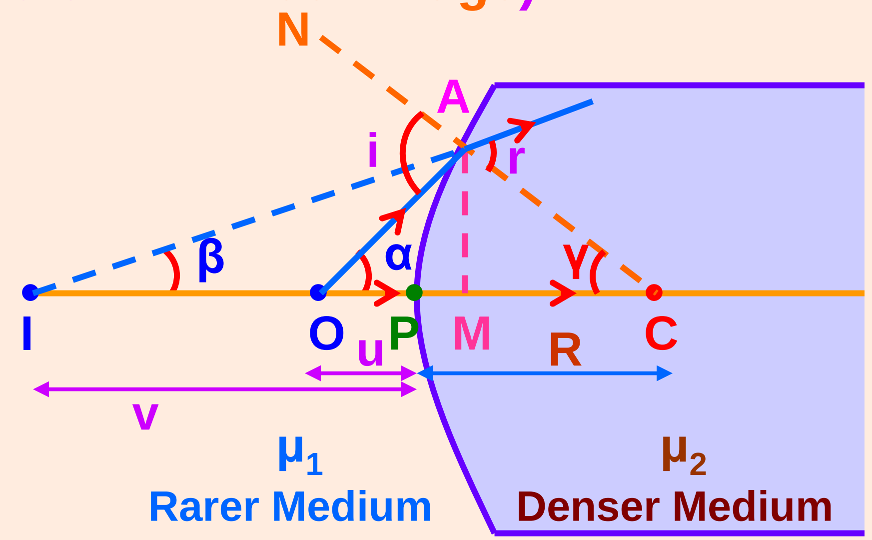
PO = - u, PI = + v and PC = + R

$$\boxed{\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

Refraction at Convex Surface:

(From Rarer Medium to Denser Medium - Virtual Image)

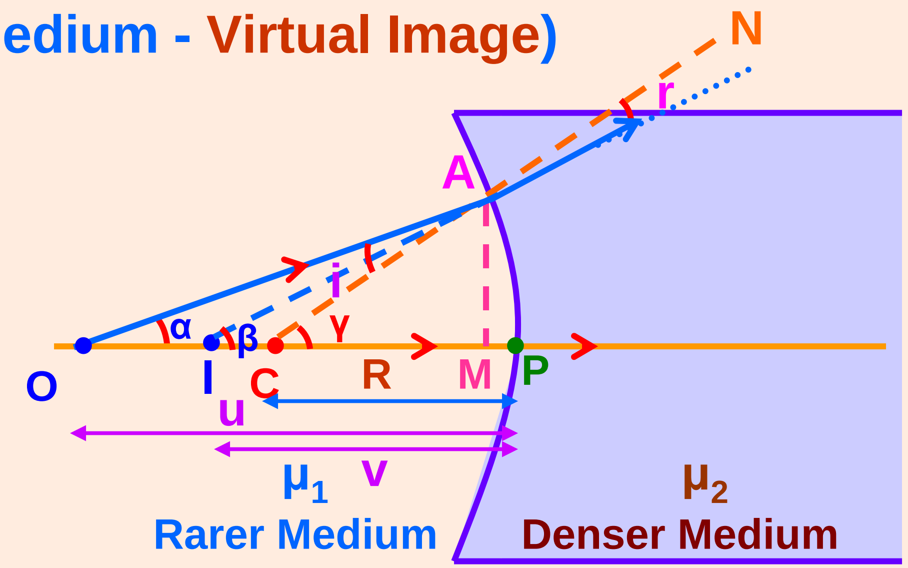
$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



Refraction at Concave Surface:

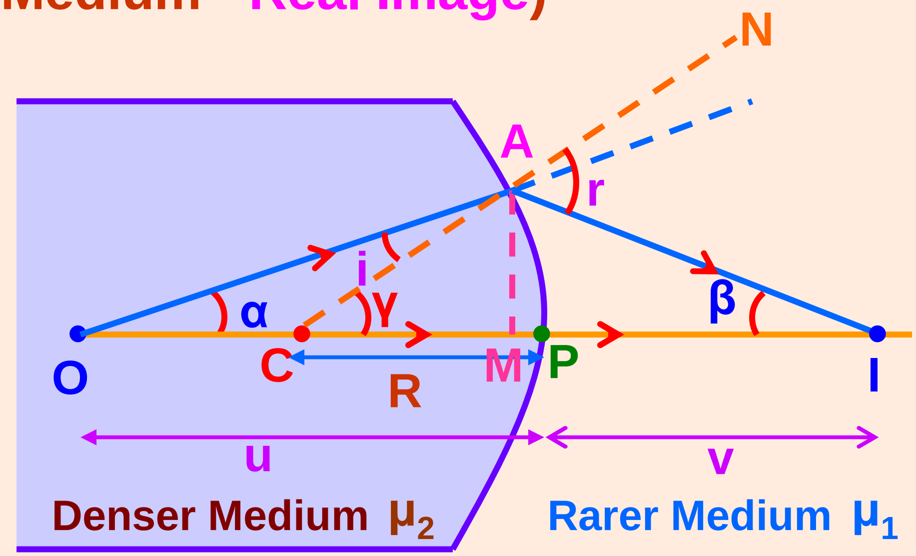
(From Rarer Medium to Denser Medium - Virtual Image)

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



Refraction at Convex Surface:
(From Denser Medium to Rarer Medium - Real Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$



Refraction at Convex Surface:
(From Denser Medium to Rarer Medium - Virtual Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Refraction at Concave Surface:
(From Denser Medium to Rarer Medium - Virtual Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Note:

1. Expression for ‘object in rarer medium’ is same for whether it is real or virtual image or convex or concave surface.

$$\boxed{\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

2. Expression for ‘object in denser medium’ is same for whether it is real or virtual image or convex or concave surface.

$$\boxed{\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}}$$

3. However the values of u, v, R, etc. must be taken with proper sign conventions while solving the numerical problems.

4. The refractive indices μ_1 and μ_2 get interchanged in the expressions.

Lens Maker's Formula:

