3. Pair of Linear Equations in Two Variables

Exercise 3.1

1. Question

Akhila went to a fair in her village. She wanted to enjoy rides on the Giant Wheel and play Hoopla (a game in which you throw a rig on the items kept in the stall, and if the ring covers any object completely you get it). The number of times she played Hoopla is half the number of rides she had on the Giant Wheel. Each ride costs Rs 3, and a game of Hoopla costs Rs 4. If she spent Rs 20 in the fair, represent this situation algebraically and graphically.

Answer

Let the number of times Akhila played Hoopla be x and number of times she played Giant wheel be y.

Given, number of times she played Hoopla is half the number of rides she had on the Giant Wheel.

⇒ $x = 2y$

⇒ $x - 2y = 0$For above equation, we have following table

x	0	2	4
y	0	1	2

Each ride costs Rs 3, and a game of Hoopla costs Rs 4 and she spent Rs 20 in the fair.

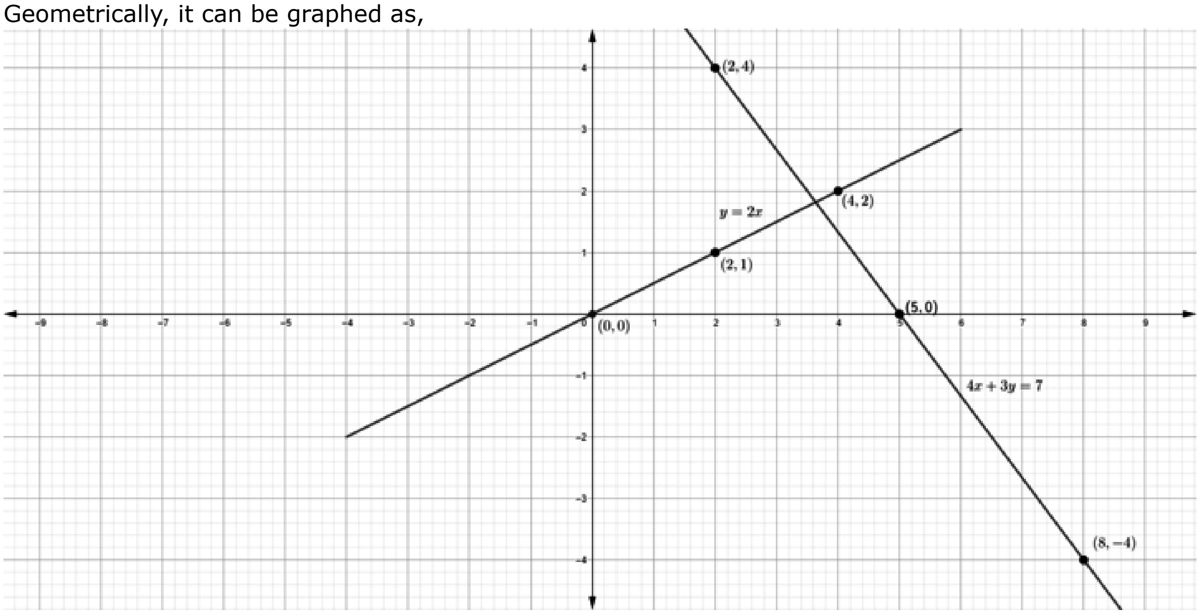
⇒ $4x + 3y = 20$

x	5	2	8
y	0	4	-4

Thus, algebraically it is represented by :

$x - 2y = 0$ and $4x + 3y = 0$

Geometrically, it can be graphed as,



2. Question

Aftab tells his daughter, 'Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Is not this interesting? Represent this situation algebraically and graphically.

Answer

Let the present age of father be x and present age of daughter be y.

Aftab tells his daughter, 'Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be."

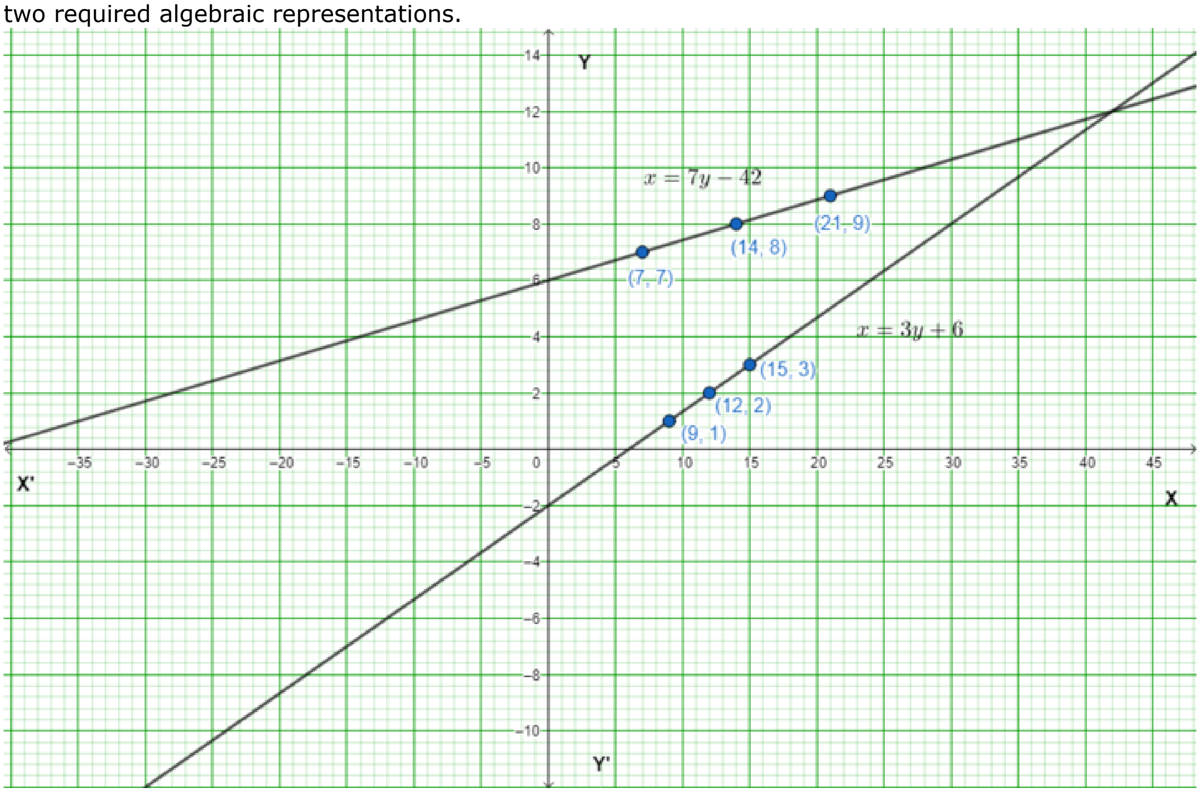
$\therefore$ x − 7 = 7(y − 7)⇒ x - 7 = 7y - 49⇒ x − 7y + 42 = 0⇒ x = 7y - 42⇒ x = 7(y - 6)   [1]The coordinates satisfying above equation are

x	7	14	21
y	7	8	9

Also, x + 3 = 3(y + 3)⇒ x + 3 = 3y + 9⇒ x − 3y − 6 = 0⇒ x = 3y + 6⇒ x = 3(y + 2)   [2]

x	9	12	15
y	1	2	3

The graph of both the equation is the required graphical representation and the equations [1] and [2] are two required algebraic representations.



3. Question

The path of a train A is given by the equation 3x + 4y − 12 = 0 and the path of another train B is given by the equation 6x + 8y − 48 = 0. Represent this situation graphically.

Answer

Given equations are 3x + 4y − 12 = 0 and 6x + 8y − 48 = 0

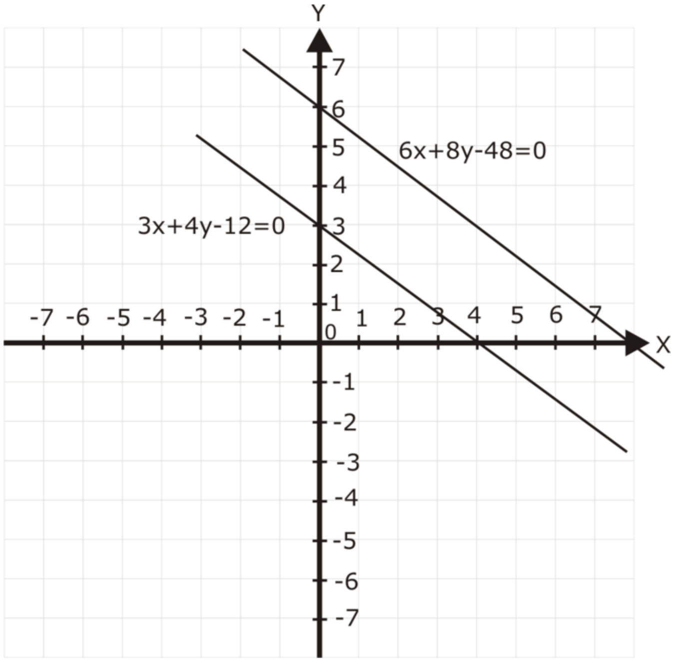
Let us plot the given equations. For this we will need some points to plot for the equation.

3 x + 4 y - 12 = 0

4 y = 12 - 3 xNow let us take random values of x and obtain the corresponding values of y by putting in the equation.at x = 0, 4 y = 12 - 3 x 04 y = 12y = 3Now at y = 0,3 x + 4 x 0 = 123 x = 12x = 4So we have two points for the equation 3 x + 4 y - 12 = 0 and those are (0, 3) and (4, 0)6 x + 8 y - 48 = 0Now let us find points for this equationat x = 0,6 x 0 + 8 y

= 488 y = 48y = 6at y = 0,6 x + 8 x 0 = 486 x = 48x = 8So we have two points for the equation 6 x + 8 y - 48 = 0 and those are (8, 0) and (0, 6)3x + 4y − 12 = 0 passes through (4, 0) and (0, 3)

6x + 8y − 48 = 0 passes through (8, 0) and (0, 6)

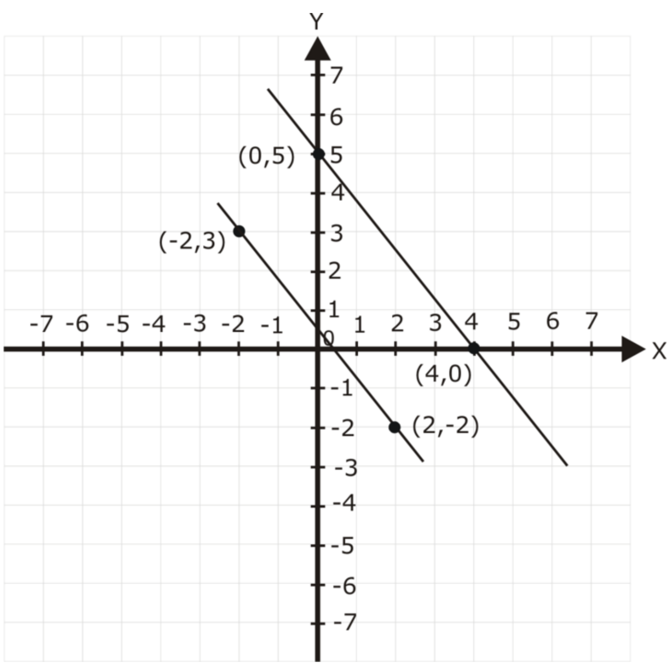


4. Question

Gloria is walking along the path joining (-2, 3) and (2, -2), while Suresh is walking along the path joining (0, 5) and (4, 0). Represent this situation graphically.

Answer

Given, Gloria is walking along the path joining (-2, 3) and (2, -2), while Suresh is walking along the path joining (0, 5) and (4, 0).



5. Question

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ *and* $\frac{c_1}{c_2}$, and without drawing them, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincide:

(i) $5x - 4y + 8 = 0$

$7x+6y-9=0$

(ii) $9x+3y+12=0$

$18x+6y+24=0$

(iii) $6x-3y+10=0$

$2x-y+9=0$

Answer

Two lines, a1x + b1y + c1 = 0 and a2x + b2y + c2 = 0

If $\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$, then the lines coincide

If $\frac{a1}{a2}=\frac{b1}{b2}\neq\frac{c1}{c2}$, then the lines are parallel

If $\frac{a1}{a2}\neq\frac{b1}{b2}\neq\frac{c1}{c2}$, then the lines intersect

(i) $5x-4y+8=0$

$7x+6y-9=0$

$$\Rightarrow\frac{a1}{a2}=\frac{5}{7},\frac{b1}{b2}=-\frac{4}{6},\frac{c1}{c2}=\frac{8}{-9}$$

Thus, $\frac{a1}{a2}\neq\frac{b1}{b2}\neq\frac{c1}{c2}$.

The lines intersect.

(ii) $9x+3y+12=0$

$18x+6y+24=0$

$$\Rightarrow\frac{a1}{a2}=\frac{9}{18}=\frac{1}{2},\frac{b1}{b2}=\frac{3}{6}=\frac{1}{2},\frac{c1}{c2}=\frac{12}{24}=\frac{1}{2}$$

Thus, $\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$

The lines are coincident.

(iii) $6x-3y+10=0$

$2x-y+9=0$

$$\Rightarrow\frac{a1}{a2}=\frac{6}{2}=3,\frac{b1}{b2}=-\frac{3}{-1}=3,\frac{c1}{c2}=\frac{10}{9}$$

Thus, $\frac{a1}{a2}=\frac{b1}{b2}\neq\frac{c1}{c2}$

The lines are parallel.

6. Question

Given the linear equation $2x+3y-8=0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i) intersecting lines (ii) parallel lines

(iii) coincident lines.

Answer

Two lines, a1x + b1y + c1 = 0 and a2x + b2y + c2 = 0

If $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, then the lines coincide

If $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, then the lines are parallel

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$, then the lines intersect

Given the linear equation $2x + 3y - 8 = 0$.

An intersecting line is x + 2y – 4 =0

A parallel line is 4x + 6y – 12 = 0

A coincident line is 4x + 6y – 16 = 0

7. Question

The cost of 2 kg of apples and l kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Answer

Let the cost of 1 kg of apple be x and cost of 1 kg of grape be y.

Given, cost of 2 kg of apples and l kg of grapes on a day was found to be Rs 160.

∴ 2x + y = 160

Also, after a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300.

∴ 4x + 2y = 300Now to present these equations graphically plot the points of respective lines,For 2x + y = 160,y = 160 - 2xWhen x = 50, y = 60When x=60, y = 40Table is :

x	50	60
y	60	40

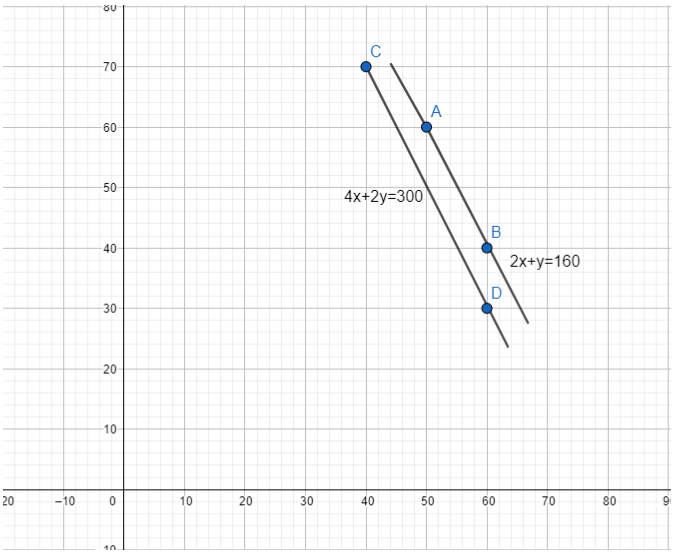
Plot the points A(50,60) and B(60,40)

For 4x + 2y = 300, $y = \frac{300 - 4x}{2}$

⇒ y = 150 - 2xWhen x = 40, y = 70When x = 60, y = 30Table is :

x	40	60
y	70	30

Plot the points C(40,-30) and D(60,30)



Exercise 3.2

1. Question

Solve the following systems of equations graphically:

$x + y = 3$
$2x + 5y = 12$

Answer

For the solutions first we make graph for the equations.

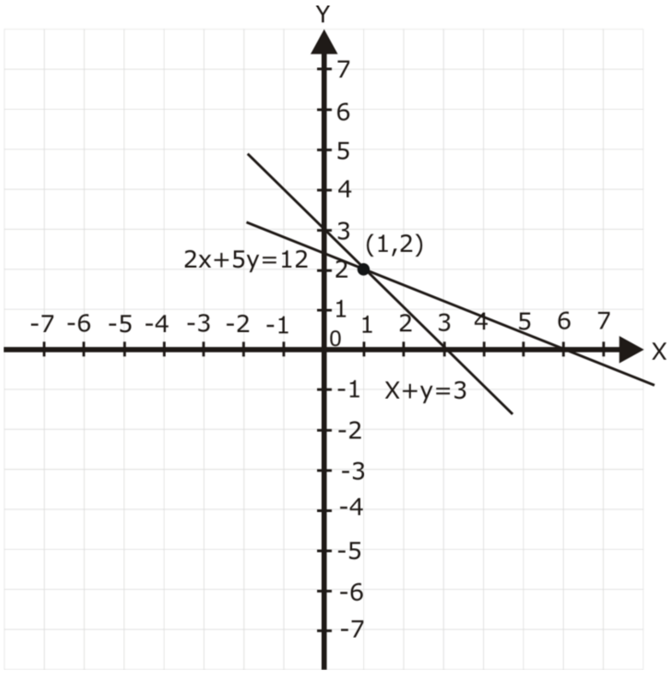
For x+y=3,

x	0	3
y	3	0

x + y = 3 passes through (3,0) and (0,3)For 2x+5y=12,

x	6	0
y	0	2.4

2x + 5y = 12 passes through (6, 0) and (0, 2.4)Now we plot the points and join them to make graph.Wherever, they intersect that is common solution to both equations.



They meet at point x = 1 and y = 2.So (1,2) is solution.

2. Question

Solve the following systems of equations graphically:

$x - 2y = 5$
$2x + 3y = 10$

Answer

Given: The system of equations: $x - 2y = 5$, $2x + 3y = 10$

To find: The solution of above system.

Solution:

For x - 2y = 5,Put x = 5, we get y = 0Put x = 1, we get y = -2Table for x - 2y = 5

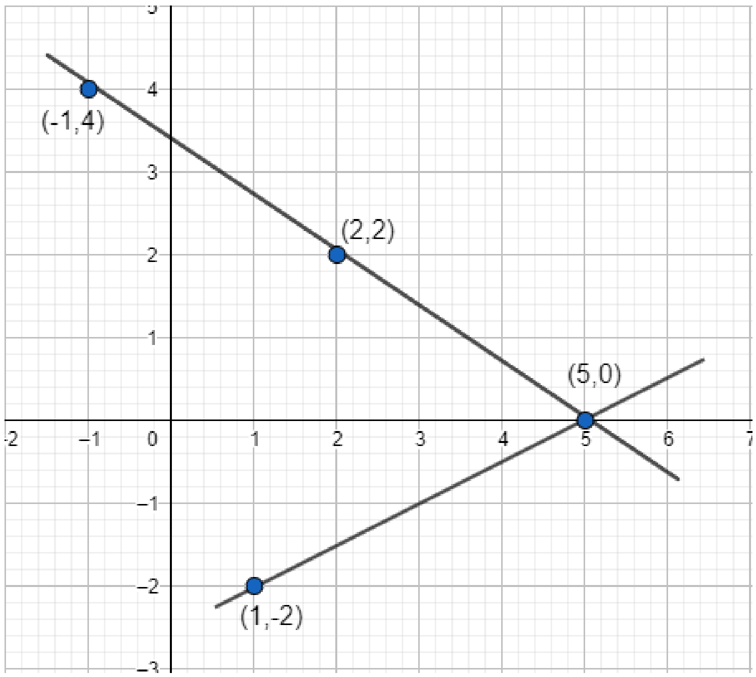
x	5	1
y	0	-2

x - 2y = 5, passes through (0,5) and (1,-2)

For 2x + 3y =10 Put x = -1 , we get y = 4Put x = 2, we get y = 2Table for 2x + 3y =10

x	-1	2
y	4	2

2x + 3y =10 passes through (0,5) and (2,2)Plot the points in the graph,



The lines meet at the point (5,0)So x=5 and y=0.

3. Question

Solve the following systems of equations graphically:

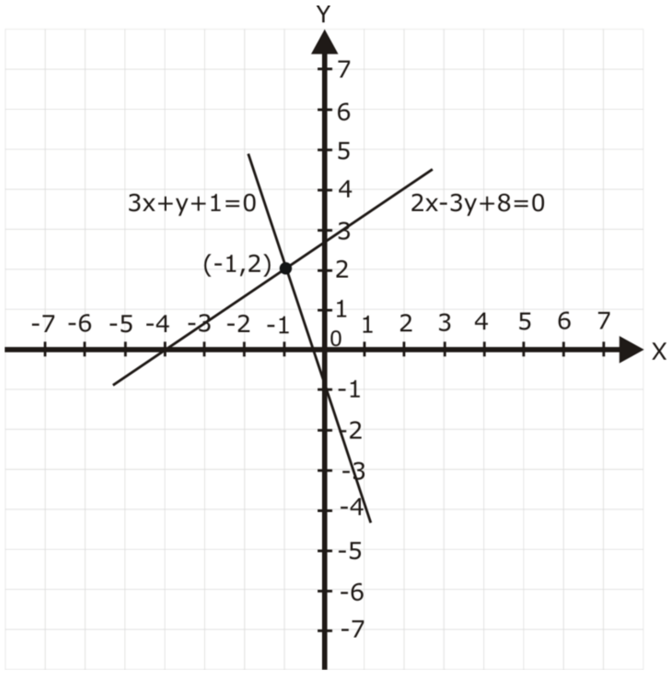
$3x + y + 1 = 0$
$2x - 3y + 8 = 0$

Answer

3x + y + 1= 0 passes through (-1/3, 0) and (0, -1)

2x – 3y + 8 = 0 passes through (-4, 0) and (0, 8/3)

They meet at point x = -1 and y = 2



4. Question

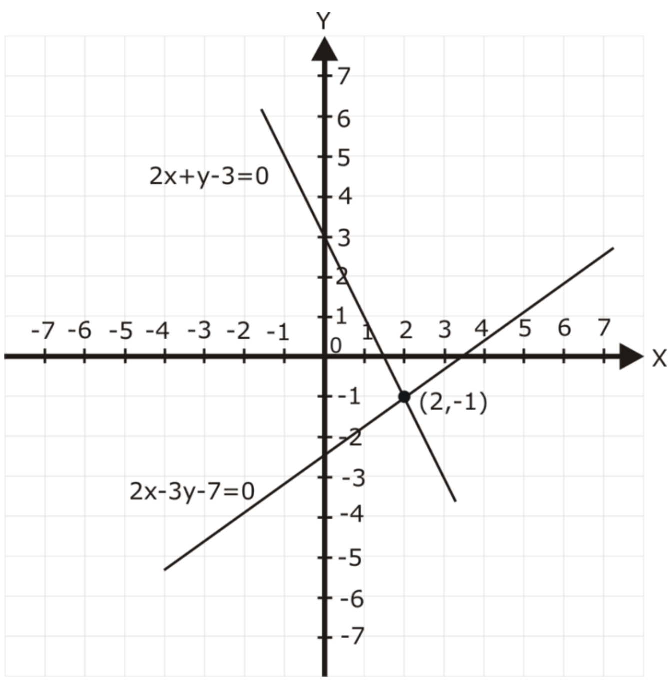
Solve the following systems of equations graphically:

$2x + y - 3 = 0$
$2x - 3y - 7 = 0$

Answer

2x + y − 3 = 0 passes through (3/2, 0) and (0, 3)

2x − 3y − 7 = 0 passes through (7/2, 0) and (0, - 7/3)



They meet at point x = 2, y = -1

5. Question

Solve the following systems of equations graphically:

$x + y = 6$
$x - y = 2$

Answer

Given: $x + y = 6$
$x - y = 2$

To show: The given pair of equations graphically.

Solution: For x + y = 6,Put y = 0, we get x = 6

Put x = 0, we get y = 6

Table for x + y = 6

x	6	0
y	0	6

x + y = 6 passes through (6, 0) and (0, 6)

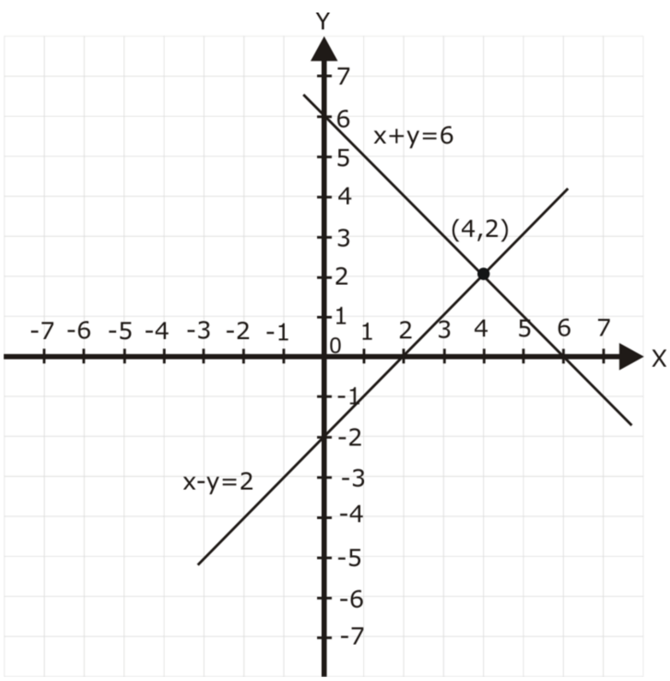
For x − y = 2 Put y = 0, we get x = 2

Put x = 0, we get y = -2

Table for x − y = 2

x	2	0
y	0	-2

x − y = 2 passes through (2, 0) and (0, - 2)Plot the points in the graph,



They meet at point x = 4, y = 2

6. Question

Solve the following systems of equations graphically:

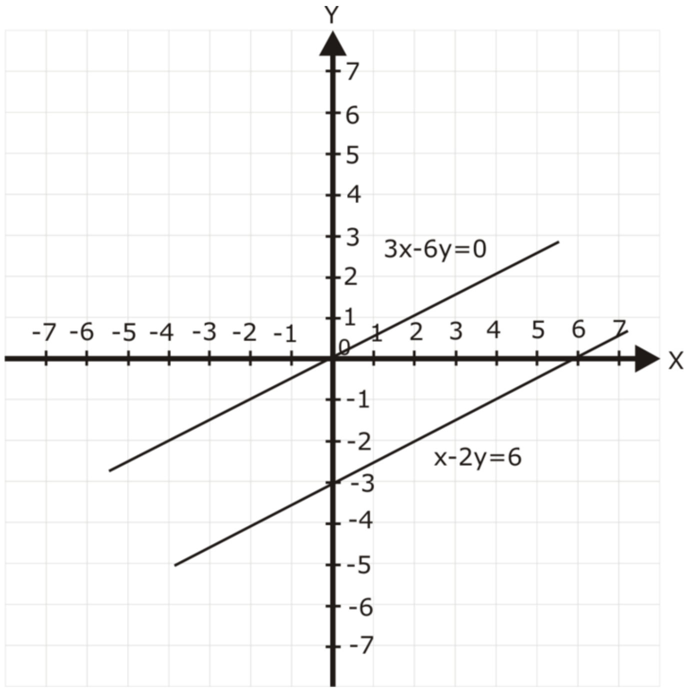
$x - 2y = 6$
$3x - 6y = 0$

Answer

X − 2y = 6 passes through (6, 0) and (0, -3)

3x – 6y = 0 passes through origin

Both lines are parallel, thus it has no solution.



7. Question

Solve the following systems of equations graphically:

$x + y = 4$
$2x - 3y = 3$

Answer

For x + y = 4,Put y = 0, we get x = 4

Put x = 0, we get y = 4

Table for x + y = 4

x	4	0
y	0	4

x + y = 4 passes through (4, 0) and (0, 4)

For 2x – 3y = 3$y = \frac{2x-3}{3}$

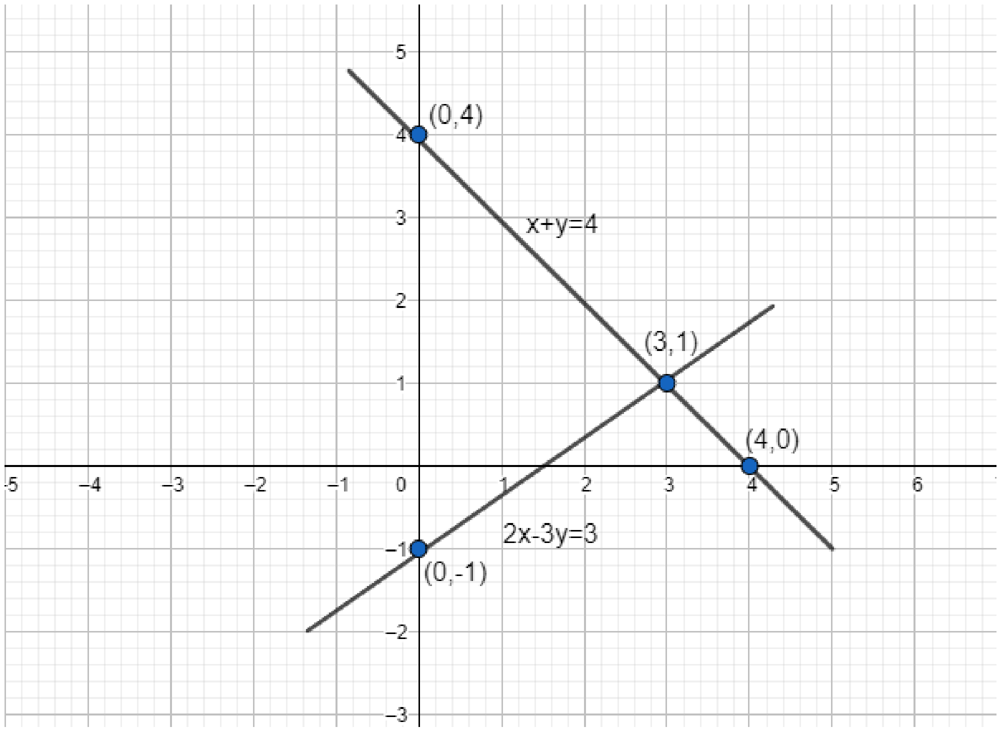
Put y = 1, we get x = 3

Put x = 0, we get y = -1

Table for x – y = 2

x	3	0
y	1	-1

2x – 3y = 3 passes through (3, 1) and (0, -)Plot the points in the graph



They meet at point (3,1)Hence x = 3, y = 1.

