Chapter

6

Work, Energy, Power and Collision

CONTENTS

Yangtze is longest river in China and the third longest in the world after the Amazon and Nile.

The 'Three Gorges Dam' on the Yangtze will be the largest hydroelectric dam in the world and the dam could cost over 70 billiion US dollar.

Hydro power must be one of the oldest methods of producing power. Huge power generators are placed inside dams. Water flowing through the dams spin turbine blades which are connected to generators.

Work, Energy, Power and Collision

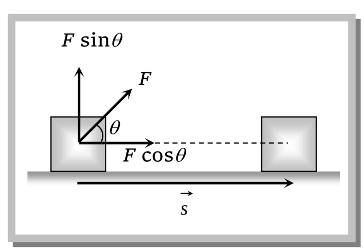
6.1 Introduction

The terms 'work', 'energy' and 'power' are frequently used in everyday language. A farmer clearing weeds in his field is said to be working hard. A woman carrying water from a well to her house is said to be working. In a drought affected region she may be required to carry it over large distances. If she can do so, she is said to have a large stamina or energy. Energy is thus the capacity to do work. The term power is usually associated with speed. In karate, a powerful punch is one delivered at great speed. In physics we shall define these terms very precisely. We shall find that there is at best a loose correlation between the physical definitions and the physiological pictures these terms generate in our minds.

Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force.

6.2 Work Done by a Constant Force

Let a constant force $\vec{F}$ be applied on the body such that it makes an angle θ with the horizontal and body is displaced through a distance s



By resolving force $\vec{F}$ into two components :

(i) $F\cos\theta$ in the direction of displacement of the body.

(ii) $F\sin\theta$ in the perpendicular direction of displacement of the body.

Since body is being displaced in the direction of $F\cos\theta$, therefore work done by the force in displacing the body through a distance s is given by

$$W = (F\cos\theta)s = Fs\cos\theta$$

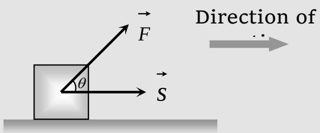
or $$W = \vec{F}.\vec{s}$$

Thus work done by a force is equal to the scalar or dot product of the force and the displacement of the body.

If a number of force $\vec{F}_1, \vec{F}_2, \vec{F}_3......\vec{F}_n$ are acting on a body and it shifts from position vector $\vec{r}_1$ to position vector $\vec{r}_2$ then $W = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...\vec{F}_n).(\vec{r}_2 - \vec{r}_1)$

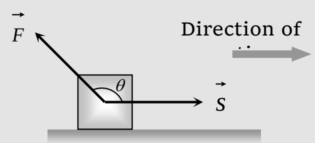
6.3 Nature of Work Done

<table>
<tr><th>Positive work</th><th>Negative work</th></tr>
<tr><td>Positive work means that force (or its component) is parallel to displacement



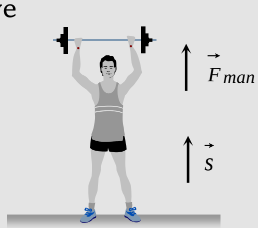
$0^o \leq \theta < 90^o$

The positive work signifies that the external force favours the motion of the body.</td><td>Negative work means that force (or its component) is opposite to displacement *i.e.*

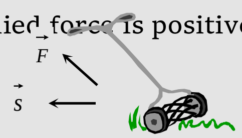


$90^o < \theta \leq 180^o$

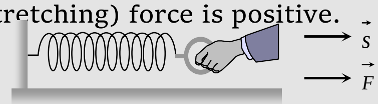
The negative work signifies that the external force opposes the motion of the body.</td></tr>
<tr><td>*Example*: (i) When a person lifts a body from the ground, the work done by the (upward) lifting force is positive

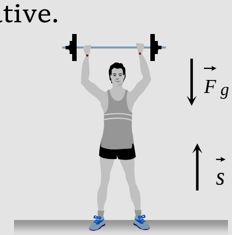


(ii) When a lawn roller is pulled by applying a force along the handle at an acute angle, work done by the applied force is positive.