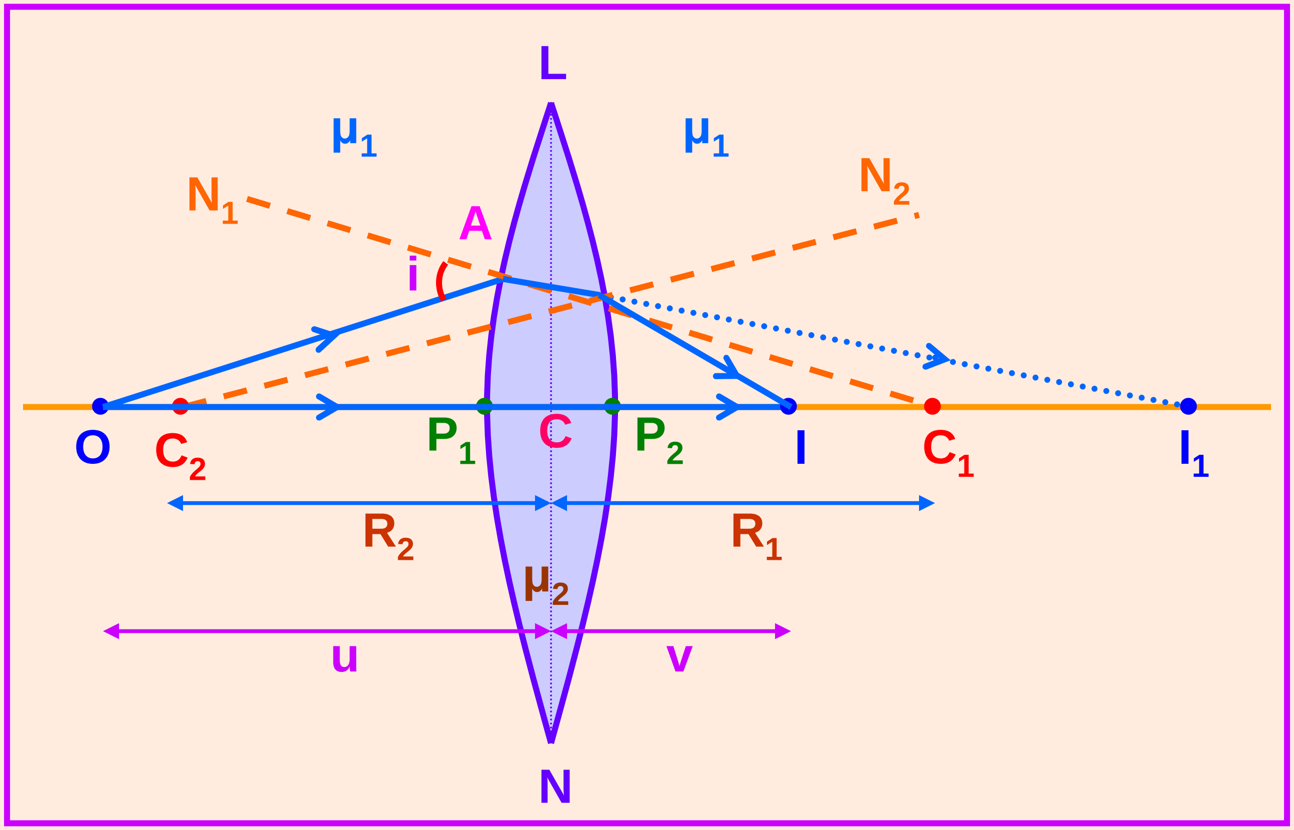
For refraction at LP_1N,

$$\frac{\mu_1}{CO} + \frac{\mu_2}{CI_1} = \frac{\mu_2 - \mu_1}{CC_1}$$

(as if the image is formed in the denser medium)

For refraction at LP_2N,

$$\frac{\mu_2}{-CI_1} + \frac{\mu_1}{CI} = \frac{-(\mu_1 - \mu_2)}{CC_2}$$



(as if the object is in the denser medium and the image is formed in the rarer medium)

Combining the refractions at both the surfaces,

$$\frac{\mu_1}{CO} + \frac{\mu_1}{CI} = (\mu_2 - \mu_1)\left(\frac{1}{CC_1} + \frac{1}{CC_2}\right)$$

Substituting the values with sign conventions,

$$\boxed{\frac{1}{-u} + \frac{1}{v} = \frac{(\mu_2 - \mu_1)}{\mu_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Since $\mu_2 / \mu_1 = \mu$

$$\frac{1}{-u} + \frac{1}{v} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or

$$\frac{1}{-u} + \frac{1}{v} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

When the object is kept at infinity, the image is formed at the principal focus.

i.e. $u = -\infty$, $v = +f$.

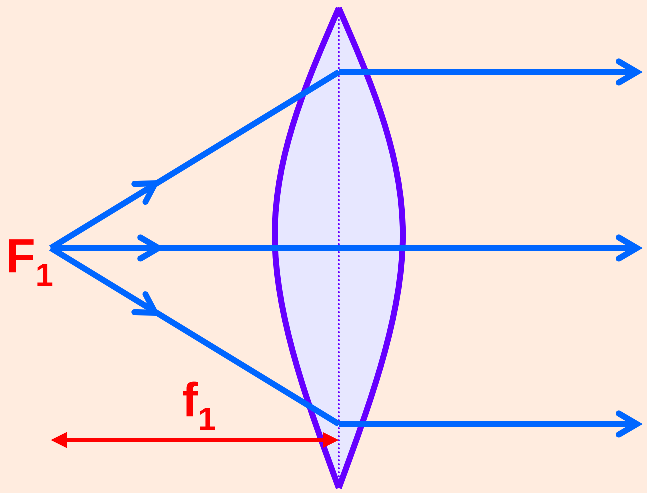
So, $$\boxed{\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

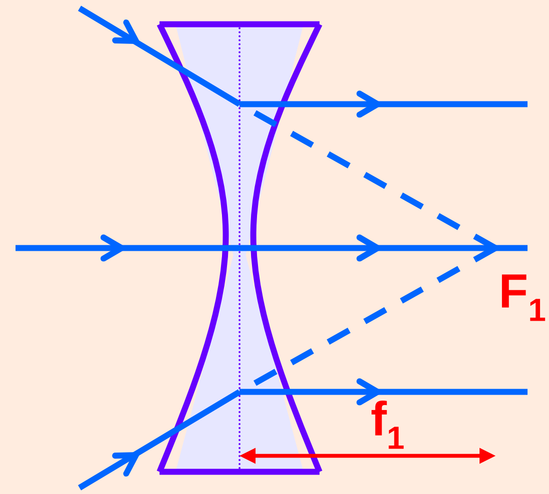
This equation is called 'Lens Maker's Formula'.