8. Question

Solve the following systems of equations graphically:

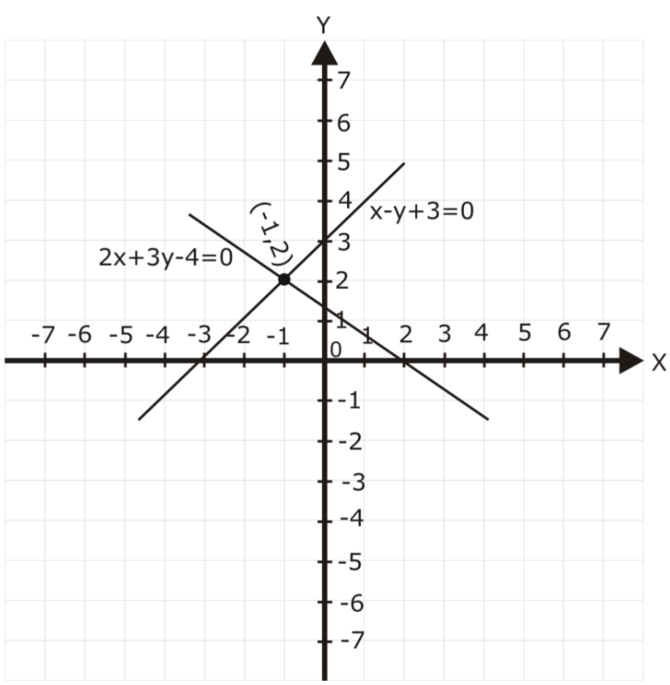
$2x + 3y = 4$
$x - y + 3 = 0$

Answer

2x + 3y = 4 passes through (2, 0) and (0, 4/3)

X – y + 3 = 0 passes through (-3, 0) and (0, 3)

They meet at point x = -1, y = 2



9. Question

Solve the following systems of equations graphically:

$$2x - 3y + 13 = 0$$
$$3x - 2y + 12 = 0$$

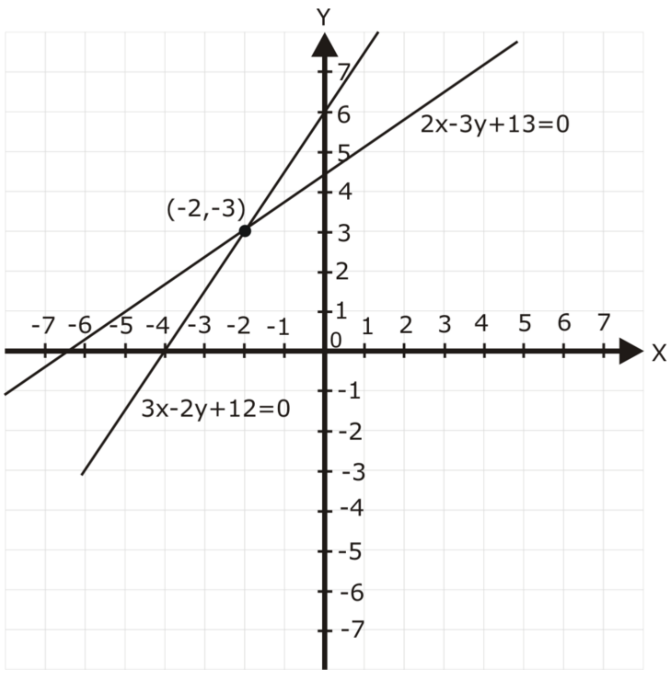
Answer

On solving graphically,

2x − 3y + 13 = 0 passes through (-6.5, 0) and (0, 13/3)

3x − 2y + 12 = 0 passes through (-4, 0) and (0, 6)

Thus the meeting point of the straight lines is, (-2, - 3)



10. Question

Solve the following systems of equations graphically:

$$2x + 3y + 5 = 0$$
$$3x - 2y - 12 = 0$$

Answer

For the equation 2x + 3y + 5 = 0, $y = \frac{-5-2x}{3}$

When x = -1 , y =- 1

When x = 5 , y =- 5

Table is :

x	-1	5
y	-1	-5

Plot the points A(-1,-1) B(-5,5) on the graph

For the equation 3x – 2y – 12 = 0 , $y = \frac{3x - 12}{2}$

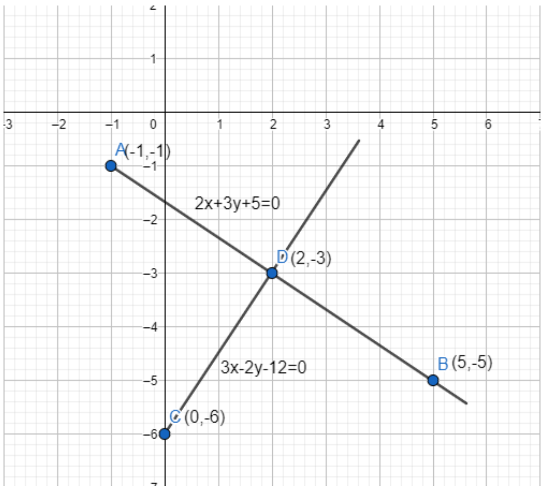
When x = 0, y = -6

When x = 2 , y = -3

Table is :

x	0	2
y	-6	-3

Plot the points C(0,-6) D(2,-3) on the graph



Since the two lines meet at the point (2,-3)Therefore, x = 2 and y = -3

11. Question

Show graphically that each one of the following systems of equations has infinitely many solutions:

$2x + 3y = 6$
$4x + 6y = 12$

Answer

Given: the system of equations: $2x + 3y = 6$, $4x + 6y = 12$

To show:systems of equations has infinitely many solutions.

Solution:

consider the equation 2x + 3y = 6

To plot its graph,we have $y = \frac{6 - 2x}{3}$

Putting x = 0 we get y = 2

putting y = 0 we get x = 3

The table for points of 2x + 3y = 6 is:

x	0	3
y	2	0

Plot A(0,2) and B(3,0) in the graph

Consider the equation 4x + 6y = 12

To plot its graph,we have $y = \frac{12 - 4x}{6}$

Putting x = 6 we get y = -2

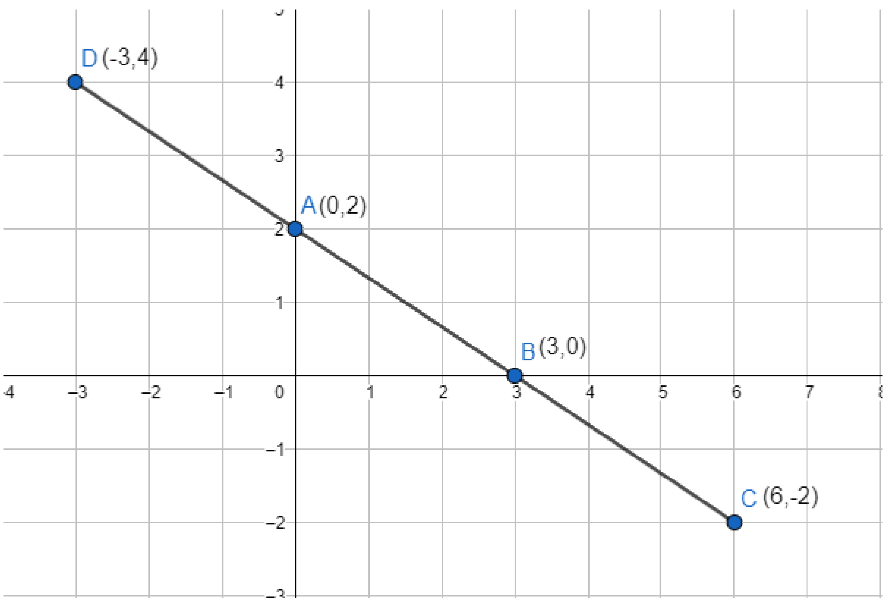
Putting y = 4 we get x = -3

The table for the points 4x + 6y = 12 is:

x	6	-3
y	-2	4

Plot C(6,-2) and D(-3, 4) in the graph

Now plot the graphs for these equations as:



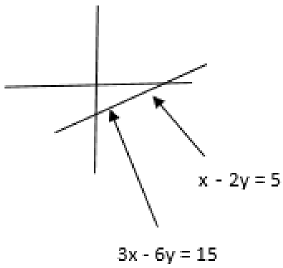
As Both the lines pass through the same points and are coinciding.There can be infinite points lying on both the lines.Hence,systems of equations has infinitely many solutions.

12. Question

Show graphically that each one of the following systems of equations has infinitely many solutions:

$x - 2y = 5$
$3x - 6y = 15$

Answer



Both pass through the same points and are coinciding.

13. Question

Show graphically that each one of the following systems of equations has infinitely many solutions:

$$3x + y = 8$$
$$6x + 2y = 16$$

Answer

Given: the system of equations: $3x + y = 8$, $6x + 2y = 16$

To show:systems of equations has infinitely many solutions.

Solution:consider the equation 3x+y=8

To plot its graph,we have y=8-3x

Putting x=0 we get y=8

putting y=2 we get x=2

The table for points of 3x+y=8 is:

x	0	2
y	8	2

Plot A(0,8) and B(2,2) in the graph

Consider the equation 6x+2y=16

To plot its graph,we have $y = \frac{16 - 2x}{6}$

Putting x=2 we get y=2

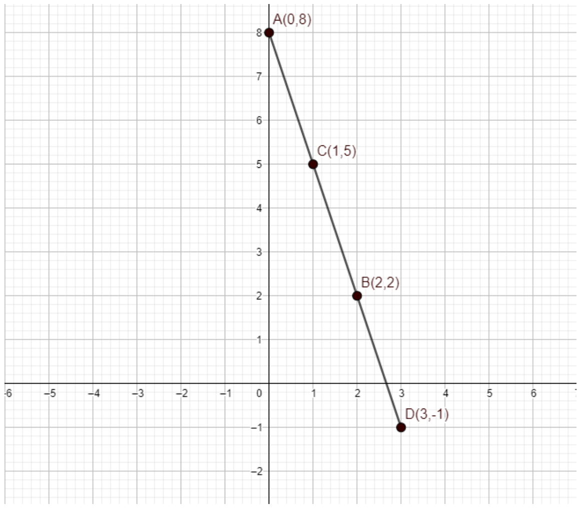
Putting y=0 we get x=8

The table for the points 6x+2y=16 is:

x	1	3
y	5	-1

Plot C(1,5) and D(3,-1) in the graph

Now plot the graphs for these equations as:



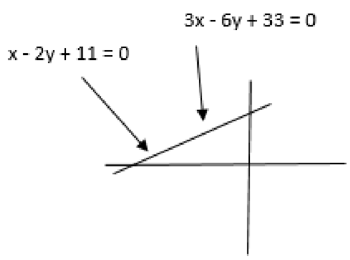
As Both the lines pass through the same points and are coinciding.There can be infinite points lying on both the lines.Hence,systems of equations has infinitely many solutions.

14. Question

Show graphically that each one of the following systems of equations has infinitely many solutions:

$x - 2y + 11 = 0$
$3x - 6y + 33 = 0$

Answer



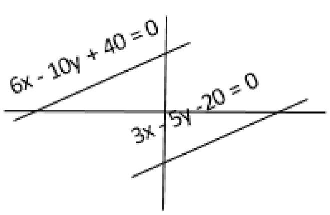
Both pass through the same points and are coinciding.

15. Question

Show graphically that each one of the following systems of equations is in-consistent (i.e. has no solution):

$3x - 5y = 20$
$6x - 10y = -40$

Answer



3x – 5y = 20 passes through (20/3, 0) and (0, -4)

6x – 10y = - 40 passes through (-20/3, 0) and (0, 4)

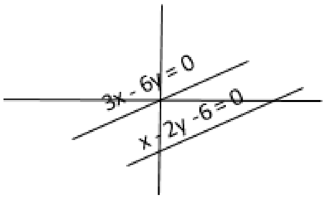
Both are parallel lines, thus system of equations is in-consistent.

16. Question

Show graphically that each one of the following systems of equations is in-consistent (i.e. has no solution):

$x - 2y = 6$
$3x - 6y = 0$

Answer



X – 2y = 6 passes through (6, 0) and (0, -3)

3x – 6y = 0 passes through origin

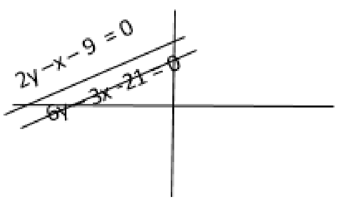
Both are parallel lines, thus system of equations is in-consistent.

17. Question

Show graphically that each one of the following systems of equations is in-consistent (i.e. has no solution):

$2y - x = 9$
$6y - 3x = 21$

Answer



2y – x = 9 passes through (-9, 0) and (0, 9/2)

6y – 3x = 21 passes through (-7, 0) and (0, 7/2)

Both are parallel lines, thus system of equations is in-consistent.

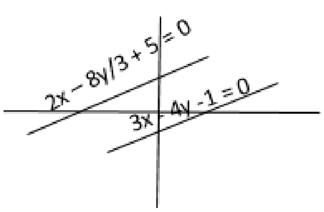
18. Question

Show graphically that each one of the following systems of equations is in-consistent (i.e. has no solution):

$3x - 4y - 1 = 0$

$2x - \frac{8}{3}y + 5 = 0$

Answer



3x – 4y – 1 = 0 passes through (1/3, 0) and (0, -1/4)

2x – 8y/3 + 5 = 0 passes through (-5/2, 0) and (0, 15/8)

Both are parallel lines, thus system of equations is in-consistent.

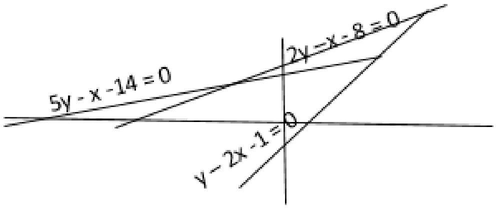
19. Question

Determine graphically the vertices of the triangle, the equations of whose sides are given below:

(i) $2y - x = 8, 5y - x = 14 \text{ and } y - 2x = 1$

(ii) $y = x, y = 0, and\ 3x + 3y = 10$

Answer



(i) $2y - x = 8, 5y - x = 14\ and\ y - 2x = 1$

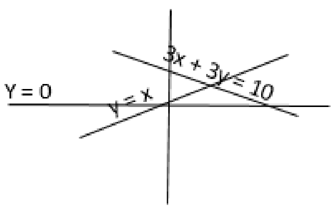
2y – x = 8 passes through (-8, 0) and (0, 4)

5y – x = 14 passes through (-14, 0) and (0, 14/5)

Y – 2x = 1 passes through (0, 1) and (-1/2, 0)

Vertices of the triangle are (-4, 2), (1, 3) and (2, 5)

(ii) $y = x, y = 0, and\ 3x + 3y = 10$



3x + 3y = 10 passes through (10/3, 0) and (0, 10/3)

Vertices of the triangle are (0, 0), (10/3, 0) and (5/3, 5/3)

20. Question

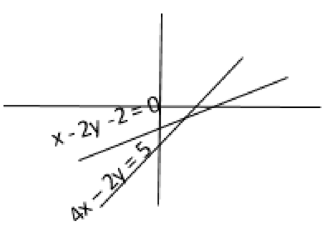
Determine, graphically whether the system of equations is consistent or in-consistent.

$x - 2y = 2, 4x - 2y = 5$

Answer

X – 2y = 2 passes through (2, 0) and (0, -1)

4x – 2y = 5 passes through (5/4, 0) and (0, - 5/2)



Thus the system of equations is consistent and it has a unique solution.

21. Question

Determine, by drawing graphs, whether the following system of linear equations has a unique solution or not:

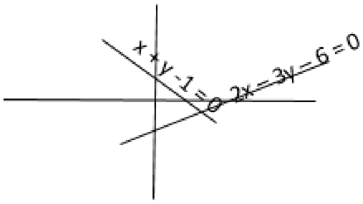
(i) $2x - 3y = 6, x + y = 1$

(ii) $2y = 4x - 6, 2x = y + 3$

Answer

(i) 2x – 3y = 6 passes through (3, 0) and (0, -2)

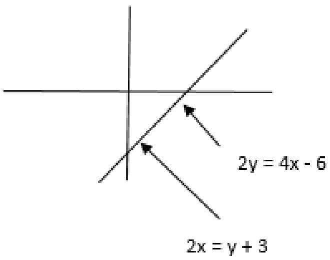
X + y = 1 passes through (0, 1) and (1, 0)



Thus it has unique solution

(ii) 2y = 4x -6 passes through (3/2, 0) and (0, -3)

2x = y + 3 passes through (3/2, 0) and (0, -3)



The lines are coincident, it has infinite solutions

22. Question

Solve graphically each of the following systems of linear equations. Also find the coordinates of the points where the lines meet axis of y.

(i) $2x - 5y + 4 = 0$

$2x + y - 8 = 0$

(ii) $3x + 2y = 12$

$5x - 2y = 4$

(iii) $2x + y - 11 = 0$

$x - y - 1 = 0$

(iv) $x + 2y - 7 = 0$

$2x - y - 4 = 0$

(v) $3x + y - 5 = 0$

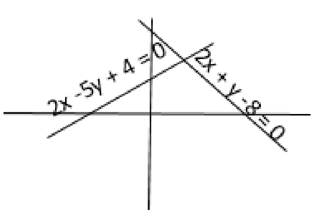
$2x - y - 5 = 0$

(vi) $2x - y - 5 = 0$

$x - y - 3 = 0$

Answer

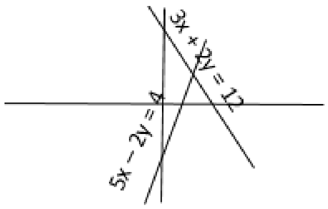
(i)



It has a unique solution at x = 3 and y = 2

Coordinates where lines meet y axis is (0, 4/5) and (0, 8)

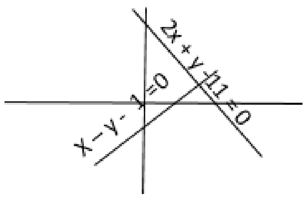
(ii)



Unique solution at x = 2 and y = 3.

Lines meet the y axis at (0, 6) and (0, - 2)

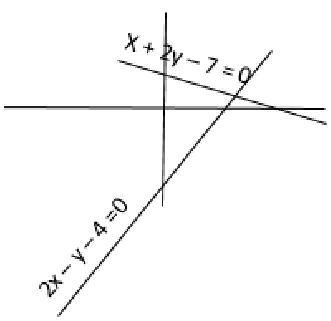
(iii)



Unique solution at x = 4 and y = 3

Lines meet the y axis at (0, 11) and (0, -1)

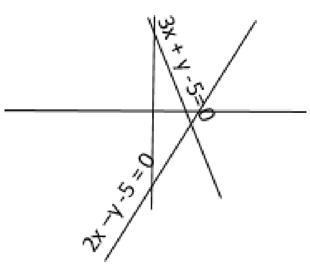
(iv)



Unique solution at x = 3 and y = 2

Lines meet y axis at (0, 7/2) and (0,- 4)

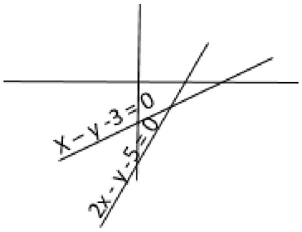
(v)



Unique solution at x = 2 and y = -1

Lines meet the y-axis at (0, 5) and (0, - 5)

(vi)



Unique solution at x = 2 and y = -1

Lines meet y-axis at (0, -5) and (0, -3)

23. Question

Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are:

(i) $y = x, y = 2x \text{ and } y + x = 6$

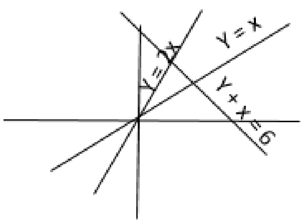
(ii) $y = x, 3y = x, x + y = 8$

Answer

(i) $y = x, y = 2x \text{ and } y + x = 6$

The triangle formed has vertices

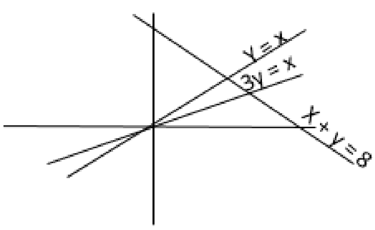
(0, 0), (2, 4) and (3,3)



(ii) $y = x, 3y = x, x + y = 8$

The triangle formed has vertices

(0, 0), (4, 4) and (6, 2)



24. Question

Solve the following system of linear equations graphically and shade the region between the two lines and x-axis:

(i) $2x + 3y = 12,$

$x - y = 1$

(ii) $3x + 2y - 4 = 0,$

$2x - 2y - 7 = 0$

(iii) $3x + 2y - 11 = 0$

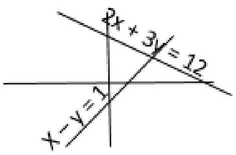
$2x - 3y + 10 = 0$

Answer

i) 2x + 3y = 12 passes through (6, 0) and (0, 4)

x – y = 1 passes through (1, 0) and (0, -1)

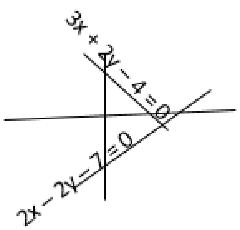
They meet at x = 3, y = 2



ii) 3x + 2y – 4 = 0 passes through, (4/3, 0) and (0, 2)

2x – 2y – 7 = 0 passes through (7/2, 0) and (0, -7/2)

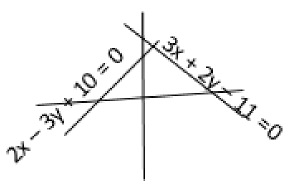
They meet at x = 2, y = -1



iii) 3x + 2y – 11 = 0 passes through (11/3, 0) and (0, 11/2)

2x – 3y + 10 = 0 passes through (-5, 0) and (0, 10/3)

They meet at x = 1, y = 4



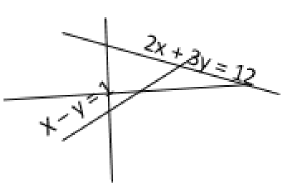
25. Question

Draw the graphs of the following equations on the same graph paper:

$2x + 3y = 12$
$x - y = 1$

Find the coordinates of the vertices of the triangle formed by the two straight lines and the y-axis.

Answer



$2x + 3y = 12$
$x - y = 1$

Lines meet at x = 3, y = 2

Coordinates of the vertices of the triangle formed with y –axis

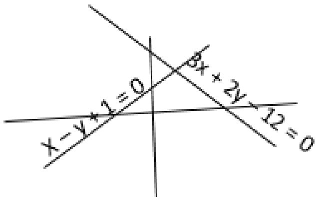
(0, - 1), (0, 4) and (3, 2)

26. Question

Draw the graphs of $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and x-axis and shade the triangular area. Calculate the area bounded by these lines and x-axis.

Answer

$x - y + 1 = 0$ and $3x + 2y - 12 = 0$



Coordinates of the vertices of triangle formed with x-axis

(2, 3), (-1, 0) and (4, 0)

Height of the triangle = 3 units

Base of the triangle = 5 units

Area = 0.5 × base × height = 7.5 sq units

27. Question

Solve graphically the system of linear equations:

$4x - 3y + 4 = 0$

$4x + 3y - 20 = 0$

Find the area bounded by these lines and x-axis.

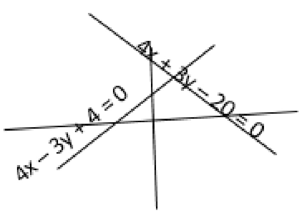
Answer

They meet at x = 2, y = 4

Height of the triangle formed = 4 units

Base of the triangle formed = 5 + 1 = 6

Area of the region bounded by these lines and x-axis



= 0.5 × 4 × 6

= 12 sq. units

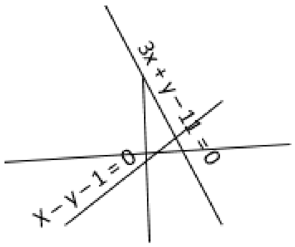
28. Question

Solve the following system of linear equations graphically:

$3x + y - 11 = 0, x - y - 1 = 0$

Shade the region bounded by these lines and y-axis. Also, find the area of the region bounded by the these lines and y-axis.

Answer



$3x + y - 11 = 0, x - y - 1 = 0$

They meet at x = 3, y = 2

Height of the triangle formed with y-axis = 3 units

Base of the triangle formed = 11 + 1 = 12 units

Area of the bounded region = 0.5 × 3 × 36 = 18 sq. units

29. Question

Solve graphically each of the following systems of linear equations. Also, find the coordinates of the points where the lines meet the axis of x in each system:

(i) $2x + y = 6$

$x - 2y = -2$

(ii) $2x - y = 2$

$4x - y = 8$

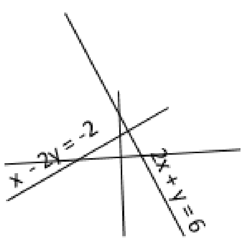
(iii) $x + 2y = 5$

$2x - 3y = -4$

(iv) $2x + 3y = 8$

$x - 2y = -3$

Answer



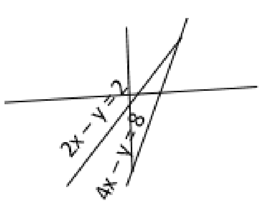
i) $2x + y = 6$ $x - 2y = -2$

Unique solution at x = 2, y = 2

Lines meet x-axis at (3, 0) and (-2, 0)

ii) $2x - y = 2$

$4x - y = 8$

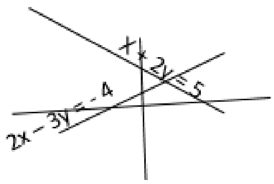


Unique solution at x = 3, y = 3

Lines meet x-axis at (1, 0) and (2, 0)

iii) $x + 2y = 5$

$2x - 3y = -4$

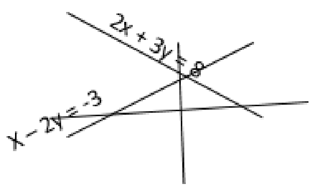


Unique solution at x =1, y = 2

Lines meet x-axis at (5, 0) and (-2, 0)

iv) $2x + 3y = 8$

$x - 2y = -3$



Unique solution at x =1, y = 2

Lines meet x-axis at (4, 0) and (-3, 0)

30. Question

Draw the graphs of the following equations:

$$2x - 3y + 6 = 0$$
$$2x + 3y - 18 = 0$$
$$y - 2 = 0$$

Find the vertices of the triangle so obtained. Also, find the area of the triangle.

Answer

For equation, 2x - 3y + 6 = 0

Coordinates satisfying graph are,

x	0	-3
y	2	0

For equation, 2x + 3y - 18 = 0

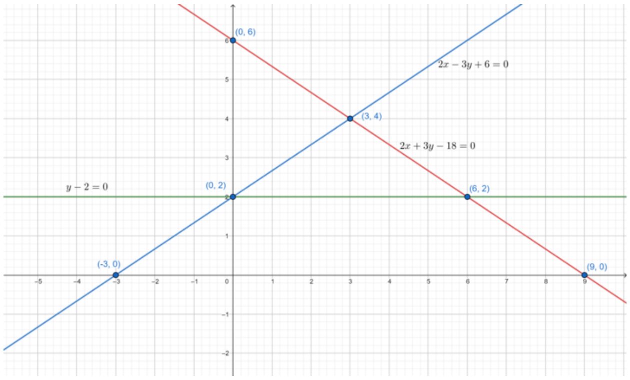
Coordinates satisfying graph are,

x	0	9
y	6	0

and y - 2 = 0

⇒ y = 2, which is a straight line parallel to x-axis intersecting the y-axis at 2

On plotting all the three line on graphs, we get the graph as follows



On observing from graph,

Vertices of the triangle obtained are

(0, 2), (3, 4) and (6, 2)

Height of triangle = 4 – 2 = 2

Base of the triangle = 6

Area of the triangle

$$= \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times 6 \times 2$$

= 6 square units

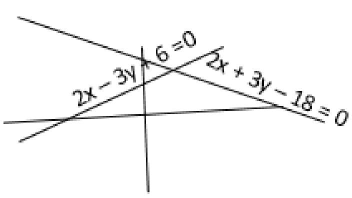
31. Question

Solve the following system of equations graphically:

$2x - 3y + 6 = 0$
$2x + 3y - 18 = 0$

Also, find the area of the region bounded by these two lines and y-axis.

