Chapter

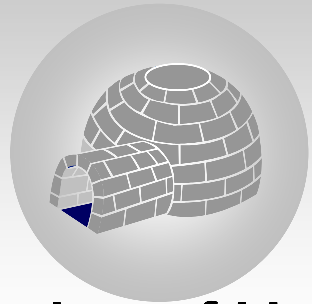
14

Transmission of Heat

Contents

In steel plants working temperature of furnace goes more than 1500ºC. For measuring this very high temperature pyrometers are used.

These are the devices used to measure the temperature by measuring the intensity of radiations received from the body. They are based on the fact that the amount of radiations emitted from a body per unit area per second is directly proportional to the fourth power of temperature.



Transmission of Heat

14.1 Introduction

Heat energy transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by one of the following modes.

Conduction	Convection	Radiation
Heat flows from hot end to cold end. Particles of the medium simply oscillate but do not leave their place.	Each particle absorbing heat is mobile	Heat flows without any intervening medium in the form of electromagnetic waves.
Medium is necessary for conduction	Medium is necessary for convection	Medium is not necessary for radiation
It is a slow process	It is also a slow process	It is a very fast process
Path of heat flow may be zig-zag	Path may be zig-zag or curved	Path is a straight line
Conduction takes place in solids	Convection takes place in fluids	Radiation takes place in gaseous and transparent media
The temperature of the medium increases through which heat flows	In this process also the temperature of medium increases	There is no change in the temperature of the medium

14.2 Conduction

The process of transmission of heat energy in which the heat is transferred from one particle to other particle without dislocation of the particle from their equilibrium position is called conduction.

(i) Conduction is a process which is possible in all states of matter.

(ii) In solids only conduction takes place.

(iii) In non-metallic solids and fluids the conduction takes place only due to vibrations of molecules, therefore they are poor conductors.

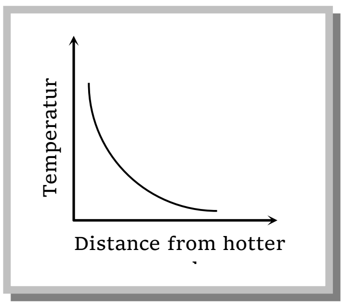
(iv) In metallic solids free electrons carry the heat energy, therefore they are good conductor of heat.

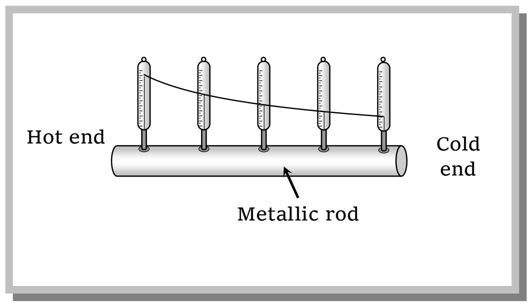
(1) **Variable and steady state**

When one end of a metallic rod is heated, heat flows by conduction from the hot end to the cold end.

In the process of conduction each cross-section of the rod receives heat from the adjacent cross-section towards the hot end. A part of this heat is absorbed by the cross-section itself whose temperature increases, another part is lost into atmosphere by convection & radiation and the rest is conducted away to the next cross-section.

Because in this state temperature of every cross-section of the rod goes on increasing, hence rod is said to exist in variable state.





After sometime, a state is reached when the temperature of every cross-section of the rod becomes constant. In this state, no heat is absorbed by the rod. The heat that reaches any cross-section is transmitted to the next except that a small part of heat is lost to surrounding from the sides by convection & radiation. This state of the rod in which no part of rod absorbs heat is called steady state.

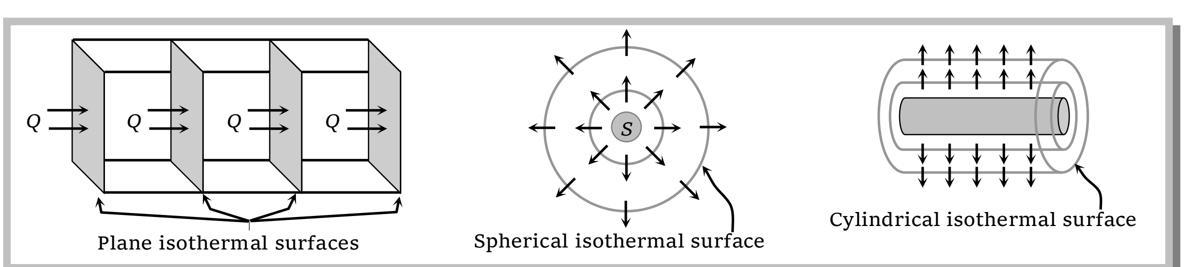
(2) **Isothermal surface**

Any surface (within a conductor) having its all points at the same temperature, is called isothermal surface. The direction of flow of heat through a conductor at any point is perpendicular to the isothermal surface passing through that point.

(i) If the material is rectangular or cylindrical rod, the isothermal surface is a plane surface.

(ii) If a point source of heat is situated at the centre of a sphere the isothermal surface will be spherical,

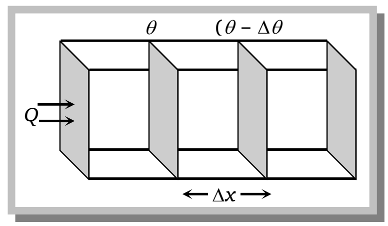
(iii) If steam passes along the axis of the hollow cylinder, heat will flow through the walls of the cylinder so that in this condition the isothermal surface will be cylindrical.



(3) **Temperature Gradient**

The rate of change of temperature with distance between two isothermal surfaces is called temperature gradient.

If the temperature of two isothermal surfaces be θ and $(\theta - \Delta\theta)$, and the perpendicular distance between them be Δx then Temperature gradient = $\frac{(\theta - \Delta\theta) - \theta}{\Delta x} = \frac{-\Delta\theta}{\Delta x}$



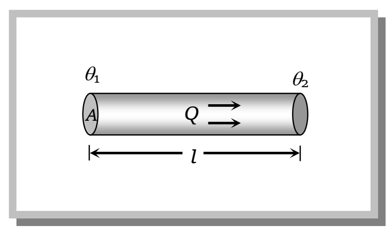
The negative sign show that temperature θ decreases as the distance x increases in the direction of heat flow.

Unit : K/m (S.I.) and Dimensions : $[L^{-1}\theta]$

(4) **Coefficient of thermal conductivity**

If L be the length of the rod, A the area of cross-section and θ_1 and θ_2 are the temperature of its two faces, then the amount of heat flowing from one face to the other face in time t is given by

$$Q = \frac{KA(\theta_1 - \theta_2)t}{l}$$



Where K is coefficient of thermal conductivity of material of rod. It is the measure of the ability of a substance to conduct heat through it.

If $A = 1m^2$, $(\theta_1 - \theta_2) = 1^oC$, $t = 1$ *sec* and $l = 1m$, then $Q = K$.

Thus, thermal conductivity of a material is the amount of heat flowing per second during steady state through its rod of length 1 m and cross-section 1 m^2 with a unit temperature difference between the opposite faces.

(i) Units : *Cal/cm-sec* ^oC (in C.G.S.), *kcal/m-sec-K* (in M.K.S.) and *W/m- K* (in S.I.)

(ii) Dimension : $[MLT^{-3}\theta^{-1}]$

(iii) The magnitude of K depends only on nature of the material.

(iv) For perfect conductors, $K = \infty$ and for perfect insulators, $K = 0$

(v) Substances in which heat flows quickly and easily are known as good conductor of heat. They possesses large thermal conductivity due to large number of free electrons. *Example* : Silver, brass *etc*.

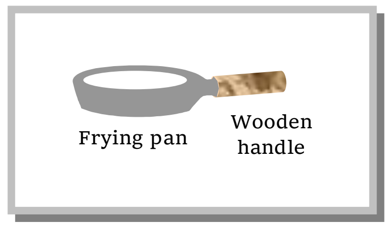
(vi) Substances which do not permit easy flow of heat are called bad conductors. They possess low thermal conductivity due to very few free electrons. *Example* : Glass, wood *etc*.

(vii) The thermal conductivity of pure metals decreases with rise in temperature but for alloys thermal conductivity increases with increase of temperature.

(viii) Human body is a bad conductor of heat (but it is a good conductor of electricity).

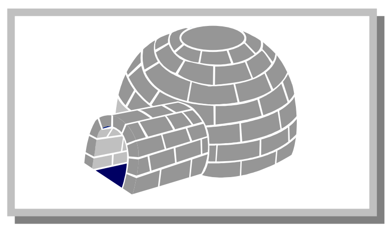
(5) **Applications of conductivity in daily life**

(i) Cooking utensils are provided with wooden handles, because wood is a poor conductor of heat. The hot utensils can be easily handled from the wooden handles and our hands are saved from burning.



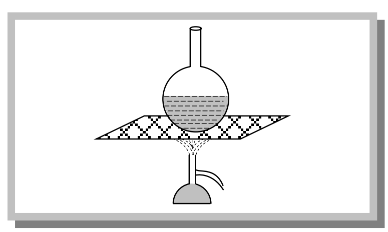
(ii) We feel warmer in a fur coat. The air enclosed in the fur coat being bad conductor heat does not allow the body heat to flow outside. Hence we feel warmer in a fur coat.

(iii) Eskimos make double walled houses of the blocks of ice. Air enclosed in between the double walls prevents transmission of heat from the house to the cold surroundings.



For exactly the same reason, two thin blankets are warmer than one blanket of their combined thickness. The layer of air enclosed in between the two blankets makes the difference.

(iv) Wire gauze is placed over the flame of Bunsen burner while heating the flask or a beaker so that the flame does not go beyond the gauze and hence there is no direct contact between the flame and the flask. The wire gauze being a good conductor of heat, absorb the heat of the flame and transmit it to the flask.



Davy's safety lamp has been designed on this principle. The gases in the mines burn inside the gauze placed around the flame of the lamp. The temperature outside the gauze is not high, so the gases outside the gauze do not catch fire.

(v) Birds often swell their feathers in winter. By doing so, they enclose more air between their bodies and the feathers. The air, being bad conductor of heat prevents the out flow of their body heat. Thus, birds feel warmer in winter by swelling their feathers.

(6) **Relation between temperature gradient and thermal conductivity**

In steady state, rate of flow of heat $\frac{dQ}{dt} = -KA\frac{d\theta}{dx}$ = – KA (Temperature gradient)

If $\frac{dQ}{dt}$ is constant then temperature gradient $\propto \frac{1}{K}$

Temperature difference between the hot end and the cold end in steady state is inversely proportional to K, *i.e.* in case of good conductors temperature of the cold end will be very near to hot end.

In ideal conductor where $K = \infty$, temperature difference in steady state will be zero.

(7) **Wiedmann-Franz law**

At a given temperature T, the ratio of thermal conductivity to electrical conductivity is constant *i.e.*, $(K/\sigma T)$= constant, *i.e.,* a substance which is a good conductor of heat (*e.g.,* silver) is also a good conductor of electricity. Mica is an exception to above law.

(8) **Thermometric conductivity or diffusivity**

It is a measure of rate of change of temperature (with time) when the body is not in steady state (*i.e.,* in variable state)

The thermometric conductivity or diffusivity is defined as the ratio of the coefficient of thermal conductivity to the thermal capacity per unit volume of the material.

Thermal capacity per unit volume = $\frac{mc}{V} = \rho c$ (As ρ is density of substance)

$\therefore$ Diffusivity (D) = $\frac{K}{\rho c}$

Unit : m^2/sec and Dimension : $[L^2T^{-1}]$

(9) **Thermal resistance**

The thermal resistance of a body is a measure of its opposition to the flow of heat through it.

It is defined as the ratio of temperature difference to the heat current (= Rate of flow of heat)

Now, temperature difference = $(\theta_1 - \theta_2)$ and heat current, $H = \frac{Q}{t}$

$\therefore$ Thermal resistance, $R = \frac{\theta_1 - \theta_2}{H} = \frac{\theta_1 - \theta_2}{Q/t} = \frac{\theta_1 - \theta_2}{KA(\theta_1 - \theta_2)/l} = \frac{l}{KA}$

Unit : $^oC \times sec/cal$ or $K \times sec/kcal$ and Dimension : $[M^{-1}L^{-2}T^3\theta]$

14.3 Electrical Analogy For Thermal Conduction

It is an important fact to appreciate that there exists an exact similarity between thermal and electrical conductivities of a conductor.