Also, from the above equations we get, $$\boxed{\frac{1}{-u} + \frac{1}{v} = \frac{1}{f}}$$

First Principal Focus:

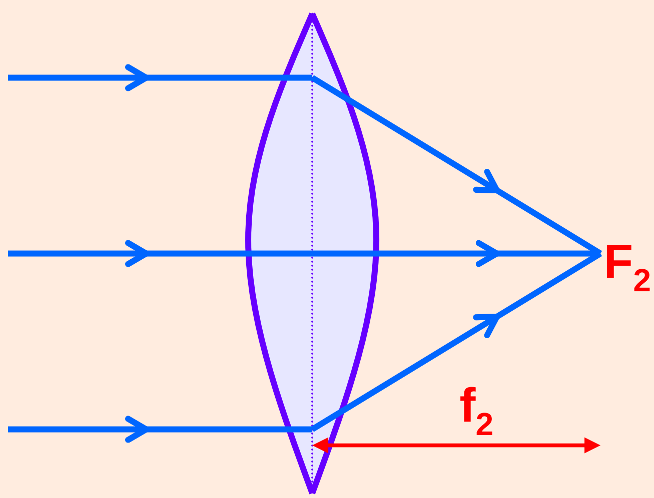
First Principal Focus is the point on the principal axis of the lens at which if an object is placed, the image would be formed at infinity.

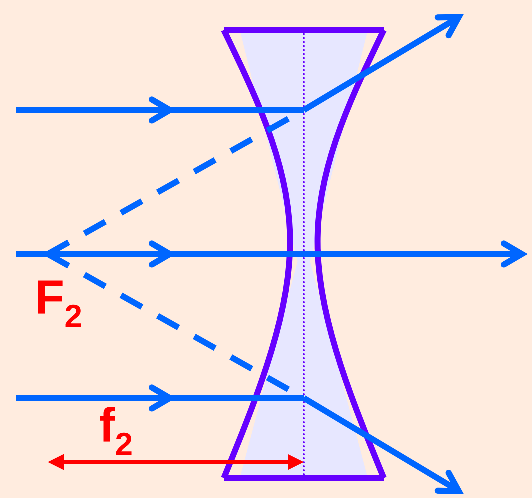




Second Principal Focus:

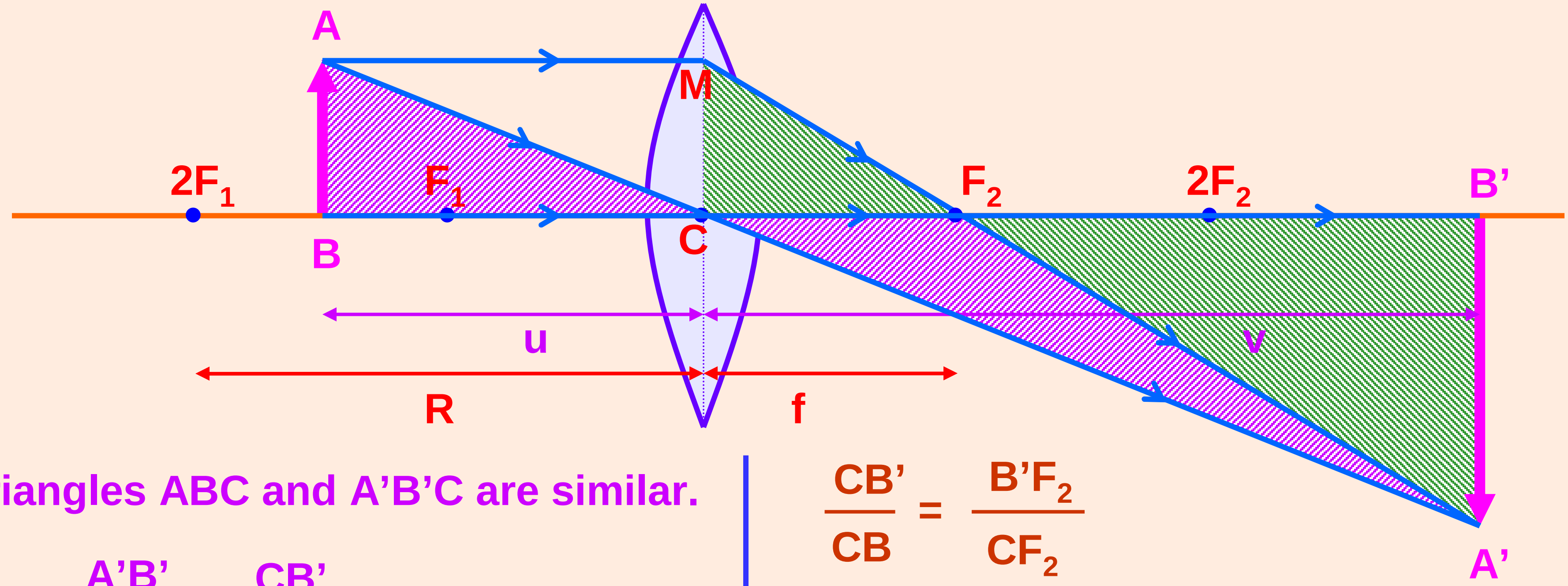
Second Principal Focus is the point on the principal axis of the lens at which the image is formed when the object is kept at infinity.





Thin Lens Formula (Gaussian Form of Lens Equation):

For Convex Lens:



Triangles ABC and A'B'C are similar.

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

Triangles MCF_2 and $A'B'F_2$ are similar.

$$\frac{A'B'}{MC} = \frac{B'F_2}{CF_2}$$

or $$\frac{A'B'}{AB} = \frac{B'F_2}{CF_2}$$

$$\frac{CB'}{CB} = \frac{B'F_2}{CF_2}$$

$$\frac{CB'}{CB} = \frac{CB' - CF_2}{CF_2}$$

According to new Cartesian sign conventions,

$CB = - u$, $CB' = + v$ and $CF_2 = + f$.

$$\therefore \boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$$

Linear Magnification:

Linear magnification produced by a lens is defined as the ratio of the size of the image to the size of the object.

$$m = \frac{I}{O}$$

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

According to new Cartesian sign conventions,

A'B' = + I, AB = - O, CB' = + v and CB = - u.

$$\frac{+I}{-O} = \frac{+v}{-u} \quad \text{or} \quad \boxed{m = \frac{I}{O} = \frac{v}{u}}$$

Magnification in terms of v and f:

$$\boxed{m = \frac{f - v}{f}}$$

Magnification in terms of v and f:

$$\boxed{m = \frac{f}{f - u}}$$

Power of a Lens:

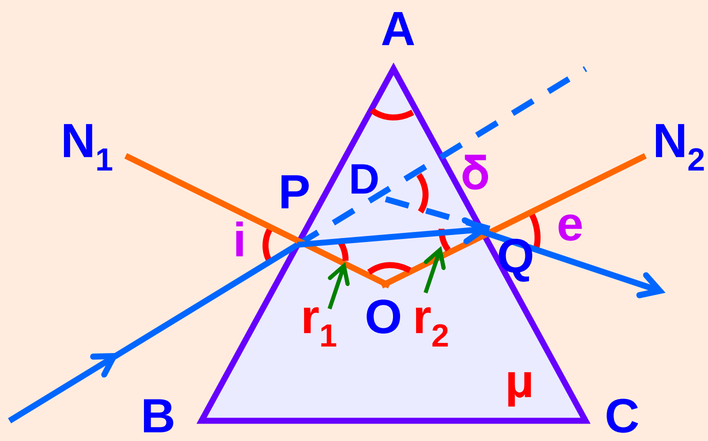
Power of a lens is its ability to bend a ray of light falling on it and is reciprocal of its focal length. When f is in metre, power is measured in Dioptre (D).

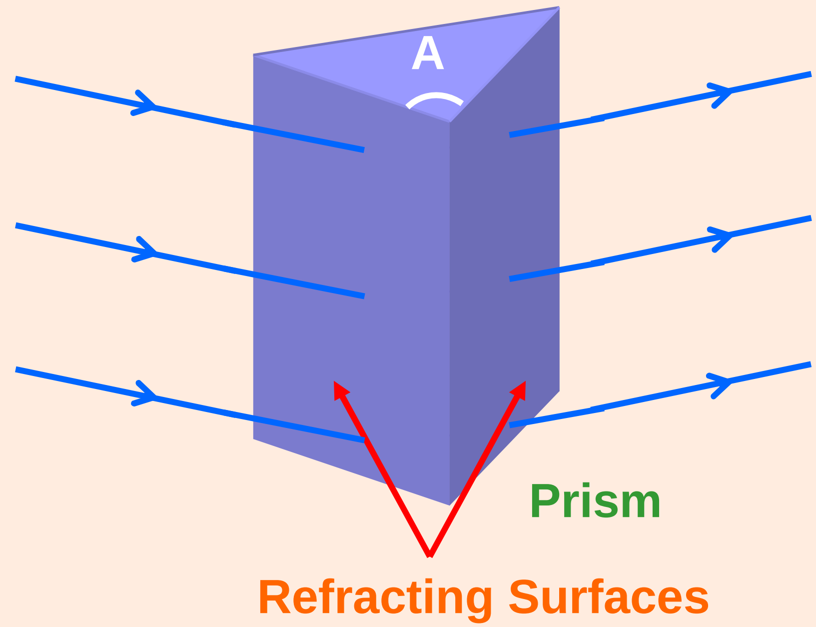
$$\boxed{P = \frac{1}{f}}$$

RAY OPTICS - II

1. Refraction through a Prism
2. Expression for Refractive Index of Prism
3. Dispersion
4. Angular Dispersion and Dispersive Power
5. Blue Colour of the Sky and Red Colour of the Sun
6. Compound Microscope
7. Astronomical Telescope (Normal Adjustment)
8. Astronomical Telescope (Image at LDDV)
9. Newtonian Telescope (Reflecting Type)
10. Resolving Power of Microscope and Telescope

Refraction of Light through Prism:





In quadrilateral APOQ,

$A + O = 180°$ …… .(1)

(since N_1 and N_2 are normal)

In triangle OPQ,

$r_1 + r_2 + O = 180°$ …… .(2)

In triangle DPQ,

$\delta = (i - r_1) + (e - r_2)$

$\delta = (i + e) - (r_1 + r_2)$ …….(3)

From (1) and (2),

$A = r_1 + r_2$

From (3),

$\delta = (i + e) - (A)$

or $\boxed{i + e = A + \delta}$

Sum of angle of incidence and angle of emergence is equal to the sum of angle of prism and angle of deviation.

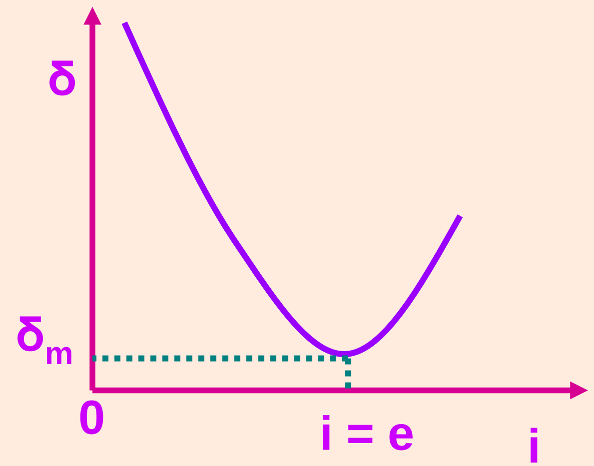
Variation of angle of deviation with angle of incidence:

When angle of incidence increases, the angle of deviation decreases.

At a particular value of angle of incidence the angle of deviation becomes minimum and is called **'angle of minimum deviation'.**

At δ_m, $i = e$ and $r_1 = r_2 = r$ (say)

After minimum deviation, angle of deviation increases with angle of incidence.



Refractive Index of Material of Prism:

$A = r_1 + r_2$

$A = 2r$

$\boxed{r = A / 2}$

$i + e = A + \delta$

$2\,i = A + \delta_m$

$\boxed{i = (A + \delta_m) / 2}$

According to Snell's law,

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin i}{\sin r}$$

$$\therefore \boxed{\mu = \frac{\sin \dfrac{(A + \delta_m)}{2}}{\sin \dfrac{A}{2}}}$$

Refraction by a Small-angled Prism for Small angle of Incidence:

$$\mu = \frac{\sin i}{\sin r_1} \quad \text{and} \quad \mu = \frac{\sin e}{\sin r_2}$$