Answer



Unique solution at x = 3 and y = 4

Vertices of triangle formed with y-axis are

(0, 2), (3, 4) and (0, 6)

Height of the triangle obtained = 3 units

Base of the triangle obtained = 6 – 2 = 4 units

Area of the triangle = 0.5 × 3 × 4 = 6 sq. units

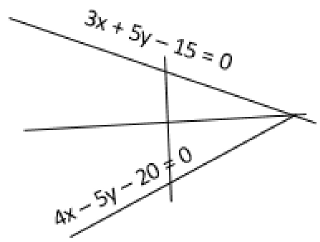
32. Question

Solve the following system of linear equations graphically:

$4x - 5y - 20 = 0$
$3x + 5y - 15 = 0$

Determine the vertices of the triangle formed by the lines representing the above equation and the y-axis.

Answer



$4x - 5y - 20 = 0$
$3x + 5y - 15 = 0$

Unique solution at x = 5 and y = 0

Vertices of the triangle with y axis are

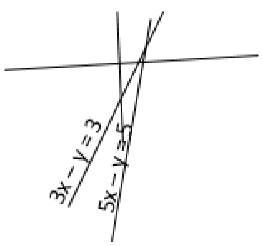
(0, 3), (0, - 4) and (5, 0)

33. Question

Draw the graphs of the equations $5x - y = 5$

$and\ 3x - y = 3$. Determine the coordinates of the vertices of the triangle formed by these lines and y-axis. Calculate the area of the triangle so formed.

Answer



$5x - y = 5$

$and\ 3x - y = 3$

Coordinates of the vertices of the triangle formed by these lines and y-axis

(1, 0), (0, -3), (0, -5)

Height of triangle = 1 unit

Base of triangle = 5 – 3 = 2 unit

Area of triangle = 0.5 × 1 × 2 = 1 sq. units

34. Question

Form the pair of linear equations in the following problems, and find their solution graphically:

(i) 10 students of class X took part in Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and a pen.

(iii) Champa went to a 'sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased.” Help her friends to find how many pants and skirts Champa bought.

Answer

i) Let the number of boys and girls be ‘a’ and ‘b’.

10 students of class X took part in Mathematics quiz. If the number of girls is 4 more than the number of boys

⇒ a + b = 10 and b = a + 4

Thus, a = 3 and b = 7

ii) Let the cost of 1 pen be a and 1 pencil be b.

5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46

⇒ 7a + 5b = 50 and 5a + 7b = 46

Solving by multiplying 1st by 7 and 2nd by 5 and subtracting

⇒ 24a = 120

⇒ a = Rs. 5

Thus, 35 + 5b = 50

⇒ b = Rs. 3

iii) Let the number of pants and skirts bought be ‘a’ and ‘b’

Champa went to a ‘sale’ to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, “The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased.”

⇒ b = 2a – 2 and b = 4a – 4

⇒ 2a – 2 = 4a – 4

⇒ a = 1

Thus, b = 2 – 2 = 0

35. Question

Solve the following system of equations graphically:

Shade the region between the lines and the y-axis

(i) $3x - 4y = 7$

$5x + 2y = 3$

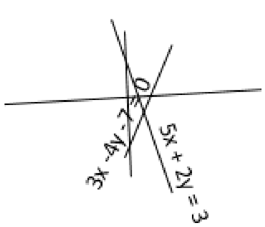
(ii) $4x - y = 4$

$3x + 2y = 14$

Answer

i) $3x - 4y = 7$

$5x + 2y = 3$

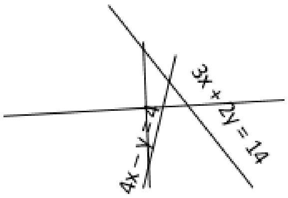


Unique solution at x = 1, y = -1

ii) $4x - y = 4$

$3x + 2y = 14$

Unique solution at x = 2, y = 4



36. Question

Represent the following pair of equations graphically and write the coordinates of points where the lines intersects y-axis

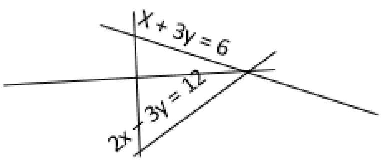
$x + 3y = 6$

$2x - 3y = 12$

Answer

$x + 3y = 6$

$2x - 3y = 12$



Lines meet y-axis at (0, 2) and (0, -4)

37. Question

Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is

(i) intersecting lines (ii) Parallel lines

(iii) coincident lines

Answer

Two lines, a1x + b1y + c1 = 0 and a2x + b2y + c2 = 0

If $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, then the lines coincide

If $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, then the lines are parallel

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$, then the lines intersect

Given line $2x + 3y - 8 = 0$

i) intersecting line : 3x + 2y – 6 = 0

ii) parallel line : 4x + 6y = 15

iii) coincident line : 4x + 6y – 16 = 0

Exercise 3.3

1. Question

Solve the following systems of equations:

$11x + 15y + 23 = 0$

$7x - 2y - 20 = 0$

Answer

$11x + 15y + 23 = 0$ [1]

$7x - 2y - 20 = 0$ [2]

Multiplying the 1st equation by 2 and 2nd equation by 15, we get22x + 30y + 46 = 0 [3]105x - 30y - 300 = 0 [4]Adding [3] and [4]

⇒ 22x + 30y + 46 + 105x − 30y − 300 = 0

⇒ 127x = 254

⇒ x = 2

Substituting value of x in eq1.

⇒ 11(2) + 15y + 23 = 0

⇒ 22 + 15y + 23 = 0

⇒ 15y = -45

⇒ y = -3

2. Question

Solve the following systems of equations:

$3x - 7y + 10 = 0$

$y - 2x - 3 = 0$

Answer

$3x - 7y + 10 = 0$ $y - 2x - 3 = 0$

Multiplying eq1 by 2 and eq2 by 3 and adding.

⇒ 6x − 14y + 20 + 3y − 6x − 9 = 0

⇒ -11y = -11

⇒ y = 1

Substituting value of y in eq1, we get x = -1

3. Question

Solve the following systems of equations:

$0.4x + 0.3y = 1.7$

$0.7x - 0.2y = 0.8$

Answer

$0.4x + 0.3y = 1.7$

$0.7x - 0.2y = 0.8$

Multiplying eq1 by 0.2 and eq2 by 0.3 and adding

⇒ 0.08x + 0.06y + 0.21x − 0.06y = 0.34 + 0.24

⇒ 0.29x = 0.58

⇒ x = 2

Substituting value of x in eq1, 0.8x + 0.3y = 1.7

⇒ y = 3

4. Question

Solve the following systems of equations:

$\frac{x}{2} + y = 0.8$

$\frac{7}{x + \frac{y}{2}} = 10$

Answer

$\frac{x}{2} + y = 0.8$

$\frac{7}{x + \frac{y}{2}} = 10$

⇒ x + y/2 = 0.7

Multiplying eq1 by 2 and subtracting eq2 from it.

⇒ x + 2y – x – y/2 = 1.6 – 0.7

⇒ 3y/2 = 0.9

⇒ y = 0.6

Thus, x/2 + 0.6 = 0.8

⇒ x = 0.4

5. Question

Solve the following systems of equations:

$7(y+3) - 2(x+2) = 14$

$4(y-2) + 3(x-3) = 2$

Answer

$7(y+3) - 2(x+2) = 14$ 7y + 21 - 2x - 4 = 147y - 2x = 14 - 21 + 4

⇒ 7y – 2x = -3 ---- (1)

$4(y-2) + 3(x-3) = 2$ 4y - 8 + 3x - 9 = 24y + 3x = 2 + 8 + 9

⇒ 4y + 3x = 19 ------ (2)

Multiplying eq1 by 3 and eq2 by 2 and adding them

3(7y – 2x) + 2(4y + 3x) = -3 (3) + 19(2)

⇒ 21y – 6x + 8y + 6x = -9 + 38⇒ 29 y = 29

⇒ y = 1

Put the value of y in (1) to get,

7 – 2x = -3- 2x = -3 - 7-2x = -10

⇒ x = 5

6. Question

Solve the following systems of equations:

$\frac{x}{7}+\frac{y}{3}=5$

$\frac{x}{2}-\frac{y}{9}=6$

Answer

$\frac{x}{7}+\frac{y}{3}=5$

⇒ 3x + 7y = 105

$\frac{x}{2}-\frac{y}{9}=6$

⇒ 9x − 2y = 108

Multiplying eq1 by 3 and subtracting eq2 from it

⇒ 9x + 21y − 9x + 2y = 315 − 108

⇒ 23y = 207

⇒ y = 9

Thus, 9x − 18 = 108

⇒ x = 14

7. Question

Solve the following systems of equations:

$\frac{x}{3}+\frac{y}{4}=11$

$\frac{5x}{6}-\frac{y}{3}=-7$

Answer

$\frac{x}{3}+\frac{y}{4}=11$

⇒ 4x + 3y = 132

$\frac{5x}{6}-\frac{y}{3}=-7$

⇒ 5x − 2y = -42

Multiplying eq1 by 2 and eq2 by 3 and adding them

⇒ 8x + 6y + 15x − 6y = 264 − 126

⇒ 23x = 138

⇒ x = 6

Thus, 24 + 3y = 132

⇒ y = 36

8. Question

Solve the following systems of equations:

$\frac{4}{x}+3y=8$

$\frac{6}{x} - 4y = -5$

Answer

$\frac{4}{x} + 3y = 8$

$\frac{6}{x} - 4y = -5$

Multiplying eq1 by 4 and eq2 by 3 and adding

⇒ 16/x + 12y + 18/x − 12y = 32 − 15⇒ 34/x = 17

⇒ x = 2

Thus, 2 + 3y = 8⇒ 3y = 6

⇒ y = 2

9. Question

Solve the following systems of equations:

$x + \frac{y}{2} = 4$

$\frac{x}{3} + 2y = 5$

Answer

$x + \frac{y}{2} = 4$ ⇒ 2x + y = 8 ...(i)

$\frac{x}{3} + 2y = 5$ ⇒ x + 6y = 15 ...(ii)

Multiplying eq2 by 2 and subtracting from eq1

⇒ 2x + y - 2x -12y = 8 − 30

⇒ -11y = -22

⇒ y = 2

Thus, 2x + 2 = 8

⇒ x = 3

10. Question

Solve the following systems of equations:

$x + 2y = \frac{3}{2}$

$2x + y = \frac{3}{2}$

Answer

$x + 2y = \frac{3}{2}$ ⇒ 2x + 4y = 3 ...(i)

$2x + y = \frac{3}{2}$ ⇒ 4x + 2y = 3 ...(ii)

Multiplying eq2 by 2 and subtracting from eq1

2x + 4y − 8x − 4y = 3 − 6

⇒ -6x = - 3

$\Rightarrow$ x = 1/2

Thus, 2 × 1/2 + 4y = 3 ⇒ y = 1/2

11. Question

Solve the following systems of equations:

$\sqrt{2x} - \sqrt{3y} = 0$

$\sqrt{3x} - \sqrt{8y} = 0$

Answer

Given: The following systems of equations:

$\sqrt{2x} - \sqrt{3y} = 0$

$\sqrt{3x} - \sqrt{8y} = 0$

To find : The values of x and y.

Solution: $\sqrt{2x} - \sqrt{3y} = 0$

$\Rightarrow \sqrt{2x} = \sqrt{3y}$

Squaring both sides to get,

2x = 3y

$\Rightarrow x = \frac{3y}{2}$

Now,

$\sqrt{3x} - \sqrt{8y} = 0 \Rightarrow \sqrt{3x} = \sqrt{8y}$

Squaring both sides to get, 3x = 8y ..... (1)

Substitute the value of x in (1),

$$3\left(\frac{3y}{2}\right) = 8y$$

$$\frac{9y}{2} = 8y$$

$$\frac{9y}{2} - 8y = 0$$

$$\frac{9y - 16y}{2} = 0$$

$$\frac{-7y}{2} = 0$$

$$-7y = 0$$

$$y = 0$$

Substitute the value of y in 2x = 3y⇒2x=3(0)⇒2x=0⇒x=0

Solution is x=0,y=0.

12. Question

Solve the following systems of equations:

$3x - \frac{Y+7}{11} + 2 = 10$

$2y + \frac{X+11}{7} = 10$

Answer

Consider $3x - \frac{Y+7}{11} + 2 = 10$,

$$\Rightarrow \frac{11(3x) - y - 7 + 2(11)}{11} = 10$$

$$\Rightarrow \frac{33x - y - 7 + 22}{11} = 10$$

⇒ 33x - y - 7 + 22 = 110⇒ 33x - y - 95 = 0⇒ 33x - 95 = y ...... (1)

Consider $2y + \frac{X+11}{7} = 10$,

$$\Rightarrow \frac{7(2y) + x + 11}{7} = 10$$

$$\Rightarrow \frac{14y + x + 11}{7} = 10$$

⇒ 14y + x + 11 = 70

⇒ 14y + x = 59 .... (2)

Put the value of y from (1) in (2) to get,14(33x-95) + x = 59 462x - 1330 + x = 59463x = 1389x = 3Put the value of x in (1) to get, 33(3) - 95 = y99 - 95 = y4 = y⇒ **x = 3 and y = 4**

13. Question

Solve the following systems of equations:

$2x - \frac{3}{y} = 9$

$3x + \frac{7}{y} = 2, y \neq 0$

Answer

$2x - \frac{3}{y} = 9$

$3x + \frac{7}{y} = 2, y \neq 0$

Multiplying eq1 by 3 and eq2 by 2 and subtracting eq1 from eq2

⇒ 6x + 21/y – 6x + 6/y = 4 – 27

⇒ 23/y = - 23

⇒ y = -1

Thus, 2x + 3 = 9

⇒ x = 3

14. Question

Solve the following systems of equations:

$0.5x + 0.7y = 0.74$

$0.3x + 0.5y = 0.5$

Answer

$0.5x + 0.7y = 0.74$

$0.3x + 0.5y = 0.5$

Multiplying eq1 by 0.3 and eq2 by 0.5 and subtracting eq1 from eq2

⇒ 0.15x + 0.25y – 0.15x – 0.21y = 0.25 – 0.222

⇒ 0.04y = 0.028

⇒ y = 0.7

Thus, x = 0.5

15. Question

Solve the following systems of equations:

$\frac{1}{7x} + \frac{1}{6y} = 3$

$$\frac{1}{2x} - \frac{1}{3y} = 5$$

Answer

Given:

systems of equations:

$\frac{1}{7x} + \frac{1}{6y} = 3$

$\frac{1}{2x} - \frac{1}{6y} = 5$

To find: The values of x and y.

Solution:

Let's take,

$$\frac{1}{x} = u, \frac{1}{y} = v$$

The equations become:

$$\frac{1}{7}u + \frac{1}{6}v = 3$$

$$\frac{6u + 7v}{42} = 3$$

6u + 7v = 126 ...... (1)

$$\frac{1}{2}u - \frac{1}{3}v = 5$$

$$\frac{3u-2v}{6} = 5$$

3u - 2v = 30 ...... (2)Now multiply eq. (2) with 2 and subtract it from eq. (1)⇒ 6u + 7v - 2(3u - 2v) = 126 - 2(30)⇒ 6u + 7v - 6u +4v = 126 - 60⇒ 11v = 66⇒ v = 6Now put the value of v in eq.

(2) to get,⇒ 3u-2(6)=30⇒ 3u-12=30⇒ 3u=42⇒ u=14

Now

$$u = \frac{1}{x} = 14$$

$$x = \frac{1}{14}$$

And

$$v = \frac{1}{y} = 6$$

$$y = \frac{1}{6}$$

16. Question

Solve the following systems of equations:

$$\frac{1}{2x} + \frac{1}{3y} = 2$$

$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$$

Answer

$$\frac{1}{2x} + \frac{1}{3y} = 2$$

$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$$

Multiplying eq 1 by 1/2 and eq2 by 1/3 and subtracting

⇒ 1/4x – 1/9x = 1 – 13/18

⇒ 5/36x = 5/18

⇒ x = 1/2

Thus, 1 + 1/3y = 2

⇒ y = 1/3

17. Question

Solve the following systems of equations:

$$\frac{x+y}{xy} = 2$$

$\frac{x-y}{xy}=6$

Answer

$\frac{x+y}{xy}=2$

$\frac{x-y}{xy}=6$

Adding both equation

⇒ 2x/xy = 8

⇒ y = 1/4

Thus, $\frac{\left(x+\frac{1}{4}\right)}{\frac{x}{4}}=2$

⇒ x + 1/4 = x/2

⇒ x = -1/2

18. Question

Solve the following systems of equations:

$\frac{15}{u}+\frac{2}{v}=17$

$\frac{1}{u}+\frac{1}{v}=\frac{36}{5}$

Answer

$\frac{15}{u}+\frac{2}{v}=17$

$\frac{1}{u}+\frac{1}{v}=\frac{36}{5}$

Multiplying eq2 by 2 and subtracting

13/u = 17 – 72/5

⇒ u = 5

Thus, 3 + 2/v = 17

⇒ v = 1/7

19. Question

Solve the following systems of equations:

$\frac{3}{x}-\frac{1}{y}=-9$

$\frac{2}{x}+\frac{3}{y}=5$

Answer

$\frac{3}{x}-\frac{1}{y}=-9$

$\frac{2}{x}+\frac{3}{y}=5$

Multiplying eq1 by 3 and adding to eq1

⇒ 11/x = -27 +5

⇒ x = -1/2

Thus, -4 + 3/y = 5

⇒ y = 1/3

20. Question

Solve the following systems of equations:

$$\frac{2}{x}+\frac{5}{y}=1$$

$$\frac{60}{x}+\frac{40}{y}=19,\ x\neq 0,\ y\neq 0$$

Answer

$$\frac{2}{x}+\frac{5}{y}=1$$

$$\frac{60}{x}+\frac{40}{y}=19,\ x\neq 0,\ y\neq 0$$

Multiplying eq1 by 8 and subtracting from eq2

⇒ 44/x = 19 – 8

⇒ x = 4

Thus, 2/4 + 5/y = 1

⇒ 5/y = 1/2

⇒ y = 10

21. Question

Solve the following systems of equations:

$$\frac{1}{5x}+\frac{1}{6y}=12$$

$$\frac{1}{3x}-\frac{3}{7y}=8,\quad x\neq 0,\ y\neq 0$$

Answer

$$\frac{1}{5x}+\frac{1}{6y}=12$$

$$\frac{1}{3x}-\frac{3}{7y}=8,\quad x\neq 0,\ y\neq 0$$

Multiplying eq1 by 1/3 and eq2 by 1/5 and subtracting

⇒ 1/18y + 3/35y = 4 - 8/5

⇒ 89/630y = 12/5

⇒ y = 89/1512

Thus, 1/5x + 1512/534 = 12

⇒ 1/5x = 816/89

⇒ x = 89/4080

22. Question

Solve the following systems of equations:

$\frac{2}{x}+\frac{3}{y}=\frac{9}{xy}$

$\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}$, *where*, $x \neq 0$, $y \neq 0$

Answer

$\frac{2}{x}+\frac{3}{y}=\frac{9}{xy}$

$$\Rightarrow \frac{2y+3x}{xy}=\frac{9}{xy}$$

⇒ 2y + 3x = 9 ----- (1)

$\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}$, *where*, $x \neq 0$, $y \neq 0$

$$\Rightarrow \frac{4y+9x}{xy}=\frac{21}{xy}$$

⇒ 4y + 9x = 21 ---- (2)

Multiplying eq1 by 2 and subtracting it from eq 2 we get,

⇒ 4y + 9x - 2(2y + 3x) = 21 - 2(9)⇒ 4y + 9x - 4y - 6x = 21 - 18⇒ 3x = 3

⇒ x = 1

Put the value of x in 1 to get,

Thus, 2y + 3(1) = 92y + 3 = 92y = 9 - 32y = 6y = 3

hence x = 1 and y = 3

23. Question

Solve the following systems of equations:

$\frac{6}{x+y}=\frac{7}{x-y}+3$

$$\frac{1}{2(x+y)}=\frac{1}{3(x-y)}$$

where, x + y ≠ 0 and x - y ≠ 0

Answer

Given : the following systems of equations:

$\frac{6}{x+y}=\frac{7}{x-y}+3$ ...... (1)

$$\frac{1}{2(x+y)}=\frac{1}{3(x-y)} \quad ...... (2)$$

To find : The value of x and y.

Solution :

Take $\frac{1}{x+y}=u\ and\ \frac{1}{x-y}=v$

The equations become:6u=7v+36u-7v=3 ...... (3)And, $\frac{1}{2}u = \frac{1}{3}v$

⇒ 3u=2v 3u-2v=0 ...... (4)To find the value of u and v,Multiply eq. 4 with 2 and subtract it from eq. 3⇒ 6u-7v-2(3u-2v)=3-0⇒ 6u-7v-6u+4v=3⇒ -3v=3⇒ v=-1Put the value of v in eq. 4 to get,⇒ 3u-2(-1)=0⇒ 3u+2=0⇒ 3u=-2 $u = \frac{-2}{3}$

Now,

$$\Rightarrow u = \frac{1}{x+y} = \frac{-2}{3}$$

$$\Rightarrow \frac{1}{x+y} = \frac{-2}{3}$$

⇒ 3 = -2(x+y)⇒ 3=-2x-2y ..... (5)

$$\Rightarrow v = \frac{1}{x-y} = -1$$

$$\Rightarrow \frac{1}{x-y} = -1$$

⇒ x-y=-1 .... (6)Multiply eq. 6 with 2 and add to eq. 5⇒ 2(x-y)+(-2x-2y)=-2+3⇒ -2y-2y-2x-2y=1⇒ -4y=1⇒ $y = \frac{-1}{4}$

Now put the value of y in eq. 6 to get $x - \left(\frac{-1}{4}\right) = -1$

$$x + \frac{1}{4} = -1$$

$$x = -1 - \frac{1}{4}$$

$$x = \frac{-4-1}{4}$$

$$\Rightarrow x = \frac{-5}{4}$$

Hence the values are $x = \frac{-5}{4},\ y = \frac{-1}{4}$.

24. Question

Solve the following systems of equations:

$\frac{xy}{x+y} = \frac{6}{5}$

$\frac{xy}{y-x} = 6$ where, x + y ≠ 0 and x - y ≠ 0

Answer

$\frac{xy}{x+y} = \frac{6}{5}$

$\frac{xy}{y-x} = 6$

Dividing the two equations

⇒ (y –x)/(x + y) = 1/5

⇒ 5y – 5x = x + y

⇒ y = 3x/2

Thus, $x \times \frac{\frac{3x}{2}}{x+\frac{3x}{2}} = \frac{6}{5}$

⇒ 3x/5 = 6/5

⇒ x = 2

Thus, y = 3

25. Question

Solve the following systems of equations:

$\frac{22}{x+y} + \frac{15}{x-y} = 5$

$\frac{55}{x+y} + \frac{45}{x-y} = 14$

Answer

$\frac{22}{x+y} + \frac{15}{x-y} = 5$

$\frac{55}{x+y} + \frac{45}{x-y} = 14$

Multiplying eq1 by 3 and subtracting from eq2

⇒ -11/(x + y) = -1

⇒ x + y = 11 ---------- (3)

Multiplying eq1 by 5 and eq2 by 2 and subtracting

⇒ 15/(x –y) = 3

⇒ x – y = 5 ------ (4)

(3) + (4)

⇒ 2x = 16

⇒ x = 8

Thus, y = 3

26. Question

Solve the following systems of equations:

$\frac{5}{x+y} - \frac{2}{x-y} = -1$

$\frac{15}{x+y} + \frac{7}{x-y} = 10$

Answer

$$\frac{5}{x+y}-\frac{2}{x-y}=-1$$

$$\frac{15}{x+y}+\frac{7}{x-y}=10$$

Multiplying eq1 by 3 and subtracting from eq2

⇒ 13/(x − y) = 13

⇒ x − y = 1 ------ (3)

Multiplying eq1 by 7 and eq2 by 2 and adding

⇒ 65/(x + y) = 13

⇒ x + y = 5 ------- (4)

Thus, 2x = 6

⇒ x = 3

Y = x − 1

⇒ y = 2

27. Question

Solve the following systems of equations:

$$\frac{3}{x+y}+\frac{2}{x-y}=2$$

$$\frac{9}{x+y}-\frac{4}{x-y}=1$$

Answer

$$\frac{3}{x+y}+\frac{2}{x-y}=2$$

$$\frac{9}{x+y}-\frac{4}{x-y}=1$$

Multiplying eq1 by 2 and adding to eq2

⇒ 15/(x + y) = 5

⇒ x + y = 3 ------- (3)

Multiplying eq1 by 3 and subtracting eq2 from it

⇒ 10/(x − y) = 5

⇒ x − y = 2 ---- (4)

Adding the two equation

⇒ 2x = 5

⇒ x = 5/2

Thus, y = 1/2

28. Question

Solve the following systems of equations:

$$\frac{1}{2(x+2y)}+\frac{5}{3(3x-2y)}=\frac{-3}{2}$$

$$\frac{5}{4(x+2y)} - \frac{4}{5(3x-2y)} = \frac{73}{60}$$

Answer

$$\frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} = \frac{-3}{2}$$

$$\frac{5}{4(x+2y)} - \frac{4}{5(3x-2y)} = \frac{73}{60}$$

Multiplying eq1 by 5/2 and subtracting from eq2

$$\Rightarrow -\frac{4}{5(3x-2y)} - \frac{25}{6(3x-2y)} = \frac{73}{60} + \frac{15}{4}$$

$$\Rightarrow -\frac{149}{30(3x-2y)} = \frac{298}{60}$$

⇒ 3x – 2y = - 1 ------- (3)

Multiplying eq1 by 12/25 and adding to eq2

$$\Rightarrow \frac{6}{25(x+2y)} + \frac{5}{4(x+2y)} = -\frac{18}{25} + \frac{73}{60}$$

$$\Rightarrow \frac{149}{100(x+2y)} = \frac{149}{300}$$

⇒ x + 2y = 3 ------- (4)

Solving (3) and (4)

We get, x = 1/2 and y = 5/4

29. Question

Solve the following systems of equations:

$$\frac{5}{x+1} - \frac{2}{y-1} = \frac{1}{2}$$

$$\frac{10}{x+1} + \frac{2}{y-1} = \frac{5}{2} \quad \textit{where, } x \neq -1,\ y \neq 1$$

Answer

$$\frac{5}{x+1} - \frac{2}{y-1} = \frac{1}{2}$$

$$\frac{10}{x+1} + \frac{2}{y-1} = \frac{5}{2}$$

Adding eq1 and eq2

⇒ 15/(x + 1) = 3

⇒ x + 1 = 5

⇒ x = 4

Thus, 5/5 – 2/(y – 1) = 12

⇒ y – 1 = 4

⇒ y = 5

30. Question

Solve the following systems of equations:

$x + y = 5xy$

$x + 2y = 13xy, \ x \neq 0, y \neq 0$

Answer

$x + y = 5xy$

$x + 2y = 13xy, \ x \neq 0, y \neq 0$

Subtracting the two eq.