Electrical conduction	**Thermal conduction**
Electric charge flows from higher potential to lower potential	Heat flows from higher temperature to lower temperature
The rate of flow of charge is called the electric current, *i.e.* $I = \frac{dq}{dt}$	The rate of flow of heat may be called as heat current *i.e.* $H = \frac{dQ}{dt}$
The relation between the electric current and the potential difference is given by Ohm's law, that is $I = \frac{V_1 - V_2}{R}$ where R is the electrical resistance of the conductor	Similarly, the heat current may be related with the temperature difference as $H = \frac{\theta_1 - \theta_2}{R}$ where R is the thermal resistance of the conductor
The electrical resistance is defined as $R = \frac{\rho l}{A} = \frac{l}{\sigma A}$	The thermal resistance may be defined as $R = \frac{l}{KA}$
where ρ = Resistivity and σ = Electrical conductivity $\frac{dq}{dt} = I = \frac{V_1 - V_2}{R} = \frac{\sigma A}{l}(V_1 - V_2)$	where K = Thermal conductivity of conductor $\frac{dQ}{dt} = H = \frac{\theta_1 - \theta_2}{R} = \frac{KA}{l}(\theta_1 - \theta_2)$

Sample problems based on Conduction

***Problem* 1.** The heat is flowing through a rod of length 50 *cm* and area of cross-section 5cm^2. Its ends are respectively at 25°*C* and 125°*C*. The coefficient of thermal conductivity of the material of the rod is 0.092 *kcal*/*m*×*s*×°*C*. The temperature gradient in the rod is

(a) 2 °*C* / *cm* (b) 2 °*C* / *m* (c) 20 °*C* / *cm* (d) 10 °*C* /*m*

Solution : (a) Temperature gradient $= \frac{\Delta\theta}{\Delta x} = \frac{\theta_2 - \theta_1}{\Delta x} = \frac{125 - 25}{50} = 2^o\, C / cm$.

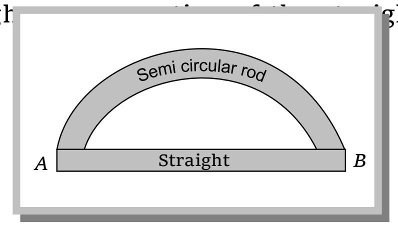
***Problem* 2.** Consider two rods of same length and different specific heats (s_1 and s_2), conductivities K_1 and K_2 and areas of cross-section (A_1 and A_2) and both giving temperature T_1 and T_2 at their ends. If the rate of heat loss due to conduction is equal, then

(a) $K_1A_1 = K_2A_2$ (b) $K_2A_1 = K_1A_2$ (c) $\frac{K_1A_1}{s_1} = \frac{K_2A_2}{s_2}$ (d) $\frac{K_2A_1}{s_2} = \frac{K_1A_2}{s_1}$

Solution : (a) According to problem, rate of heat loss in both rods are equal *i.e.* $\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2$

$\Rightarrow \frac{K_1A_1\Delta\theta_1}{l_1} = \frac{K_2A_2\Delta\theta_2}{l_2}$ $\therefore K_1A_1 = K_2A_2$ [As $\Delta\theta_1 = \Delta\theta_2 = (T_1 - T_2)$ and $l_1 = l_2$ given]

Problem 3. Two rods (one semi-circular and other straight) of same material and of same cross-sectional area are joined as shown in the figure. The points A and B are maintained at different temperature. The ratio of the heat transferred through a cross-section of a semi-circular rod to the heat transferred through a cross section of the straight rod in a given time is **[UPSEAT 2002]**



(a) $2 : \pi$

(b) 1 : 2

(c) $\pi : 2$

(d) 3 : 2

Solution : (a) $\frac{dQ}{dt} = \frac{KA\Delta\theta}{l}$, For both rods K, A and $\Delta\theta$ are same $\therefore$ $\frac{dQ}{dt} \propto \frac{1}{l}$

So $\frac{(dQ/dt)_{semi\ circular}}{(dQ/dt)_{straight}} = \frac{l_{straight}}{l_{semicircular}} = \frac{2r}{\pi r} = \frac{2}{\pi}$.

Problem 4. For cooking the food, which of the following type of utensil is most suitable

[MNR 1986; MP PET 1990; CPMT 1991; SCRA 1998;MP PMT/PET 1998, 2000; RPET 2001]

(a) High specific heat and low conductivity (b) High specific heat and high conductivity

(c) Low specific heat and low conductivity (d) Low specific heat and high conductivity

Solution : (d) Cooking utensil should conduct maximum and absorb minimum heat so it should possess high conductivity and low specific heat.

Problem 5. A heat flux of 4000 J/s is to be passed through a copper rod of length 10 cm and area of cross-section 100 cm^2. The thermal conductivity of copper is 400 W/m^oC. The two ends of this rod must be kept at a temperature difference of

(a) 1 ^oC (b) 10 ^oC (c) 100 ^oC (d) 1000 ^oC

Solution : (c) From $\frac{dQ}{dt} = \frac{KA\,\Delta\theta}{l}$ $\Rightarrow$ $\Delta\theta = \frac{l}{K \times A} \times \frac{dQ}{dt}$ $= \frac{0.1}{400 \times (100 \times 10^{-4})} \times 4000$ $= 100^oC$

Problem 6. The coefficients of thermal conductivity of copper, mercury and glass are respectively K_c, K_m and K_g such that $K_c > K_m > K_g$. If the same quantity of heat is to flow per second per unit area of each and corresponding temperature gradients are respectively X_c, X_m and X_g then

(a) $X_c = X_m = X_g$ (b) $X_c > X_m > X_g$ (c) $X_c < X_m < X_g$ (d) $X_m < X_c < X_g$

Solution : (c) $\frac{dQ/dt}{A} = K\left(\frac{\Delta\theta}{\Delta x}\right)$ $\Rightarrow$ Rate of flow of heat per unit area = Thermal conductivity $\times$ Temperature gradient

Temperature gradient (X) $\propto \frac{1}{\text{Thermal conductivity (K)}}$ [As $\frac{dQ/dt}{A} = \text{constant}$]

As $K_C > K_m > K_g$ therefore $X_C < X_m < X_g$.

Problem 7. A room is maintained at 20°*C* by a heater of resistance 20 *ohm* connected to 200 *volt* mains. The temperature is uniform through out the room and heat is transmitted through a glass window of area 1 m^2 and thickness 0.2 *cm*. What will be the temperature outside? Given that thermal conductivity *K* for glass is 0.2 *cal/m* × °*C* × *sec* and *J* = 4.2 *J/cal*

(a) 15.24 °*C* (b) 15.00 °*C* (c) 24.15 °*C* (d) None of the above

Solution : (a) As the temperature of room remain constant therefore the rate of heat generation from the heater should be equal to the rate of flow of heat through a glass window

$$\frac{1}{J}\left(\frac{V^2}{R}t\right) = KA\frac{\Delta\theta}{l}.t$$

$$\Rightarrow \frac{1}{4.2}\times\frac{(200)^2}{20} = \frac{0.2\times1\times(20-\theta)}{0.2\times10^{-2}} \Rightarrow \theta = 15.24°C$$ [where θ = temperature of outside]

Problem 8. A point source of heat of power *P* is placed at the centre of a spherical shell of mean radius *R*. The material of the shell has thermal conductivity *K*. If the temperature difference between the outer and the inner surface of the shell is not to exceed *T*, then the thickness of the shell should not be less than

(a) $\frac{2\pi R^2 KT}{P}$ (b) $\frac{4\pi R^2 KT}{P}$ (c) $\frac{\pi R^2 KT}{P}$ (d) $\frac{\pi R^2 KT}{4P}$

Solution : (b) Rate of flow of heat or power $(P) = \frac{KA\Delta\theta}{\Delta x} = \frac{K4\pi R^2 T}{\Delta x}$

$\therefore$ Thickness of shell $\Delta x = \frac{4\pi R^2 KT}{P}$.

Problem 9. There are three thermometers – one in contact with the skin of the man, other in between the vest and the shirt and third in between the shirt and coat. The readings of the thermometers are 30°*C*, 25°*C* and 22°*C* respectively. If the vest and shirt are of the same thickness, the ratio of their thermal conductivities is

(a) 9 : 25 (b) 25 : 9 (c) 5 : 3 (d) 3 : 5

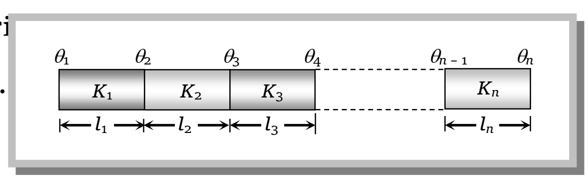
Solution : (d) Rate of flow of heat will be equal in both vest and shirt

$$\therefore \frac{K_{vest}A.\Delta\theta_{vest}t}{l} = \frac{K_{shirt}A\Delta\theta_{shirt}t}{l} \Rightarrow \frac{K_{vest}}{K_{shirt}} = \frac{\Delta\theta_{shirt}}{\Delta\theta_{vest}} \Rightarrow \frac{K_{vest}}{K_{shirt}} = \frac{25-22}{30-25} = \frac{3}{5}.$$

14.4 Combination of Conductors

(1) **Series combination :**

Let *n* slabs each of cross-sectional area *A*, lengths $l_1, l_2, l_3, ...l_n$ and conductivities $K_1, K_2, K_3......K_n$ respectively be connected in the series



Heat current is the same in all the conductors.

i.e., $\frac{Q}{t} = H_1 = H_2 = H_3 .... = H_n$

$$\frac{K_1 A(\theta_1 - \theta_2)}{l_1} = \frac{K_2 A(\theta_2 - \theta_3)}{l_2} = \frac{K_3 A(\theta_3 - \theta_4)}{l_3} = ... = \frac{K_n A(\theta_{n-1} - \theta_n)}{l_n}$$

(i) Equivalent resistance $R = R_1 + R_2 + R_3 + ...R_n$

(ii) If K_s is equivalent conductivity, then from relation $R = \frac{l}{KA}$

$$\frac{l_1 + l_2 + l_3 + ...l_n}{K_s} = \frac{l_1}{K_1 A} + \frac{l_2}{K_2 A} + \frac{l_3}{K_3 A} + ... + \frac{l_n}{K_n A}$$

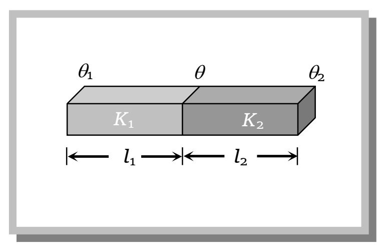
$$\therefore \quad K_s = \frac{l_1 + l_2 + l_3 + ... l_n}{\frac{l_1}{K_1} + \frac{l_2}{K_2} + \frac{l_3}{K_3} + ... \frac{l_n}{K_n}}$$

(iii) Equivalent thermal conductivity for n slabs of equal length $K = \frac{n}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + ... \frac{1}{K_n}}$

For two slabs of equal length, $K = \frac{2K_1 K_2}{K_1 + K_2}$

(iv) Temperature of interface of composite bar : Let the two bars are arranged in series as shown in the figure.

Then heat current is same in the two conductors.



i.e., $\frac{Q}{t} = \frac{K_1 A(\theta_1 - \theta)}{l_1} = \frac{K_2 A(\theta - \theta_2)}{l_2}$

By solving we get $\theta = \frac{\frac{K_1}{l_1}\theta_1 + \frac{K_2}{l_2}\theta_2}{\frac{K_1}{l_1} + \frac{K_2}{l_2}}$

If $(l_1 = l_2 = l)$ then $\theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$

(2) **Parallel Combination**

Let n slabs each of length l, areas $A_1, A_2, A_3, ...A_n$ and thermal conductivities $K_1, K_2, K_3, ...K_n$ are connected in parallel then.

