# Probability

- **Experimental Probability:** The probability obtained from the result of an experiment when we actually perform the experiment is called experimental (or empirical) probability.
- **Theoretical Probability:** The probability we find through the theoretical approach without actually performing the experiment is called theoretical probability.
- The theoretical probability (or classical probability) of an event E, is denoted by P(E) and is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

- Experimental probability may or may not be equal to the theoretical probability.
- Formula of theoretical probability can be used to find the probabilities of various events.

**Example 1:** A dice is thrown once. What is the probability of getting 1 on the dice?

**Solution:** When a dice is thrown once, the possible outcomes are 1, 2, 3, 4, 5, 6.
Let A be the event of getting 1 on the dice.

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Number of all possible outcomes}} = \frac{1}{6}$$

**Example 2:** A box contains 3 white, 5 green, and 6 red balls. A ball is drawn at random out of the box. Find the probability of drawing a red ball.

**Solution:** Total number of balls = 3 + 5 + 6 = 14
Therefore, total number of possible outcomes = 14
Let E be the event of drawing a red ball.
Number of outcomes favourable to E = 6

$$\therefore P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes}} = \frac{6}{14} = \frac{3}{7}$$

**Example 3:** A card is drawn at random from a deck of 52 cards. Find the probability that the card drawn is,

- 
  - A black card
  - An ace
  - Neither a black card nor an ace

**Solution:**
Since there are 52 cards in a deck, the number of all possible outcomes is 52.

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  - Number of black cards in the deck = 26

Therefore, the probability that the drawn card is black $= \frac{26}{52} = \frac{1}{2}$

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- Number of aces in the deck = 4

  Therefore, the probability that the drawn card is an ace $= \frac{4}{52} = \frac{1}{13}$

- Number of cards which are neither black nor an ace = 26 – 2 = 24

  Therefore, the probability that the drawn card is neither black nor an ace $= \frac{24}{52} = \frac{6}{13}$

- **Complementary events**

  For an event E such that $0 \le P(E) \le 1$ of an experiment, the event $\bar{E}$ represents 'not E' , which is called the complement of the event E. We say, E and $\bar{E}$ are **complementary** events.

  $P(E) + P(\bar{E}) = 1$

  $\Rightarrow P(\bar{E}) = 1 - P(E)$

  **Example:**
  A pair of dice is thrown once. Find the probability of getting a different number on each die.
  **Solution:**
  When a pair of dice is thrown, the possible outcomes of the experiment can be listed as:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (1, 1) | (1, 2) | (1, 3) | (1, 4) | (1, 5) | (1, 6) |
| 2 | (2, 1) | (2, 2) | (2, 3) | (2, 4) | (2, 5) | (2, 6) |
| 3 | (3, 1) | (3, 2) | (3, 3) | (3, 4) | (3, 5) | (3, 6) |
| 4 | (4, 1) | (4, 2) | (4, 3) | (4, 4) | (4, 5) | (4, 6) |
| 5 | (5, 1) | (5, 2) | (5, 3) | (5, 4) | (5, 5) | (5, 6) |
| 6 | (6, 1) | (6, 2) | (6, 3) | (6, 4) | (6, 5) | (6, 6) |

  The number of all possible outcomes = 6 × 6 = 36
  Let E be the event of getting the same number on each die.
  Then, $\bar{E}$ is the event of getting different numbers on each die.

  Now, the number of outcomes favourable to E is 6.
  $\therefore P(\bar{E}) = 1 - P(E) = 1 - \frac{6}{36} = \frac{5}{6}$

  Thus, the required probability is $\frac{5}{6}$.