# ADHIKAANSH 

## ACADEMY

## (IITJEE NEET IX X XI XII)

RUN BY:
DEEPAK SAINI SIR
B.TECH, M.TECH (N.S.I.T. DELHI UNIVERSITY)

Ex. Faculty of
Resonance Kota, Career Point Kota
Aakash Institute Mumbai
MATHS NOTES
(CLASS 11 ${ }^{\text {TH }}$ )


## FOR MORE FREE QUESTION BANKS AND SAMPLE PAPER WT'S UP ON : 7665186856

## A Best Faculty Group in Meerut....

## ADH|KAANSH ACADEMY

 सही समय और सही दिशा में कड़ी मेहनत, तो सफलता पक्की!| So why to wait... | Class $11^{\text {th }}$ से बी थुखू करेंे, Board के साथ-साथ, <br>  |
| :---: | :---: |
|  | Special Classes For Students Studying in std. 9th \& 10th |
| DIRECTOR <br> DEEPAK SAINI (DSA SIR) | EX. FACULTY OF RESONANCE KOTA, AAKASH INSTITUTE MUMBAI |
| B.TECH, M.TECH | $\bigcirc$ 225/5, 1ST FLOOR, PANCHSHEEL COLONY, BEHIND PINWACLE TOWER, GARH ROAD, MEERUT. © 8057870069 |
| NSIT DELHI UNIVERSITY | Qwww.adhikanshiitjeemedical.com \| Madhikanshiitjeemedical@gmail.com |

Take Dummy Admission in Class 11th \& Prepare For..
IITJEE \& NEET

For Free Counselling \& Career Guidance contact on:

$$
7665186856
$$

## Chapter 4

## PRINCIPLE OF MATHEMATICAL INDUCTION

## INTRODUCTION

To prove certain results or statements in Algebra, that are formulated in terms of n , where n is a natural number, we use a specific technique called principle of mathematical induction (P.M.I)

## Steps of P.M.I

Step I - Let $\mathrm{p}(\mathrm{n})$ : result or statement formulated in terms of n (given question)
Step II - Prove that $\mathrm{P}(1)$ is true
Step III - Assume that $\mathrm{P}(\mathrm{k})$ is true
Step IV - Using step III prove that $\mathrm{P}(\mathrm{k}+1)$ is true
Step V - Thus $\mathrm{P}(1)$ is true and $\mathrm{P}(\mathrm{k}+1)$ is true whenever $\mathrm{P}(\mathrm{k})$ is true.
Hence by P.M.I, $\mathrm{P}(\mathrm{n})$ is true for all natural numbers n

## Type I

Eg: Ex 4.1

1) Prove that
$1+3+3^{2}+\ldots \ldots \ldots+3^{n-1}=\frac{3^{n}-1}{2}$
Solution:-
Step I : Let $P(n): 1+3+3^{2}+\ldots \ldots \ldots+3^{n-1}=\frac{3^{n}-1}{2}$
Step II: P(1):

$$
\begin{aligned}
& \text { LHS }=1 \\
& \text { RHS }==\frac{3-1}{2}=\frac{2}{2}=1
\end{aligned}
$$

## LHS=RHS

Therefore $\mathrm{p}(1)$ is true.
Step III: Assume that $\mathrm{P}(\mathrm{k})$ is true

$$
\begin{equation*}
\text { i.e } 1+3+3^{2}+\ldots \ldots \ldots+3^{k-1}=\frac{3^{k}-1}{2} \tag{1}
\end{equation*}
$$

Step IV: we have to prove that $\mathrm{P}(\mathrm{k}+1)$ is true.
ie to prove that $1+3+3^{2}+\ldots \ldots \ldots+3^{k-1}+3=\frac{3^{k+1}-1}{2}$