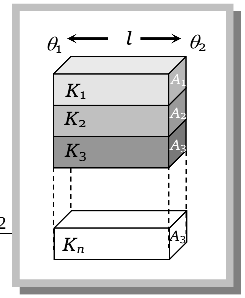
(i) Equivalent resistance $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... \frac{1}{R_n}$

(ii) Temperature gradient across each slab will be same.

(iii) Heat current in each slab will be different. Net heat current will be the sum of heat currents through individual slabs. *i.e.,* $H = H_1 + H_2 + H_3 + ...H_n$

$$\frac{K(A_1 + A_2 + A_3 + ... + A_n)(\theta_1 - \theta_2)}{l} = \frac{K_1 A_1(\theta_1 - \theta_2)}{l} + \frac{K_2 A_2(\theta_1 - \theta_2)}{l} + \frac{K_3 A_3(\theta_1 - \theta_2)}{l} .... + \frac{K_n A_n(\theta_1 - \theta_2)}{l}$$

$\therefore\ K = \dfrac{K_1A_1 + K_2A_2 + K_3A_3 + ...K_nA_n}{A_1 + A_2 + A_3 + ...A_n}$



For n slabs of equal area $K = \dfrac{K_1 + K_2 + K_3 + ...K_n}{n}$

Equivalent thermal conductivity for two slabs of equal area $K = \dfrac{K_1 + K_2}{2}$

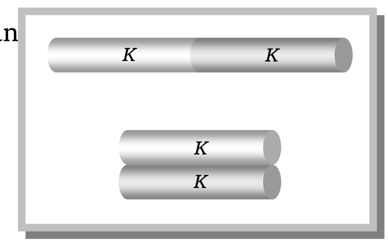
Sample problems based on Combination of conductors

***Problem* 10.** Two rods of same length and material transfer a given amount of heat in 12 seconds, when they are joined end to end. But when they are joined lengthwise, then they will transfer same heat in same conditions in

[BHU 1998; UPSEAT 2002]

(a) 24 s (b) 3 s (c) 1.5 s (d) 48 s

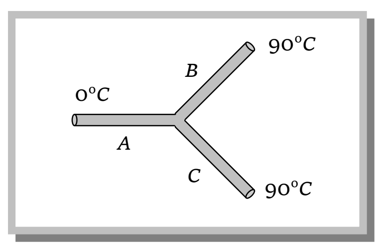
Solution : (b) $Q = KA\dfrac{\Delta\theta}{l}.t$ $\therefore t \propto \dfrac{l}{A}$ [As Q, K and $\Delta\theta$ are constan



$$\frac{t_1}{t_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \left(\frac{l_1}{l_1/2}\right) \times \left(\frac{2A_1}{A_1}\right)$$

$$\frac{t_1}{t_2} = 4 \Rightarrow t_2 = \frac{t_1}{4} = \frac{12}{4} = 3$$

***Problem* 11.** Three rods made of the same material and having the same cross section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0^oC and 90^o respectively. The temperature of the junction of the three rods will be



(a) 45° C

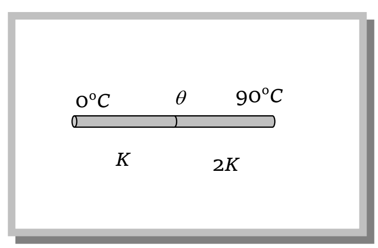
(b) 60° C

(c) 30° C

(d) 20° C

Solution : (b) Let the conductivity of each rod is K. By considering the rods B and C are in parallel, effective thermal conductivity of B and C will be $2K$.

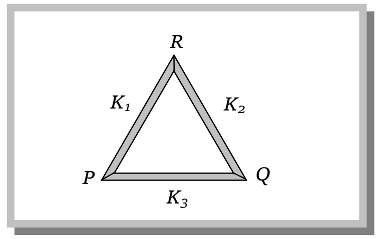
Now with the help of given formula



Temperature of interface $\theta = \dfrac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$

$$\theta = \frac{K \times 0 + 2K \times 90}{K + 2K} = \frac{180}{3} = 60\,^oC\,.$$

***Problem* 12.** Three rods of same dimensions are arranged as shown in figure they have thermal conductivities K_1, K_2 and K_3. The points P and Q are maintained at different temperatures for the heat to flow at the same rate along *PRQ* and *PQ* then which of the following option is correct **[KCET (Engg. & Med.) 2001]**



(a) $K_3 = \frac{1}{2}(K_1 + K_2)$

(b) $K_3 = K_1 + K_2$

(c) $K_3 = \frac{K_1 K_2}{K_1 + K_2}$

(d) $K_3 = 2(K_1 + K_2)$

Solution : (c) Rate of flow of heat along *PQ* $\left(\frac{dQ}{dt}\right)_{PQ} = \frac{K_3 A \Delta\theta}{l}$ .....(i)

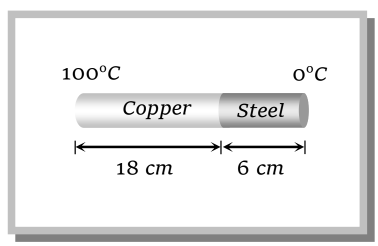
Rate of flow of heat along *PRQ* $\left(\frac{dQ}{dt}\right)_{PRQ} = \frac{K_s A \Delta\theta}{2l}$

Effective conductivity for series combination of two rods of same length $K_s = \frac{2K_1K_2}{K_1 + K_2}$

So $\left(\frac{dQ}{dt}\right)_{PRQ} = \frac{2K_1K_2}{K_1 + K_2} \cdot \frac{A\Delta\theta}{2l} = \frac{K_1K_2}{K_1 + K_2} \cdot \frac{A\Delta\theta}{l}$ .....(ii)

Equating (i) and (ii) $K_3 = \frac{K_1K_2}{K_1 + K_2}$

Problem **13.** The coefficient of thermal conductivity of copper is nine times that of steel. In the composite cylindrical bar shown in the figure. What will be the temperature at the junction of copper and steel **[MP PMT 2000]**



(a) 75°C

(b) 67°C

(c) 33°C

(d) 25°C

Solution : (a) $K_1 = 9K_2$, $l_1 = 18\ cm$, $l_2 = 6\ cm$, $\theta_1 = 100^\circ C$, $\theta_2 = 0^\circ C$

Temperature of the junction $\theta = \dfrac{\frac{K_1}{l_1}\theta_1 + \frac{K_2}{l_2}\theta_2}{\frac{K_1}{l_1} + \frac{K_2}{l_2}} \Rightarrow \theta = \dfrac{\frac{9K_2}{18}100 + \frac{K_2}{6} \times 0}{\frac{9K_2}{18} + \frac{K_2}{6}} = \dfrac{50 + 0}{8/12} = 75\,^oC$

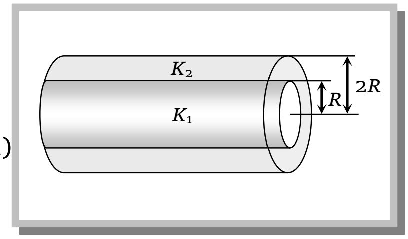
Problem **14.** A cylinder of radius *R* made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius *R* and outer radius 2*R* made of material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is

[IIT-JEE 1988; MP PMT 1994, 97; SCRA 1998]

(a) $K_1 + K_2$ (b) $\dfrac{K_1K_2}{K_1 + K_2}$ (c) $\dfrac{K_1 + 3K_2}{4}$ (d) $\dfrac{3K_1 + K_2}{4}$

Solution : (c) We can consider this arrangement as a parallel combination of two materials having different thermal conductivities K_1 and K_2

For parallel combination $K = \dfrac{K_1A_1 + K_2A_2}{A_1 + A_2}$

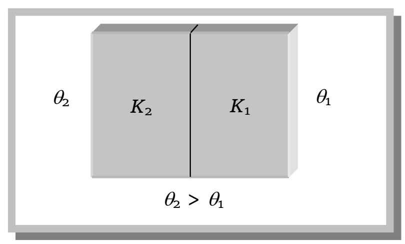


A_1 = Area of cross-section of internal cylinder = πR^2,

A_2 = Area of cross-section of outer cylinder = $\pi(2R)^2 - \pi(R)$

$$\therefore \quad K = \frac{K_1.\pi R^2 + K_2\ 3\pi R^2}{\pi R^2 + 3\pi R^2} = \frac{K_1 + 3K_2}{4}$$

Problem **15.** The temperature of the interface of a compound wall as shown in the figure, in terms of their thermal resistances R_1 and R_2 is



(a) $\dfrac{\theta_1 + \theta_2}{2}$

(b) $\dfrac{R_1\theta_2 + R_2\theta_1}{R_1 + R_2}$

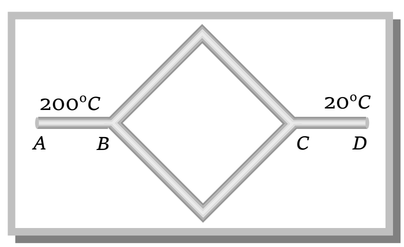
(c) $\dfrac{R_1\theta_1 + R_2\theta_2}{R_1 + R_2}$

(d) $\dfrac{R_2\theta_1 + R_1\theta_2}{\theta_1 + \theta_2}$

Solution : (b) Temperature of interface $\theta = \dfrac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$

Substituting $K_1 = \dfrac{l}{R_1A}$ and $K_2 = \dfrac{l}{R_2A}$ we get $\theta = \dfrac{R_1\theta_2 + R_2\theta_1}{R_1 + R_2}$.

Problem **16.** Six identical conducting rods are joined as shown in figure. Points A and D are maintained at temperatures 200°C and 20°C respectively. The temperature of junction B will be



(a) 120°C

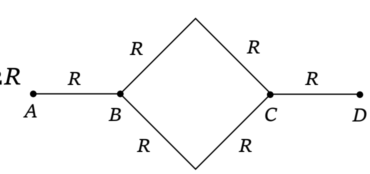
(b) 100°C

(c) 140°C

(d) 80°C

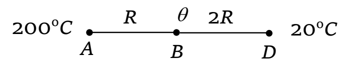
Solution : (c) Let the thermal resistance of each rod is R

Effective thermal resistance between B and D = $2R$



Temperature of interface $\theta = \dfrac{R_1\theta_2 + R_2\theta_1}{R_1 + R_2}$

$$\theta = \frac{R \times 20 + 2R \times 200}{R + 2R} = \frac{420}{3} = 140\ ^{o}C.$$



14.5 Ingen-Hauz Experiment

It is used to compare thermal conductivities of different materials. If l_1 and l_2 are the lengths of wax melted on rods, then the ratio of thermal conductivities is $\frac{K_1}{K_2} = \frac{l_1^2}{l_2^2}$

i.e., in this experiment, we observe Thermal conductivity $\propto$ (length)^2

14.6 Searle's Experiment

It is a method of determination of K of a metallic rod. Here we are not much interested in the detailed description of the experimental setup. We will only understand its essence, which is the essence of solving many numerical problems.