⇒ -y = -8xy

⇒ x = 1/8

Thus, 1/8 + y = 5y/8

⇒ y = 1/3

31. Question

Solve the following systems of equations:

$x + y = 2xy$

$\frac{x - y}{xy} = 6, \ x \neq 0, y \neq 0$

Answer

x + y = 2xy

x – y = 6xy

Adding the two equation

⇒ 2x = 8xy

⇒ y = 1/4

Thus, x + 1/4 = 2 × x × 1/4

⇒ x/2 = -1/4

⇒ x = -1/2

32. Question

Solve the following systems of equations:

$2(3u - \upsilon) = 5u\upsilon$

$2(u + 3\upsilon) = 5u\upsilon$

Answer

$2(3u - \upsilon) = 5u\upsilon$

$2(u + 3\upsilon) = 5u\upsilon$

Equating both equations

⇒ 6u – 2v = 2u + 6v

⇒ u = 2v

Substituting value of u

⇒ 2(6v – v) = 5 × 2v × v

⇒ v = 1

Thus, 2(3u – 1) = 5u

⇒ 6u – 2 = 5u

⇒ u = 2

33. Question

Solve the following systems of equations:

$$\frac{2}{3x+2y}+\frac{3}{3x-2y}=\frac{17}{5}$$

$$\frac{5}{3x+2y}+\frac{1}{3x-2y}=2$$

Answer

$$\frac{2}{3x+2y}+\frac{3}{3x-2y}=\frac{17}{5}$$

$$\frac{5}{3x+2y}+\frac{1}{3x-2y}=2$$

Multiplying eq1 by 5 and eq2 by 2 and subtracting

⇒ 13/(3x – 2y) = 13

⇒ 3x – 2y = 1------- (3)

Multiplying eq2 by 3 and subtracting eq1 from it

⇒ 13/(3x +2y) = 13/5

⇒ 3x + 2y = 5 ------ (4)

(3) + (4)

⇒ 6x = 6

⇒ x = 1

Thus, 3 + 2 = 5

⇒ y = 1

34. Question

Solve the following systems of equations:

$$\frac{4}{x}+3y=14$$

$$\frac{3}{x}-4y=23$$

Answer

$$\frac{4}{x}+3y=14$$

$$\frac{3}{x}-4y=23$$

Multiply eq1 by 4 and eq2 by 3 and adding

⇒ 25/x = 125

⇒ x = 1/5

Thus, 20 + 3y = 14

⇒ $y = -2$

35. Question

Solve the following systems of equations:

$$99x + 101y = 499$$

$$101x + 99y = 501$$

Answer

$$99x + 101y = 499$$

$$101x + 99y = 501$$

Adding both equations

⇒ $200x + 200y = 1000$

⇒ $x + y = 5$ ------ (1)

Subtracting both equation

⇒ $2y - 2x = -2$

⇒ $x - y = 1$ ----- (2)

Adding 1 and 2

⇒ $2x = 6$

⇒ $x = 3$

Thus, $3 + y = 5$

⇒ $y = 2$

36. Question

Solve the following systems of equations:

$$23x - 29y = 98$$

$$29x - 23y = 110$$

Answer

$$23x - 29y = 98$$

$$29x - 23y = 110$$

Adding both equation

⇒ $52x - 52y = 208$

⇒ $x - y = 4$ ----- (1)

Subtracting both equation

⇒ $-6x - 6y = -12$

⇒ $x + y = 12$ ----- (2)

(1) + (2)

⇒ $2x = 16$

⇒ $x = 8$

Thus, $y = 4$

37. Question

Solve the following systems of equations:

$x - y + z = 4$

$x - 2y + 3z = 9$

$2x + y + 3z = 1$

Answer

$x - y + z = 4$

$x - 2y + 3z = 9$

$2x + y + 3z = 1$

Subtracting eq2 and eq3

⇒ x + 3y = 0

⇒ x = -3y

Substituting in eq1

⇒ -3y – y + z = 4

⇒ z = 4 + 4y

Substituting in eq2

⇒ -3y – 2y + 12 + 12y = 1

⇒ 7y = -11

⇒ y = -11/7

Thus, x = 33/7

And z = 4 – 44/7 = - 16/7

38. Question

Solve the following systems of equations:

$x - y + z = 4$

$x + y + z = 2$

$2x + y - 3z = 0$

Answer

$x - y + z = 4$

$x + y + z = 2$

$2x + y - 3z = 0$

Adding eq 1 and eq2

⇒ 2(x + z) = 6

⇒ x = 3 – z

(3) – (2) – (1)

⇒ y – 5z = - 4

⇒ y = -4 + 5z

Substituting in eq2

⇒ 3 − z - 4 + 5z + z = 2

⇒ 5z = 3

⇒ z = 3/5

Thus, x = 3 − z

⇒ x = 12/5

Y = 5z − 4

⇒ y = -1

39. Question

Solve the following systems of equations:

$$\frac{44}{x+y}+\frac{30}{x-y}=10$$

$$\frac{55}{x+y}+\frac{40}{x-y}=13$$

Answer

$$\frac{44}{x+y}+\frac{30}{x-y}=10$$

$$\frac{55}{x+y}+\frac{40}{x-y}=13$$

Multiplying eq1 by 5 and eq2 by 4 and subtracting

⇒ -10/(x − y) = -2

⇒ x − y = 5

Multiply eq1 by 4 and eq2 by 3 and subtracting

⇒ 11/(x + y) = 1

⇒ (x + y) = 11

Thus, 2x = 16

⇒ x = 8

∴ y = 8 − 5 = 3

40. Question

Solve the following systems of equations:

$$\frac{4}{x}+5y=7$$

$$\frac{3}{x}+4y=5$$

Answer

$$\frac{4}{x}+5y=7$$

$$\frac{3}{x}+4y=5$$

Multiply eq1 by 4 and eq2 by 5 and subtracting

⇒ 1/x = 3

⇒ x = 1/3

Thus, 12 + 5y = 7

⇒ y = 1

41. Question

Solve the following systems of equations:

$\frac{2}{x}+\frac{3}{y}=13$

$\frac{5}{x}-\frac{4}{y}=-2$

Answer

$\frac{2}{x}+\frac{3}{y}=13$

$\frac{5}{x}-\frac{4}{y}=-2$

Multiplying eq1 by 4 and eq2 by 3 and adding

⇒ 23/x = 46

⇒ x = 1/2

Thus, 4 + 3/y = 13

⇒ y = 1/3

42. Question

Solve the following systems of equations:

$\frac{5}{x-1}+\frac{1}{y-2}=2$

$\frac{6}{x-1}-\frac{3}{y-2}=1$

Answer

Given : The system of equations: $\frac{5}{x-1}+\frac{1}{y-2}=2$

$\frac{6}{x-1}-\frac{3}{y-2}=1$

To find: The values of x and y

Solution: Let $\frac{1}{x-y}=u \ and \ \frac{1}{y-2}=v$

Now system of equations become:

5u + v = 2 .... (1) 6u - 3v = 1 .... (2)

Now multiply equation (1) by 3 and add to equation (2),3(5u + v)+ 6u - 3v = 3(2) +115u +3v + 6u -3v = 6+121u = 7

$u=\frac{7}{21}$

$u=\frac{1}{3}$

so, $u=\frac{1}{x-1}=\frac{1}{3}$

⇒x - 1 =3⇒x = 3 + 1⇒x = 4Now put $u=\frac{1}{3}$ in the equation (1) to get v,

$$\frac{5}{3}+v=2$$

$$v=2-\frac{5}{3}$$

$$v=\frac{6-5}{3}$$

$$v=\frac{1}{3}$$

So, $v=\frac{1}{y-2}=\frac{1}{3}$

⇒y - 2 = 3⇒y = 3 + 2⇒y=5Hence the values are x= 4 and y=5.

43. Question

Solve the following systems of equations:

$$\frac{10}{x+y}+\frac{2}{x-y}=4$$

$$\frac{15}{x+y}-\frac{9}{x-y}=-2$$

Answer

$$\frac{10}{x+y}+\frac{2}{x-y}=4$$

$$\frac{15}{x+y}-\frac{9}{x-y}=-2$$

Multiplying eq1 by 9 and eq2 by 2 and adding

⇒ 120/(x + y) = 32

⇒ x + y = 15/4 ---- (1)

Multiplying eq1 by 3 and eq2 by 2 and subtracting

⇒ 24/(x − y) = 16

⇒ x − y = 3/2 ---- (2)

Adding (1) and (2)

⇒ 2x = 21/4

⇒ x = 21/8

Thus, 21/8 − y = 3/2

⇒ y = 9/8

44. Question

Solve the following systems of equations:

$$\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$$

$$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$$

Answer

$$\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$$

$$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$$

Multiplying eq2 by 2 and adding to eq1

⇒ 2/(3x + y) = 3/4 - 2/8

⇒ 3x + y = 4 ---- (3)

Multiplying eq2 by 2 and subtracting from eq1

⇒ 2/(3x − y) = 1

⇒ 3x − y = 2 ----- (4)

Adding (3) and (4)

⇒ 6x = 6

⇒ x = 1

Thus, y = 3 − 2 = 1

45. Question

Solve the following systems of equations:

$$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$$

$$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$$

Answer

$$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$$

$$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$$

Multiplying eq1 by 3 and adding to eq2

⇒ 10/√x = 5

⇒ √x = 2

⇒ x = 4

Thus,

⇒ 2/√4 + 3/√y = 2

⇒ y = 9

46. Question

Solve the following systems of equations:

$$\frac{7x-2y}{xy}=5$$

$$\frac{8x+7y}{xy}=15$$

Answer

$$\frac{7x-2y}{xy}=5$$

⇒ 7/y – 2/x = 5 ---- (1)

$$\frac{8x+7y}{xy}=15$$

⇒ 8/y + 7/x = 15 ---- (2)

Multiplying eq1 by 7 and eq2 by 2 and adding

⇒ 65/y = 65

⇒ y = 1

Thus, 7 – 2/x = 5

⇒ x = 1

47. Question

Solve the following systems of equations:

$$152x-378y=-74$$

$$-378x+158y=-604$$

Answer

$$152x-378y=-74$$

$$-378x+158y=-604$$

Adding (1) and (2)

⇒ -226x – 226y = -678

⇒ x + y = 3----- (3)

(1) – (2)

⇒ 530x – 530y = 530

⇒ x – y = 1 ------ (4)

Adding (3) and (4)

⇒ 2x = 4

⇒ x = 2

Thus, y = 1

Exercise 3.4

1. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$$x+2y+1=0$$

$$2x-3y-12=0$$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $x + 2y + 1 = 0$

$2x - 3y - 12 = 0$

$$\frac{x}{-24 + 3} = \frac{y}{2 + 12} = \frac{1}{-3 - 4}$$

⇒ x = -21/-7 = 3

⇒ y = 14/-7 = -2

2. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$3x + 2y + 25 = 0$

$2x + y + 10 = 0$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $3x + 2y + 25 = 0$

$2x + y + 10 = 0$

$$\therefore \frac{x}{2\times10 - 1\times25} = \frac{y}{2\times25 - 3\times10} = \frac{1}{3\times1 - 2\times2}$$

$$\Rightarrow \frac{x}{-5} = \frac{y}{20} = -1$$

⇒ x= 5 and y = -20

3. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$2x + y = 35$

$3x + 4y = 65$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $2x + y = 35$

$3x + 4y = 65$

$$\frac{x}{-65+140} = \frac{y}{-105+130} = \frac{1}{8-3}$$

⇒ x = 75/5 = 15

⇒ y = 25/5 = 5

4. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$2x - y = 6$

$x - y = 2$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $2x - y = 6$

$x - y = 2$

$$\frac{x}{2-6} = \frac{y}{-6+4} = \frac{1}{-2+1}$$

⇒ x = -4/-1 = 4

⇒ y = -2/-1 = 2

5. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{x+y}{xy} = 2, \ \frac{x-y}{xy} = 6$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given,

$\frac{x+y}{xy} = 2, \ \frac{x-y}{xy} = 6$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = 2, -\frac{1}{x} + \frac{1}{y} = 6$$

$$\frac{\frac{1}{x}}{-6+2} = \frac{\frac{1}{y}}{2+6} = \frac{1}{1+1}$$

⇒ 1/x = -2

⇒ x = -1/2

Also, 1/y = 4

⇒ y = 1/4

6. Question

Solve each of the following systems of equations by the method of cross-multiplication:

ax + by = a – b

$bx - ay = a + b$

Answer

Method of cross multiplication

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Given,

ax + by = a – b

$bx - ay = a + b$

This can be written as:

ax+by-(a-b)=0bx-ay-(a+b)=0So,

$$\frac{x}{(b)\big(-(a+b)\big)-(-a)\big(-(a-b)\big)} = \frac{-y}{a(-(a+b))-b(-(a-b))} = \frac{1}{a(-a)-b(b)}$$

$$\frac{x}{(b)(-a-b)+a(b-a)} = \frac{-y}{a(-a-b)-b(b-a)} = \frac{1}{-a^2-b^2}$$

$$\frac{x}{-ba-b^2+ab-a^2} = \frac{-y}{-a^2-ab-b^2+ab} = \frac{1}{-a^2-b^2}$$

$$\frac{x}{-b^2-a^2} = \frac{-y}{-a^2-b^2} = \frac{1}{-a^2-b^2}$$

$$\frac{x}{-b^2-a^2} = \frac{y}{a^2+b^2} = \frac{1}{-a^2-b^2}$$

$$\frac{x}{-b^2-a^2} = \frac{1}{-a^2-b^2} \ and \ \frac{y}{a^2+b^2} = \frac{1}{-a^2-b^2}$$

$$x = \frac{-b^2-a^2}{-a^2-b^2} \ and \ y = \frac{a^2+b^2}{-a^2-b^2}$$

$$x = \frac{-b^2 - a^2}{-a^2 - b^2}\ and\ y = \frac{-(-a^2 - b^2)}{-a^2 - b^2}$$

⇒ x = 1 and y = -1

7. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$x + ay = b$

$ax - by = c$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $x + ay = b$

$ax - by = c$

$$\frac{x}{-ac - b^2} = \frac{y}{-ab + c} = \frac{1}{-b - a^2}$$

$$\Rightarrow x = \frac{ac + b^2}{a^2 + b}$$

$$\Rightarrow y = \frac{ab + c}{a^2 + b}$$

8. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$ax + by = a^2$

$bx + ay = b^2$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $ax + by = a^2$

$bx + ay = b^2$

$$\Rightarrow \frac{x}{-b^3 + a^3} = \frac{y}{-ba^2 + ab^2} = \frac{1}{a^2 - b^2}$$

$$\Rightarrow x = \frac{a^3 - b^3}{a^2 - b^2}$$

$$\Rightarrow x = \frac{(a-b)(a^2+ab+b^2)}{(a-b)(a+b)} = \frac{a^2+ab+b^2}{a+b}$$

$$y = \frac{ab(b-a)}{(a-b)(a+b)} = -\frac{ab}{a+b}$$

9. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$$\frac{x}{a} + \frac{y}{b} = 2$$

$$ax - by = a^2 - b^2$$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given,

$$\frac{x}{a} + \frac{y}{b} = 2$$

$$ax - by = a^2 - b^2$$

$$\frac{x}{\frac{b^2-a^2}{b} - 2b} = \frac{y}{-2a + \frac{a^2-b^2}{a}} = \frac{1}{-\frac{b}{a} - \frac{a}{b}}$$

$$\Rightarrow \frac{x}{\frac{-a^2-b^2}{b}} = \frac{y}{\frac{-a^2-b^2}{a}} = \frac{1}{\frac{-a^2-b^2}{ab}}$$

⇒ x = a and y = b

10. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$$\frac{x}{a} + \frac{y}{b} = a + b$$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $\frac{x}{a} + \frac{y}{b} = a + b$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

$$\frac{x}{-\frac{2}{b}+\frac{a+b}{b^2}}=\frac{y}{-\frac{a+b}{a^2}+\frac{2}{a}}=\frac{1}{\frac{1}{ab^2}-\frac{1}{a^2b}}$$

$$\Rightarrow \frac{x}{\frac{a-b}{b^2}}=\frac{y}{\frac{a-b}{a^2}}=\frac{1}{\frac{1}{a^2b^2}(a-b)}$$

⇒ $x = a^2$, $y = b^2$

11. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{x}{a}=\frac{y}{b}$

$ax+by=a^2+b^2$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given, $\frac{x}{a}=\frac{y}{b}$

$ax+by=a^2+b^2$

$$\Rightarrow \frac{x}{\frac{a^2+b^2}{b}}=\frac{y}{\frac{a^2+b^2}{a}}=\frac{1}{\frac{b}{a}+\frac{a}{b}}$$

$$\Rightarrow \frac{x}{\frac{a^2+b^2}{b}}=\frac{y}{\frac{a^2+b^2}{a}}=\frac{ab}{a^2+b^2}$$

⇒ x = a and y = b

12. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{5}{x+y}-\frac{2}{x-y}=-1$

$\frac{15}{x+y}+\frac{7}{x-y}=10$ *where* $x \neq 0$ *and* $y \neq 0$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given,

$$\frac{5}{x+y}-\frac{2}{x-y}=-1$$

$$\frac{15}{x+y}+\frac{7}{x-y}=10$$

Multiplying eq1 by 3 and subtracting from eq2

$$\Rightarrow \frac{13}{x-y} = 13$$

⇒ x − y = 1 ------- (3)

Multiplying eq2 by 2 and eq1 by 7 and adding

⇒ 65/(x + y) = 13

⇒ x + y = 5 ---------- (4)

Thus,

$$\frac{x}{5+1}=\frac{y}{-1+5}=\frac{1}{1+1}$$

⇒ x = 6/2 = 3 and y = 4/2 = 2

13. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$$\frac{2}{x}+\frac{3}{y}=13$$

$$\frac{5}{x}-\frac{4}{y}=-2 \quad \textit{where } x \neq 0 \textit{ and } y \neq 0$$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given, $\frac{2}{x}+\frac{3}{y}=13$

$$\frac{5}{x}-\frac{4}{y}=-2$$

$$\Rightarrow \frac{\frac{1}{x}}{6-52}=\frac{\frac{1}{y}}{-65-4}=\frac{1}{-8-15}$$

⇒ 1/x = 2

⇒ x = 1/2

Also, 1/y = 3

⇒ y = 1/3

14. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$$ax+by=\frac{a+b}{2}$$

3x + 5y = 4

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given,

$$ax + by = \frac{a+b}{2}$$

3x + 5y = 8

$$\Rightarrow \frac{x}{-4b + \frac{(5a+5b)}{2}} = \frac{y}{\frac{-3a-3b}{2} + 4a} = \frac{1}{5a - 3b}$$

$$\Rightarrow \frac{2x}{5a-3b} = \frac{2y}{5a-3b} = \frac{1}{5a-3b}$$

⇒ x = 1/2 , y = 1/2

15. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$2ax + 3by = a + 2b$

$3ax + 2by = 2a + b$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $2ax + 3by = a + 2b$

$3ax + 2by = 2a + b$

$$\Rightarrow \frac{x}{-6ab - 3b^2 + 2ab + 4b^2} = \frac{y}{-3a^2 - 6ab + 4a^2 + 2ab} = \frac{1}{4ab - 9ab}$$

$$\Rightarrow \frac{x}{b^2 - 4ab} = \frac{y}{a^2 - 4ab} = \frac{1}{-5ab}$$

$$\Rightarrow x = \frac{4a-b}{5a}, y = \frac{4b-a}{5b}$$

16. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$5ax + 6by = 28$

$3ax + 4by = 18$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $5ax + 6by = 28$

$3ax + 4by = 18$

$$\frac{x}{-108b + 112b} = \frac{y}{-84a + 90a} = \frac{1}{20ab - 18ab}$$

$$\Rightarrow \frac{x}{4b} = \frac{y}{6a} = \frac{1}{2ab}$$

⇒ x = 2/a and y = 3/b

17. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$(a + 2b)x + (2a - b)y = 2$

(a – 2b)x + (2a + b)y = 3

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

$(a + 2b)x + (2a - b)y = 2$

(a – 2b)x + (2a + b)y = 3

$$\Rightarrow \frac{x}{3b - 6a + 4a + 2b} = \frac{y}{3a + 6b - 2a + 4b}$$
$$= \frac{1}{(2a + b)(a + 2b) - (a - 2b)(2a - b)}$$

$$\Rightarrow \frac{x}{5b - 2a} = \frac{y}{a + 10b} = \frac{1}{2a^2 + ab + 4ab + 2b^2 - (2a^2 - ab - 4ab + 2b^2)}$$
$$= \frac{1}{10ab}$$

$$\Rightarrow x = \frac{5b - 2a}{10ab}, y = \frac{a + 10b}{10ab}$$

18. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$$x\left(a - b + \frac{ab}{a - b}\right) = y\left(a + b - \frac{ab}{a - b}\right)$$

$x + y = 2a^2$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

$$x\left(a - b + \frac{ab}{a - b}\right) = y\left(a + b - \frac{ab}{a - b}\right)$$

$$x + y = 2a^2$$

$$\Rightarrow \frac{x}{2a^2 \times \frac{a^2 - b^2 - ab}{a - b}} = \frac{y}{2a^2 \times \frac{a^2 + b^2 - ab}{a - b}} = \frac{1}{\left(\frac{a^2 + b^2 - ab}{a - b} + \frac{a^2 - b^2 - ab}{a - b}\right)}$$

$$\Rightarrow \frac{x}{2a^2(a^2 - b^2 - ab)} = \frac{y}{2a^2(a^2 + b^2 - ab)} = \frac{1}{2a(a - b)}$$

$$\Rightarrow x = \frac{a(a^2 - b^2 - ab)}{a - b}, y = \frac{a(a^2 + b^2 - ab)}{a - b}$$

19. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$$bx + cy = a + b \quad ax\left(\frac{1}{a - b} - \frac{1}{a + b}\right) + cy\left(\frac{1}{b - a} - \frac{1}{b + a}\right) = \frac{2a}{a + b}$$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

$$bx + cy = a + b \quad ax\left(\frac{1}{a - b} - \frac{1}{a + b}\right) + cy\left(\frac{1}{b - a} - \frac{1}{b + a}\right) = \frac{2a}{a + b}$$

$$\Rightarrow \frac{x}{-\frac{2ac}{a + b} + (a + b) \times c \times \frac{2a}{b^2 - a^2}} = \frac{y}{-(a + b) \times a \times \frac{2b}{a^2 - b^2} + \frac{2ab}{a + b}}$$
$$= \frac{1}{-bc \times \frac{2a}{a^2 - b^2} - ac \times \frac{2b}{a^2 - b^2}}$$

$$\Rightarrow \frac{x}{-\frac{2ac}{a + b} - \frac{2ac}{a - b}} = \frac{y}{\frac{2ab}{a + b} - \frac{2ab}{a - b}} = \frac{1}{-\frac{4abc}{a^2 - b^2}}$$

$$\Rightarrow \frac{x}{-2ac \times \frac{2a}{a^2 - b^2}} = \frac{y}{2ab \times -\frac{2b}{a^2 - b^2}} = \frac{1}{-\frac{4abc}{a^2 - b^2}}$$

$$\Rightarrow x = \frac{a}{b}, y = \frac{b}{c}$$

20. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$(a-b)x+(a+b)y=2a^2-2b^2$

$(a+b)(x+y)=4ab$

Answer

Method of cross multiplication:For the system of equations:

$a_1x + b_1y + c_1 = 0$$a_2x + b_2y + c_2 = 0$

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

Here,$a_1 = (a - b)$, $b_1 = (a+b)$, $c_1 = 2b^2 - 2a^2$$a_2 = (a+b)$, $b_2 = (a+b)$, $c_2 = -4ab$So,

$$\Rightarrow \frac{x}{(a+b)(-4ab)-(a+b)(2b^2-2a^2)}=\frac{y}{(2b^2-2a^2)(a+b)-(-4ab)(a-b)}=\frac{1}{(a-b)(a+b)-(a+b)(a+b)}$$

$$\Rightarrow \frac{x}{(a+b)(-4ab)-2(a+b)(b^2-a^2)}=\frac{y}{2(b^2-a^2)(a+b)+4ab(a-b)}=\frac{1}{(a+b)(a-b-a-b)}$$

$$\Rightarrow \frac{x}{2(a+b)\left[-2ab-(b^2-a^2)\right]}=\frac{y}{2(b-a)(b+a)(a+b)+4ab(a-b)}=\frac{1}{(a+b)(-2b)}$$

$$\Rightarrow \frac{x}{-2(a+b)\left[2ab+b^2-a^2\right]}=\frac{y}{2(b-a)\left[(a+b)^2-2ab\right]}=\frac{1}{(a+b)(-2b)}$$

$$\Rightarrow \frac{x}{-2(a+b)\left[2ab+b^2-a^2\right]}=\frac{y}{2(b-a)\left[a^2+b^2+2ab-2ab\right]}=\frac{1}{(a+b)(-2b)}$$

$$\Rightarrow \frac{x}{-2(a+b)\left[2ab+b^2-a^2\right]}=\frac{y}{2(b-a)\left[a^2+b^2\right]}=\frac{1}{(a+b)(-2b)}$$

$$\Rightarrow x=\frac{-2(a+b)\left[2ab+b^2-a^2\right]}{(a+b)(-2b)}\ and\ y=\frac{2(b-a)\left[a^2+b^2\right]}{(a+b)(-2b)}$$

$$\Rightarrow x=\frac{\left[2ab+b^2-a^2\right]}{b}\ and\ y=\frac{(a-b)\left[a^2+b^2\right]}{b(a+b)}$$

21. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$a^2x+b^2y=c^2$

$b^2x+a^2y=a^2$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

$a^2x + b^2y = c^2$

$b^2x + a^2y = a^2$

$$\Rightarrow \frac{x}{-b^2a^2 + a^2c^2} = \frac{y}{-b^2c^2 + a^4} = \frac{1}{a^4 - b^4}$$

$$\Rightarrow x = \frac{a^2c^2 - b^2a^2}{a^4 - b^4}, y = \frac{a^4 - b^2c^2}{a^4 - b^4}$$

22. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{57}{x+y} + \frac{6}{x-y} = 5$

$\frac{38}{x+y} + \frac{21}{x-y} = 9$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

$\frac{57}{x+y} + \frac{6}{x-y} = 5$

$\frac{38}{x+y} + \frac{21}{x-y} = 9$

$$\Rightarrow \frac{\frac{1}{x+y}}{-6\times 9 + 5\times 21} = \frac{\frac{1}{x-y}}{-38\times 5 + 57\times 9} = \frac{1}{57\times 21 - 6\times 38}$$

⇒ x + y = 19 and x – y = 8

Solving the above two equation

⇒ x = 11 and y = 8

23. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$2(ax - by) + a + 4b = 0$

$2(bx + ay) + b - 4a = 0$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given,

$2(ax - by) + a + 4b = 0$

2ax - 2by + a + 4b = 0

$2(bx + ay) + b - 4a = 0$

2bx + 2ay + b - 4a = 0

$$\Rightarrow \frac{x}{(-2b)(b-4a)-2a(a+4b)} = \frac{y}{(2b)(a+4b)-(2a)(b-4a)} = \frac{1}{(2a)(2a)-(-2b)(2b)}$$

$$\Rightarrow \frac{x}{-2b^2+8ab-2a^2-8ab} = \frac{y}{2ab+8b^2-2ab+8a^2} = \frac{1}{4a^2+4b^2}$$

$$\Rightarrow \frac{x}{-2b^2-2a^2} = \frac{y}{8b^2+8a^2} = \frac{1}{4a^2+4b^2}$$

$$\Rightarrow \frac{x}{-2(b^2+a^2)} = \frac{y}{8(b^2+a^2)} = \frac{1}{4(a^2+b^2)}$$

$$\Rightarrow x = \frac{-2(b^2+a^2)}{4(a^2+b^2)} \ and \ y = \frac{8(a^2+b^2)}{4(a^2+b^2)}$$

$$\Rightarrow x = \frac{-1}{2} \ and \ y = 2$$

24. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$6(ax + by) = 3a + 2b$

$6(bx - ay) = 3b - 2a$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

$6(ax + by) = 3a + 2b$

$6(bx - ay) = 3b - 2a$

$$\Rightarrow \frac{x}{6b(2a-3b)-6a(3a+2b)} = \frac{y}{-6b(3a+2b)+6a(3b-2a)} = \frac{1}{-36a^2-3b^2}$$

$$\Rightarrow \frac{x}{-18a^2-18b^2} = \frac{y}{-12a^2-12b^2} = \frac{1}{-36a^2-36b^2}$$

⇒ x = 1/2, y = 1/3

25. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{a^2}{x} - \frac{b^2}{y} = 0$

$\frac{a^2b}{x}+\frac{b^2a}{y}=a+b,\ x,y\neq 0$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given,

$\frac{a^2}{x}-\frac{b^2}{y}=0$

$\frac{a^2b}{x}+\frac{b^2a}{y}=a+b,\ x,y\neq 0$

$$\Rightarrow \frac{\frac{1}{x}}{b^2(a+b)}=\frac{\frac{1}{y}}{a^2(a+b)}=\frac{1}{a^3b^2+a^2b^3}$$

$$\Rightarrow \frac{\frac{1}{x}}{b^2(a+b)}=\frac{\frac{1}{y}}{a^2(a+b)}=\frac{1}{a^2b^2(a+b)}$$

⇒ $x = a^2$, $y = b^2$

26. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$mx-ny=m^2+n^2$

$x+y=2m$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2-b2c1}=\frac{y}{a2c1-a1c2}=\frac{1}{a1b2-a2b1}$$

Given, $mx-ny=m^2+n^2$

$x+y=2m$

$$\Rightarrow \frac{x}{2mn+m^2+n^2}=\frac{y}{-m^2-n^2+2m^2}=\frac{1}{m+n}$$

$$\Rightarrow \frac{x}{(m+n)^2}=\frac{y}{m^2-n^2}=\frac{1}{m+n}$$

⇒ x = m + n and y = m − n

27. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$\frac{ax}{b}-\frac{by}{a}=a+b$

$ax - by = 2ab$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $\frac{ax}{b} - \frac{by}{a} = a + b$

$ax - by = 2ab$

$$\Rightarrow \frac{x}{2b^2 - ab - b^2} = \frac{y}{-a^2 - ab + 2a^2} = \frac{1}{-a + b}$$

$$\Rightarrow \frac{x}{b(b-a)} = \frac{y}{a(a-b)} = \frac{1}{b-a}$$

$$\Rightarrow x = b, y = -a$$

28. Question

Solve each of the following systems of equations by the method of cross-multiplication:

$$\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2$$

$x + y = 2ab$

Answer

Method of cross multiplication

a1x + b1y + c1 = 0

a2x + b2y + c2 = 0

$$\frac{x}{b1c2 - b2c1} = \frac{y}{a2c1 - a1c2} = \frac{1}{a1b2 - a2b1}$$

Given, $\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2$

$x + y = 2ab$

$$\frac{x}{-2a^2 + a^2 + b^2} = \frac{y}{-a^2 - b^2 + 2b^2} = \frac{1}{\frac{b}{a} - \frac{a}{b}}$$

$$\Rightarrow \frac{x}{b^2 - a^2} = \frac{y}{b^2 - a^2} = \frac{ab}{b^2 - a^2}$$

⇒ x = y = ab

Exercise 3.5

1. Question

In each of the following systems of equations determine whether the system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it:

$x - 3y = 3$

$3x - 9y = 2$

Answer

a1x + b1y + c1 = 0

a2x + b2x + c2 = 0

if $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, infinite solution

$if \frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, no solution

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$, unique solution

X – 3y = 3

3x – 9y = 2

Thus, $\frac{1}{3} = \frac{1}{3} \neq \frac{3}{2}$

The system has no solution

2. Question

In each of the following systems of equations determine whether the system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it:

$2x + y = 5$

$4x + 2y = 10$

Answer

a1x + b1y + c1 = 0

a2x + b2x + c2 = 0

if $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, infinite solution

$if \frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, no solution

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$, unique solution

2x + y = 5

4x + 2y = 10

$Thus, \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

Infinitely many solution

3. Question

In each of the following systems of equations determine whether the system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it:

$3x - 5y = 20$

$6x - 10y = 40$

Answer

a1x + b1y + c1 = 0

a2x + b2x + c2 = 0

if $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, infinite solution

$if \frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, no solution

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$,unique solution

3x – 5y = 20

6x – 10y = 40

$$Thus, \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$$

Infinitely many solution

4. Question

In each of the following systems of equations determine whether the system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it:

$x - 2y = 8$

$5x - 10y = 10$

Answer

a1x + b1y + c1 = 0

a2x + b2x + c2 = 0

if $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$, infinite solution

$if \frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, no solution

If $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$,unique solution

$x - 2y = 8$

$5x - 10y = 10$

$$Thus, \frac{1}{5} = \frac{1}{5} \neq \frac{4}{5}$$

No solution.