Proof

$$
\begin{aligned}
\text { LHS } & =\left(1+3+3^{2}+\ldots \ldots \ldots+3^{k-1}\right)+3 \\
& =\frac{3^{k}-1}{2}+3^{\mathrm{k}} \text { from eq }(1) \\
& =\frac{3^{\mathrm{k}}-1+2.3^{\mathrm{k}}}{2} \\
& =\frac{3.3^{\mathrm{k}}-1}{2}=\frac{3^{\mathrm{k}+1}-1}{2}=\text { RHS }
\end{aligned}
$$

Therefore $\mathrm{P}(\mathrm{k}+1)$ is true
Step V:Thus $\mathrm{P}(1)$ is true and $\mathrm{P}(\mathrm{k}+1)$ is true whenever $\mathrm{P}(\mathrm{k})$ is true. Hence by P.M.I, $\mathrm{P}(\mathrm{n})$ is true for all natural number n .

## Text book

## Ex 4.1

Q. $1,2,3^{* *}(\mathrm{HOT}), 4,5^{*}, 6^{*}, 7,8,9,10^{*}, 11^{* *}, 12,13^{* *}, 14^{* *}, 15,16^{* *}, 17^{* *}$, eg 1 , eg 3

## Type 2

Divisible / Multiple Questions like Q. 20**,21,22**,23 of Ex 4.1 eg 4, eg $6^{* *}$ (HOT) Q 22. Prove that $3^{2 n+2}-8 n-9$ is divisible by 8 for all natural number $n$. Solution
Step I: Let $\mathrm{p}(\mathrm{n}): 3^{2 \mathrm{n}+2}-8 \mathrm{n}-9 \quad$ is divisible by 8
Step II: P(1): $3^{4}-8-9=81-17=64$ which is divisible by 8 Therefore $\mathrm{p}(1)$ is true
Step III: Assume that $\mathrm{p}(\mathrm{k})$ is true
i.e $3^{2 k+2}-8 k-9=8 m ; \quad m$ is a natural number.
i.e $3^{2 \mathrm{k}} .9=8 \mathrm{~m}+8 \mathrm{k}+9$
ie $3^{2 \mathrm{k}}=\underline{8 \mathrm{~m}+8 \mathrm{k}+9}$

Step IV: To prove that $\mathrm{p}(\mathrm{k}+1)$ is true.
ie to prove that $3^{2 k+4}-8(k+1)-9$ is divisible by 8 .
Proof: $3^{2 k+4}-8 \mathrm{k}-17=3^{2 \mathrm{k}} \cdot 3^{4}-8 \mathrm{k}-17=\left(\frac{8 m+8 k+9}{9}\right) \times 3^{4}-8 \mathrm{k}-17($ from eqn (1))
$=(8 \mathrm{~m}+8 \mathrm{k}+9) 9-8 \mathrm{k}-17=72 \mathrm{~m}+72 \mathrm{k}+81-8 \mathrm{k}-17=72 \mathrm{~m}-64 \mathrm{k}+64=8[9 \mathrm{~m}-$ $8 \mathrm{k}+8]$ is divisible by 8 .

Step V: Thus $\mathrm{P}(1)$ is true and $\mathrm{P}(\mathrm{k}+1)$ is true whenever $\mathrm{P}(\mathrm{k})$ is true. hence by P.M.I, $\quad \mathrm{P}(\mathrm{n})$ is true for all natural numbers n .

Type III: Problems based on Inequations

## Ex 4.1

$$
\text { Q. 18,14, eg } 7
$$

(Q 18) Prove that $1+2+3+\ldots \ldots+\mathrm{n}<\frac{(2 n+1)^{2}}{8}$
Step I : Let P(n): $1+2+3+\ldots \ldots+\mathrm{n}<\frac{(2 n+1)^{2}}{8}$
Step II: $\mathrm{P}(1): 1<\frac{9}{8}$ which is true, therefore $\mathrm{p}(1)$ is true.
Step III: Assume that $\mathrm{P}(\mathrm{k})$ is true. ie $1+2+3+\ldots \ldots+\mathrm{k}<\frac{(2 k+1)^{2}}{8}$ $\qquad$