In this experiment a temperature difference $(\theta_1 - \theta_2)$ is maintained across a rod of length l and area of cross section A. If the thermal conductivity of the material of the rod is K, then the amount of heat transmitted by the rod from the hot end to the cold end in time t is given by, $Q = \frac{KA(\theta_1 - \theta_2)t}{l}$ ......(i)

In Searle's experiment, this heat reaching the other end is utilized to raise the temperature of certain amount of water flowing through pipes circulating around the other end of the rod. If temperature of the water at the inlet is θ_3 and at the outlet is θ_4, then the amount of heat absorbed by water is given by, $Q = mc(\theta_4 - \theta_3)$ ......(ii)

Where, m is the mass of the water which has absorbed this heat and temperature is raised and c is the specific heat of the water

Equating (i) and (ii), K can be determined *i.e.,* $K = \frac{mc(\theta_4 - \theta_3)l}{A(\theta_1 - \theta_2)t}$

Note : ❑ In numericals we may have the situation where the amount of heat travelling to the other end may be required to do some other work *e.g.,* it may be required to melt the given amount of ice. In that case equation (i) will have to be equated to mL.

i.e. $mL = \frac{KA(\theta_1 - \theta_2)t}{l}$

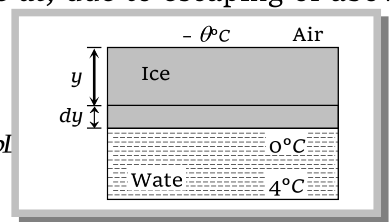
14.7 Growth of Ice on Lake

Water in a lake starts freezing if the atmospheric temperature drops below $0^{o}C$. Let y be the thickness of ice layer in the lake at any instant t and atmospheric temperature is $-\theta^{o}C$. The temperature of water in contact with lower surface of ice will be zero. If A is the area of lake, heat escaping through ice in time dt is

$$dQ_1 = \frac{KA[0-(-\theta)]dt}{y}$$

Now, suppose the thickness of ice layer increases by dy in time dt, due to escaping of above heat. Then

$$dQ_2 = mL = \rho(dy\ A)L$$



As $dQ_1 = dQ_2$, hence, rate of growth of ice will be $(dy/dt) = (K\theta/\rho L$

So, the time taken by ice to grow to a thickness y is $t = \frac{\rho L}{K\theta}\int_0^y y\,dy = \frac{\quad}{2K\theta} y$

If the thickness is increased from y_1 to y_2 then time taken $t = \frac{\rho L}{K\theta}\int_{y_1}^{y_2} y dy = \frac{\rho L}{2K\theta}(y_2^2 - y_1^2)$

(i) Take care and do not apply a negative sign for putting values of temperature in formula and also do not convert it to absolute scale.

(ii) Ice is a poor conductor of heat, therefore the rate of increase of thickness of ice on ponds decreases with time.

(iii) It follows from the above equation that time taken to double and triple the thickness, will be in the ratio of

$$t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2, \textit{ i.e., } t_1 : t_2 : t_3 :: 1 : 4 : 9$$

(iv) The time intervals to change the thickness from 0 to y, from y to $2y$ and so on will be in the ratio

$$\Delta t_1 : \Delta t_2 : \Delta t_3 :: (1^2 - 0^2) : (2^2 - 1^2) : (3^2 : 2^2);\ \Delta t_1 : \Delta t_2 : \Delta t_3 :: 1 : 3 : 5$$

Sample problems (miscellaneous) based on Conduction

***Problem* 17.** If the ratio of coefficient of thermal conductivity of silver and copper is 10 : 9, then the ratio of the lengths upto which wax will melt in Ingen Hausz experiment will be

(a) 6 : 10 (b) $\sqrt{10} : 3$ (c) 100 : 81 (d) 81 : 100

Solution : (b) According to Ingen Hausz, $K \propto l^2$ $\therefore \frac{l_1}{l_2} = \sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{10}{9}} = \frac{\sqrt{10}}{3}$.

***Problem* 18.** An ice box used for keeping eatables cool has a total wall area of 1 *metre*^{2} and a wall thickness of 5.0 *cm*. The thermal conductivity of the ice box is K= 0.01 *J/m°C*. It is filled with ice at 0°*C* along with eatables on a day when the temperature is 30°*C*. The latent heat of fusion of ice is 334×10^3 *J/kg*. The amount of ice melted in one day is (1 *day* = 86,400 *seconds*) **[MP PMT 1995]**

(a) 776 *g* (b) 7760 *g* (c) 11520 *g* (d) 1552 *g*

Solution : (d) Quantity of heat transferred through wall will be utilized in melting of ice.

$$Q = \frac{KA\Delta\theta\, t}{\Delta x} = mL \quad \therefore \text{ Amount of ice melted } m = \frac{KA\Delta\theta\, t}{\Delta x\ L}$$

$$\therefore \quad m = \frac{0.01 \times 1 \times (30-0) \times 86400}{5 \times 10^{-2} \times 334 \times 10^{3}} = 1.552\ kg \text{ or } 1552 g$$

Problem 19. Ice starts forming in lake with water at 0°*C* and when the atmospheric temperature is – 10°*C*. If the time taken for 1 *cm* of ice be 7 *hr*, then the time taken for the thickness of ice to change from 1 *cm* to 2 *cm* is

[NCERT 1971; MP PMT / PET 1988]

(a) 7 *hrs* (b) 14 *hrs* (c) Less than 7 *hrs* (d) More than 7 *hrs*

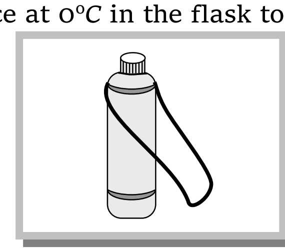
Solution : (d) Time required in increment of thickness from y_1 to y_2 $t = \frac{\rho L}{2K\theta}(y_2^2 - y_1^2)$

In first condition $y_1 = 0,\ y_2 = 1\ cm$ then $\Delta t_1 \propto (1^2 - 0^2)$

In second condition $y_1 = 1\ cm,\ y_2 = 2\ cm$ then $\Delta t_2 \propto (2^2 - 1^2)$

$$\therefore \quad \frac{\Delta t_1}{\Delta t_2} = \frac{1}{3} \Rightarrow \Delta t_2 = 3 \times \Delta t_1 = 3 \times 7 = 21\ hrs.$$

Problem 20. The only possibility of heat flow in a thermos flask is through its cork which is 75 cm^2 in area and 5 *cm* thick. Its thermal conductivity is 0.0075 *cal/cm sec°C*. The outside temperature is 40°*C* and latent heat of ice is 80 $cal\ g^{-1}$. Time taken by 500 *g* of ice at 0°*C* in the flask to melt into water at 0°*C* is **[CPMT 1974, 78; MNR 1983]**



(a) 2.47 *hr*

(b) 4.27 *hr*

(c) 7.42 *hr*

(d) 4.72 *hr*.

Solution : (a) $mL = \frac{KA\Delta\theta\ t}{\Delta x} \Rightarrow 500 \times 80 = \frac{0.0075 \times 75 \times (40-0)t}{5} \Rightarrow t = 8.9 \times 10^3\ sec = 2.47\ hr.$

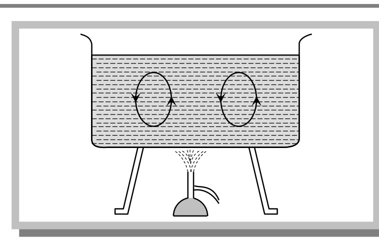
Problem 21. There is ice formation in a tank of water of thickness 10 *cm*. How much time it will take to have a layer of 0.1 *cm* below it ? The outer temperature is – 5°*C*, the thermal conductivity of ice *K* = 0.005 *cal/cm-sec°C*, the latent heat of ice is 80 *cal/gm* and the density of ice is 0.91 *gm/cc*

(a) 46.39 *minutes* (b) 47.63 *minutes* (c) 48.77 *minutes* (d) 49.31 *minutes*

Solution : (c) $t = \frac{\rho l}{2k\theta}\left(y_2^2 - y_1^2\right) = \frac{0.91 \times 80}{2 \times 0.005 \times 5}[(10.1)^2 - (10)^2] = 2926\ sec = 48.77 \text{ min}.$

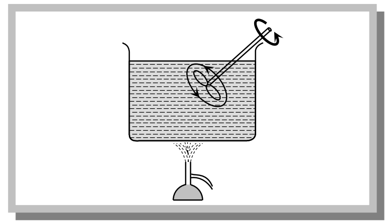
14.8 Convection

Mode of transfer of heat by means of migration of material particles of medium is called convection. It is of two types.



(1) **Natural convection :** This arise due to difference of densities at two places and is a consequence of gravity because on account of gravity the hot light particles rise up and cold heavy particles try setting down. It mostly occurs on heating a liquid/fluid.

(2) **Forced convection :** If a fluid is forced to move to take up heat from a hot body then the convection process is called forced convection. In this case Newton's law of cooling holds good. According to which rate of loss of heat from a hot body due to moving fluid is directly proportional to the surface area of body and excess temperature of body over its surroundings



i.e. $$\frac{Q}{t} \propto A(T - T_0)$$

$$\frac{Q}{t} = h\,A(T - T_0)$$

where h = Constant of proportionality called convection coefficient,

T = Temperature of body and T_0 = Temperature of surrounding

Convection coefficient (h) depends on properties of fluid such as density, viscosity, specific heat and thermal conductivity.

(i) Natural convection takes place from bottom to top while forced convection in any direction.

(ii) In case of natural convection, convection currents move warm air upwards and cool air downwards. That is why heating is done from base, while cooling from the top.

(iii) Natural convection plays an important role in ventilation, in changing climate and weather and in forming land and sea breezes and trade winds.

(iv) Natural convection is not possible in a gravity free region such as a free falling lift or an orbiting satellite.

(v) The force of blood in our body by heart helps in keeping the temperature of body constant.

(vi) If liquids and gases are heated from the top (so that convection is not possible) they transfer heat (from top to bottom) by conduction.

(vii) Mercury though a liquid is heated by conduction and not by convection.

14.9 Radiation

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation.

Precisely it is electromagnetic energy transfer in the form of electromagnetic wave through any medium. It is possible even in vacuum.

For example, the heat from the sun reaches the earth through radiation.

Properties of thermal radiation

(1) The wavelength of thermal radiations ranges from $7.8 \times 10^{-7}\, m$ to $4 \times 10^{-4}\, m$. They belong to *infra-red* region of the electromagnetic spectrum. That is why thermal radiations are also called *infra-red radiations*.

Radiation	Frequency	Wavelength
Cosmic rays	$> 10^{21}\, Hz$	$< 10^{-13}\, m$
Gamma rays	10^{18} - $10^{21}\, Hz$	10^{-13} - $10^{-10}\, m$
X-rays	10^{16} - $10^{19}\, Hz$	10^{-11} - $10^{-8}\, m$ (0.1 Å - 100 Å)
Ultraviolet rays	7.5×10^{14} - $2 \times 10^{6}\, Hz$	1.4×10^{-8} - $4 \times 10^{-7}\, m$ (140 Å - 4000 Å)
Visible rays	4×10^{14} - $7.5 \times 10^{14}\, Hz$	4×10^{-7} - $7.8 \times 10^{-7}\, m$ (4000 Å - 7800 Å)
☞ Infrared rays (Heat)	3×10^{11} - $4 \times 10^{14}\, Hz$	7.8×10^{-7} - 10^{-3} (7800 Å - 3×10^{5} Å) ☜
Microwaves	3×10^{8} - $3 \times 10^{11}\, Hz$	$10^{-3}\, m$ - $0.1\, m$
Radio waves	10^{4} - $3 \times 10^{9}\, Hz$	$0.1\, m$ - $10^{4}\, m$

(2) Medium is not required for the propagation of these radiations.

(3) They produce sensation of warmth in us but we can't see them.

(4) Every body whose temperature is above zero Kelvin emits thermal radiation.

(5) Their speed is equal to that of light *i.e.* $(= 3 \times 10^{8}\, m/s)$.

(6) Their intensity is inversely proportional to the square of distance of point of observation from the source (*i.e.* $I \propto 1/d^2$).

(7) Just as light waves, they follow laws of reflection, refraction, interference, diffraction and polarisation.

(8) When these radiations fall on a surface then exert pressure on that surface which is known as radiation pressure.

(9) While travelling these radiations travel just like photons of other electromagnetic waves. They manifest themselves as heat only when they are absorbed by a substance.

(10) Spectrum of these radiations can not be obtained with the help of glass prism because it absorbs heat radiations. It is obtained by quartz or rock salt prism because these materials do

not have free electrons and interatomic vibrational frequency is greater than the radiation frequency, hence they do not absorb heat radiations.

14.10 Some Definition About Radiations

(1) **Diathermanous Medium :** A medium which allows heat radiations to pass through it without absorbing them is called diathermanous medium. Thus the temperature of a diathermanous medium does not increase irrespective of the amount of the thermal radiations passing through it *e.g.*, dry air, SO_2, rock salt ($NaCl$).

