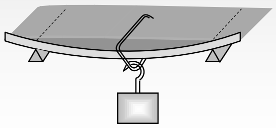
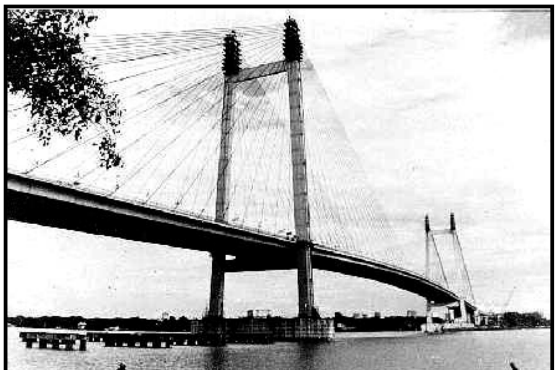


Chapter 9

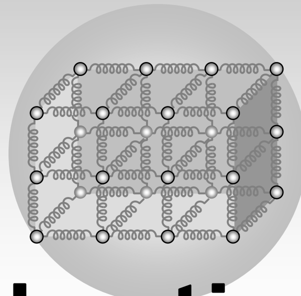
Elasticity

Contents



The bridges are designed in such a way that they do not bend much or break under the load of heavy traffic, force of strongly blowing wind and its own weight.

In a suspension bridge, there is a stretch in the ropes by the load of the bridge. Due to which length of rope changes, hence young's modulus of elasticity is involved.



Elasticity

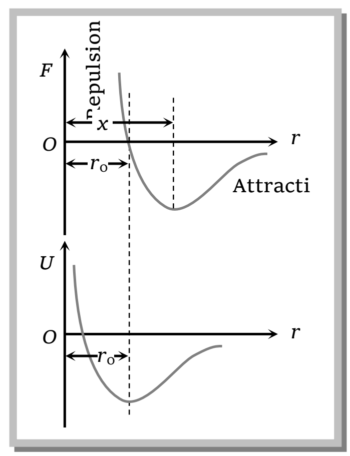
9.1 Interatomic Forces

The forces between the atoms due to electrostatic interaction between the charges of the atoms are called interatomic forces. These forces are electrical in nature and these are active if the distance between the two atoms is of the order of atomic size *i.e.* 10^{-10} *metre*.

(1) Every atom is electrically neutral, the number of electrons (negative charge) orbiting around the nucleus is equal to the number of proton (positive charge) in the nucleus. So if two atoms are placed at a very large distance from each other then there will be a very small (negligible) interatomic force working between them.

(2) When these two atoms are brought close to each other to a distance of the order of 10^{-10} *m*, the distances between their positive nuclei and negative electron clouds get disturbed, and due to this, attractive interatomic force is produced between two atoms.

(3) This attractive force increases continuously with decrease in r and becomes maximum for one value of r called critical distance, represented by x (as shown in the figure). Beyond this the attractive force starts decreasing rapidly with further decrease in the value of r.



(4) When the distance between the two atoms becomes r_0, the interatomic force will be zero. This distance r_0 is called normal or equilibrium distance.

(r_0 = 0.74 Å for hydrogen).

(5) When the distance between the two atoms further decreased, the interatomic force becomes repulsive in nature and increases very rapidly with decrease in distance between two atoms.

(6) The potential energy U is related with the interatomic force F by the following relation.

$$F = \frac{-dU}{dr}$$

(i) When two atoms are at very large distance, the potential energy is negative and becomes more negative as r is decreased.

(ii) When the distance between the two atoms becomes r_0, the potential energy of the system of two atoms becomes minimum (*i.e.* attains maximum negative value). As the state of minimum potential energy is the state of equilibrium, hence the two atoms at separation r_0 will be in a state of equilibrium.

($U_0 = -7.2 \times 10^{-19}$ *Joule* for hydrogen).

(iii) When the distance between the two atoms is further decreased (*i.e.* $r < r_0$) the negative value of potential energy of the system starts decreasing. It becomes zero and then attains positive value with further decrease in r (as shown in the figure).

9.2 Intermolecular Forces

The forces between the molecules due to electrostatic interaction between the charges of the molecules are called intermolecular forces. These forces are also called Vander Waal forces and are quite weak as compared to inter-atomic forces. These forces are also electrical in nature and these are active if the separation between two molecules is of the order of molecular size *i.e.* $\approx 10^{-9}$ *m*.

(1) It is found that the force of attraction between molecules varies inversely as seventh power of the distance between them *i.e.*

$$F_{att} \propto \frac{1}{r^7} \quad \text{or} \quad F_{att} = \frac{-a}{r^7}$$

The negative sign indicates that the force is attractive in nature.

(2) When the distance between molecules becomes less than r_0, the forces becomes repulsive in nature and is found to vary inversely as ninth power of the distance between them *i.e.*

$$F_{rep} \propto \frac{1}{r^9} \quad \text{or} \quad F_{rep} = \frac{b}{r^9}.$$

Therefore force between two molecules is given by $F = F_{att} + F_{rep} = \frac{-a}{r^7} + \frac{b}{r^9}$

The value of constants a and b depend upon the structure and nature of molecules.

(3) Intermolecular forces between two molecules has the same general nature as shown in the figure for interatomic forces.

(4) Potential Energy : Potential energy can be approximately expressed by the formula $U = \frac{A}{r^n} - \frac{B}{r^m}$

where the term $\frac{A}{r^n}$ represents repulsive contribution and term $\frac{B}{r^m}$ represents the attractive contribution. Constants A, B and numbers m and n are different for different molecules.

For majority of solids $n = 12$ and $m = 6$.

So potential energy can be expressed as $U = \frac{A}{r^{12}} - \frac{B}{r^6}$

9.3 Comparison Between Interatomic and Intermolecular Forces

(1) **Similarities**

(i) Both the forces are electrical in origin.

(ii) Both the forces are active over short distances.

(iii) General shape of force-distance graph is similar for both the forces.

(iv) Both the forces are attractive up to certain distance between atoms/molecules and become repulsive when the distance between them become less than that value.

(2) **Dissimilarities**

(i) Interatomic force depends upon the distance between the two atoms, whereas the intermolecular force depends upon the distance between the two molecules as well as their relative orientation.

(ii) Interatomic forces are about 50 to100 times stronger than intermolecular forces.

(iii) The value of r_0 for two atoms is smaller than the corresponding value for the molecules. Therefore one molecule is not restricted to attract only one molecule, but can attract many molecule. It is not so incase of atoms, since the atoms of one molecule cannot bind the atoms of other molecules.

9.4 States of Matter

The three states of matter differ from each other due to the following two factors.

(1) The different magnitudes of the interatomic and intermolecular forces.

(2) The extent of random thermal motion of atoms and molecules of a substance (which depends upon temperature).

Comparison Chart of Solid, Liquid and Gaseous States

Property	Solid	Liquid	Gas
Shape	Definite	Not definite	Not definite
Volume	Definite	Definite	Not definite
Density	Maximum	Less than solids but more than gases.	Minimum
Compressibility	Incompressible	Less than gases but more than solids.	Compressible
Crystallinity	Crystalline	Non-crystalline	
Interatomic or intermolecular distance	Constant	Not constant	Not constant

Relation between kinetic energy K and potential energy (U)	$K < U$	$K > U$	$K >> U$
Intermolecular force	Strongest	Less than solids but more than gases.	Weakest
Freedom of motion	Molecules vibrate about their mean position but cannot move freely.	Molecules have limited free motion.	Molecules are free to move.
Effect of temperature	Matter remains in solid form below a certain temperature.	Liquids are found at temperatures more than that of solid.	These are found at temperatures greater than that of solids and liquids.

Note : ❑ The fourth state of matter in which the medium is in the form of positive and negative ions, is known as plasma. Plasma occurs in the atmosphere of stars (including the sun) and in discharge tubes.

9.5 Types of Solids

A solid is that state of matter in which its constituent atoms or molecules are held strongly at the position of minimum potential energy and it has a definite shape and volume. The solids can be classified into two categories, crystalline and glassy or amorphous solids.

Comparison chart of Crystalline and Amorphous Solids

Crystalline solids	**Amorphous or glassy solids**
The constituent atoms, ions or molecules are arranged in a regular repeated three dimensional pattern, within the solid.	The constituent atoms, ions or molecules are not arranged in a regular repeated three dimensional pattern, within the solid.
Definite external geometric shape.	No regularity in external shape.
All the bonds in ions, or atoms or molecules are equally strong.	All the bonds are not equally strong.
They are anisotropic.	They are isotropic.
They have sharp melting point.	They don't have no sharp melting point.
They have a long-range order of atoms or ions or molecules in them.	They don't have a long-range order.
They are considered true and stable solids.	They are not regarded as true and stable solids.

9.6 Elastic Property of Matter

(1) **Elasticity :** The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming force is called elasticity.

(2) **Plasticity :** The property of matter by virtue of which it does not regain its original shape and size after the removal of deforming force is called plasticity.

(3) **Perfectly elastic body :** If on the removal of deforming forces the body regain its original configuration completely it is said to be perfectly elastic.

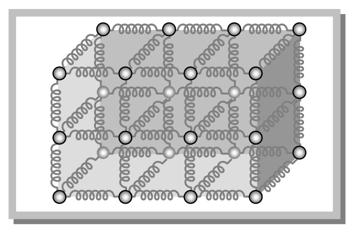
A quartz fibre and phosphor bronze (an alloy of copper containing 4% to 10% tin, 0.05% to 1% phosphorus) is the nearest approach to the perfectly elastic body.

(4) **Perfectly plastic body :** If the body does not have any tendency to recover its original configuration, on the removal of deforming force, it is said to be perfectly plastic.

Paraffin wax, wet clay are the nearest approach to the perfectly plastic body.

Practically there is no material which is either perfectly elastic or perfectly plastic and the behaviour of actual bodies lies between the two extremes.

(5) **Reason of elasticity :** In a solids, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to neighbouring molecules. These forces are known as intermolecular forces.



For simplicity, the two molecules in their equilibrium positions (at inter-molecular distance $r = r_0$) (see graph in article 9.1) are shown by connecting them with a spring.

In fact, the spring connecting the two molecules represents the inter-molecular force between them. On applying the deforming forces, the molecules either come closer or go far apart from each other and restoring forces are developed. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium position ($r = r_0$) and hence the body regains its original form.

(6) **Elastic limit :** Elastic bodies show their property of elasticity upto a certain value of deforming force. If we go on increasing the deforming force then a stage is reached when on removing the force, the body will not return to its original state. The maximum deforming force upto which a body retains its property of elasticity is called elastic limit of the material of body.

Elastic limit is the property of a body whereas elasticity is the property of material of the body.

(7) **Elastic fatigue :** The temporary loss of elastic properties because of the action of repeated alternating deforming force is called elastic fatigue.

It is due to this reason

(i) Bridges are declared unsafe after a long time of their use.

(ii) Spring balances show wrong readings after they have been used for a long time.

(iii) We are able to break the wire by repeated bending.

(8) **Elastic after effect :** The time delay in which the substance regains its original condition after the removal of deforming force is called elastic after effect. It is the time for which restoring forces are present after the removal of the deforming force it is negligible for perfectly elastic substance, like quartz, phosphor bronze and large for glass fibre.

9.7 Stress

When a force is applied on a body there will be relative displacement of the particles and due to property of elasticity an internal restoring force is developed which tends to restore the body to its original state.

The internal restoring force acting per unit area of cross section of the deformed body is called stress.

At equilibrium, restoring force is equal in magnitude to external force, stress can therefore also be defined as external force per unit area on a body that tends to cause it to deform.

If external force F is applied on the area A of a body then,

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

Unit : N/m^2 (S.I.) , $dyne/cm^2$ (C.G.S.)

Dimension : $[ML^{-1}T^{-2}]$

Stress developed in a body depends upon how the external forces are applied over it.

On this basis there are two types of stresses : Normal and Shear or tangential stress

(1) **Normal stress :** Here the force is applied normal to the surface.

It is again of two types : Longitudinal and Bulk or volume stress

(i) *Longitudinal stress*

(a) It occurs only in solids and comes in picture when one of the three dimensions *viz*. length, breadth, height is much greater than other two.

(b) Deforming force is applied parallel to the length and causes increase in length.

(c) Area taken for calculation of stress is area of cross section.

(d) Longitudinal stress produced due to increase in length of a body under a deforming force is called tensile stress.

(e) Longitudinal stress produced due to decrease in length of a body under a deforming force is called compressional stress.

(ii) *Bulk or Volume stress*

(a) It occurs in solids, liquids or gases.

(b) In case of fluids only bulk stress can be found.

(c) It produces change in volume and density, shape remaining same.

(d) Deforming force is applied normal to surface at all points.

(e) Area for calculation of stress is the complete surface area perpendicular to the applied forces.

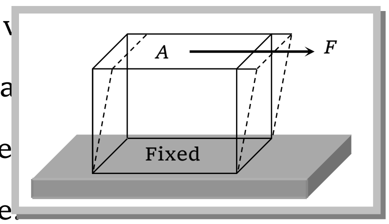
(f) It is equal to change in pressure because change in pressure is responsible for change in volume.

(2) **Shear or tangential stress :** It comes in picture when successive layers of solid move on each other *i.e.* when there is a relative displacement between v

(i) Here deforming force is applied tangential to one of the fa

(ii) Area for calculation is the area of the face on which force

(iii) It produces change in shape, volume remaining the same.



Difference between Pressure and Stress	
Pressure	**Stress**
Pressure is always normal to the area.	Stress can be normal or tangential.
Always compressive in nature.	May be compressive or tensile in nature.

Sample problems based on Stress

***Problem* 1.** A and B are two wires. The radius of A is twice that of B. they are stretched by the same load. Then the stress on B is

(a) Equal to that on A (b) Four times that on A

(c) Two times that on A (d) Half that on A

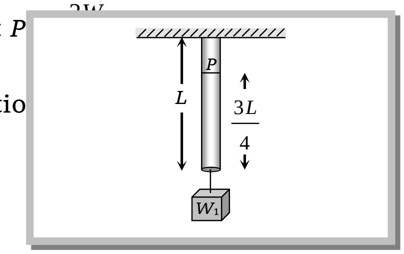
Solution : (b) Stress $= \frac{\text{Force}}{\text{Area}} = \frac{F}{\pi r^2}$

$\therefore$ Stress $\propto \frac{1}{r^2} \Rightarrow \frac{(\text{Stress})_B}{(\text{Stress})_A} = \left(\frac{r_A}{r_B}\right)^2 = (2)^2 \Rightarrow (\text{Stress})_B = 4 \times (\text{stress})_A$ [As F = constant]

***Problem* 2.** One end of a uniform wire of length L and of weight W is attached rigidly to a point in the roof and a weight W_1 is suspended from its lower end. If S is the area of cross-section of the wire, the stress in the wire at a height $3L/4$ from its lower end is

(a) $\frac{W_1}{S}$ (b) $\frac{W_1 + (W/4)}{S}$ (c) $\frac{W_1 + (3W/4)}{S}$ (d) $\frac{W_1 + W}{S}$

Solution : (c) As the wire is uniform so the weight of wire below point P



$\therefore$ Total force at point $P = W_1 + \frac{3W}{4}$ and area of cross-sectio

$\therefore$ Stress at point $P = \frac{\text{Force}}{\text{Area}} = \frac{W_1 + \frac{3W}{4}}{S}$

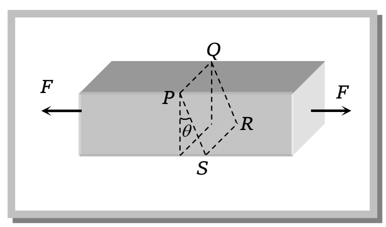
***Problem* 3.** On suspending a weight *Mg*, the length *l* of elastic wire and area of cross-section *A* its length becomes double the initial length. The instantaneous stress action on the wire is

(a) Mg/A (b) $Mg/2A$ (c) $2Mg/A$ (d) $4Mg/A$

Solution : (c) When the length of wire becomes double, its area of cross section will become half because volume of wire is constant $(V = AL)$.

So the instantaneous stress = $\frac{\text{Force}}{\text{Area}} = \frac{Mg}{A/2} = \frac{2Mg}{A}$.