5. Question

Find the value of k for which the following system of equations has a unique solution:

$kx + 2y = 5$

$3x + y = 1$

Answer

Kx + 2y = 5

3x + y = 1

Multiplying eq2 by 2 and subtracting from eq1

(k – 6)x = 3

⇒ x = 3/(k – 6)

Thus, for k ≠ 6 the system has unique solution

6. Question

Find the value of k for which the following system of equations has a unique solution:

$4x + ky + 8 = 0$

$2x + 2y + 2 = 0$

Answer

$4x + ky + 8 = 0$

$2x + 2y + 2 = 0$

Multiplying eq2 by 2 and subtracting from eq1

⇒ (k − 4)y + 6 = 0

⇒ y = 6/(4 − k)

Thus, for k ≠ 4 it has a unique solution

7. Question

Find the value of k for which the following system of equations has a unique solution:

$4x - 5y = k$

$2x - 3y = 12$

Answer

Multiplying eq2 by 2 and subtracting from eq1

⇒ -5y + 6y = k − 24

⇒ y = k − 24

Thus, for any value of k it has a unique solution

8. Question

Find the value of k for which the following system of equations has a unique solution:

$x + 2y = 3$

$5x + ky + 7 = 0$

Answer

$x + 2y = 3$

$5x + ky + 7 = 0$

Multiplying eq1 by 5 and subtracting from eq2

⇒ (k − 10)y = -22

Thus for all k except 10 it has a unique solution

9. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$2x + 3y - 5 = 0$

$6x + ky - 15 = 0$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$2x + 3y - 5 = 0$

$6x + ky - 15 = 0$

$$\Rightarrow \frac{2}{6} = \frac{3}{k} = \frac{5}{15}$$

⇒ k = 3 × 3 = 9

10. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$4x + 5y = 3$

$kx + 15y = 9$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

4x + 5y = 3

Kx + 15y = 9

$$\Rightarrow \frac{4}{k} = \frac{5}{15} = \frac{3}{9}$$

⇒ k = 4 × 3 = 12

11. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$kx - 2y + 6 = 0$

$4x - 3y + 9 = 0$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

kx – 2y + 6 = 0

4x – 3y + 9 = 0

$$\frac{k}{4} = \frac{2}{3} = \frac{6}{9}$$

⇒ k = 4 × 2/3 = 8/3

12. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$8x + 5y = 9$

$kx + 10y = 18$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

8x + 5y = 9

Kx + 10y = 18

$$\frac{8}{k} = \frac{5}{10} = \frac{9}{18}$$

⇒ k = 2 × 8 = 16

13. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$2x - 3y = 7$

$(k+2)x - (2k+1)y = 3(2k-1)$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

2x − 3y = 7

(k + 2)x − (2k + 1)y = 3(2k − 1)

$$\frac{2}{k+2} = \frac{3}{2k+1} = \frac{7}{3(2k-1)}$$

⇒ 4k + 2 = 3k + 6

⇒ k = 4

14. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$2x + 3y = 2$

(k+1)x + 9y = k + 1

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

2x + 3y = 2

(k + 1)x + 9y = k + 1

$$\Rightarrow \frac{2}{k+1} = \frac{3}{9} = \frac{2}{k+1}$$

⇒ k + 1 = 6

⇒ k = 5

15. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$x + (k + 1)y = 4$

$(k + 1)x + 9y = 5k + 2$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

x + (k + 1)y = 4

(k + 1)x + 9y = 5k + 2

$$\frac{1}{k+1} = \frac{k+1}{9} = \frac{4}{5k+2}$$

⇒ 5k + 2 = 4k + 4

⇒ k = 2

16. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$kx + 3y = 2k + 1$

$2(k + 1)x + 9y = 7k + 1$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

Kx + 3y = 2k + 1

2(k + 1) + 9y = 7k + 1

$$\frac{k}{2(k+1)} = \frac{3}{9} = \frac{2k+1}{7k+1}$$

⇒ 9k = 6k + 6

⇒ 3k = 6

⇒ k = 2

17. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$2x + (k - 2)y = k$

$6x + (2k - 1)y = 2k + 5$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

2x + (k − 2)y = k

6x + (2k − 1y = 2k + 5

$$\frac{2}{6} = \frac{k-2}{2k-1} = \frac{k}{2k+5}$$

⇒ 4k – 2 = 6k – 12

⇒ 2k = 10

⇒ k = 5

18. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$2x + 3y = 7$

$(k+1)x + (2k-1)y = 4k+1$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$2x + 3y = 7$

$(k+1)x + (2k-1)y = 4k+1$

$$\frac{2}{k+1} = \frac{3}{2k-1} = \frac{7}{4k+1}$$

⇒ 4k – 2 = 3k + 3

⇒ k = 5

19. Question

Find the value of k for which each of the following systems of equations have infinitely many solution:

$2x + 3y = k$

$(k-1)x + (k+2)y = 3k$

Answer

Infinitely many solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$2x + 3y = k$

$(k-1)x + (k+2)y = 3k$

$$\frac{2}{k-1} = \frac{3}{2+k} = \frac{k}{3k}$$

⇒ 4 + 2k = 3k – 3

⇒ k = 7

20. Question

Find the value of k for which the following system of equations has no solution:

$kx - 5y = 2$

$6x + 2y = 7$

Answer

No solution will be when

$$\frac{a1}{a2}=\frac{b1}{b2}\neq\frac{c1}{c2}$$

$kx-5y=2$

$6x+2y=7$

$$\frac{k}{6}=-\frac{5}{2}$$

⇒ k = -15

21. Question

Find the value of k for which the following system of equations has no solution:

$x+2y=0$

$2x+ky=5$

Answer

No solution will be when

$$\frac{a1}{a2}=\frac{b1}{b2}\neq\frac{c1}{c2}$$

$x+2y=0$

$2x+ky=5$

$$\frac{1}{2}=\frac{2}{k}$$

⇒ k = 4

22. Question

Find the value of k for which the following system of equations has no solution:

$3x-4y+7=0$

$kx+3y-5=0$

Answer

No solution will be when

$$\frac{a1}{a2}=\frac{b1}{b2}\neq\frac{c1}{c2}$$

$3x-4y+7=0$

$kx+3y-5=0$

$$\frac{3}{k}=-\frac{4}{3}$$

⇒ k = -9/4

23. Question

Find the value of k for which the following system of equations has no solution:

$2x-ky+3=0$

$3x+2y-1=0$

Answer

No solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$2x - ky + 3 = 0$

$3x + 2y - 1 = 0$

$$\frac{2}{3} = -\frac{k}{2}$$

⇒ k = -4/3

24. Question

Find the value of k for which the following system of equations has no solution:

$2x + ky = 11$

$5x - 7y = 5$

Answer

No solution will be when

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$2x + ky = 11$

$5x - 7y = 5$

$$\frac{2}{5} = -\frac{k}{7}$$

⇒ k = -14/5

25. Question

Find the value of k for which the following system of equations has no solution:

$kx + 3y = 3$

$12x + ky = 6$

Answer

No solution will be when

a1/b1 = b1/b2 ≠ C1/C2

$kx + 3y = 3$

$12x + ky = 6$

k/12 = 3/k

⇒ k = ± 6

26. Question

For what value of k the following system of equations will be inconsistent?

$4x + 6y = 11$
$2x + ky = 7$

Answer

System will be inconsistent will be when

$$\frac{a1}{a2}=\frac{b1}{b2}\neq\frac{c1}{c2}$$

$$4x+6y=11$$
$$2x+ky=7$$

$$\frac{4}{2}=\frac{6}{k}\neq\frac{11}{7}$$

$$\frac{4}{2}=\frac{6}{k}$$

⇒ k = 3

27. Question

For what value of a, the system of equations is inconsistent

ax + 3y = a - 3

12x + ay = a

Answer

System will be inconsistent will be when

a1/a2 = b1/b2 ≠ c1/c2

ax + 3y = a - 3

12x + ay = a

a/12 = 3/1 ≠ a-3/a

⇒ a = ± 6

But +6 doesn't satisfy

a/12 = 3/1 ≠ 1-3/a

Thus, a = - 6

28. Question

Find the value of k for which the system

$$kx+2y=5$$
$$3x+y=1$$

has (i) a unique solution, and (ii) no solution.

Answer

$$kx+2y=5$$
$$3x+y=1$$

i) For unique solution:

$$\frac{a1}{a2}\neq\frac{b1}{b2}\neq\frac{c1}{c2}$$

$$\Rightarrow\frac{k}{3}\neq\frac{2}{1}$$

⇒ k ≠ 6

ii) For no solution

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$$\Rightarrow \frac{k}{3} = \frac{2}{1}$$

⇒ k = 6

29. Question

Prove that there is a value of c (≠ 0) for which the system

$$6x + 3y = c - 3$$
$$12x + cy = c$$

Has infinitely many solutions. Find this value.

Answer

For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$$6x + 3y = c - 3$$
$$12x + cy = c$$

$$\frac{6}{12} = \frac{3}{c}$$

⇒ c = 6

30. Question

Find the values of k for which the system

$$2x + ky = 1$$
$$3x - 5y = 7$$

Will have (i) a unique solution and (ii) no solution. Is there a value of k for which the system has infinitely many solutions?

Answer

$$2x + ky = 1$$
$$3x - 5y = 7$$

i) For unique solution:

$$\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$$

$$\frac{2}{3} \neq \frac{k}{-5}$$

⇒ k ≠ -10/3

ii) For no solution

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$$\frac{2}{3} = \frac{k}{-5}$$

⇒ k = -10/3

For infinite solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$$\Rightarrow \frac{2}{3} = \frac{k}{-5} \neq \frac{1}{7}$$

Thus, for no value of k will the system have infinite solution

31. Question

For what value of k, the following system of equations will represent the coincident lines?

$$x + 2y + 7 = 0$$
$$2x + ky + 14 = 0$$

Answer

For coincident lines

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$$x + 2y + 7 = 0$$
$$2x + ky + 14 = 0$$

$$\frac{1}{2} = \frac{2}{k} = \frac{7}{14}$$

⇒ k = 4

32. Question

Obtain the condition for the following system of linear equations to have a unique solution

$$ax + by = c$$
$$lx + my = n$$

Answer

For unique solution:

$$\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$$ **Given**

$$ax + by = c$$
$$lx + my = n$$

$$\frac{a}{l} \neq \frac{b}{m}$$

⇒ am ≠ lbHence the system of given linear equations will have unique solution when am ≠ lb.

33. Question

Determine the values of a and b so that the following system of linear equations have infinitely many solutions:

$$(2a - 1)x + 3y - 5 = 0$$

3x + (b − 1)y − 2 = 0

Answer

For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$(2a - 1)x + 3y - 5 = 0$

3x + (b − 1)y − 2 = 0

$$\frac{2a - 1}{3} = \frac{3}{b - 1} = \frac{5}{2}$$

⇒ 2a – 1 = 15/2

⇒ a = 17/4

6 = 5b – 5

⇒ b = 11/5

34. Question

Find the values of a and b for which the following system of linear equations has infinite number of solutions:

$2x - 3y = 7$

$(a + b)x - (a + b - 3)y = 4a + b$

Answer

For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$2x - 3y = 7$

$(a + b)x - (a + b - 3)y = 4a + b$

$$\frac{2}{a + b} = \frac{3}{a + b - 3} = \frac{7}{4a + b}$$

⇒ 2a + 2b − 6 = 3a + 3b and 12a + 3b = 7a + 7b − 21

⇒ a + b = -6 ----- (1) and 5a – 4b = - 21 ------- (2)

Multiplying eq1 by 4 and adding to eq2

⇒ 9a = -45

⇒ a = -5

Thus, b = -1

35. Question

Find the values of p and q for which the following system of linear equations has infinite number of solutions:

$2x + 3y = 9$

$(p + q)x - (2p - q)y = 3(p + q + 1)$

Answer

For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$2x + 3y = 9$

$(p + q)x - (2p - q)y = 3(p + q + 1)$

$$\frac{2}{p+q} = \frac{3}{-(2p-q)} = \frac{9}{p+q+1}$$

⇒ -4p + 2q = 3p + 3q and 2p + 2q + 2 = 9p + 9q

⇒ q = -7p and 7p + 7q = 2

⇒ 7q − q = 2

⇒ q = 1/3

Thus, p = -1/21

36. Question

Find the values of a and b for which the following system of equations has infinitely many solutions:

(i) $(2a-1)x - 3y = 5$

$3x + (b-2)y = 3$

(ii) $2x - (2a+5y)y = 5$

$(2b+1)x - 9y = 15$

(iii) $(a-1)x + 3y = 2$

$6x + (1-2b)y = 6$

(iv) $3x + 4y = 12$

$(a+b)x + 2(a-b)y = 5a-1$

(v) $2x + 3y = 7$

$(a-b)x + (a+b)y = 3a+b-2$

(vi) $2x + 3y - 7 = 0$

$(a-1)x + (a+1)y = (3a-1)$

(vii) $2x + 3y = 7$

$(a-1)x + (a+2)y = 3a$

Answer

For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$$\frac{2a-1}{3} = -\frac{3}{b-2} = \frac{5}{3}$$

⇒ 6a − 3 = 15 and -9 = 5b − 10

⇒ a = 3 and b = 1/5

(ii) For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$$\frac{2}{2b+1} = -\frac{2a+5}{-9} = \frac{5}{15}$$

$$\frac{2}{2b+1} = \frac{2a+5}{9} = \frac{1}{3}$$

⇒ 6 = 2b + 1 and 6a + 15 = 9

⇒ b = 5/2 and a = -1

(iii) For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$$\frac{a-1}{6} = \frac{3}{1-2b} = \frac{2}{6}$$

$$\Rightarrow \frac{a-1}{6} = \frac{3}{1-2b} = \frac{1}{3}$$

⇒ 3a – 3 = 6 and 9 = 1 – 2b

⇒ a = 3 and b = -4

(iv) For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$$\frac{3}{a+b} = \frac{4}{2(a-b)} = \frac{12}{5a-1}$$

⇒ 15a – 3 = 12a + 12b and 20a – 4 = 24a – 24b

⇒ 3a – 12b = 3 ------ (1) and 6b – a = 1 ------ (2)

Multiplying eq2 by 3 and adding to eq1

⇒ 6b = 6

⇒ b = 1

Thus, 3a – 12 = 3

⇒ a = 5

(v) For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$$\frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

⇒ 2a + 2b = 3a – 3b and 6a + 2b – 4 = 7a – 7b

⇒ a = 5b and 9b – a = 4

Thus, 9b – 5b = 4

⇒ b = 1

⇒ a = 5

(vi) For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$2x + 3y - 7 = 0$

$(a-1)x + (a+1)y = (3a-1)$

$$\frac{2}{a-1} = \frac{3}{a+1} = -\frac{7}{-(3a-1)}$$

⇒ 2a + 2 = 3a – 3

⇒ a = 5

(vii) For infinitely many solution

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

$$\Rightarrow \frac{2}{a-1} = \frac{3}{a+2} = \frac{7}{3a}$$

⇒ 2a + 4 = 3a – 3

⇒ a = 7

Exercise 3.6

1. Question

5 pens and 6 pencils together cost Rs 9 and 3 pens and 2 pencils cost Rs 5. Find the cost of 1 pen and 1 pencil.

Answer

Let the cost of 1 pen be ‘a’ and the cost of 1 pencil be ‘b’.

Given, 5 pens and 6 pencils together cost Rs 9.

5a + 6b = 9 --------- (1)

Given, 3 pens and 2 pencils cost Rs 5.

3a + 2b = 5 -------- (2)

Multiplying eq2 by 3 and subtracting eq1 from it.3(3a + 2b) - (5a + 6b) = 5(3) - 9⇒ 9a + 6b – 5a – 6b = 15 – 9

⇒ 4a = 6

⇒ a = 3/2

Substituting value of 'a' in (1)

⇒ 15/2 + 6b = 9

⇒ 6b = 3/2

⇒ b = 1/4

2. Question

7 audio cassettes and 3 video cassettes cost Rs 1110, while 5 audio cassettes and 4 video cassettes cost Rs 1350. Find the cost of an audio cassette and a video cassette.

Answer

Let the cost of an audio cassette be ‘a’ and a video cassette be ‘b’.

Given, 7 audio cassettes and 3 video cassettes cost Rs 1110.

⇒ 7a + 3b = 1110 -------- (1)

Also, 5 audio cassettes and 4 video cassettes cost Rs 1350.

⇒ 5a + 4b = 1350 -------- (2)

Multiplying eq1 by 4 and eq2 by 3 and subtracting eq2 from eq1.

⇒ 28a + 12b – 15a – 12b = 4440 – 4050

⇒ $13a = 390$

⇒ $a =$ Rs. 30

Substituting value of a in eq1

⇒ $7 \times 30 + 3b = 1110$

⇒ $3b = 900$

⇒ $b =$ Rs. 300

3. Question

Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then number of pencils would become 4 times the number of pens. Find the original number of pens and pencils.

Answer

Let the number of pens be ‘a’ and number of pencils be ‘b’.

Given, Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then number of pencils would become 4 times the number of pens.

⇒ $a + b = 40$ ------ (1)

Also, $b + 5 = 4(a - 5)$

⇒ $b = 4a - 25$ ------- (2)

Subtracting eq2 from eq1

⇒ $a + b - b = 40 - 4a + 25$

⇒ $5a = 65$

⇒ $a = 13$

Substituting the value of 'a' in equation (1), we get⇒ $13 + b = 40$⇒ $b = 27$

Hence, Reena has 13 pens and 27 pencils.

4. Question

4 tables and 3 chairs, together, cost Rs 2,250 and 3 tables and 4 chairs cost Rs 1950. Find the cost of 2 chairs and 1 table.

Answer

Let the cost of 1 table be ‘a’ and cost of 1 chair be ‘b’.

Given, 4 tables and 3 chairs, together, cost Rs 2,250 and 3 tables and 4 chairs cost Rs 1950.

⇒ $4a + 3b = 2250$ -------- (1)

Also, $3a + 4b = 1950$ --------- (2)

Multiplying eq1 by 3 and eq2 by 4 and subtracting eq2 from eq1.

⇒ $12a + 9b - 12a - 16b = 6750 - 7800$

⇒ $7b = 1050$

⇒ $b = 150$

Substituting ‘b’ in eq1

⇒ $4a + 450 = 2250$

⇒ $a =$ Rs. 450

Cost of 2 chair and 1 table = $2b + a =$ Rs. 750

5. Question

3 bags and 4 pens together cost Rs 257 whereas 4 bags and 3 pens together cost Rs 324. Find the total cost of 1 bag and 10 pens.

Answer

Let the cost of 1 bag be 'a' and 1 pen be 'b'.

Given, 3 bags and 4 pens together cost Rs 257 whereas 4 bags and 3 pens together cost Rs 324.

⇒ $3a + 4b = 257$ ----------- (1) and $4a + 3b = 324$ ---------- (2)

Multiplying eq1 by 4 and eq2 by 3 and subtracting eq2 from eq1.

⇒ $12a + 16b – 12a – 9b = 1028 – 972$

⇒ $7b = 56$

⇒ b = Rs. 8

Substituting in eq1

⇒ $3a + 32 = 257$

⇒ a = Rs. 75

Cost of 1 bag and 10 pens = $75 + 10 \times 8$ = Rs. 155

6. Question

5 books and 7 pens together cost Rs 79 whereas 7 books and 5 pens together cost Rs 77. Find the total cost of 1 book and 2 pens.

Answer

Let the cost of 1 book be 'a' and cost of 1 pen be 'b'.

Given, 5 books and 7 pens together cost Rs 79 whereas 7 books and 5 pens together cost Rs 77.

⇒ $5a + 7b = 79$ ------ (1) and $7a + 5b = 77$

Multiplying eq1 by 7 and eq2 by 5 and subtracting eq2 from eq1.

⇒ $35a + 49b – 35a – 25b = 553 – 385$

⇒ $24b = 168$

⇒ b = Rs. 7

Substituting value of b in eq1, we get

⇒ $5a + 49 = 79$

⇒ a = Rs. 6

Cost of 1 book and 2 pens = $6 + 2 \times 7$ = Rs. 20

7. Question

A and B each have a certain number of mangoes. A says to B, "if you give 30 of your mangoes, I will have twice as many as left with you." B replies, "if you give me 10, I will have thrice as many as left with you." How many mangoes does each have?

Answer

Let the number of mangoes A has be 'a' and number of mangoes B has be 'b'.

Given, . A says to B, "if you give 30 of your mangoes, I will have twice as many as left with you." B replies, "if you give me 10, I will have thrice as many as left with you"

⇒ $a + 30 = 2(b – 30)$

⇒ $a = 2b – 90$ --------- (1)

Also, b + 10 = 3(a – 10)

⇒ b = 3a – 40 ---------- (2)

Substituting value of a from eq1 in eq2

⇒ b = 6b – 270 – 40

⇒ 5b = 310

⇒ b = Rs. 62

Substituting value of b in eq1

⇒ a = 124 – 90

⇒ a = Rs. 34

8. Question

On selling a T.V. at 5% gain and a fridge at 10% gain, a shopkeeper gains Rs 2000. But if he sells the T.V. at 10% gain and the fridge at 5% loss. He gains Rs 1500 on the transaction. Find the actual prices of T.V. and fridge.

Answer

Let the actual price of Tv be 'a' and actual price of fridge be 'b'.

Given, on selling a T.V. at 5% gain and a fridge at 10% gain, a shopkeeper gains Rs 2000.

∴ 5% of a + 10% of b = 2000

$$\Rightarrow \frac{5}{100}a + \frac{10}{100}b = 2000$$

⇒ 5a + 10b = 200000 ----------- (1)

Also, if he sells the T.V. at 10% gain and the fridge at 5% loss.

∴ 10% of a – 5% of b = 1500

$$\Rightarrow \frac{10}{100}a - \frac{5}{100}b = 1500$$

⇒ 10a – 5b = 150000 -------- (2)

Multiplying eq2 by 2 and adding eq1 to it

⇒ 5a + 10b + 20a – 10b = 200000 + 300000

⇒ 25a = 500000

⇒ a = Rs. 20000

Thus, 5 × 20000 + 10b = 200000

⇒ b = Rs. 10000

9. Question

The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, he buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

Answer

Let the cost of each bat be 'a' and each ball be 'b'.

Given, coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, he buys 3 bats and 5 balls for Rs 1750

⇒ 7a + 6b = 3800 ---- (1) and

3a + 5b = 1750 ---- (2)

Multiplying eq1 by 3 and eq2 by 7 and subtracting eq2 from eq1.

⇒ 3(7a + 6b) - 7(3a + 5b) = 3(3800) - 7(1750)

⇒ 21a + 18b – 21a – 35b = 11400 – 12250

⇒ -17b = -850

⇒ 17b = 850

⇒ b = 50Putting this in eq 1, we get7a + 6(50) = 3800⇒ 7a + 300 = 3800⇒ 7a = 3500⇒ a = 500Hence, Each bat cost a = 500 Rupees and Each ball costs b = 50 Rupees.

10. Question

One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you".Tell me what is the amount of their respective capital?

Answer

Given: One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you".

To find: the amount of their respective capital.

Solution: Let the capital of two friends be 'a'and 'b' respectively.

Given, one says, "Give me a hundred, friend! I shall then become twice as rich as you.It means if one is giving Rs. 100 other is losing the Rs. 100.Since a is gaining 100 and b is losing 100.⇒ a + 100 = 2(b – 100)

⇒ a + 100 = 2b – 200

⇒ a - 2b = -200 - 100⇒ a - 2b = -300 ...... (1)In another condition 2nd friend replies, "Give me a ten, I shall be six times as rich as you".It means if one person is gaining 10 other person is losing 10.

Here b is gaining Rs 10 and a is losing Rs 10.⇒ b + 10 = 6(a – 10)⇒ b + 10 = 6a – 60⇒ 6a - b = -60-10⇒ 6a - b = -70 ...... (2)Now solve equations (1) and (2) to get the amount a and b.Multiply eq. (1) with 6 and subtract we. (2) from it.⇒6(a - 2b)-(6a - b) = 6 (-300) -70⇒6a - 12b-6a+b = -1800 -70⇒ -11b= -1870⇒ b = 170put the value of b in the eq.(1) to get value of a,⇒a - 2(170) = -300⇒a - 340 = -300 ⇒ a = -300 + 340⇒ a= 40

Hence the amount of capital of two friends is Rs 40 and Rs 170.

11. Question

A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Answer

Let the fixed charge and the charge for each extra day be 'a'and 'b' respectively.

Given, a lending library has a fixed charge for the first three days and an additional charge for each day thereafter

Saritha paid Rs 27 for a book kept for seven days⇒ a + 4b = 27 -------- (1)

Susy paid Rs 21 for the book she kept for five days⇒ a + 2b = 21 --------- (2)

Subtracting eq2 from eq1

⇒ 2b = 6

⇒ b = 3

Putting this value in eq(1), we get⇒ a + 4(3) = 27⇒ a = 27 - 12⇒ a = 15 Therefore, fixed charge, a = 15 Rupees and charge thereafter, b = 3 Rupees per day

⇒ a = Rs. 15

Exercise 3.7

1. Question

The sum of two numbers is 8. If their sum is four times their difference, find the number.

Answer

Let the numbers be 'a' and 'b'.

Given, sum of two numbers is 8. If their sum is four times their difference, find the number.

⇒ a + b = 8 ------ (1)

Also,

a + b = 4(a – b)

⇒ a + b = 4a – 4b

⇒ a = 5b/3

Substituting value of a in eq1

⇒ 5b/3 + b = 8

⇒ 8b/3 = 8

⇒ b = 3

Thus, a = 5

2. Question

The sum of digits of a two digit number is 13. If the number is subtracted from the one obtained by interchanging the digits, the result is 45. What is the number?

Answer

Let the one's digit be 'a' and ten's digit be 'b'

Given, sum of digits of a two digit number is 13. If the number is subtracted from the one obtained by interchanging the digits, the result is 45.

⇒ a + b = 13 ----- (1) and,

10a + b – (10b + a) = 45

⇒ 9a – 9b = 45

⇒ a – b = 5 ---------- (2)

Adding (1) and (2)

⇒ 2a = 18

⇒ a = 9

Thus, b = 4

Number is 10b + a

⇒ Number = 49

3. Question

A number consists of two digits whose sum is five. When the digits are reversed, the number becomes greater by nine. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'

Given, number consists of two digits whose sum is five. When the digits are reversed, the number becomes greater by nine.

⇒ a+ b = 5 ----- (1) and

10a + b − (10b + a) = 9

⇒ a − b = 1 ------ (2)

Adding (1) and (2)

Thus, 2a = 6

⇒ a = 3

∴ b = 2

Number is 23.

4. Question

The sum of digits of a two digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number.

Answer

Let the unit’s digit be a and tens digit be b.

Number = 10b + a

Reverse of the number = 10a + b

Given, sum of digits of a two digit number is 15.

⇒ a + b = 15 ------ (1)

Also, the number obtained by reversing the order of digits of the given number exceeds the given number by 9.

⇒ 10a + b − 10b − a = 9

⇒ a − b = 1 ------- (2)

Solving (1) and (2), we get

⇒ a = 8 and b = 7

The number is 78.

5. Question

The sum of two-digit number and the number formed by reversing the order of digits is 66. If the two digits differ by 2, find the number. How many such numbers are there?

Answer

Given : The sum of two-digit number and the number formed by reversing the order of digits is 66 and the two digits differ by 2.

To find: the number. How many such numbers are there?

Solution: Let the one’s digit be ‘a’ and ten’s digit be ‘b’.The 2 digit number is formed as (10×number on tens' digit + number on one's digit)So the number is 10 b + a,If we reverse the digits one's digit will be 'b' and tens' digit will be 'a'.So number after reversing will become 10 a + b.

As it is given that sum of two-digit number and the number formed by reversing the order of digits is 66.

⇒ 10 b + a + 10 a + b = 66

⇒ 11 b + 11 a = 66

⇒ a + b = 6 .... (1)

Also, digits differ by 2.

⇒ a − b = 2 ...... (2)or b − a = 2 ..... (3)

Adding eq 1 and 2 we geta + b + a - b = 6 + 2⇒ 2a = 8⇒ a = 4Putting value of a in 1 we get,4 + b = 6 ⇒ b = 6 - 4⇒ b = 2Adding eq 1 and 3 we geta + b + b - a = 6 + 2⇒ 2b = 8

⇒ b = 4Putting value of a in 1 we get,a + 4 = 6 ⇒ a = 6 - 4⇒ a = 2

Thus, a = 4 and b = 2 or a = 2 and b = 4Now 2 digit number is 10 b + aFor a = 4 and b = 2 The 2 digit number is 10 (2) + 4 = 20 + 4 = 24For a = 2 and b = 4The 2 digit number is 10 (4) + 2 = 40 + 2 = 42

Hence Numbers can be 24 or 42

6. Question

The sum of two numbers is 1000 and the difference between their squares is 256000. Find the numbers.