(i) Dry air does not get heated in summers by absorbing heat radiations from sun. It gets heated through convection by receiving heat from the surface of earth.

(ii) In winters heat from sun is directly absorbed by human flesh while the surrounding air being diathermanous is still cool. This is the reason that sun's warmth in winter season appears very satisfying to us.

(2) **Athermanous medium :** A medium which partly absorbs heat rays is called a thermous medium As a result temperature of an athermanous medium increases when heat radiations pass through it *e.g.*, wood, metal, moist air, simple glass, human flesh *etc*.

Glass and water vapours transmit shorter wavelengths through them but reflects longer wavelengths. This concept is utilised in Green house effect. Glass transmits those waves which are emitted by a source at a temperature greater than $100°C$. So, heat rays emitted from sun are able to enter through glass enclosure but heat emitted by small plants growing in the nursery gets trapped inside the enclosure.

(3) **Reflectance, Absorptance and transmittance**

When thermal radiations (Q) fall on a body, they are partly reflected, partly absorbed and partly transmitted.

(i) Reflectance or reflecting power (r) : It is defined as the ratio of the amount of thermal radiations reflected (Q_r) by the body in a given time to the total amount of thermal radiations incident on the body in that time.

(ii) Absorptance or absorbing power (a) : It is defined as the ratio of the amount of thermal radiations absorbed (Q_a) by the body in a given time to the total amount of thermal radiatons incident on the body in that time.

(iii) Transmittance or transmitting power (t) : It is defined as the ratio of the amount of thermal radiations transmitted (Q_t) by the body in a given time to the total amount of thermal radiations incident on the body in that time.

From the above definitions $r = \frac{Q_r}{Q}$, $a = \frac{Q_a}{Q}$ and $t = \frac{Q_t}{Q}$

By adding we get $r + a + t = \frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q} = \frac{(Q_r + Q_a + Q_t)}{Q} = 1$

$\therefore \quad r + a + t = 1$

(a) r, a and t all are the pure ratios so they have no unit and dimension.

(b) For perfect reflector : $r = 1$, $a = 0$ and $t = 0$

(c) For perfect absorber : $a = 1$, $r = 0$ and $t = 0$ (Perfectly black body)

(d) For perfect transmitter : $t = 1$, $a = 0$ and $r = 0$

(e) If body does not transmit any heat radiation, $t = 0$ $\therefore\ r + a = 1$ or $a = 1 - r$

So if r is more, a is less and vice-versa. It means good reflectors are bad absorbers.

(4) **Monochromatic Emittance or Spectral emissive power**

For a given surface it is defined as the radiant energy emitted per sec per unit area of the surface with in a unit wavelength around λ *i.e.* lying between $\left(\lambda - \frac{1}{2}\right)$ to $\left(\lambda + \frac{1}{2}\right)$.

Spectral emissive power $(E_\lambda) = \frac{\text{Energy}}{\text{Area} \times \text{time} \times \text{wavelength}}$

Unit : $\frac{Joule}{m^2 \times sec \times Å}$ and Dimension : $[ML^{-1}T^{-3}]$

(5) **Total emittance or total emissive power**

It is defined as the total amount of thermal energy emitted per unit time, per unit area of the body for all possible wavelengths. $E = \int_0^\infty E_\lambda d\lambda$

Unit : $\frac{Joule}{m^2 \times \text{sec}}$ or $\frac{Watt}{m^2}$ and Dimension : $[MT^{-3}]$

(6) **Monochromatic absorptance or spectral absorptive power**

It is defined as the ratio of the amount of the energy absorbed in a certain time to the total heat energy incident upon it in the same time, both in the unit wavelength interval. It is dimensionless and unit less quantity. It is represented by a_λ.

(7) **Total absorptance or total absorpting power :** It is defined as the total amount of thermal energy absorbed per unit time, per unit area of the body for all possible wavelengths.

$$a = \int_0^\infty a_\lambda d\lambda$$

It is also unit less and dimensionless quantity.

(8) **Emissivity (e) :** Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the body ($E_{\text{practical}}$) to the total emissive power of a perfect black body (E_{black}) at that temperature.

i.e. $$e = \frac{E_{practical}}{E_{black}}$$

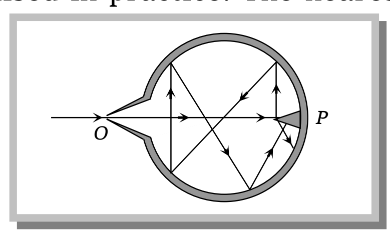
e = 1 for perfectly black body but for practical bodies emissivity (e) lies between zero and one ($0 < e < 1$).

(9) **Perfectly black body :** A perfectly black body is that which absorbs completely the radiations of all wavelengths incident on it. As a perfectly black body neither reflects nor transmits any radiation, therefore the absorptance of a perfectly black body is unity *i.e.* $t = 0$ and $r = 0$ $\quad \therefore a = 1$.

We know that the colour of an opaque body is the colour (wavelength) of radiation reflected by it. As a black body reflects no wavelength so, it appears black, whatever be the colour of radiations incident on it.

When perfectly black body is heated to a suitable high temperature, it emits radiation of all possible wavelengths. For example, temperature of the sun is very high (6000 K approx.) it emits all possible radiation so it is an example of black body.

(10) **Ferry's black body :** A perfectly black body can't be realised in practice. The nearest example of an ideal black body is the Ferry's black body. It is a doubled walled evacuated spherical cavity whose inner wall is blackened. There is a fine hole in it. All the radiations incident upon this hole are absorbed by this black body. If this black body is heated to high temperature then it emits radiations of all wavelengths.



Sample problems based on Radiation

***Problem* 22.** An ideal black body at room temperature is thrown into a furnace. It is observed that

(a) Initially it is the darkest body and at later times the brightest

(b) It is the darkest body at all times

(c) It cannot be distinguished at all times

(d) Initially it is the darkest body and at later times is cannot be distinguished

Solution : (d)

***Problem* 23.** A body is in thermal equilibrium with the surrounding

(a) It will stop emitting heat radiation

(b) Amount of radiations emitted and absorbed by it will be equal

(c) It will emit heat radiations at faster rate

(d) It will emit heat radiations slowly

Solution : (b)

Problem **24.** If transmission power of a surface is 1/6 and reflective power is 1/3, then its absorptive power will be

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

Solution : (b) $t = \frac{1}{6}$, $r = \frac{1}{3}$ and we know $a + r + t = 1$

$\therefore$ $a = 1 - r - t = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$.

14.11 Prevots Theory of Heat Exchange

(1) Every body emits heat radiations at all finite temperature (Except 0 *K*) as well as it absorbs radiations from the surroundings.

(2) Exchange of energy along various bodies takes place via radiation.

(3) The process of heat exchange among various bodies is a continuous phenomenon.

(4) If the amount of radiation absorbed by a body is greater than that emitted by it then the temperature of body increases and it appears hotter.

(5) If the amount of radiation absorbed by a body is less than that emitted by it, then the temperature of the body decreases and consequently the body appears colder.

(6) If the amount of radiation absorbed by a body is equal to that emitted by the body, then the body will be in thermal equilibrium and the temperature of the body remains constant.

(7) At absolute zero temperature (0 *K* or – 273°*C*) this law is not applicable because at this temperature the heat exchange among various bodies ceases.

14.12 Kirchoff's Law

The ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

Thus if $a_{\text{practical}}$ and $E_{\text{practical}}$ represent the absorptive and emissive power of a given surface, while a_{black} and E_{black} for a perfectly black body, then according to law $\frac{E_{\text{practical}}}{a_{\text{practical}}} = \frac{E_{\text{black}}}{a_{\text{black}}}$

But for a perfectly black body $a_{\text{black}} = 1$ so $\frac{E_{\text{practical}}}{a_{\text{practical}}} = E_{\text{black}}$

If emissive and absorptive powers are considered for a particular wavelength λ, $\left(\frac{E_\lambda}{a_\lambda}\right)_{\text{practical}} = (E_\lambda)_{\text{black}}$

Now since $(E_\lambda)_{\text{black}}$ is constant at a given temperature, according to this law if a surface is a good absorber of a particular wavelength it is also a good emitter of that wavelength.

This in turn implies that a good absorber is a good emitter (or radiator)

Applications of Kirchoff's law

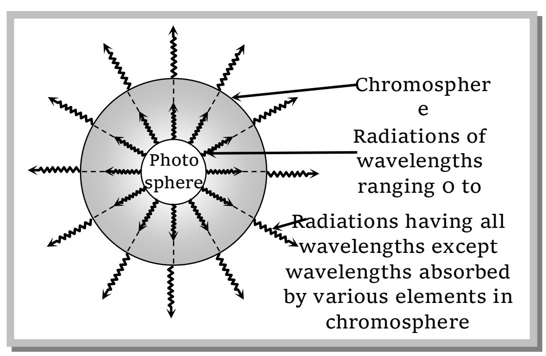
(1) Sand is rough black, so it is a good absorber and hence in deserts, days (when radiation from the sun is incident on sand) will be very hot. Now in accordance with Kirchoff's law, good absorber is a good emitter so nights (when sand emits radiation) will be cold. This is why days are hot and nights are cold in desert.

(2) Sodium vapours, on heating, emit two bright yellow lines. These are called D_1, D_2 lines of sodium. When continuos white light from an arc lamp is made to pass through sodium vapours at low temperature, the continuous spectrum is intercepted by two dark lines exactly in the same places as D_1 and D_2 lines. Hence sodium vapours when cold, absorbs the same wavelength, as they emit while hot. This is in accordance with Kirchoff's law.

(3) When a shining metal ball having some black spots on its surface is heated to a high temperature and is seen in dark, the black spots shine brightly and the shining ball becomes dull or invisible. The reason is that the black spots on heating absorb radiation and so emit these in dark while the polished shining part reflects radiations and absorb nothing and so does not emit radiations and becomes invisible in the dark.

(4) When a green glass is heated in furnace and taken out, it is found to glow with red light. This is because red and green are complimentary colours. At ordinary temperatures, a green glass appears green, because it transmits green colour and absorb red colour strongly. According to Kirchoff's law, this green glass, on heating must emit the red colour, which is absorbed strongly. Similarly when a red glass is heated to a high temperature it will glow with green light.

(5) Kirchoff' law also explains the existence of Fraunhoffer lines. These are some dark lines observed in the otherwise spectrum of the sun. According to Fraunhoffer, the central portion of the sun, called photosphere, is at a very high temperature and emits continuous light of all wavelengths. Before reaching us, the light passes through outer portion of the sun, called chromosphere. The chromosphere has some terrestrial elements in vapour form at lower temperature than that of photosphere. These elements absorb those wavelength which they would emit while hot. These absorbed wavelengths, which are missing appear as dark lines in the spectrum of the sun.

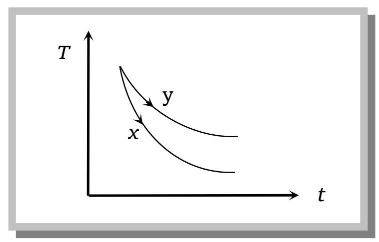


But during total solar eclipse these lines appear bright because the gases and vapour present in the chromosphere start emitting those radiation which they had absorbed.

(6) A person with black skin experiences more heat and more cold as compared to a person of white skin because when the outside temperature is greater, the person with black skin absorbs more heat and when the outside temperature is less the person with black skin radiates more energy.

Sample problems based on Kirrchoff's law

Problem **25.** The graph. Shown in the adjacent diagram, represents the variation of temperature (T) of two bodies, x and y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity (e) and absorptivity (a) of the two bodies **[IIT-JEE (Screening) 2003]**



(a) $e_x > e_y$ & $a_x < a_y$

(b) $e_x < e_y$ & $a_x > a_y$

(c) $e_x > e_y$ & $a_x > a_y$

(d) $e_x < e_y$ & $a_x < a_y$

Solution : (c) From the graph it is clear that initially both the bodies are at same temperature but after that at any instant temperature of body x is less then the temperature of body y. It means body x emits more heat *i.e.* emissivity of body x is more than body y $\therefore e_x > e_y$

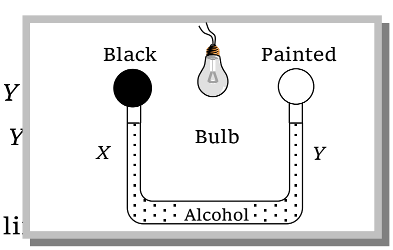
and according to Kirchoff's law good emitter are also good absorber so $a_x > a_y$.

