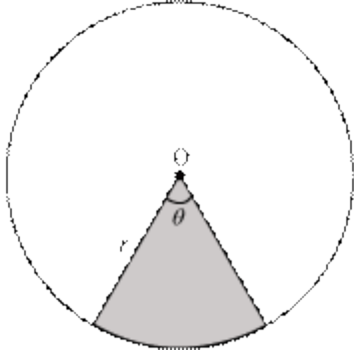


## Areas Related to Circles

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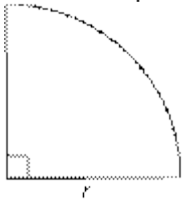
- **Area of sector:**

Area of the sector of angle  $\theta = \frac{\theta}{360^\circ} \times \pi r^2$ , where  $r$  is the radius of the circle.



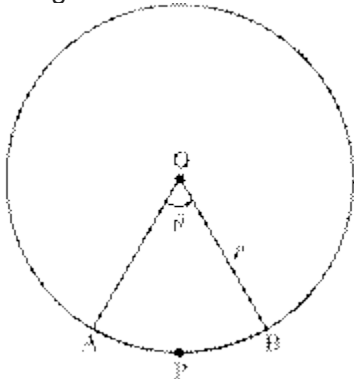
- **Area of quadrant:**

Area of a quadrant of a circle with radius  $r = \frac{\pi r^2}{4}$        $\left[ \theta = 90^\circ \Rightarrow \frac{\theta}{360^\circ} = \frac{90^\circ}{360^\circ} = \frac{1}{4} \right]$



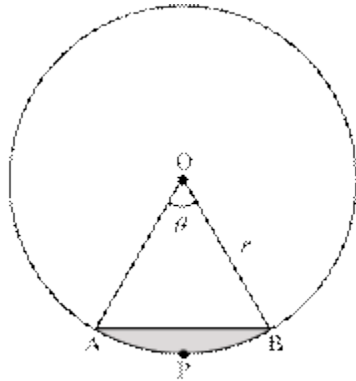
- **Area of a semicircle**  $= \frac{180^\circ}{360^\circ} \times \pi r^2 = \frac{1}{2} \pi r^2$
- **Length of an arc:**

Length of an arc of a sector of angle  $\theta = \frac{\theta}{360^\circ} \times 2\pi r$ , where  $r$  is the radius of the circle



**Perimeter of a Sector**  $= l + 2r$

- **Area of the segment of a circle**



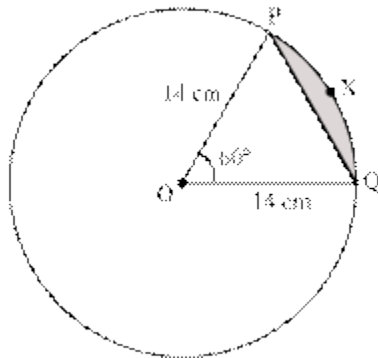
Area of segment APB

$$= \text{Area of sector OAPB} - \text{Area of } \triangle OAB$$

$$= \frac{\theta}{360} \times \pi r^2 - \text{area of } \triangle OAB$$

**Example:**

In the given figure, the radius of the circle is 14 cm, and  $\angle POQ = 60^\circ$ . Find the area of the segment P X Q.



**Solution:**

$$\text{Area of segment PXQ} = \text{Area of sector OPXQ} - \text{Area of } \triangle OPQ \quad \text{--- (1)}$$

Area of sector OPXQ

$$= \frac{60^\circ}{360^\circ} \times \pi \times 14 \times 14 \text{ cm}^2 \quad \left[ \text{Area of sector of angle } \theta \text{ and radius } r = \frac{\theta}{360} \times \pi r^2 \right]$$

$$= \frac{1}{6} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2$$

$$= \frac{22}{3} \times 14 = \frac{308}{3} \text{ cm}^2$$

In  $\triangle OPQ$ , we have

$$OP = OQ \quad \text{[radii of the same circle]}$$

$$\Rightarrow \angle OPQ = \angle OQP = \frac{1}{2}(180 - 60^\circ) = 60^\circ$$

$\triangle OPQ$  is an equilateral triangle.