Answer

Let the numbers be 'a' and 'b'

Given, of two numbers is 1000 and the difference between their squares is 256000

⇒ a + b = 1000 ---- (1) and

⇒ $a^2 - b^2 = 25600$

⇒ (a + b)(a − b) = 25600 [From 1]

⇒ 1000(a - b) = 256000

⇒ a − b = 256 ----- (2)

Adding (1) and (2)

⇒ 2a = 1256

⇒ a = 628

From (1) 628 + b = 1000⇒ b = 1000 - 628 = 372

7. Question

The sum of a two digit number and the number obtained by reversing the order of its digits is 99. If the digits differ by 3, find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'

Given, sum of two-digit number and the number formed by reversing the order of digits is 99.

⇒ 10a + b + 10b + a = 99

⇒ a + b = 9

Also, digits differ by 3.

⇒ a − b = 3 or b − a = 3

Adding both equation

2a = 12 or 2b = 12

⇒ a = 6 and b = 3 or a = 3 and b = 6

Number can be 36 or 63

8. Question

A two-digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'

Given, number is 4 times the sum of its digits.

10b + a = 4(a + b)

⇒ a = 2b -----(1)

Also, if 18 is added to the number, the digits are reversed.

⇒ 10b + a + 18 = 10a + b

⇒ a – b = 2 ----- (2)

Substituting a from eq1 in eq2

⇒ b = 2

Thus, a = 4

Number is 24

9. Question

A two-digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'

Then, the number is = 10b + a

Given, the number is 3 more than 4 times the sum of its digits.

⇒ 10b + a = 4(a + b) + 3

⇒ 10b + a = 4a + 4b + 3

⇒ 3a = 6b - 3

⇒ a = 2b - 1 -----(1)

Also, if 18 is added to the number, the digits are reversed.

⇒ 10b + a + 18 = 10a + b

⇒ 9a - 9b = 18

⇒ a – b = 2using (1)

⇒ 2b - 1 - b = 2⇒ b = 3

Thus, again from (1)a = 6 – 1 = 5

Hence, Number is 35.

10. Question

A two-digit number is 4 more than 6 times the sum of its digits. If 18 is subtracted from the number, the digits are reversed. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'

Given, two-digit number is 4 more than 6 times the sum of its digits.

⇒ 10b + a = 6(a + b) + 4

⇒ 4b = 5a + 4 ------- (1)

Also, if 18 is subtracted from the number, the digits are reversed.

⇒ 10b + a – 18 = 10a + b

⇒ b – a = 2 ------- (2)

Multiplying eq2by 4 and subtracting from eq1

⇒ 4b – 5a – 4b + 4a = 4 – 8

⇒ a = 4

Thus, b = 6

Number is 64

11. Question

A two-digit number is 4 times the sum of its digits and twice the product of the digits. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'.

Given, two digit number is 4 times the sum of its digits and twice the product of the digits

⇒ 10b + a = 4(a + b)

⇒ a = 2b

Also, 10b + a = 2ab

Substituting value of a.

⇒ 10b + 2b = 2 × 2b × b

⇒ b = 3

Thus, a = 6

Number is 36

12. Question

A two-digit number is such that the product of its digits is 20. If 9 is added to the number, the digits interchange their places. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'.

Given, two-digit number is such that the product of its digits is 20.

⇒ ab = 20 ----- (1)

Also, if 9 is added to the number, the digits interchange their places.

⇒ 10b + a + 9 = 10a + b

⇒ a – b = 1 ----- (2)

Substituting value of a from eq1 in to eq2

⇒ 20/b – b = 1

⇒ b^2 + b – 20 = 0

⇒ (b + 5)(b – 4) = 0

Thus, b = 4 and a = b + 1 = 5

Number is 45.

13. Question

The difference between two numbers is 26 and one number is three times the other. Find them.

Answer

Let the numbers be a and b.

Given, difference between two numbers is 26 and one number is three times the other.

⇒ a − b = 26 [1] and a = 3b [2]From [1] and [2], we get

⇒ 3b - b = 26

⇒ 2b = 26

⇒ b = 13

Thus, a = 3b = 3(13) = 39

14. Question

The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Answer

Let the one's digit be 'a' and ten's digit be 'b'.therefore no is = 10b + aReversed no = 10a + b

Given, Sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits.

⇒ a + b = 9 ----- (1)

And 9(10b + a) = 2(10a + b)

⇒ 90b + 9a = 20a + 2b

⇒ 88b = 11a

⇒ a = 8b

Substituting value of a in eq1

⇒ 8b + b = 9

⇒ 9b = 9

⇒ b = 1

Thus, a = 8(1) = 9

Hence, no is 10(1) + 8 = 18

15. Question

Seven times a two-digit number is equal to four times the number obtained by reversing the digits. If the difference between the digits is 3. Find the number.

Answer

Given: Seven times a two-digit number is equal to four times the number obtained by reversing the digits and the difference between the digits is 3.

To find: the number.

Solution: Let the one's digit be 'a' and ten's digit be 'b'.The the number will be 10b+aIf the digits are reversed the number will be 10a+b

Given, seven times a two-digit number is equal to four times the number obtained by reversing the digits.

⇒ 7(10b + a) = 4(10a + b)⇒ 7(10b) + 7a = 4(10a)+ 4b⇒ 70b + 7a = 40a + 4b⇒ 70b - 4b = 40a - 7a

⇒ 66b = 33a

⇒ a = 2b ----- (1)

Also, difference between the digits is 3

⇒ a − b = 3

Substitute value of a from (1)

⇒ 2b − b = 3

Thus, b = 3

⇒ a = 2 × 3 = 6

The number is 10b+a = 10(3)+6 = 36

∴ Number is 36.

Exercise 3.8

1. Question

The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.

Answer

Let the numerator be a and denominator be b.

Given, numerator of a fraction is 4 less than the denominator.

⇒ a = b − 4 --------- (1)

Also, if the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator.

⇒ b + 1 = 8(a − 2)

⇒ b = 8a − 17 -------- (2)

Substituting value of b from (2) in (1).

⇒ a = 8a − 17 − 4

⇒ a = 3

Thus, b = 3 + 4 = 7

Fraction is 3/7.

2. Question

A fraction becomes 9/11 if 2 is added to both numerator and the denominator, it becomes 5/7 if 2 is subtracted from both numerator and the denominator. Find the fraction.

Answer

Let the numerator be 'a' and denominator be 'b'

Given, fraction becomes 9/11 if 2 is added to both numerator and the denominator, it becomes 5/7 if 2 is subtracted from both numerator and the denominator

$$\frac{\Rightarrow (a+2)}{b+2} = \frac{9}{11}$$

⇒ 11a + 22 = 9b + 18

⇒ 11a = 9b − 4 ------ (1)

Also, $\frac{a-2}{b-2} = \frac{5}{7}$

⇒ 7a − 14 = 5b − 10

⇒ 7a – 5b = 4 ------- (2)

Multiplying eq1 by 7 and eq2 by 11 and subtracting eq2 from eq1

⇒ 77a – 63b – 77a + 55b = -28 – 44

⇒ 8b = 72

⇒ b = 9

Thus, 7a – 45 = 4

⇒ a = 7

Fraction is 7/9.

3. Question

A fraction becomes 1/3 if 1 is subtracted from both its numerator and denominator. If 1 is added to both the numerator and denominator, it becomes 1/2. Find the fraction.

Answer

Let the numerator be 'a' and denominator be 'b'

Given, fraction becomes 1/3 if 1 is subtracted from both its numerator and denominator. If 1 is added to both the numerator and denominator, it becomes 1/2.

$$\Rightarrow \frac{a-1}{b-1} = \frac{1}{3}$$

⇒ 3a – 3 = b – 1

⇒ b = 3a – 2 ------ (1)

Also, if 1 is added to both the numerator and denominator, it becomes 1/2.

$$\Rightarrow \frac{a+1}{b+1} = \frac{1}{2}$$

⇒ b = 2a + 1 ---------- (2)

Equating (1) and (2)

⇒ 3a – 2 = 2a + 1

⇒ a = 3

Thus, b = 7

Fraction is 3/7

4. Question

If we add 1 to the numerator and subtract 1 from the denominator, a fraction becomes 1. It also becomes 1/2 if we only add 1 to the denominator. What is the fraction?

Answer

Let the numerator be 'a' and denominator be 'b'

Given, if add 1 to the numerator and subtract 1 from the denominator, a fraction becomes 1.

$$\Rightarrow \frac{a+1}{b-1} = 1$$

⇒ a = b – 2 -------- (1)

Also, it becomes 1/2 if we only add 1 to the denominator.

$$\Rightarrow \frac{a}{b+1} = \frac{1}{2}$$

⇒ 2a = b + 1 --------- (2)

Subtracting eq 1 from eq2

⇒ 2a – a = b + 1 – b + 2

⇒ a = 3

Thus, b = 5

Fraction is 3/5

5. Question

If the numerator of a fraction is multiplied by 2 and the denominator is reduced by 5 the fraction becomes 6/5. And, if the denominator is doubled and the numerator is increased by 8, the fraction becomes 2/5. Find the fraction.

Answer

Let the numerator be 'a' and denominator be 'b'

Given, if the numerator of a fraction is multiplied by 2 and the denominator is reduced by 5 the fraction becomes 6/5

$$\Rightarrow \frac{2a}{b-5} = \frac{6}{5}$$

⇒ 10a = 6b – 30

⇒ 5a = 3b – 15 ---------- (1)

Also, if the denominator is doubled and the numerator is increased by 8, the fraction becomes 2/5

$$\Rightarrow \frac{a+8}{2b} = \frac{2}{5}$$

⇒ 5a + 40 = 4b -------- (2)

Subtracting eq2 from eq1

⇒ 5a – 5a – 40 = 3b – 15 – 4b

⇒ b = 25

Thus, 5a = 75 – 15

⇒ a = 12

Fraction is 12/25

6. Question

When 3 is added to the denominator and 2 is subtracted from the numerator a fraction becomes 1/4. And, when 6 is added to numerator and the denominator is multiplied by 3, it becomes 2/3. Find the fraction.

Answer

Let the numerator be 'a' and denominator be 'b'

Given, 3 is added to the denominator and 2 is subtracted from the numerator a fraction becomes 1/4.

$$\Rightarrow \frac{a-2}{b+3} = \frac{1}{4}$$

⇒ 4a – 8 = b + 3

⇒ 4a – b = 11 ------ (1)

Also, when 6 is added to numerator and the denominator is multiplied by 3, it becomes 2/3.

$$\Rightarrow \frac{a+6}{3b} = \frac{2}{3}$$

⇒ 3a + 18 = 2b

⇒ a + 6 = 2b -------- (2)

Multiplying eq2 by 4 and subtracting from eq1

⇒ 4a – b – 4a – 24 = 11 – 8b

⇒ 7b = 35

⇒ b = 5

Thus, a = 4

Fraction is 4/5

7. Question

The sum of a numerator and denominator of a fraction is 18. If the denominator is increased by 2, the fraction reduces to 1/3. Find the fraction.

Answer

Let the numerator be 'a' and denominator be 'b'

Given, sum of a numerator and denominator of a fraction is 18.

⇒ a + b = 18 ----- (1)

Also, if the denominator is increased by 2, the fraction reduces to 1/3.

⇒ a/(b + 2) = 1/3

⇒ 3a - b = 2 ------ (2)

Adding (1) and (2)

⇒ 4a = 20

⇒ a = 5

Thus, b = 13

Fraction is 5/13

8. Question

If 2 is added to the numerator of a fraction, it reduces to 1/2 and if 1 is subtracted from the denominator, it reduces to 1/3. Find the fraction.

Answer

Let the numerator be 'a' and denominator be 'b'.

Given, if 2 is added to the numerator of a fraction, it reduces to 1/2 and if 1 is subtracted from the denominator, it reduces to 1/3

⇒ (a + 2)/b = 1/2

⇒ 2a + 4 = b ------ (1)

Also, a/(b – 1) = 1/3

⇒ 3a + 1 = b ------- (2)

Subtracting eq2 from eq1

⇒ 2a + 4 – 3a – 1 = b – b

⇒ a = 3

Thus, b = 2a + 4 = 10

Fraction is 3/10

9. Question

The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction.

Answer

Let the numerator be ‘a’ and denominator be ‘b’.

Given, sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3.

⇒ a + b = 2a + 4

⇒ b – a = 4 -------- (1)

Also, $\frac{a+3}{b+3} = \frac{2}{3}$

⇒ 3a + 9 = 2b + 6

⇒ 3a – 2b = - 3 ------(2)

Multiplying eq1 by 2 and adding to eq1

⇒ 2b – 2a + 3a – 2b = 8 - 3

⇒ a = 5

Thus b = 9

Fraction is 5/9

10. Question

The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Determine the fraction.

Answer

Let the numerator be ‘a’ and denominator be ‘b’.

Given, sum of the numerator and denominator of a fraction is 3 less than twice the denominator.

⇒ a + b = 2b – 3

⇒ a = b – 3 ------ (1)

Also, if the numerator and denominator are decreased by 1, the numerator becomes half the denominator.

⇒ a – 1 = (b – 1)/2

⇒ b = 2a - 1 ------ (2)

Substituting value of b in eq1.

⇒ a = 2a – 1 – 3

⇒ a = 4.

Thus, b = 2 × 4 – 1 = 7

11. Question

The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes 1/2. Find the fraction.

Answer

Let the numerator be ‘a’ and denominator be ‘b’.

Given, sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes 1/2.

$\Rightarrow a + b = 12$ ------ (1)

Also, $a/(b + 3) = 1/2$

$\Rightarrow 2a - b = 3$ ------- (2)

Adding eq1 and eq2

$\Rightarrow a + b + 2a - b = 15$

$\Rightarrow 3a = 15$

$\Rightarrow a = 5$

Thus, $b = 7$

Fraction is 5/7.

Exercise 3.9

1. Question

A father is three times as old as his son. After twelve years, his age will be twice as that of his son then. Find their present ages.

Answer

Given: A father is three times as old as his son. After twelve years, his age will be twice as that of his son.**To find:** The present age of father and his son.**Solution:** Let the present age of father be ‘a’ and present age of son be b.

Given, father is three times as old as his son.

$\Rightarrow a = 3b$ ---- (1)

Also, after twelve years, his age will be twice as that of his son then.12 years later age of father will be a+12 and son will be b+12.

$\Rightarrow a + 12 = 2(b + 12) \Rightarrow a+12 = 2b + 24$

$\Rightarrow a = 2b + 12$

Put the value of a from eq. (1)

$\Rightarrow 3b = 2b + 12$

$\Rightarrow 3b - 2b = 12$

$\Rightarrow b = 12$

Put the value of a in eq. 1 to get,a=3ba=3(12)a=36

Thus, present age of father is 36 years and present age of son is 12 years.

2. Question

Ten years later, A will be twice as old as B and five years ago, A was three times as old as B. What are the present ages of A and B?

Answer

Let the present ages of A and B be ‘a’ and ‘b’ respectively.

Given, ten years later, A will be twice as old as B and five years ago, A was three times as old as B

$\Rightarrow a + 10 = 2(b + 10)$

$\Rightarrow a = 2b + 10$ ------- (1)

Also, a – 5 = 3(b – 5)

⇒ a = 3b – 10 -------- (2)

Equating eq2 and eq1

⇒ 2b + 10 = 3b – 10

⇒ b = 20

Thus, a = 2b + 10 = 50

3. Question

A is elder to B by 2 years. A’s father F is twice as old as A and B is twice as old as his sister S. If the ages of the father and sister differ by 40 years, find the age of A.

Answer

Let the present ages of A, B, F and S be ‘a’, ‘b’, ‘c’ and ‘d’ respectively.

Given, A is elder to B by 2 years

⇒ a = b + 2 -------- (1)

A’s father F is twice as old as A and B is twice as old as his sister S

⇒ c = 2a and b = 2d ------- (2)

Also, the ages of the father and sister differ by 40 years.

⇒ c – d = 40 ------- (3)

From [2], we have

⇒ 2a – b/2 = 40

$$2a - \frac{(a-2)}{2} = 40$$

$$\Rightarrow \frac{4a - a + 2}{2} = 40$$

$$\Rightarrow 3a + 2 = 80$$

$$\Rightarrow 3a = 78$$

$$\Rightarrow a = 26$$

Hence, Age of 'A' is 26 years.

4. Question

Six years hence a man’s age will be three times the age of his son and three years ago he was nine times as old as his son. Find their present ages.

Answer

Let the present age of man be ‘a’ and present age of son be b.

Given, Six years hence a man’s age will be three times the age of his son and three years ago he was nine times as old as his son

⇒ a + 6 = 3(b + 6)

⇒ a = 3b + 12 ------ (1)

Also, three years ago he was nine times as old as his son

$\Rightarrow a - 3 = 9(b - 3)$

$\Rightarrow a = 9b - 24$ ----- (2)

Equating eq1 and eq2

$\Rightarrow 3b + 12 = 9b - 24$

$\Rightarrow b = 6$

Thus, $a = 3b + 12 = 30$

Hence age of man is 30 yrs and age of son is 6 yrs.

5. Question

Ten years ago, a father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be then. Find their present ages.

Answer

Let the present age of father be 'a' and present age of son be b.

Given, ten years ago, a father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be then.

$\Rightarrow a - 10 = 12(b - 10)$

$\Rightarrow a = 12b + 110$ ------ (1)

Also, $a + 10 = 2(b + 10)$

$\Rightarrow a = 2b + 10$ ------- (2)

Equating eq1 and eq2.

$\Rightarrow 12b + 110 = 2b + 10$

$\Rightarrow b = 10$

Thus, $a = 30$

6. Question

The present age of a father is three years more than three times the age of the son. Three years hence father's age will be 10 years more than twice the age of the son. Determine their present ages.

Answer

Let the father's age be 'a' and son's age be 'b'

Given, present age of a father is three years more than three times the age of the son.

$\Rightarrow a = 3b + 3$ ---------- (1)

Also, three years hence father's age will be 10 years more than twice the age of the son.

$\Rightarrow a + 3 = 2(b + 3) + 10$

$\Rightarrow a = 2b + 13$ --------- (2)

Equating (1) and (2), we get

$3b + 3 = 2b + 13$

$\Rightarrow b = 10$ years

Father's age (a) = $3 \times 10 + 3 = 33$

7. Question

A father is three times as old as his son. In 12 years time, he will be twice as old as hi son. Find the present ages of father and the son.

Answer

Let the present age of father be 'a' and present age of son be b.

Given, father is three times as old as his son. In 12 years time, he will be twice as old as hi son.

⇒ $a = 3b$ and

$a + 12 = 2(b + 12)$

⇒ $3b + 12 = 2b + 24$

⇒ $b = 12$

Thus, $a = 36$

8. Question

Father's age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father.

Answer

Given: Father's age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children.

To find: the age of father.

Solution: Let the present age of father be 'a' and present sum of age of both sons be b.

Given, Father's age is three times the sum of ages of his two children.

⇒ $a = 3b$

After 5 years his age will be twice the sum of ages of two children.So age of father will be a + 5 and as there are two sons, age of both sons combined after 5 years will be b + 10.

⇒ $a + 5 = 2(b + 10)$

Put the value of a in above equation,

⇒ $3b + 5 = 2b + 20$

⇒ $3b - 2b = 20 - 15$

⇒ $b = 15$

Thus, $a = 3b = 3 (15) = 45$Hence father's present age is 45 years.

9. Question

Two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son. Find the present ages of father and son.

Answer

Let the present age of father be 'a' and present age of son be b.

Given, two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son.

⇒ $a – 2 = 5(b – 2)$

⇒ $a = 5b – 8$ ------ (1)

Also, $a + 2 = 3(b + 2) + 8$

⇒ $a = 3b + 12$ ----- (2)

Equating (1) and (2)

⇒ $5b – 8 = 3b + 12$

⇒ b = 10

From [1], we havea = 5(10) - 8Thus father's age, a = 50 – 8 = 42 Years

10. Question

Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Answer

Given: Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu.

To find: The age of Nuri and Sonu.

Solution: Let the present age of Nuri and his son be 'a' and 'b' respectively.

Given, Five years ago, Nuri was thrice as old as Sonu.

This implies age of Nuri and Sonu five years ago was a - 5 and b - 5.

⇒ a – 5 = 3(b – 5)

⇒ a - 5 = 3b – 15⇒ a = 3b – 15 + 5

⇒ a = 3b – 10 ------- (1)

Ten years later, Nuri will be twice as old as Sonu.This implies after ten years the age of Nuri and Sonu will be a + 10 and b + 10.

⇒ a + 10 = 2(b + 10)

⇒ a + 10 = 2b + 20⇒ a = 2b + 20 - 10

⇒ a = 2b + 10 -------- (2)

Equating (1) and (2), we get

3b – 10 = 2b + 10

⇒ 3b - 2b = 10 + 10⇒ b = 20

Thus, a = 60 – 10 = 50Hence Nuri's present age is 50 years and Sonu's age is 20 years.

11. Question

The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju as twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Answer

Let the ages of Ani, Biju, Dharam and Cathy be a, b, c and d respectively.

Given, ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju as twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years.

a – b = 3

c = 2a and b = 2d

c – d = 30

Solving the above four equations

⇒ 2a – b/2 = 30

⇒ 2a – a/2 + 3/2 = 30

⇒ a = 19

Thus, b = 16 years

Exercise 3.10

1. Question

Points A and B are 70 km a part on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours, but if they travel towards each other, they meet in one hour. Find the speed of the two cars.

Answer

Let the speed of car from A be ‘a’ and of car from B be ‘b’

Speed = distance/time

Relative speed of cars when moving in same direction = a + b

Relative speed of cars when moving in opposite direction = a – b

Given, if they travel in the same direction, they meet in 7 hours, but if they travel towards each other, they meet in one hour. A and B are 70 km a part on a highway

⇒ a + b = 70 ------ (1)

Also, a – b = 70/7 = 10 ------ (2)

Adding (1) and (2)

⇒ 2a = 80

⇒ a = 40 km/hr

Thus, b = 40 – 10 = 30 km/hr

2. Question

A sailor goes 8 km downstream in 40 minutes and returns in 1 hour. Determine the speed of the sailor in still water and the speed of the current.

Answer

Speed = distance/time

Let the speed of sailor in still water be ‘a’ and speed of the current be ‘b’.

The relative speed of sailor going upstream = a – b

The relative speed of sailor going downstream = a + b

Given, sailor goes 8 km downstream in 40 minutes and returns in 1 hour.

$$\Rightarrow a + b = \frac{8}{\frac{40}{60}}$$

⇒ a + b = 12 ----- (1)

Also,

a – b = 8/1 = 8 ------ (2)

Adding (1) and (2).

⇒ 2a = 20

⇒ a = 10km/hr

Thus, b = 12 – 10 = 2 km/hr

So, speed of sailor is 10 km/h and current is 2 km/h.

3. Question

The boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of stream and that of the boat in still water.

Answer

Given: The boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream.

To find: the speed of stream and that of the boat in still water.

Solution: Speed = distance/time

Let the speed of boat be 'a' and speed of stream be 'b'

Relative speed of boat going upstream = a – b

Relative speed of boat going downstream = a + b

Given, boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstreamand we know, $time = \frac{speed}{distance}$

$\Rightarrow \frac{30}{a-b} + \frac{44}{a+b} = 10$ ------- (1)

Also, $\frac{40}{a-b} + \frac{55}{a+b} = 13$ ------ (2)

Now take, $\frac{1}{a-b} = u, \frac{1}{a+b} = v$

Eq 1 and 2 becomes$\Rightarrow$ 30 u + 44 v = 10Take 2 common out of above equation$\Rightarrow$ 15 u + 22 v - 5 = 0 ..... (3)and40 u + 55 v - 13 = 0 ..... (4)Solve the equations by cross multiplication method

$$\frac{u}{22\times(-13)-55\times(-5)} = \frac{-v}{15\times(-13)-40\times(-5)} = \frac{1}{15\times(55)-40\times(22)}$$

$$\frac{u}{-286+275} = \frac{-v}{-195+200} = \frac{1}{825-880}$$

$$\frac{u}{-11} = \frac{-v}{5} = \frac{1}{-55}$$

$$u = \frac{-11}{-55},\ v = \frac{-5}{-55}$$

$$u = \frac{1}{5},\ v = \frac{1}{11}$$

$\Rightarrow$ a- b = 5 .... (4) a + b = 11 .... (5)Add eq 4 and 5 to geta - b + a + b = 5 + 11$\Rightarrow$ 2a = 16$\Rightarrow$ a = 8 Substitute value of a in eq 4 we get,8 - b = 5$\Rightarrow$ b = 8 - 5$\Rightarrow$ b = 3Speed of boat = 8 km/hr and speed of stream = 3 km/hr

4. Question

A boat goes 24 km upstream and 28 km downstream in 6 hrs. It goes 30 km upstream and 21 km downstream in $6\frac{1}{2}$ hrs. Find the speed of the boat in still water and also speed of the stream.

Answer

Speed = distance/time

Let the speed of boat be 'a' and speed of stream be 'b'

Relative speed of boat going upstream = a – b

Relative speed of boat going downstream = a + b

Given, boat goes 24 km upstream and 28 km downstream in 6 hrs. It goes 30 km upstream and 21 km downstream in $6\frac{1}{2}$ hrs

$\Rightarrow \frac{24}{a-b} + \frac{28}{a+b} = 6$ ------- (1)

Also, $\frac{30}{a-b} + \frac{21}{a+b} = \frac{13}{2}$ ------ (2)

Multiplying eq1 by 3 and eq2 by 4 and (1) – (2)

$$\Rightarrow \frac{72}{a-b} - \frac{120}{a-b} = -8$$

⇒ a - b = 6 ------- (3)

Substituting value of (a - b) into eq1

$$\Rightarrow \frac{24}{6} + \frac{28}{a+b} = 6$$

⇒ a + b = 14 ------ (4)

From (1) and (2), we get

a = 10 km/hr and b = 4 km/hr

5. Question

While covering distance of 30 km. Ajeet takes 2 hours more than Amit. If Ajeet doubles his speed, he would take 1 hour less than Amit. Find their speeds of walking.

Answer

Speed = distance/time

Let the speed of Ajeet be 'a' and speed of Amit be 'b'.

Given, while covering distance of 30 km. Ajeet takes 2 hours more than Amit. If Ajeet doubles his speed, he would take 1 hour less than Amit.

$$\Rightarrow \frac{30}{a} - \frac{30}{b} = 2$$

Also, $\frac{30}{b} - \frac{30}{2a} = 1$

Adding the two equations

⇒ 30/a - 15/a = 3

⇒ a = 5 km/hr

Thus, 30/5 – 30/b = 2

⇒ b = 7.5 km/hr

6. Question

A man walks a certain distance with certain speed. If he walks 1/2 km an hour faster, he takes 1 hour less. But, if he walks 1 km an hour slower, he takes 3 more hours. Find the distance covered by the man and his original rate of walking.

Answer

Given: A man walks a certain distance with certain speed. If he walks 1/2 km an hour faster, he takes 1 hour less. But, if he walks 1 km an hour slower, he takes 3 more hours. '

To find: the distance covered by the man and his original rate of walking.

Solution: Let the original speed, original time taken and distance be 'a', 't' and 'd'.As we know,distance = speed × time

⇒ d = at ------- (1)

Given, if he walks 1/2 km = 0.5 km an hour faster, he takes 1 hour less,

⇒ d = (a + 0.5)(t − 1)⇒ d = at + 0.5t − a − 0.5 ⇒ at = at + 0.5t - a - 0.5 [From 1]⇒ 0.5t - a = 0.5Multiply the above equation by 10 to get,⇒ 5t - 10a = 5⇒ 10a = 5t - 5 -------- (2)

Also if he walks 1 km an hour slower, her takes 3 more hours., ⇒d = (a − 1)(t + 3) ⇒ d = at + 3a - t - 3⇒ at = at + 3a - t - 3 [From 1]⇒ 3a - t = 3⇒ t = 3a - 3 ------- (3)Put the value of t in eq (2)⇒ 10a = 5(3a - 3) - 5⇒ 10a = 15a - 15 - 5⇒ 10a - 15a = - 15 - 5⇒ - 5a = - 20⇒ a = 4Putting back in (3), we get⇒ t = 3(4) - 3 = 9 Therefore, d = at = 4(9) = 36Hence, speed of man = a = 4 km/hrdistance covered by man = d = 36 km

7. Question

Ramesh travels 760 km to his home partly by train and partly by car. He takes 8 hours if he travels 160 km by train and the rest by car. He takes 12 minutes more if the travels 240 km by train and the rest by car. Find the speed of the train and car respectively.

Answer

Let the speed of train be 'a' and speed of car be 'b'.

Speed = distance/time

Given, Ramesh travels 760 km to his home partly by train and partly by car. He takes 8 hours if he travels 160 km by train and the rest by car. He takes 12 minutes more if the travels 240 km by train and the rest by car.

$\Rightarrow \frac{160}{a}+\frac{600}{b}=8$------- (1)

Also, $\frac{240}{a}+\frac{520}{b}=8+\frac{12}{60}$

⇒ 240/a + 520/b = 41/5 ----- (2)

Multiplying eq1 by 3 and eq2 by 2 and subtracting eq2 from eq1

$$\Rightarrow \frac{1800}{b}-\frac{1040}{b}=24-\frac{82}{5}$$

$$\Rightarrow \frac{760}{b}=\frac{38}{5}$$

⇒ b = 100 km/hr

Thus, 160/a + 6 = 8

⇒ a = 80 km/hr

8. Question

A man travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes. But, if he travels 200 km by train and the rest by car, he takes half an hour longer. Find the speed of the train and that of the car.

