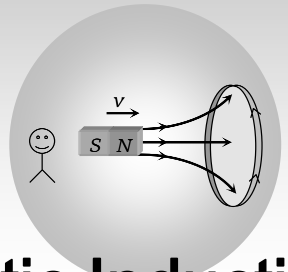
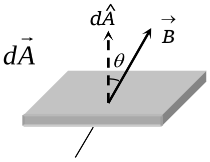


Electromagnetic Induction

Magnetic Flux

The total number of magnetic lines of force passing normally through an area placed in a magnetic field is equal to the magnetic flux linked with that area.



For elementary area dA of a surface flux linked $d\phi = B\,dA\cos\theta$ or $d\phi = \vec{B}\cdot d\vec{A}$

So, Net flux through the surface $\boldsymbol{\phi = \oint \vec{B}\cdot d\vec{A} = BA\cos\theta}$

For N-turns coil $\phi = NBA\cos\theta$

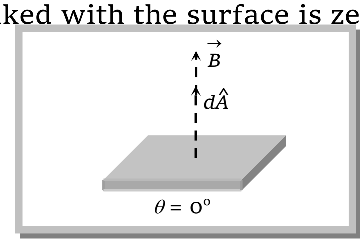
(1) **Unit and Dimension**

Magnetic flux is a scalar quantity it's S.I. unit is *weber* (*wb*), CGS unit is *Maxwell* or *Gauss* × *cm*²; $1\,wb = 10^8\,Maxwell$. Other units : *Tesla* × *m*² $= \frac{N\times m}{Amp} = \frac{Joule}{Amp} = \frac{Volt\times Coulomb}{Amp} = Volt\times sec$ = *Ohm* × *Coulomb* = *Henry* × *Amp*. It's dimensional formula $[\phi] = [ML^2T^{-2}A^{-1}]$

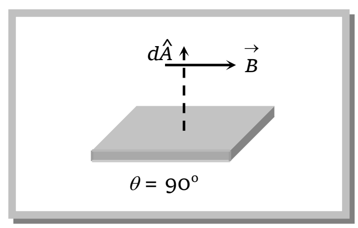
(2) **Maximum and Zero flux**

If $\theta = 0^o$, *i.e.* plane is held perpendicular to the direction of magnetic field then flux from the surface is maximum and if $\theta = 90^o$ *i.e.* plane is held parallel to the direction of magnetic field then flux linked with the surface is zero.

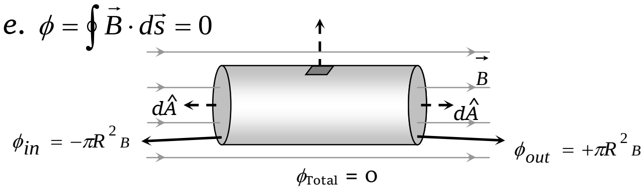
$\phi_{max} = BA$



$\phi = 0$

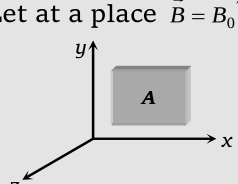


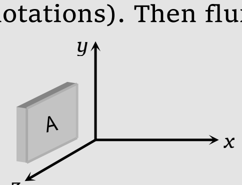
Note : ❒ In case of a body present in a field, either uniform or non-uniform, outward flux is taken to be positive while inward negative and Net flux linked with a closed surface is zero *i.e.* $\phi = \oint \vec{B}\cdot d\vec{s} = 0$

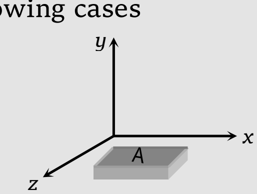


Specific example

Let at a place $\vec{B} = B_0\hat{i}$ (with usual notations). Then flux for the following cases







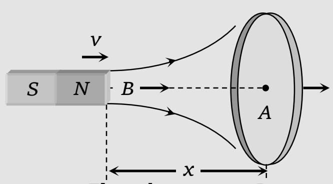
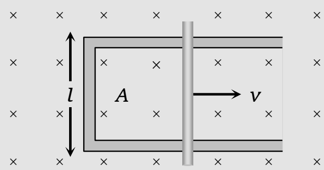
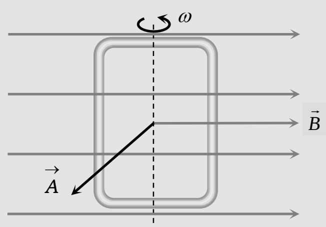
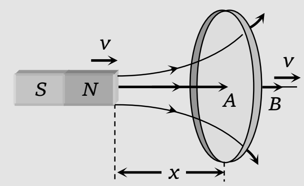
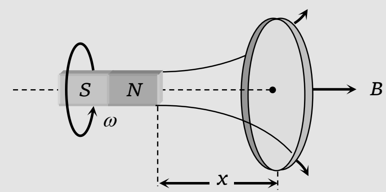
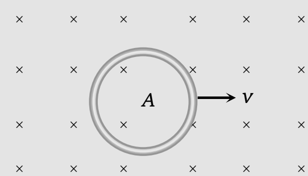
$\vec{A} = A\hat{k}$ $\quad$ $\phi = \vec{B}\cdot\vec{A} = (B_0\hat{i})\cdot A\hat{k} = 0$

$\vec{A} = A\hat{i}$ $\quad$ $\phi = \vec{B}\cdot\vec{A} = (B_0\hat{i})\cdot A\hat{i} = B_0 A$

$\vec{A} = A\hat{j}$ $\quad$ $\phi = \vec{B}\cdot\vec{A} = (B_0\hat{i})\cdot A\hat{j} = 0$

(3) **Variation of magnetic flux**

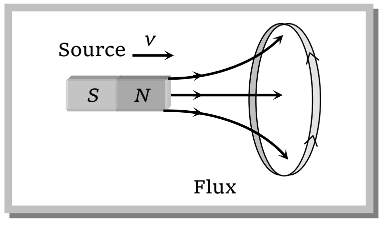
We know that magnetic flux linked with an area A is $\phi = BA\cos\theta$ *i.e.* ϕ will change if either B, A or θ will change

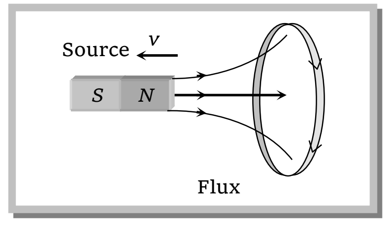
Flux changes	Flux not changes
 Flux changes as B  Flux changes as area (swept by rod)  Flux changes as θ changes	   In all these three cases flux ϕ will not change because B, A and θ doesn't change with time

Faraday's Experiment and Laws

(1) **First experiment**

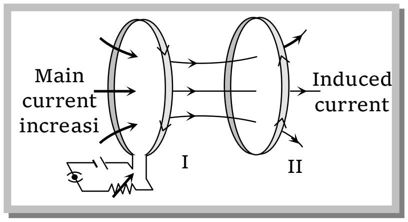
A coil is arranged to link some of the magnetic flux from a source S. If relative motion occurs between coil and source S such that flux linked with the coil changes, a current is induced in it.





(2) **Second experiment**

Two coils are arranged so that a steady current flows in one and some of its magnetic flux links with the other. If the current in the first coil changes a current is induced in the second.



(3) **Faradays first law**

Whenever the number of magnetic lines of force (magnetic flux) passing through a circuit changes (or a moving conductor cuts the magnetic flux) an emf is produced in the circuit (or emf induces across the ends of the conductor) called induced emf. The induced emf persists only as long as there is change or cutting of flux.

(4) **Faradays second law**

The induced emf is given by rate of change of magnetic flux linked with the circuit *i.e.* $e = -\frac{d\phi}{dt}$.

For N turns $e = -\frac{N\,d\phi}{dt}$; Negative sign indicates that induced emf (e) opposes the change of flux.

(i) **Other forms :** We know that $\phi = BA\cos\theta$; Hence ϕ will change if either, B, A or θ will change

$$\text{So}\quad e = -N\frac{d\phi}{dt} = -\frac{N(\phi_2 - \phi_1)}{\Delta t} = -\frac{NA(B_2 - B_1)\cos\theta}{\Delta t} = -\frac{NBA(\cos\theta_2 - \cos\theta_1)}{\Delta t}$$

Note : ❐ Term $\frac{B_2 - B_1}{\Delta t}$ = rate of change of magnetic field, it's unit is *Tesla/sec*

(ii) **Induced current :** If circuit is closed, then induced current is given by $i = \frac{e}{R} = -\frac{N}{R}.\frac{d\phi}{dt}$; where R is the resistance of circuit

(iii) **Induced charge :** If dq charge flows due to induction in time dt then $i = \frac{dq}{dt}$; $dq = i\,dt = -\frac{N}{R}\cdot d\phi$ *i.e.* the charge induced does not depend on the time interval in which flux through the circuit changes. It simply depends on the net change in flux and resistance of the circuit.

(iv) **Induced power** : It exists when the circuit is open or closed $P = ei = \frac{e^2}{R} = i^2R = \frac{N^2}{R}\left(\frac{d\phi}{dt}\right)^2$.

It depends on time and resistance

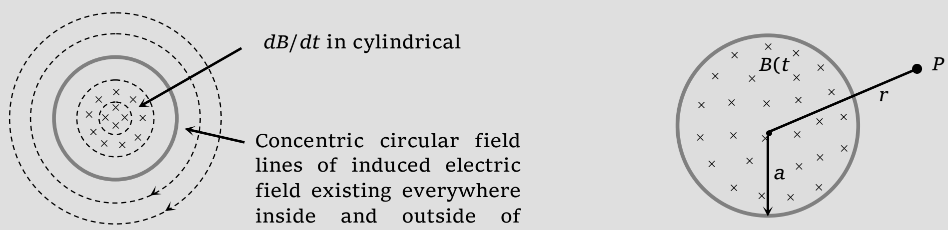
(5) **Induced electric field**

It is non-conservative and non-electrostatic in nature. Its field lines are concentric circular closed curves. A time varying magnetic field $\frac{dB}{dt}$ always produced induced electric field in all space surrounding it. Induced electric field is directly proportional to induced emf so $e = \oint \vec{E}_{in}\cdot d\vec{l}$

here $\vec{E}_{in} =$ induced electric field ......(i)

Also Induced emf from Faraday laws of EMI $e = -\frac{d\phi}{dt}$ ......(ii)

From (i) and (ii) $e = \oint \vec{E}_{in}.d\vec{l} = -\frac{d\phi}{dt}$ This is known as integral form of Faraday's laws of EMI.



A uniform but time varying magnetic field $B(t)$ exists in a circular region of radius 'a' and is directed into the plane of the paper as shown, the magnitude of the induced electric field (E_{in}) at point P lies at a distance r from the centre of the circular region is calculated as follows.

So $\oint \vec{E}_{in} d\vec{l} = e = \frac{d\phi}{dt} = A\frac{dB}{dt}$ *i.e.* $E(2\pi r) = \pi a^2 \frac{dB}{dt}$ where $r \geq a$ or $E = \frac{a^2}{2r}\frac{dB}{dt}$; $\boldsymbol{E_{\text{in}} \propto \frac{1}{r}}$

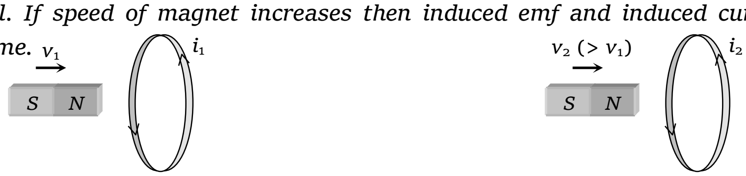
(6) **Change in induced parameter (*e*, *i* and *q*) with change in θ**

Suppose a coil having *N turns,* area of each turn is *A* placed in a transverse magnetic field *B* such that it's plane is perpendicular to the direction of magnetic field *i.e.* initially $\theta_1 = 0°$. If *R* is the resistance of entire circuit and $\phi_1 = NBA \cos 0° = NBA$, is initial flux linked with the coil then.

Change	**Final flux (ϕ_2)**	**Change in flux** $\Delta\phi = (\phi_2 - \phi_1)$	**Time taken (Δt)**	**Induced emf** $e = -\frac{\Delta\phi}{\Delta t}$	**Induced current** $i = \frac{e}{R}$	**Induced charge** $q = i\Delta t$
Coil turn through 180° (end to end)	$-NBA$	$-2NBA$	t	$\frac{2NBA}{t}$	$\frac{2NBA}{Rt}$	$\frac{2NBA}{R}$
Turn through 90°	Zero	$-NBA$	t	$\frac{NBA}{t}$	$\frac{NBA}{Rt}$	$\frac{NBA}{R}$
Taken out of the field	Zero	$-NBA$	t	$\frac{NBA}{t}$	$\frac{NBA}{Rt}$	$\frac{NBA}{R}$

Concepts

☞ *If a bar magnet moves towards a fixed conducting coil, then due to the flux changes an emf, current and charge induces in the coil. If speed of magnet increases then induced emf and induced current increases but induced charge remains same.*

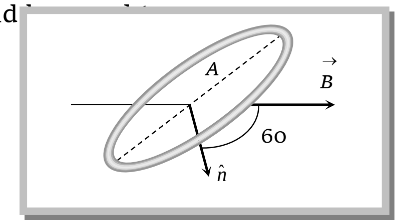


Induced parameter : e_1, i_1, q_1 *Induced parameter :* $e_2\ (> e_1), i_2(> i_1), q_2\ (= q_1)$

- *Can ever electric lines of force be closed curve ? Yes, when produced by a changing magnetic field.*
- *It should be kept in mind that the total induced emf in a loop is not confined to any particular point but it is distributed around the loop in direct proportion to the resistance of it's parts.*

Example

***Example*: 1** A coil of area $A = 0.5\ m^2$ is situated in a uniform magnetic field $B = 4.0\ wb/m^2$ and area vector makes an angle of 60° with respect to the magnetic field as shown in figure. The value of the magnetic flux through the area A would be equal to



(a) 2 *weber*

(b) 1 *weber*

(c) 3 *weber*

(d) $\frac{3}{2}$ *weber*

Solution: (b) Angle between normal to the plane of the coil and direction of magnetic field is $\theta = 60°$

$\therefore$ Flux linked with coil $\phi = BA\cos\theta = 4.0 \times 0.5 \times \cos 60° \Rightarrow \phi = 1$ *weber*

***Example*: 2** A coil of N turns and area A is rotated at the rate of n rotations per second in a magnetic field of intensity B, the magnitude of the maximum magnetic flux will be

(a) NAB (b) nAB (c) $NnAB$ (d) $2\pi nNAB$

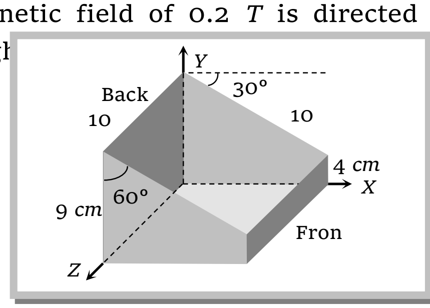
Solution: (a) Since $\phi = NBA\cos\theta$; For ϕ to be maximum; $\cos\theta$ = max = 1 so $\phi_{max} = NBA$.

***Example*: 3** A square coil of $10^{-2}\ m^2$ area is placed perpendicular to a uniform magnetic field of intensity $10^3\ wb/m^2$. The magnetic flux through the coil is **[MP PMT 1990, 2001]**

(a) 10 *weber* (b) 10^{-5} *weber* (c) 10^5 *weber* (d) 100 *weber*

Solution: (a) By using $\phi = BA\cos\theta$; here $\theta = 0°$ $\therefore \phi = BA = 10^3 \times 10^{-2} = 10$ *weber*

***Example*: 4** Consider the following figure, a uniform magnetic field of 0.2 T is directed along the positive x-axis. What is the magnetic flux through the top surface



(a) Zero

(b) 0.8 *m-wb*

(c) 1.0 *m-wb*

(d) – 1.8 *m-wb*

Solution: (c) Magnetic flux $\phi = BA\cos\theta$ for the top surface, the angle between normal to the surface and the x-axis is $\theta = 60°$

$\therefore \phi = 0.2 \times (10 \times 10 \times 10^{-4}) \times \cos 60° = 10^{-3}\ wb = 1 m\text{-}wb$

***Example*: 5** A coil of area 100 cm^2 has 500 turns. Magnetic field of 0.1 *weber/metre*^{2} is perpendicular to the coil. The field is reduced to zero in 0.1 *sec*. The induced emf in the coil is

(a) 1 V (b) 5 V (c) 50 V (d) Zero

Solution: (b) By using $e = -\frac{N(B_2 - B_1)A}{t}$; $e = -\frac{500\,(0 - 0.1) \times 100 \times 10^{-4}}{0.1} = 5V$.

***Example*: 6** A coil has 1000 turns and 500 cm^2 as it's area. The plane of the coil is placed at right angles to a magnetic induction field of 2×10^{-5} wb/m^2. The coil is rotated through 180° in 0.2 *sec*. The average emf induced in the coil in *mV* is

(a) 5 (b) 10 (c) 15 (d) 20

Solution: (b) $e = -\frac{NBA(\cos\theta_2 - \cos\theta_1)}{t}$

Initially θ_1 = 0° and finally θ_2 = 180° so $e = -\frac{1000 \times 2 \times 10^{-5} \times 500 \times 10^{-4}(\cos 180^o - \cos 0^o)}{0.2} = 10^{-2}V =$ 10 *mV*.

***Example*: 7** A coil having 500 square loops each of side 10 *cm* is placed normal to a magnetic field which increases at a rate of 1*T*/*s*. The induced emf in *volt* is

(a) 0.1 (b) 0.5 (c) 1 (d) 5

Solution: (d) By using $e = -\frac{N(B_2 - B_1)A\cos\theta}{\Delta t}$; Given θ = 0°, N = 500, $A = 100 \times 10^{-4}$ m^2, $\frac{B_2 - B_1}{\Delta t} = 1\frac{T}{\sec}$

$\therefore\ e = -500 \times 1 \times 10^{-2} \times \cos 0^o = -5V,\ |\,e\,| = 5V$

***Example*: 8** The magnetic field of 2×10^{-2} *Tesla* acts at right angle to a coil of area 100 cm^2 with 50 turns. The average emf induced in the coil is 0.1 V when it is removed from the field in time t. The value of t is

[CBSE 1992; CPMT 2001]

(a) 0.1 s (b) 0.01 s (c) 1 s (d) 20 s

Solution: (a) Given $B_1 = 2 \times 10^{-2}$ T, B_2 = 0, θ = 0°, N = 50, e = 0.1 V and $A = 100 \times 10^{-4}$ m^2

By using $e = -\frac{N(B_2 - B_1)A\cos\theta}{\Delta t}$; $0.1 = \frac{-50 \times (0 - 2 \times 10^{-2}) \times 10^{-2} \times \cos 0^o}{t}$ $\Rightarrow t = 0.1\,s$

***Example*: 9** A circular coil of 500 turns of a wire has an enclosed area of 0.1 m^2 per turn. It is kept perpendicular to a magnetic field of induction 0.2 T and rotated by 180° about a diameter perpendicular to the field in 0.1 *sec*. How much charge will pass when the coil is connected to a galvanometer with a combined resistance of 50 *ohms*

(a) 0.2 C (b) 0.4 C (c) 2 C (d) 4 C

Solution: (b) Given N = 500, A = 0.1 m^2, θ_1 = 0°, θ_2 = 180°, B = 0.2 T, Δt = 0.1 *sec*, R = 50Ω

By using $q = \frac{N}{R}.d\phi = -\frac{N}{R}BA(\cos\theta_2 - \cos\theta_1)$; $q = 0.4\,C$.

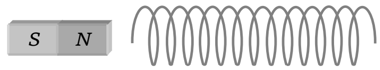
***Example*: 10** Flux ϕ (in *weber*) in a closed circuit of resistance 10 *ohm* varies with time t (in *sec*) according to the equation

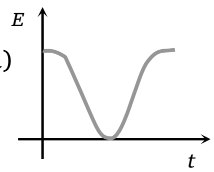
$\phi = 6t^2 - 5t + 1$. What is the magnitude of the induced current at t = 0.25 s ?

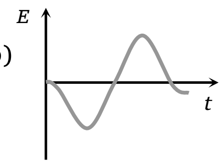
(a) 1.2 A (b) 0.8 A (c) 0.6 S (d) 0.2 A

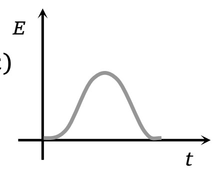
Solution: (d) By using $i = \frac{e}{R} = -\frac{1}{R}\frac{d\phi}{dt}$; $i = -\frac{1}{10}\frac{d}{dt}(6t^2 - 5t + 1) = -\frac{1}{10}(12t - 5)$; $i = -\frac{1}{10}(12 \times 0.25 - 5) = 0.2\,A$

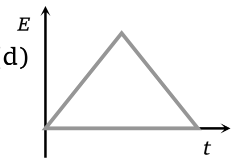
***Example*: 11** The variation of induced emf (E) with time (t) in a coil if a short bar magnet is moved along its axis with a constant velocity is best represented as



(a)


(b)


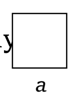
(c)


(d)


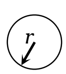
Solution: (b) As the magnet moves towards the coil, the magnetic flux increases (nonlinearly). Also there is a change in polarity of induced emf when the magnet passes on to the other side of the coil.