Problem **26.** Certain substance emits only the wavelengths $\lambda_1, \lambda_2, \lambda_3$ and λ_4 when it is at a high temperature. When this substance is at a colder temperature, it will absorb only the following wavelengths **[MP PET 1990]**

(a) λ_1 (b) λ_2 (c) λ_1 and λ_2 (d) $\lambda_1, \lambda_2, \lambda_3$ and λ_4

Solution : (d) If a body emits wavelength $\lambda_1, \lambda_2, \lambda_3$ and λ_4 at a high temperature then at a lower temperature it will absorbs the radiation of same wavelength. This is in accordance with Kirchoff's law.

Problem **27.** The following figure shows two air-filled bulbs connected by a U-tube partly filled with alcohol. What happens to the levels of alcohol in the limbs X and Y when an electric bulb placed midway between the bulbs is lighted



(a) The level of alcohol in limb X falls while that in limb Y

(b) The level of alcohol in limb X rises while that in limb Y

(c) The level of alcohol falls in both limbs

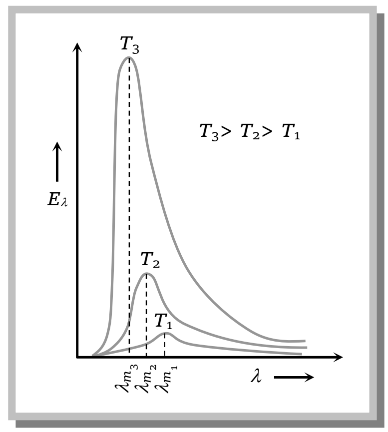
(d) There is no change in the levels of alcohol in the two li

Solution : (a) Black bulb absorbs more heat in comparison with painted bulb. So air in black bulb expands more. Hence the level of alcohol in limb X falls while that in limb Y rises.

14.13 Distribution of Energy in The Spectrum of Black Body

Langley and later on Lummer and Pringsheim investigated the distribution of energy amongst the different wavelengths in the thermal spectrum of a black body radiation. The results obtained are shown in figure. From these curves it is clear that

(1) At a given temperature energy is not uniformly distributed among different wavelengths.



(2) At a given temperature intensity of heat radiation increases with wavelength, reaches a maximum at a particular wavelength and with further increase in wavelength it decreases.

(3) With increase in temperature wavelength λ_m corresponding to most intense radiation decreases in such a way that $\lambda_m \times T = \text{constant.}$ [Wien's law]

(4) For all wavelengths an increase in temperature causes an increase in intensity.

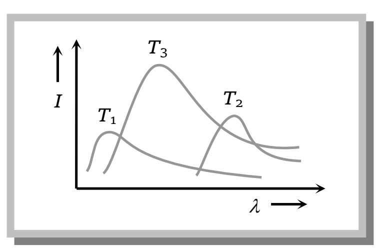
(5) The area under the curve $= \int E_\lambda d\lambda$ will represent the total intensity of radiation at a particular temperature. This area increases with rise in temperature of the body. It is found to be directly proportional to the fourth power of absolute temperature of the body, *i.e.*,

$$E = \int E_\lambda d\lambda \propto T^4 \qquad \text{[Stefan's law]}$$

(6) The energy (E_{max}) emitted corresponding to the wavelength of maximum emission (λ_m) increases with fifth power of the absolute temperature of the black body *i.e.*, $E_{max} \propto T^5$

Sample problems based on Energy distribution graph

***Problem* 28.** The plots of intensity versus wavelength for three black bodies at temperatures T_1, T_2 and T_3 respectively are as shown. Their temperature are such that



(a) $T_1 > T_2 > T_3$

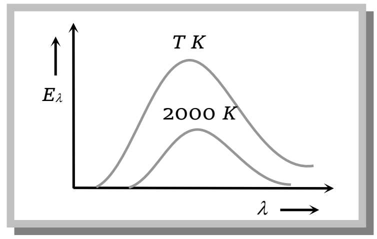
(b) $T_1 > T_3 > T_2$

(c) $T_2 > T_3 > T_1$

(d) $T_3 > T_2 > T_1$

Solution : (b) According to Wien's law $\lambda_m \propto \frac{1}{T}$ and from the figure $(\lambda_m)_1 < (\lambda_m)_3 < (\lambda_m)_2$ therefore $T_1 > T_3 > T_2$.

Problem 29. The adjoining diagram shows the spectral energy density distribution E_λ of a black body at two different temperatures. If the areas under the curves are in the ratio 16 : 1, the value of temperature T is **[DCE 1999]**



(a) 32,000 K

(b) 16,000 K

(c) 8,000 K

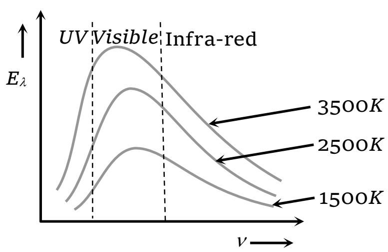
(d) 4,000 K

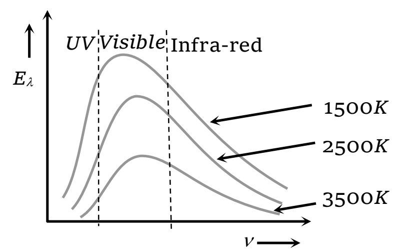
Solution : (d) Area under curve represents the emissive power of the body $\frac{E_T}{E_{2000}} = \frac{A_T}{A_{2000}} = \frac{16}{1}$ (given) ....(i)

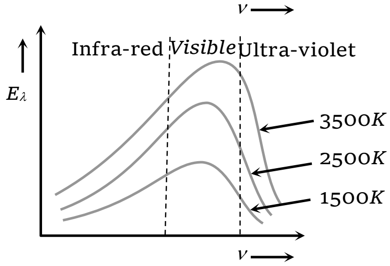
But from Stefan's law $E \propto T^4$ $\therefore \frac{E_T}{E_{2000}} = \left(\frac{T}{2000}\right)^4$ ....(ii)

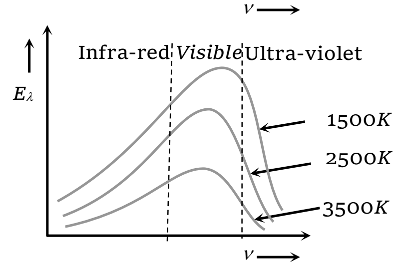
From (i) and (ii) $\left(\frac{T}{2000}\right)^4 = \frac{16}{1} \Rightarrow \frac{T}{2000} = 2 \Rightarrow T = 4000K$.

Problem 30. Following graph shows the correct variation in intensity of heat radiations by black body and frequency at a fixed temperature

(a)


(b)


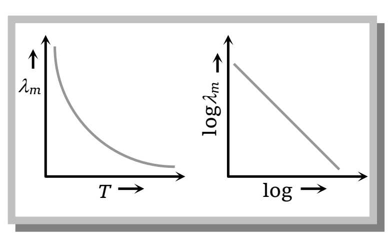
(c)


(d)


Solution : (c) As the temperature of body increases, frequency corresponding to maximum energy in radiation (ν_m) increases this is shown in graph (c).

14.14 Wien's Displacement Law

When a body is heated it emits radiations of all wavelength. However the intensity of radiations of different wavelength is different.



According to Wien's law the product of wavelength corresponding to maximum intensity of radiation and temperature of body (in Kelvin) is constant, *i.e.*

$\lambda_m T = b = \text{constant}$

Where b is Wien's constant and has value $2.89 \times 10^{-3} m\text{-}K$.

This law is of great importance in 'Astrophysics' as through the analysis of radiations coming from a distant star, by finding λ_m the temperature of the star $T(= b/\lambda_m)$ is determined.

Sample problems based on Wien's displacement law

***Problem* 31.** A black body at 200 K is found to emit maximum energy at a wavelength of 14 μm. When its temperature is raised to 1000 K, the wavelength at which maximum energy is emitted is **[MP PET 1991; BVP 2003]**

(a) 14 μm (b) 70 μF (c) 2.8 μm (d) 2.8 mm

Solution : (c) $\lambda_m T = \text{constant}$ $\Rightarrow \frac{(\lambda_m)_2}{(\lambda_m)_1} = \frac{T_1}{T_2} = \frac{200}{1000} = \frac{1}{5} \Rightarrow (\lambda_m)_2 = \frac{(\lambda_m)_1}{5} = \frac{14\,\mu m}{5} = 2.8\,\mu m$.

***Problem* 32.** The energy spectrum of a black body exhibits a maximum around a wavelength λ_0. The temperature of the black body is now changed such that the energy is maximum around a wavelength $\frac{3\lambda_0}{4}$. The power radiated by the black body will now increase by a factor of

(a) 256/81 (b) 64/27 (c) 16/9 (d) 4/3

Solution : (a) According to Wien's law wavelength corresponding to maximum energy decreases. When the temperature of black body increases *i.e.* $\lambda_m T =$ constant $\Rightarrow \frac{T_2}{T_1} = \frac{\lambda_1}{\lambda_2} = \frac{\lambda_0}{3\lambda_0/4} = \frac{4}{3}$

Now according to Stefan's law $\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{4}{3}\right)^4 = \frac{256}{81}$.

***Problem* 33.** The wavelength of maximum energy released during an atomic explosion was $2.93 \times 10^{-10} m$. Given that Wien's constant is $2.93 \times 10^{-3} m - K$, the maximum temperature attained must be of the order of

[Haryana CEE 1996; MH CET 2002]

(a) 10^{-7} K (b) 10^{7} K (c) 10^{-13} K (d) $5.86 \times 10^{7} K$

Solution : (b) From Wien's displacement law $\lambda_m T = b$ $\therefore$ $T = \frac{b}{\lambda_m} = \frac{2.93 \times 10^{-3}}{2.93 \times 10^{-10}} = 10^7 K$.

***Problem* 34.** Consider the following statements

Assertion (A) : Blue star is at higher temperature than red star

Reason (R) : Wien's displacement law states that

$$T \propto \frac{1}{\lambda_m}$$

Of these statements **[AIIMS 2002]**

(a) Both A and R are true and the R is a correct explanation of the A

(b) Both A and R are true but the R is not a correct explanation of the A

(c) A is true but the R is false

(d) Both A and R are false

(e) A is false but the R is true

Solution : (a) Wavelength of radiation from blue star is less than that of red star. So its temperature will be higher than red star.

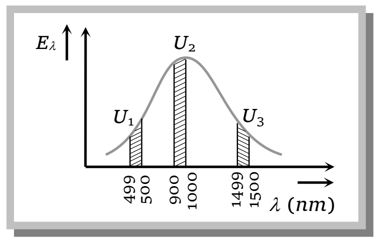
***Problem* 35.** A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wien's constant $b = 2.88 \times 10^6 nm\ K$. Then

(a) $U_1 = 0$ (b) $U_3 = 0$ (c) $U_1 > U_2$ (d) $U_2 > U_1$

Solution : (d) According to Wien's displacement law $\lambda_m T = b$

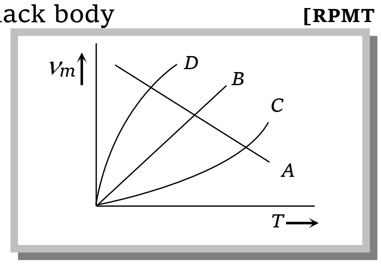
$$\therefore \quad \lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6\ nm\text{-}K}{2880\ nm} = 1000\ nm$$

i.e. energy corresponding to wavelength 1000 nm will be maximum *i.e.* U_2 will be maximum $U_1 < U_2 > U_3$



Energy distribution graph with wavelength will be as follows

***Problem* 36.** Which of the following is the ν_m-T graph for a perfectly black body **[RPMT 1996]**



(a) A

(b) B

(c) C

(d) D

Solution : (b) Wien's law $\lambda_m \propto \frac{1}{T}$ or $\nu_m \propto T$

ν_m increases with temperature. So the graph will be straight line.