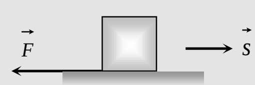


(iii) When a spring is stretched, work done by the external (stretching) force is positive.

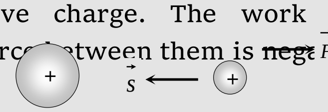

</td><td>*Example*: (i) When a person lifts a body from the ground, the work done by the (downward) force of gravity is negative.



(ii) When a body is made to slide over a rough surface, the work done by the frictional force is negative.



(iii) When a positive charge is moved towards another positive charge. The work done by electrostatic force between them is negative.


</td></tr>
<tr><td>Maximum work : $W_{max} = F s$</td><td>Minimum work : $W_{min} = -F s$</td></tr>
</table>

When $\cos\theta = \text{maximum} = 1$ *i.e.* $\theta = 0^o$	When $\cos\theta = \text{minimum} = -1$ *i.e* $\theta = 180^o$
It means force does maximum work when angle between force and displacement is zero.	It means force does minimum [maximum negative] work when angle between force and displacement is 180°.

Zero work

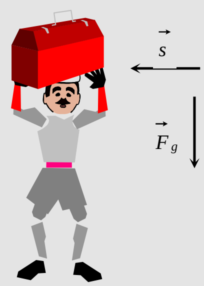
Under three condition, work done becomes zero $W = Fs\cos\theta = 0$

(1) **If the force is perpendicular to the displacement** $[\vec{F} \perp \vec{s}]$

Example: (i) When a coolie travels on a horizontal platform with a load on his head, work done against gravity by the coolie is zero.

(ii) When a body moves in a circle the work done by the centripetal force is always zero.

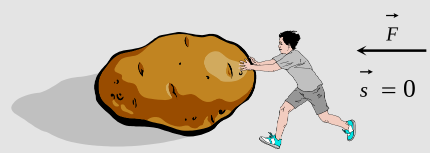
(iii) In case of motion of a charged particle in a magnetic field as force $[\vec{F} = q(\vec{v} \times \vec{B})]$ is always perpendicular to motion, work done by this force is always zero.



(2) **If there is no displacement [s = 0]**

Example: (i) When a person tries to displace a wall or heavy stone by applying a force then it does not move, the work done is zero.

(ii) A weight lifter does work in lifting the weight off the ground but does not work in holding it up.



(3) **If there is no force acting on the body [F = 0]**

Example: Motion of an isolated body in free space.

Sample Problems based on work done by constant force

Problem 1. A body of mass 5 *kg* is placed at the origin, and can move only on the *x*-axis. A force of 10 *N* is acting on it in a direction making an angle of 60^o with the *x*-axis and displaces it along the *x*-axis by 4 *metres*. The work done by the force is

(a) 2.5 *J* (b) 7.25 *J* (c) 40 *J* (d) 20 *J*

Solution : (d) Work done $= \vec{F}.\vec{s} = F\,s\cos\theta = 10 \times 4 \times \cos 60^o = 20\,J$

Problem 2. A force $F = (5\hat{i} + 3\hat{j})\,N$ is applied over a particle which displaces it from its origin to the point $r = (2\hat{i} - 1\hat{j})$ *metres*. The work done on the particle is

(a) –7 *J* (b) +13 *J* (c) +7 *J* (d) +11 *J*

Solution : (c) Work done $= \vec{F}.\vec{r} = (5i + 3\hat{j}).(2\hat{i} - \hat{j}) = 10 - 3 = +7\,J$

Problem 3. A horizontal force of 5 *N* is required to maintain a velocity of 2 *m/s* for a block of 10 *kg* mass sliding over a rough surface. The work done by this force in one minute is

(a) 600 *J* (b) 60 *J* (c) 6 *J* (d) 6000 *J*

Solution : (a) Work done = Force × displacement = $F \times s = F \times v \times t = 5 \times 2 \times 60 = 600\,J$.