***Problem* 4.** A bar is subjected to equal and opposite forces as shown in the figure. *PQRS* is a plane making angle θ with the cross-section of the bar. If the area of cross-section be '*A*', then what is the tensile stress on *PQRS*



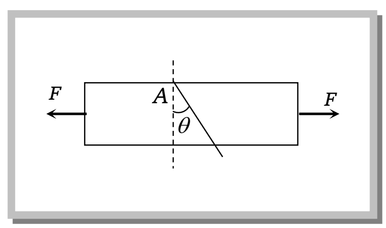
(a) F/A

(b) $F\cos\theta/A$

(c) $F\cos^2\theta/A$

(d) $F/A\cos\theta$

Solution : (c) As tensile stress = $\frac{\text{Normal force}}{\text{Area}} = \frac{F_N}{A_N}$



and here $A_N = (A/\cos\theta)$, F_N = Normal force = $F\cos\theta$

So, Tensile stress $= \frac{F\cos\theta}{A/\cos\theta} = \frac{F\cos^2\theta}{A}$

***Problem* 5.** In the above question, what is the shearing stress on *PQ*

(a) $F/A\cos\theta$ (b) $F\sin 2\theta/2A$ (c) $F/2A\sin 2\theta$ (d) $F\cos\theta/A$

Solution : (b) Shear stress $= \frac{\text{Tangential force}}{\text{Area}} = \frac{F\sin\theta}{(A/\cos\theta)} = \frac{F\sin\theta\cos\theta}{A} = \frac{F\sin 2\theta}{2A}$

***Problem* 6.** In the above question, when is the tensile stress maximum

(a) $\theta = 0^o$ (b) $\theta = 30^o$ (c) $\theta = 45^o$ (d) $\theta = 90^o$

Solution : (a) Tensile stress $= \frac{F\cos^2\theta}{A}$. It will be maximum when $\cos^2\theta = \max$. *i.e.* $\cos\theta = 1 \Rightarrow \theta = 0^o$.

Problem 7. In the above question, when is the shearing stress maximum

(a) $\theta = 0^o$ (b) $\theta = 30^o$ (c) $\theta = 45^o$ (d) $\theta = 90^o$

Solution : (c) Shearing stress $= \frac{F\sin 2\theta}{2A}$. It will be maximum when $\sin 2\theta = \max$ *i.e.* $\sin 2\theta = 1 \Rightarrow 2\theta = 90^o \Rightarrow \theta = 45^o$.

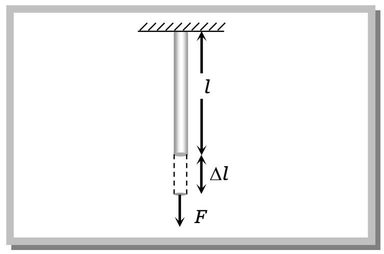
9.8 Strain

The ratio of change in configuration to the original configuration is called strain.

Being the ratio of two like quantities, it has no dimensions and units.

Strain are of three types :

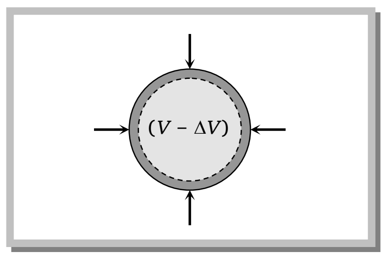
(1) **Linear strain :** If the deforming force produces a change in length alone, the strain produced in the body is called linear strain or tensile strain.



$$\text{Linear strain} = \frac{\text{Change in length}(\Delta l)}{\text{Original length}(l)}$$

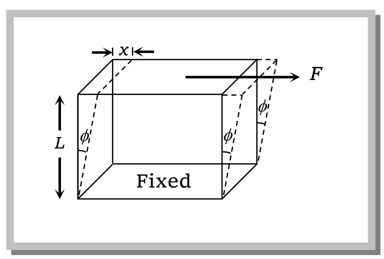
Linear strain in the direction of deforming force is called longitudinal strain and in a direction perpendicular to force is called lateral strain.

(2) **Volumetric strain :** If the deforming force produces a change in volume alone the strain produced in the body is called volumetric strain.



$$\text{Volumetric strain} = \frac{\text{Change in volume}(\Delta V)}{\text{Original v olume}(V)}$$

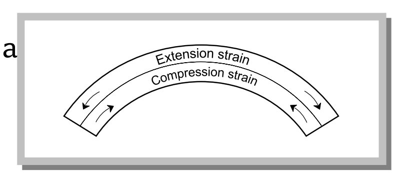
(3) **Shearing strain :** If the deforming force produces a change in the shape of the body without changing its volume, strain produced is called shearing strain.



It is defined as angle in radians through which a plane perpendicular to the fixed surface of the cubical body gets turned under the effect of tangential force.

$$\phi = \frac{x}{L}$$

Note : ❑ When a beam is bent both compression strain as well a extension strain is produced.



Sample problems based on Strain

Problem **8.** A cube of aluminium of sides 0.1 *m* is subjected to a shearing force of 100 *N*. The top face of the cube is displaced through 0.02 *cm* with respect to the bottom face. The shearing strain would be **[MP PAT 1990]**

(a) 0.02 (b) 0.1 (c) 0.005 (d) 0.002

Solution : (d) Shearing strain $\phi = \frac{x}{L} = \frac{0.02\,cm}{0.1m} = 0.002$

Problem **9.** A wire is stretched to double its length. The strain is

(a) 2 (b) 1 (c) Zero (d) 0.5

Solution : (b) Strain $= \frac{\text{Change in length}}{\text{Original length}} = \frac{2L - L}{L} = 1$

Problem **10.** The length of a wire increases by 1% by a load of 2 *kg-wt*. The linear strain produced in the wire will be

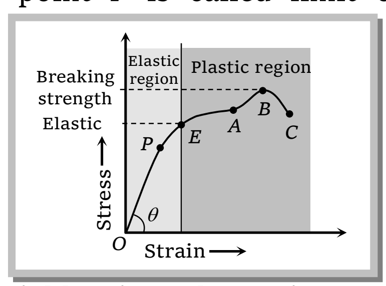
(a) 0.02 (b) 0.001 (c) 0.01 (d) 0.002

Solution : (c) Strain $= \frac{\text{Change in length}}{\text{Original length}} = \frac{1\%\text{ of }L}{\text{L}} = \frac{L/100}{L} = 0.01$

9.9 Stress-strain Curve

If by gradually increasing the load on a vertically suspended metal wire, a graph is plotted between stress (or load) and longitudinal strain (or elongation) we get the curve as shown in figure. From this curve it is clear that :

(1) When the strain is small (< 2%) (*i.e.*, in region *OP*) stress is proportional to strain. This is the region where the so called Hooke's law is obeyed. The point *P* is called limit of proportionality and slope of line *OP* gives the Young's modulus *Y* of the material of the wire. If θ is the angle of *OP* from strain axis then $Y = \tan\theta$.



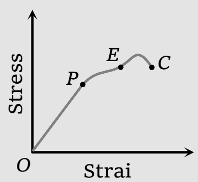
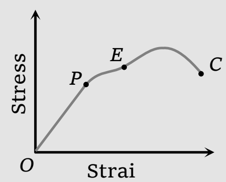
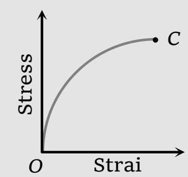
(2) If the strain is increased a little bit, *i.e.*, in the region *PE*, the stress is not proportional to strain. However, the wire still regains its original length after the removal of stretching force. This behaviour is shown up to point *E* known as elastic limit or yield-point. The region *OPE* represents the elastic behaviour of the material of wire.

(3) If the wire is stretched beyond the elastic limit *E*, *i.e.*, between *EA*, the strain increases much more rapidly and if the stretching force is removed the wire does not come back to its natural length. Some permanent increase in length takes place.

(4) If the stress is increased further, by a very small increase in it a very large increase in strain is produced (region *AB*) and after reaching point *B*, the strain increases even if the wire is unloaded and ruptures at *C*. In the region *BC* the wire literally flows. The maximum stress

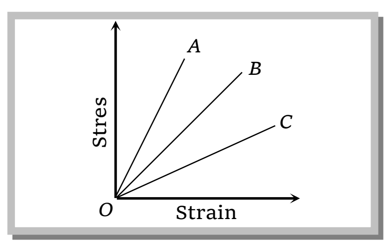
corresponding to B after which the wire begins to flow and breaks is called breaking or tensile strength. The region $EABC$ represents the plastic behaviour of the material of wire.

(5) Stress-strain curve for different materials.

Brittle material	Ductile material	Elastomers
		
The plastic region between E and C is small for brittle material and it will break soon after the elastic limit is crossed.	The material of the wire have a good plastic range and such materials can be easily changed into different shapes and can be drawn into thin wires	Stress strain curve is not a straight line within the elastic limit for elastomers and strain produced is much larger than the stress applied. Such materials have no plastic range and the breaking point lies very close to elastic limit. Example rubber

Sample problems based on Stress-strain curve

***Problem* 11.** The stress-strain curves for brass, steel and rubber are shown in the figure. The lines A, B and C are for



(a) Rubber, brass and steel respectively

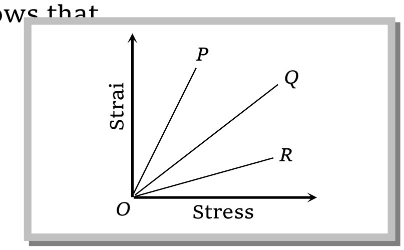
(b) Brass, steel and rubber

(c) Steel, brass and rubber respectively

(d) Steel, rubber and brass

Solution : (c) From the graph $\tan\theta_C < \tan\theta_B < \tan\theta_A \Rightarrow Y_C < Y_B < Y_A \quad \therefore Y_{\text{Rubber}} < Y_{\text{Brass}} < Y_{\text{Steel}}$

***Problem* 12.** The strain stress curves of three wires of different materials are shown in the figure. P, Q and R are the elastic limits of the wires. The figure shows that



(a) Elasticity of wire P is maximum

(b) Elasticity of wire Q is maximum

(c) Tensile strength of R is maximum

(d) None of the above is true

Solution : (d) On the graph stress is represented on X- axis and strain Y-axis

So from the graph $Y = \cot\theta = \frac{1}{\tan\theta} \propto \frac{1}{\theta}$ [where θ is the angle from stress axis]

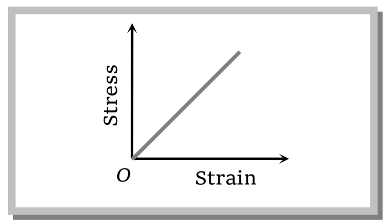
$\therefore Y_P < Y_Q < Y_R$ [As $\theta_P > \theta_Q > \theta_R$]

We can say that elasticity of wire P is minimum and R is maximum.

9.10 Hooke's law and Modulus of Elasticity

According to this law, within the elastic limit, stress is proportional to the strain.

i.e. stress $\propto$ strain or $\frac{\text{stress}}{\text{strain}} = \text{constant} = E$



The constant E is called modulus of elasticity.

(1) It's value depends upon the nature of material of the body and the manner in which the body is deformed.

(2) It's value depends upon the temperature of the body.

(3) It's value is independent of the dimensions (length, volume etc.) of the body.

There are three modulii of elasticity namely Young's modulus (Y), Bulk modulus (K) and modulus of rigidity (η) corresponding to three types of the strain.

9.11 Young's Modulus (Y)

It is defined as the ratio of normal stress to longitudinal strain within limit of proportionality.

$$Y = \frac{\text{Normal stress}}{\text{longitudinal strain}} = \frac{F/A}{l/L} = \frac{FL}{Al}$$

If force is applied on a wire of radius r by hanging a weight of mass M, then

$$Y = \frac{MgL}{\pi r^2 l}$$

Important points

(i) If the length of a wire is doubled,

Then longitudinal strain = $\frac{\text{change in length}(l)}{\text{initial length}(L)} = \frac{\text{final length} - \text{initial length}}{\text{Initial length}} = \frac{2L - L}{L} = 1$

$\therefore$ Young's modulus = $\frac{\text{stress}}{\text{strain}}$ $\Rightarrow$ Y = stress [As strain = 1]

So young's modulus is numerically equal to the stress which will double the length of a wire.

(ii) Increment in the length of wire $l = \frac{FL}{\pi r^2 Y}$ $\left[\text{As } Y = \frac{FL}{Al}\right]$

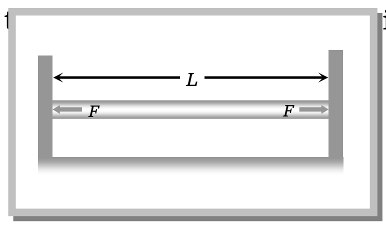
So if same stretching force is applied to different wires of same material, $l \propto \frac{L}{r^2}$ [As F and Y are constant]

i.e., greater the ratio $\frac{L}{r^2}$, greater will be the elongation in the wire.

(iii) **Elongation in a wire by its own weight** : The weight of the wire Mg act at the centre of gravity of the wire so that length of wire which is stretched will be $L/2$.

$\therefore$ Elongation $l = \frac{FL}{AY} = \frac{Mg(L/2)}{AY} = \frac{MgL}{2AY} = \frac{L^2 dg}{2Y}$ [As mass (M) = volume (AL) × density (d)]

(iv) **Thermal stress** : If a rod is fixed between two rigid supports, due to change in temperature its length will change and so it will exert a normal stress (compressive if temperature increases and tensile if temperature decreases) on the supports. This is called thermal stress.



As by definition, coefficient of linear expansion $\alpha = \frac{l}{L\Delta\theta}$

$\Rightarrow$ thermal strain $\frac{l}{L} = \alpha\Delta\theta$

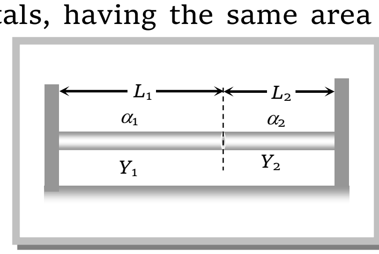
So thermal stress = $Y\alpha\,\Delta\theta$ [As Y = stress/strain]

And tensile or compressive force produced in the body = $YA\alpha\,\Delta\theta$

Note : ❑ In case of volume expansion Thermal stress = $K\gamma\Delta\theta$

Where K = Bulk modulus, γ = coefficient of cubical expansion

(v) **Force between the two rods** : Two rods of different metals, having the same area of cross section A, are placed end to end between two massive walls as shown in figure. The first rod has a length L_1, coefficient of linear expansion α_1 and young's modulus Y_1. The corresponding quantities for second rod are L_2, α_2 and Y_2. If the temperature of both the rods is now raised by T degrees.



Increase in length of the composite rod (due to heating) will be equal to

$l_1 + l_2 = [L_1\alpha_1 + L_2\alpha_2]T$ [As $l = L\,\alpha\,\Delta\theta$]

and due to compressive force F from the walls due to elasticity,

decrease in length of the composite rod will be equal to $\left[\frac{L_1}{Y_1} + \frac{L_2}{Y_2}\right]\frac{F}{A}$ $\left[\text{As } l = \frac{FL}{AY}\right]$

as the length of the composite rod remains unchanged the increase in length due to heating must be equal to decrease in length due to compression *i.e.* $\frac{F}{A}\left[\frac{L_1}{Y_1}+\frac{L_2}{Y_2}\right]=[L_1\alpha_1+L_2\alpha_2]T$

or $$F=\frac{A[L_1\alpha_1+L_2\alpha_2]T}{\left[\frac{L_1}{Y_1}+\frac{L_2}{Y_2}\right]}$$

(vi) **Force constant of wire** : Force required to produce unit elongation in a wire is called force constant of material of wire. It is denoted by k.

$\therefore$ $$k=\frac{F}{l} \quad .....(i)$$

but from the definition of young's modulus $Y=\frac{F/A}{l/L} \Rightarrow \frac{F}{l}=\frac{YA}{L}$ .....(ii)

from (i) and (ii) $k=\frac{YA}{L}$

It is clear that the value of force constant depends upon the dimension (length and area of cross section) and material of a substance.

(vii) Actual length of the wire : If the actual length of the wire is L, then under the tension T_1, its length becomes L_1 and under the tension T_2, its length becomes L_2.

$L_1=L+l_1 \Rightarrow L_1=L+\frac{T_1}{k}$ ......(i) and $L_2=L+l_2 \Rightarrow L_2=L+\frac{T_2}{k}$ .....(ii)

From (i) and (ii) we get $L=\frac{L_1T_2-L_2T_1}{T_2-T_1}$

Sample problems based on Young's modulus

***Problem* 13.** The diameter of a brass rod is 4 *mm* and Young's modulus of brass is $9\times10^{10}\,N/m^2$. The force required to stretch by 0.1% of its length is

(a) 360 πN (b) 36 N (c) $144\,\pi\times10^3\,N$ (d) $36\pi\times10^5\,N$

Solution : (a) $r=2\times10^{-3}m,\ Y=9\times10^{10}\,N/m^2,\ l=0.1\%\,L \Rightarrow \frac{l}{L}=0.001$

As $Y=\frac{F}{A}\frac{L}{l}$ $\quad\therefore F=YA\frac{l}{L}=9\times10^{10}\times\pi(2\times10^{-3})^2\times0.001=360\,\pi\,N$

***Problem* 14.** A wire of length 2*m* is made from 10 cm^3 of copper. A force F is applied so that its length increases by 2 *mm*. Another wire of length 8 *m* is made from the same volume of copper. If the force F is applied to it, its length will increase by

(a) 0.8 *cm* (b) 1.6 *cm* (c) 2.4 *cm* (d) 3.2 *cm*

Solution : (d) $l=\frac{FL}{AY}=\frac{FL^2}{VY}$ $\quad\therefore l\propto L^2$ [As V, Y and F are constant]

$\frac{l_2}{l_1}=\left[\frac{L_2}{L_1}\right]^2=\left(\frac{8}{2}\right)^2=16 \Rightarrow l_2=16\,l_1=16\times2\,mm=32\,mm=3.2\,cm$

***Problem* 15.** A wire of length L and radius r is rigidly fixed at one end. On stretching the other end of the wire with a force F, the increase in its length is l. If another wire of same material but of length $2L$ and radius $2r$ is stretched with a force of $2F$, the increase in its length will be

[AIIMS 1980; MP PAT 1990; MP PET 1989, 92; MP PET/PMT 1988; MP PMT 1996, 2002; UPSEAT 2002]

(a) l (b) $2l$ (c) $\frac{l}{2}$ (d) $\frac{l}{4}$

Solution : (a) $l = \frac{FL}{\pi r^2 Y} \Rightarrow \frac{l_2}{l_1} = \frac{F_2}{F_1}\frac{L_2}{L_1}\left(\frac{r_1}{r_2}\right)^2 = 2 \times 2 \times \left(\frac{1}{2}\right)^2 = 1 \quad \therefore l_2 = l_1$ *i.e.* the increment in length will be same.