14.15 Law of Distribution of Energy

The theoretical explanation of black body radiation was done by Planck.

If the walls of hollow enclosure are maintained at a constant temperature, then the inside of enclosure are filled with the electromagnetic radiation.

The radiation coming out from a small hole in the enclosure are called black body radiation. According to Max Planck, the radiation inside the enclosure may be assumed to be produced by a number of harmonic oscillators.

A harmonic oscillator oscillating with frequency ν can possesses energies, which are integral multiples of $h\nu$. Where h is a constant, called Planck's constant. Thus the harmonic oscillator can posses energies given by $E = nh\nu$ where n is an integer.

According to Planck's law $E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{[e^{hc/\lambda KT}-1]}d\lambda$

This law is valid for radiations of all wavelengths ranging from zero to infinite.

For radiations of short wavelength $\left(\lambda << \frac{hc}{KT}\right)$

Planck's law reduces to *Wien's energy distribution law* $E_\lambda d\lambda = \frac{A}{\lambda^5}e^{-B/\lambda T}d\lambda$

For radiations of long wavelength $\left(\lambda >> \frac{hc}{KT}\right)$

Planck's law reduces to *Rayleigh-Jeans energy distribution law* $E_\lambda d\lambda = \frac{8\pi KT}{\lambda^4}d\lambda$

14.16 Stefan's Law

According to it the radiant energy emitted by a perfectly black body per unit area per sec (*i.e.* emissive power of black body) is directly proportional to the fourth power of its absolute temperature,

i.e. $E \propto T^4$ or $E = \sigma T^4$

where σ is a constant called Stefan's constant having dimension $[MT^{-3}\theta^{-4}]$ and value $5.67 \times 10^{-8} W/m^2K^4$.

(i) If e is the emissivity of the body then $E = e\sigma T^4$

(ii) If Q is the total energy radiated by the body then $E = \frac{Q}{A \times t} = e\sigma T^4 \Rightarrow Q = Ate\sigma T^4$

(iii) If a body at temperature T is surrounded by a body at temperature T_0, then Stefan's law may be put as

$$E = e\sigma(T^4 - T_0^4)$$

(iv) Cooling by radiation : If a body at temperature T is in an environment of temperature $T_0 (< T)$, the body is loosing as well as receiving so net rate of loss of energy

$$\frac{dQ}{dt} = eA\sigma(T^4 - T_0^4) \quad .....(i)$$

Now if m is the mass of body and c its specific heat, the rate of loss of heat at temperature T must be

$$\frac{dQ}{dt} = mc\frac{dT}{dt} \quad .....(ii)$$

From equation (i) and (ii) $mc\frac{dT}{dt} = eA\sigma(T^4 - T_0^4)$

$\therefore$ Rate of fall of temperature or rate of cooling, $\frac{dT}{dt} = \frac{eA\sigma}{mc}(T^4 - T_0^4)$ .....(iii)

i.e. when a body cools by radiation the rate of cooling depends on

(a) Nature of radiating surface *i.e.* greater the emissivity, faster will be the cooling.

(b) Area of radiating surface, *i.e.* greater the area of radiating surface, faster will be the cooling.

(c) Mass of radiating body *i.e.* greater the mass of radiating body slower will be the cooling.

(d) Specific heat of radiating body *i.e.* greater the specific heat of radiating body slower will be cooling.

(e) Temperature of radiating body *i.e.* greater the temperature of body faster will be cooling.

(f) Temperature of surrounding *i.e.* greater the temperature of surrounding slower will be cooling.

Sample problems based on Stefan's law

Problem **37.** Two black metallic spheres of radius 4m, at 2000 *K* and 1m at 4000 *K* will have ratio of energy radiation as

[RPET 2000; AIEEE 2002]

(a) 1 : 1 (b) 4 : 1 (c) 1 : 4 (d) 2 : 1

Solution : (a) $Q = \sigma A\, t\, T^4 \Rightarrow \frac{Q_1}{Q_2} = \frac{A_1}{A_2}\left(\frac{T_1}{T_2}\right)^4 = \frac{\pi r_1^2}{\pi r_2^2}\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{4}{1}\right)^2 \times \left(\frac{2000}{4000}\right)^4 = 16 \times \frac{1}{16} = 1:1$.

Problem **38.** Two identical metal balls at temperature 200°*C* and 400°*C* kept in air at 27°*C*. The ratio of net heat loss by these bodies is

(a) 1/ 4 (b) 1/ 2 (c) 1/ 16 (d) $\frac{473^4 - 300^4}{673^4 - 300^4}$

Solution : (d) Emissive power of a body (*T*) in a surrounding (T_0), $E = \sigma(T^4 - T_0^4)$ or $Q \propto (T_1^4 - T_0^4)$

$$\Rightarrow \quad \frac{Q_1}{Q_2} = \frac{(T_1^4 - T_0^4)}{(T_2^4 - T_0^4)} = \frac{(473)^4 - (300)^4}{(673)^4 - (300)^4}.$$

Problem **39.** Two spheres made of same material have radii in the ratio 1 : 2. Both are at same temperature. Ratio of heat radiation energy emitted per second by them is

(a) 1 : 2 (b) 1 : 8 (c) 1 : 4 (d) 1 : 16

Solution : (c) $P = \frac{Q}{t} = A\sigma T^4 \quad \therefore \frac{P_1}{P_2} = \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ [If *T* = constant]

Problem **40.** Two spherical black bodies of radii r_1 and r_2 and with surface temperature T_1 and T_2 respectively radiate the same power. Then the ratio of r_1 and r_2 will be

(a) $\left(\frac{T_2}{T_1}\right)^2$ (b) $\left(\frac{T_2}{T_1}\right)^4$ (c) $\left(\frac{T_1}{T_2}\right)^2$ (d) $\left(\frac{T_1}{T_2}\right)^4$

Solution : (a) $P = A\,\sigma T^4 = 4\pi r^2 \sigma T^4 \Rightarrow P \propto r^2 T^4$ or $r^2 \propto \frac{1}{T^4}$ [As P = constant] $\therefore \frac{r_1}{r_2} = \left(\frac{T_2}{T_1}\right.$

Problem **41.** The rectangular surface of area 8 cm × 4 cm of a black body at a temperature of 127°C emits energy at the rate of E per second. If the length and breadth of the surface are each reduced to half of the initial value and the temperature is raised to 327°C, the rate of emission of energy will become **[MP PET 2000]**

(a) $\frac{3}{8}E$ (b) $\frac{81}{16}E$ (c) $\frac{9}{16}E$ (d) $\frac{81}{64}E$

Solution : (d) Energy radiated by body per second $\frac{Q}{t} = A\sigma T^4$ or $\frac{Q}{t} \propto l \times b \times T^4$ [Area = $l \times b$]

$$\therefore \frac{E_2}{E_1} = \frac{l_2}{l_1} \times \frac{b_2}{b_1} \times \left(\frac{T_2}{T_1}\right)^4 = \frac{(l_1/2)}{l_1} \times \frac{(b_1/2)}{b_1} \times \left(\frac{600}{400}\right)^4 = \frac{1}{2} \times \frac{1}{2} \times \left(\frac{3}{2}\right)^4 \Rightarrow E_2 = \frac{81}{64}E$$

Problem **42.** A solid copper cube of edges 1 cm is suspended in an evacuated enclosure. Its temperature is found to fall from 100°C to 99°C in 100s. Another solid copper cube of edges 2 cm, with similar surface nature, is suspended in a similar manner. The time required for this cube to cool from 100°C to 99°C will be approximately

(a) 25 s (b) 50 s (c) 200 s (d) 400 s

Solution : (c) $\frac{dT}{dt} = \frac{eA\sigma}{mc}\left(T^4 - T_0^4\right) = \frac{e(6a^2)\sigma}{(a^3 \times \rho)c}(T^4 - T_0^4) \Rightarrow$ For the same fall in temperature, time $dt \propto a$

$\frac{dt_2}{dt_1} = \frac{a_2}{a_1} = \frac{2cm}{1cm} \Rightarrow dt_2 = 2 \times dt_1 = 2 \times 100\ sec = 200\ sec$ [As $A = 6a^2$ and $m = V \times \rho = a^3 \times \rho$]

Problem **43.** Two metallic spheres S_1 and S_2 are made of the same material and have identical surface finish. The mass of S_1 is three times that of S_2. Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is

(a) 1/ 3 (b) $(1/3)^{1/3}$ (c) $1/\sqrt{3}$ (d) $\sqrt{3}/1$

Solution : (b) $\frac{dT}{dt} = \frac{e\,A\sigma}{mc}(T^4 - T_0^4)$ $\therefore$ Rate of cooling $R \propto \frac{4\pi r^2}{\frac{4}{3}\pi r^3 \times \rho} \propto \frac{1}{r} \Rightarrow \frac{R_1}{R_2} = \frac{r_2}{r_1}$

But according to problem $m_1 = 3m_2 \Rightarrow \frac{4}{3}\pi r_1^3 \times \rho = 3\left(\frac{4}{3}\pi r_2^3 \times \rho\right) \Rightarrow r_1^3 = 3r_2^3 \Rightarrow \left(\frac{r_2}{r_1}\right) = \left(\frac{1}{3}\right)^{1/3}$

$\therefore$ Ratio of rate of cooling $\frac{R_1}{R_2} = \left(\frac{1}{3}\right)^{1/3}$.

Problem **44.** A sphere, a cube and a thin circular plate, all made of same substance and all have same mass. These are heated to 200°C and then placed in a room, then the

(a) Temperature of sphere drops to room temperature at last

(b) Temperature of cube drops to room temperature at last

(c) Temperature of thin circular plate drops to room temperature at last

(d) Temperature of all the three drops to room temperature at the same time

Solution : (a) $\frac{dT}{dt} = \frac{eA\sigma}{mc}(T^4 - T_0^4) = \frac{eA\sigma}{V\rho c}(T^4 - T_0^4)$ $\therefore$ Rate of cooling $R \propto A$

[As masses are equal then volume of each body must be equal because materiel is same]

i.e. rate of cooling depends on the area of cross-section and we know that for a given volume the area of cross-section will be minimum for sphere. It means the rate of cooling will be minimum in case of sphere.

So the temperature of sphere drops to room temperature at last.