Problem 4. A box of mass 1 *kg* is pulled on a horizontal plane of length 1 *m* by a force of 8 *N* then it is raised vertically to a height of 2*m*, the net work done is

(a) 28 *J* (b) 8 *J* (c) 18 *J* (d) None of above

Solution : (a) Work done to displace it horizontally = $F \times s = 8 \times 1 = 8\,J$

Work done to raise it vertically $F \times s = mgh = 1 \times 10 \times 2 = 20\,J$

∴ Net work done = 8 +20 = 28 *J*

Problem 5. A 10 *kg* satellite completes one revolution around the earth at a height of 100 *km* in 108 *minutes*. The work done by the gravitational force of earth will be

(a) $108 \times 100 \times 10\ J$ (b) $\frac{108 \times 10}{100}\ J$ (c) $\frac{100 \times 10}{108}\ J$ (d) Zero

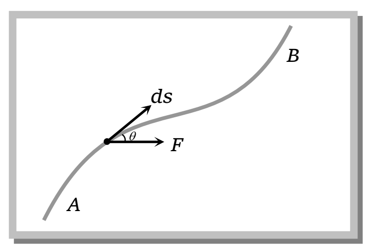
Solution : (d) Work done by centripetal force in circular motion is always equal to zero.

6.4 Work Done by a Variable Force

When the magnitude and direction of a force varies with position, the work done by such a force for an infinitesimal displacement is given by $dW = \vec{F}.d\vec{s}$

The total work done in going from *A* to *B* as shown in the figure is

$$W = \int_A^B \vec{F}.d\vec{s} = \int_A^B (F\cos\theta)ds$$



In terms of rectangular component $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\therefore\ W = \int_A^B (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

or $W = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$

Sample Problems based on work done by variable force

Problem 6. A position dependent force $\vec{F} = (7 - 2x + 3x^2)N$ acts on a small abject of mass 2 *kg* to displace it from $x = 0$ to $x = 5m$. The work done in joule is

(a) 70 *J* (b) 270 *J* (c) 35 *J* (d) 135 *J*

Solution : (d) Work done $= \int_{x_1}^{x_2} F\,dx = \int_0^5 (7 - 2x + 3x^2)dx = [7x - x^2 + x^3]_0^5 = 35 - 25 + 125 = 135\ J$

Problem 7. A particle moves under the effect of a force $F = Cx$ from $x = 0$ to $x = x_1$. The work done in the process is

[CPMT 1982]

(a) Cx_1^2 (b) $\frac{1}{2}Cx_1^2$ (c) Cx_1 (d) Zero

Solution : (b) Work done $= \int_{x_1}^{x_2} F\,dx = \int_0^{x_1} Cx\,dx = C\left[\frac{x^2}{2}\right]_0^{x_1} = \frac{1}{2}C\,x_1^2$

Problem **8.** The vessels *A* and *B* of equal volume and weight are immersed in water to a depth *h*. The vessel *A* has an opening at the bottom through which water can enter. If the work done in immersing *A* and *B* are W_A and W_B respectively, then

(a) $W_A = W_B$ (b) $W_A < W_B$ (c) $W_A > W_B$ (d) $W_A >=< W_B$

Solution : (b) When the vessels are immersed in water, work has to be done against up-thrust force but due to opening at the bottom in vessel *A*, up-thrust force goes on decreasing. So work done will be less in this case.

Problem **9.** Work done in time *t* on a body of mass *m* which is accelerated from rest to a speed *v* in time t_1 as a function of time *t* is given by

(a) $\frac{1}{2}m\frac{v}{t_1}t^2$ (b) $m\frac{v}{t_1}t^2$ (c) $\frac{1}{2}\left(\frac{mv}{t_1}\right)^2 t^2$ (d) $\frac{1}{2}m\frac{v^2}{t_1^2}t^2$

Solution : (d) Work done $= F.s = ma.\left(\frac{1}{2}at^2\right) = \frac{1}{2}m\,a^2\,t^2 = \frac{1}{2}m\left(\frac{v}{t_1}\right)^2 t^2$ $\left[\text{As acceleration } (a) = \frac{v}{t_1} \text{ given}\right]$

6.5 Dimension and Units of Work

Dimension : As work = Force × displacement

∴ [*W*] = [Force] × [Displacement]