***Problem* 16.** Two wires A and B are of same materials. Their lengths are in the ratio 1 : 2 and diameters are in the ratio 2 : 1 when stretched by force F_A and F_B respectively they get equal increase in their lengths. Then the ratio F_A/F_B should be

(a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) 8 : 1

Solution : (d) $Y = \frac{FL}{\pi r^2 l} \qquad \therefore F = Y\pi r^2 \frac{l}{L}$

$$\frac{F_A}{F_B} = \frac{Y_A}{Y_B}\left(\frac{r_A}{r_B}\right)^2\left(\frac{l_A}{l_B}\right)\left(\frac{L_B}{L_A}\right) = 1 \times \left(\frac{2}{1}\right)^2 \times (1) \times \left(\frac{2}{1}\right) = 8$$

***Problem* 17.** A uniform plank of Young's modulus Y is moved over a smooth horizontal surface by a constant horizontal force F. The area of cross-section of the plank is A. the compressive strain on the plank in the direction of the force is

(a) $\frac{F}{AY}$ (b) $\frac{2F}{AY}$ (c) $\frac{1}{2}\left(\frac{F}{AY}\right)$ (d) $\frac{3F}{AY}$

Solution : (a) Compressive strain $= \frac{\text{Stress}}{\text{Young's modulus}} = \frac{F/A}{Y} = \frac{F}{AY}$

***Problem* 18.** A wire is stretched by 0.01 m by a certain force F. Another wire of same material whose diameter and length are double to the original wire is stretched by the same force. Then its elongation will be

[EAMCET (Engg.) 1995; CPMT 2001]

(a) 0.005 m (b) 0.01 m (c) 0.02 m (d) 0.002 m

Solution : (a) $l = \frac{FL}{\pi r^2 Y} \quad \therefore l \propto \frac{L}{r^2}$ [As F and Y are constants]

$$\frac{l_2}{l_1} = \left(\frac{L_2}{L_1}\right)\left(\frac{r_1}{r_2}\right)^2 = (2) \times \left(\frac{1}{2}\right)^2 = \frac{1}{2} \Rightarrow l_2 = \frac{l_1}{2} = \frac{0.01}{2} = 0.005\, m\,.$$

***Problem* 19.** The length of an elastic string is a *metres* when the longitudinal tension is 4 N and b *metres* when the longitudinal tension is 5 N. The length of the string in *metres* when the longitudinal tension is 9 N is

[EAMCET 2001]

(a) $a - b$ (b) $5b - 4a$ (c) $2b - \frac{1}{4}a$ (d) $4a - 3b$

Solution : (b) Let the original length of elastic string is L and its force constant is k.

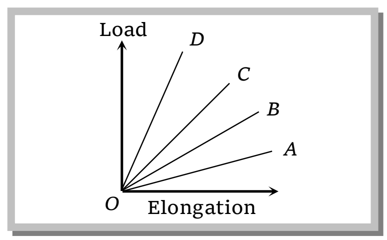
When longitudinal tension $4N$ is applied on it $L + \frac{4}{k} = a$ .....(i)

and when longitudinal tension $5N$ is applied on it $L + \frac{5}{k} = b$

By solving (i) and (ii) we get $k = \frac{1}{b-a}$ and $L = 5a - 4b$

Now when longitudinal tension $9N$ is applied on elastic string then its length = $L + \frac{9}{k} = 5a - 4b + 9(b-a) = 5b - 4a$

Problem **20.** The load versus elongation graph for four wires of the same material is shown in the figure. The thickest wire is represented by the line



(a) OD

(b) OC

(c) OB

(d) OA

Solution : (a) Young's modulus $Y = \frac{FL}{Al}$ $\therefore\ l \propto \frac{1}{A}$ (As Y, L and F are constant)

From the graph it is clear that for same load elongation is minimum for graph OD.

As elongation (l) is minimum therefore area of cross-section (A) is maximum.

So thickest wire is represented by OD.

Problem **21.** A 5 m long aluminum wire $(Y = 7 \times 10^{10}\ N/m^2)$ of diameter 3 mm supports a 40 kg mass. In order to have the same elongation in a copper wire $(Y = 12 \times 10^{10}\ N/m^2)$ of the same length under the same weight, the diameter should now be, in mm

(a) 1.75 (b) 2.0 (c) 2.3 (d) 5.0

Solution : (c) $l = \frac{FL}{\pi r^2 Y} = \frac{4FL}{\pi d^2 Y}$ [As $r = d/2$]

If the elongation in both wires (of same length) are same under the same weight then $d^2 Y$ = constant

$$\left(\frac{d_{Cu}}{d_{Al}}\right)^2 = \frac{Y_{Al}}{Y_{Cu}} \Rightarrow d_{Cu} = d_{Al} \times \sqrt{\frac{Y_{Al}}{Y_{Cu}}} = 3 \times \sqrt{\frac{7 \times 10^{10}}{12 \times 10^{10}}} = 2.29\ mm$$

Problem **22.** On *applying* a stress of $20 \times 10^8\ N/m^2$ the length of a perfectly elastic wire is doubled. Its Young's modulus will be

(a) $40 \times 10^8 N/m^2$ (b) $20 \times 10^8 N/m^2$ (c) $10 \times 10^8 N/m^2$ (d) $5 \times 10^8 N/m^2$

Solution : (b) When strain is unity then Young's modulus is equal to stress.

Problem **23.** The dimensions of four wires of the same material are given below. In which wire the increase in length will be maximum when the same tension is applied

[IIT-JEE 1981; NCERT 1976; CPMT 1983, 90; MP PMT 1992, 94, 97; MP PET/PMT 1998; MP PET 1989, 90, 99]

(a) Length 100 *cm*, diameter 1 *mm* (b) Length 200 *cm*, diameter 2 *mm*

(c) Length 300 *cm*, diameter 3 *mm* (d) Length 50 *cm*, diameter 0.5 *mm*

Solution : (d) If same force is applied on four wires of same material then elongation in each wire depends on the length and diameter of the wire and given by $l \propto \frac{L}{d^2}$ and the ratio of $\frac{L}{d^2}$ is maximum for (d) option.

Problem **24.** The Young's modulus of a wire of length L and radius r is Y N/m^2. If the length and radius are reduced to $L/2$ and $r/2$, then its Young's modulus will be

(a) $Y/2$ (b) Y (c) $2Y$ (d) $4Y$

Solution : (b) Young's modulus do not depend upon the dimensions of wire. It is constant for a given material of wire.

Problem **25.** A fixed volume of iron is drawn into a wire of length L. The extension x produced in this wire by a constant force F is proportional to

(a) $\frac{1}{L^2}$ (b) $\frac{1}{L}$ (c) L^2 (d) L

Solution : (c) $l = \frac{FL}{AY} = \frac{FL^2}{ALY} = \frac{FL^2}{VY}$ for a fixed volume $l \propto L^2$

Problem **26.** A rod is fixed between two points at $20^o C$. The coefficient of linear expansion of material of rod is $1.1 \times 10^{-5} / {}^o C$ and Young's modulus is $1.2 \times 10^{11} N/m^2$. Find the stress developed in the rod if temperature of rod becomes $10^o C$

(a) $1.32 \times 10^7 N/m^2$ (b) $1.10 \times 10^{15} N/m^2$ (c) $1.32 \times 10^8 N/m^2$ (d) $1.10 \times 10^6 N/m^2$

Solution : (a) Thermal stress $\frac{F}{A} = Y\alpha\Delta\theta = 1.2 \times 10^{11} \times 1.1 \times 10^{-5} \times (20 - 10) = 1.32 \times 10^7 N/m^2$

Problem **27.** The coefficient of linear expansion of brass and steel are α_1 and α_2. If we take a brass rod of length L_1 and steel rod of length L_2 at $0^o C$, their difference in length $(L_2 - L_1)$ will remain the same at any temperature if

[EAMCET (Med.) 1995]

(a) $\alpha_1 L_2 = \alpha_2 L_1$ (b) $\alpha_1 L_2^2 = \alpha_2 L_1^2$ (c) $\alpha_1^2 L_1 = \alpha_2^2 L_2$ (d) $\alpha_1 L_1 = \alpha_2 L_2$

Solution : (d) Difference in lengths of rods will remain same if expansion is same in both the rods.

If expansion in first rod is $l_1 = L_1\alpha_1\Delta\theta$ and expansion in second rod is $l_2 = L_2\alpha_2\Delta\theta$

then $L_1\alpha_1\Delta\theta = L_2\alpha_2\Delta\theta \quad \therefore L_1\alpha_1 = L_2\alpha_2$

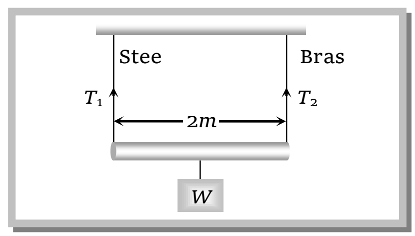
***Problem* 28.** The force required to stretch a steel wire of 1 cm^2 cross-section to 1.1 times its length would be ($Y = 2\times10^{11}\ Nm^{-2}$) **[MP PET 1992]**

(a) $2\times10^{6}\ N$ (b) $2\times10^{3}\ N$ (c) $2\times10^{-6}\ N$ (d) $2\times10^{-7}\ N$

Solution : (a) $L_2 = 1.1\,L_1 \qquad \therefore \text{Strain} = \frac{l}{L_1} = \frac{L_2 - L_1}{L_1} = \frac{1.1L_1 - L_1}{L_1}$ = 0.1.

$$F = YA\frac{l}{L} = 2\times10^{11}\times1\times10^{-4}\times0.1 = 2\times10^{6}\ N.$$

***Problem* 29.** A two *metre* long rod is suspended with the help of two wires of equal length. One wire is of steel and its cross-sectional area is 0.1 cm^2 and another wire is of brass and its cross-sectional area is 0.2 cm^2. If a load W is suspended from the rod and stress produced in both the wires is same then the ratio of tensions in them will be



(a) Will depend on the position of W

(b) $T_1 / T_2 = 2$

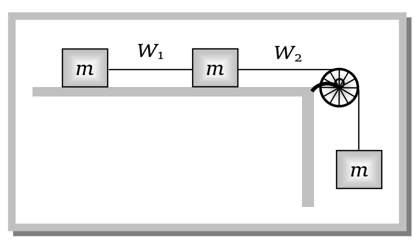
(c) $T_1 / T_2 = 1$

(d) $T_1 / T_2 = 0.5$

Solution : (d) Stress = $\frac{\text{Tension}}{\text{Area of cross-section}}$ = constant

$$\therefore \frac{T_1}{A_1} = \frac{T_2}{A_2} \Rightarrow \frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{0.1}{0.2} = \frac{1}{2} = 0.5.$$

***Problem* 30.** Three blocks, each of same mass m, are connected with wires W_1 and W_2 of same cross-sectional area a and Young's modulus Y. Neglecting friction the strain developed in wire W_2 is



(a) $\frac{2}{3}\frac{mg}{aY}$

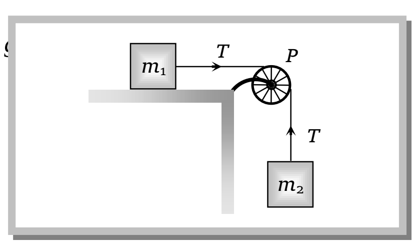
(b) $\frac{3\,mg}{2\,aY}$

(c) $\frac{1}{3}\frac{mg}{aY}$

(d) $\frac{3mg}{aY}$

Solution : (a) If the system moves with acceleration a and T is the tension in the string W_2 then by comparing this condition from standard case $T = \frac{m_1 m_2}{m_1 + m_2}g$

In the given problem $m_1 = (m+m) = 2m$ and $m_2 = m$



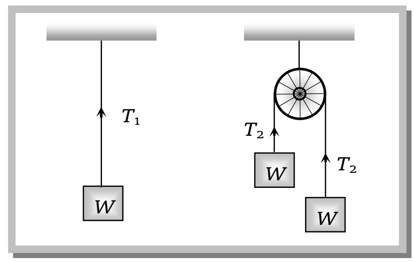
$\therefore$ Tension $= \dfrac{m.2m.g}{m+2m} = \dfrac{2}{3}mg$

$\therefore$ Stress $= \dfrac{T}{a} = \dfrac{2}{3a}mg$ and Strain $= \dfrac{\text{Stress}}{\text{Young's modulus}} = \dfrac{2}{3}\dfrac{mg}{aY}$

***Problem* 31.** A wire elongates by 1.0 *mm* when a load *W* is hanged from it. If this wire goes over a pulley and two weights *W* each are hung at the two ends, the elongation of the wire will be

(a) 0.5 *m* (b) 1.0 *mm* (c) 2.0 *mm* (d) 4.0 *mm*

Solution : (b) Elongation in the wire $\propto$ Tension in the wire



In first case $T_1 = W$ and in second case $T_2 = \dfrac{2W \times W}{W+W} = W$

As $\dfrac{T_1}{T_2} = 1 \quad \therefore \dfrac{l_1}{l_2} = 1 \Rightarrow l_2 = l_1 = 1.0mm$

***Problem* 32.** The Young's modulus of three materials are in the ratio 2 : 2 : 1. Three wires made of these materials have their cross-sectional areas in the ratio 1 : 2 : 3. For a given stretching force the elongation's in the three wires are in the ratio

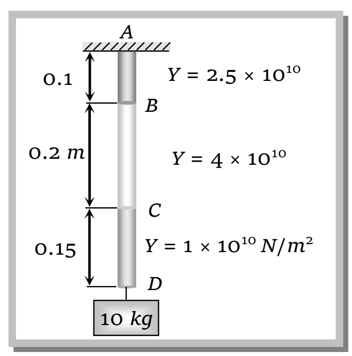
(a) 1 : 2 : 3 (b) 3 : 2 : 1 (c) 5 : 4 : 3 (d) 6 : 3 : 4

Solution : (d) $l = \dfrac{FL}{AY}$ and for a given stretching force $l \propto \dfrac{1}{AY}$

Let three wires have young's modulus $2Y$, $2Y$ and Y and their cross sectional areas are A, $2A$ and $3A$ respectively.

$$l_1 : l_2 : l_3 = \frac{1}{A_1Y_1} : \frac{1}{A_2Y_2} : \frac{1}{A_3Y_3} = \frac{1}{A \times 2Y} : \frac{1}{2A \times 2Y} : \frac{1}{3A \times Y} = \frac{1}{2} : \frac{1}{4} : \frac{1}{3} = 6 : 3 : 4 .$$

***Problem* 33.** A light rod with uniform cross-section of $10^{-4}\,m^2$ is shown in the adjoining figure. The rod consists of three different materials whose lengths are 0.1 *m*, 0.2 *m* and 0.15 *m* respectively and whose Young's modulii are $2.5 \times 10^{10}\,N/m^2$, $4 \times 10^{10}\,N/m^2$ and $1 \times 10^{10}\,N/m^2$ respectively. The displacement of point *B* will be



(a) $24 \times 10^{-6} m$

(b) $9 \times 10^{-6} m$

(c) $4 \times 10^{-6} m$

(d) $1 \times 10^{-6} m$

Solution : (c) Increment in the length $AB = \dfrac{MgL}{AY} = \dfrac{10 \times 10 \times 0.1}{10^{-4} \times 2.5 \times 10^{10}} = 4 \times 10^{-6} m$

$\therefore$ Displacement of point $B = 4 \times 10^{-6} m$

***Problem* 34.** In the above problem, displacement of point *C* will be

(a) $24 \times 10^{-6} m$ (b) $9 \times 10^{-6} m$ (c) $4 \times 10^{-6} m$ (d) $1 \times 10^{-6} m$

Solution : (b) Increment in the length $BC = \frac{MgL}{AY} = \frac{10 \times 10 \times 0.2}{10^{-4} \times 4 \times 10^{10}} = 5 \times 10^{-6} m$

$\therefore$ Displacement of point $C = 4 \times 10^{-6} + 5 \times 10^{-6} = 9 \times 10^{-6} m$

***Problem* 35.** In the above problem, the displacement of point D will be

(a) $24 \times 10^{-6} m$ (b) $9 \times 10^{-6} m$ (c) $4 \times 10^{-6} m$ (d) $1 \times 10^{-6} m$

Solution : (a) Increment in the length $CD = \frac{MgL}{AY} = \frac{10 \times 10 \times 0.15}{10^{-4} \times 1 \times 10^{10}} = 15 \times 10^{-6} m$

$\therefore$ Displacement of point $D = 4 \times 10^{-6} + 5 \times 10^{-6} m + 15 \times 10^{-6} = 24 \times 10^{-6} m$.