***Problem* 45.** A solid copper sphere (density ρ and specific heat capacity c) of radius r at an initial temperature 200K is suspended inside a chamber whose walls are at almost 0K. The time required (in μs) for the temperature of the sphere to drop to 100 K is

(a) $\frac{72}{7}\frac{r\rho c}{\sigma}$ (b) $\frac{7}{72}\frac{r\rho c}{\sigma}$ (c) $\frac{27}{7}\frac{r\rho c}{\sigma}$ (d) $\frac{7}{27}\frac{r\rho c}{\sigma}$

Solution : (b) $\frac{dT}{dt} = \frac{\sigma A}{mcJ}(T^4 - T_0^4)$ [In the given problem fall in temperature of body $dT = (200 - 100) = 100K$

Temperature of surrounding T_0 = 0K, Initial temperature of body T = 200K]

$\frac{100}{dt} = \frac{\sigma 4\pi r^2}{\frac{4}{3}\pi r^3 \rho c J}(200^4 - 0^4) \Rightarrow dt = \frac{r\rho c J}{48\sigma} \times 10^{-6}s = \frac{r\rho c}{\sigma}.\frac{4.2}{48} \times 10^{-6} = \frac{7}{80}\frac{r\rho c}{\sigma}\mu s \simeq \frac{7}{72}\frac{r\rho c}{\sigma}\mu s$ [As J = 4.2]

***Problem* 46.** A sphere and a cube of same material and same volume are heated upto same temperature and allowed to cool in the same surroundings. The ratio of the amounts of radiations emitted will be

(a) 1 : 1 (b) $\frac{4\pi}{3}:1$ (c) $\left(\frac{\pi}{6}\right)^{1/3}:1$ (d) $\frac{1}{2}\left(\frac{4\pi}{3}\right)^{2/3}:1$

Solution : (c) $Q = \sigma A\, t\,(T^4 - T_0^4)$

If T, T_0, σ and t are same for both bodies then $\frac{Q_{sphere}}{Q_{cube}} = \frac{A_{sphere}}{A_{cube}} = \frac{4\pi r^2}{6a^2}$ .....(i)

But according to problem, volume of sphere = Volume of cube $\Rightarrow \frac{4}{3}\pi r^3 = a^3 \Rightarrow a = \left(\frac{4}{3}\pi\right)^{1/3} r$

Substituting the value of a in equation (i) we get

$$\frac{Q_{sphere}}{Q_{cube}} = \frac{4\pi r^2}{6a^2} = \frac{4\pi r^2}{6\left\{\left(\frac{4}{3}\pi\right)^{1/3} r\right\}^2} = \frac{4\pi r^2}{6\left(\frac{4}{3}\pi\right)^{2/3} r^2} = \left(\frac{\pi}{6}\right)^{1/3} :1$$

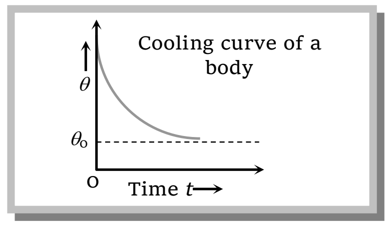
14.17 Newton's Law of Cooling

If in case of cooling by radiation the temperature T of body is not very different from that of surrounding

i.e. $T = T_0 + \Delta T$

$$T^4 - T_0^4 = [(T_0 + \Delta T)^4 - T_0^4] = T_0^4\left[\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1\right] = T_0^4\left(1 + \frac{4\Delta T}{T_0} - 1\right)$$ [Using Binomial theorem]

$$= 4T_0^3 \Delta T \qquad .....(i)$$



By Stefan's law, $\frac{dT}{dt} = \frac{eA\sigma}{mc}[T^4 - T_0^4]$

From equation (i), $\frac{dT}{dt} = \frac{eA\sigma}{mc} 4T_0^3 \Delta T$

So $\frac{dT}{dt} \propto \Delta T$ or $\frac{d\theta}{dt} \propto \theta - \theta_0$

i.e., if the temperature of body is not very different from surrounding, rate of cooling is proportional to temperature difference between the body and its surrounding. This law is called Newton's law of cooling.

(1) Practical examples

(i) Hot water loses heat in smaller duration as compared to moderate warm water.

(ii) Adding milk in hot tea reduces the rate of cooling.

(2) Greater the temperature difference between body and its surrounding greater will be the rate of cooling.

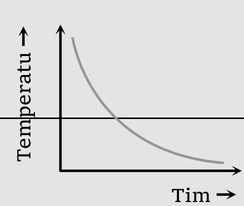
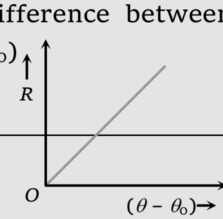
(3) If $\theta = \theta_0$, $\frac{d\theta}{dt} = 0$ *i.e.* a body can never be cooled to a temperature lesser than its surrounding by radiation.

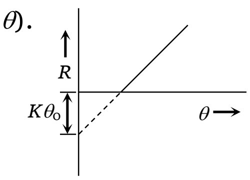
(4) If a body cools by radiation from $\theta_1^o C$ to $\theta_2^o C$ in time t, then $\frac{d\theta}{dt} = \frac{\theta_1 - \theta_2}{t}$ and $\theta = \theta_{av} = \frac{\theta_1 + \theta_2}{2}$

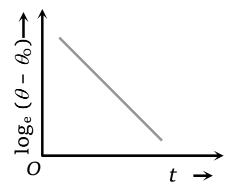
The Newton's law of cooling becomes $\left[\frac{\theta_1 - \theta_2}{t}\right] = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$

This form of law helps in solving numericals.

(5) Cooling curves :

Curve between temperature of body θ and time.	Curve between rate of cooling (R) and temperature difference between body (θ) and surrounding (θ_0)
	

<table>
<tr>
<td>$\theta - \theta_0 = Ae^{-kt}$, which indicates temperature decreases exponentially with increasing time.</td>
<td>$R \propto (\theta - \theta_0)$. This is a straight line passing through origin.</td>
</tr>
<tr>
<td>Curve between the rate of cooling (R) and body temperature (θ).


$R = K(\theta - \theta_0) = K\theta - K\theta_0$
This is a straight line intercept R-axis at $-K\theta_0$</td>
<td>Curve between log(θ – θ_0) and time


As $\frac{d\theta}{dt} \propto -(\theta - \theta_0) \Rightarrow \frac{d\theta}{(\theta - \theta_0)} = -Kdt$
Integrating $\log_e(\theta - \theta_0) = -Kt + C$
$\log_e(\theta - \theta_0) = -Kt + \log_e A$
This is a straight line with negative slope</td>
</tr>
</table>

(6) Determination of specific heat of a liquid : If volume, radiating surface area, nature of surface, initial temperature and surrounding of water and given liquid are equal and they are allowed to cool down (by radiation) then rate of loss of heat and fall in temperature of both will be same.

i.e. $$\left(\frac{dQ}{dt}\right)_{\text{water}} = \left(\frac{dQ}{dt}\right)_{\text{liquid}}$$

$$(ms + W)\frac{(\theta_1 - \theta_2)}{t_1} = (m_1 s_1 + W)\frac{(\theta_1 - \theta_2)}{t_2}$$

or $$\left[\frac{ms + W}{t_1}\right] = \left[\frac{m_1 s_1 + W}{t_2}\right]$$ [where W = water equivalent of calorimeter]

If density of water and liquid is ρ and ρ' respectively then $m = V\rho$ and $m' = V\rho'$

Sample problems based on Newton's law of cooling

***Problem* 47.** A bucket full of hot water cools from 75°*C* to 70°*C* in time T_1, from 70°*C* to 65°*C* in time T_2 and from 65°*C* to 60°*C* in time T_3, then

(a) $T_1 = T_2 = T_3$ (b) $T_1 > T_2 > T_3$ (c) $T_1 < T_2 < T_3$ (d) $T_1 > T_2 < T_3$

Solution : (c) According to Newton's law of cooling rate of cooling depends upon the difference of temperature between the body and the surrounding. It means that when the difference of temperature between the body and the surrounding is small then time required for same fall in temperature is more in comparison with the same fall at higher temperature difference between the body and surrounding. So according to problem $T_1 < T_2 < T_3$.

***Problem* 48.** A cup of tea cools from 80°C to 60°C in one minute. The ambient temperature is 30°C. In cooling from 60°C to 50°C it will take

(a) 30 *Seconds* (b) 60 *Seconds* (c) 90 *Seconds* (d) 50 *Seconds*

Solution : (d) According to Newton's law of cooling $\frac{\theta_1 - \theta_2}{t} \propto \left[\frac{\theta_1 + \theta_2}{2} - \theta\right]$

For first condition $\frac{80-60}{60} \propto \left[\frac{80+60}{2} - 30\right]$ .....(i)

and for second condition $\frac{60-50}{t} \propto \left[\frac{60+50}{2} - 30\right]$ ....(ii)

By solving (i) and (ii) we get $t = 48$ *sec* $\simeq 50$ *sec.*

***Problem* 49.** A body takes T minutes to cool from 62°C to 61°C when the surrounding temperature is 30°C. The time taken by the body to cool from 46°C to 45.5°C is

(a) Greater than T minutes (b) Equal to T minutes (c) Less than T minutes(d)

Solution : (b) According to Newton's law of cooling $\frac{\theta_1 - \theta_2}{t} \propto \left[\frac{\theta_1 + \theta_2}{2} - \theta\right]$

For first condition $\frac{62-61}{T} \propto \left[\frac{62+61}{2} - 30\right]$ .....(i)

and for second condition $\frac{46-45.5}{t} \propto \left[\frac{46+45.5}{2} - 30\right]$ ....(ii)

By solving (i) and (ii) we get $t = T$ *sec.*

***Problem* 50.** The rates of cooling of two different liquids put in exactly similar calorimeters and kept in identical surroundings are the same if

(a) The masses of the liquids are equal

(b) Equal masses of the liquids at the same temperature are taken

(c) Different volumes of the liquids at the same temperature are taken

(d) Equal volumes of the liquids at the same temperature are taken

Solution : (d) $\frac{dT}{dt} = \frac{\sigma A}{mc}(T^4 - T_0^4)$. If the liquids put in exactly similar calorimeters and identical surrounding then we can consider T_0 and A constant then $\frac{dT}{dt} \propto \frac{(T^4 - T_0^4)}{mc}$

If we consider that equal masses of liquid (m) are taken at the same temperature then $\frac{dT}{dt} \propto \frac{1}{c}$

So for same rate of cooling c should be equal which is not possible because liquids are of different nature.

Again from (i) equation $\frac{dT}{dt} \propto \frac{(T^4 - T_0^4)}{mc} \Rightarrow \frac{dT}{dt} \propto \frac{(T^4 - T_0^4)}{V\rho c}$

Now if we consider that equal volume of liquid (V) are taken at the same temperature then $\frac{dT}{dt} \propto \frac{1}{\rho c}$.

So for same rate of cooling multiplication of $\rho \times c$ for two liquid of different nature can be possible. So option (d) may be correct.

***Problem* 51.** Hot water cools from 60°*C* to 50°*C* in the first 10 *minutes* and to 42°*C* in the next 10 *minutes*. The temperature of the surrounding

(a) 5°*C* (b) 10°*C* (c) 15°*C* (d) 20°*C*

Solution : (b) $\frac{\theta_1 - \theta_2}{t} \propto \left[\frac{\theta_1 + \theta_2}{2} - \theta\right]$

For first condition $\frac{60 - 50}{10} \propto \left[\frac{60 + 50}{2} - \theta\right] \Rightarrow 1 = K\,[55 - \theta]$ ......(i)

For second condition $\frac{50 - 42}{10} \propto \left[\frac{50 + 42}{2} - \theta\right] \Rightarrow 0.8 = K\,(46 - \theta)$ .....(ii)

From (i) and (ii) we get $\theta = 10°C$

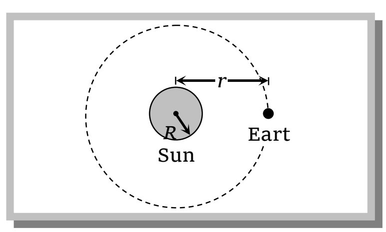
14.18 Temperature of The Sun and Solar Constant

If R is the radius of the sun and T its temperature, then the energy emitted by the sun per sec through radiation in accordance with Stefan's law will be given by

$$P = eA\,\sigma T^4 = 4\pi R^2 \sigma T^4$$

In reaching earth this energy will spread over a sphere of radius r (= average distance between sun and earth); so the intensity of solar radiation at the surface of earth (called solar constant S) will be given by

$$S = \frac{P}{4\pi r^2} = \frac{4\pi R^2 \sigma T^4}{4\pi r^2}$$



i.e. $$T = \left[\left(\frac{r}{R}\right)^2 \frac{S}{\sigma}\right]^{1/4} = \left[\left(\frac{1.5 \times 10^8}{7 \times 10^5}\right)^2 \times \frac{1.4 \times 10^3}{5.67 \times 10^{-8}}\right]^{1/4} \simeq 5800\ K$$

As $r = 1.5 \times 10^8\ km,\ R = 7 \times 10^5\ km,\ S = 2\frac{cal}{cm^2 min} = 1.4\frac{kW}{m^2}$ and $\sigma = 5.67 \times 10^{-8}\frac{W}{m^2 K^4}$

This result is in good agreement with the experimental value of temperature of sun, *i.e.*, 6000 *K*.

The difference in the two values is attributed to the fact that sun is not a perfectly black body.