If i is assumed to be small, then r_1, r_2 and e will also be very small.
So, replacing sines of the angles by angles themselves, we get

$$\mu = \frac{i}{r_1} \quad \text{and} \quad \mu = \frac{e}{r_2}$$

$$i + e = \mu (r_1 + r_2) = \mu A$$

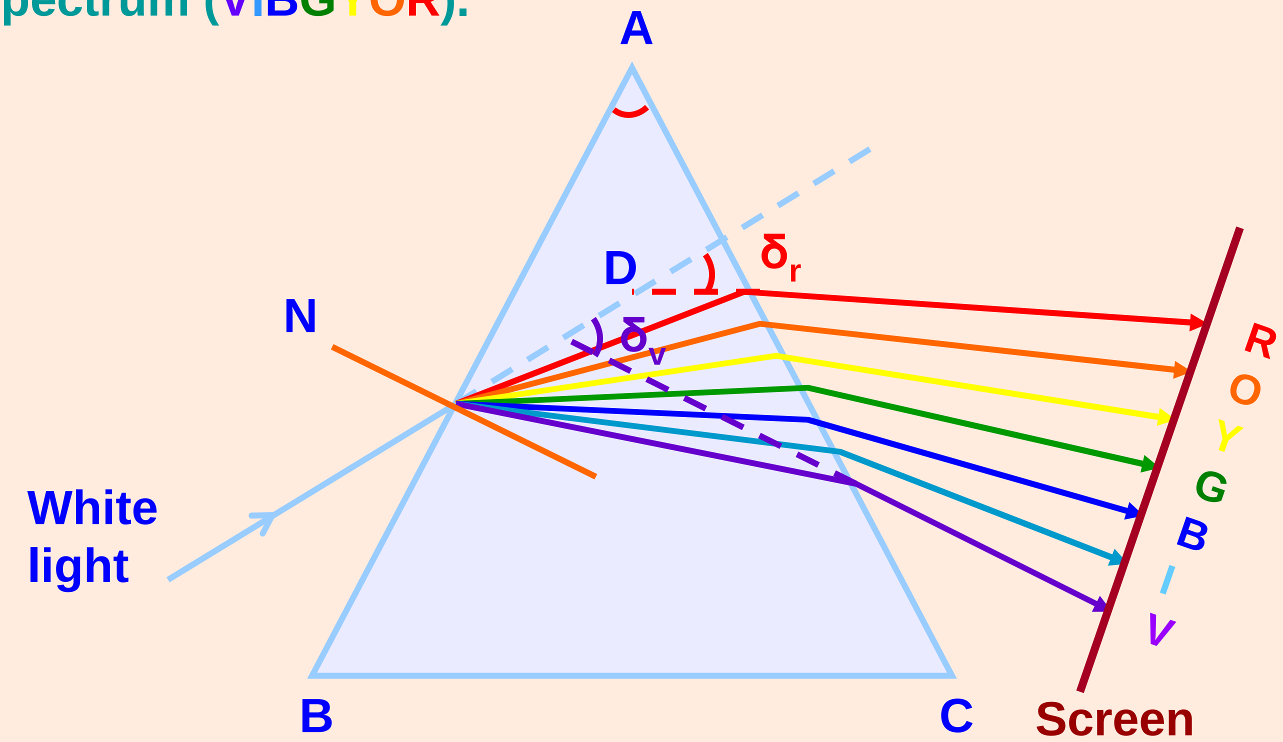
But $i + e = A + \delta$

So, $A + \delta = \mu A$

or $$\boxed{\delta = A (\mu - 1)}$$

Dispersion of White Light through Prism:

The phenomenon of splitting a ray of white light into its constituent colours (wavelengths) is called dispersion and the band of colours from violet to red is called spectrum (VIBGYOR).



Cause of Dispersion:

$$\mu_v = \frac{\sin i}{\sin r_v} \quad \text{and} \quad \mu_r = \frac{\sin i}{\sin r_r}$$

Since $\mu_v > \mu_r$, $r_r > r_v$

So, the colours are refracted at different angles and hence get separated.

Dispersion can also be explained on the basis of Cauchy's equation.

$$\mu = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}$$ (where a, b and c are constants for the material)

Since $\lambda_v < \lambda_r$, $\mu_v > \mu_r$

But $\delta = A(\mu - 1)$

Therefore, $\delta_v > \delta_r$

So, the colours get separated with different angles of deviation.
Violet is most deviated and Red is least deviated.

Angular Dispersion:

1. The difference in the deviations suffered by two colours in passing through a prism gives the angular dispersion for those colours.
2. The angle between the emergent rays of any two colours is called angular dispersion between those colours.
3. It is the rate of change of angle of deviation with wavelength. ($\Phi = d\delta / d\lambda$)

$\Phi = \delta_v - \delta_r$ or $\boxed{\Phi = (\mu_v - \mu_r) A}$

Dispersive Power:

The dispersive power of the material of a prism for any two colours is defined as the ratio of the angular dispersion for those two colours to the mean deviation produced by the prism.

It may also be defined as dispersion per unit deviation.

$$\omega = \frac{\Phi}{\delta}$$ where δ is the mean deviation and $$\delta = \frac{\delta_v + \delta_r}{2}$$

Also $$\omega = \frac{\delta_v - \delta_r}{\delta}$$ or $$\omega = \frac{(\mu_v - \mu_r) A}{(\mu_y - 1) A}$$ or $$\omega = \frac{(\mu_v - \mu_r)}{(\mu_y - 1)}$$