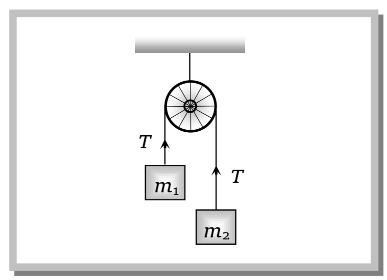
***Problem* 36.** Two blocks of masses m_1 and m_2 are joined by a wire of Young's modulus Y via a massless pulley. The area of cross-section of the wire is S and its length is L. When the system is released, increase in length of the wire is

(a) $\frac{m_1 m_2 gL}{YS\,(m_1 + m_2)}$

(b) $\frac{2m_1 m_2 gL}{YS\,(m_1 + m_2)}$

(c) $\frac{(m_1 - m_2)gL}{YS\,(m_1 + m_2)}$

(d) $\frac{4m_1 m_2 gL}{YS\,(m_1 + m_2)}$



Solution : (b) Tension in the wire $T = \frac{2m_1 m_2}{m_1 + m_2} g$ $\therefore$ stress in the wire $= \frac{T}{S} = \frac{2m_1 m_2 g}{S(m_1 + m_2)}$

$\therefore$ Strain $\frac{l}{L} = \frac{\text{Stress}}{\text{Y}} = \frac{2m_1 m_2 g}{YS\,(m_1 + m_2)} \Rightarrow l = \frac{2m_1 m_2 gL}{YS\,(m_1 + m_2)}$

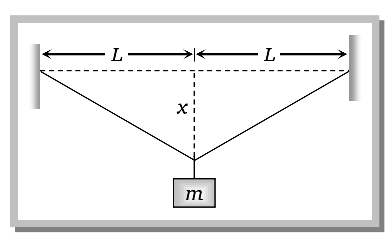
***Problem* 37.** A steel wire of diameter d, area of cross-section A and length $2L$ is clamped firmly at two points A and B which are $2L$ *metre* apart and in the same plane. A body of mass m is hung from the middle point of wire such that the middle point sags by x lower from original position. If Young's modulus is Y then m is given by

(a) $\frac{1}{2}\frac{YAx^2}{gL^2}$

(b) $\frac{1}{2}\frac{YAL^2}{gx^2}$

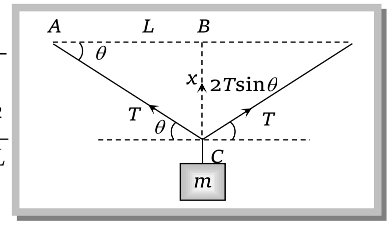
(c) $\frac{YAx^3}{gL^3}$

(d) $\frac{YAL^3}{gx^2}$



Solution : (c) Let the tension in the string is T and for the equilibrium of mass m

$2T\sin\theta = mg \Rightarrow T = \frac{mg}{2\sin\theta} = \frac{mgL}{2x}$ [As θ is small then $\sin\theta = \frac{x}{L}$]



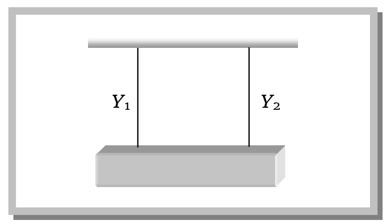
Increment in the length $l = AC - AB = \sqrt{L^2 + x^2} - L = (L^2 + x^2)^{1/2} - L$

$$= L\left[\left(1 + \frac{x^2}{L^2}\right)^{1/2} - 1\right] = L\left[1 + \frac{1}{2}\frac{x^2}{L^2} - 1\right] = \frac{x^2}{2L}$$

As Young's modulus $Y = \frac{T}{A}\frac{L}{l}$ $\therefore T = \frac{YAl}{L}$

Substituting the value of T and l in the above equation we get $\frac{mgL}{2x} = \frac{YA}{L}.\frac{x^2}{2L}$ $\therefore m = \frac{YAx^3}{gL^3}$

***Problem* 38.** Two wires of equal length and cross-section are suspended as shown. Their Young's modulii are Y_1 and Y_2 respectively. The equivalent Young's modulus will be



(a) $Y_1 + Y_2$

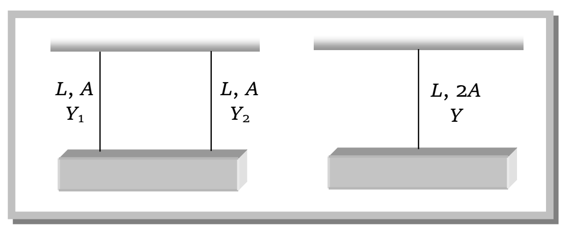
(b) $\frac{Y_1 + Y_2}{2}$

(c) $\frac{Y_1 Y_2}{Y_1 + Y_2}$

(d) $\sqrt{Y_1 Y_2}$

Solution : (b) Let the equivalent young's modulus of given combination is Y and the area of cross section is $2A$.

For parallel combination $k_1 + k_2 = k_{eq.}$



$$\frac{Y_1 A}{L} + \frac{Y_2 A}{L} = \frac{Y 2A}{L}$$

$$Y_1 + Y_2 = 2Y \text{ , } \therefore Y = \frac{Y_1 + Y_2}{2}$$

***Problem* 39.** If a load of 9*kg* is suspended on a wire, the increase in length is 4.5 *mm*. The force constant of the wire is

(a) $0.49 \times 10^4\ N/m$ (b) $1.96 \times 10^4\ N/m$ (c) $4.9 \times 10^4\ N/m$ (d) $0.196 \times 10^4\ N/m$

Solution : (b) Force constant $k = \frac{F}{l} = \frac{mg}{l} = \frac{9 \times 9.8}{4.5 \times 10^{-3}} \Rightarrow k = 1.96 \times 10^4\ N/m$

***Problem* 40.** One end of a long metallic wire of length L, area of cross-section A and Young's modulus Y is tied to the ceiling. The other end is tied to a massless spring of force constant k. A mass m hangs freely from the free end of the spring. It is slightly pulled down and released. Its time period is given by

(a) $2\pi\sqrt{\frac{m}{K}}$ (b) $2\pi\sqrt{\frac{mYA}{KL}}$ (c) $2\pi\sqrt{\frac{mK}{YA}}$ (d) $2\pi\sqrt{\frac{m(KL+YA)}{KYA}}$

Solution : (d) Force constant of wire $k_1 = \frac{F}{l} = \frac{YA}{L}$ and force constant of spring $k_2 = k$ (given)

Equivalent force constant for given combination $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{L}{YA} + \frac{1}{k} \Rightarrow k_{eq} = \frac{kYA}{kL+YA}$

$\therefore$ Time period of combination $T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m(kL+YA)}{kYA}}$

Problem **41.** Two wires A and B have the same length and area of cross section. But Young's modulus of A is two times the Young's modulus of B. Then the ratio of force constant of A to that of B is

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\sqrt{2}$

Solution : (b) Force constant of wire $k = \frac{YA}{L} \Rightarrow \frac{k_A}{k_B} = \frac{Y_A}{Y_B} = 2$ [As L and A are same]

9.12 Work Done in Stretching a Wire

In stretching a wire work is done against internal restoring forces. This work is stored in the wire as elastic potential energy or strain energy.

If a force F acts along the length L of the wire of cross-section A and stretches it by x then

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{x/L} = \frac{FL}{Ax} \Rightarrow F = \frac{YA}{L}.x$$

So the work done for an additional small increase dx in length, $dw = Fdx = \frac{YA}{L}x.dx$

Hence the total work done in increasing the length by l,
$W = \int_0^l dW = \int_0^l Fdx = \int_0^l \frac{YA}{L}.x\,dx = \frac{1}{2}\frac{YA}{L}l^2$

This work done is stored in the wire.

$\therefore$ Energy stored in wire $U = \frac{1}{2}\frac{YAl^2}{L} = \frac{1}{2}Fl$ $\left[\text{As } F = \frac{YAl}{L}\right]$

Dividing both sides by volume of the wire we get energy stored in per unit volume of wire.

$U_V = \frac{1}{2}\times\frac{F}{A}\times\frac{l}{L} = \frac{1}{2}\times\text{stress}\times\text{strain} = \frac{1}{2}\times Y\times(\text{strain})^2 = \frac{1}{2Y}(\text{stress})^2$ [As AL = volume of wire]

Total energy stored in wire (U)	**Energy stored in per unit volume of wire (U_V)**

$\frac{1}{2}Fl$	$\frac{1}{2}\frac{Fl}{\text{volume}}$
$\frac{1}{2}\times\text{stress}\times\text{strain}\times\text{volume}$	$\frac{1}{2}\times\text{stress}\times\text{strain}$
$\frac{1}{2}\times Y\times(\text{strain})^2\times\text{volume}$	$\frac{1}{2}\times Y\times(\text{strain})^2$
$\frac{1}{2Y}\times(\text{stress})^2\times\text{volume}$	$\frac{1}{2Y}\times(\text{stress})^2$

Note : ❑ If the force on the wire is increased from F_1 to F_2 and the elongation in wire is l then energy stored in the wire $U=\frac{1}{2}\frac{(F_1+F_2)}{2}l$

❑ Thermal energy density = Thermal energy per unit volume = $\frac{1}{2}$ × Thermal stress × strain

$$=\frac{1}{2}\frac{F}{A}\frac{l}{L}=\frac{1}{2}(Y\alpha\Delta\theta)(\alpha\Delta\theta)=\frac{1}{2}Y\alpha^2(\Delta\theta)^2$$

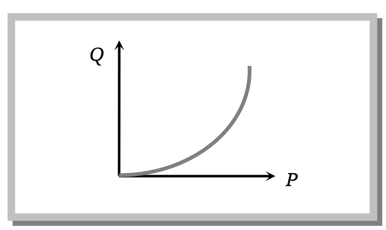
Sample problems based on Work done in Stretching a Wire

Problem 42. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm, then the elastic energy stored in the wire is **[AIEEE 2003]**

(a) 0.1 J (b) 0.2 J (c) 10 J (d) 20 J

Solution : (a) Elastic energy stored in wire = $U=\frac{1}{2}Fl=\frac{1}{2}\times 200\times 1\times 10^{-3}=0.1J$

Problem 43. The graph shows the behaviour of a length of wire in the region for which the substance obeys Hooke's law. P and Q represent



(a) P = applied force, Q = extension

(b) P = extension, Q = applied force

(c) P = extension, Q = stored elastic energy

(d) P = stored elastic energy, Q = extension

Solution : (c) The graph between applied force and extension will be straight line because in elastic range applied force ∝ extension, but the graph between extension and stored elastic energy will be parabolic in nature.

As $U=\frac{1}{2}kx^2$ or $U\propto x^2$

Problem 44. When a 4 *kg* mass is hung vertically on a light spring that obeys Hooke's law, the spring stretches by 2 *cms*. The work required to be done by an external agent in the stretching this spring by 5 *cms* will be (g = 9.8 m/s^2)

[MP PMT 1995]

(a) 4.900 *J* (b) 2.450 *J* (c) 0.495 *J* (d) 0.245 *J*

Solution : (b) When a 4 *kg* mass is hung vertically on a spring, it stretches by 2 *cm*

$$\therefore k = \frac{F}{x} = \frac{4 \times 9.8}{2 \times 10^{-2}} = 1960 \; N/m$$

Now work done in stretching this spring by 5 *cms*

$$U = \frac{1}{2}kx^2 = \frac{1}{2} \times 1960\,(5 \times 10^{-2})^2 = 2.45\,J.$$

Problem 45. A rod of iron of Young's modulus $Y = 2.0 \times 10^{11}\,N/m^2$ just fits the gap between two rigid supports 1*m* apart. If the rod is heated through 100^oC the strain energy of the rod is ($\alpha = 18 \times 10^{-6}\,{}^oC^{-1}$ and area of cross-section $A = 1\,cm^2$)

(a) 32.4 *J* (b) 32.4 *mJ* (c) 26.4 *J* (d) 26.4 *mJ*

Solution : (a) $U = \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{volume} = \frac{1}{2} \times Y(\alpha\,\Delta\theta)^2 \times A \times L$ $\left(\text{Thermal strain } \frac{l}{L} = \alpha\,\Delta\theta\right)$

$$= \frac{1}{2} \times (2 \times 10^{11}) \times (18 \times 10^{-6} \times 100)^2 \times 1 \times 10^{-4} \times 1 = 324 \times 10^{-1} = 32.4\,J.$$

Problem 46. Which of the following cases will have the greatest strain energy (F is the stretching force, A is the area of cross section and s is the strain)

(a) F = 10 N, A = 1 cm^2, s = 10^{-3} (b) F = 15 N, A = 2 cm^2, s = 10^{-3}

(c) F = 10 N, A = $\frac{1}{2}cm^2$, s = 10^{-4} (d) F = 5 N, A = 3 cm^2, s = 10^{-3}

Solution: (b) Strain energy $= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} = \frac{1}{2} \times \frac{F}{A} \times \text{strain} \times AL = \frac{1}{2} \times F \times \text{strain} \times L$

For wire (a) $U = \frac{1}{2} \times 10 \times 10^{-3} \times L = 5 \times 10^{-3}L$; For wire (b) $U = \frac{1}{2} \times 15 \times 10^{-3} \times L = 7.5 \times 10^{-3}L$

For wire (c) $U = \frac{1}{2} \times 10 \times 10^{-4} \times L = 0.5 \times 10^{-3}L$; For wire (d) $U = \frac{1}{2} \times 5 \times 10^{-3} = 2.5 \times 10^{-3}L$

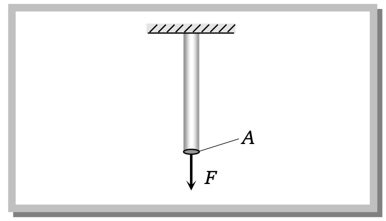
For a given length wire (b) will have greatest strain energy.

9.13 Breaking of Wire

When the wire is loaded beyond the elastic limit, then strain increases much more rapidly. The maximum stress corresponding to B (see stress-strain curve) after which the wire begin to flow and breaks, is called breaking stress or tensile strength and the force by application of which the wire breaks is called the breaking force.

(i) Breaking force depends upon the area of cross-section of the wire *i.e.*, Breaking force $\propto A$

$\therefore$ Breaking force = $P \times A$



Here P is a constant of proportionality and known as breaking stress.

(ii) Breaking stress is a constant for a given material and it does not depends upon the dimension (length or thickness) of wire.

(iii) If a wire of length L is cut into two or more parts, then again it's each part can hold the same weight. Since breaking force is independent of the length of wire.

(iv) If a wire can bear maximum force F, then wire of same material but double thickness can bear maximum force $4F$ because Breaking force $\propto \pi r^2$.

(v) The working stress is always kept lower than that of a breaking stress.

So that safety factor = $\frac{\text{breaking stress}}{\text{working stress}}$ may have large value.

(vi) Breaking of wire under its own weight.

Breaking force = Breaking stress $\times$ Area of cross section

Weight of wire = $Mg = ALdg = PA$ [As mass = volume $\times$ density = ALd]

$$\Rightarrow \quad Ldg = P \quad \therefore \quad L = \frac{P}{dg}$$

This is the length of wire if it breaks by its own weight.