$= [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

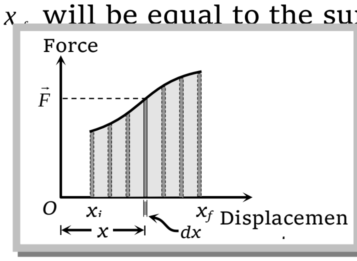
Units : The units of work are of two types

Absolute units	Gravitational units
Joule [S.I.]: Work done is said to be one *Joule*, when 1 *Newton* force displaces the body through 1 *meter* in its own direction. From *W* = *F.s* 1 *Joule* = 1 *Newton* × 1 *metre*	*kg-m* [S.I.]: 1 *Kg-m* of work is done when a force of 1*kg-wt*. displaces the body through 1*m* in its own direction. From *W* = *F s* 1 *kg-m* = 1 *kg-wt* × 1 *metre* = 9.81 *N* × 1 *metre* = 9.81 *Joule*
Erg [C.G.S.] : Work done is said to be one *erg* when 1 *dyne* force displaces the body through 1 *cm* in its own direction. From *W* = *F s* 1 *Erg* = 1*Dyne* × 1*cm* *Relation between Joule and erg* 1 *Joule* = 1 *N* × 1 *m* = 10^5 *dyne* × 10^2 *cm* = 10^7 *dyne* × *cm* = 10^7 *Erg*	*gm-cm* [C.G.S.] : 1 *gm-cm* of work is done when a force of 1*gm-wt* displaces the body through 1*cm* in its own direction. From *W* = *F s* 1 *gm-cm* = 1*gm-wt* × 1*cm*. = 981 *dyne* × 1*cm* = 981 *erg*

6.6 Work Done Calculation by Force Displacement Graph

Let a body, whose initial position is x_i, is acted upon by a variable force (whose magnitude is changing continuously) and consequently the body acquires its final position x_f.

Let $\vec{F}$ be the average value of variable force within the interval *dx* from position *x* to (*x* + *dx*) *i.e.* for small displacement *dx*. The work done will be the area of the shaded strip of width *dx*. The work done on the body in displacing it from position x_i to x_f will be equal to the sum of areas of all the such strips



$$dW = \vec{F}\,dx$$

$$\therefore W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} \vec{F}\,dx$$

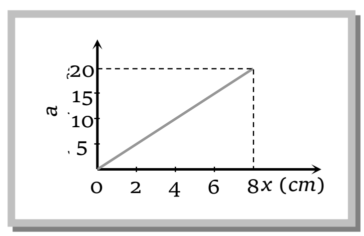
$$\therefore W = \int_{x_i}^{x_f} (\text{Area of strip of width } dx)$$

$$\therefore W = \text{Area under curve Between } x_i \text{ and } x_f$$

i.e. Area under force displacement curve with proper algebraic sign represents work done by the force.

Sample problems based on force displacement graph

***Problem* 10.** A 10 *kg* mass moves along *x*-axis. Its acceleration as a function of its position is shown in the figure. What is the total work done on the mass by the force as the mass moves from $x = 0$ to $x = 8\,cm$ **[AMU (Med.) 2000]**



(a) $8 \times 10^{-2}\ J$

(b) $16 \times 10^{-2}\ J$

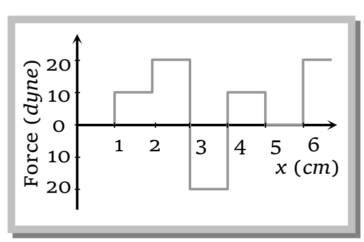
(c) $4 \times 10^{-4}\ J$

(d) $1.6 \times 10^{-3}\ J$

Solution : (a) Work done on the mass = mass × covered area between the graph and displacement axis on *a-t* graph.

$$= 10 \times \frac{1}{2}(8 \times 10^{-2}) \times 20 \times 10^{-2} = 8 \times 10^{-2}\ J.$$

***Problem* 11.** The relationship between force and position is shown in the figure given (in one dimensional case). The work done by the force in displacing a body from $x = 1$ *cm* to $x = 5$ *cm* is **[CPMT 1976]**



(a) 20 *ergs*

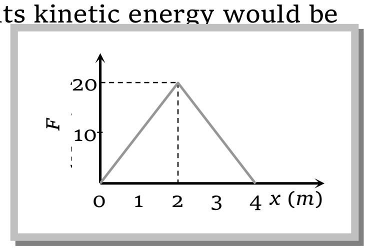
(b) 60 *ergs*

(c) 70 *ergs*

(d) 700 *ergs*

Solution : (a) Work done = Covered area on force-displacement graph = 1 × 10 + 1 × 20 – 1 × 20 + 1 × 10 = 20 *erg*.