$$\text{Area of } \triangle OPQ = \frac{\sqrt{3}}{4} \times (14)^2 \text{ cm}^2$$

$$= 49\sqrt{3} \text{ cm}^2$$

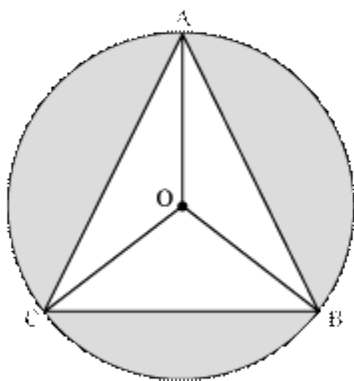
$$\therefore \text{From (1), Area of segment PXQ} = \left( \frac{308}{3} - 49\sqrt{3} \right) \text{ cm}^2$$

- Areas of Combination of Plane Figures

**Example:**

In the given figure A, B, and C are points on the circle with centre O, such that

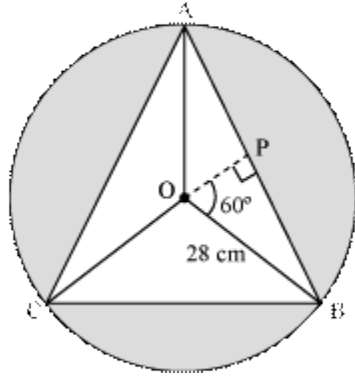
$\angle AOB = \angle AOC = \angle BOC$ . If the radius of the circle is 28 cm. Find the area of the shaded region.



**Solution:**

Area of the shaded region = Area of the circle – Area of  $\triangle ABC$

$$\text{Area of the circle} = \pi \times (\text{Radius})^2 = \frac{22}{7} \times 28 \times 28 \text{ cm}^2 = 22 \times 4 \times 28 \text{ cm}^2$$



Since  $\angle AOB = \angle BOC = \angle AOC$

$$\angle AOB + \angle BOC + \angle AOC = 360^\circ$$

$$\Rightarrow \angle AOB = \angle BOC = \angle AOC = \frac{1}{3} \times 360^\circ = 120^\circ$$

It can be easily shown that

$$\triangle AOB \cong \triangle AOC \cong \triangle BOC$$

$$\Rightarrow AB = BC = CA$$

$\therefore \triangle ABC$  is an equilateral triangle

Draw  $OP \perp AB$

Then, in  $\triangle OAP$  and  $\triangle OBP$ , we have

$$\angle OPA = \angle OPB = 90^\circ$$

$$OA = OB \quad \text{[radii of the same circle]}$$

$$OP = OP \quad \text{[common]}$$

$$\therefore \triangle OAP \cong \triangle OBP \quad \text{[by RHS congruency criterion]}$$

$$\Rightarrow \angle AOP = \angle BOP \quad \text{[C.P.C.T]}$$

$$\therefore \angle BOP = \frac{1}{2} \times 120^\circ = 60^\circ$$

$$\text{Now, } \frac{PB}{OB} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore PB = \frac{\sqrt{3}}{2} \times 28 = 14\sqrt{3}$$

$$\therefore AB = AP + PB = 2PB = 2 \times 14\sqrt{3} = 28\sqrt{3} \text{ cm}$$

$$\therefore \text{Area of } \triangle ABC = \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times 28\sqrt{3} \times 28\sqrt{3} \text{ cm}^2 = 7 \times 28 \times 3\sqrt{3} \text{ cm}^2$$

Thus, area of the shaded region

$$= (22 \times 4 \times 28 - 7 \times 28 \times 3\sqrt{3}) \text{ cm}^2$$

$$= 28(88 - 21\sqrt{3}) \text{ cm}^2$$