Sample problems based on Breaking of Wire

***Problem* 47.** A wire of diameter 1 *mm* breaks under a tension of 1000 *N*. Another wire of same material as that of the first one, but of diameter 2 *mm* breaks under a tension of

(a) 500 *N* (b) 1000 *N* (c) 10000 *N* (d) 4000 *N*

Solution : (d) Breaking force $\propto$ area of cross-section $(\pi r^2) \propto d^2$

$$\frac{F_2}{F_1} = \left(\frac{d_2}{d_1}\right)^2 \Rightarrow \frac{F_2}{1000} = \left(\frac{2mm}{1mm}\right)^2 \Rightarrow F_2 = 1000 \times 4 = 4000\ N.$$

***Problem* 48.** In steel, the Young's modulus and the strain at the breaking point are $2 \times 10^{11}\ Nm^{-2}$ and 0.15 respectively. The stress at the breaking point for steel is therefore

(a) $1.33 \times 10^{11}\ Nm^{-2}$ (b) $1.33 \times 10^{12}\ Nm^{-2}$ (c) $7.5 \times 10^{-13}\ Nm^{-2}$ (d) $3 \times 10^{10}\ Nm^{-2}$

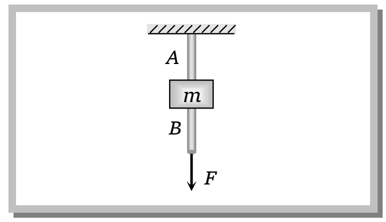
Solution : (d) $Y = \frac{\text{Stress}}{\text{Strain}}$ $\therefore$ Stress = Y × Strain $= 2 \times 10^{11} \times 0.15 = 0.3 \times 10^{11} = 3 \times 10^{10}\ N/m^2$

***Problem* 49.** To break a wire, a force of $10^6\ N/m^2$ is required. If the density of the material is $3 \times 10^3\ kg/m^3$, then the length of the wire which will break by its own weight will be

(a) 34 *m* (b) 30 *m* (c) 300 *m* (d) 3 *m*

Solution : (a) Length of the wire which will break by its own weight $L = \frac{P}{dg} = \frac{10^6}{3 \times 10^3 \times 10} = \frac{100}{3} = 33.3\ m \simeq$ 34 m.

Problem 50. The wires A and B shown in the figure are made of the same material and have radii r_A and r_B respectively. The block between them has a mass m. When the force F is $mg/3$, one of the wires break



(a) A will break before B if $r_A = r_B$

(b) A will break before B if $r_A < 2r_B$

(c) Either A or B may break if $r_A = 2r_B$

(d) The lengths of A and B must be known to predict which wire will break

Solution :(a,b,c) When force $F = \frac{mg}{3}$ is applied at the lower end then

Stress in wire $B = \frac{F}{\pi r_B^2} = \frac{mg}{3\pi r_B^2}$ and stress in wire $A = \frac{F + mg}{\pi r_A^2} = \frac{\frac{mg}{3} + mg}{\pi r_A^2} = \frac{4}{3}\frac{mg}{\pi r_A^2}$

(i) if $r_A = r_B = r$ (Let) then stress in wire $B = \frac{mg}{3\pi r^2}$ and stress in wire $A = \frac{4}{3} \cdot \frac{mg}{\pi r^2}$

i.e. stress in wire $A >$ stress in wire B so the A will break before B

(ii) if $r_B = r$, (let) then $r_A = 2r$

Stress in wire $B = \frac{mg}{3\pi r^2}$ and Stress in wire $A = \frac{4mg}{3\pi (2r)^2} = \frac{mg}{3\pi r^2}$

i.e. stress in wire A = stress in wire B. It means either A or B may break.

(iii) If $r_A < 2r_B$ then stress in A will be more than B. *i.e.* A will break before B.

Problem 51. A body of mass 10 kg is attached to a wire 0.3 m long. Its breaking stress is $4.8 \times 10^7\ N/m^2$. The area of cross-section of the wire is $10^{-6} m^2$. What is the maximum angular velocity with which it can be rotated in the horizontal circle

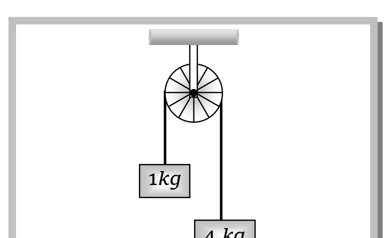
(a) 1 rad/sec (b) 2 rad/sec (c) 4 rad/sec (d) 8 rad/sec

Solution : (c) Breaking force = centrifugal force

Breaking stress × area of cross-section $= m\omega^2 l$

$4.8 \times 10^7 \times 10^{-6} = 10 \times \omega^2 \times 0.3 \Rightarrow \omega^2 = 16 \quad \Rightarrow \omega = 4 rad/\sec$

Problem 52. Two block of masses 1 kg and 4 kg are connected by a metal wire going over a smooth pulley as shown in the figure. The breaking stress of the metal is $3.18 \times 10^{10}\ N/m^2$. The minimum radius of the wire so it will not break is



(a) $1 \times 10^{-5} m$

(b) $2 \times 10^{-5} m$

(c) $3 \times 10^{-5} m$

(d) $4 \times 10^{-5} m$

Solution : (d) Tension in the wire $T = \dfrac{2m_1 m_2}{m_1 + m_2} g \Rightarrow T = \dfrac{2 \times 1 \times 4}{1+4} \times 10 \Rightarrow T = 16\, N$

Breaking force = Breaking stress × Area of cross-section

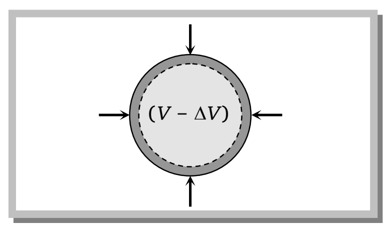
Tension in the wire = $3.18 \times 10^{10} \times \pi r^2$

$16 = 3.18 \times 10^{10} \times \pi r^2 \Rightarrow r = \sqrt{\dfrac{16}{3.18 \times 10^{10} \times 3.14}} = 4 \times 10^{-5} m.$

9.14 Bulk Modulus

When a solid or fluid (liquid or gas) is subjected to a uniform pressure all over the surface, such that the shape remains the same, then there is a change in volume.

Then the ratio of normal stress to the volumetric strain within the elastic limits is called as Bulk modulus. This is denoted by K.



$$K = \frac{\text{Normal stress}}{\text{volumetric strain}}$$

$$K = \frac{F/A}{-\Delta V/V} = \frac{-pV}{\Delta V}$$

where p = increase in pressure; V = original volume; ΔV = change in volume

The negative sign shows that with increase in pressure p, the volume decreases by ΔV *i.e.* if p is positive, ΔV is negative. The reciprocal of bulk modulus is called compressibility.

C = compressibility = $\dfrac{1}{K} = \dfrac{\Delta V}{pV}$

S.I. unit of compressibility is $N^{-1}m^2$ and C.G.S. unit is $dyne^{-1}\, cm^2$.

Gases have two bulk moduli, namely isothermal elasticity E_θ and adiabatic elasticity E_ϕ .

(1) **Isothermal elasticity (E_θ) :** Elasticity possess by a gas in isothermal condition is defined as isothermal elasticity.

For isothermal process, PV = constant (Boyle's law)

Differentiating both sides $PdV + VdP = 0 \Rightarrow PdV = -VdP$

$$P = \frac{dP}{(-dV/V)} = \frac{\text{stress}}{\text{strain}} = E_\theta$$

$\therefore \qquad E_\theta = P$

i.e., Isothermal elasticity is equal to pressure.

(2) **Adiabatic elasticity (E_ϕ) :** Elasticity possess by a gas in adiabatic condition is defined as adiabatic elasticity.

For adiabatic process, PV^γ = constant (Poisson's law)

Differentiating both sides, $P\gamma V^{\gamma-1}dV + V^\gamma dP = 0 \Rightarrow \gamma PdV + VdP = 0$

$$\gamma P = \frac{dP}{\left(\frac{-dV}{V}\right)} = \frac{\text{stress}}{\text{strain}} = E_\phi$$

$\therefore \qquad E_\phi = \gamma P$

i.e., adiabatic elasticity is equal to γ times pressure. [where $\gamma = \frac{C_p}{C_v}$]

Note : ❑ Ratio of adiabatic to isothermal elasticity $\frac{E_\phi}{E_\theta} = \frac{\gamma P}{P} = \gamma > 1 \qquad \therefore E_\phi > E_\theta$

i.e., adiabatic elasticity is always more than isothermal elasticity.

9.15 Density of Compressed Liquid

If a liquid of density ρ, volume V and bulk modulus K is compressed, then its density increases.

As density $\rho = \frac{m}{V}$ so $\frac{\Delta\rho}{\rho} = \frac{-\Delta V}{V}$ .....(i)

But by definition of bulk modulus $K = \frac{-V\Delta P}{\Delta V} \Rightarrow -\frac{\Delta V}{V} = \frac{\Delta P}{K}$ .....(ii)

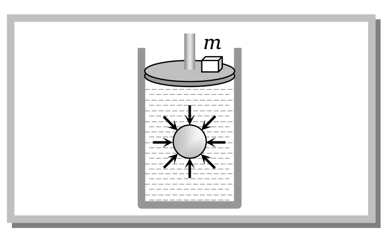
From (i) and (ii) $\frac{\Delta\rho}{\rho} = \frac{\rho' - \rho}{\rho} = \frac{\Delta P}{K}$ [As $\Delta\rho = \rho' - \rho$]

or $\rho' = \rho\left[1 + \frac{\Delta P}{K}\right] = \rho[1 + C\Delta P]$ $\left[\text{As } \frac{1}{K} = C\right]$

9.16 Fractional Change in the Radius of Sphere

A solid sphere of radius R made of a material of bulk modulus K is surrounded by a liquid in a cylindrical container.

A massless piston of area A floats on the surface of the liquid.



Volume of the spherical body $V = \frac{4}{3}\pi R^3$

$$\frac{\Delta V}{V} = 3\frac{\Delta R}{R}$$

$\therefore \qquad \frac{\Delta R}{R} = \frac{1}{3}\frac{\Delta V}{V}$ .....(i)

Bulk modulus $K = -V\frac{\Delta P}{\Delta V}$

$\therefore \qquad \left|\frac{\Delta V}{V}\right| = \frac{\Delta P}{K} = \frac{mg}{AK}$ .....(ii) $\left[\text{As}\, \Delta P = \frac{mg}{A}\right]$

Substituting the value of $\frac{\Delta V}{V}$ from equation (ii) in equation (i) we get $\frac{\Delta R}{R} = \frac{1}{3}\frac{mg}{AK}$

Sample problems based on Bulk modulus

***Problem* 53.** When a pressure of 100 atmosphere is applied on a spherical ball of rubber, then its volume reduces to 0.01%. The bulk modulus of the material of the rubber in *dyne/cm*^{2} is

(a) 10×10^{12} (b) 100×10^{12} (c) 1×10^{12} (d) 20×10^{12}

Solution : (c) 1 *atm* $= 10^5\, N/m^2$ $\therefore 100\, atm = 10^7\, N/m^2$ and $\Delta V = 0.01\% V$ $\therefore \frac{\Delta V}{V} = 0.0001$

$$K = \frac{P}{\Delta V / V} = \frac{10^7}{0.0001} = 1 \times 10^{11}\, N/m^2 = 1 \times 10^{12}\, \frac{Dyne}{cm^2}.$$

***Problem* 54.** Coefficient of isothermal elasticity E_θ and coefficient of adiabatic elasticity E_ϕ are related by $(\gamma = C_p / C_v)$

[MP PET 2000]

(a) $E_\theta = \gamma E_\phi$ (b) $E_\phi = \gamma E_\theta$ (c) $E_\theta = \gamma / E_\phi$ (d) $E_\theta = \gamma^2 E_\phi$

Solution : (b) Adiabatic elasticity $= \gamma \times$ isothermal elasticity $\Rightarrow E_\phi = \gamma E_\theta$.

***Problem* 55.** A uniform cube is subjected to volume compression. If each side is decreased by 1%,then bulk strain is

[EAMCET (Engg.) 1995; DPMT 2000]

(a) 0.01 (b) 0.06 (c) 0.02 (d) 0.03

Solution : (d) Volume of cube $V = L^3$ $\therefore$ Percentage change in $V = 3 \times$ (percentage change in *L*)= $3(1\%) = 3\%$

$\therefore \Delta V = 3\%$ of V $\Rightarrow$ Volumetric strain $= \frac{\Delta V}{V} = \frac{3}{100} = 0.03$

***Problem* 56.** A ball falling in a lake of depth 200 *m* shows 0.1% decrease in its volume at the bottom. What is the bulk modulus of the material of the ball

(a) $19.6 \times 10^8\, N/m^2$ (b) $19.6 \times 10^{-10}\, N/m^2$ (c) $19.6 \times 10^{10}\, N/m^2$ (d) $19.6 \times 10^{-8}\, N/m^2$

Solution : (a) $K = \frac{P}{\Delta V / V} = \frac{hdg}{\Delta V / V} = \frac{200 \times 10^3 \times 9.8}{0.001} = 19.6 \times 10^8\, N/m^2$

Problem 57. The ratio of the adiabatic to isothermal elasticities of a triatomic gas is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 1 (d) $\frac{5}{3}$

Solution : (b) For triatomic gas $\gamma = 4/3$ $\therefore$ Ratio of adiabatic to isothermal elasticity $\gamma = \frac{4}{3}$.

Problem 58. A gas undergoes a change according to the law $P = P_0 e^{\alpha V}$. The bulk modulus of the gas is

(a) P (b) αPV (c) αP (d) $\frac{PV}{\alpha}$

Solution : (b) $P = P_o e^{\alpha V} \Rightarrow \frac{dP}{dV} = P_o e^{\alpha V} \alpha = P\alpha$ [As $P = P_o e^{\alpha V}$]

$\frac{dP}{dV} V = P\alpha V \Rightarrow \frac{dP}{(dV/V)} = P\alpha V$ $\therefore K = P\alpha V$

Problem 59. The ratio of two specific heats of gas C_p/C_v for argon is1.6 and for hydrogen is 1.4. Adiabatic elasticity of argon at pressure P is E. Adiabatic elasticity of hydrogen will also be equal to E at the pressure

(a) P (b) $\frac{8}{7}P$ (c) $\frac{7}{8}P$ (d) 1.4 P

Solution : (b) Adiabatic elasticity = γ (pressure)

For Argon $(E_\phi)_{Ar} = 1.6P$ and for Hydrogen $(E_\phi)_{H_2} = 1.4P'$

According to problem $(E_\phi)_{H_2} = (E_\phi)_{Ar} \Rightarrow 1.4P' = 1.6P \Rightarrow P' = \frac{16}{14}P = \frac{8}{7}P$.

Problem 60. The pressure applied from all directions on a cube is P. How much its temperature should be raised to maintain the original volume ? The volume elasticity of the cube is β and the coefficient of volume expansion is α

(a) $\frac{P}{\alpha\beta}$ (b) $\frac{P\alpha}{\beta}$ (c) $\frac{P\beta}{\alpha}$ (d) $\frac{\alpha\beta}{P}$

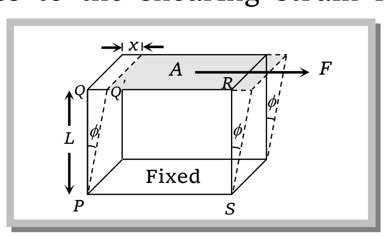
Solution : (a) Change in volume due to rise in temperature $\Delta V = V\alpha\,\Delta\theta$

$\therefore$ volumetric strain $= \frac{\Delta V}{V} = \alpha\,\Delta\theta$

But bulk modulus $\Rightarrow \beta = \frac{\text{stress}}{\text{strain}} = \frac{P}{\alpha\,\Delta\theta}$ $\therefore \Delta\theta = \frac{P}{\alpha\beta}$

9.17 Modulus of Rigidity

Within limits of proportionality, the ratio of tangential stress to the shearing strain is called modulus of rigidity of the material of the body and is denoted by η, *i.e.* $\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$



In this case the shape of a body changes but its volume remains unchanged.

Consider a cube of material fixed at its lower face and acted upon by a tangential force F at its upper surface having area A. The shearing stress, then, will be

$$\text{Shearing stress} = \frac{F_{\parallel}}{A} = \frac{F}{A}$$

This shearing force causes the consecutive horizontal layers of the cube to be slightly displaced or sheared relative to one another, each line such as PQ or RS in the cube is rotated through an angle ϕ by this shear. The shearing strain is defined as the angle ϕ in radians through which a line normal to a fixed surface has turned. For small values of angle,

$$\text{Shearing strain} = \phi = \frac{QQ'}{PQ} = \frac{x}{L}$$

So $$\eta = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\phi} = \frac{F}{A\phi}$$

Only solids can exhibit a shearing as these have definite shape.

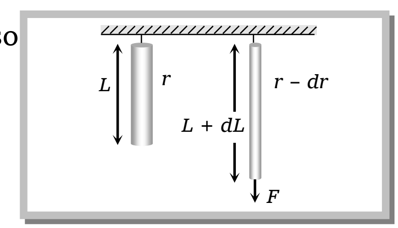
9.18 Poisson's Ratio

When a long bar is stretched by a force along its length then its length increases and the radius decreases as shown in the figure.

Lateral strain : The ratio of change in radius to the original radius is called lateral strain.

Longitudinal strain : The ratio of change in length to the original length is called longitudinal strain.

The ratio of lateral strain to longitudinal strain is called Poisso



i.e. $$\sigma = \frac{\text{Lateral strain}}{\text{Longitudin al strain}}$$

$$\sigma = \frac{-dr/r}{dL/L}$$

Negative sign indicates that the radius of the bar decreases when it is stretched.