Problem **12.** The graph between the resistive force F acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 *kg* and initial velocity is 2 *m/s*. When the distance covered by the body is $5m$, its kinetic energy would be



(a) 50 *J*

(b) 40 *J*

(c) 20 *J*

(d) 10 *J*

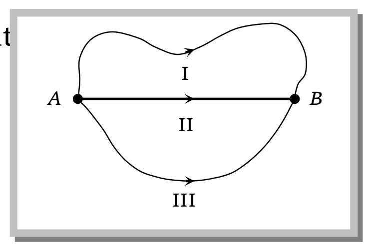
Solution : (d) Initial kinetic energy of the body $= \frac{1}{2}mu^2 = \frac{1}{2} \times 25 \times (2)^2 = 50\ J$

Final kinetic energy = Initial energy – work done against resistive force (Area between graph and displacement axis)

$$= 50 - \frac{1}{2} \times 4 \times 20 = 50 - 40 = 10\ J.$$

6.7 Work Done in Conservative and Non-Conservative Field

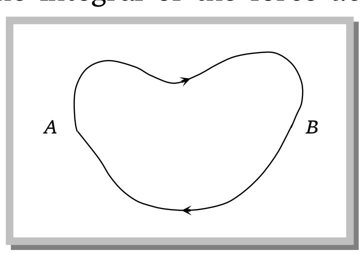
(1) In conservative field work done by the force (line integral of the force *i.e.* $\int \vec{F}.\vec{dl}$) is independent of the path followed between any two points.



$$\underset{\text{Path I}}{W_{A\to B}} = \underset{\text{Path II}}{W_{A\to B}} = \underset{\text{Path III}}{W_{A\to B}}$$

or $$\underset{\text{Path I}}{\int \vec{F}.\vec{dl}} = \underset{\text{Path II}}{\int \vec{F}.\vec{dl}} = \underset{\text{Path III}}{\int \vec{F}.\vec{dl}}$$

(2) In conservative field work done by the force (line integral of the force *i.e.* $\int \vec{F}.\vec{dl}$) over a closed path/loop is zero.



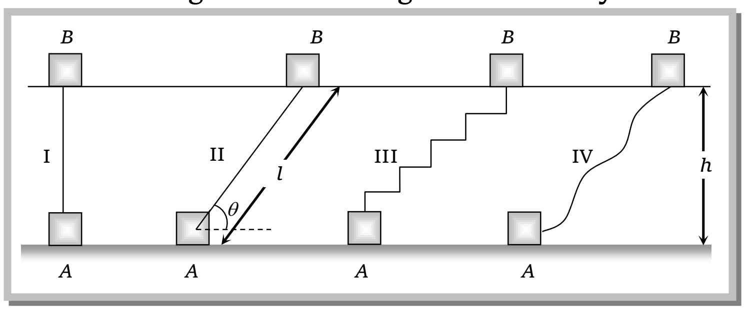
$$W_{A\to B} + W_{B\to A} = 0$$

or $$\oint \bar{F}.d\bar{l} = 0$$

Conservative force : The forces of these type of fields are known as conservative forces.

Example : Electrostatic forces, gravitational forces, elastic forces, magnetic forces *etc* and all the central forces are conservative in nature.