***Example*: 12** A square loop of side '*a*' and resistance R is placed in a transverse uniform magnetic field B. If it suddenly changes into circular form in time t then magnitude of induced charge will be

(a) $\frac{Ba^2}{R}(4\pi - 1)$ (b) $\frac{Ba^2}{R}\left(1 - \frac{1}{4\pi}\right)$ (c) $\frac{Ba^2}{R}\left(\frac{1}{4\pi} - 1\right)$ (d) $\frac{Ba^2}{R}\left(\frac{4}{\pi} - 1\right)$

Solution: (d) Initially


It's area $A_1 = a^2$; and flux linked $\phi_1 = BA_1$

Finally


$4a = 2\pi r$

It's area $A_2 = \pi r^2 = \pi\left(\frac{2a}{\pi}\right)^2 = \frac{4a^2}{\pi}$ and flux linked $\phi_2 = BA_2$

Induced emf $|e| = \frac{\Delta\phi}{\Delta t} = \frac{\phi_2 - \phi_1}{\Delta t} = \frac{B(A_2 - A_1)}{\Delta t} = \frac{Ba^2}{t}\left(\frac{4}{\pi} - 1\right)$ so induced charged $|q| = \frac{|e|}{R}.t$

$= \boldsymbol{\frac{Ba^2}{R}\left(\frac{4}{\pi} - 1\right)}$

***Example*: 13** A circular coil and a bar magnet placed near by are made to move in the same direction. The coil covers a distance of 1 *m* in 0.5 *sec* and the magnet a distance of 2 *m* in 1 *sec*. The induced emf produced in the coil

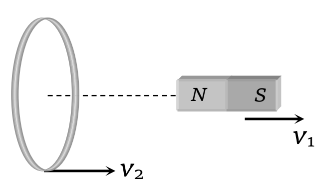
(a) Zero (b) 1 *V*

(c) 0.5 *V* (d) Cannot be determined from the given information

Solution: (a)

Speed of the magnet $v_1 = \frac{2}{1} = 2\,m/s$

Speed of the coil $v_2 = \frac{1}{0.5} = 2\,m/s$



Relative speed between coil and magnet is zero, so there is no induced emf in the coil.

Example: 14 A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would

(a) Halved (b) The same (c) Doubled (d) Quadrupled

Solution: (b) Power $P = \frac{e^2}{R}$; Here e = induced *emf* = $-\frac{d\phi}{dt} = -NA\left(\frac{dB}{dt}\right)$

$\because\ R \propto \frac{l}{r^2}$; where R = resistance, r = radius of wire, l = length of wire $\propto$ number of turns N

(if area of each turn is constant) $\Rightarrow P \propto \frac{N^2 r^2}{l} \propto Nr^2 \Rightarrow \frac{P_1}{P_2} = 1$.

Example: 15 A conducting circular loop is placed in a uniform magnetic field B = 40 mT with its plane perpendicular to the field. If the radius of the loop starts shrinking at a constant rate 0.2 mm/s, then the induced emf in the loop at an instant when its radius is 1.0 cm is

(a) 0.1 $\pi\mu V$ (b) 0.2 $\pi\mu V$ (c) 1.0 $\pi\mu V$ (d) 0.16 $\pi\mu V$

Solution: (d) $e = -B\frac{dA}{dt} = -B\frac{d}{dt}(\pi r^2) = -B\left(2\pi r\frac{dr}{dt}\right) = 2\pi Br\left(-\frac{dr}{dt}\right) \Rightarrow e = 2\times\pi\times 40\times 10^{-3}\times 10^{-2}\times(0.2\times 10^{-3}) = 0.16\pi\mu V$.

Example: 16 A solenoid has 2000 turns wound over a length of 0.314 m. Around its central section a coil of 100 turns and area of cross-section $1\times 10^{-3}\ m^2$ is wound. If an initial current of 2 A in the solenoid is reversed in 0.25 sec, the emf induced in the coil is equal to

(a) $6\times 10^{-4}\ V$ (b) 12.8 mV (c) $6\times 10^{-2}\ V$ (d) 12.8 V

Solution: (b) Magnetic field at the centre of the solenoid is given by $B = \mu_0 ni = \frac{\mu_0 Ni}{l} = 4\times 3.14\times 10^{-7}\times\frac{2000}{0.314}\times 2 = 16\times 10^{-3}\,T$. This magnetic field is perpendicular to the plane of the coil

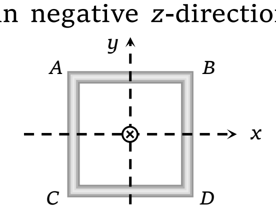
$\therefore$ Magnetic flux linked with coil $\phi = N'BA$

$\therefore$ Induced emf $e = \frac{-d\phi}{dt} = -\frac{d}{dt}(N'BA) = -N'A\frac{dB}{dt} = -N'A\frac{(-B-B)}{dt} = \frac{2N'BA}{dt}$

$\therefore\ e = \frac{2\times 100\times 16\times 10^{-3}\times 1\times 10^{-3}}{0.25} = 12.8\,mV$.

Tricky example: 1

A square coil $ABCD$ lying in x-y plane with it's centre at origin. A long straight wire passing through origin carries a current $i = 2t$ in negative z-direction. The induced current in the coil is



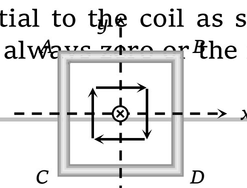
(a) Clockwise

(b) Anticlockwise

(c) Alternating

(d) Zero

Solution : (d) Magnetic lines are tangential to the coil as shown in figure. Thus net magnetic flux passing through the coil is always zero or the induced current will be zero.



Tricky example: 2

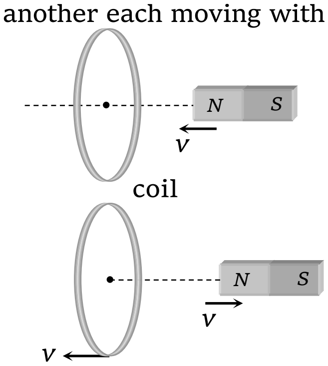
In the following figure, the magnet is moved towards the coil with a speed v and induced *emf* is e. If magnet and coil recede away from one another each moving with speed v, the induced *emf* in the coil will be

(a) e

(b) $2e$

(c) $e/2$

(d) $4e$



Solution : (b) $\left(\frac{d\phi}{dt}\right)_{\text{In first case}} = e$

$$\left(\frac{d\phi}{dt}\right)_{\text{relative velocity} 2v} = 2\left(\frac{d\phi}{dt}\right)_{\text{I case}} = 2e$$

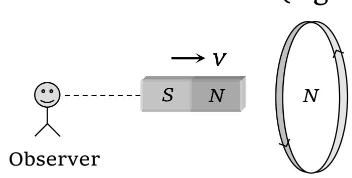
Lenz's law

This law gives the direction of induced emf/induced current. According to this law, the direction of induced emf or current in a circuit is such as to oppose the cause that produces it. This law is based upon law of conservation of energy. To understand the Lenz's law consider the followings.

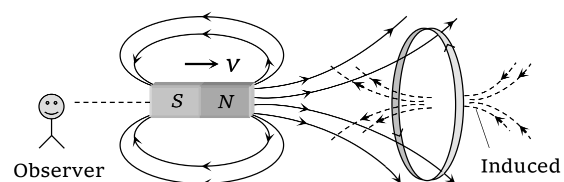
(1) **Motion of bar magnet towards a coil**

When N-pole of a bar magnet moves towards the coil, the flux associated with loop increases and an emf is induced in it. Since the circuit of loop is closed, induced current also flows in it.

Cause of this induced current, is approach of north pole and therefore to oppose the cause, *i.e.*, to repel the approaching north pole, the induced current in loop is in such a direction so that the front face of loop behaves as north pole. Therefore induced current as seen by observer O is in anticlockwise direction. (figure (i))



(i)



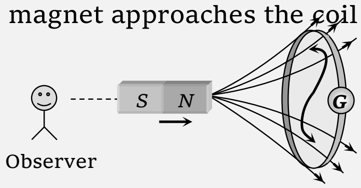
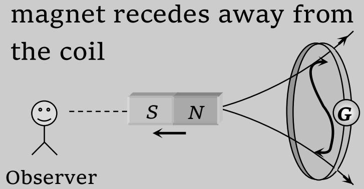
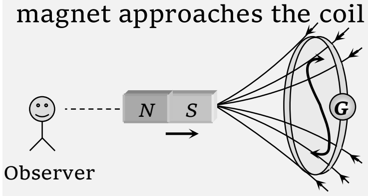
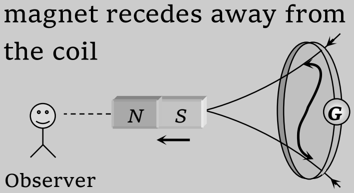
(ii)

In other words when N-pole of bar magnet moves towards the coil, inward magnetic lines of force (*i.e.* (×)) linked with coil (as viewed from left) increases. To oppose this change some dots (·) must be produced i.e. direction of induced current is anticlockwise. (figure (ii))

In this example, If the loop is free to move the cause of induced emf in the coil can also be termed as relative motion. Therefore to oppose the cause, the relative motion between the two approaching magnet and the loop should be opposed. For this, the loop will itself start moving in the direction of motion of the magnet.

Note :
- ❒ It is important to remember that whenever cause of induced emf is relative motion, the new motion is always in the direction of motion of the cause.
- ❒ In the above discussion, If once the coil is of *Cu* and once of brass and magnet approaches the coil with same velocity in both the case, then induced current in *Cu* will be greater (because of lesser resistance) and more energy conversion takes place in case of *Cu* coil.

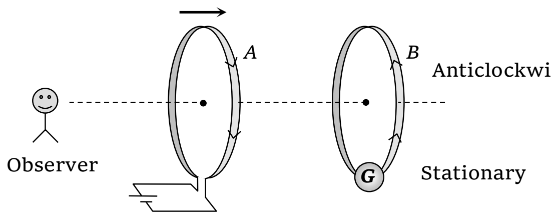
(2) **The various positions of relative motion between the magnet and the coil**

Position of magnet	**Direction of induced current**	**Behaviour of face of the coil**	**Type of magnetic force opposed**	**Magnetic field linked with the coil and it's progress as viewed from left**
When the north pole of magnet approaches the coil 	Anticlockwise direction	As a north pole	Repulsive force	Cross (×), Increases
When the north pole of magnet recedes away from the coil 	Clockwise direction	As a south pole	Attractive force	Cross (×), Decreases
When the south pole of magnet approaches the coil 	Clockwise direction	As a south pole	Repulsive force	Dots (·) Increases
When the south pole of magnet recedes away from the coil 	Anticlockwise direction	As a north pole	Attractive force	Dots (·) Decreases

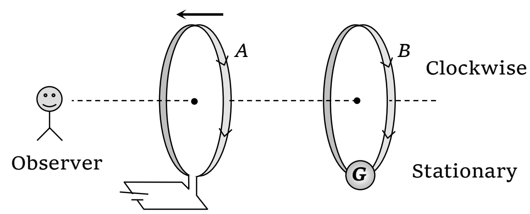
Some Standard Cases for Questions Based on Direction

(1) **Relative motion between co-axial circular coils**

(i) When a current carrying coil moves towards/away from a stationary coil



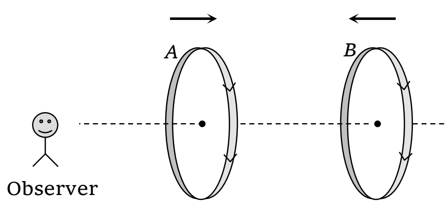
Induced current in coil B is opposite to the main current



Induced current in coil B is in the same direction to the main

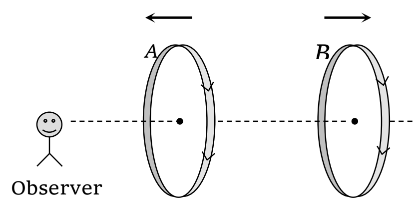
(ii) When two current carrying coils carries currents in the same direction and

Moves towards each other



Induced current in both the coils opposite to that of main current so current through

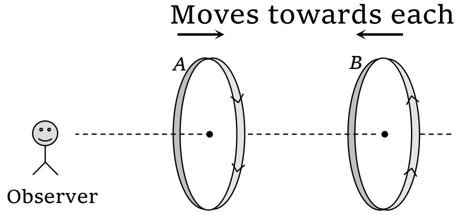
Moves away from each other



Induced current in both the coils assist the main current so current through each

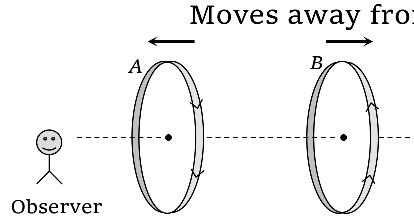
(iii) When two current carrying coils carries currents in the opposite direction and

Moves towards each other



Induced current in coil A is clockwise and that in coil B is anti-clockwise *i.e.* in both the coils induced current flows in the direction of main current. Hence current through both the coil

Moves away from each other

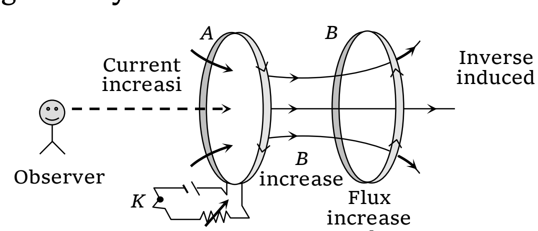


Induced current in coil A is anti-clockwise and that in coil B is clockwise *i.e.* in both the coils induced current flows in the direction opposite to main current. Hence current

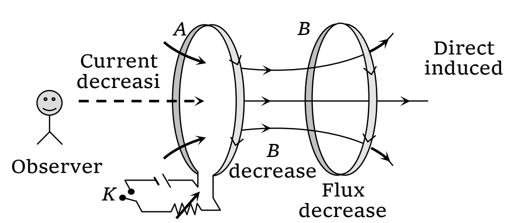
(2) **When the inductive circuits are closed or opened**

If two coils A and B (primary and secondary) are arranged as shown in the figure and if the primary circuit is closed or opened then the direction of induced current in secondary will be as follows

(i) Current increases in coil A by pressing the key (ii) Current decreases in coil A by opening the key



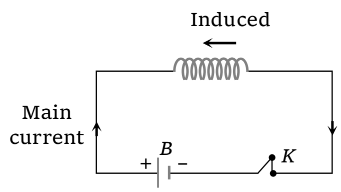
Direction of induced current in the secondary coil is opposite to that in



Direction of induced current in the secondary coil is same as that in the

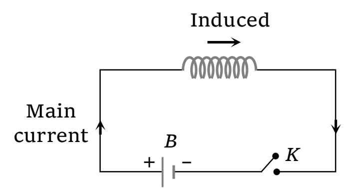
(3) **Increasing and decreasing of current in current carrying coil**

(i) When current increases by pressing the key



Direction of induced current in the coil will be in a direction opposite to that

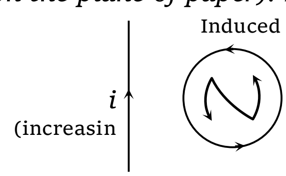
(ii) When current decreases by opening the key



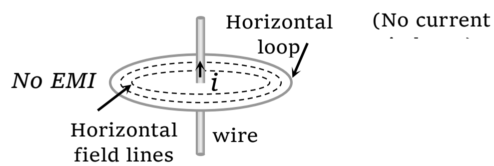
Direction of induced current in the coil will be same as that of the

Concepts

☞ *To apply Lenz's law, you can remember* ***RIN*** *(when the loop lies on the plane of paper). In* ***RIN****,* ***R*** *stands for right,* ***I*** *stands for increasing and* ***N*** *for north pole (anticlockwise). It means, if a loop is placed on the right side of a straight current carrying conductor and the current i in the conductor is increasing, then induced current in the loop is anticlockwise ( )*



☞ *No flux cutting* *No EMI*



Example

***Example*: 17** Consider a metal ring kept on a horizontal plane. A bar magnet is held above the ring with its length along the central axis of the ring. If the magnet is now dropped freely, the acceleration of the falling magnet is (g is acceleration due to gravity)

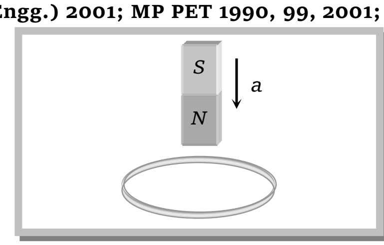
Kerala (Engg.) 2001; MP PET 1990, 99, 2001; MP PMT 2001]

(a) More than g

(b) Equal to g

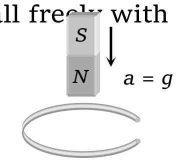
(c) Less than g

(d) Depends on mass of magnet

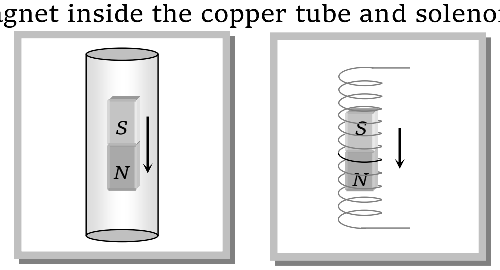


Solution: (c) When the magnet is allowed to fall vertically along the axis of loop with its north pole towards the ring. The upper face of the ring will become north pole in an attempt to oppose the approaching north pole of the magnet. Therefore the acceleration in the magnet is less than g.

Note : ❒ If the coil is broken at any point then induced *emf* will be generated in it but no induced current will flow. In this condition the coil will not oppose the motion of magnet and the magnet will fall freely with acceleration g. (i.e. $a = g$)



***Example*: 18** A bar magnet is falling freely inside a long copper tube and a solenoid as shown in figure (i) and (ii) respectively then acceleration of magnet inside the copper tube and solenoid are respectively (acceleration due to gravity = g)



(a) g, g

(b) Greater than g, lesser than g

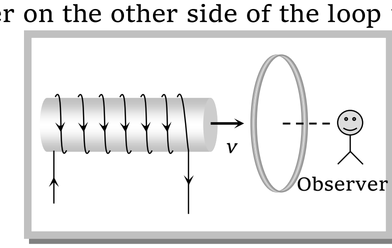
(c) Greater than g, g

(d) Zero, lesser than g

Solution: (d) If bar magnet is falling vertically through the hollow region of long vertical copper tube then the magnetic flux linked with the copper tube (due to 'non-uniform' magnetic field of magnet) changes and eddy currents are generated in the body of the tube by Lenz's law the eddy currents opposes the falling of the magnet which therefore experience a retarding force. The retarding force increases with increasing velocity of the magnet and finally equals the weight of the magnet. The magnet then attains a constant final terminal velocity *i.e.* magnet ultimately falls with zero acceleration in the tube.

The resistance of copper solenoid is much higher than that of copper tube, hence the induced current in it, due to motion of magnet, will be much less than that in the tube. Consequently the opposition to the motion of magnet will be less and the magnet will fall with an acceleration (a) less than g. (*i.e.* $a < g$).

***Example*: 19** A current carrying solenoid is approaching a conducting loop as shown in the figure. The direction of induced current as observed by an observer on the other side of the loop will be



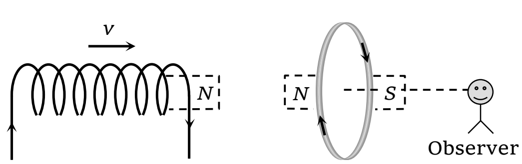
(a) Anticlockwise

(b) Clockwise

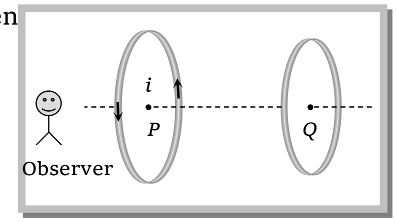
(c) East

(d) West

Solution: (b) The direction of current in the solenoid is anti-clockwise as seen by observer. On displacing it towards the loop a current in the loop will be induced in a direction so as to oppose the approach of solenoid. Therefore the direction of induced current as observed by the observer will be clockwise.



***Example*: 20** Two coils P and Q are lying a little distance apart coaxially. If an anticlockwise current i is suddenly set up in the coil P then the direction of curren



(a) Clockwise

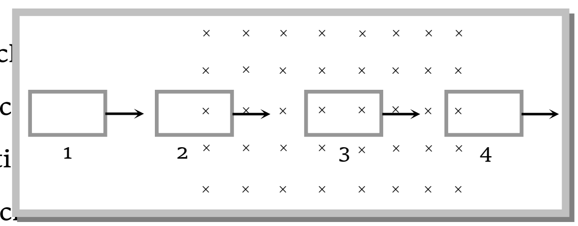
(b) Towards north

(c) Towards south

(d) Anticlockwise

Solution: (a) Since current setup in the coil P is anticlockwise which increases the dot's linked with coil Q hence induced current in coil Q will be clockwise.

***Example*: 21** A rectangular loop is drawn from left to right across a uniform magnetic field perpendicular into the plane of the loop



(a) The direction of current in position 1 is cloc

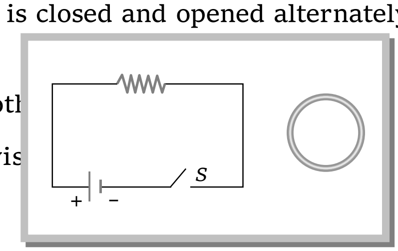
(b) The direction of current in position 2 is cloc

(c) The direction of current in position 3 is anti

(d) The direction of current in position 4 is cloc

Solution: (d) No current is induced in position 1, anticlockwise current is induced in position 2 because it is a case of increase of flux, no current in position 3 as there is no change of flux, clockwise current is produced in position 4 because it is a case of decrease of flux.

***Example*: 22** A small loop lies outside a circuit. The key of the circuit is closed and opened alternately. The closed loop will show



(a) Clockwise pulse followed by anoth

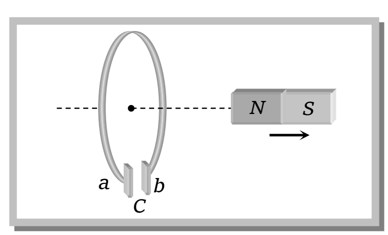
(b) Anticlockwise pulse followed by another anticlockwis

(c) Anticlockwise pulse followed by a clockwise pulse

(d) Clockwise pulse followed by an anticlockwise pulse

Solution: (d) When key is closed dots are linked with closed loop (*i.e.* increases from zero to a certain value) so induced current will be clockwise when key is opened dots linked with loop decreases (from a certain value to zero) so induced current will be anticlockwise in direction.