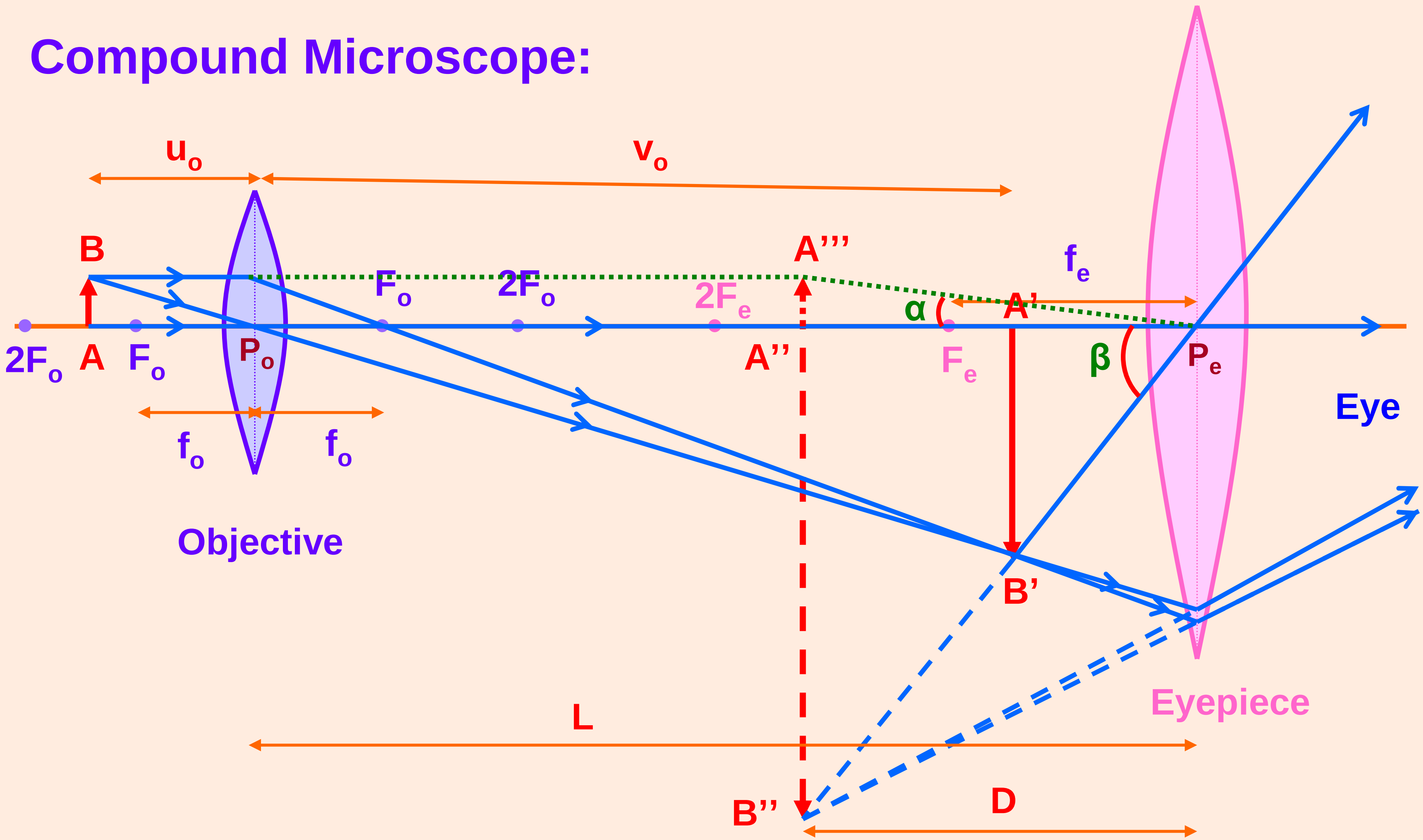
Scattering of Light – Blue colour of the sky and Reddish appearance of the Sun at Sun-rise and Sun-set:

The molecules of the atmosphere and other particles that are smaller than the longest wavelength of visible light are more effective in scattering light of shorter wavelengths than light of longer wavelengths. The amount of scattering is inversely proportional to the fourth power of the wavelength. (Rayleigh Effect)

Light from the Sun near the horizon passes through a greater distance in the Earth's atmosphere than does the light received when the Sun is overhead. The correspondingly greater scattering of short wavelengths accounts for the reddish appearance of the Sun at rising and at setting.

When looking at the sky in a direction away from the Sun, we receive scattered sunlight in which short wavelengths predominate giving the sky its characteristic bluish colour.

Compound Microscope:



Objective: The converging lens nearer to the object.

Eyepiece: The converging lens through which the final image is seen.

Both are of short focal length. Focal length of eyepiece is slightly greater than that of the objective.

Angular Magnification or Magnifying Power (M):

Angular magnification or magnifying power of a compound microscope is defined as the ratio of the angle β subtended by the final image at the eye to the angle α subtended by the object seen directly, when both are placed at the least distance of distinct vision.

$$\boxed{M = \frac{\beta}{\alpha}}$$

Since angles are small,
$\alpha = \tan\alpha$ and $\beta = \tan\beta$

$$M = \frac{\tan\beta}{\tan\alpha}$$

$$M = \frac{A''B''}{D} \times \frac{D}{A''A'''}$$

$$M = \frac{A''B''}{D} \times \frac{D}{AB}$$

$$M = \frac{A''B''}{AB}$$

$$M = \frac{A''B''}{A'B'} \times \frac{A'B'}{AB}$$

$$M = M_e \times M_o$$

$$M_e = 1 - \frac{v_e}{f_e} \quad \text{or} \quad M_e = 1 + \frac{D}{f_e}$$

(v_e = - D = - 25 cm)

and $M_o = \dfrac{v_o}{-u_o}$

$$\therefore \quad \boxed{M = \frac{v_o}{-u_o}\left(1 + \frac{D}{f_e}\right)}$$

Since the object is placed very close to the principal focus of the objective and the image is formed very close to the eyepiece,
$u_o \approx f_o$ and $v_o \approx L$

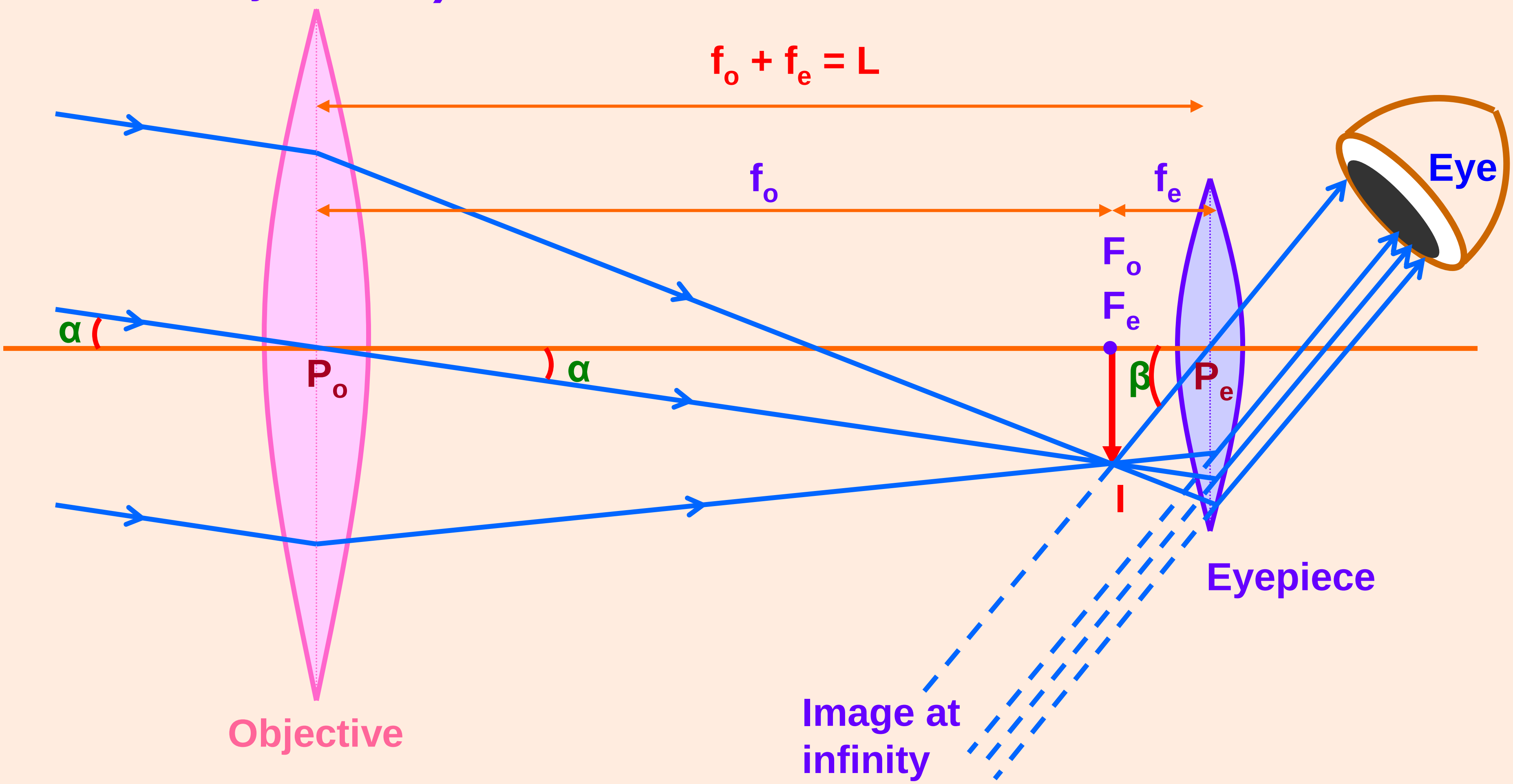
$$\boxed{M = \frac{-L}{f_o}\left(1 + \frac{D}{f_e}\right)}$$