If a body of man *m* lifted to height *h* from the ground level by different path as shown in the figure



Work done through different paths

$$W_I = F.s = mg \times h = mgh$$

$$W_{II} = F.s = mg\ \sin\theta \times l = mg\ \sin\theta \times \frac{h}{\sin\theta} = mgh$$

$$W_{III} = mgh_1 + 0 + mgh_2 + 0 + mgh_3 + 0 + mgh_4 = mg(h_1 + h_2 + h_3 + h_4) = mgh$$

$$W_{IV} = \int \vec{F}.d\vec{s} = mgh$$

It is clear that $W_I = W_{II} = W_{III} = W_{IV} = mgh$.

Further if the body is brought back to its initial position *A,* similar amount of work (energy) is released from the system it means $W_{AB} = mgh$

and $W_{BA} = -mgh$.

Hence the net work done against gravity over a round strip is zero.

$$W_{Net} = W_{AB} + W_{BA}$$

$$= mgh + (-mgh) = 0$$

i.e. the gravitational force is conservative in nature.

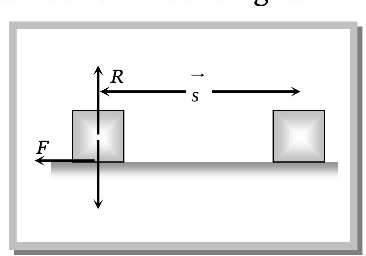
Non-conservative forces : A force is said to be non-conservative if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions and for complete cycle this work done can never be a zero.

Example: Frictional force, Viscous force, Airdrag *etc*.

If a body is moved from position *A* to another position *B* on a rough table, work done against frictional force shall depends on the length of the path between *A* and *B* and not only on the position *A* and *B*.

$$W_{AB} = \mu mgs$$

Further if the body is brought back to its initial position *A,* work has to be done against the



frictional force, which always opposes the motion. Hence the net work done against the friction over a round trip is not zero.

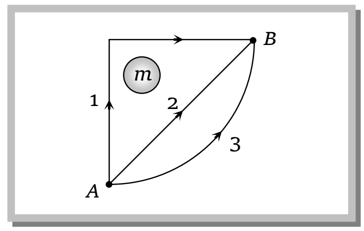
$$W_{BA} = \mu mgs.$$

$$\therefore W_{Net} = W_{AB} + W_{BA} = \mu mgs + \mu mgs = 2\mu mgs \neq 0.$$

i.e. the friction is a non-conservative force.

Sample problems based on work done in conservative and non-conservative field

***Problem* 13.** If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation



(a) $W_1 > W_2 > W_3$

(b) $W_1 = W_2 = W_3$

(c) $W_1 < W_2 < W_3$

(d) $W_2 > W_1 > W_3$

Solution : (b) As gravitational field is conservative in nature. So work done in moving a particle from A to B does not depends upon the path followed by the body. It always remains same.

***Problem* 14.** A particle of mass 0.01 *kg* travels along a curve with velocity given by $4\hat{i} + 16\hat{k}\ ms^{-1}$. After some time, its velocity becomes $8\hat{i} + 20\hat{j}\ ms^{-1}$ due to the action of a conservative force. The work done on particle during this interval of time is

(a) 0.32 *J* (b) 6.9 *J* (c) 9.6 *J* (d) 0.96 *J*

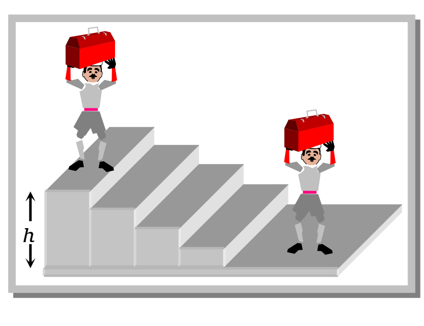
Solution : (d) $v_1 = \sqrt{4^2 + 16^2} = \sqrt{272}$ and $v_2 = \sqrt{8^2 + 20^2} = \sqrt{464}$

Work done = Increase in kinetic energy $= \frac{1}{2} m[v_2^2 - v_1^2] = \frac{1}{2} \times 0.01[464 - 272] = 0.96\ J$.

6.8 Work Depends on Frame of Reference

With change of frame of reference (inertial) force does not change while displacement may change. So the work done by a force will be different in different frames.