***Example*: 23** Consider the arrangement shown in figure in which the north pole of a magnet is moved away from a thick conducting loop containing capacitor. Then excess positive charge will arrive on



(a) Plate a

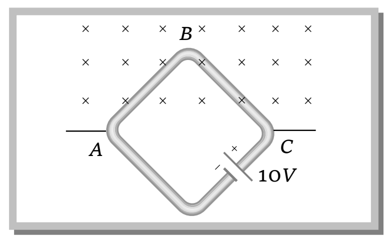
(b) Plate b

(c) On both plates *a* and *b*

(d) On neither *a* nor *b* plates

Solution: (b) When north pole of the magnet is moved away, then south pole is induced on the face of the loop in front of the magnet *i.e.* as seen from the magnet side, a clockwise induced current flows in the loop. This makes free electrons to move in opposite *i.e.* direction, to plate *b* to *a* inside the loop. Thus excess positive charge appear on plate *b*.

***Example*: 24** A square loop of side 1*m* is placed in a perpendicular magnetic field. Half of the area of the loop inside the magnetic field. A battery of emf 10 *V* and negligible internal resistance is connected in the loop. The magnetic field changes with time according to relation *B* = 0.01 – 2*t Tesla*. The resultant emf in the loop will be



(a) 1 *V*

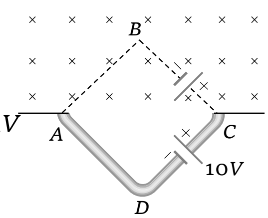
(b) 11 *V*

(c) 10 *V*

(d) 9 *V*

Solution: (d) Given *B* = 0.01 – 2*t Tesla* ; $\frac{dB}{dt} = -2\,Tesla/sec$,

Induced emf $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -A\frac{dB}{dt} = -\frac{1}{2}(1^2)\times(-2) \Rightarrow e = 1V$



Since magnetic field (×) decreasing so according to Lenz's law direction of induced current in upper part of square will be clockwise *i.e.* from *A* to *C* or in other words emf induces in a direction opposite to the main emf so resultant emf = 10 – 1 = 9*V*.

Tricky example: 3

A short magnet is allowed to fall along the axis of a horizontal metallic ring. Starting from rest, the distance fallen by the magnet in one second may be

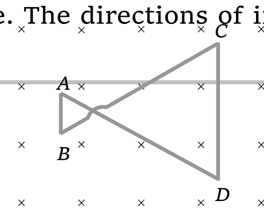
(a) 4 *m* (b) 5 *m* (c) 6 *m* (d) 7 *m*

Solution : (a) We know that in this case acceleration of falling magnet will be lesser than *g*. If '*g*' would have been acceleration, then distance covered $= \frac{1}{2}gt^2 = 5m$.

Now the distance covered will be less than 5 *m*. hence only option (a) is correct.

Tricky example: 4

A conducting wire frame is placed in a magnetic field which is directed into the paper. The magnetic field is increasing at a constant rate. The directions of induced current in wires *AB* and *CD* are



(a) B to A and D to C

(b) A to B and C to D

(c) A to B and D to C

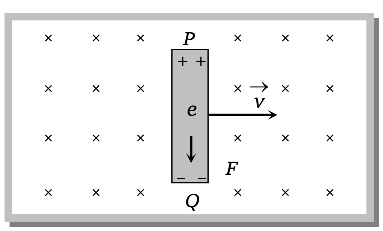
(d) B to A and C to D

Solution : (a) Inward magnetic field (×) increasing. Therefore, induced current in both the loops should be anticlockwise. But as the area of loop on right side is more, induced *emf* in this will be more compared to the left side loop $\left(e = -\frac{d\phi}{dt} = -A.\frac{dB}{dt}\right)$. Therefore net current in the complete loop will be in a direction shown below. Hence only option (a) is correct.

Dynamic (Motional) EMI Due to Translatory Motion

When a conducting rod moves in a magnetic field, it cuts the magnetic field lines, this process is called flux cutting. Due to this a potential difference developed across the ends of the rod called Dynamic (motional) emf.

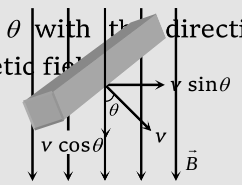
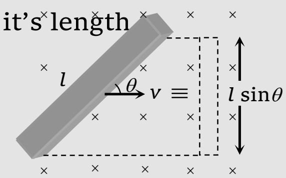
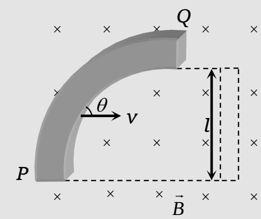
Consider a conducting rod of length l moving with a uniform velocity $\vec{v}$ perpendicular to a uniform magnetic field $\vec{B}$, directed into the plane of the paper. Let the rod be moving to the right as shown in figure. The conducting electrons also move to the right as they are trapped within the rod.



Conducting electrons experiences a magnetic force $\vec{F}_m = -e(\vec{v} \times \vec{B})$. In the present situation they experiences force towards Q, so they move from P to Q within the rod. The end P of the rod becomes positively charged while end Q becomes negatively charged, hence an electric field is set up within the rod which opposes the further downward movement of electrons *i.e.* an equilibrium is reached and in equilibrium electric force = magnetic force *i.e.* $eE = evB$ or $E = vB$

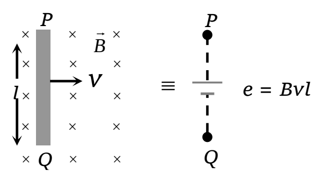
$\Rightarrow$ **Induced emf** $\boldsymbol{e = El = Bvl}$ $\quad [E = \frac{V}{l}]$

Important cases

If the rod does not translate in a plane perpendicular to the magnetic field or in other words rod is moving in a direction which is making an angle θ with the direction of magnetic field  This situation is equivalent to a straight conductor moving perpendicular to the magnetic field with a induced emf $e = B(v\sin\theta)l$ $\Rightarrow$ $\boldsymbol{e = Bvl\sin\theta}$	If the rod is moving perpendicular to the magnetic field but it's direction of motion is making an angle θ with it's length  This situation is equivalent to a straight rod of length $l\sin\theta$ perpendicular to it's direction of motion so induced emf across the rod $e = Bv(l\sin\theta) \Rightarrow \boldsymbol{e = Bvl\sin\theta}$	An arbitrary shaped conducting rod translating in a uniform magnetic field.  This rod can be replaced by a straight conductor whose length is equal to the projected length of the conductor on to a plane perpendicular to the direction of motion (dotted rod) so induced emf between P and Q $\boldsymbol{e = Bvl}$

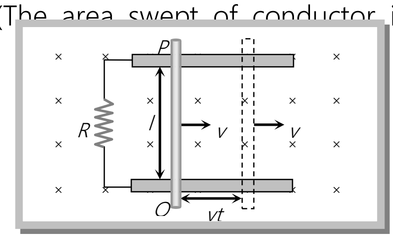
Note :❒ Vector form of motional emf : $e = (\vec{v} \times \vec{B}).\vec{l}$

❒ While solving the problems, flux cutting conducting rod can be treated as a single cell.



(1) Induced current

If conducting rod moves on two parallel conducting rails as shown in following figure then phenomenon of induced emf can also be understand by the concept of generated area (The area swept of conductor in magnetic field, during it's motion)



As shown in figure in time t distance travelled by conductor = vt

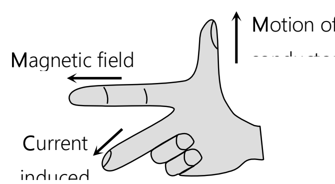
Area generated $A = lvt$

Flux linked with this area $\phi = BA = Blvt$

Hence induced emf $|e| = \frac{d\phi}{dt} = Bvl$ induced current $i = \frac{e}{R}$; $\boldsymbol{i = \frac{Bvl}{R}}$

Direction of induced current can be found with the help of Flemings right hand rule.

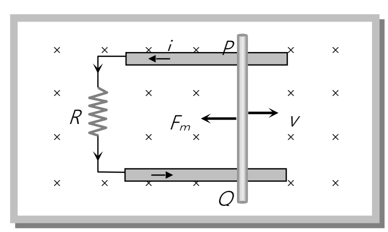
Fleming's right hand rule : According to this law, if we stretch the right hand thumb and two nearby fingers perpendicular to one another and first finger points in the direction of magnetic field and the thumb in the direction of motion of the conductor then the central finger will point in the direction of the induced current.



Note : ❐ Here it is worthy to note that the rod PQ is acting as a source of emf and inside a source of emf direction of current is from lower potential to higher potential; so the point P of the rod is at higher potential than Q though the current in the rod PQ is from Q to P.

(2) Magnetic force on conductor

Now current is set up in circuit (conductor). As we know when a current carrying conductor moves in a magnetic field, it experiences a force $F_m = Bil$ (maximum) whose direction can be find with the help of Flemings left hand rule.

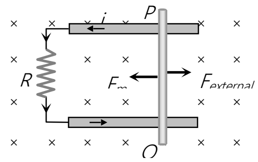


So, here conductor PQ experiences a magnetic force $F_m = Bil$ in opposite direction of it's motion and $F_m = Bil = B\left(\frac{Bvl}{R}\right)l$; $F_m = \frac{B^2vl^2}{R}$

(As a result of this force (F_m) speed of rod decreases as time passes.)

Note : ❒ To move the rod with uniform velocity some external mechanical force is required and this is $F_{ext} = -F_m$

$\Rightarrow |F_{ext}| = \frac{B^2 v l^2}{R}$



(3) **Power dissipated in moving the conductor**

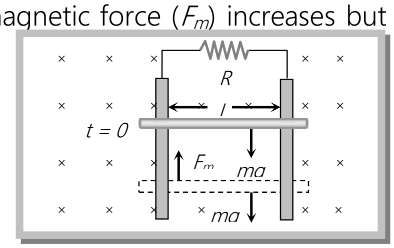
For uniform motion of rod PQ, the rate of doing mechanical work by external agent or mech. Power delivered by external source is given as $P_{mech} = P_{ext} = \frac{dW}{dt} = F_{ext}.v = \frac{B^2 v l^2}{R} \times v \Rightarrow P_{mech} = \frac{B^2 v^2 l^2}{R}$

Also electrical power dissipated in resistance or rate of heat dissipation across resistance is given as

$$P_{thermal} = \frac{H}{t} = i^2 R = \left(\frac{Bvl}{R}\right)^2 .R \; ; \quad P_{thermal} = \frac{B^2 v^2 l^2}{R}$$

Note : ❒ It is clear that $P_{mech.} = P_{thermal}$ which is consistent with the principle of conservation of energy.

(4) **Motion of conductor rod in a vertical plane :** If conducting rod released from rest (at $t = 0$) as shown in figure then with rise in it's speed (v), induces emf (e), induced current (i), magnetic force (F_m) increases but it's weight remains constant.

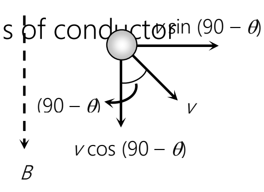


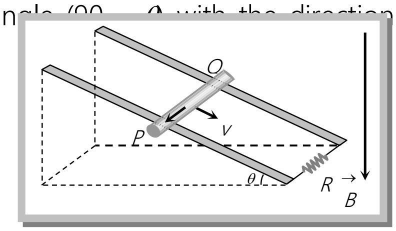
Rod will achieve a constant maximum (terminal) velocity v_T if $F_m = mg$

So $$\frac{B^2 v_T^2 l^2}{R} = mg$$

$\Rightarrow$ $$v_T = \frac{mgR}{B^2 l^2}$$

(5) **Motion of conducting rod on an inclined plane :** When conductor start sliding from the top of an inclined plane as shown, it moves perpendicular to it's length but at an angle $(90 - \theta)$ with the direction of magnetic field. Hence induced emf across the ends of conductor



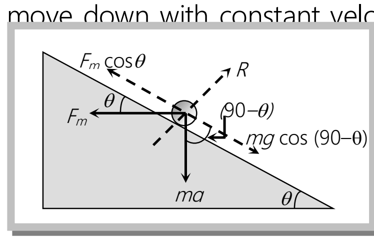


$$e = Bv \sin(90 - \theta) l = Bvl \cos\theta$$

So induced current $i = \frac{Bvl \cos\theta}{R}$

(directed from Q to P).

The forces acting on the bar are shown in following figure. The rod will move down with constant velocity only if

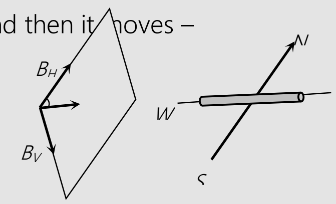
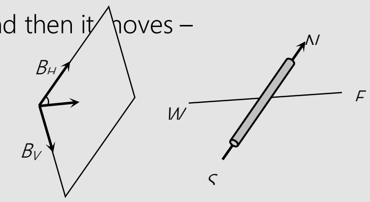
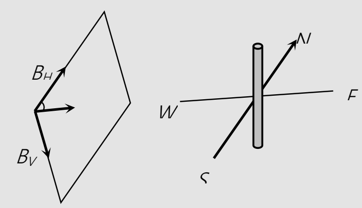


$$F_m \cos\theta = mg \cos(90 - \theta) = mg \sin\theta$$

$$Bil\cos\theta = mg\sin\theta$$

$$B\left(\frac{Bv_T l\cos\theta}{R}\right)l\cos\theta = mg\sin\theta \Rightarrow v_T = \frac{mgR\sin\theta}{B^2l^2\cos^2\theta}$$

(6) **Motion of a conducting rod in earth's magnetic field** : Suppose a conducting rod of length l, executes translatory motion with speed v in earth's magnetic field with

<table>
<tr><th colspan="3">Position I</th><th colspan="3">Position II</th><th colspan="3">Position III</th></tr>
<tr><td colspan="3">When conductor is held horizontal with it's length along E-W direction and then it moves –
</td><td colspan="3">When conductor is held horizontal with it's length along N-S direction and then it moves –
</td><td colspan="3">When conductor is held vertical and then it moves –
</td></tr>
<tr><td>Towards East or West</td><td>Towards North or South</td><td>Vertically up or down</td><td>Towards East or West</td><td>Towards North or South</td><td>Vertically up or down</td><td>Towards East or West</td><td>Towards North or South</td><td>Vertically up or down</td></tr>
<tr><td>In this condition conductor is moving along it's length, so generated area $A = 0$ hence $e = 0$</td><td>Vertical Component (B_V) is cut by the conductor perpendicularly so $\boldsymbol{e = B_V vl}$</td><td>Conductor cuts, perpendicularly horizontal component (B_H) so $\boldsymbol{e = B_H vl}$</td><td>Conductor cuts, the vertical component perpendicularly so $\boldsymbol{e = B_V vl}$</td><td>Conductor is moving along its length so $\boldsymbol{e = 0}$</td><td>Conductor moves in magnetic meridian i.e. No component is cut by the conductor so $e = 0$</td><td>Conductor cut's the horizontal component perpendicularly so $\boldsymbol{e = B_H vl}$</td><td>Conductor moves in magnetic meridian so $\boldsymbol{e = 0}$</td><td>Conductor moves along it's length so $\boldsymbol{e = 0}$</td></tr>
</table>

(7) **Movement of train in earth's magnetic field :** When a train moves on rails, then a potential difference between the ends of the axle of the wheels is induced because the axle of the wheels of the train cuts the vertical component B_V of earth's magnetic field and so the magnetic flux linked with it changes and the potential difference or emf is induced. $e = B_V lv$ where l is the length of the axle and v is the speed of the train.

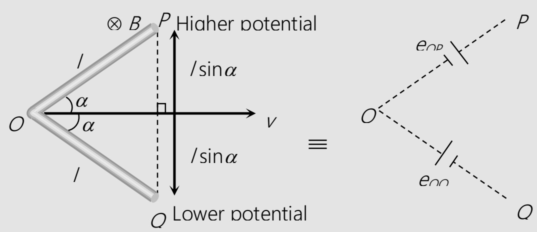
(8) **Motion of aeroplane in earth's magnetic field** : A potential difference or emf across the wings of an aeroplane flying horizontally at a definite height is also induced because aeroplane cuts the vertical component

B_V of earth's magnetic field. Thus induced emf $e = B_V l v$ *volt* where l is the length of the wings of an aeroplane and v is the speed of the aeroplane.

(9) **Orbital satellite :** If the orbital plane of an artificial satellite of metallic surface is coincident with equatorial plane of the earth, then no emf will be induced. If orbital plane makes an angle with the equatorial plane, then emf will be induced on it.

(10) **Translatory motion of metallic frame in uniform/non-uniform magnetic field**

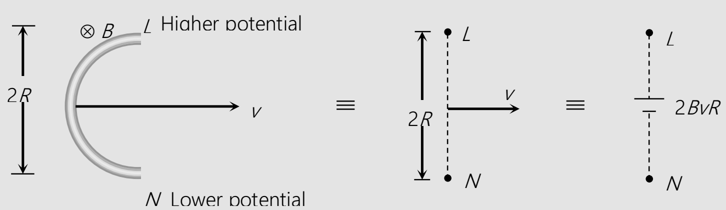
(i) Metal frame of different shape moves in uniform magnetic field



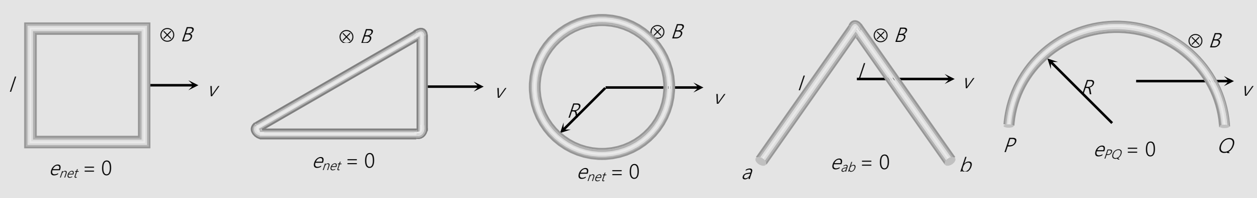
For part OP $e_{OP} = V_P - V_O = Bv\,(l\sin\alpha)$

For part QO $e_{QO} = V_O - V_Q = Bv\,(l\sin\alpha)$

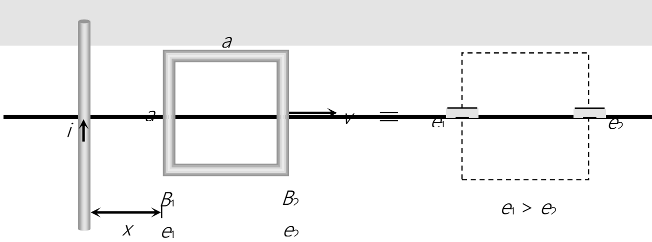
$$e_{QP} = V_P - V_Q = 2Bv\,(l\sin\alpha)$$



$$e_{LN} = 2BvR$$



(ii) Moving metal frame in non-uniform magnetic field



$e_1 = B_1 a v$ $e_2 = B_2 a v$ For loop $e_{net} = (e_1 - e_2)$

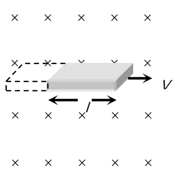
$e_{net} = av(B_1 - B_2)$

Now $B_1 = \dfrac{\mu_0 i}{2\pi x}$ and $B_2 = \dfrac{\mu_0 i}{2\pi(x+a)}$

$$e_{net} = \frac{\mu_0 iav}{2\pi}\left[\frac{1}{x} - \frac{1}{x+a}\right] = \frac{\mu_0 ia^2 v}{2\pi(x)(x+a)}$$

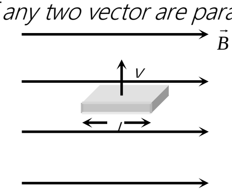
Concepts

- *In motional emf $\vec{B}, \vec{v}$ and $\vec{l}$ are three vectors. If any two vector are parallel – No flux cutting.*



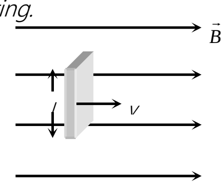
$\vec{v} \parallel \vec{l}$ so $e = 0$

or Generated area $A = 0$



$\vec{B} \parallel \vec{l}$ so $e = 0$

or Normal to generated area makes an angle 90° with $\vec{B}$



$\vec{v} \parallel \vec{B}$ so $e = 0$

or Normal to generated area makes an angle 90° with $\vec{B}$

- *A piece of metal and a piece of non-metal are dropped from the same height near the surface of the earth. The non-metallic piece will reach the ground first because there will be no induced current in it.*
- *If an aeroplane is landing down or taking off and its wings are in the east-west direction, then the potential difference or emf will be induced across the wings. If an aeroplane is landing down or taking off and its wings are in the north-south direction, then no potential difference or emf will be induced.*
- *When a conducting rod moving horizontally on equator of earth no emf induces because there is no vertical component of earth's magnetic field. But at poles B_V is maximum so maximum flux cutting hence emf induces.*
- *When a conducting rod falling freely in earth's magnetic field such that it's length lies along East - West direction then induced emf continuously increases w.r.t. time and induced current flows from West - East.*

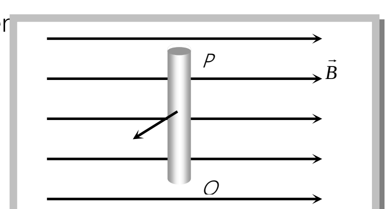
Examples

Example: 25 A two *metre* wire is moving with a velocity of 1*m/sec* perpendicular to a magnetic field of 0.5 *weber/m*^{2}. The emf induced in it will be

(a) 0.5 *volt* (b) 0.1 *volt* (c) 1 *volt* (d) 2 *volt*

Solution: (c) $e = Bvl = 0.5 \times 1 \times 2 = 1 volt$

Example: 26 A *Cu* rod *PQ* is drawn out normally through the magnetic field $\vec{B}$ then



(a) Equal potential on both P and Q occurs and will be positive

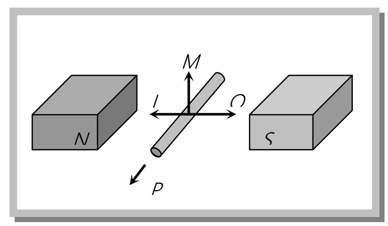
(b) Equal potential on both P and Q occurs and will be negative

(c) The potential at P will be greater then at Q

(d) The potential at P will be lesser than at Q

Solution: (c) By Fleming's right hand rule direction of induced current in rod PQ is from Q to P, hence P is at higher potential.