Poisson's ratio is a dimensionless and a unitless quantity.

9.19 Relation Between Volumetric Strain, Lateral Strain and Poisson's Ratio

If a long bar have a length L and radius r then volume $V = \pi r^2 L$

Differentiating both the sides $dV = \pi r^2 dL + \pi 2rL\,dr$

Dividing both the sides by volume of bar $\dfrac{dV}{V} = \dfrac{\pi r^2 dL}{\pi r^2 L} + \dfrac{\pi 2rL\,dr}{\pi r^2 L} = \dfrac{dL}{L} + 2\dfrac{dr}{r}$

$\Rightarrow$ Volumetric strain = longitudinal strain + 2(lateral strain)

$\Rightarrow \quad \frac{dV}{V} = \frac{dL}{L} + 2\sigma\frac{dL}{L} = (1+2\sigma)\frac{dL}{L}$ $\quad \left[\text{As } \sigma = \frac{dr/r}{dL/L} \Rightarrow \frac{dr}{r} = \sigma\frac{dL}{L}\right]$

or $\quad \sigma = \frac{1}{2}\left[1 - \frac{dV}{AdL}\right]$ [where A = cross-section of bar]

Important points

(i) If a material having σ = – 0.5 then $\frac{dV}{V} = [1+2\sigma]\frac{dL}{L}$ = 0

$\therefore$ Volume = constant or $K = \infty$ *i.e.*, the material is incompressible.

(ii) If a material having σ = 0, then lateral strain is zero *i.e.* when a substance is stretched its length increases without any decrease in diameter *e.g.* cork. In this case change in volume is maximum.

(iii) Theoretical value of Poisson's ratio $-1 < \sigma < 0.5$.

(iv) Practical value of Poisson's ratio $0 < \sigma < 0.5$

9.20 Relation between *Y*, *k*, η and σ

Moduli of elasticity are three, *viz.* *Y*, *K* and η while elastic constants are four, *viz*, *Y*, *K*, η and σ. Poisson's ratio σ is not modulus of elasticity as it is the ratio of two strains and not of stress to strain. Elastic constants are found to depend on each other through the relations : $Y = 3K(1-2\sigma)$ and $Y = 2\eta(1+\sigma)$

Eliminating σ or Y between these, we get $Y = \frac{9K\eta}{3K+\eta}$ and $\sigma = \frac{3K-2\eta}{6K+2\eta}$

Sample problems based on relation between Y, k, η and σ

Problem 61. Minimum and maximum values of Poisson's ratio for a metal lies between

(a) – ∞ to + ∞ (b) 0 to 1 (c) – ∞ to 1 (d) 0 to 0.5

Solution : (d)

Problem 62. For a given material, the Young's modulus is 2.4 times that of rigidity modulus. Its Poisson's ratio is

[EAMCET 1990; RPET 2001]

(a) 2.4 (b) 1.2 (c) 0.4 (d) 0.2

Solution : (d) $Y = 2\eta(1+\sigma) \Rightarrow 2.4\eta = 2\eta(1+\sigma) \Rightarrow 1.2 = 1+\sigma \Rightarrow \sigma = 0.2$

Problem 63. There is no change in the volume of a wire due to change in its length on stretching. The Poisson's ratio of the material of the wire is

(a) + 0.50 (b) – 0.50 (c) + 0.25 (d) – 0.25

Solution : (b) $\frac{dV}{V} = \frac{dL}{L} + 2\sigma\frac{dL}{L} = (1+2\sigma)\frac{dL}{L}$ = 0 [As there is no change in the volume of the wire]

$\therefore 1 + 2\sigma = 0 \Rightarrow \sigma = -\frac{1}{2}$

***Problem* 64.** The values of Young's and bulk modulus of elasticity of a material are $8\times10^{10}\ N/m^2$ and $10\times10^{10}\ N/m^2$ respectively. The value of Poisson's ratio for the material will be

(a) 0.25 (b) – 0.25 (c) 0.37 (d) – 0.37

Solution : (c) $Y = 3K(1-2\sigma) \Rightarrow 8\times10^{10} = 3\times10\times10^{10}(1-2\sigma) \Rightarrow \sigma = 0.37$

***Problem* 65.** The Poisson's ratio for a metal is 0.25. If lateral strain is 0.0125, the longitudinal strain will be

(a) 0.125 (b) 0.05 (c) 0.215 (d) 0.0125

Solution : (b) $\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$ $\therefore$ Longitudinal strain $= \frac{\text{Lateral strain}}{\sigma} = \frac{0.0125}{0.25} = 0.05$

***Problem* 66.** The 'σ' of a material is 0.20. If a longitudinal strain of 4.0×10^{-3} is caused, by what percentage will the volume change

(a) 0.48% (b) 0.32% (c) 0.24% (d) 0.50%

Solution : (c) Longitudinal strain $= 4\times10^{-3}$ or 0.4%

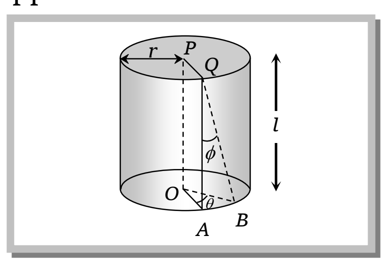
Lateral strain $= \sigma\times0.4\% = 0.2\times0.4\% = 0.08\%$

$\therefore$ Volumetric strain = longitudinal strain – 2 lateral strain $= 0.4 - 2\times(0.08)$ = 0.24%

$\therefore$ Volume will change by 0.24%.

9.21 Torsion of Cylinder

If the upper end of a cylinder is clamped and a torque is applied at the lower end the cylinder gets twisted by angle θ. Simultaneously shearing strain ϕ is produced in the cylinder.



(i) The angle of twist θ is directly proportional to the distance from the fixed end of the cylinder.

At fixed end $\theta = 0^o$ and at free end θ = maximum.

(ii) The value of angle of shear ϕ is directly proportional to the radius of the cylindrical shell.

At the axis of cylinder ϕ = 0 and at the outermost shell ϕ = maximum.

(iii) Relation between angle of twist (θ) and angle of shear (ϕ)

$$AB = r\theta = \phi l \qquad \therefore\ \phi = \frac{r\theta}{l}$$

(iv) Twisting couple per unit twist or torsional rigidity or torque required to produce unit twist.

$$C = \frac{\pi \eta r^4}{2l} \qquad \therefore\ C \propto r^4 \propto A^2$$

(v) Work done in twisting the cylinder through an angle θ is $W = \frac{1}{2} C\theta^2 = \frac{\pi \eta r^4 \theta^2}{4l}$

Sample problems based on Torsion

Problem 67. Mark the wrong statement

(a) Sliding of molecular layer is much easier than compression or expansion

(b) Reciprocal of bulk modulus of elasticity is called compressibility

(c) It is difficult to twist a long rod as compared to small rod

(d) Hollow shaft is much stronger than a solid rod of same length and same mass

Solution : (c)

Problem 68. A rod of length l and radius r is joined to a rod of length $l / 2$ and radius $r / 2$ of same material. The free end of small rod is fixed to a rigid base and the free end of larger rod is given a twist of θ, the twist angle at the joint will be

(a) $\theta / 4$ (b) $\theta / 2$ (c) $5\theta / 6$ (d) $8\theta / 9$

Solution : (d) If torque τ is applied at the free end of larger rod and twist θ is given to it then twist at joint is θ_1 and twist at the upper end (fixed base) θ_2

$$\tau = \frac{\pi \eta r^4 (\theta - \theta_1)}{2l} = \frac{\pi \eta \left(\frac{r}{2}\right)^4 (\theta_1 - \theta_2)}{2(l/2)}$$

$$\Rightarrow \qquad (\theta - \theta_1) = \frac{(\theta_1 - 0)}{8} \qquad [\text{As } \theta_2 = 0]$$

$$\Rightarrow \qquad 8\theta - 8\theta_1 = \theta_1 \Rightarrow 9\theta_1 = 8\theta \Rightarrow \theta_1 = \frac{8\theta}{9}.$$

Problem 69. The upper end of a wire of radius 4 *mm* and length 100 *cm* is clamped and its other end is twisted through an angle of 30^o. Then angle of shear is **[NCERT 1990; MP PMT 1996]**

(a) 12^o (b) 0.12^o (c) 1.2^o (d) 0.012^o

Solution : (b) $L\phi = r\theta \qquad \therefore \phi = \frac{r\theta}{L} = \frac{4 \times 10^{-3} \times 30^o}{1} = 0.12^o$

Problem 70. Two wires A and B of same length and of the same material have the respective radii r_1 and r_2. Their one end is fixed with a rigid support, and at the other end equal twisting couple is applied. Then the ratio of the angle of twist at the end of A and the angle of twist at the end of B will be **[AIIMS 1980]**

(a) $\frac{r_1^2}{r_2^2}$ (b) $\frac{r_2^2}{r_1^2}$ (c) $\frac{r_2^4}{r_1^4}$ (d) $\frac{r_1^4}{r_2^4}$

Solution : (c) $\tau_1 = \tau_2 \Rightarrow \frac{\pi\eta r_1^4\theta_1}{2l_1} = \frac{\pi\eta r_2^4\theta_2}{2l_2} \Rightarrow \frac{\theta_1}{\theta_2} = \left(\frac{r_2}{r_1}\right)^4$

Problem **71.** The work done in twisting a steel wire of length 25 *cm* and radius 2*mm* through 45^o will be $(\eta = 8\times10^{10}\ N/m^2)$

(a) 2.48 J (b) 3.1 J (c) 15.47 J (d) 18.79 J

Solution : (a) $W = \frac{1}{2}C\theta^2 = \frac{\pi\eta r^4\theta^2}{4l} = \frac{3.14\times8\times10^{10}\times(2\times10^{-3})^4\times(\pi/4)^2}{4\times25\times10^{-2}} = 2.48\ J$

9.22 Interatomic Force Constant

Behaviour of solids with respect to external forces is such that if their atoms are connected to springs. When an external force is applied on a solid, this distance between its atoms changes and interatomic force works to restore the original dimension.

The ratio of interatomic force to that of change in interatomic distance is defined as the interatomic force constant. $K = \frac{F}{\Delta r}$

It is also given by $K = Y\times r_0$ [Where Y = Young's modulus, r_0 = Normal distance between the atoms of wire]

Unit of interatomic force constant is N/m and Dimension MT^{-2}

Note : ❑ The number of atoms having interatomic distance r_0 in length l of a wire, $N = l/r_0$.

❑ The number of atoms in area A of wire having interatomic separation r_0 is $N = A/r_0^2$.

Sample problems based on Interatomic Force Constant

Problem **72.** The mean distance between the atoms of iron is $3\times10^{-10}m$ and interatomic force constant for iron is 7 N/m. The Young's modulus of elasticity for iron is

(a) $2.33\times10^5\ N/m^2$ (b) $23.3\times10^{10}\ N/m^2$ (c) $233\times10^{10}\ N/m^2$ (d) $2.33\times10^{10}\ N/m^2$

Solution : (d) $Y = \frac{k}{r_o} = \frac{7}{3\times10^{-10}} = 2.33\times10^{10}\ N/m^2$.

Problem **73.** The Young's modulus for steel is $Y = 2\times10^{11}\ N/m^2$. If the inter-atomic distance is 3.2$Å$, the inter atomic force constant in $N/Å$ will be

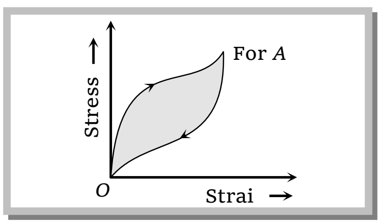
(a) 6.4×10^{9} (b) 6.4×10^{-9} (c) 3.2×10^{9} (d) 3.2×10^{-9}

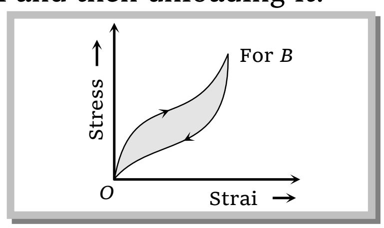
Solution : (b) $k = Y \times r_0 = 2 \times 10^{11} \times 3.2 \times 10^{-10} = 6.4 \times 10^{1}\ N/m = 6.4 \times 10^{-9}\ N/Å$.

9.23 Elastic Hysteresis

When a deforming force is applied on a body then the strain does not change simultaneously with stress rather it lags behind the stress. The lagging of strain behind the stress is defined as elastic hysteresis. This is the reason why the values of strain for same stress are different while increasing the load and while decreasing the load.

Hysteresis loop : The area of the stress-strain curve is called the hysteresis loop and it is numerically equal to the work done in loading the material and then unloading it.





If we have two tyres of rubber having different hysteresis loop then rubber *B* should be used for making the car tyres. It is because of the reason that area under the curve *i.e.* work done in case of rubber *B* is lesser and hence the car tyre will not get excessively heated and rubber *A* should be used to absorb vibration of the machinery because of the large area of the curve, a large amount of vibrational energy can be dissipated.

9.24 Factors Affecting Elasticity

(1) Hammering and rolling : Crystal grains break up into smaller units by hammering and rolling. This result in increase in the elasticity of material.

(2) Annealing **:** The metals are annealed by heating and then cooling them slowly. Annealing results in decrease in the elasticity of material.

(3) Temperature **:** Intermolecular forces decreases with rise in temperature. Hence the elasticity decreases with rise in temperature but the elasticity of invar steel (alloy) does not change with change of temperature.

(4) Impurities **:** Due to impurities in a material elasticity can increase or decrease. The type of effect depends upon the nature of impurities present in the material.

9.25 Important Facts About Elasticity

(1) The body which requires greater deforming force to produce a certain change in dimension is more elastic.

Example : Ivory and steel balls are more elastic than rubber.

(2) When equal deforming force is applied on different bodies then the body which shows less deformation is more elastic.

Example : (i) For same load, more elongation is produced in rubber wire than in steel wire hence steel is more elastic than rubber.

(ii) Water is more elastic than air as volume change in water is less for same applied pressure.

(iii) Four identical balls of different materials are dropped from the same height then after collision balls rises upto different heights.

The order of their height can be given by $h_{\text{ivory}} > h_{\text{steel}} > h_{\text{rubber}} > h_{\text{clay}}$ because $Y_{\text{ivory}} > Y_{\text{steel}} > Y_{\text{rubber}} > Y_{\text{clay}}$.

(3) The value of moduli of elasticity is independent of the magnitude of the stress and strain. It depends only on the nature of material of the body.

(4) For a given material there can be different moduli of elasticity depending on the type of stress applied and resulting strain.

Name of substance	**Young's modulus (Y) $10^{10} N/m^2$**	**Bulk modulus (K) $10^{10} N/m^2$**	**Modulus of rigidity (η) $10^{10} N/m^2$**
Aluminium	6.9	7.0	2.6
Brass	9.0	6.7	3.4
Copper	11.0	13.0	4.5
Iron	19.0	14.0	4.6
Steel	20.0	16.0	8.4
Tungsten	36.0	20.0	15.0
Diamond	83.0	55.0	34.0
Water	–	0.22	–
Glycerin	–	0.45	–
Air	–	1.01	–

(5) The moduli of elasticity has same dimensional formula and units as that of stress since strain is dimensionless. $\therefore$ Dimensional formula $ML^{-1}T^{-2}$ while units $dyne/cm^2$ or $Newton/m^2$.

(6) Greater the value of moduli of elasticity more elastic is the material. But as $Y \propto (1/l)$, $K \propto (1/\Delta V)$ and $\eta \propto (1/\phi)$ for a constant stress, so smaller change in shape or size for a given stress corresponds to greater elasticity.

(7) The moduli of elasticity Y and η exist only for solids as liquids and gases cannot be deformed along one dimension only and also cannot sustain shear strain. However K exist for all states of matter *viz*. solid, liquid or gas.

(8) Gases being most compressible are least elastic while solids are most *i.e.* the bulk modulus of gas is very low while that for liquids and solids is very high. $K_{solid} > K_{liquid} > K_{gas}$

(9) For a rigid body l, ΔV or ϕ = 0 so Y, K or η will be ∞, *i.e.* elasticity of a rigid body is infinite.

Diamond and carborundum are nearest approach to rigid bodies.

(10) In a suspension bridge there is a stretch in the ropes by the load of the bridge. Due to which length of rope changes. Hence Young's modulus of elasticity is involved.

(11) In an automobile tyre as the air is compressed, volume of the air in tyre changes, hence the bulk modulus of elasticity is involved.

(12) In transmitting power, an automobile shaft is sheared as it rotates, so shearing strain is set up, hence modulus of rigidity is involved.

(13) The shape of rubber heels changes under stress, so modulus of rigidity is involved.

9.26 Practical Applications of Elasticity

(i) The metallic parts of machinery are never subjected to a stress beyond elastic limit, otherwise they will get permanently deformed.