Answer

Given: A man travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes. But, if he travels 200 km by train and the rest by car, he takes half an hour longer.

To find: the speed of the train and that of the car.

Solution: Let the speed of train be 'a' and speed of car be 'b'.

Speed = distance/time

Given, man travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes.

$\Rightarrow \frac{400}{a} + \frac{200}{b} = 6.5$------- (1)

But, if he travels 200 km by train and the rest by car, he takes half an hour longer.

$\frac{200}{a} + \frac{400}{b} = 7$------- (2)

Multiplying eq1 by 2 and subtracting from eq2

$$\left(\frac{200}{a} + \frac{400}{b}\right) - 2\left(\frac{400}{a} + \frac{200}{b}\right) = 7 - 2(6.5)$$

$$\frac{200}{a} + \frac{400}{b} - \frac{800}{a} - \frac{400}{b} = 7 - 13$$

$$\Rightarrow \frac{200}{a} - \frac{800}{a} = 7 - 13 \Rightarrow \frac{-600}{a} = -6$$

⇒ -600 = -6a

⇒ a = 100 km/hr

Thus, 4 + 200/b = 6.5200/b = 6.5-4

⇒ 200/b = 2.5 ⇒ 2.5b = 200

⇒ b = 80 km/hr

9. Question

Places A and B are 80 km apart form each other on a highway. A car starts from A and other from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite directions, they meet in 1 hour and 20 minutes. Find the speeds of the cars.

Answer

Given: places A and B are 80 km apart form each other on a highway. A car starts from A and other from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite directions, they meet in 1 hour and 20 minutes.

To find: the speeds of the cars.

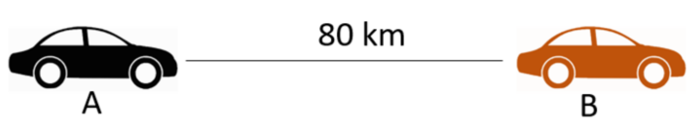
Solution: Let the speed of car from A be 'a' and of car from B be 'b'.

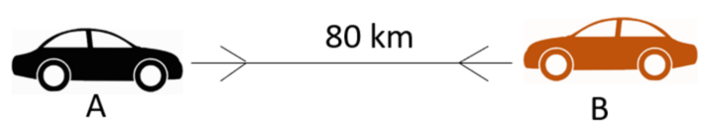
Speed = distance/time

Relative speed of cars when moving in same direction = a + b

Relative speed of cars when moving in opposite direction = a – b

Given, places A and B are 80 km apart form each other on a highway. A car starts from A and other from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite directions, they meet in 1 hour and 20 minutes.





⇒ a – b = 80/8 = 10 ------ (1)

$$Also, a + b = \frac{80}{1 + \frac{20}{60}} = 60 ------(2)$$

Adding (1) and (2)⇒ a - b + a + b = 10 + 60

⇒ 2a = 70

⇒ a = 35 km/hr

Put the value of a in (1).

Thus, b = 35 – 10 = 25 km/hr

Hence, speed of two cars are 35km/h and 25 km/h

10. Question

A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time. Find the speed of the oat in still water and the speed of the stream.

Answer

Speed = distance/time

Let the speed of boat be 'a' and speed of stream be 'b'

Relative speed of boat going upstream = a – b

Relative speed of boat going downstream = a + b

Given, boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time

⇒ $\frac{12}{a-b} + \frac{40}{a+b} = 8$ ------- (1)

Also, $\frac{16}{a-b} + \frac{32}{a+b} = 8$ ------ (2)

Equating (1) and (2), we get

⇒ 8/(a + b) = 4/(a – b)

⇒ a = 3b

Substitute value of a in eq1

⇒ 12/2b + 40/4b = 8

⇒ 16/b = 8

⇒ b = 2 km/hr

Thus, a = 6 km/hr.

11. Question

Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus respectively.

Answer

Let the speed of train be 'a' and speed of bus be 'b'.

Speed = distance/time

Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer

⇒ $\frac{60}{a} + \frac{240}{b} = 4$------- (1)

Also, $\frac{100}{a}+\frac{200}{b}=4+\frac{10}{60}$

$\frac{100}{a}+\frac{200}{b}=\frac{25}{6}$ ------- (2)

Multiplying eq1 by 5 and eq2 by 3 and subtracting eq2 from eq1

$\Rightarrow \frac{1200}{b}-\frac{600}{b}=20-\frac{25}{2}$

⇒ b = 80 km/hr

Thus, 60/a + 3 = 4

⇒ a = 60 km/hrHence speed of train is 60 km/h and bus is 80 km/h.

12. Question

Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

Answer

Let the speed of her rowing in still water be 'a' and speed of current be 'b.'We know, $\text{speed}=\frac{\text{distance}}{\text{time}}$

Relative speed of boat going upstream = a – b

Relative speed of boat going downstream = a + b

Given, Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours

⇒ a + b = 20/2 = 10 ---- (1) and a – b = 4/2 = 2 ------ (2)

Adding eq1 and eq2

⇒ 2a = 12

⇒ a = 6 km/hr

Also, b = a – 2 = 4 km/hr

Therefore, her speed of rowing in still water is 6 km/h and the speed of the current is 4 km/h

13. Question

A takes 3 hours more than B to walk a distance of 30 km. But, if A doubles his pace (speed) he is ahead of B by $1\frac{1}{2}$ hours. Find the speeds of A and B.

Answer

Let the speed of A be 'a' and speed of B be 'b'.

Given, A takes 3 hours more than B to walk a distance of 30 km. But, if A doubles his pace (speed) he is ahead of B by $1\frac{1}{2}$ hours

$\Rightarrow \frac{30}{a}-\frac{30}{b}=3$ -------- (1)

$Also, \frac{30}{b}-\frac{30}{2a}=1.5$ ------ (2)

Adding (1) and (2)

⇒ 15/a = 4.5

⇒ a = 150/45 = 10/3 km/hr

Thus,

$$\frac{30}{\frac{10}{3}} - \frac{30}{b} = 3$$

⇒ b = 5 km/hr

14. Question

Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 260 km by train and 240 km by taxi he takes 6 minutes longer. Find the speed of the train and that of the taxi.

Answer

Let the speed of train be 'a' and speed of taxi be 'b'.

Speed = distance/time

Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 260 km by train and 240 km by taxi he takes 6 minutes longer.

⇒ $\frac{300}{a} + \frac{200}{b} = 5.5$------- (1)

Also, $\frac{260}{a} + \frac{240}{b} = 5.5 + \frac{6}{60}$

$\frac{260}{a} + \frac{240}{b} = 5.6$ ------- (2)

Multiplying eq1 by 6 and eq2 by 5 and subtracting eq2 from eq1

$$\Rightarrow \frac{1800}{a} - \frac{1300}{a} = 33 - 28$$

⇒ a = 100 km/hr

Thus, 3 + 200/b = 5.5

⇒ b = 80 m/hr

15. Question

A train covered a certain distance at a uniform speed. If the train could have been 10 km/hr. faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hours more than the scheduled time. Find the distance covered by train.

Answer

Let the speed of train be 's', scheduled time be 't' and distance be 'd'.

Speed = distance/time

Given, covered a certain distance at a uniform speed. If the train could have been 10 km/hr. faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hours more than the scheduled time.

⇒ d = st ------- (1)

d = (s + 10)(t – 2) ------ (2)

d = (s – 10)(t + 3) ------ (3)

Solving the above three equation we get, d = 600km, s = 50 km/hr and t = 12 hours

16. Question

Places A and B are 100 km apart on a highway. One car starts form A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of two cars?

Answer

Let the speed of car from A be ‘a’ and of car from B be ‘b’

Speed = distance/time

Relative speed of cars when moving in same direction = a + b

Relative speed of cars when moving in opposite direction = a − b

Given, places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour

⇒ a − b = 100/5 = 20   (1)

Also, a + b = 100/1 = 100   (2)

Adding (1) and (2)

a - b + a + b = 20 + 100

⇒ 2a = 120

⇒ a = 60 km/hr

Putting value of a in (1) we get,

Thus, b = 60 − 20 = 40 km/hrSpeed of two cars are 60 km/h and 40 km/h

Exercise 3.11

1. Question

If in a rectangle, the length is increased and breadth reduced each by 2 units, the area is reduced by 28 square units. If, however the length is reduced by 1 unit and the breadth increased by 2 units, the area increases by 33 square units. Find the area of the rectangle.

Answer

Area of a rectangle = l × b

Given, if in a rectangle, the length is increased and breadth reduced each by 2 units, the area is reduced by 28 square units. If, however the length is reduced by 1 unit and the breadth increased by 2 units, the area increases by 33 square units.

⇒ (l + 2)(b − 2) = lb − 28

⇒ 2b − 2l = - 24 ------ (1)

Also, (l − 1)(b + 2) = lb + 33

⇒ 2l − b = 35 ------- (2)

Adding (1) and (2), we get

⇒ b = 11

Thus, 2l = 46, l = 23

Area of the rectangle = lb = 253 square units.

2. Question

The area of a rectangle remains the same if the length is increased by 7 metres and the breadth is decreased by 3 metres. If the length is decreased by 7 metres and breadth is increased by 4 metres, the area is decreased by 21 sq. metres. Find the dimensions of the rectangle.

Answer

Area of a rectangle = l × b

Given, area of a rectangle remains the same if the length is increased by 7 metres and the breadth is decreased by 3 metres. If the length is decreased by 7 metres and breadth is increased by 4 metres, the area is decreased by 21 sq.

metres.

⇒ (l + 7)(b − 3) = lblb - 3l + 7b - 21 = lb

⇒ 7b − 3l = 21 ------ (1)

Also, (l − 7)(b + 4) = lb - 21

lb + 4l - 7b - 28 = lb - 21

⇒ 4l − 7b = 7 ------- (2)

Adding (1) and (2), we get7b - 3l + 4l - 7b = 21 + 7

⇒ l = 28 m

put the value of l in 1 to get7b - 3(28) = 21

Thus, 7b − 84 = 21

7b = 21 + 847b = 105

⇒ b = 15 mHence the length is 28 m and breadth is 15 m.

3. Question

In a rectangle, if the length is increased by 3 metres and breadth is decreased by 4 metres, the area of the rectangle is reduced by 67 square metres. If length is reduced by 1 metre and breadth is increased by 4 metres, the area is increased by 89 sq. metres. Find the dimensions of the rectangle.

Answer

Area of a rectangle = l × b

Given, if the length is increased by 3 metres and breadth is decreased by 4 metres, the area of the rectangle is reduced by 67 square metres. If length is reduced by 1 metre and breadth is increased by 4 metres, the area is increased by 89 sq. metres.

⇒ (l + 3)(b − 4) = lb − 67

⇒ 3b − 4l = -55 ------ (1)

Also, (l − 1)(b + 4) = lb + 89

⇒ 4l − b = 93 ------- (2)

Adding (1) and (2), we get

⇒2 b = 38

⇒ b = 19

Thus, 4l − 19 = 93

⇒ l = 28m

4. Question

The incomes of X and Y are in the ratio of 8 : 7 and their expenditures are in the ratio 19 : 16. If each saves Rs 1250, find their incomes.

Answer

Given, incomes of X and Y are in the ratio of 8 : 7 and their expenditures are in the ratio 19 : 16.

Let the incomes of X and Y be 8a and 7a

Let the expenditures of X and Y be 19b and 16b

Each saves Rs. 1250

⇒ 8a − 19b = 1250 -------- (1) and 7a − 16b = 1250 ------- (2)

Equating both equation

⇒ 8a – 19b = 7a – 16b

⇒ a = 3b

Substituting value of a in eq1

⇒ 24b – 19b = 1250

⇒ b = 250

Thus, a = 750

X's income = 8a = Rs. 6000

Y's income = 7a = Rs. 5250

5. Question

A and B each has some money. If A gives Rs 30 to B, then B will have twice the money left with A. But, if B gives Rs 10 to A, then a will have thrice as much as is left with B. How much money does each have?

Answer

Let the amount of money which A has be 'a' and which B has be 'b'.

Given, if A gives Rs 30 to B, then B will have twice the money left with A.

⇒ b + 30 = 2(a – 30)⇒ b + 30 = 2a – 60⇒ b = 2a-60-30⇒ b = 2a – 90 --------- (1)But, if B gives Rs 10 to A, then A will have thrice as much as is left with B

a + 10 = 3(b - 10)a+10=3b-30a=3b-30-10

⇒ a = 3b - 40 --------- (2)

Substituting 'a' from eq2 in eq1b = 2(3b-40) – 90

⇒ b = 6b - 80 – 90

⇒ b - 6b =- 80 – 90⇒ -5b = -170⇒ b = -170/-5

⇒ b = Rs. 34

Substitute value of b in eq. 2,a=3(34)-40

Thus, a = 102 – 40 = Rs. 62Hence, A has Rs 62 and B has Rs 34.

6. Question

There are two examination rooms A and B. If 10 candidates are sent from A to B, the number of students in each room is same. If 20 candidates are sent from B to A, the number of students in A is double the number of students in B. Find the number of students in each room.

Answer

Let the number of candidates in rooms A and B be 'a' and 'b' respectively.

Given, if 10 candidates are sent from A to B, the number of students in each room is same.

⇒ a – 10 = b + 10

⇒ a – b = 20 ----- (1)

Also, if 20 candidates are sent from B to A, the number of students in A is double the number of students in B.

⇒ a + 20 = 2(b – 20)

⇒ a – 2b = -60 ------- (2)

Subtracting eq2 from eq1

⇒ a – b – a + 2b = 20 + 60

⇒ b = 80

Thus, a = 80 + 20 = 100

7. Question

2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys. How long would it take one man and one boy to do it?

Answer

Given:2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys. **To find:** How long would it take one man and one boy to do it.**Solution:**Let the number of days in which 1 man and 1 boy can do the work be 'a' and 'b' respectively.

In 1 day,In a days 1 man can work⇒ In 1 day 1/a man can do work⇒ 2 men can do work in 2/a daysSimilarlyIn b days 1 boy can work⇒ In 1 day 1/b boy can do work⇒ 7 boys can do work in 7/b daysGiven, 2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys.

$\Rightarrow \frac{2}{a}+\frac{7}{b}=\frac{1}{4}$ -------- (1)

$\Rightarrow \frac{4}{a}+\frac{4}{b}=\frac{1}{3}$ -------- (2)

Multiplying eq1 by 2 and subtracting eq2 from eq1

$$\Rightarrow 2\left(\frac{2}{a}\right)+2\left(\frac{7}{b}\right)-\frac{4}{a}-\frac{4}{b}=2\left(\frac{1}{4}\right)-\frac{1}{3}$$

$$\Rightarrow \frac{4}{a}+\frac{14}{b}-\frac{4}{a}-\frac{4}{b}=\frac{2}{4}-\frac{1}{3}$$

$$\Rightarrow \frac{14}{b}-\frac{4}{b}=\frac{1}{2}-\frac{1}{3}$$

$$\Rightarrow \frac{14-4}{b}=\frac{3-2}{6}$$

⇒ 10/b = 1/6

⇒ b = 60 days

Substitute the value of b in eq.2 $\Rightarrow \frac{4}{a}+\frac{4}{60}=\frac{1}{3}$

$$\Rightarrow \frac{4}{a}+\frac{\cancel{4}^{1}}{\cancel{60}_{15}}=\frac{1}{3}$$

Thus, 4/a + 1/15 = 1/3

$$\Rightarrow \frac{4}{a}=\frac{1}{3}-\frac{1}{15}$$

$$\Rightarrow \frac{4}{a}=\frac{5-1}{15}$$

$$\Rightarrow \frac{4}{a}=\frac{4}{15}$$

⇒ 4/a = 4/15

⇒ a = 15 days

thus 1 man can do work in 15 days and 1 boy can do work in 60 days.

8. Question

In a ΔABC, $\angle A = x°$, $\angle B = (3x - 2)°$,

$\angle C = y°$. Also, $\angle C - \angle B = 9°$. Find the three angles.

Answer

Sum of angles of a triangle = 180°

Given, $\angle A = x°$, $\angle B = (3x - 2)°$,

$\angle C = y°$.

⇒ x + 3x − 2 + y = 180

⇒ 4x + y = 182 ------ (1)

Also, $\angle C - \angle B = 9°$.

⇒ y − 3x + 2 = 9

⇒ y − 3x = 7 ------- (2)

(1) − (2)

⇒ 4x + y − y + 3x = 182 − 7

⇒ 7x = 175

⇒ x = 25°

Thus, y = 75 + 7 = 82°

∠A = 25°

∠B = 3X − 2 = 73°

∠C = 82°

9. Question

In a cyclic quadrilateral ABCD $\angle A = (2x + 4)°$, $\angle B = (y + 3)°$, $\angle C = (2y + 10)°$, $\angle D = (4x - 5)°$. Find the four angles.

Answer

Opposite angles of a cyclic quadrilateral are supplementary

∠A + ∠C = 180°

∠B + ∠D = 180°

Given, $\angle A = (2x + 4)°$, $\angle B = (y + 3)°$, $\angle C = (2y + 10)°$, $\angle D = (4x - 5)°$

2x + 4 + 2y + 10 = 180

⇒ x + y = 83 ------- (1)

y + 3 + 4x − 5 = 180

⇒ y + 4x = 182 -------- (2)

(1) − (2)

⇒ x − 4x = 83 − 182

⇒ x = 33

Thus, y = 50

$\angle A = 2x + 4 = 70°$

$\angle B = y + 3 = 53°$

$\angle C = 2y + 10 = 110°$

$\angle D = 4x - 5 = 127°$

10. Question

A railway half ticket costs half the full fare and the reservation charge is the same on half ticket as on full ticket. One reserved first class ticket from Mumbai to Ahmedabad costs Rs 216 and one full and one half reserved first class tickets cost Rs 327. What is the basic first class full fare and what is the reservation charge?

Answer

Let the basic full fare be 'a', half far be 'b' and reservation charges be 'r'.

Given, railway half ticket costs half the full fare and the reservation charge is the same on half ticket as on full ticket.

a = 2b ----- (1)

Also, reserved first class ticket from Mumbai to Ahmedabad costs Rs 216 and one full and one half reserved first class tickets cost Rs 327.

⇒ a + r = 216 ------ (2)

Also, a + b + 2r = 327 ----- (3)

Substituting value of a in eq2 and eq3

⇒ 2b + r = 216 and 3b + 2r = 327

Solving the above equations, we get

r = Rs. 6 and b = Rs. 105

Thus a = Rs. 210

11. Question

In a ΔABC, $\angle A = x°$, $\angle B = 3x°$, $\angle C = y°$. If 3y - 5x = 30, prove that the triangle is right angled.

Answer

Sum of angles of a triangle = 180°

Given, $\angle A = x°$, $\angle B = 3x°$, $\angle C = y°$

∴ x + 3x + y = 180

⇒ y + 4x = 180 ------- (1)

Also, 3y − 5x = 30 ------- (2)

Multiplying eq1 by 5 and eq2 by 4 and adding them

⇒ 5y + 12y = 900 + 120

⇒ y = 60

Thus, 60 + 4x = 180

⇒ x = 30

$\angle A = 30°$, $\angle B = 90°$ and $\angle C = 60°$

Triangle is right angled at B.

12. Question

The car hire charges in a city computerise of a fixed charges together with the charge for the distance covered. For a journey of 12 km, the charge paid is Rs 89 and for a journey of 20 km, the charge paid is Rs 145. What will a person have to pay for travelling a distance of 30 km?

Answer

Let the fixed charge be 'a' and the charge per km travelled be 'b'

Given, for a journey of 12 km, the charge paid is Rs 89 and for a journey of 20 km, the charge paid is Rs 145

⇒ a + 12b = 89 ----- (1)

and a + 20b = 145 ----- (2)

(1) – (2)

⇒ -8b = -56

⇒ b = 7

Thus, a + 84 = 89

⇒ a = 5

For a distance of 30km, amount paid = distance travelled x charge per kilometre + fixed charge

= 30 × 7 + 5 = Rs. 215

13. Question

A part of monthly hostel charges in a college are fixed and the remaining depend on the number of days one has taken food in the mess. When a student a takes food for 20 days, he has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charge and the cost of food per day.

Answer

Let the fixed charges be 'a' and the cost of food per day be 'b'

Given, when a student a takes food for 20 days, he has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges.

⇒ a + 20b = 1000

a + 26b = 1180

Subtracting one from another

⇒ 6b = 180

⇒ b = Rs. 30

Thus, a + 600 = 1000

⇒ a = Rs. 400

14. Question

Half the perimeter of a garden, whose length is 4 more than its width is 36 m. Find the dimensions of the garden.

Answer

Given, half the perimeter of a garden, whose length is 4 more than its width is 36 m.

⇒ l = b + 4 and l + b = 36

Solving these two equations, we get

Length = 20 m, Width = 16 m

15. Question

The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Answer

Given: The larger of two supplementary angles exceeds the smaller by 18 degrees.

To find: The measure of both angles.

solution: Let the larger angle be 'a' and the smaller angle be 'b'.

Given, larger of two supplementary angles exceeds the smaller by 18 degrees.

⇒ a = b + 18 ⇒ a - b = 18 ...... (1)The sum of supplementary angles is 180°.⇒a + b = 180.....(2)

Adding the equations 1 and 2 we get,

a-b+a+b=18+180

⇒ 2a = 198

⇒ a = 99°Put the value of b in eq. 1 to get, 99-b=18⇒ -b=18-99⇒ -b=-81⇒ b=81

Thus, b = 81°Hence the measure of smaller angle is 81° and larger angle is 99°.

16. Question

2 Women and 5 men can together finish a piece of embroidery in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the embroidery, and that taken by 1 man alone.

Answer

Let the number of days in which 1 woman and 1 man can do the work be 'a' and 'b' days respectively.

In 1 day, 1 woman completes 1/a part while 1 man does 1/b part

Given, 2 Women and 5 men can together finish a piece of embroidery in 4 days, while 3 women and 6 men can finish it in 3 days.

$\Rightarrow \frac{2}{a}+\frac{5}{b}=\frac{1}{4}$ ------ (1)

$\frac{3}{a}+\frac{6}{b}=\frac{1}{3}$ -------(2)

Multiplying eq1 by 3 and eq2 by 2

$$\frac{6}{a}+\frac{15}{b}=\frac{3}{4} \qquad (3)$$

$$\frac{6}{a}+\frac{12}{b}=\frac{2}{3} \qquad (4)$$

subtracting eq3 from eq4

$$\Rightarrow \frac{15}{b}-\frac{12}{b}=\frac{3}{4}-\frac{2}{3}$$

⇒ 3/b = 1/12

⇒ b = 36 days

Thus, 3/a + 1/6 = 1/3

⇒ 3/a = 1/6

⇒ a = 18 daysHence, A women can finish the embroidery in 18 days and A single man can finish the embroidery in 36 days

17. Question

A wizard having powers of mystic in candations and magical medicines seeing a cock, fight going on, spoke privately to both the owners of cocks. To one he said; if your bird wins, than you give me your stake-money, but if you do not win, I shall give you two third of that. Going to the other, he promised in the same way to give three fourths. From both of them his gain would be only 12 gold coins. Find the stake of money each of the cock-owners have.

Answer

Let the stake money be 'a' and 'b' respectively.

Given, to one he said; if your bird wins, than you give me your stake-money, but if you do not win, I shall give you two third of that. Going to the other, he promised in the same way to give three fourths.

From both of them his gain would be only 12 gold coins

If the 1st one wins

⇒ a - 3b/4 = 12 ----- (1)

If the 2nd one wins

⇒ b – 2a/3 = 12 ----- (2)

Equating 1 and 2

⇒ 5a/3 = 7b/4

⇒ a = 21b/20

Thus, 21b/20 – 3b/4 = 12

⇒ 6b/20 = 12

⇒ b = 40

Thus, a = 21b/20 = Rs. 42

18. Question

Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many note Rs 50 and Rs 100 she received.

Answer

Given: Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all.

To find: how many notes of Rs 50 and Rs 100 she received.

Solution: Let the number of Rs. 50 and Rs. 100 notes be 'a' and 'b'.

Given, Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only.

⇒ 50a + 100b = 2000

⇒ a + 2b = 40 ------- (1)

Also Meena got 25 notes in all.

⇒ a + b = 25 ----- (2)

(1) – (2)a+2b-(a+b)=40-25

⇒ a+2b-a-b=40-25

⇒ b = 15

Put the value of b in eq. 1 to get,a+15=25⇒ a=25-15

Thus, a = 10Therefore, Meena received 10 notes of denomination 50 and 15 notes of denomination 100.

19. Question

Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

Answer

Given: Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks.

To find: How many questions were there in the test?

Solution: Let the number of right answers be 'a' and number of wrong answers be 'b'

Given, Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Deducted marks are represented by "-" sign.

$\Rightarrow 3a - b = 40$ ------ (1)Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks

$\Rightarrow 4a - 2b = 50$ ---- (2)

Multiplying eq1 by 2 and subtracting eq2 from it$\Rightarrow 4a - 2b - 2(3a - b) = 50 - 2(40) \Rightarrow 4a - 2b - 6a + 2b = 50 - 80 \Rightarrow 4a - 6a = 50 - 80$

$\Rightarrow 6a - 4a = 80 - 50$

$\Rightarrow 2a = 30$

$\Rightarrow a = 15$Put the value of a in eq. 1 to get,$3(15) - b = 40 \Rightarrow 45 - b = 40 \Rightarrow -b = 40 - 45 \Rightarrow -b = -5 \Rightarrow b = 5$

Total number of questions in the test = right answers + wrong answers= $a + b = 15 + 5 = 20$

20. Question

The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row there would be 2 rows more. Find the number of students in the class.

Answer

Let the number of students in a row be 'a' and number of rows be 'b'.

Given, 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row there would be 2 rows more.

Number of students remain constant.

$\Rightarrow ab = (a + 3)(b - 1)$ $ab = (a - 3)(b + 2)$

So,$ab = ab - a + 3b$ $-3a - 3b = -3$ ...... (1) and$ab = ab + 2a - 3b - 6$$2a - 3b = 6$ ...... (2)Subtract 1 from 2 to get,$2a - 3b - (a - 3b) = 6 - (-3)$$2a - 3b - a + 3b = 6 + 3$$a = 9$Put the value of a in 1 to get,$9 - 3b = -3$$-3b = -3 -9$$-3b = -12$$b = 4$

Solving the above equations we get , a = 9 and b = 4

Thus, number of students = ab = (9)(4) = 36

21. Question

One says, "give me hundred, friend! I shall then become twice as rich as you" The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their respective capital?

Answer

Given: One says, "give me hundred, friend! I shall then become twice as rich as you" The other replies, "If you give me ten, I shall be six times as rich as you."

To find: the amount of their respective capital.

Solution: Let the capitals be 'a' and 'b'.

Given, one says, "give me hundred, friend! I shall then become twice as rich as you" .Lets assume "b" gives hundred to "a".According to given condition

$a + 100 = 2(b - 100) \Rightarrow a + 100 = 2b - 200$

$\Rightarrow a = 2b - 200 - 100$

$\Rightarrow a = 2b - 300 \Rightarrow a - 2b = -300$ ------ (1)Now The other replies, "If you give me ten, I shall be six times as rich as you."

Which means "a" gives 10 to "b".

So, b + 10 = 6(a − 10)⇒ b + 10 = 6a − 60⇒ b = 6a − 60 - 10

⇒ b = 6a − 70⇒ 6a - b = 70 ----- (2)

Multiplying eq1 by 6 and subtract from eq2

⇒ 6a - b - 6 (a - 2b) = 70 - 6 (- 300)⇒ 6a - b - 6 a + 12b = 70 + 1800⇒ 11b = 1870⇒ b = 170substitute the value of b in eq 1 to get, a - 2(170) = -300⇒ a - 340 = -300⇒ a = -300 + 340⇒ a = 40The amount of their respective capital is Rs 40 and Rs 170.

22. Question

In a cyclic quadrilateral ABCD $\angle A = (2x+4)^\circ, \angle B = (y+3)^\circ, \angle C = (2y+10)^\circ, \angle D = (4x-5)^\circ$. Find the four angles.

Answer

In a cyclic quadrilateral sum of opposite angles is 180°.

Given, in a cyclic quadrilateral ABCD $\angle A = (2x+4)^\circ, \angle B = (y+3)^\circ, \angle C = (2y+10)^\circ, \angle D = (4x-5)^\circ$

∴ ∠A + ∠C = 180° and ∠B + ∠D = 180°

⇒ 2x + 4 + 2y + 10 = 180 and y + 3 + 4x − 5 = 180

⇒ x + y = 83 --------- (1) and y + 4x = 182 ---------- (2)

Subtracting eq1 from eq2.

⇒ y + 4x − x − y = 182 − 83

⇒ 3x = 99

⇒ x = 33

Substituting in eq1.

⇒ y = 50

∠A = 2 × 33 + 4 = 70°

∠B = 50 + 3 = 53°

∠C = 2 × 50 + 10 = 110°

∠D = 4 × 33 – 5 = 127°

CCE - Formative Assessment

1. Question

Write the value of k for which the system of equations x + y − 4 = 0 and 2x + ky − 3 = 0 has no solution.

Answer

Given:

Equation 1: x + y = 4

Equation 2: 2x + ky = 3

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have no solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 1$

$a_2 = 2$

$b_1 = 1$

$b_2 = k$

$c_1 = 4$

$c_2 = 3$

Putting the above values in equation (i) we get:

$$\frac{1}{2} = \frac{1}{k}$$

⇒ k = 2

Also we find $\frac{1}{2} \neq \frac{4}{3}$

The value of k for which the system of equations has no solution is k = 2

2. Question

Write the value of k for which the system of equations

2x – y = 5

6x + ky = 15

has infinitely many solutions.