***Example*: 27** An electric potential difference will be induced between the ends of the conductor shown in fig. when conductor moves in the direction **[AIIMS 1982]**



(a) P

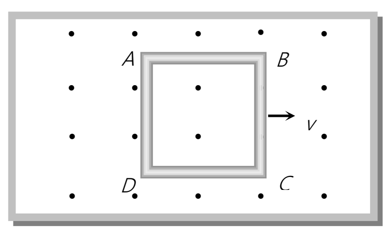
(b) Q

(c) L

(d) M

Solution: (d) When conductor moves either in the direction P, Q or L it will not cut the magnetic lines of force, so emf will induced only when conductor moves in the direction M

***Example*: 28** A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. An electric field is induced **[IIT-JEE (Screening) 2001]**



(a) In AD, but not in BC

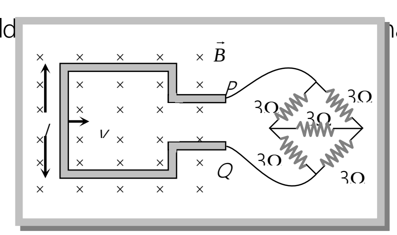
(b) In BC, but not in AD

(c) Neither in AD nor in BC

(d) In both AD and BC

Solution: (d) Both AD and BC are straight conductors moving in a uniform magnetic field and emf will be induced in both. This will cause electric field in both but no net current flows in the circuit.

***Example*: 29** A square metallic wire loop of side 0.1 m and resistance of 1Ω is moved with a constant velocity in a magnetic field of 2 wb/m^2 as shown in figure. The magnetic field is perpendicular to the plane of the loop, loop is connected to a network of resistances. What should have a steady current of 1mA in loop



(a) 1 *cm/sec*

(b) 2 *cm/sec*

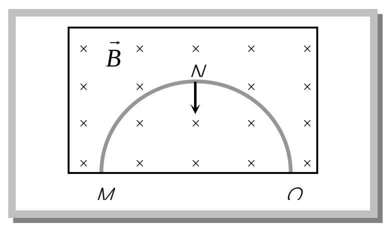
(c) 3 *cm/sec*

(d) 4 *cm/sec*

Solution: (b) Equivalent resistance of the given Wheatstone bridge circuit (balanced) is 3Ω so total resistance in circuit is $R = 3 + 1 = 4\Omega$. The emf induces in the loop $e = Bvl$.

So induced current $i = \frac{e}{R} = \frac{Bvl}{R} \Rightarrow 10^{-3} = \frac{2 \times v \times (10 \times 10^{-2})}{4} \Rightarrow v = 2cm / sec.$

Example: 30 A thin semicircular conducting ring of radius *R* is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$ (fig.). At the position *MNQ* the speed of the ring is *v* and the potential difference development across the ring is [IIT-JEE 1996]



(a) Zero

(b) $\frac{Bv\pi R^2}{2}$ and *M* is at higher potential

(c) πRBv and *Q* is at higher potential

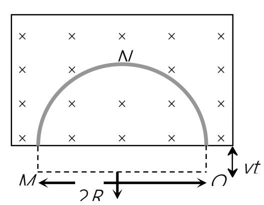
(d) $2RBv$ and *Q* is at higher potential

Solution: (d) Suppose in time *t* vertical distance travelled by ring is $v \times t$ so change in area $\frac{dA}{dt} = 2Rvt$

$\therefore \quad e = -\frac{d\phi}{dt} = -B\frac{dA}{dt}$

$\Rightarrow \quad |e| = 2RBv$

and by Flemings right hand rule. *Q* is at higher potential.

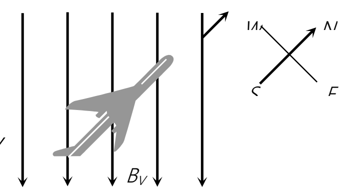


Example: 31 A metal aeroplane having a distance of 50 *meter* between the edges of its wings is flying horizontally with a speed of 360 *km/hour*. At the place of flight, the earth's total magnetic field is 4.0×10^{-5} *weber/meter*^{2} and the angle of dip is 30°. The induced potential difference between the edges of its wings will be

(a) 0.1 *V* (b) 0.01 *V* (c) 1 *V* (d) 10 *V*

Solution: (a) The flying of the aeroplane is shown in the adjoining figure. Its wings are cutting flux-lines due to the vertical component of earth's magnetic field. So, a potential difference *V* (say) in induced between the edges of its wings.

If the earth's total magnetic field be B and the angle of dip θ, then the vertical component of earth's magnetic field is $B_V = B\sin\theta = (4.0 \times 10^{-5}) \times \sin 30^\circ = 2.0 \times 10^{-5}$ *wb/metre*^{2}.



We know that when a conductor of length l *meter* moves with velocity v *meter/second* perpendicular to a magnetic field of B *weber/meter*^{2}, then the potential difference induced in the conductor is given by $V = B_V vl\ volt$

Here l = 50 *meter*, v = 360 *km/hour* $= \dfrac{360 \times 1000}{60 \times 60} = 100\ meter/second$

and $B = B_V = 2.0 \times 10^{-5}$ *weber/meter*^{2} ; $V = (2.0 \times 10^{-5}) \times 100 \times 50 = 0.1\,volt$

Example: 32 The two rails of a railway track, insulated from each other and the ground, are connected to a *millivoltmeter*. What is the reading of the *millivoltmeter* when a train travels at a speed of 20 *m/sec* along the track, given that the vertical component of earth's magnetic field is 0.2×10^{-4} *wb/m*^{2} and the rails are separated by 1 *metre*

[CPMT 1981]

(a) 4 *mV* (b) 0.4 *mV* (c) 80 *mV* (d) 10 *mV*

Solution: (b) When a train runs on the rails, it cuts the magnetic flux lines of the vertical component of earth's magnetic field. Hence a potential difference is induced between the ends of it's axle. Distance between the rails l = 1*m*.

Speed of train $v = 36\dfrac{km}{hour} = \dfrac{36 \times 1000}{3600} = 10\ m/sec$

By using $e = Bvl$; $e = 0.2 \times 10^{-4} \times 20 \times 1 = 4 \times 10^{-4}$ *volt* = 0.4 *mV*

Example: 33 The horizontal component of the earth's magnetic field at a place is 3×10^{-4} *T* and the dip is $\tan^{-1}\left(\dfrac{4}{3}\right)$. A metal rod of length 0.25 *m* placed in the north-south position and is moved at a constant speed of 10 *cm/s* towards the east. The emf induced in the rod will be

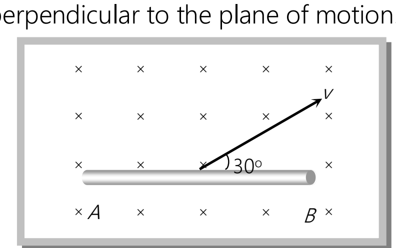
(a) Zero (b) 1μV (c) 5 μV (d) 10 μV

Solution: (d) Rod is moving towards east, so induced emf across it's end will be $e = B_V vl$

B_V = vertical component of Earth's magnetic field = $B_H \tan\phi$ (B_H – horizontal component of earth's magnetic field; ϕ – angle of dip), $B_V = 3 \times 10^{-4} \times \dfrac{4}{3} = 4 \times 10^{-4}\,T$

$\therefore\ e = 4 \times 10^{-4} \times (10 \times 10^{-2}) \times 0.25 = 10^{-5}\,V = 10\mu V$

Example: 34 A conducting rod AB of length l = 1 *m* is moving at a velocity v = 4 *m/s* making an angle 30° with it's length. A uniform magnetic field B = 2 *T* exists in a direction perpendicular to the plane of motion. Then



(a) $V_A - V_B = 8$ V

(b) $V_A - V_B = 4$ V

(c) $V_B - V_A = 8$ V

(d) $V_B - V_A = 4$ V

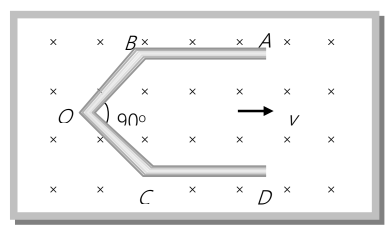
Solution: (b) The emf induced across the rod *AB* is $e = Bv_{\perp}l$

Here $v_{\perp} = v \sin 30^{o}$ = component of velocity perpendicular to length.

Hence $e = Bvl \sin 30^{o} = (2)(4)(1)\left(\frac{1}{2}\right) = 4V$

The free electrons of the rod shift towards right due to the force $q(\vec{v} \times \vec{B})$. Thus the left side of the rod is at higher potential. or $V_A - V_B = 4V$

Example: 35 A conductor *ABOCD* moves along its bisector with a velocity of 1 *m/s* through a perpendicular magnetic field of 1 wb/m^2, as shown in fig. If all the four sides are of 1*m* length each, then the induced emf between points *A* and *D* is



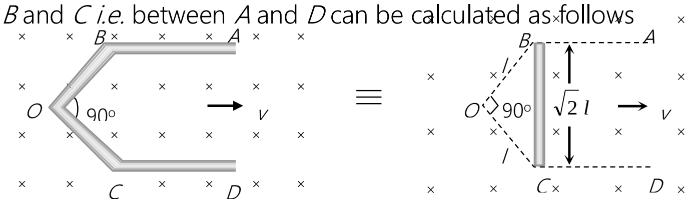
(a) 0

(b) 1.41 *volt*

(c) 0.71 *volt*

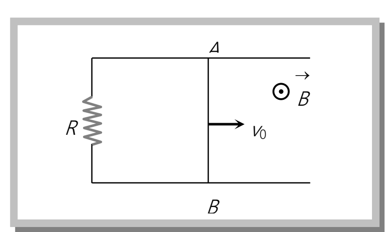
(d) None of the above

Solution: (b) There is no induced emf in the part *AB* and *CD* because they are moving along their length while emf induces between *B* and *C* *i.e.* between *A* and *D* can be calculated as follows



Induced emf between *B* and *C* = Induced emf between *A* and *B* = $Bv(\sqrt{2}\,l) = 1 \times 1 \times 1 \times \sqrt{2} = 1.41\ volt.$

Example: 36 Two long parallel metallic wires with a resistance *R* forms a horizontal plane. A conducting rod *AB* is on the wires as shown here. The space has a magnetic field pointing vertically upwards. The rod is given an initial



velocity v_0. There is no friction and no resistance in the wires and the rod. After a time t, the velocity of the rod will be v such that [MP PMT 1995]

(a) $v > v_0$

(b) $v < v_0$

(c) $v = v_0$

(d) $v = -v_0$

Solution: (b) When rod AB starts it's motion, current induces in it from A to B, due to which rod experiences a magnetic force towards left (Flemings left hand rule) which opposes the motion of the rod. Hence $v < v_0$.

Example: 37 A player with $3m$ long iron rod runs towards east with a speed of 30 km/hr. Horizontal component of earth's magnetic field is 4×10^{-5} wb/m^2. If he is running with rod in horizontal (East-west) and vertical positions, then the potential difference induced between the two ends of the rod in two cases will be

(a) Zero in vertical position and 1×10^{-3} V in horizontal position

(b) 1×10^{-3} V in vertical position and zero is horizontal position

(c) Zero in both cases

(d) 1×10^{-3} V in both cases

Solution: (b) In horizontal position rod is moving along it's length so $e = 0$

In vertical position, horizontal component of earth's magnetic field is cut by rod so induced emf $e = B_H vl$

$$\therefore \quad e = 4 \times 10^{-5} \times 30 \times \frac{1000}{3600} \times 3 = 10^{-3}\, V$$

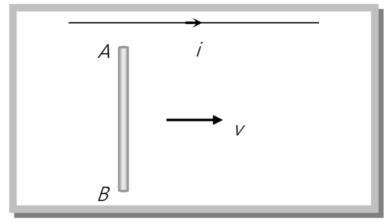
Example: 38 At a place the value of horizontal component of the earth's magnetic field H is 3×10^{-5} $weber/m^2$. A metallic rod AB of length 2 m placed in east-west direction, having the end A towards east, falls vertically downwards with a constant velocity of 50 m/s. Which end of the rod becomes positively charged and what is the value of induced potential difference between the two ends

(a) End A, 3×10^{-3} mV (b) End A, 3 mV (c) End B, 3×10^{-3} mV (d) End B, 3 mV

Solution: (b) According to Flemings right hand rule direction of induced current in rod AB is from B to A *i.e*. end A becomes positively charged. *i.e.* end A becomes positively charged.

emf induces across the ends of the rod $e = Hvl = 3 \times 10^{-5} \times 50 \times 2 = 3 \times 10^{-3}$ $volt = 3mV$.

Example: 39 The current carrying wire and the rod AB are in the same plane. The rod moves parallel to the wire with a velocity v. Which one of the following statements is true about induced emf in the rod



(a) End A will be at lower potential with respect to B

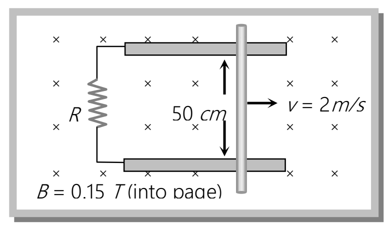
(b) A and B will be at the same potential

(c) There will be no induced emf in the rod

(d) Potential at A will be higher than that at B

Solution: (d) According to Fleming's right hands rule direction of induced current in rod is from B to A i.e. end A is at higher potential.

Example: 40 As shown in the figure a metal rod makes contact and complete the circuit. The circuit is perpendicular to the magnetic field with $B = 0.15$ *Tesla*. If the resistance is 3Ω, force needed to move the rod as indicated with a constant speed of $2\,m/sec$ is



(a) $3.75 \times 10^{-3}\ N$

(b) $3.75 \times 10^{-2}\ N$

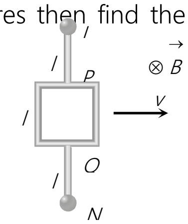
(c) $3.75 \times 10^{2}\ N$

(d) $3.75 \times 01^{-4}\ N$

Solution: (a) Force needed to move the rod is $F = \dfrac{B^2vl^2}{R} = \dfrac{(0.15)^2 \times 2 \times (0.5)^2}{3} = 3.75 \times 10^{-3}\ N$

Tricky example: 5

A sphere frame of metallic wire is moving in a uniform magnetic field $(\vec{B})$ acting perpendicular to the paper inward as shown. LP and QN are also metallic wires then find the potential difference between L and N



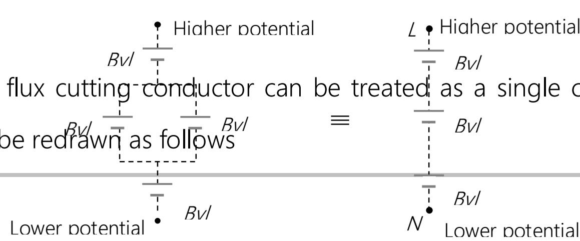
(a) Zero

(b) Bvl

(c) $2Bvl$

(d) $3Bvl$

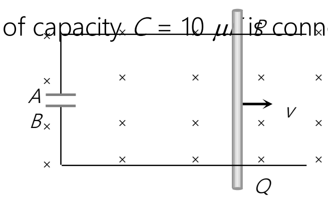
Solution : (d) We know that a flux cutting conductor can be treated as a single cell of emf $e = Bvl$. Hence the given figure can be redrawn as follows



$\Rightarrow V_{LN} = 3Bvl$

Tricky example: 6

A conducting rod PQ of length L = 1.0 m is moving with a uniform speed v = 2 m/s in a uniform magnetic field $B = 4.0\ T$ directed into the paper. A capacitor of capacity $C = 10\ \mu F$ is connected as shown in figure. Then



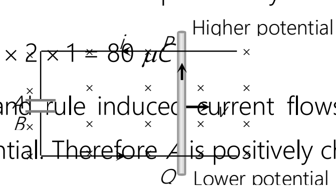
(a) $q_A = +80\ \mu C$ and $q_B = -80\ \mu C$

(b) $q_A = -80\ \mu C$ and $q_B = +80\ \mu C$

(c) $q_A = 0 = q_B$

(d) Charge stored in the capacitor increases exponentially with time

Solution : (a) $Q = CV = C(Bvl) = 10 \times 10^{-6} \times 4 \times 2 \times 1 = 80\ \mu C$

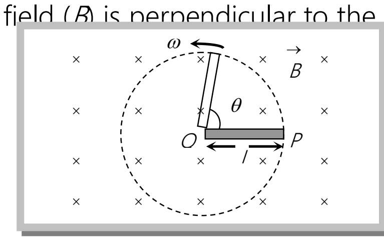


According to Fleming's right hand rule induced current flows from Q to P. Hence P is at higher potential and Q is at lower potential. Therefore A is positively charged and B is negatively charged.

Motional EMI Due to Rotational Motion

(1) **Conducting rod**

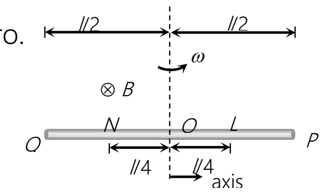
A conducting rod of length l whose one end is fixed, is rotated about the axis passing through it's fixed end and perpendicular to it's length with constant angular velocity ω. Magnetic field (B) is perpendicular to the plane of the paper.



emf induces across the ends of the rod $e = \frac{1}{2} Bl^2 \omega = Bl^2 \pi \nu = \frac{Bl^2 \pi}{T}$

where ν = frequency (revolution per sec) and T = Time period.

Note : ❐ If above metallic rod rotated about its axis of rotation, then induced potential difference between any pair of identical located points of rod, is always zero.



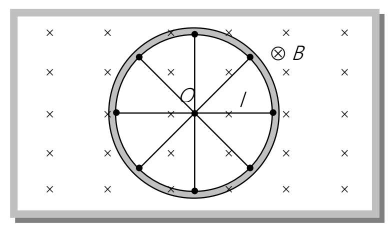
It is clear parts OP and OQ are identical hence

$e_{OP} = e_{OQ}$ *i.e.* $e_{PQ} = 0$

Similarly $e_{LN} = 0 (V_L = V_N)$

(2) **Cycle wheel**

A conducting wheel each spoke of length l is rotating with angular velocity ω in a given magnetic field as shown below in fig.



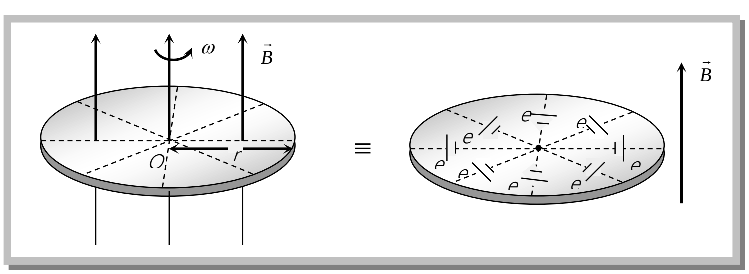
Due to flux cutting each metal spoke becomes identical cell of emf e (say), all such identical cells connected in parallel fashion $e_{net} = e$ (emf of single cell). Let N be the number of spokes hence $e_{net} = \frac{1}{2} Bwl^2 ; \omega = 2\pi\nu$

Here $e_{net} \propto N^o$ *i.e.* total emf does not depends on number of spokes 'N.

Note : ❐ Here magnetic field (may be component of Earth's magnetic field) some times, depends on plane of motion of wheel. If wheel rotates in horizontal plane, then $B = B_V$ used; If wheel rotates in vertical plane, then $B = B_H$ used (B_H-horizontal component of earth's magnetic field while B_V-vertical component)

(3) **Faraday copper disc generator**

During rotational motion of disc, it cuts away magnetic field lines.



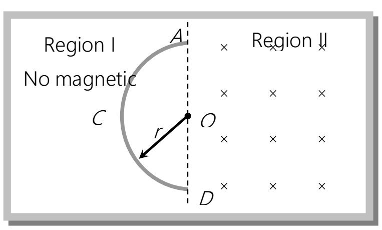
A metal disc can be assumed to made of uncountable radial conductors when metal disc rotates in transverse magnetic field these radial conductors cuts away magnetic field lines and because of this flux cutting all becomes identical cells each of emf 'e' where $e = \frac{1}{2}B\omega r^2$, as shown in following fig. and periphery of disc becomes equipotential.

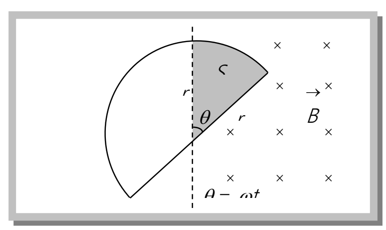
All identical cells connected in parallel fashion, So net emf for disc $e_{net} = e = \frac{1}{2}B\omega r^2 = B(\pi r^2)\nu$

Note : ❒ If a galvanometer is connected between two peripheral points or diametrical opposite ends it's reading will be zero.

(4) **Semicircular conducting loop**

For the given figure a semi-circular conducting loop (*ACD*) of radius '*r*' with centre at *O*, the plane of loop being in the plane of paper. The loop is now made to rotate with a constant angular velocity ω, about an axis passing through *O* and perpendicular to the plane of paper. The effective resistance of the loop is *R*.



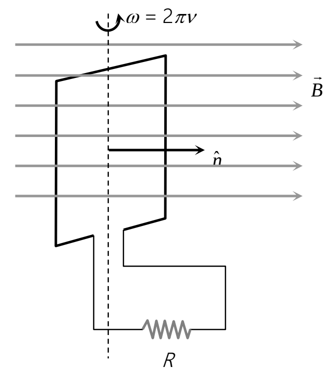


In time *t* the area swept by the loop in the field *i.e.* region II $A = \frac{1}{2}r(r\theta) = \frac{1}{2}r^2\omega t$; $\frac{dA}{dt} = \frac{r^2\omega}{2}$

Flux link with the rotating loop at time t $\phi = BA$

Hence induced emf in the loop in magnitude $|e| = \frac{d\phi}{dt} = B\frac{dA}{dt} = \frac{B\omega r^2}{2}$ and induced current $i = \frac{|e|}{R} = \frac{B\omega r^2}{2R}$

Suppose a rectangular coil having N turns placed initially in a magnetic field such that magnetic field is perpendicular to it's plane as shown.