or

$$\boxed{M \approx \frac{-L}{f_o} \times \frac{D}{f_e}}$$

(Normal adjustment i.e. image at infinity)

Astronomical Telescope: (Image formed at infinity – Normal Adjustment)



Focal length of the objective is much greater than that of the eyepiece.

Aperture of the objective is also large to allow more light to pass through it.

Angular magnification or Magnifying power of a telescope in normal adjustment is the ratio of the angle subtended by the image at the eye as seen through the telescope to the angle subtended by the object as seen directly, when both the object and the image are at infinity.

$$\boxed{M = \frac{\beta}{\alpha}}$$

Since angles are small, $\alpha = \tan\alpha$ and $\beta = \tan\beta$

$$M = \frac{\tan\beta}{\tan\alpha}$$

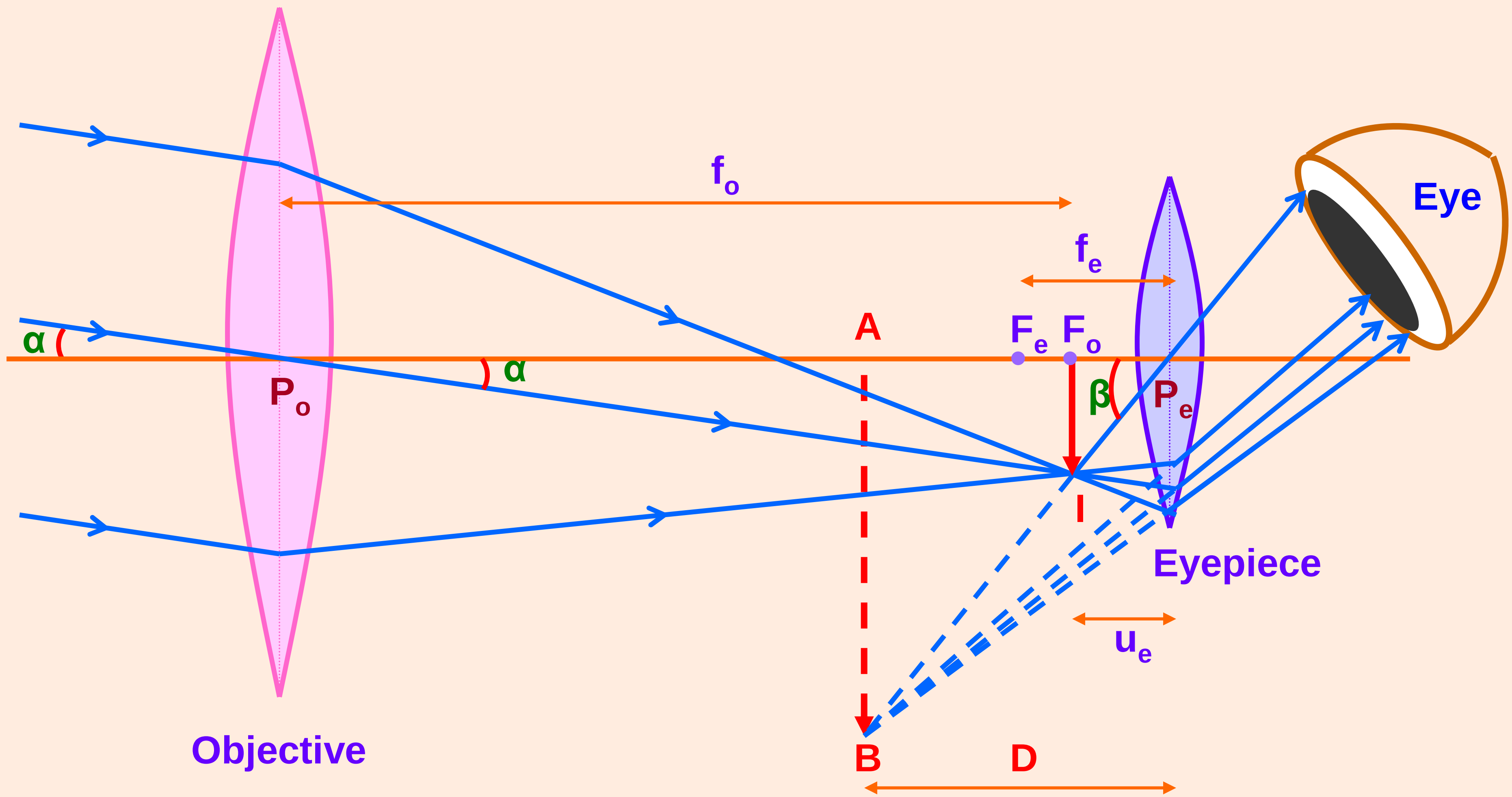
$$M = \frac{F_e I}{P_e F_e} \Big/ \frac{F_e I}{P_o F_e}$$

$$M = \frac{-I}{-f_e} \Big/ \frac{-I}{f_o}$$

$$\boxed{M = \frac{-f_o}{f_e}}$$

($f_o + f_e = L$ is called the length of the telescope in normal adjustment).