(ii) The thickness of the metallic rope used in the crane in order to lift a given load is decided from the knowledge of elastic limit of the material of the rope and the factor of safety.

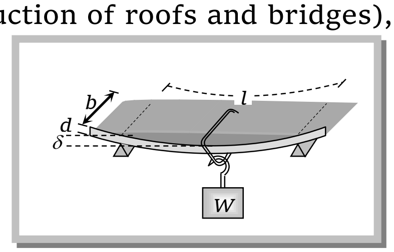
(iii) The bridges are declared unsafe after long use because during its long use, a bridge under goes quick alternating strains continuously. It results in the loss of elastic strength.

(iv) Maximum height of a mountain on earth can be estimated from the elastic behaviour of earth.

At the base of the mountain, the pressure is given by $P = h\rho g$ and it must be less than elastic limit (K) of earth's supporting material.

$$K > P > h\rho g \qquad \therefore\ h < \frac{K}{\rho g} \qquad \text{or} \qquad h_{max} = \frac{K}{\rho g}$$

(v) In designing a beam for its use to support a load (in construction of roofs and bridges), it is advantageous to increase its depth rather than the breadth of the beam because the depression in rectangular beam.



$$\delta = \frac{Wl^3}{4Ybd^3}$$

To minimize the depression in the beam, it is designed as I-shaped girder.

(vi) For a beam with circular cross-section depression is given by $\delta = \dfrac{WL^3}{12\pi r^4 Y}$

(vii) A hollow shaft is stronger than a solid shaft made of same mass, length and material.

Torque required to produce a unit twist in a solid shaft $\tau_{\text{solid}} = \frac{\pi\eta r^4}{2l}$ ......(i)

and torque required to produce a unit twist in a hollow shaft $\tau_{\text{hollow}} = \frac{\pi\eta(r_2^4 - r_1^4)}{2l}$ .....(ii)

From (i) and (ii), $\frac{\tau_{\text{hollow}}}{\tau_{\text{solid}}} = \frac{r_2^4 - r_1^4}{r^4} = \frac{(r_2^2 + r_1^2)(r_2^2 - r_1^2)}{r^4}$ .....(iii)

Since two shafts are made from equal volume $\therefore\ \pi r^2 l = \pi(r_2^2 - r_1^2)l \Rightarrow r^2 = r_2^2 - r_1^2$

Substituting this value in equation (iii) we get, $\frac{\tau_{\text{hollow}}}{\tau_{\text{solid}}} = \frac{r_2^2 + r_1^2}{r^2} > 1 \quad \therefore\ \tau_{\text{hollow}} > \tau_{\text{solid}}$

i.e., the torque required to twist a hollow shaft is greater than the torque necessary to twist a solid shaft of the same mass, length and material through the same angle. Hence, a hollow shaft is stronger than a solid shaft.

Practice Problems

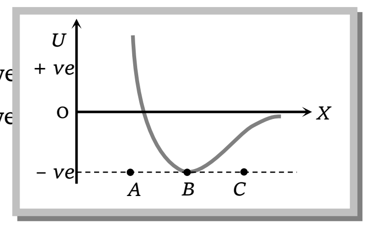
Problems based on Interatomic and Intermolecular forces

1. In solids, inter-atomic forces are

(a) Totally repulsive (b) Totally attractive

(c) Combination of (a) and (b) (d) None of these

2. The potential energy U between two molecules as a function of the distance X between them has been shown in the figure. The two molecules are



(a) Attracted when x lies between A and B and are repelled when X lies betwe

(b) Attracted when x lies between B and C and are repelled when X lies betwe

(c) Attracted when they reach B

(d) Repelled when they reach B

3. The nature of molecular forces resembles with the nature of the

(a) Gravitational force (b) Nuclear force (c) Electromagnetic force (d) Weak force

Problems based on Stress

4. The ratio of radius of two wire of same material is 2 : 1. Stretched by same force, then the ratio of stress is **[PET 1991]**

(a) 2 : 1 (b) 1 : 2 (c) 1 : 4 (d) 4 : 1

5. If equal and opposite forces applied to a body tend to elongate it, the stress so produced is called

(a) Tensile stress (b) Compressive stress (c) Tangential stress (d) Working stress

6. A vertical hanging bar of length l and mass m per unit length carries a load of mass M at the lower end, its upper end is clamped to a rigid support. The tensile force at a distance x from support is

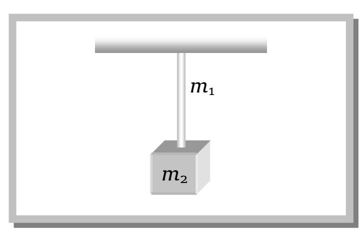
(a) $Mg + mg(l - x)$ (b) Mg (c) $Mg + mgl$ (d) $(M+m)g\dfrac{x}{l}$

7. One end of a uniform rod of mass m_1 and cross-sectional area A is hung from a ceiling. The other end of the bar is supporting mass m_2. The stress at the midpoint is

(a) $\dfrac{g(m_2+2m_1)}{2A}$

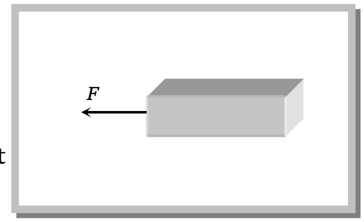
(b) $\dfrac{g(m_2+m_1)}{2A}$

(c) $\dfrac{g(2m_2+m_1)}{2A}$



(d) $\frac{g(m_2+m_1)}{A}$

8. A uniform bar of square cross-section is lying along a frictionless horizontal surface. A horizontal force is applied to pull it from one of its ends then



(a) The bar is under same stress throughout its length

(b) The bar is not under any stress because force has been applied only at

(c) The bar simply moves without any stress in it

(d) The stress developed reduces to zero at the end of the bar where no force is applied

Problems based on Strain

9. Which one of the following quantities does not have the unit of force per unit area

(a) Stress (b) Strain

(c) Young's modulus of elasticity (d) Pressure

10. The reason for the change in shape of a regular body is **[EAMCET 1980]**

(a) Volume stress (b) Shearing strain (c) Longitudinal strain (d) Metallic strain

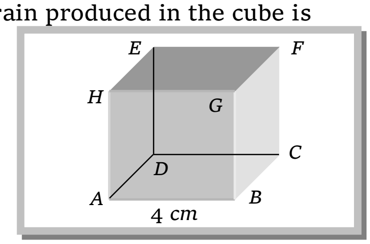
11. When a spiral spring is stretched by suspending a load on it, the strain produced is called

(a) Shearing (b) Longitudinal (c) Volume (d) Transverse

12. The longitudinal strain is only possible in

(a) Gases (b) Fluids (c) Solids (d) Liquids

13. The face *EFGH* of the cube shown in the figure is displaced 2 *mm* parallel to itself when forces of $5\times10^5 N$ each are applied on the lower and upper faces. The lower face is fixed. The strain produced in the cube is



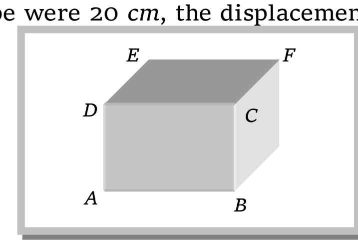
(a) 2

(b) 0.5

(c) 0.05

(d) 1.2×10^8

14. Forces of $10^5 N$ each are applied in opposite direction on the upper and lower faces of a cube of side 10 *cm*, shifting the upper face parallel to itself by 0.5 *cm*. If the side of the cube were 20 *cm*, the displacement would be



(a) 1 *cm*

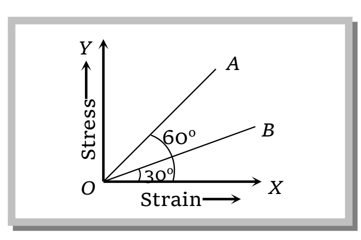
(b) 0.5 *cm*

(c) 0.25 *cm*

(d) 0.125 *cm*

Problems based on Stress strain curve

15. The stress versus strain graphs for wires of two materials *A* and *B* are as shown in the figure. If Y_A and Y_B are the Young's modulii of the materials, then



(a) $Y_B = 2Y_A$

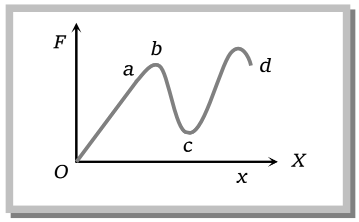
(b) $Y_A = Y_B$

(c) $Y_B = 3Y_A$

(d) $Y_A = 3Y_B$

16. The graph is drawn between the applied force F and the strain (x) for a thin uniform wire. The wire behaves as a liquid in the part

[CPMT 1988]



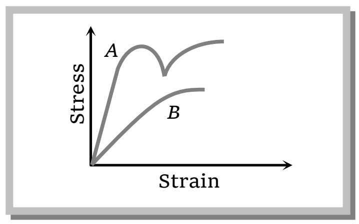
(a) ab

(b) bc

(c) cd

(d) oa

17. The diagram shows stress v/s strain curve for the materials A and B. From the curves we infer that



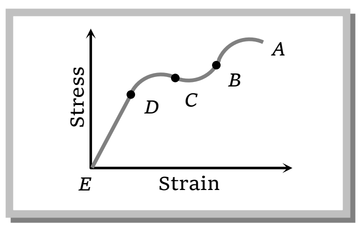
(a) A is brittle but B is ductile

(b) A is ductile and B is brittle

(c) Both A and B are ductile

(d) Both A and B are brittle

18. The figure shows the stress-strain graph of a certain substance. Over which region of the graph is Hooke's law obeyed



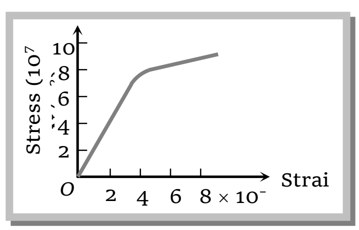
(a) AB

(b) BC

(c) CD

(d) ED

19. Which one of the following is the Young's modulus (in N/m^2) for the wire having the stress-strain curve shown in the figure



(a) 24×10^{11}

(b) 8.0×10^{11}

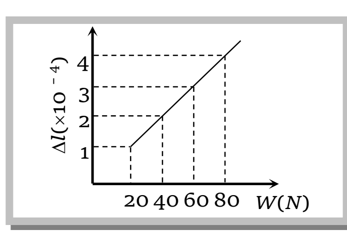
(c) 10×10^{11}

(d) 2.0×10^{11}

Problems based on Young's Modulus

20. The adjacent graph shows the extension (Δl) of a wire of length 1m suspended from the top of a roof at one end with a load W connected to the other end. If the cross sectional area of the wire is $10^{-6} m^2$, calculate the young's modulus of the material of the wire

[IT-JEE (Screening) 2003]



(a) $2\times10^{11}\ N/m^2$

(b) $2\times10^{-11}\ N/m^2$

(c) $3\times10^{-12}\ N/m^2$

(d) $2\times10^{-13}\ N/m^2$

21. In the Young's experiment, if length of wire and radius both are doubled then the value of Y will become **[RPET 2003]**

(a) 2 times (b) 4 times (c) Remains same (d) Half

22. A rubber cord catapult has cross-sectional area $25 mm^2$ and initial length of rubber cord is $10 cm$. It is stretched to $5 cm$. and then released to project a missile of mass $5 gm$. Taking $Y_{rubber} = 5\times10^8\ N/m^2$ velocity of projected missile is **[CPMT 2002]**

(a) $20\ ms^{-1}$ (b) $100\ ms^{-1}$ (c) $250\ ms^{-1}$ (d) $200\ ms^{-1}$

23. Consider the following statements

Assertion (*A*) : Stress is the internal force per unit area of a body.

Reason (*R*) : Rubber is more elastic than steel.

Of these statements **[AIIMS 2002]**

(a) Both A and R are true and the R is a correct explanation of the A

(b) Both A and R are true but the R is not a correct explanation of the A

(c) A is true but the R is false

(d) Both A and R are false

(e) A is false but the R is true

24. The area of cross-section of a steel wire $(Y = 2.0\times10^{11}\ N/m^2)$ is $0.1\ cm^2$. The force required to double its length will be

[MP PET 2002]

(a) $2\times10^{12}\ N$ (b) $2\times10^{11}\ N$ (c) $2\times10^{10}\ N$ (d) $2\times10^{6}\ N$

25. A metal bar of length L and area of cross-section A is clamped between two rigid supports. For the material of the rod, its Young's modulus is Y and coefficient of linear expansion is α. If the temperature of the rod is increased by $\Delta t^o C$, the force exerted by the rod on the supports is

(a) $Y\,AL\,\Delta t$ (b) $Y\,A\,\alpha\,\Delta t$ (c) $\frac{YL\,\alpha\Delta t}{A}$ (d) $Y\alpha\,AL\,\Delta t$

26. Which one of the following substances possesses the highest elasticity **[MP PMT 1992; RPMT 1999; RPET 2000; MH CET (M**

(a) Rubber (b) Glass (c) Steel (d) Copper

27. There are two wires of same material and same length while the diameter of second wire is 2 times the diameter of first wire, then ratio of extension produced in the wires by applying same load will be

(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 4 : 1

28. Consider the following statements

Assertion (*A*) : Rubber is more elastic than glass.

Reason (*R*) : The rubber has higher modulus of elasticity than glass.

Of these statements **[AIIMS 2000]**

(a) Both A and R are true and the R is a correct explanation of the A

(b) Both A and R are true but the R is not a correct explanation of the A

(c) A is true but the R is false

(d) Both A and R are false

(e) A is false but the R is true

29. The longitudinal extension of any elastic material is very small. In order to have an appreciable change, the material must be in the form of

(a) Thin block of any cross section (b) Thick block of any cross section

(c) Long thin wire (d) Short thin wire

30. In suspended type moving coil galvanometer, quartz suspension is used because

(a) It is good conductor of electricity (b) Elastic after effects are negligible

(c) Young's modulus is greater (d) There is no elastic limit

31. You are given three wires *A*, *B* and *C* of the same length and cross section. They are each stretched by applying the same force to the ends. The wire *A* is stretched least and comes back to its original length when the stretching force is removed. The wire *B* is stretched more than *A* and also comes back to its original length when the stretching force is removed. The wire *C* is stretched most and remains stretched even when stretching force is removed. The greatest Young's modulus of elasticity is possessed by the material of wire

(a) *A* (b) *B* (c) *C* (d) All have the same elasticity

32. The ratio of diameters of two wires of same material is $n : 1$. The length of wires are 4 *m* each. On applying the same load, the increase in length of thin wire will be

(a) n^2 times (b) n times (c) $2n$ times (d) None of the above

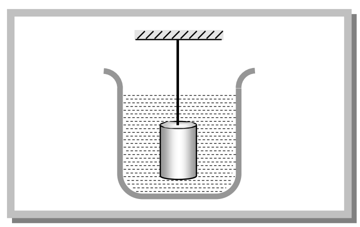
33. A wire of radius *r*, Young's modulus *Y* and length *l* is hung from a fixed point and supports a heavy metal cylinder of volume *V* at its lower end. The change in length of wire when cylinder is immersed in a liquid of density ρ is in fact

(a) Decrease by $\frac{Vl\rho g}{Y\pi r^2}$

(b) Increase by $\frac{Vr\rho g}{Y\pi l^2}$

(c) Decrease by $\frac{V\rho g}{Y\pi r}$

(d) $\frac{V\rho g}{Y\pi}$



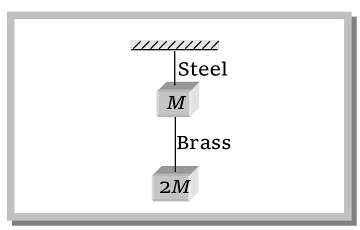
34. If the ratio of lengths, radii and Young's modulii of steel and brass wires in the figure are *a*, *b* and *c* respectively. Then the corresponding ratio of increase in their lengths would be

(a) $\frac{2a^2c}{b}$

(b) $\frac{3a}{2b^2c}$

(c) $\frac{2ac}{b^2}$

(d) $\frac{3c}{2ab^2}$

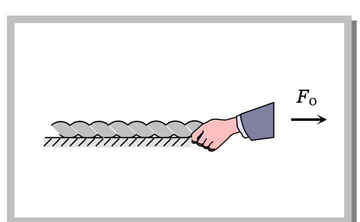


35. A uniform heavy rod of weight *W*, cross sectional area *A* and length *L* is hung from a fixed support. Young's modulus of the material of the rod is *Y*. If lateral contraction is neglected, the elongation of the rod under its own weight is

(a) $\frac{2WL}{AY}$ (b) $\frac{WL}{AY}$ (c) $\frac{WL}{2AY}$ (d) Zero

36. A constant force F_0 is applied on a uniform elastic string placed over a smooth horizontal surface as shown in figure. Young's modulus of string is *Y* and area of cross-section is *S*. The strain produced in the string in the direction of force is