ω – Angular speed

ν – Frequency of rotation of coil

R – Resistance of coil

For uniform rotational motion with ω, the flux linked with coil at any time t

$\phi = NBA \cos\theta = NBA \cos\omega t$ (as $\theta = \omega t$)

$\phi = \phi_0 \cos\omega t$ where $\phi_0 = NBA$ = flux amplitude or maximum flux

(This relation shows that the flux changes in periodic nature)

(1) **Induced emf in coil**

Induced emf also changes in periodic manner that's why this phenomenon called periodic EMI

$$e = -\frac{d\phi}{dt} = NBA\,\omega \sin\omega t \Rightarrow e = e_0 \sin\omega t \text{ where } e_0 = \text{emf amplitude or max. emf} = NBA\,\omega = \phi_0\omega$$

(2) **Induced current**

At any time t, $i = \dfrac{e}{R} = \dfrac{e_0}{R}\sin\omega t = i_0 \sin\omega t$ where i_0 = current amplitude or max. current $i_0 = \dfrac{e_0}{R} = \dfrac{NBA\,\omega}{R} = \dfrac{\phi_0\omega}{R}$

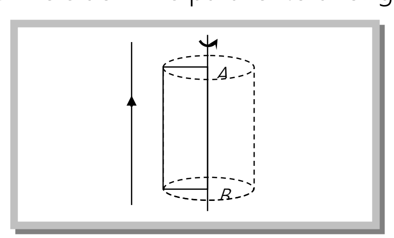
Note : ❐ For rotating coil, induced emf and linked flux keep a phase difference of $\dfrac{\pi}{2}$ *i.e.* when plane of coil is perpendicular to magnetic field $\vec{B}$ so flux linked will be max. and induced emf $e = 0$ and when plane of coil parallel to $\vec{B}$ flux linked $\phi_{min} = 0$ and induced emf will be maximum $e_{max} = e_0$

❐ Frequency of induced any parameter = Frequency of rotation of coil = ν

❐ Both emf and current changes their value *w.r.t.* time according to sine function hence they called as sinusoidal induced quantities.

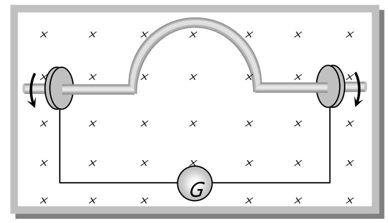
(3) **Special cases**

(i) A rectangular coil rotates at a constant speed about one of its sides AB. The side AB is parallel to a long, straight current carrying conductor.



The current carrying conductor is in the plane of the page and the magnetic field due to it at coil is perpendicular to the plane of the paper. The emf induced in the coil rotating in this field is minimum when the coil is perpendicular to the field that is in the plane of the conductor. The emf will be maximum, when the coil is perpendicular to the plane of the conductor.

(ii) A stiff wire bent into a semicircle of radius '*r*' is rotated at a frequency ν in a uniform field of magnetic induction $\vec{B}$ as shown in figure. If resistance of the entire circuit is *R* then



Current amplitude given as $i_0 = \frac{BA\omega}{R} = \frac{B(2\pi\nu)}{R}\frac{(\pi r^2)}{2} = \frac{\pi^2 r^2 B\nu}{R}$

Area of loop $= \frac{\pi r^2}{2}$

(Frequency of induced current = frequency of rotation of loop = ν)

Examples

Example: 41 A metal conductor of length 1 *m* rotates vertically about one of its ends at angular velocity 5 radians per second. If the horizontal component of earth's magnetic field is $0.2\times10^{-4}\,T$, then the e.m.f. developed between the two ends of the conductor is

(a) $5\,mV$ (b) $50\,\mu V$ (c) $5\,\mu V$ (d) $50\,mV$

Solution: (b) Induced emf $e = \frac{1}{2}B_H\,l^2\omega = \frac{1}{2}\times0.2\times10^{-4}\times(1)^2\times5 = 5\times10^{-5}\,V = 50\,\mu V$

Example: 42 A rectangular coil of 300 turns has an average area of 25 *cm* × 10 *cm*. The coil rotates with a speed of 50 *cps* in a uniform magnetic field of strength $4\times10^{-2}\,T$ about an axis perpendicular to the field. The peak value of the induced *emf* is (in *volt*)

(a) 3000 π (b) 300 π (c) 30 π (d) 3 π

Solution: (c) Peak value of emf = $e_0 = \omega NBA = 2\pi\nu\, NBA = 2\pi\times50\times300\times4\times10^{-2}\times(25\times10^{-2}\times10\times10^{-2}) = 30\,\pi$ *volt*

Example: 43 A wheel with ten metallic spokes each 0.50 *m* long is rotated with a speed of 120 *rev/min* in a place normal to the earth's magnetic field at the place. If the magnitude of the field is 0.04 *G*, the induced emf between the axle and the rim of the wheel is equal to

[AMU 2002]

(a) $1.256\times10^{-3}\,V$ (b) $6.28\times10^{-3}\,V$ (c) $1.256\times10^{-4}\,V$ (d) $6.28\times10^{-6}\,V$

Solution: (d) $e = \frac{1}{2}Bl^2\omega = Bl^2\pi\nu = (0.04\times10^{-4})\times(0.5)^2\times3.14\times\frac{120}{60} = 6.28\times10^{-6}\,V.$

Example: 44 A copper disc of radius 0.1 *m* rotates about its centre with 10 revolutions per second in a uniform magnetic field of 0.1 *Tesla*. The emf induced across the radius of the disc is

(a) $\frac{\pi}{10}V$ (b) $\frac{2\pi}{10}V$ (c) 10 π *mV* (d) 20 π *mV*

Solution: (c) The induced emf between centre and rim of the rotating disc is

$$E=\frac{1}{2}B\omega R^2=\frac{1}{2}\times 0.1\times 2\pi\times 10\times(0.1)^2=10\pi\times 10^{-3}\,volt$$

Example: 45 A rectangular coil having dimensions 10 *cm* × 5 *cm* has 100 turns. It is moving at right angles to a field of 5 *Tesla* at angular speed of 314 *rad/sec*. The emf at instant when flux passing the coils is half the maximum value is

(a) 785 V (b) $\frac{785}{2}V$ (c) $\frac{785\sqrt{3}}{2}V$ (d) 0

Solution: (c) $\phi=\phi_0\cos\theta\Rightarrow\frac{\phi_0}{2}=\phi_0\cos\theta;\ \ \theta=60^{\circ}$

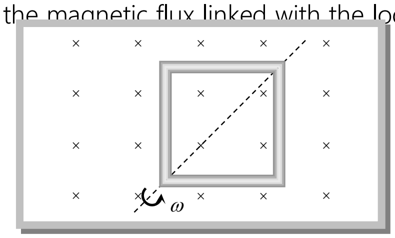
$\therefore\ \ e=e_0\sin\theta\Rightarrow e_0=\omega NBA=314\times 100\times 5\times 50\times 10^{-4}=785\ V\Rightarrow e=785\sin 60^{\circ}=\frac{785\sqrt{3}}{2}V$

Example: 46 A loop of area 0.1 m^2 rotates with a speed of 60 *rev/sec* with the axis of rotation perpendicular to a magnetic field B = 0.4 T. If there are 100 turns in the loop, the maximum voltage induced in the loop is [MP PMT 199

(a) 15.07 *V* (b) 150.7 *V* (c) 1507 *V* (d) 250 *V*

Solution: (c) Maximum voltage $e_0=\omega NBA=2\pi\nu NBA=2\times 3.14\times 60\times 100\times 0.4\times 0.1=1507\ V$

Example: 47 A square loop of side a is rotating about its diagonal with angular velocity ω in a perpendicular magnetic field as shown in the figure. If the number of turns in it is 10 then the magnetic flux linked with the loop at any instant will be



(a) 10 $Ba^2\cos\omega t$

(b) 10 Ba

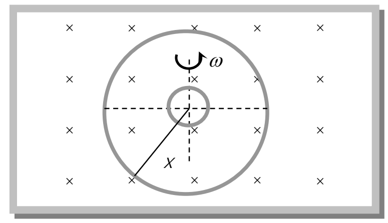
(c) 10 Ba^2

(d) 20 Ba^2

Solution: (a) The magnetic flux linked with the loop at any instant of time t is given by $\phi=BAN\cos\omega t$ or $\phi=10\,Ba^2\cos\omega t$

Here N = 10, $A=a^2$

Example: 48 A very small circular loop of area A and the resistance R and negligible inductance is initially coplanar and concentric with a much larger fixed loop of radius x. A constant current i is passed in the bigger loop and the smaller loop is rotated with constant angular velocity ω about a diameter then induced current in the smaller loop as a function of time will be



(a) $\dfrac{\mu_0 iA}{2xR}\sin\omega t$

(b) $\dfrac{\mu_0 iA\omega}{2xR}\sin\omega t$

(c) $\dfrac{\mu_0 iA\omega}{2xR}\sin 2\omega t$

(d) 0

Solution: (b) At any instant t flux linked with smaller loop $\phi = BA\cos wt$ where B = magnetic field produced by larger loop at it's centre $= \dfrac{\mu_0 i}{2x}$. So $\phi = \dfrac{\mu_0 iA}{2x}\cos\omega t$; $e = -\dfrac{d\phi}{dt} = \dfrac{\mu_0 i}{2x}\omega A\sin\omega t \Rightarrow i = \dfrac{e}{R} = \dfrac{\mu_0 i\omega A}{2xR}\sin\omega t$.

Example: 49 In periodic motion of a coil in an uniform magnetic field if induced emf at any instant t is given by $e = 10\sin 314\,t$ then induced emf at $t = \dfrac{1}{300}\,sec$ will be

(a) 5 V (b) $5\sqrt{2}\,V$ (c) $5\sqrt{3}\,volt$ (d) None of these

Solution: (c) $\because\ e = 10\sin 314\,t = 10\sin(3.14\times 100)t = 10\sin 100\,\pi t$

Putting $t = \dfrac{1}{300}\,sec;\ e = 10\sin\dfrac{\pi}{3} = 10\sin 60^{o} = \dfrac{10\sqrt{3}}{2} = 5\sqrt{3}\,V.$

Example: 50 In the previous question at what time t instantaneous induced emf will be half of maximum induced emf

(a) $\dfrac{1}{300}\,sec$ (b) $\dfrac{1}{400}\,sec$ (c) $\dfrac{1}{500}\,sec$ (d) $\dfrac{1}{600}\,sec$

Solution: (d) Given $e = 10\sin 314\,t = 10\sin 100\,\pi t$; $\dfrac{10}{2} = 10\sin 314\,t \Rightarrow \dfrac{1}{2} = \sin 100\,\pi t \Rightarrow 100\,\pi t = \sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6}$, $t = \dfrac{1}{600}\,sec$

Example: 51 In a region of uniform magnetic induction $B = 10^{-2}$ *Tesla*, a circular coil of radius 30 *cm* and resistance π^2 ohm is rotated about an axis which is perpendicular to the direction of B and which forms a diameter of the coil. If the coil rotates at 200 *rpm* the amplitude of the alternating current induced in the coil is [CBSE 1990]

(a) $4\pi^2$ *mA* (b) 30 *mA* (c) 6 *mA* (d) 200 *mA*

Solution: (c) $i_0 = \dfrac{BAN\omega}{R} = \dfrac{10^{-2}\times\pi\times(30\times 10^{-2})^2\times 1\times 2\pi\times 200}{\pi^2\times 60} = 6\times 10^{-3}\,A = 6\,mA.$

Static EMI

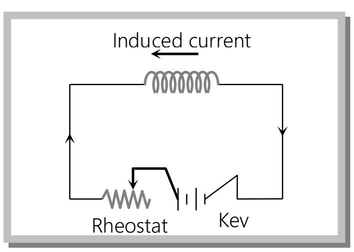
Inductance is that property of electrical circuits which opposes any change in the current in the circuit.

Inductance is inherent property of electrical circuits. It will always be found in an electrical circuit whether we want it or not. The circuit in which a large emf is induced when the current in the circuit changes is said to have greater inductance. A straight wire carrying current with no iron part in the circuit will have lesser value of inductance while if the circuit contains a circular coil having many number of turns, the induced emf to oppose the cause will be greater and the circuit is therefore said to have greater value of inductance.

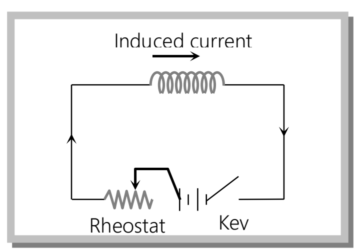
Inductance is called electrical inertia : Inductance is analogous to inertia in mechanics, because we know that due to inertia a body at rest opposes any attempt which tries to bring it in motion and a body in motion opposes any attempt which tries to bring it to rest. Inductance of an electrical circuit opposes any change of current in the circuit thus it is also called electrical inertia.

(1) **Self-Induction**

Whenever the electric current passing through a coil or circuit changes, the magnetic flux linked with it will also change. As a result of this, in accordance with Faraday's laws of electromagnetic induction, an emf is induced in the coil or the circuit which opposes the change that causes it. This phenomenon is called 'self induction' and the emf induced is called back emf, current so produced in the coil is called induced current.



Main current increasing



Main current

(i) **Coefficient of self-induction :** If no magnetic materials are present near the coil, number of flux linkages with the coil is proportional to the current *i*. *i.e.* $N\phi \propto i$ or $N\phi = Li$ (*N* is the number of turns in coil and $N\phi$ – total flux linkage) where $\boldsymbol{L} = \dfrac{\boldsymbol{N\phi}}{\boldsymbol{i}}$ = coefficient of self induction.

If i = 1*amp*, N = 1 then, $L = \phi$ *i.e.* the coefficient of self induction of a coil is equal to the flux linked with the coil when the current in it is 1 *amp*.

By Faraday's second law induced emf $e = -N\frac{d\phi}{dt}$. Which gives $\boldsymbol{e = -L\frac{di}{dt}}$; If $\frac{di}{dt} = 1\,Amp\ /sec$ then $|e| = L$.

Hence coefficient of self induction is equal to the emf induced in the coil when the rate of change of current in the coil is unity.

Note : ❒ Here we must note that if we are asked to calculate the induced emf in an inductor, then we have $e = -L\frac{di}{dt}$. But when we are asked to calculate the voltage (V) across the inductor then $V = |e| = \frac{di}{dt} \times L$

(ii) **Units and dimensional formula of 'L'**

S.I. unit : $\frac{weber}{Amp} = \frac{Tesla \times m^2}{Amp} = \frac{N \times m}{Amp^2} = \frac{Joule}{Amp^2} = \frac{Coulomb \times volt}{Amp^2} = \frac{volt \times \sec}{amp} = ohm \times \sec$

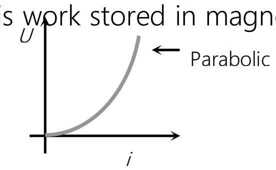
But practical unit is henry (H). It's dimensional formula $[L] = [ML^2T^{-2}A^{-2}]$

Note : ❒ 1 *henry* = 10^9 *emu* of inductance or 10^9 *ab-henry*.

(iii) **Dependence of self inductance (L) :** 'L' does not depend upon current flowing or change in current flowing but it depends upon number of turns (N), Area of cross section (A) and permeability of medium (μ). (Soft iron has greater permeability. Hence greater self inductance L)

'L' does not play any role till there is a constant current flowing in the circuit. 'L' comes in to the picture only when there is a change in current.

(iv) **Magnetic potential energy of inductor :** In building a steady current in the circuit, the source emf has to do work against of self inductance of coil and whatever energy consumed for this work stored in magnetic field of coil this energy called as magnetic potential energy (U) of coil



$U = \int_0^i Li\,di = \frac{1}{2}Li^2$; Also $U = \frac{1}{2}(Li)i = \frac{N\phi i}{2}$

Note : ❒ Energy density is given as $U = \frac{1}{2}\frac{B^2}{\mu_0}$.

(v) **Calculation of self inductance for a current carrying coil :** If a coil of any shape having N turns, carries a current i, then total flux linked with coil $N\phi = Li$

Also $\phi = BA\cos\theta$; where B = magnetic field produced at the centre of coil due to it's current; A = Area of each turn; θ = Angle between normal to the plane of coil and direction of magnetic field.

$\therefore\ L = \dfrac{N\phi}{i} = \dfrac{NBA\cos\theta}{i}$; If $\theta = 0^{\circ}$, $\phi_{\max} = BA$ So $\boldsymbol{L = \dfrac{NBA}{i}}$

Circular coil

If a circular coil of *N* turns carrying current *i* and its each turn is of radius *r* then its self inductance can be calculated as follows as

Magnetic field at the centre of coil due to its own current $B = \dfrac{\mu_0}{4\pi}.\dfrac{2\pi Ni}{r}$ $\Rightarrow$

$$L = \frac{N\left(\dfrac{\mu_0}{4\pi}.\dfrac{2\pi Ni}{r}\right)(\pi r^2)}{i} = \frac{\mu_0 \pi N^2 r}{2}$$

$\Rightarrow$ $\boldsymbol{L \propto N^2}$ $\Rightarrow$ $\dfrac{L_1}{L_2} = \left(\dfrac{N_1}{N_2}\right)^2$ (For constant *r*)

Note : ❐ If radius is doubled so self inductance will also doubles. $(L \propto r)$ (If *N* = constant)

- ❐ If area across section is doubled (*N* = constant) *i.e.* $A' = 2A \Rightarrow \pi r'^2 = 2 \times \pi r^2 \Rightarrow r' = \sqrt{2}\, r$

  So $L' = \sqrt{2}\, L$ *i.e.* increase in self induction is 41.4%.

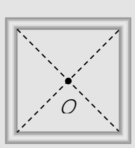
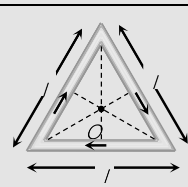
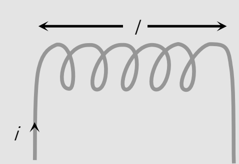
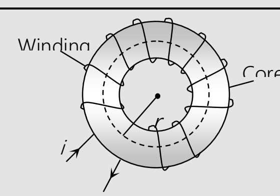
- ❐ If a current carrying wire of constant length is bend into circular coil of *N*-turns then $N(2\pi r) = l$; $N \propto \dfrac{l}{r}$

  Now as $L \propto N^2 r \longrightarrow$ *If N - given* *If r - given*

  $L \propto N^2\left(\dfrac{l}{N}\right) \Rightarrow \boldsymbol{L \propto N}$ $L \propto \dfrac{l}{r^2}(r) \Rightarrow \boldsymbol{L \propto \dfrac{l}{r}}$

  e.g. If a wire of length *l* first bent in single turn circular coil then in double turn (concentric coplanar) coil so by using $L \propto N$ we can say that *L* in second case twice that in first case.

Other important cases

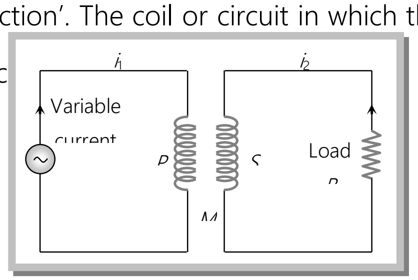
Square coil	Triangular coil	Solenoid	Toroid
			

$B = \frac{\mu_0}{4\pi}\cdot\frac{8\sqrt{2}\,i}{a}N$ $L = \frac{N\left(\frac{\mu_0}{4\pi}\cdot\frac{8\sqrt{2}Ni}{a}\right)a^2}{i}$ $L = \frac{2\sqrt{2}\mu_0 N^2 a}{\pi} \Rightarrow L \propto N^2$	$B = \frac{\mu_0}{4\pi}\cdot\frac{18\,Ni}{l}$ $L = \frac{N\left(\frac{\mu_0}{4\pi}\frac{18\,Ni}{l}\right)\times\left(\frac{\sqrt{3}}{4}l^2\right)}{i}$ $L = \frac{9\sqrt{3}\,\mu_0 N^2 l}{8\pi} \Rightarrow L \propto N^2$	$B = \mu_0 ni = \frac{\mu_0 Ni}{l}$ $L = \frac{N\left(\frac{\mu_0 Ni}{l}\right).A}{i}$; $L = \frac{\mu_0 N^2 A}{l} \Rightarrow L \propto N^2$ For iron cored solenoid $L = \frac{\mu_0\mu_r N^2 A}{l} = \frac{\mu N^2 A}{l}(\mu = \mu_0\mu_r)$	$B = \frac{\mu_0 Ni}{2\pi r}$ $L = \frac{N\left(\frac{\mu_0 Ni}{2\pi r}\right)\pi r^2}{i} = \frac{\mu_0 N^2 r}{2}$

Note : ❑ Inductance at the ends of a solenoid is half of it's the inductance at the centre. $\left(L_{end} = \frac{1}{2}L_{centre}\right)$.

(2) **Mutual Induction**

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence an emf will be induced in the neighbouring coil or circuit. This phenomenon is called 'mutual induction'. The coil or circuit in which the current changes is called 'primary' while the other in which emf is set up is c



In case of mutual inductance for two coils situated close to each other, total flux linked with the secondary due to current in the primary is $N_2\phi_2$ and $N_2\phi_2 \propto i_1 \Rightarrow \boldsymbol{N_2\phi_2 = Mi_1}$ where N_1 - Number of turns in primary; N_2 - Number of turns in secondary; ϕ_2 - Flux linked with each turn of secondary; i_1 - Current flowing through primary; M-Coefficient of mutual induction or mutual inductance.