Step IV: We have to prove that $\mathrm{P}(\mathrm{k}+1)$ is true. ie to

$$
\text { prove that } 1+2+3+\ldots . .+\mathrm{k}+(\mathrm{k}+1)<\frac{(2 k+3)^{2}}{8}
$$

Proof: Adding ( $\mathrm{k}+1$ ) on both sides of inequation (1)

$$
\begin{aligned}
1+2+3+\ldots .+\mathrm{k}+(\mathrm{k}+1) & <\frac{(2 \mathrm{k}+1)^{2}}{8}+(k+1) \\
& =\frac{\left(4 \mathrm{k}^{2}+4 \mathrm{k}+1\right)+8 \mathrm{k}+8}{8} \\
& =\frac{4 \mathrm{k}^{2}+12 \mathrm{k}+9}{8}
\end{aligned}
$$

$$
=\frac{(2 k+3)^{2}}{8}
$$

$$
\text { Therefore } 1+2+3+\ldots . .+k+(k+1)<\frac{(2 k+3)^{2}}{8}
$$

$$
\mathrm{P}(\mathrm{k}+1) \text { is true. }
$$

Step V : Thus $\mathrm{P}(1)$ is true and $\mathrm{P}(\mathrm{k}+1)$ is true whenever $\mathrm{P}(\mathrm{k})$ is true. Hence by P.M.I, $\mathrm{P}(\mathrm{n})$ is true for all natural number n .

## HOT/EXTRA QUESTIONS

Prove by mathematical induction that for all natural numbers $n$.

1) $a^{2 n-1}-1$ is divisible by a-1 (type II)
2) $\frac{n^{7}}{7}+\frac{n^{5}}{5}+\frac{2 n^{3}}{3}-\frac{n}{105}$ is an integer(HOT)
3) $\sin x+\sin 3 x+\ldots \ldots+\sin (2 n-1) x=\underline{\sin ^{2} n x} \quad$ (HOT Type 1) $\sin x$
4) $3^{2 n-1}+3^{n}+4$ is divisible by 2 (type II)
5) Let $P(n): n^{2}+n-19$ is prime, state whether $P(4)$ is true or false
6) $2^{2 n+3} \leq(n+3)$ ! (type III)
7) What is the minimum value of natural number $n$ for which $2^{n}<n$ ! holds true?
8) $7^{2 \mathrm{n}}+2^{3 \mathrm{n}-3} \cdot 3^{\mathrm{n}-1}$ is divisible by 25 (type II)

## Answers

5) false
6) 4

## FOR MORE FREE QUESTION BANKS AND SAMPLE PAPER WT'S UP ON : 7665186856

## A Best Faculty Group in Meerut....

## ADH|KAANSH ACADEMY

 सही समय और सही दिशा में कड़ी मेहनत, तो सफलता पक्की!| So why to wait... | Class $11^{\text {th }}$ से बी थुखू करेंे, Board के साथ-साथ, <br>  |
| :---: | :---: |
|  | Special Classes For Students Studying in std. 9th \& 10th |
| DIRECTOR <br> DEEPAK SAINI (DSA SIR) | EX. FACULTY OF RESONANCE KOTA, AAKASH INSTITUTE MUMBAI |
| B.TECH, M.TECH | $\bigcirc$ 225/5, 1ST FLOOR, PANCHSHEEL COLONY, BEHIND PINWACLE TOWER, GARH ROAD, MEERUT. © 8057870069 |
| NSIT DELHI UNIVERSITY | Qwww.adhikanshiitjeemedical.com \| Madhikanshiitjeemedical@gmail.com |

Take Dummy Admission in Class 11th \& Prepare For..
IITJEE \& NEET

For Free Counselling \& Career Guidance contact on:

$$
7665186856
$$