(a) $\frac{F_0 Y}{S}$



(b) $\frac{F_0}{SY}$

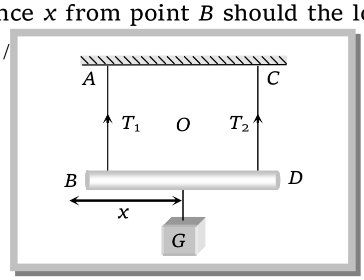
(c) $\frac{F_0}{2SY}$

(d) $\frac{F_0 Y}{2S}$

37. A uniform rod of length L has a mass per unit length λ and area of cross section A. The elongation in the rod is l due to its own weight if it is suspended from the ceiling of a room. The Young's modulus of the rod is

(a) $\frac{2\lambda gL^2}{Al}$ (b) $\frac{\lambda gL^2}{2Al}$ (c) $\frac{2\lambda gL}{Al}$ (d) $\frac{\lambda gl^2}{AL}$

38. AB is an iron wire and CD is a copper wire of same length and same cross-section. BD is a rod of length 0.8 m. A load G = 2kg-wt is suspended from the rod. At what distance x from point B should the load be suspended for the rod to remain in a horizontal position $(Y_{Cu} = 11.8 \times 10^{10} N /$



(a) 0.1 m

(b) 0.3 m

(c) 0.5 m

(d) 0.7 m

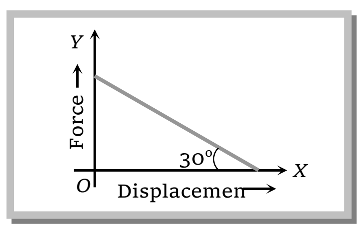
39. A slightly conical wire of length L and end radii r_1 and r_2 is stretched by two forces F, F applied parallel to length in opposite directions and normal to end faces. If Y denotes the Young's modulus, then extension produced is

(a) $\frac{FL}{\pi r_1^2 Y}$ (b) $\frac{FL}{\pi r_1 Y}$ (c) $\frac{FL}{\pi r_1 r_2 Y}$ (d) $\frac{FLY}{\pi r_1 r_2}$

40. The force constant of wire is K and its area of cross-section is A. If the force F is applied on it, then the increase in its length will be

(a) KA (b) FKA (c) $\frac{F}{K}$ (d) $\frac{FK}{AL}$

41. The value of force constant between the applied elastic force F and displacement will be



(a) $\sqrt{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{2}$

(d) $\frac{\sqrt{3}}{2}$

42. The force constant of a wire does not depend on

(a) Nature of the material (b) Radius of the wire (c) Length of the wire (d)

43. A metal wire of length L, area of cross-section A and Young's modulus Y behaves as a spring. The equivalent spring constant will be

(a) $\frac{Y}{AL}$ (b) $\frac{YA}{L}$ (c) $\frac{YL}{A}$ (d) $\frac{L}{AY}$

44. A highly rigid cubical block A of small mass M and side L is fixed rigidly onto another cubical block B of the same dimensions and modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force is applied perpendicular to one of the sides faces of A. After the force is withdrawn, block A execute small oscillations the time period of which is given by

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\frac{M\eta}{L}}$ (c) $2\pi\sqrt{\frac{ML}{\eta}}$ (d) $2\pi\sqrt{\frac{M}{\eta L}}$

Problems based on Stretching a wire

45. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. It is stretched by an amount x. The work done is

(a) $\frac{YxA}{2L}$ (b) $\frac{Yx^2A}{L}$ (c) $\frac{Yx^2A}{2L}$ (d) $\frac{2Yx^2A}{L}$

46. Two wires of same diameter of the same material having the length l and $2l$. If the force F is applied on each, the ratio of the work done in the two wires will be

(a) 1 : 2 (b) 1 : 4 (c) 2 : 1 (d) 1 : 1

47. If the potential energy of a spring is V on stretching it by 2 cm, then its potential energy when it is stretched by 10 cm will be

[CPMT 1976]

(a) $V/25$ (b) $5V$ (c) $V/5$ (d) $25V$

48. The strain energy stored in a body of volume V due to shear S and shear modulus η is

(a) $\frac{S^2V}{2\eta}$ (b) $\frac{SV^2}{2\eta}$ (c) $\frac{S^2V}{\eta}$ (d) $\frac{1}{2}\eta S^2V$

49. K is the force constant of a spring. The work done in increasing its extension from l_1 to l_2 will be

(a) $K(l_2 - l_1)$ (b) $\frac{K}{2}(l_2 + l_1)$ (c) $K(l_2^2 - l_1^2)$ (d) $\frac{K}{2}(l_2^2 - l_1^2)$

Problems based on Breaking of wire

50. The breaking stress of a wire depends upon **[AIIMS 2002]**

(a) Length of the wire (b) Radius of the wire (c) Material of the wire (d) Shape of the cross section

51. An aluminium rod has a breaking strain of 0.2%. The minimum cross sectional area of the rod, in m^2, in order to support a load of 10^4 N is $(Y = 7\times10^9\ N/m^2)$

(a) 1.4×10^{-4} (b) 7.1×10^{-4} (c) 1.4×10^{-3} (d) 7.1×10^{-5}

52. A cable is replaced by another one of the same length and material but of twice the diameter. The maximum load that the new wire can support without exceeding the elastic limit, as compared to the load that the original wire could support, is

(a) Half (b) Double (c) Four times (d) One-fourth

53. A heavy mass is attached to a thin wire and is whirled in a vertical circle. The wire is most likely to break

(a) When the mass is at the highest point (b) When the mass is at the lowest point
(c) When the wire is horizontal (d) At an angle of $\cos^{-1}(1/3)$ from the upward vertical

54. A heavy uniform rod is hanging vertically from a fixed support. It is stretched by its own weight. The diameter of the rod is

(a) Smallest at the top and gradually increases down the rod
(b) Largest at the top and gradually decreases down the rod
(c) Uniform everywhere

(d) Maximum in the middle

Problems based on Bulk modulus

55. The isothermal bulk modulus of a gas at atmospheric pressure is

(a) 1 *mm* of *Hg* (b) 13.6 *mm* of *Hg* (c) $1.013 \times 10^5\ N/m^2$ (d) $2.026 \times 10^5\ N/m^2$

56. The specific heat at constant pressure and at constant volume for an ideal gas are C_p and C_v and its adiabatic and isothermal elasticities are E_ϕ and E_θ respectively. The ratio of E_ϕ to E_θ is

(a) C_v / C_p (b) C_p / C_v (c) $C_p C_v$ (d) $1/C_p C_v$

57. If a rubber ball is taken at the depth of 200 *m* in a pool. Its volume decreases by 0.1%. If the density of the water is $1 \times 10^3\ kg/m^3$ and $g = 10\ m/s^2$, then the volume elasticity in N/m^2 will be

(a) 10^8 (b) 2×10^8 (c) 10^9 (d) 2×10^9

58. The compressibility of water is 4×10^{-5} per unit atmospheric pressure. The decrease in volume of 100 *cubic centimetre* of water under a pressure of 100 atmosphere will be

(a) 0.4 *cc* (b) 4×10^{-5} *cc* (c) 0.025 *cc* (d) 0.004 *cc*

59. An ideal gas of mass *m*, volume *V*, pressure *p* and temperature *T* undergoes a small change in state at constant temperature. Its adiabatic exponent *i.e.*, $\frac{C_p}{C_v}$ is γ. The bulk modulus of the gas at the constant temperature process called isothermal process is

(a) p (b) γp (c) $\frac{m\gamma p}{T}$ (d) $\frac{\gamma p V}{T}$

60. An ideal gas of mass *m*, volume *V*, pressure *p* and temperature *T* undergoes a small change under a condition that heat can neither enter into it from outside nor can it leave the system. Such a process is called adiabatic process. The bulk modulus of the gas $\left(\gamma = \frac{C_p}{C_v}\right)$ is

(a) p (b) γp (c) $\frac{m\gamma p}{T}$ (d) $\frac{\gamma p V}{T}$

61. An ideal gas whose adiabatic exponent is γ is expanded according to the law $p = \alpha V$ where α is a constant. For this process the bulk modulus of the gas is

(a) p (b) $\frac{p}{\alpha}$ (c) αp (d) $(l - \alpha)p$

62. 1 *c.c.* of water is taken from the top to the bottom of a 200 *m* deep lake. What will be the change in its volume if *K* of water is $2.2 \times 10^9\ N/m^2$

(a) 8.8×10^{-6} *c.c.* (b) 8.8×10^{-2} *c.c.* (c) 8.8×10^{-4} *c.c.* (d) 8.8×10^{-1} *c.c*

Problems based on Modulus of rigidity

63. Modulus of rigidity of a liquid

(a) Non zero constant (b) Infinite (c) Zero (d) Cannot be predicted

64. The Young's modulus of the material of a wire is $6 \times 10^{12}\ N/m^2$ and there is no transverse strain in it, then its modulus of rigidity will be

(a) $3 \times 10^{12}\ N/m^2$ (b) $2 \times 10^{12}\ N/m^2$ (c) $10^{12}\ N/m^2$ (d) None of the above

Problems based on relation between Y, η, K and σ

65. The value of Poisson's ratio lies between **[AIIMS 1985; MP PET 1986; DPMT 2002]**

(a) -1 to $\frac{1}{2}$ (b) $-\frac{3}{4}$ to $-\frac{1}{2}$ (c) $-\frac{1}{2}$ to 1 (d) 1 to 2

66. Which of the following will be σ if $Y = 2.4\eta$ **[RPET 2001]**

(a) -1 (b) 0.2 (c) 0.1 (d) -0.25

67. Which is correct relation **[RPET 2001]**

(a) $Y < \sigma$ (b) $Y > \sigma$ (c) $Y = \sigma$ (d) $\sigma = +1$

68. The relationship between Young's modulus Y, bulk modulus K and modulus of rigidity η is

(a) $Y = \frac{9\eta K}{\eta + 3K}$ (b) $\eta = \frac{9yK}{Y + 3K}$ (c) $Y = \frac{9\eta K}{3\eta + K}$ (d) $Y = \frac{3\eta K}{9\eta + K}$

69. The Poisson's ratio cannot have the value **[EAMCET 1989]**

(a) 0.7 (b) 0.2 (c) 0.1 (d) 0.5

70. Which of the following relations is true **[CPMT 1984]**

(a) $3Y = K(1 - \sigma)$ (b) $K = \frac{9\eta Y}{Y + \eta}$ (c) $\sigma = (6K + \eta)\, Y$ (d) $\sigma = \frac{0.5Y - \eta}{\eta}$

71. The wrong relation for modulus of rigidity (η) is

(a) $\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$ (b) Unit of η is N/m^2 (c) $\eta = \frac{Y}{2(1-\sigma)}$ (d) $\eta = \frac{Y}{2(1+\sigma)}$

Problems based on Torsion

72. A rod of $2m$ length and radius 1 *cm* is twisted at one end by 0.8 *rad* with respect to other end being clamped. The shear strain developed in its rod will be

(a) 0.002 (b) 0.004 (c) 0.008 (d) 0.016

73. The upper end of a wire 1 *metre* long and 2 *mm* in radius is clamped. The lower end is twisted through an angle of 45^o. The angle of shear is

(a) 0.09^o (b) 0.9^o (c) 9^o (d) 90^o

74. The end of a wire of length $0.5m$ and radius $10^{-3}m$ is twisted through 0.80 *radian*. The shearing strain at the surface of wire will be

(a) 1.6×10^{-3} (b) 1.6×10^{3} (c) 16×10^{3} (d) 16×10^{6}

75. Two cylinders A and B of the same material have same length, their radii being in the ratio of 1 : 2 respectively. The two are joined in series. The upper end of A is rigidly fixed. The lower end of B is twisted through an angle θ, the angle of twist of the cylinder A is **fig.**

(a) $\frac{15}{16}\theta$ (b) $\frac{16}{15}\theta$ (c) $\frac{16}{17}\theta$ (d) $\frac{17}{16}\theta$

Problems based on Interatomic force constant

76. If the interatomic spacing in a steel wire is 3.0Å and $Y_{steel} = 20 \times 10^{10}\ N/m^2$, then force constant is

(a) $6 \times 10^{-2}\ N/Å$ (b) $6 \times 10^{-9}\ N/Å$ (c) $4 \times 10^{-5}\ N/Å$ (d) $6 \times 10^{-5}\ N/Å$

77. The Young's modulus of a metal is $1.2 \times 10^{11}\ N/m^2$ and the inter-atomic force constant is $3.6 \times 10^{-9} N/Å$. The mean distance between the atoms of the metal is

(a) 2Å (b) 3 Å (c) 4.5 Å (d) 5 Å

78. The interatomic distance for a metal is $3 \times 10^{-10} m$. If the interatomic force constant is $3.6 \times 10^{-9}\ N/Å$, then the Young's modulus in N/m^2 will be

(a) 1.2×10^{11} (b) 4.2×10^{11} (c) 10.8×10^{-19} (d) 2.4×10^{10}

Miscellaneous problems

79. A particle of mass m is under the influence of a force F which varies with the displacement x according to the relation $F = -kx + F_0$ in which k and F_0 are constants. The particle when disturbed will oscillate

(a) About $x = 0$, with $\omega \neq \sqrt{k/m}$ (b) About $x = 0$, with $\omega = \sqrt{k/m}$

(c) About $x = F_0/k$ with $\omega = \sqrt{k/m}$ (d) About $x = F_0/k$ with $\omega \neq \sqrt{k/m}$

80. The extension in a string obeying Hooke's law is x. The speed of sound in the stretched string is v. If the extension in the string is increased to $1.5x$, the speed of sound will be

(a) 1.22 v (b) 0.61 v (c) 1.50 v (d) 0.75 v

81. Railway lines and girders for buildings, are I shaped, because

(a) The bending of a girder is inversely proportional to depth, hence high girder bends less

(b) The coefficient of rigidity increases by this shape

(c) Less volume strain is caused

(d) This keeps the surface smooth

82. If Young's modulus for a material is zero, then the state of material should be

(a) Solid (b) Solid but powder (c) Gas (d) None of the above

83. The elasticity of *invar*

(a) Increases with temperature rise (b) Decreases with temperature rise

(c) Does not depend on temperature (d) None of the above

84. For the same cross-sectional area and for a given load, the ratio of depressions for the beam of square cross-section and circular cross-section is

(a) $\pi : 3$ (b) $\pi : 1$ (c) $3 : \pi$ (d) $1 : \pi$

85. A uniform rod of mass m, length L, area of cross-section A is rotated about an axis passing through one of its ends and perpendicular to its length with constant angular velocity ω in a horizontal plane. If Y is the Young's modulus of the material of rod, the increase in its length due to rotation of rod is

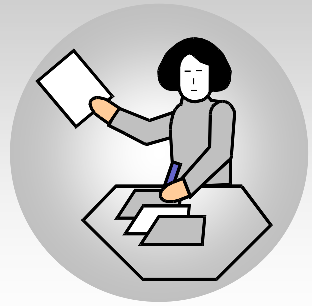
(a) $\frac{m\omega^2L^2}{AY}$ (b) $\frac{m\omega^2L^2}{2AY}$ (c) $\frac{m\omega^2L^2}{3AY}$ (d) $\frac{2m\omega^2L^2}{AY}$

86. A steel wire is suspended vertically from a rigid support. When loaded with a weight in air, it extends by l_a and when the weight is immersed completely in water, the extension is reduced to l_w. Then the relative density of the material of the weight is

(a) $\frac{l_a}{l_w}$ (b) $\frac{l_a}{l_a - l_w}$ (c) $\frac{l_a}{l_a - l_w}$ (d) $\frac{l_w}{l_a}$

87. The twisting couple per unit twist for a solid cylinder of radius 4.9 cm is 0.1 N-m. The twisting couple per unit twist for a hollow cylinder of same material with outer and inner radii of 5 cm and 4 cm respectively, will be

(a) 0.64 N-m (b) 0.64×10^{-1} N-m (c) 0.64×10^{-2} N-m (d) 0.64×10^{-3} N-m



Answer Sheet (Practice problems)

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
c	b	c	c	a	a	c	b	b	b
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
a	c	c	c	d	b	b	d	d	a
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
c	c	c	d	b	c	d	d	c	b
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
a	a	a	b	c	c	b	b	c	c
41.	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**	**48.**	**49.**	**50.**
b	d	b	d	c	a	d	d	d	c
51.	**52.**	**53.**	**54.**	**55.**	**56.**	**57.**	**58.**	**59.**	**60.**
b	c	b	a	c	b	d	a	a	b
61.	**62.**	**63.**	**64.**	**65.**	**66.**	**67.**	**68.**	**69.**	**70.**
a	c	c	a	a	b	b	a	a	d
71.	**72.**	**73.**	**74.**	**75.**	**76.**	**77.**	**78.**	**79.**	**80.**
c	b	a	a	c	b	b	a	c	a
81.	**82.**	**83.**	**84.**	**85.**	**86.**	**87.**			
a	b	c	c	c	b	b			