According to Faraday's second law emf induces in secondary $e_2 = -N_2 \frac{d\phi_2}{dt}$; $\boldsymbol{e_2} = -\boldsymbol{M}\frac{\boldsymbol{di_1}}{\boldsymbol{dt}}$; If $\frac{di_1}{dt} = \frac{1 Amp}{sec}$ then $|e_2| = M$. Hence coefficient of mutual induction is equal to the emf induced in the secondary coil when rate of change of current in primary coil is unity.

Units and dimensional formula of M are similar to self-inductance (L)

(i) **Dependence of mutual inductance**

(a) Number of turns (N_1, N_2) of both coils

(b) Coefficient of self inductances (L_1, L_2) of both the coils

(c) Area of cross-section of coils

(d) Magnetic permeability of medium between the coils (μ_r) or nature of material on which two coils are wound

(e) Distance between two coils (As $d\uparrow = M\downarrow$)

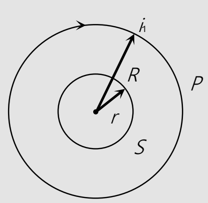
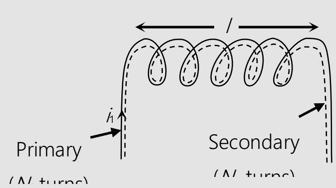
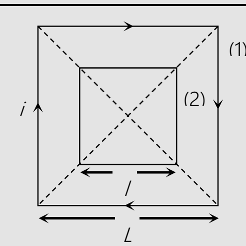
(f) Orientation between primary and secondary coil (for 90° orientation no flux relation $M = 0$)

(g) Coupling factor 'K' between primary and secondary coil

(ii) **Calculation of mutual inductance between two coils**

If two coils (1 and 2) also called primary and secondary coils are placed close to each other (maximum coupling); N_1 and N_2 = Number of turns in primary and secondary coils respectively, ϕ_2 = Flux linked with each turn of secondary, $N_2\phi_2$ = Total flux linkage with secondary coils; M = Mutual inductance between two coil

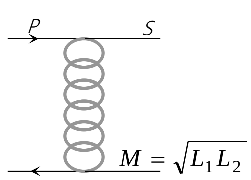
So $N_2\phi_2 = Mi_1 \Rightarrow N_2(B_1A_2) = Mi_1 \Rightarrow \boldsymbol{M} = \frac{\boldsymbol{B_1 N_2 A_2}}{\boldsymbol{i_1}}$

Two concentric coplaner circular coils	Two Solenoids	Two concentric coplaner square coils
		

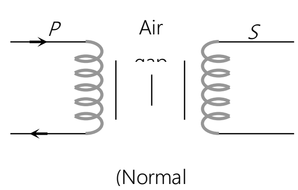
Magnetic field at the centre due to current in outer coil is $B_1 = \frac{\mu_0}{4\pi}.\frac{2\pi N_1 i_1}{R}$, From the above formula $M = \frac{\pi\mu_0 N_1 N_2 r^2}{2R} \Rightarrow$ $\boldsymbol{M \propto \frac{r^2}{R}}$	If two air cored solenoid tightly wound to each other as shown : Magnetic field inside the primary solenoid $B_1 = \mu_0 n_1 i_1$ $\left\{ n_1 = \frac{N_1}{l} \right.$ From the above formula {A = Area of each solenoid) $\boldsymbol{M = \frac{\mu_0 N_1 N_2 A}{l}}$	Magnetic field at the centre due to current in outer coil is $B_1 = \frac{\mu_0}{4\pi}.\frac{8\sqrt{2}\,N_1 i_1}{L}$ From the above formula $M = \frac{\mu_0 2\sqrt{2}\,N_1 N_2 l^2}{\pi L} \Rightarrow$ $\boldsymbol{M \propto \frac{l^2}{L}}$

(iii) **Relation between M, L_1 and L_2**

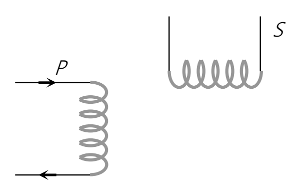
For two magnetically coupled coils $M = k\sqrt{L_1 L_2}$; where k – coefficient of coupling or coupling factor which is defined as $k = \frac{\text{magnetic flux linked in secondary}}{\text{magnetic flux linked in primary}}$; $0 \le k \le 1$



If coils are tightly coupled (k = 1)



If coils are loosely coupled (0 < k < 1)

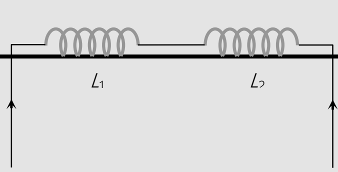
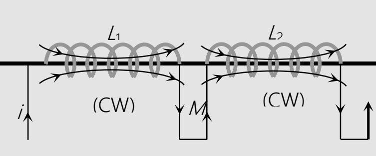
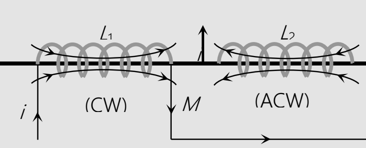


No coupling (k = 0)

Note : ❐ Specially for Transformer in ideal case $M = \frac{N_2}{N_1} L_1$ and $M = \frac{N_1}{N_2} L_2$; $\boldsymbol{\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2}$

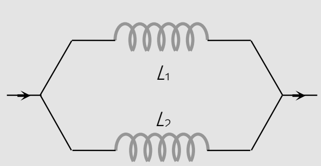
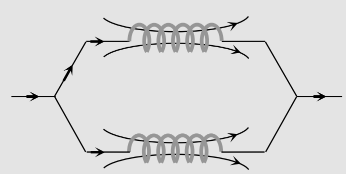
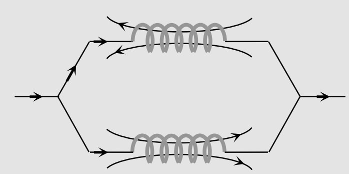
(3) **Combination of inductance**

(i) **Series combination**

Mutual induction is absent (k = 0)	Mutual induction is present and favours self inductance of coils	Mutual induction is present and opposes self inductance of coils
		

$L_{eq} = L_1 + L_2$	Current in same direction Winding nature same Their flux assist each other $L_{eq} = L_1 + L_2 + 2M$	Current in opposite direction Opposite winding nature Their flux opposes each other $L_{eq} = L_1 + L_2 - 2M$

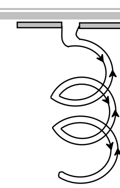
(ii) **Parallel combination**

Mutual induction is absent ($k = 0$)	Mutual induction is present and favours self inductance of coils	Mutual induction is present and opposes self inductance of coils
		
$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$	$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$	$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

Note : ❒ If nothing is said then it is to be considered that mutual inductance between the coils is absent.

Concepts

☞ *A thin long wire made up of material of high resistivity behaves predominantly as a resistance. But it has some amount of*



inductance as well as capacitance in it. It is thus difficult to obtain pure resistor. Similarly it is difficult to obtain pure capacitor as well as pure inductor.

- *Due to inherent presence of self inductance in all electrical circuits, a resistive circuit with no capacitive or inductive element in it, also has some inductance associated with it.*

  The effect of self-inductance can be eliminated as in the coils of a resistance box by doubling back the coil on itself. The coil is placed in space as shown in figure below

- *It is not possible to have mutual inductance without self inductance but it may or may not be possible self inductance without mutual inductance.*
- *If main current through a coil increases (i↑) so* $\frac{di}{dt}$ *will be positive (+ve), hence induced emf e will be negative (i.e. opposite emf)* $\Rightarrow E_{net} = E - e$
- *If main current through a circuit (coil) decreases (i ↓) so* $\frac{di}{dt}$ *will be negative (– ve), hence induced emf e will be positive (i.e. same directed emf)* $\Rightarrow E_{net} = E + e$
- *Sometimes at sudden opening of key, because of high inductance of circuit a high momentarily induced emf* ***produced*** *and a sparking occurs at key position. To avoid sparking a capacitor is connected across the key.*
- *One can have resistance with or without inductance but one can't have inductance without having resistance.*
- *In checking balancing of Wheatstone bridge, always firstly pressed cell key and after–wards galvanometer key, so that momentarily induced current produced, because of self inductance of coil of galvanometer becomes almost zero or disappear.*
- *The circuit behaviour of an inductor is quite different from that of a resistor. while a resistor opposes the current i, an inductor opposes the change* $\frac{di}{dt}$ *in the circuit*

Examples

Example: 52 A circular coil of radius 5 *cm* has 500 turns of a wire. The approximate value of the coefficient of self induction of the coil will be

(a) 25 *millihenry* (b) 25 × 10^{-3} *millihenry* (c) 50 × 10^{-34} *millihenry* (d) 50 × 10^{-3} *henry*

Solution: (a) By using $L = \frac{\pi\mu_0 N^2 r}{2}$; $L = \frac{(3.14)\times 4\times(3.14)\times 10^{-7}\times(500)^2\times 5\times 10^{-2}}{2} \approx 25\times 10^{-3} H \approx 25 mH$

Example: 53 A solenoid has 2000 turns wound over a length of 0.30 *metre*. The area of its cross-section is 1.2 × 10^{-3} m^2. Around its central section, a coil of 300 turns is wound. If an initial current of 2*A* in the solenoid is reversed in 0.25 *sec*, then the emf induced in the coil is [NCERT 1982]

(a) 6 × 10^{-4} *V* (b) 4 × 10^{-3} *V* (c) 6 × 10^{-2} *V* (d) 48 *mV*

Solution: (d) By using $M = \frac{\mu_0 N_1 N_2 A}{l}$ and $|e| = M\frac{di}{dt}$; $M = 3.01\times 10^{-3} H \Rightarrow e = 3.01\times 10^{-3}\times\frac{\{2-(-2)\}}{0.25}$; $e = 48\ mV$.

Example: 54 The coefficient of self inductance of a solenoid is 0.18 *mH*. If a crode of soft iron of relative permeability 900 is inserted, then the coefficient of self inductance will become nearly

(a) 5.4 *mH* (b) 162 *mH* (c) 0.006 *mH* (d) 0.0002 *mH*

Solution: (b) We know for air cored solenoid $L = \frac{\mu_0 N^2 A}{l}$

In case of soft of iron core it's self inductance $L' = \frac{\mu_0\mu_r N^2 A}{l}$; $L' = \mu_r L$. So here L' = 900 × 0.18 = 162 *mH*

Note : ❒ The self-inductance of a solenoid may be increased by inserting a soft iron core. The function of the core is to improve the flux linkage between the turns of the coil.

Example: 55 The current in an inductor is given by i = 2 + 3t *amp* where t is in second. The self induced emf in it is 9 *mV* the energy stored in the inductor at t= 1 second is

(a) 10 *mJ* (b) 37.5 *mJ* (c) 75 *mJ* (d) Zero

Solution: (b) At t = 1 *sec*, i = 2 + 3 × 1 = 5*A* and $|e| = L\frac{di}{dt} \Rightarrow 9\times 10^{-6} = L\times\frac{d}{dt}(2+3t) \Rightarrow L = 3\times 10^{-3}\ H$

So energy $U = \frac{1}{2}Li^2 = \frac{1}{2}(3\times 10^{-3})\times(5)^2$ = 37.5 *mJ*.

Example: 56 The number of turns in two coils *A* and *B* are 300 and 400 respectively. They are placed close to each other. Co-efficient of mutual induction between them is 24 *mH*. If the current passing through the coil *A* is 2 *Amp* then the flux linkage with coil *B* will be

(a) 24 *mwb* (b) 12×10^{-5} *wb* (c) 48 *mwb* (d) 48×10^{-5} *wb*

Solution: (c) Flux linkage $= N_2\phi_2 = Mi_1 = 24 \times 2 = 48\ mwb$

Example: 57 A coil of wire of a certain radius has 600 turns and a self-inductance of 108 *mH*. The self-inductance of another similar coil of 500 turns will be [MP PMT 1990]

(a) 74 *mH* (b) 75 *mH* (c) 76 *mH* (d) 77 *mH*

Solution: (b) $\because\ L \propto N^2 \Rightarrow \frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2 \Rightarrow \frac{108}{L_2} = \left(\frac{600}{500}\right)^2;\ L_2 = 75\,mH$

Example: 58 Two different coils have self-inductance $L_1 = 8mH$, $L_2 = 2mH$. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same rate. At a certain instant of time, the power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then choose incorrect option

(a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = 4$ (c) $\frac{W_2}{W_1} = 4$ (d) $\frac{V_2}{V_1} = \frac{1}{4}$

Solution: (b) By $|e| = L\frac{di}{dt} \Rightarrow \frac{e_1}{e_2} = \frac{L_1}{L_2}\ \left\{\frac{di}{dt} - \text{same}\right\} \Rightarrow \frac{V_1}{V_2} = \frac{8}{2} = 4$

Power $P = ei \Rightarrow i \propto \frac{1}{e}\ \{P - \text{same}\} \Rightarrow \frac{i_1}{i_2} = \frac{e_2}{e_1} = \frac{V_2}{V_1} = \frac{L_2}{L_1} = \frac{2}{8} = \frac{1}{4}$

Energy stored $W = \frac{1}{2}Li^2;\ \frac{W_1}{W_2} = \frac{L_1}{L_2} \times \left(\frac{i_1}{i_2}\right)^2 = 4 \times \left(\frac{1}{4}\right)^2 = \frac{1}{4}$.

Example: 59 A current increases uniformly from zero to one ampere in 0.01 second, in a coil of inductance 10 *mH* it. The induced emf will be

(a) 1 *V* (b) 2 *V* (c) 3 *V* (d) 4 *V*

Solution: (a) $e = -L\frac{dI}{dt} = -10 \times 10^{-3}\frac{1.0}{0.01} = -1\,volt\ \therefore\ |e| = 1\,volt$

Example: 60 The current in a coil varies *w.r.t.* to time *t* as $I = 3t^2 + 2t$. If the inductance of coil be 10 *mH*, the value of induced emf at $t = 2s$ will be

(a) 0.14 *V* (b) 0.12 *V* (c) 0.1 *V* (d) 0.13 *V*

Solution: (a) $e = -L\frac{dI}{dt} = -\frac{d}{dt}[3t^2 + 2t] = -L[6t + 2] = -10 \times 10^{-3}[6t + 2]$

$(e)_{\text{at } t=2} = -10 \times 10^{-3}(6 \times 2 + 2) \quad = -10 \times 10^{-3}(14) = -0.14\, volt$; $|e| = 0.14\, volt$

Example: 61 What inductance would be needed to store 1 *KWh* of energy in a coil carrying a 200 *A* current

(a) 100 *H* (b) 180 *H* (c) 200 *H* (d) 450 *H*

Solution: (b) U = 1KWH = 3.6 × 10^6 *J*. By using $U = \frac{1}{2}Li^2$ $\Rightarrow$ 3.6 × 10^6 = $\frac{1}{2} \times L \times (200)^2$ $\Rightarrow L$ = 180 *H*

Example: 62 The self inductance of a coil is *L*, keeping the length and area same, the number of turns in the coil is increased to four times. The self inductance of the coil will now be [MP PMT 1997]

(a) $\frac{1}{4}L$ (b) *L* (c) 4 *L* (d) 16 *L*

Solution: (d) $L \propto N^2$

Example: 63 The mutual inductance between a primary and secondary circuit is 0.5 *H*. The resistance of the primary and the secondary circuits are 20 *ohms* and 5 *ohms* respectively. To generate a current of 0.4 *A* in the secondary, current in the primary must be changed at the rate of

(a) 4.0 *A/s* (b) 16.0 *A/s* (c) 1.6 *A/s* (d) 8.0 *A/s*

Solution: (a) By using $|e_2| = M\frac{di_1}{dt}$; $i_2 = \frac{e_2}{R_2} = \frac{M}{R_2}\frac{di_1}{dt} \Rightarrow 0.4 = \frac{0.5}{5} \times \frac{di_1}{dt}$; $\frac{di_1}{dt} = 4\, A/sec$

Example: 64 The average emf induced in a coil in which a current changes from 0 to 2 *A* in 0.05 *s* is 8 *V*. The self inductance of the coil is [CPMT 1999]

(a) 0.1 *H* (b) 0.2 *H* (c) 0.4 *H* (d) 0.8 *H*

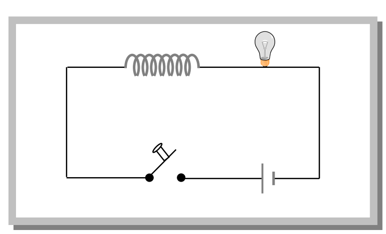
Solution: (b) By using $|e| = L\frac{di_1}{dt}$; $8 = L \times \frac{(2-0)}{0.05} \Rightarrow L = 0.2H$

Example: 65 A coil of *Cu* wire (radius-*r*, self inductance-*L*) is bent in two concentric turns each having radius is $\frac{r}{2}$. The self inductance now

(a) 2*L* (b) *L* (c) 4 *L* (d) *L* / 2

Solution: (a) $\because\ L \propto N^2 r$; $\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2 \times \frac{r_1}{r_2} \Rightarrow \frac{L}{L_2} = \left(\frac{1}{2}\right)^2 \times \left(\frac{r}{r/2}\right) = \frac{1}{2}$; L_2 = 2*L*

Example: 66 In the following circuit, the bulb will become suddenly bright if [CBSE 1989]



(a) Contact is made or broken

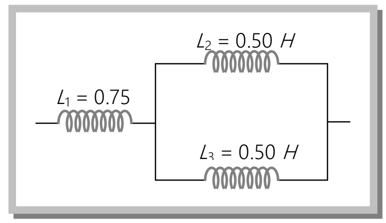
(b) Contact is made

(c) Contact is broken

(d) Won't becomes bright at all

Solution: (c) When contact is broken induced current flows in the same direction of main current. So bulb suddenly glows more brightly.

Example: 67 Three inductances are connected as shown below. Assuming no coupling, the resultant inductance will be



(a) 0.25 *H*

(b) 0.75 *H*

(c) 0.01 *H*

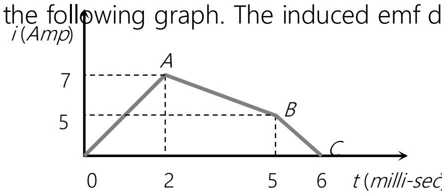
(d) 1 *H*

Solution: (d) L_2 and L_3 are in parallel. Thus their combination gives $L' = \dfrac{L_2 L_3}{L_2 + L_3} = 0.25\ H$

The L' and L_1 are in series, thus the equivalent inductance is $L = L_1 + L' = 0.75 + 0.25 = 1H$.

Tricky example: 7

The current through a 4.6 *H* inductor is shown on the following graph. The induced emf during the time interval *t* = 5 *milli-sec* to 6 *milli-sec* will be



(a) $10^3\ V$

(b) $-23 \times 10^3\ V$

(c) $23 \times 10^3\ V$

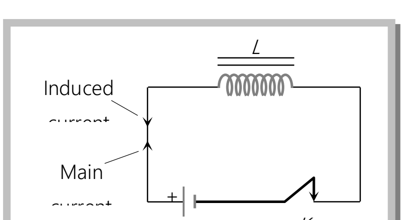
(d) Zero

Solution : (c) Rate of decay of current between *t* = 5 *ms* to 6 *ms* $= \dfrac{di}{dt} = -(\text{Slope of the line } BC)$

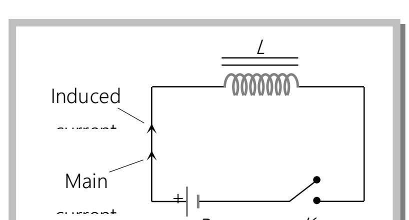
$= -\left(\dfrac{5}{1 \times 10^{-3}}\right) = -5 \times 10^3\ A/s.$ Hence induced emf $e = -L\dfrac{di}{dt} = -4.6 \times (-5 \times 10^3) = 23 \times 10^3\ V.$

Growth and Decay of Current in *LR*-Circuit

If a circuit containing a pure inductor *L* and a resistor *R* in series with a battery and a key then on closing the circuit current through the circuit rises exponentially and reaches up to a certain maximum value (steady state). If circuit is opened from it's steady state condition then current through the circuit decreases exponentially.



Growth of current



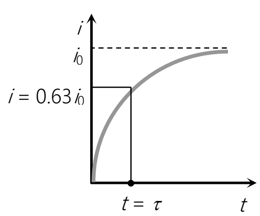
Decay of current

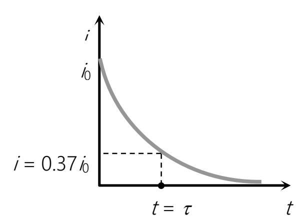
The value of current at any instant of time t after closing the circuit (*i.e.* during the rising of current) is given by $\boldsymbol{i = i_0\left[1 - e^{-\frac{R}{L}t}\right]}$; where $i_0 = i_{\max} = \dfrac{E}{R}$ = steady state current.

The value of current at any instant of time t after opening from the steady state condition (*i.e.* during the decaying of current) is given by $\boldsymbol{i = i_0 e^{-\frac{R}{L}t}}$

(1) **Time constant (τ)**

In this circuit $\tau = \dfrac{L}{R}$; It's unit is *second*. In other words the time interval, during which the current in an inductive circuit rises to 63% of its maximum value at make, is defined as time constant or it is the time interval, during which the current after opening an inductive circuit falls to 37% of its maximum value.





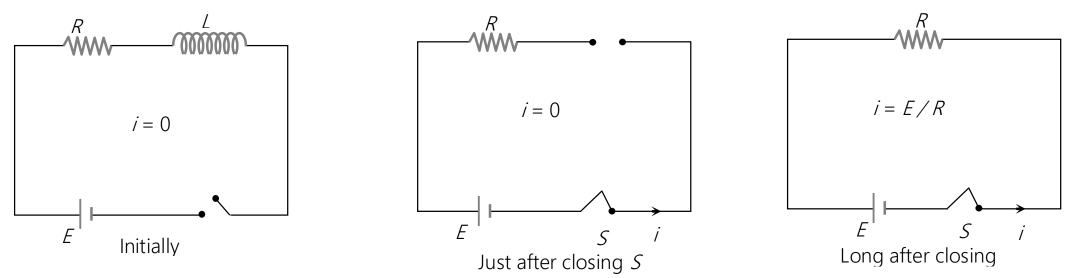
Note : ❒ The dimensions of $\dfrac{L}{R}$ are same as those of time *i.e.* $M^0L^0T^1$

❒ *Half life* (T) : In this time current reduces to 50% of its initial max value *i.e.* if $t = T$ then $i = \dfrac{i_0}{2}$ and again half life obtained as $T = 0.693\ \dfrac{L}{R}$ or T = 70% of time constant.