Astronomical Telescope: (Image formed at LDDV)



Angular magnification or magnifying power of a telescope in this case is defined as the ratio of the angle β subtended at the eye by the final image formed at the least distance of distinct vision to the angle α subtended at the eye by the object lying at infinity when seen directly.

$$\boxed{M = \frac{\beta}{\alpha}}$$

Since angles are small,
$\alpha = \tan\alpha$ and $\beta = \tan\beta$

$$M = \frac{\tan\beta}{\tan\alpha}$$

$$M = \frac{F_o I}{P_e F_o} \Big/ \frac{F_o I}{P_o F_o}$$

$$M = \frac{P_o F_o}{P_e F_o} \quad \text{or} \quad M = \frac{+f_o}{-u_e}$$

Lens Equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$ becomes

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

or $$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

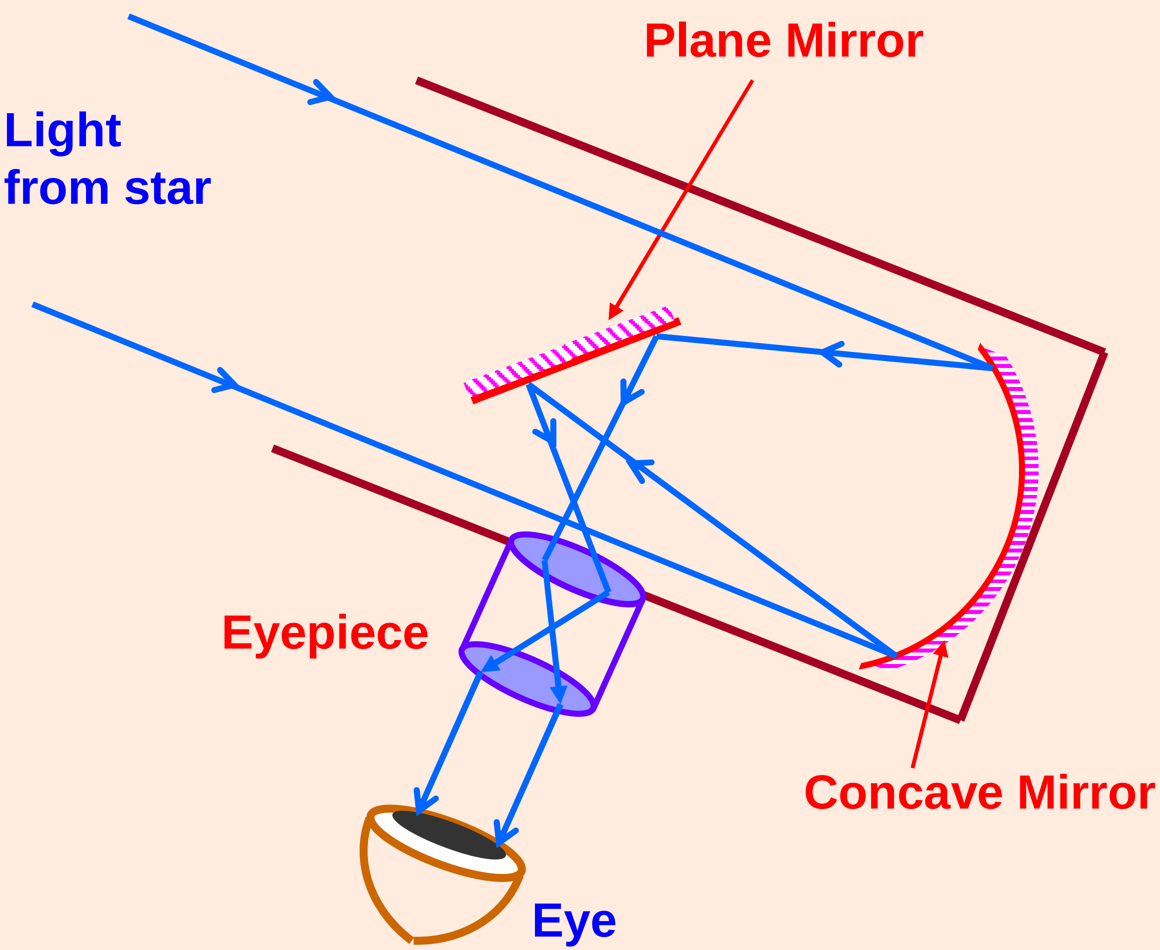
Multiplying by f_o on both sides and rearranging, we get

$$\boxed{M = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right)}$$

Clearly focal length of objective must be greater than that of the eyepiece for larger magnifying power.

Also, it is to be noted that in this case M is larger than that in normal adjustment position.

Newtonian Telescope: (Reflecting Type)



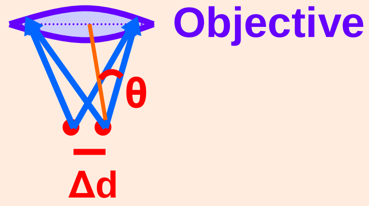
Magnifying Power:

$$M = \frac{f_o}{f_e}$$

Resolving Power of a Microscope:

The resolving power of a microscope is defined as the reciprocal of the distance between two objects which can be just resolved when seen through the microscope.

$$\text{Resolving Power} = \frac{1}{\Delta d} = \boxed{\frac{2\,\mu \sin\theta}{\lambda}}$$

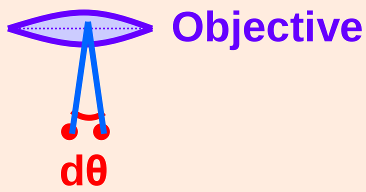


Resolving power depends on i) wavelength λ, ii) refractive index of the medium between the object and the objective and iii) half angle of the cone of light from one of the objects θ.

Resolving Power of a Telescope:

The resolving power of a telescope is defined as the reciprocal of the smallest angular separation between two distant objects whose images are seen separately.

$$\text{Resolving Power} = \frac{1}{d\theta} = \boxed{\frac{a}{1.22\,\lambda}}$$



Resolving power depends on i) wavelength λ, ii) diameter of the objective a.