Answer

Given:

Equation 1: 2x – y = 5

Equation 2: 6x + ky = 15

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have infinitely many solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 2$

$a_2 = 6$

$b_1 = -1$

$b_2 = k$

$c_1 = 5$

$c_2 = 15$

Putting the above values in equation (i) we get:

$$\frac{2}{6} = \frac{-1}{k}$$

⇒ 2k = – 6 ⇒ k = – 3

Also we find $\frac{2}{6} = \frac{-1}{-3} = \frac{5}{15}$

The value of k for which the system of equations has infinitely many solution is k = – 3

3. Question

Write the value of k for which the system of equations 3x – 2y = 0 and kx + 5y = 0 has infinitely many solutions.

Answer

Given:

Equation 1: 3x – 2y = 0

Equation 2: kx + 5y = 0

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have infinitely many solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 3$

$a_2 = k$

$b_1 = -2$

$b_2 = 5$

$c_1 = 0$

$c_2 = 0$

In this problem since c_1 & $c_2 = 0$ so $\frac{c_1}{c_2} = \frac{0}{0}$ which is undefined.

So for this problem the system of linear equations will have infinite solutions if

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$ ....(ii)

Putting the above values in equation (ii) we get:

$$\frac{3}{k} = \frac{-2}{5}$$

$\Rightarrow -2k = 15$

$\Rightarrow k = -\frac{15}{2}$

The value of k for which the system of equations has infinitely many solution is k = $-\frac{15}{2}$

4. Question

Write the values of k for which the system of equations x + ky = 0, 2x – y = 0 has unique solution.

Answer

Given:

Equation 1: x + ky = 0

Equation 2: 2x – y = 0

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

According to the problem:

$a_1 = 1$

$a_2 = 2$

$b_1 = k$

$b_2 = -1$

$c_1 = 0$

$c_2 = 0$

So for this problem the system of linear equations will have unique solution if

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ ....(i)

Putting the above values in equation (i) we get:

$$\frac{1}{2} \neq \frac{k}{-1}$$

$\Rightarrow k \neq -\frac{1}{2}$

The value of k for which the system of equations has unique solution is k$\neq -\frac{1}{2}$

5. Question

Write the set of values of a and b for which the following system of equations has infinitely many solutions.

2x + 3y = 7

2ax + (a + b) y = 28

Answer

Given:

Equation 1: 2x + 3y = 7

Equation 2: 2ax + (a + b)y = 28

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

<u>For the system of linear equations to have infinitely many solutions we must have</u>

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 2$

$a_2 = 2a$

$b_1 = 3$

$b_2 = (a + b)$

$c_1 = 7$

$c_2 = 28$

Putting the above values in equation (i) we get:

$\frac{2}{2a} = \frac{3}{a+b} = \frac{7}{28}$ **...(ii)**

To obtain the value of a & b we need to solve the above equality. First we solve the extreme left and extreme right of the equality to obtain the value of a.

$\Rightarrow \frac{2}{2a} = \frac{7}{28} \Rightarrow 2a*7 = 2*28 \Rightarrow 14a = 56 \Rightarrow a = 4$

After obtaining the value of a we again solve the extreme left and middle portion of the equality (ii)

$\frac{2}{2*4} = \frac{3}{4+b} \Rightarrow 2*(4 + b) = 3*2*4 \Rightarrow b + 4 = 12 \Rightarrow b = 8$

<u>**The value of a & b for which the system of equations has infinitely many solution is a = 4 & b = 8**</u>

6. Question

For what value of k, the following pair of linear equations has infinitely many solutions?

10x + 5y – (k – 5) = 0

20x + 10y – k = 0

Answer

Given:

Equation 1: $10x + 5y = (k - 5)$ Equation 2: $20x + 10y = k$

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have infinitely many solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 10$

$a_2 = 20$

$b_1 = 5$

$b_2 = 10$

$c_1 = k - 5$

$c_2 = k$

Putting the above values in equation (i) we get:

$\frac{10}{20} = \frac{5}{10} = \frac{k-5}{k}$ **...(ii)**

On solving the equality (ii) we get

$5k = 10(k - 5) \Rightarrow 5k = 10k - 50 \Rightarrow 5k = 50 \Rightarrow k = 10$

The value of k for which the system of equations has infinitely many solution is k = 10

7. Question

Write the number of solutions of the following pair of linear equations:

$x + 2y - 8 = 0$

$2x + 4y = 16$

Answer

Given:

Equation 1: $x + 2y = 8$ Equation 2: $2x + 4y = 16$

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

The system of linear equations needs to be analyzed by checking the nature of ratios of each coefficients in the above two equations.

According to the problem:

$a_1 = 1$

$a_2 = 2$

$b_1 = 2$

$b_2 = 4$

$c_1 = 8$

$c_2 = 16$

Comparing the ratios of the coefficients we see:

$$\Rightarrow \frac{a_1}{a_2} = \frac{1}{2} \ldots \text{(i)}$$

$$\Rightarrow \frac{b_1}{b_2} = \frac{2}{4}$$

$$\Rightarrow \frac{b_1}{b_2} = \frac{1}{2} \ldots \text{(ii)}$$

$$\Rightarrow \frac{c_1}{c_2} = \frac{8}{16}$$

$$\Rightarrow \frac{c_1}{c_2} = \frac{1}{2} \ldots \text{(iii)}$$

On seeing equation (i), (ii) and (iii) we find

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Conclusion: The system of linear equations have infinite number of solution.

The given system of linear equations will have infinite number of solutions for all values of x and y

8. Question

Write the number of solutions of the following pair of linear equations:

$x + 3y - 4 = 0$

$2x + 6y = 7$

Answer

Given:

Equation 1: $x + 3y = 4$ Equation 2: $2x + 6y = 7$

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

According to the problem:

$a_1 = 1$

$a_2 = 2$

$b_1 = 3$

$b_2 = 6$

$c_1 = 4$

$c_2 = 7$

Comparing the ratios of the coefficients we see:

$$\Rightarrow \frac{a_1}{a_2} = \frac{1}{2} \ldots(i)$$

$$\Rightarrow \frac{b_1}{b_2} = \frac{3}{6} \Rightarrow \frac{b_1}{b_2} = \frac{1}{2} \ldots(ii)$$

$$\Rightarrow \frac{c_1}{c_2} = \frac{4}{7} \ldots(iii)$$

On seeing equation (i), (ii) and (iii) we find

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Conclusion: The system of linear equations has no solutions.

The given system of linear equations will have no solution for all values of x and y

1. Question

The value of k for which the system of equations

$kx - y = 2$

$6x - 2y = 3$

has a unique solution, is

A. = 3

B. ≠ 3

C. ≠ 0

D. = 0

Answer

Given:

Equation 1: $kx - y = 2$

Equation 2: $6x - 2y = 3$

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have unique solution we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \ldots\ldots\ldots(i)$$

According to the problem:

$a_1 = k$

$a_2 = 6$

$b_1 = -1$

$b_2 = -2$

$c_1 = 2$

$c_2 = 3$

Putting the above values in equation (i) we get

$\frac{k}{6} \neq \frac{-1}{-2} \Rightarrow k \neq \frac{-6}{-2} \Rightarrow k \neq 3$

<u>The</u> <u>value of k for which the system of equations has unique solution is</u> <u>k</u>≠3

2. Question

The value of k for which the system of equations

$2x + 3y = 5$

$4x + ky = 10$

has infinite number of solutions, is

A. 1

B. 3

C. 6

D. 0

Answer

Given:

Equation 1: $2x + 3y = 5$

Equation 2: $4x + ky = 10$

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

<u>For the system of linear equations to have infinitely many solutions we must have</u>

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 2$

$a_2 = 4$

$b_1 = 3$

$b_2 = k$

$c_1 = 5$

$c_2 = 10$

Putting the above values in equation (i) and solving the extreme left and middle portion of the equality we get the value of k

$$\frac{2}{4} = \frac{3}{k}$$

$\Rightarrow 2k = 12 \Rightarrow k = 6$

Also we find $\frac{2}{4} = \frac{3}{6} = \frac{5}{10}$

<u>The value of k for which the system of equations has infinitely many solution is k = 6</u>

3. Question

The value of k for which the system of equations x + 2y – 3 = 0 and 5x + ky + 7 = 0 has no solution, is

A. 10

B. 6

C. 3

D. 1

Answer

Given:

Equation 1: x + 2y = 3

Equation 2: 5x + ky = – 7

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

<u>For the system of linear equations to have no solutions we must have</u>

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 1$

$a_2 = 5$

$b_1 = 2$

$b_2 = k$

$c_1 = 3$

$c_2 = -7$

Putting the above values in equation (i) and solving we get:

$$\frac{1}{5} = \frac{2}{k}$$

$\Rightarrow k = 10$

Also we find $\frac{1}{5} = \frac{2}{10} \neq \frac{3}{-7}$

<u>The value of k for which the system of equations has no solution is k = 10</u>

4. Question

The value of k for which the system of equations 3x + 5y = 0 and kx + 10y = 0 has a non-zero solution, is

A. 0

B. 2

C. 6

D. 8

Answer

Given:

Equation 1: 3x + 5y = 0

Equation 2: kx + 10y = 0

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

According to the problem:

$a_1 = 3$

$a_2 = k$

$b_1 = 5$

$b_2 = 10$

$c_1 = 0$

$c_2 = 0$

<u>The equation will have a non zero solution only when it will satisfy a non trivial solution i.e. the equations should satisfy with values other than x = 0 & y = 0 .For the given system of equations the equation will have a non zero solution if</u>

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$ ....(i)

Putting the above values in equation (i) we get:

$$\frac{3}{k} = \frac{5}{10}$$

⇒ 5k = 30 ⇒ k = 6

<u>The value of k for which the system of equations has non zero solution is k = 6</u>

5. Question

If the system of equations

2x + 3y = 7

(a + b) x + (2a – b) y = 21

has infinitely many solutions, then

A. a = 1, b = 5

B. a = 5, b = 1

C. a = – 1, b = 5

D. a = 5, b = – 1

Answer

Given:

Equation 1: 2x + 3y = 7

Equation 2: (a + b)x + (2a –b)y = 21

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have infinitely many solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 2$

$a_2 = (a + b)$

$b_1 = 3$

$b_2 = (2a – b)$

$c_1 = 7$

$c_2 = 21$

Putting the above values in equation (i) we get:

$$\frac{2}{a + b} = \frac{3}{2a - b} = \frac{7}{21}$$

On cross multiplication and solving the above equalities we get two sets of linear equation with the variables a & b.

⇒ 7(a + b) = 15*2 ⇒ 7a + 7b = 42 ....(ii)

⇒ 7(2a – b) = 15*3 ⇒ 14a – 7b = 63 ....(iii)

Equation (ii) and (iii) are two linear equations with a and b as variables. To solve this two set of linear equations we use the elimination technique.

In the elimination technique one variable is eliminated by equating it's coefficient with the other equation. From equation (ii) and (iii) we first eliminate the variable b and find the value of a. Since the coefficient of b are equal but of opposite signs so we add equations (ii) and (iii) On adding we get

(14 + 7)a = 42 + 63 ⇒ 21a = 105 ⇒ a = 5

Putting the value of a in equation (ii) we get

$7*5 + 7b = 42 \Rightarrow 7b = 42 - 35 \Rightarrow b = 1$

The value of a & b for which the system of equations has infinitely many solutions is a = 5 & b = 1.

6. Question

If the system of equations

$3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

is inconsistent, then k =

A. 1

B. 0

C. – 1

D. 2

Answer

Given:

Equation 1: $3x + y = 1$

Equation 2: $(2k - 1)x + (k - 1)y = (2k + 1)$

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have no solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 3$

$a_2 = (2k - 1)$

$b_1 = 1$

$b_2 = (k - 1)$

$c_1 = 1$

$c_2 = (2k + 1)$

Putting the above values in equation (i) and solving we get:

$$\frac{3}{2k - 1} = \frac{1}{k - 1}$$

$\Rightarrow 3(k - 1) = 2k - 1 \Rightarrow 3k - 3 = 2k - 1 \Rightarrow k = 3 - 1 \Rightarrow k = 2$

Therefore $\frac{c_1}{c_2} = \frac{1}{2k + 1} \Rightarrow \frac{c_1}{c_2} = \frac{1}{5}$

Putting the value of k we calculate $\frac{a_1}{a_2}, \frac{b_1}{b_2} \& \frac{c_1}{c_2}$

After comparing the ratio we find $\frac{1}{1} = \frac{1}{1} \neq \frac{1}{5}$

So the given system of equations are inconsistent.
<u>The value of k for which the system of equations is inconsistent is k = 2</u>

7. Question

If am ≠ bl, then the system of equations

ax + by = c

lx + my = n

A. has a unique solution

B. has no solution

C. has infinitely many solutions

D. may or may not have a solution.

Answer

Given:

Equation 1: ax + by = c

Equation 2: lx + my = n

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

According to the problem:

$a_1 = a$

$a_2 = l$

$b_1 = b$

$b_2 = m$

$c_1 = c$

$c_2 = n$

According to the question the condition given is
am $\neq$ bl …(i)

To develop a relationship between the coefficients we divide both sides of the equation by l*m
After dividing we get

$$\frac{a}{l} \neq \frac{b}{m}$$

Since $a_1 = a$,$a_2 = l$,$b_1 = b$,$b_2 = m$

So $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

We know from our properties of linear equations that if the ratio of the coefficients of x and y are not equal then their exists a unique solution.

<u>The given system of equation has a unique solution for all values of x and y</u>

8. Question

If the system of equations

2x + 3y = 7

2ax + (a + b)y = 28

has infinitely many solutions, then

A. a = 2b

B. b = 2a

C. a + 2b = 0

D. 2a + b = 0

Answer

Given:

Equation 1: 2x + 3y = 7

Equation 2: 2ax + (a + b)y = 28

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

<u>For the system of linear equations to have infinitely many solutions we must have</u>

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 2$

$a_2 = 2a$

$b_1 = 3$

$b_2 = (a + b)$

$c_1 = 7$

$c_2 = 28$

Putting the above values in equation (i) and solving the extreme left and extreme right portion of the equality we get the value of a

$$\frac{2}{2a} = \frac{7}{28}$$

⇒ $14a = 56 \Rightarrow a = 4$

We now put the value of a and solve for b

$\frac{3}{a+b} = \frac{7}{28} \Rightarrow \frac{3}{a+b} = \frac{1}{4} \Rightarrow a + b = 12 \Rightarrow b = 8$

So $b = 2a$

The correct relationship between a & b for which the system of equations has infinitely many solution is $b = 2a$

9. Question

The value of k for which the system of equations

x + 2y = 5

3x + ky + 15 = 0

has no solution is

A. 6

B. – 6

C. 3/2

D. None of these

Answer

Given:

Equation 1: x + 2y = 5

Equation 2: 3x + ky = – 15

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have no solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 1$

$a_2 = 3$

$b_1 = 2$

$b_2 = k$

$c_1 = 5$

$c_2 = -15$

Putting the above values in equation (i) and solving we get:

$$\frac{1}{3} = \frac{2}{k}$$

⇒ k = 6

Also we find $\frac{1}{3} = \frac{2}{6} \neq \frac{5}{-15}$

The value of k for which the system of equations has no solution is k = 6

10. Question

If 2x – 3y = 7 and (a + b) x – (a + b – 3) y = 4a + b represent coincident lines, then a and b satisfy the equation

A. a + 5b = 0

B. 5a + b = 0

C. a – 5b = 0

D. 5a – b = 0

Answer

Given:

Equation 1: 2x – 3y = 7

Equation 2:(a + b)x + (a + b – 3)y = 4a + b

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

When two sets of linear equations which are coincident then they will have infinite number of solutions since both the equations represent the same line .So we have to use the conditions for the infinitely many number of solution.

For the system of linear equations to have infinitely many solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 2$

$a_2 = a + b$

$b_1 = -3$

$b_2 = a + b - 3$

$c_1 = 7$

$c_2 = 4a + b$

Putting the above values in equation (i) and solving the extreme left and middle portion of the equality

$$\frac{2}{a + b} = \frac{-3}{a + b - 3}$$

⇒ – 3(a + b) = 2(a + b – 3) ⇒ 5a + 5b = 6 …(ii)

Again We Solve for the extreme left and right side of the equality

$$\frac{2}{a + b} = \frac{7}{4a + b}$$

⇒ 8a + 2b = 7a + 7b ⇒ a = 5b (iii)

We solve for a & b from Equation (ii) & (iii).We substitute the value of a from equation (iii) in equation (ii)

After substituting we get

5*5b + 5b = 6 ⇒ 30b = 6⇒ b = $\frac{1}{5}$

Putting the value of b in equation (iii) we get a = 1

So 5b = a

The relationship between a and b for which the two equations represent coincident line is a = 5b or a – 5b = 0

11. Question

If a pair of linear equations in two variables is consistent, then the lines represented by two equations are

A. intersecting

B. parallel

C. always coincident

D. intersecting or coincident

Answer

A pair of linear equations is called inconsistent when the lines doesn't have any solution. It means both the lines are parallel to each other.

A pair of linear equations is called consistent when they have infinite number of solutions or they have a unique solution.

An intersecting line will always have a unique solution.

A coincident line will have infinite number of solutions.

So the line represented by a pair of linear equations in two variables is always intersecting or coincident if the system of equation is consistent.

Let $a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ be two lines where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have infinitely many solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ....(i)

The system of linear equations will have unique solution if

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ ....(ii)

For the system of linear equations to be consistent either condition (i) or (ii) must be satisfied.

If the equations are consistent then they are either intersecting or coincident

12. Question

The area of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is

A. ab

B. 2ab

C. □ $\frac{1}{2}$ab

D. $\frac{1}{4}$ab

Answer

Given:

$$\frac{x}{a} + \frac{y}{b} = 1$$

The given linear equation is in the slope intercept form. Intercept means the distance at which the given equation cuts or meets the coordinate axis. In this problem a & b are the intercepts on the x and y axis respectively.

The triangle formed by a straight line with the coordinate axis is a right angled triangle where the angle subtended at origin is 90^0. So the length of the x intercepts becomes the perpendicular and y intercept becomes the base of the triangle.

We know Area Of a triangle = $\frac{1}{2}$ * (perpendicular length) * (base length)

So Area of the triangle becomes $\frac{1}{2}$ab

The Area of the triangle is $\frac{1}{2}$ab

13. Question

The area of the triangle formed by the lines y = x, x = 6 and y = 0 is

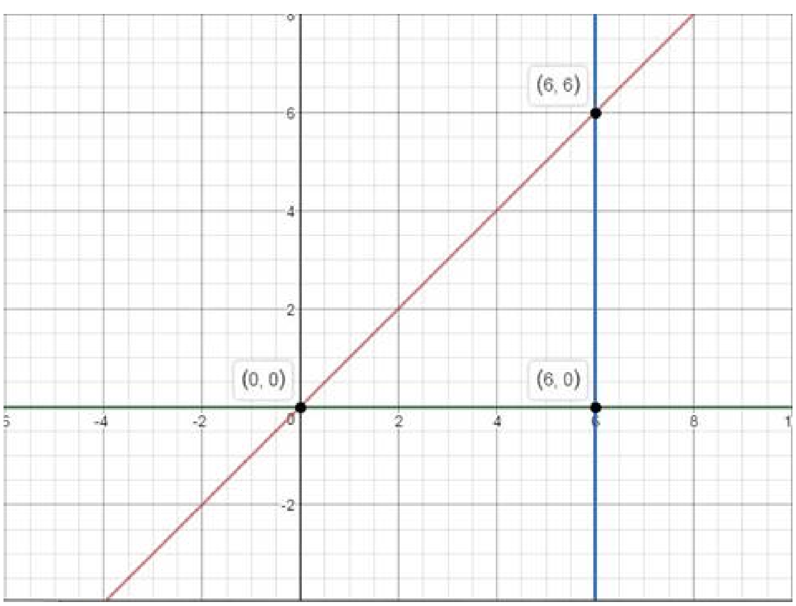
A. 36 sq. units

B. 18 sq. units

C. 9 sq. units

D. 72 sq. units

Answer



Given:

Equation 1: $y = x$ Equation 2: $x = 6$ Equation 3: $y = 0$

According to the given question Equation 1 , 2 & 3 cuts each other at three different points creating a triangle and we need to calculate the area of this triangle formed by these lines.

Now Equation 2 is a line parallel to Y axis at a distance of 6 units. Equation 3 is the equation of the x axis.

So we can say that equation 2 & 3 are mutually perpendicular to each other and the triangle formed by these 3 equations is a right angled triangle .

We solve equation 1 & 2 by substitution method

After solving we get $x = 6$ & $y = 6$

So the perpendicular height of the triangle turns out to be 6 units.

Since Equation 2 is at a distance of 6 units from the origin so the length of the base turns out to be 6 units.

Perpendicular height = 6 units

Base Length = 6 units.

Area of the Triangle = $\frac{1}{2}$ * (perpendicular length) * (base length)

Area of the triangle becomes = $\frac{1}{2} * 6 * 6 = 18$ sq.units

The Area of the triangle is 18 sq. units

14. Question

If the system of equations $2x + 3y = 5$, $4x + ky = 10$ has infinitely many solutions, then k =

A. 1

B. 1/2

C. 3

D. 6

Answer

Given:

Equation 1: $2x + 3y = 5$

Equation 2: $4x + ky = 10$

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have infinitely many solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = 2$

$a_2 = 4$

$b_1 = 3$

$b_2 = k$

$c_1 = 5$

$c_2 = 10$

Putting the above values in equation (i) and solving the extreme left and extreme right portion of the equality we get the value of a

$$\frac{2}{4} = \frac{3}{k}$$

$\Rightarrow 2k = 12 \Rightarrow k = 6$

The value of k for which the system of equations has infinitely many solution is k = 6

15. Question

If the system of equations kx – 5y = 2, 6x + 2y = 7 has no solution, then k =

A. – 10

B. – 5

C. – 6

D. – 15

Answer

Given:

Equation 1: kx – 5y = 2

Equation 2: 6x + 2y = 7

Both the equations are in the form of :

$a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ where

a_1 & a_2 are the coefficients of x

b_1 & b_2 are the coefficients of y

c_1 & c_2 are the constants

For the system of linear equations to have no solutions we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ ....(i)

According to the problem:

$a_1 = k$

$a_2 = 6$

$b_1 = -5$

$b_2 = 2$

$c_1 = 2$

$c_2 = 7$

Putting the above values in equation (i) and solving we get:

$$\frac{k}{6} = \frac{-5}{2}$$

$\Rightarrow$ **k** $= \frac{-5*6}{2}$ $\Rightarrow$ **k = – 15**

Also we find $\frac{-15}{6} = \frac{-5}{2} \neq \frac{2}{7}$

<u>The value of k for which the system of equations has no solution is k = – 15</u>

16. Question

The area of the triangle formed by the lines x = 3, y = 4 and x = y is

A. 1/2 sq. unit

B. 1 sq. unit

C. 2 sq. unit

D. None of these

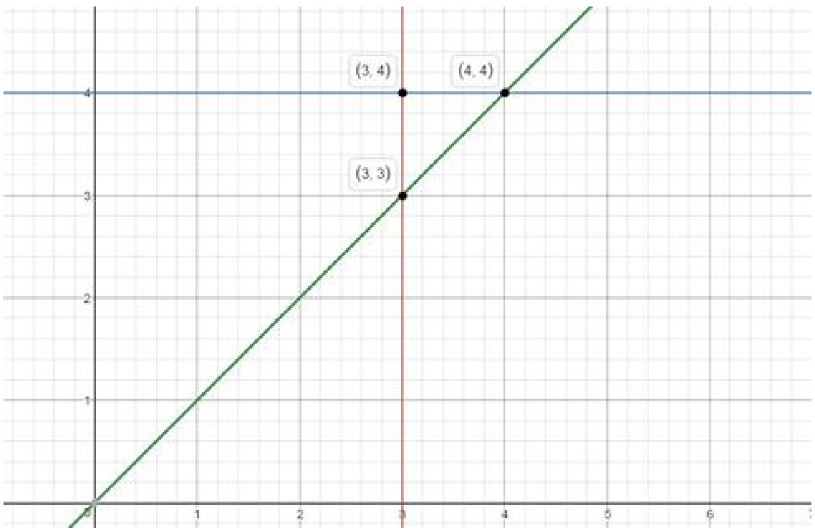
Answer

Given :

Equation 1: x = 3

Equation 2: y = 4

Equation 3: x = y



Equation 1 is a line parallel to y axis

Equation 2 is a line parallel to x axis

So Equation 1 & 2 are mutually perpendicular to each other.

Hence the triangle formed is a right angled triangle.

First we solve the three lines simultaneously by method of substitution and get the three points of intersection or three coordinates of the triangle.

Solving Equation 1 & 2 we get the coordinate (3, 4). Let this Coordinate name be P_1

Solving Equation 2 & 3 we get the coordinate (4 ,4). Let this Coordinate name be P_2

Solving Equation 3 & 1 we get the coordinate (3 ,3). Let this Coordinate name be P_3

We now use the formula for Area of a triangle through 3 given points

Area = $\frac{1}{2}$ *| x_1 * (y_2 – y_3) + x_2 * (y_3 – y_1) + x_3 * (y_1 – y_2) |

Where x_1 ,y_1 are the coordinates of P_1

x_2, y_2 are the coordinates of P_2

x_3 ,y_3 are the coordinates of P_3

Area of the Given Triangle = $\frac{1}{2}$ *| 3* (4– 3) + 4 * (3 – 4) + 3* (4– 4) |

Area = $\frac{1}{2}$ *| 3* (1) + 4 * (– 1) + 3* (0) |

Area = $\frac{1}{2}$ *| 3– 4 |⇒ Area = $\frac{1}{2}$ sq. units

The Area of the triangle is $\frac{1}{2}$ sq. units

17. Question

The area of the triangle formed by the lines 2x + 3y = 12, x – y – 1 = 0 and x = 0 (as shown in Fig. 3.23) , is

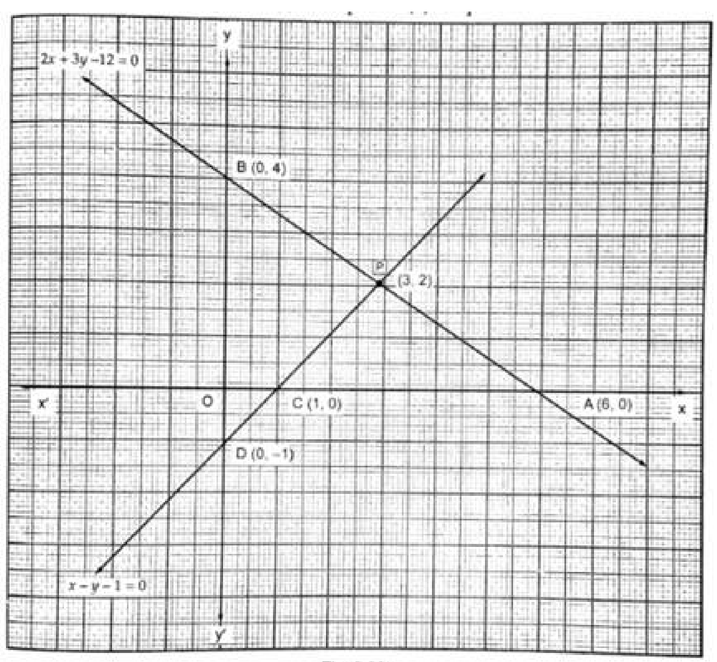


Fig. 3.23

A. 7 sq. units

B. 7.5 sq. units

C. 6.5 sq. units

D. 6 sq. units

Answer

Given

Equation 1: 2x + 3y = 12 Equation 2: x – y = 1 Equation 3: x = 0

To calculate the Area at first we solve the Equation 1 & 2 Simultaneously by method of substitution.

We substitute the value of x from Equation 2 in Equation 1 to get the value of y

Equation 2: x – y = 1 ⇒ x = y + 1

Equation 1: 2x + 3y = 12

Substituting the value from equation 2 we get

2(y + 1) + 3y = 12 ⇒ 2y + 2 + 3y = 12 ⇒ 5y = 10 ⇒ y = 2

Putting the value in Equation 1 we get

x = 2 + 1 ⇒ x = 3

So both this lines passes through (3 , 2) Let this Coordinate name be P_1

Equation 3 is the equation for y axis

Equation 1 meets Y axis at (0 ,4) which is calculated by substituting x = 0 in Equation 1. Let this Coordinate name be P_2

Equation 2 meets Y axis at (0 , – 1) which is calculated by substituting x = 0 in Equation 2. Let this Coordinate name be P_3

So Area of the triangle = $\frac{1}{2} * | x_1 * (y_2 - y_3) + x_2 * (y_3 - y_1) + x_3 * (y_1 - y_2) |$

Where x_1 ,y_1 are the coordinates of P_1

x_2, y_2 are the coordinates of P_2

x_3 ,y_3 are the coordinates of P_3

⇒ Area of the Given Triangle = $\frac{1}{2} * | 3 * (4 + 1) + 0 * (- 1 - 2) + 0 * (2 - 4) |$

⇒ Area of the Given Triangle = $\frac{1}{2} * | 3 * 5 |$

⇒ Area = $\frac{15}{2}$ ⇒ Area = 7.5 Sq. Units

<u>Area of the triangle is 7.5 sq. Units</u>