Now from $U = \frac{1}{2}Li^2$ so in half life time current changes from $i_0 \to \frac{i_0}{2}$ hence energy changes from $U_0 \to \frac{U_0}{4}$

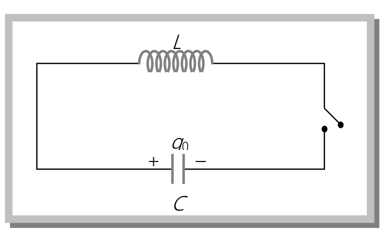
(2) **Behaviour of inductor**

The current in the circuit grows exponentially with time from 0 to the maximum value $i\left(=\frac{E}{R}\right)$. Just after closing the switch as i = 0, inductor act as open circuit *i.e.* broken wires and long after the switch has been closed as $i = i_0$, the inductor act as a short circuit *i.e.* a simple connecting wire.



LC Oscillation

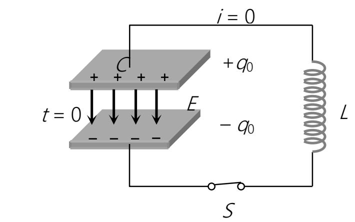
When a charged capacitor C having an initial charge q_0 is discharged through an inductance L, the charge and current in the circuit start oscillating simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat. We also assume an idealized situation in which energy is not radiated away from the circuit. The total energy associated with the circuit is constant.



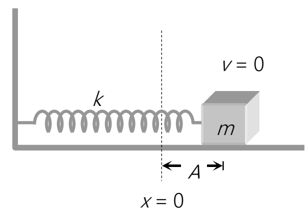
Frequency of oscillation is given by

$\omega = \frac{1}{\sqrt{LC}} \frac{rad}{sec}$ or $\nu = \frac{1}{2\pi\sqrt{LC}} Hz$

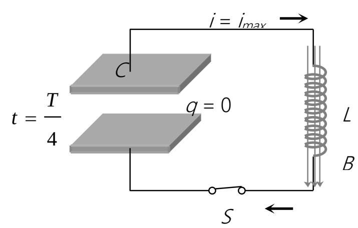
The oscillation of the *LC* circuit are an electromagnetic analog to the mechanical oscillation of a block-spring system.



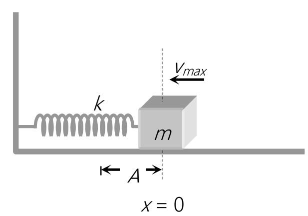
At $t = 0$, capacitor is ready to discharge



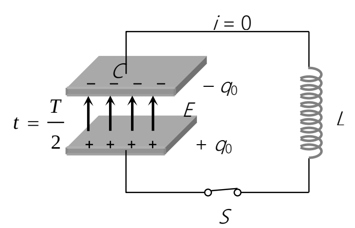
At $t = 0$, block is ready to move



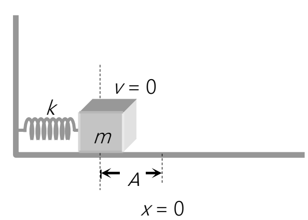
At $t = \frac{T}{4}$, capacitor is fully discharged *i.e.* charge $q = 0$ and current through the circuit



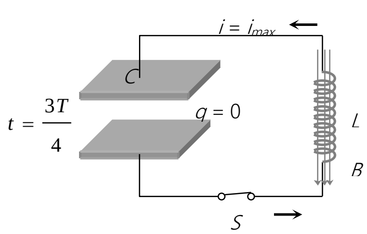
At $t = \frac{T}{4}$, block comes in it's mean position *i.e.* $x = 0$and velocity of block becomes



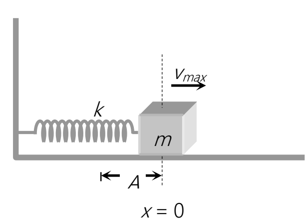
At $t = \frac{T}{2}$, capacitor is again recharged with reverse polarity and $i = 0$



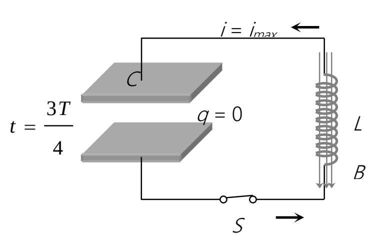
At $t = \frac{T}{2}$, block reaches it's extreme position other side and $v = 0$



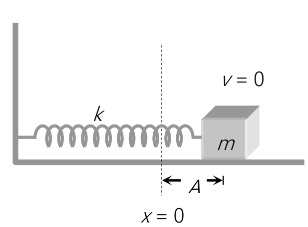
At $t = \frac{3T}{4}$, capacitor again discharges completely $i = i_{max}$



At $t = \frac{3T}{4}$, block again reaches it's mean position and it's velocity becomes maximum



At $t = \frac{3T}{4}$, capacitor again discharges completely $i = i_{max}$



At $t = \frac{3T}{4}$, block again reaches it's mean position and it's velocity becomes maximum

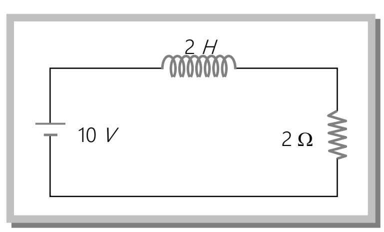
Concepts

☞ *Comparison of oscillation of a mass spring system and an LC circuit*

Mass spring system	*v/s*	*LC circuit*
Displacement (x)		*Charge (q)*
Velocity (v)		*Current (i)*
Acceleration (a)		*Rate of change of current* $\left(\frac{di}{dt}\right)$
Mass (m) [Inertia]		*Inductance (L) [Inertia of electricity]*
Momentum (p = mv)		*Magnetic flux (* $\phi = Li$ *)*
Retarding force $\left(-m\frac{dv}{dt}\right)$		*Self induced emf* $\left(-L\frac{di}{dt}\right)$
Equation of free oscillations: $\frac{d^2x}{dt^2} = -\omega^2 x$; *where* $\omega = \sqrt{\frac{K}{m}}$		*Equation of free oscillations :* $\frac{d^2q}{dt^2} = -\left(\frac{1}{LC}\right).q$; *where* $\omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$
Force constant K,		Capacitance *C*
Kinetic energy $= \frac{1}{2}mv^2$		*Magnetic energy* $= \frac{1}{2}Li^2$
Elastic potential energy $= \frac{1}{2}Kx^2$		*Electrical potential energy* $= \frac{1}{2}\frac{q^2}{C}$

Examples

Example: 68 In the figure magnetic energy stored in the coil is [RPET 2000]



(a) Zero

(b) Infinite

(c) 25 *J*

(d) None of these

Solution: (c) $U = \frac{1}{2}Li^2 = \frac{1}{2} \times 2\left(\frac{10}{2}\right)^2 = 25\ J$

Example: 69 An emf of 15 *volt* is applied in a circuit containing 5 *henry* inductance and 10 *ohm* resistance. The ratio of the currents at time $t = \infty$ and at $t = 1$ *second* is [MP PMT 1994]

(a) $\dfrac{e^{1/2}}{e^{1/2}-1}$ (b) $\dfrac{e^{2}}{e^{2}-1}$ (c) $1-e^{-1}$ (d) e^{-1}

Solution: (b) By using $i = i_0\left(1-e^{-\frac{Rt}{L}}\right)$; At $t = \infty$, $i = i_0$ and at $t = 1$ *sec* $i = i_0\left(1-e^{-\frac{10\times 1}{5}}\right)$; $i = i_0(1-e^{-2}) = i_0\left(\dfrac{e^2-1}{e^2}\right)$;

$\dfrac{i_0}{i} = \dfrac{e^2}{e^2-1}$

Example: 70 An ideal coil of 10 *henry* is joined in series with a resistance of 5 *ohm* and a battery of 5 *volt*. 2 *second* after joining, the current flowing in *ampere* in the circuit will be [MP PET 1995]

(a) e^{-1} (b) $(1-e^{-1})$ (c) $(1-e)$ (d) e

Solution: (b) By using $i = i_0\left(1-e^{-\frac{Rt}{L}}\right)$; $i = \dfrac{5}{5}\left(1-e^{-\frac{5\times 2}{10}}\right)$ or $i = (1-e^{-1})$

Example: 71 A coil of self inductance 50 *henry* is joined to the terminals of a battery of emf 2 *volts* through a resistance of 10 *ohm* and a steady current is flowing through the circuit. If the battery is now disconnected, the time in which the current will decay to $1/e$ of its steady value is

(a) 500 *seconds* (b) 50 *seconds* (c) 5 *seconds* (d) 0.5 *seconds*

Solution: (c) In decaying if $t = \tau = \dfrac{L}{R}$ current becomes $\dfrac{1}{e}$ times of it's initial value i.e. i_0. So $t = \dfrac{50}{10} = 5\, sec.$

Example: 72 A solenoid has an inductance of 50 *mH* and a resistance of 0.025 Ω. If it is connected to a battery, how long will it take for the current to reach one half of its final equilibrium value

(a) 1.34 *ms* (b) 1.2 *ms* (c) 6.32 *ms* (d) 0.23 *ms*

Solution: (a) $i = i_0\left(1-e^{-\frac{t}{\tau}}\right)$ where $i = \dfrac{1}{2}i_0$ and $\tau = \dfrac{L}{R}$. Thus $\dfrac{1}{2}i_0 = i_0\left(1-e^{-\frac{t}{\tau}}\right)$ or $\dfrac{1}{2} = e^{-\frac{t}{\tau}}$ or $2 = e^{+\frac{t}{\tau}}$

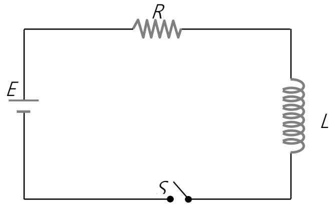
Thus $t = \tau \log_e 2 = \dfrac{50\times 10^{-3}}{0.025}\times 0.693 = 1.34\times 10^{-3}\, s = 1.34$ *milli second*.

Example: 73 A 50 *volt* potential difference is suddenly applied to a coil with $L = 5\times 10^{-3}$ *henry* and $R = 180$ *ohm*. The rate of increase of current after 0.001 *second* is

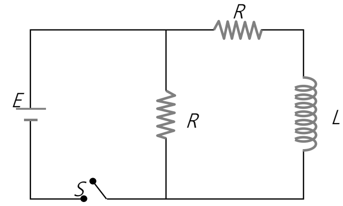
(a) 27.3 *amp/sec* (b) 27.8 *amp/sec* (c) 2.73 *amp/sec* (d) None of these

Solution: (d)

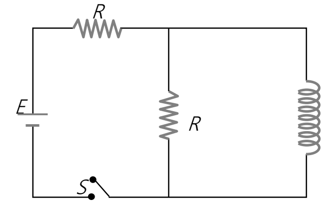
Example: 74 In which of the following circuit is the current maximum just after the switch *S* is closed



(i)



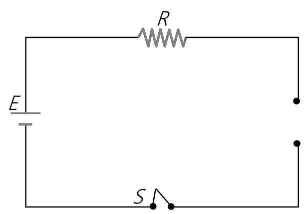
(ii)



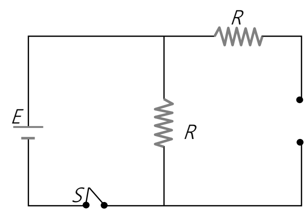
(iii)

(a) (i) (b) (ii) (c) (iii) (d) Both (ii) and (iii)

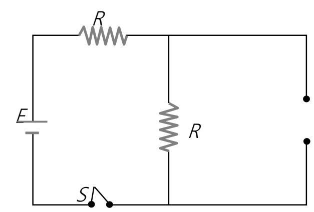
Solution: (b) At $t = 0$ current through L is zero so it acts as open circuit. The given figures can be redrawn as follow.



(i)



(ii)



(iii)

$$i_1 = 0 \qquad i_2 = \frac{E}{R} \qquad i_3 = \frac{E}{2R}$$

Hence $i_2 > i_3 > i_1$

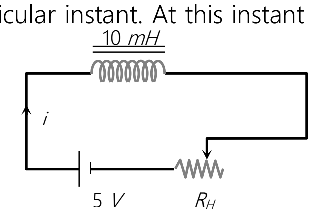
Example: 75 An oscillator circuit consists of an inductance of 0.5 mH and a capacitor of $20\,\mu F$. The resonant frequency of the circuit is nearly [Kerala PET 2002]

(a) 15.92 Hz (b) 159.2 Hz (c) 1592 Hz (d) 15910 Hz

Solution: (b)

Tricky example: 8

The resistance in the following circuit is increased at a particular instant. At this instant the value of resistance is 10Ω. The current in the circuit will be now



(a) $i = 0.5\ A$

(b) $i > 0.5\ A$

(c) $i < 0.5\ A$

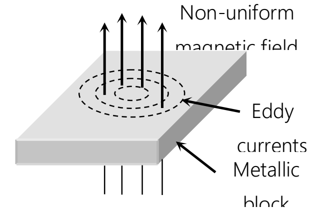
(d) $i = 0$

Solution : (b) If resistance is constant (10Ω) then steady current in the circuit $i = \frac{5}{10} = 0.5\ A$. But resistance is increasing it means current through the circuit start decreasing. Hence inductance comes in picture which induces a current in the circuit in the same direction of main current. So $i > 0.5\ A$.

Application of EMI

(1) **Eddy current**

When a changing magnetic flux is applied to a bulk piece of conducting material then circulating currents called eddy currents are induced in the material. Because the resistance of the bulk conductor is usually low, eddy currents often have large magnitudes and heat up the conductor.



These are circulating currents like eddies in water

Experimental concept given by Focault hence also named as “Focault current”

(i) **Disadvantages of eddy currents**

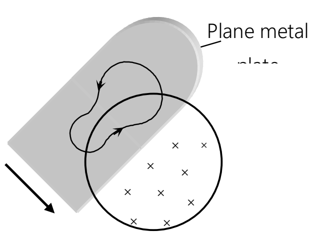
(a) The production of eddy currents in a metallic block leads to the loss of electric energy in the form of heat.

(b) The heat produced due to eddy currents breaks the insulation used in the electrical machine or appliance.

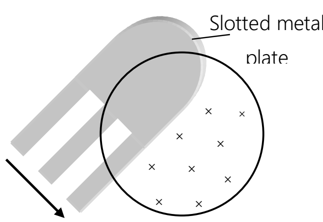
(c) Eddy currents may cause unwanted damping effect.

(ii) **Minimisation of losses due to eddy currents**

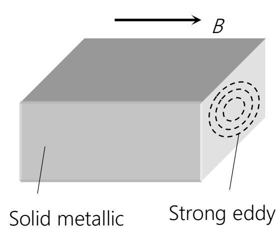
By Lamination, slotting processes the resistance path for circulation of eddy current increases, resulting in to weakening them and also reducing losses causes by them (slots and lamination intercept the conducting paths and decreases the magnitude of eddy currents and reduces possible paths of eddy currents)

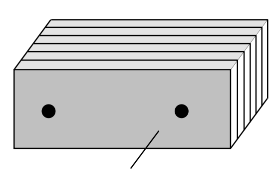


Strong eddies produced
Cause excessive electro



Feeble eddies
Gradual damping





Laminated core
(Feeble eddy currents)

(iii) **Application of eddy currents :** Though most of the times eddy currents are undesirable but they find some useful applications as enumerated below

(a) **Dead-beat galvanometer :** A dead beat galvanometer means one whose pointer comes to rest in the final equilibrium position immediately without any oscillation about the equilibrium position when a current is passed in its coil.

We know that the coil of a moving coil galvanometer is wound over a light aluminium frame. When the coil moves due to the torque produced by the current being measured, the aluminium frame also moves in the field. As a result the flux associated with the frame changes and eddy currents are induced in the frame. Eddy currents induced in aluminium frame as per Lenz's law always oppose the cause that produces them. Hence they damp the oscillation about the final steady position.

(b) **Electric-brakes :** When the train is running its wheel is moving in air and when the train is to be stopped by electric breaks the wheel is made to move in a field created by electromagnet. Eddy currents induced in the wheels due to the changing flux oppose the cause and stop the train.

(c) **Induction furnace :** Here a large amount of heat is to be generated so as to melt metal in it. To produce such a large amount of heat, a solid core of the furnace is taken (as against laminated core in situations where the heat produced is to be minimized).

(d) **Speedometer :** In the speedometer of an automobile, a magnet is geared to the main shaft of the vehicle and it rotates according to the speed of the vehicle. The magnet is mounted in an aluminium cylinder with the help of hair springs. When the magnet rotates, it produces eddy currents in the drum and drags it through an angle, which indicates the speed of the vehicle on a calibrated scale.

(e) **Diathermy :** Eddy currents have been used for deep heat treatment called diathermy.

(f) **Energy meter :** In energy meters, the armature coil carries a metallic aluminium disc which rotates between the poles of a pair of permanent horse shoe magnets. As the armature rotates, the current induced in the disc tends to oppose the motion of the armature coil. Due to this braking effect, deflection is proportional to the energy consumed.

(2) **dc motors**

It is an electrical machine which converts electrical energy into mechanical energy.

(i) **Principle :** It is based on the fact that a current carrying coil placed in the magnetic field experiences a torque. This torque rotates the coil.

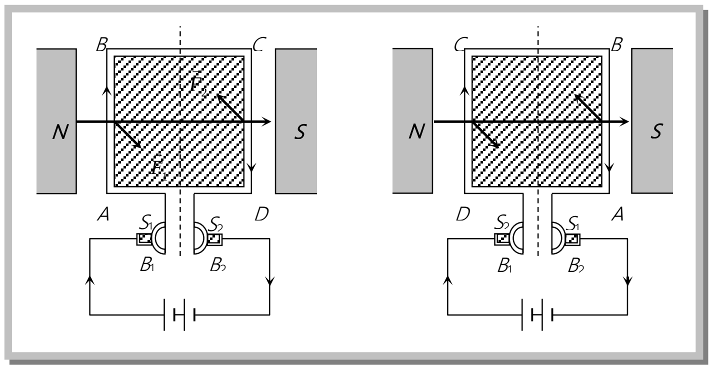
(ii) **Construction :** It consists of the following components figure.

ABCD = Armature coil

S_1, S_2 = split ring comutators

B_1, B_2 = Carbon brushes

N, *S* = Strong magnetic poles



(iii) **Working :** Force on any arm of the coil is given by $\vec{F} = i(\vec{l} \times \vec{B})$ in fig., force on *AB* will be perpendicular to plane of the paper and pointing inwards. Force on *CD* will be equal and opposite. So coil rotates in clockwise sense when viewed from top in fig. The current in *AB* reverses due to commutation keeping the force on *AB* and *CD* in such a direction that the coil continues to rotate in the same direction.

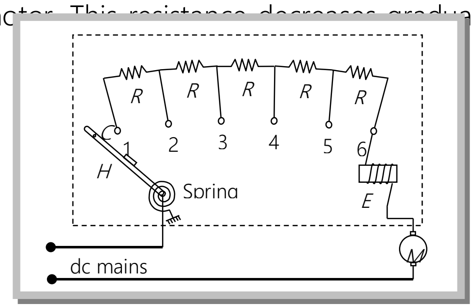
(iv) **Back emf in motor :** When the armature coil rotates in the magnetic field, an induced emf is set up in its windings. According to Lenz's law, this induced emf opposes the motion of the coil and its direction is opposite to the applied emf in the motor circuit. Hence the induced emf is known as back emf $\boldsymbol{e = E - iR}$

Value of back emf directly depends upon the angular velocity ω of armature and magnetic field *B*. But for constant magnetic field *B*, value of back emf *e* is given by $e \propto \omega$ or $\boldsymbol{e = k\omega}$ $(e = NBA\omega \sin \omega t)$

Let *e* = Magnitude of induced emf, *E* = Magnitude of the supply voltage, *R* = Resistance of the armature coil, *i* = Current in the armature. According to Ohm's law $i = \frac{E + (-e)}{R} = \frac{E - e}{R}$ or $iR = E - e$

(v) **Current in the motor :** $i = \frac{E-e}{R} = \frac{E-k\omega}{R}$; When motor is just switched on *i.e.* $\omega = 0$ so $e = 0$ hence $i = \frac{E}{R} = \text{maximum}$ and at full speed, ω is maximum so back emf e is maximum and i is minimum. Thus, maximum current is drawn when the motor is just switched on which decreases when motor attains the speed.

Hence a starter is used for starting a dc motor safely. Its function is to introduce a suitable resistance in the circuit at the time of starting of the motor. This resistance decreases gradually and reduces to zero when the motor runs at full sped.



The value of starting resistance is maximum at time t = 0 and its value is controlled by spring and electromagnetic system and is made to zero when the motor attains its safe speed.

Note : ❒ Small motor tends to have higher resistance then the large ones and do not normally need a starter.

(vi) **Mechanical power and Efficiency of dc motor :** Power supplied to the motor, $P_{in} = Ei$

and the power dissipated in the form of heat = i^2R

So remaining power = $Ei - i^2R$. This power is known as the mechanical power developed in the motor.

Hence mechanical power, $P_{mech.} = (E - iR)\, i = ei$

Efficiency of dc motor $\eta = \frac{P_{mechanical}}{P_{supplied}} = \frac{P_{out}}{P_{in}} = \frac{e}{E} = \frac{\text{Back e.m.f.}}{\text{Supply voltage}}$

Note : ❒ η will be maximum if ei = maximum. which obtained when $e = \frac{E}{2}$. So $\eta_{max.} = \frac{E/2}{E} \times 100 = 50\%$

(vii) **Uses of dc motors :** They are used in electric locomotives, electric ears, rolling mills, electric cranes, electric lifts, dc drills, fans and blowers, centrifugal pumps and air compressors, *etc*.

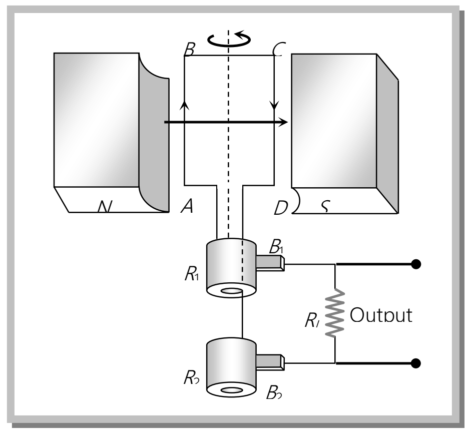
(3) **ac generator/Alternator/Dynamo**

An electrical machine used to convert mechanical energy into electrical energy is known as ac generator/alternator.

(i) **Principle :** It works on the principle of electromagnetic induction *i.e.*, when a coil is rotated in uniform magnetic field, an induced emf is produced in it.

(ii) **Construction :** The main components of ac generator are

(a) **Armature :** Armature coil (*ABCD*) consists of large number of turns of insulated copper wire wound over a soft iron core.



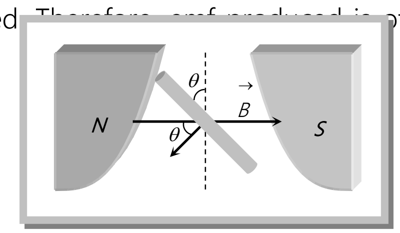
(b) **Strong field magnet :** A strong permanent magnet or an electromagnet whose poles (*N* and *S*) are cylindrical in shape in a field magnet. The armature coil rotates between the pole pieces of the field magnet. The uniform magnetic field provided by the field magnet is perpendicular to the axis of rotation of the coil.

(c) **Slip rings :** The two ends of the armature coil are connected to two brass slip rings R_1 and R_2. These rings rotate along with the armature coil.

(d) **Brushes :** Two carbon brushes (B_1 and B_2), are pressed against the slip rings. The brushes are fixed while slip rings rotate along with the armature. These brushes are connected to the load through which the output is obtained.

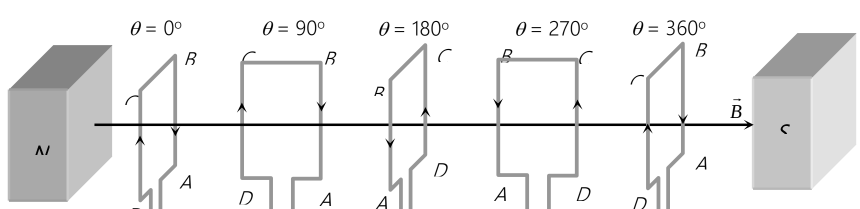
(iii) **Working :** When the armature coil *ABCD* rotates in the magnetic field provided by the strong field magnet, it cuts the magnetic lines of force. Thus the magnetic flux linked with the coil changes and hence induced emf is set up in the coil. The direction of the induced emf or the current in the coil is determined by the Fleming's right hand rule.

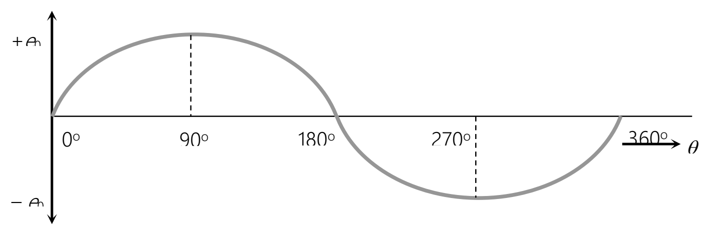
The current flows out through the brush B_1 in one direction of half of the revolution and through the brush B_2 in the next half revolution in the reverse direction. This process is repeated. Therefore, emf produced is of alternating nature.



$$e = -\frac{Nd\phi}{dt} = NBA\omega \sin \omega t = e_0 \sin \omega t \quad \text{where } e_0 = NBA\omega$$

$$i = \frac{e}{R} = \frac{e_0}{R}\sin \omega t = i_0 \sin \omega t \qquad R \to \text{Resistance of the circuit}$$





Note : ❐ Frequency of ac produced given by $[f_{AC}] = \frac{NP}{2}$, where P = Number of magnetic poles of field, N = Rotational frequency of armature coil in *rps* (rotations per *seconds*)

❐ For (a) Simple generator $P = 2 \Rightarrow f_{ac} = N$ (b) Multiple generator $P > 2 \Rightarrow f_{ac} > N$

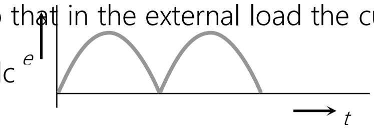
❐ To produce ac of given frequency, multiple generator is prove to be economical.

(4) **dc generator**

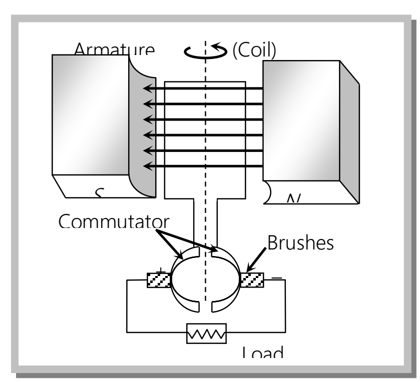
If the current produced by the generator is direct current, then the generator is called dc generator.

dc generator consists of (i) Armature (coil) (ii) Magnet (iii) Commutator (iv) Brushes

In dc generator commutator is used in place of slip rings. The commutator rotates along with the coil so that in every cycle when direction of '*e*' reverses, the commutator also reverses or makes contact with the other brush so that in the external load the current remains in the some direction giving dc



(Output of a single loop dc generator for one cycle of rotation)



Note : ❒ Practical efficiencies of big generators are about 92% to 95%.

Concepts

- *dc motor is a highly versatile energy conversion device. It can meet the demand of loads requiring high starting torque, high accelerating and decelerating torque.*
- *Constructionally there is no basic difference between a dc generator and a dc motor. Infect the same dc machine can be used interchangeably as a generator or as a motor.*
- *All rating marked on dynamos and motors are for full loads. For example a 5 kW, 100 V, 1000 rpm dynamo delivers 5 kW electrical power at 100 V terminal voltage and it's speed of rotation at full load is 1000 rpm.*

Examples

Example: 76 The armature of dc motor has 20 Ω resistance. It draws current of 1.5 *ampere* when run by 220 *volts* dc supply. The value of back emf induced in it will be [MP PMT 1999]

(a) 150 *V* (b) 170 *V* (c) 180 *V* (d) 190

Solution: (d) $e = E - iR = 220 - 1.5 \times 20 = 190\ V$.

Example: 77 A simple electric motor has an armature resistance of one *ohm* and runs from a dc source of 12 *volts*. When unloaded it draws a current of 2 *amperes*. When a certain load is connected, its speed becomes one-half of its unloaded value. Then the current in *ampere* it draws is

(a) 7 *amp* (b) 6 *amp* (c) 2 *amp* (d) 4 *amp*

Solution: (a) Back emf $e \propto$ speed, $e = E - iR = 12 - 2 \times 1 = 10\ V$

$$e' = \frac{e}{2} = E - i'R \Rightarrow 5 = 12 - i' \times 1 \Rightarrow i' = 7\ amp.$$

Example: 78 If the rotational velocity of a dynamo armature is doubled, then the induced emf will

(a) Become half (b) Become double (c) Become quadruple (d) Remain unchanged

Solution: (b) $e \propto \omega$ when ω doubles, 'e' gets doubled.

Example: 79 In an ac dynamo, the peak value of emf is 60 *volts*, then the induced emf in the position, when armature makes an angle of 30° with the magnetic field perpendicular with the coil, will be

(a) 20 *volts* (b) $30\sqrt{3}$ *volts* (c) 30 *volts* (d) 45 *volts*

Solution: (c) $e = e_0 \sin \omega t = e_0 \sin \theta = 60 \sin 30° = 30$ *volts*

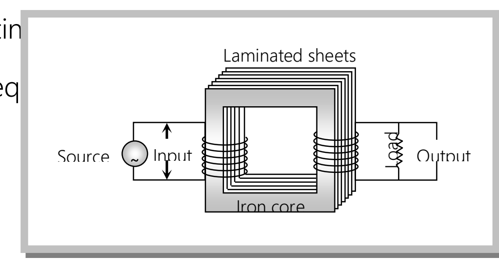
Example: 80 In an ac dynamo, the number of turns in the armature are made four times and the angular velocity 9 times, then the peak value of induced emf will become

(a) 36 times (b) 12 times (c) 6 times (d) 18 times

Solution: (a) $e = e_0 \sin \omega t$ where $e_0 = \omega NBA = e_0' = (9\omega)(4N)BA = 36\, e_0$

Transformer

It is a device which raises or lowers the voltage in ac circuits through mutual induction. It consists of two coils wound on the same core. The coil which is connected to the source (*i.e.*, to which input is applied) is called primary while the other which is connected to the load (*i.e.*, from which output is taken) is called secondary. The alternating current passing through the primary creates a continuously changing flux through the core. This changing flux induces an alternatin es of force are closed curves, the flux per turn of the primary must be eq *i.e.*,



(i) Transformer works on ac only and never on dc.

(ii) It can increase or decrease either voltage or current but not both simultaneously.

(iii) Transformer does not change the frequency of input ac.

(iv) There is no electrical connection between the winding but they are linked magnetically.

(v) Effective resistance between primary and secondary winding is infinite.

(vi) The flux per turn of each coil must be same *i.e.* $\phi_S = \phi_S$; $-\frac{d\phi_S}{dt} = -\frac{d\phi_P}{dt}$

(vii) If Suppose for a transformer –

N_P = number of turns in primary ; N_S = number of turns in secondary

V_P = applied (input) voltage to primary; V_S = Voltage across secondary (load voltage or output)

e_P = induced emf in primary ; e_S = induced emf in secondary

ϕ = flux linked with primary as well as secondary

i_P = current in primary; i_S = current in secondary (or load current)

R_P = resistance of primary; R_S = resistance of secondary

t_P = thickness of turn in primary; t_S = thickness of turn in secondary

As in an ideal transformer there is no loss of power *i.e.* $P_{out} = P_{in}$ and $e = V$

So $V_S i_S = V_P i_P$ and $V_P \approx e_P$, $V_S \approx e_S$

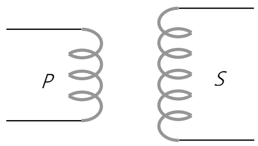
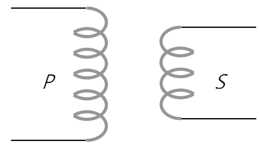
According to Faraday's law $e_S = -N_S \frac{d\phi}{dt}, e_P = -N_P \frac{d\phi}{dt}$

Hence $\frac{\boldsymbol{e_S}}{\boldsymbol{e_P}} = \frac{\boldsymbol{N_S}}{\boldsymbol{N_P}} = \frac{\boldsymbol{V_S}}{\boldsymbol{V_P}} = \frac{\boldsymbol{i_P}}{\boldsymbol{i_S}} = \boldsymbol{k}$; k = Transformation ratio (or turn ratio)

From above discussions, it is clear that in transformers the side having greater number of turns will have greater voltage and lesser current. Since in increasing the voltage level, the current level decreases, therefore it can be concluded that voltage increases at the cost of current.

(viii) **Types of transformer** : Transformer is of two type

Step up transformer	Step down transformer

It increases voltage and decreases current	It decreases voltage and increases current
$V_S > V_P$	$V_S < V_P$
$N_S > N_P$	$N_S < N_P$
$E_S > E_P$ 	$E_S < E_P$ 
$i_S < i_P$	$i_S > i_P$
$R_S > R_P$	$R_S < R_P$
$t_S > t_P$	$t_S > t_P$
$k > 1$	$k < 1$

(ix) **Efficiency of transformer (η) :** Efficiency is defined as the ratio of output power and input power

i.e. $$\eta\% = \frac{P_{out}}{P_{in}} \times 100 = \frac{V_S i_S}{V_P i_P} \times 100$$

For an ideal transformer $P_{out} = P_{in}$ so $\eta = 100\,\%$ (But efficiency of practical transformer lies between 70% – 90%)

For practical transformer $P_{in} = P_{out} + P_{losses}$ so $\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{P_{out}}{(P_{out} + P_L)} \times 100 = \frac{(P_{in} - P_L)}{P_{in}} \times 100$

(x) **Losses in transformer :** In transformers some power is always lost due to, heating effect, flux leakage eddy currents, hysteresis and humming.

(a) ***Cu* loss ($i^2 R$) :** When current flows through the transformer windings some power is wasted in the form of heat $(H = i^2 Rt)$. To minimize this loss windings are made of thick *Cu* wires (To reduce resistance)

(b) **Iron loss :** If is further divided in two types

Eddy current loss : Some electrical power is wasted in the form of heat due to eddy currents, induced in core, to minimize this loss transformers core are laminated and silicon is added to the core material as it increases the resistivity. The material of the core is then called silicon-iron (steel).

Hystersis loss : The alternating current flowing through the coils magnetises and demagnetises the iron core again and again. Therefore, during each cycle of magnetisation, some energy is lost due to hysteresis. However, the loss of energy can be minimised by selecting the material of core, which has a narrow hysterisis loop. Therefore core of transformer is made of soft iron. Now a days it is made of "Permalloy" (*Fe*-22%, *Ni*-78%).

(c) **Magnetic flux leakage :** Magnetic flux produced in the primary winding is not completely linked with secondary because few magnetic lines of force complete their path in air only. To minimize this loss secondary winding is kept inside the primary winding.

(d) **Humming losses :** Due to the passage of alternating current, the core of the transformer starts vibrating and produces humming sound. Thus, some part (may be very small) of the electrical energy is wasted in the form of humming sounds produced by the vibrating core of the transformer.

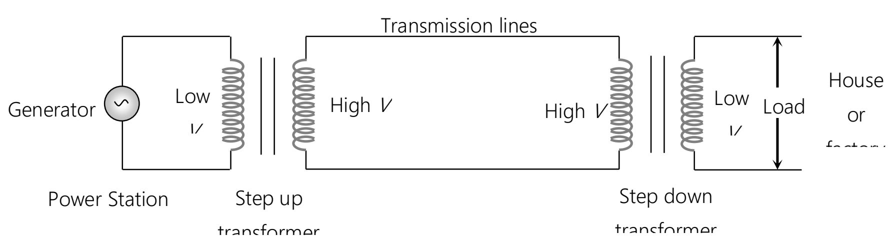
(xi) **Uses of transformer :** A transformer is used in almost all ac operations *e.g.*

(a) In voltage regulators for TV, refrigerator, computer, air conditioner etc.

(b) In the induction furnaces.

(c) Step down transformer is used for welding purposes.

(d) In the transmission of ac over long distance.



(e) Step down and step up transformers are used in electrical power distribution.

(f) Audio frequency transformers are used in radiography, television, radio, telephone *etc*.

(g) Radio frequency transformers are used in radio communication.

(h) Transformers are also used in impedance matching.

(xii) **Relation between primary and secondary resistances :** However if one end of primary and one end of secondary are connected together and a source of emf is connected across the two remaining ends, ohm's law can still be applied.

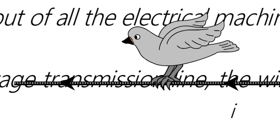
Thus if voltage across primary winding alone is increased, the primary current will increase. Similarly if voltage across the secondary winding alone is increased, the secondary current will increases. But interestingly in transformers the side having greater voltage has lesser current. We know that if voltage in high voltage (*H.V.*) winding is k times greater the current in it is k times smaller. It is possible only when the resistance of the *H.V.* winding is k^2 times the resistance of the low voltage (*L.V.*) winding. Thus, $R_{H.V.} = k^2 R_{L.V.}$ (where, $k > 1$)

Thus purposely the *H.V.* turns are kept thinner and larger in number.

Similarly the *L.V.* turns are kept thicker and lesser in number. This may be remembered by the fact that amount of copper used in making both *H.V.* and *L.V.* windings is same.

Concepts

- *When a source of emf is connected across the two ends of the primary winding alone or across the two ends of secondary winding alone, ohm's law can be applied. But in the transformer as a whole, ohm's law should not be applied because primary winding and secondary winding are not connected electrically.*
- *Even when secondary circuit of the transformer is open it also draws some current called no load primary current for supplying no load Cu and iron loses.*
- *Transformer has highest possible efficiency out of all the electrical machines.*
- *When current is passing through a high voltage transmission line, the wings of a bird sitting on it are repelled due to induction which makes it fly away.*



Examples

Example: 81 An ideal transformer has 500 and 5000 turns in primary and secondary windings respectively. If the primary voltage is connected to a 6V battery then the secondary voltage is [Orissa JEE 2003]

(a) 0 (b) 60 *V* (c) 0.6 *V* (d) 6.0 *V*

Solution: (a) Zero, because transformer works on *ac* only.

Example: 82 In a step-down transformer, the transformation ratio is 0.1, current in primary is 10 *mA*. The current in secondary is

(a) 10 *mA* (b) 1 *mA* (c) 1 *mA* (d) 0.1 *A*

Solution: (d) We know that, the transformation of current or voltage from primary to secondary or vice-versa in an ideal transformer takes place according to transformation ratio. Since, it is a step-down transformer, the turns in secondary are smaller in number. Hence current in secondary must be larger. Therefore the secndary current must be $\frac{1}{0.1}$ times the primary current. Hence $I_s = 10 \times 10\ mA = 100\ mA = 0.1\,amp$

Example: 83 How much current is drawn by primary of a transformer connected to 220 *V* supply, when it power to a 110 *V* and 550 *W* refrigerator

(a) 2.5 *A* (b) 0.4 *A* (c) 4 *A* (d) 25 *A*

Solution: (a) $V_p = 220\ V, V_s = 110\ V, V_s I_s = 550\ W,$ Now $V_p I_p = V_s I_s$ or $I_p = \frac{V_s I_s}{V_p} = \frac{550}{220} = 2.5A$

Example: 84 A step down transformer is connected to main supply 200 *V* to operate a 6 *V*, 30 *W* bulb. The current in primary is

(a) 3 *amp* (b) 1.5 *amp* (c) 0.3 *amp* (d) 0.15 *amp*

Solution: (d) $V_p = 200\ V,\ V_s = 6V \Rightarrow P_{out} = V_s i_s \Rightarrow 30 = 6 \times i_s \Rightarrow i_s = 5A$

From $\frac{V_s}{V_p} = \frac{i_p}{i_s} \Rightarrow \frac{6}{200} = \frac{i_p}{5} \Rightarrow i_p = 0.15\,A$

Example: 85 An ideal transformer steps down 220 *V* to 22 *V* in order to operate a device with an impedance of 220 Ω. The current in the primary is

(a) 0.01 *A* (b) 0.1 *A* (c) 0.5 *A* (d) 1.0 *A*

Solution: (a) $V_p = 220\ V,\ V_s = 22\ V,\ R_s = 220\ \Omega$ secondary current $i_s = \frac{V_s}{R_s} = \frac{22}{220} = \frac{1}{10}\,amp$

So by using the relation $\frac{V_p}{V_s} = \frac{i_s}{i_p}, i_p = 0.01\ A$

Example: 86 Primary voltage is V_p, resistance of the primary winding is R_p. Turns in primary and secondary are respectively N_p and N_s then secondary current in terms of primary voltage and secondary voltage respectively will be

(a) $\frac{V_p N_p}{R_p N_s}, \frac{V_s N_p^2}{R_p N_s^2}$ (b) $\frac{V_p N_p^2}{R_p N_s}, \frac{V_s^2 N_p^2}{R_p N_s^2}$ (c) $\frac{V_p N_p}{R_p^2 N_s}, \frac{V_s N^2}{R_p^2 N_s^2}$ (d) $\frac{V_p N_p^2}{R_p N_s^2}, \frac{V_s^2 N_p}{R_p^2 N_s}$

Solution: (a) $\frac{i_s}{i_p} = \frac{N_p}{N_s}$ Now, according to the information given in the problem, i_p can be calculated by using the formula, $V = iR$ so $i_s = \frac{V_p}{R_p} \times \frac{N_p}{N_s}$ (This is the secondary current in terms of V_p)

Now to rearrange the result obtained above, in terms of secondary voltage, we must replace the term of V_p in the above result by V_s. We know that $\frac{V_p}{V_s} = \frac{N_p}{N_s}$; $V_p = \frac{V_s N_p}{N_s}$, Substituting this in equation (i)

$$i_s = \frac{V_s}{R_p} \frac{N_p^2}{N_s^2}$$

Example: 87 A transformer is used to light 140 *watt*, 24 *volt* lamp from 240 *volts* ac mains. If the current in the mains is 0.7 *A*, then the efficiency of transformer is

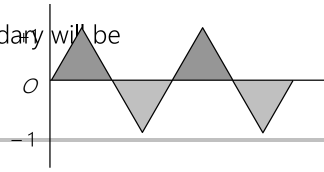
(a) 63.8% (b) 84% (c) 83.3% (d) 48%

Solution: (c) $P_{out} = V_s i_s = 140\ W, V_s = 24\ V, V_p = 240\ V, i_p = 0.7 A$

$$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{P_{out}}{V_p i_p} \times 100 = \frac{140}{240 \times 0.7} \times 100 = 83.3\%.$$

Tricky example: 9

An alternating current of frequency 200 *rad/sec* and peak value 1*A* as shown in the figure, is applied to the primary of a transformer. If the coefficient of mutual induction between the primary and the secondary is 1.5 *H*, the voltage induced in the secondary will be



(a) 300 *V*

(b) 191 V

(c) 220 V

(d) 471 V

Solution : (b) $e = -M\dfrac{di}{dt} = -1.5\dfrac{(1-0)}{(T/4)} = -\dfrac{6}{T}$

Also $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{200} = \dfrac{\pi}{100} \Rightarrow |e| = \dfrac{600}{\pi} = 190.9\ V \simeq 191\ V$