Examples : (1) If a porter with a suitcase on his head moves up a staircase, work done by the upward lifting force relative to him will be zero (as displacement relative to him is zero) while relative to a person on the ground will be *mgh*.



(2) If a person is pushing a box inside a moving train, the work done in the frame of train will $\vec{F}.\vec{s}$ while in the frame of earth will be $\vec{F}.(\vec{s} + \vec{s}_0)$ where $\vec{s}_0$ is the displacement of the train relative to the ground.

6.9 Energy

The energy of a body is defined as its capacity for doing work.

(1) Since energy of a body is the total quantity of work done therefore it is a scalar quantity.

(2) Dimension: $[ML^2T^{-2}]$ it is same as that of work or torque.

(3) Units : *Joule* [S.I.], *erg* [C.G.S.]

Practical units : *electron volt* (*eV*), Kilowatt hour (*KWh*), Calories (*Cal*)

Relation between different units: 1 *Joule* = 10^7 *erg*

1 *eV* = 1.6×10^{-19} *Joule*

1 *KWh* = 3.6×10^{6} *Joule*

1 *Calorie* = 4.18 *Joule*

(4) Mass energy equivalence : Einstein's special theory of relativity shows that material particle itself is a form of energy.

The relation between the mass of a particle *m* and its equivalent energy is given as

$E = mc^2$ where c = velocity of light in vacuum.

If $m = 1\, amu = 1.67 \times 10^{-27}\, kg$ then $E = 931\, MeV = 1.5 \times 10^{-10}\, Joule$.

If $m = 1 kg$ then $E = 9 \times 10^{16}\, Joule$

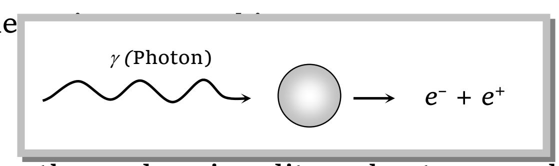
Examples : (i) Annihilation of matter when an electron (e^-) and a positron (e^+) combine with each other, they annihilate or destroy each other. The masses of electron and positron are converted into energy. This energy is released in the form of γ-rays.

$$e^- + e^+ \rightarrow \gamma + \gamma$$

Each γ photon has energy = 0.51 *MeV*.

Here two γ photons are emitted instead of one γ photon to conserve the linear momentum.

(ii) Pair production : This process is the reverse of annihilation of matter. In this case, a photon (γ) having energy equal to 1.02 *MeV* interacts with a nucleus and give rise to electron $(e^-)$and positron (e^+). This ene



(iii) Nuclear bomb : When the nucleus is split up due to mass defect (The difference in the mass of nucleons and the nucleus) energy is released in the form of γ-radiations and heat.

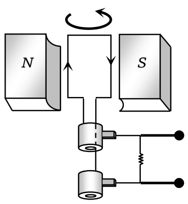
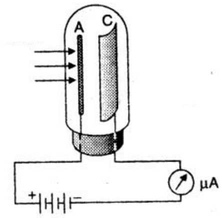
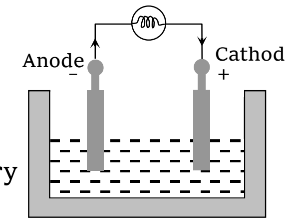
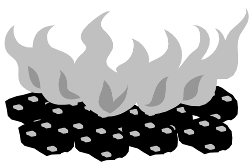
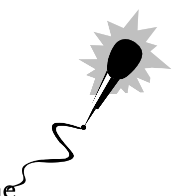
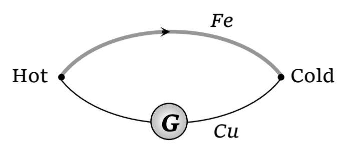
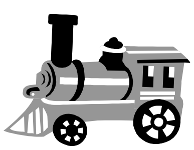
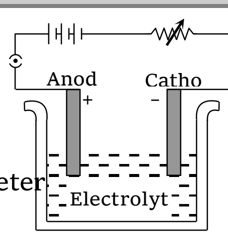
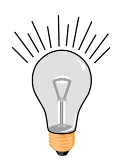
(5) Various forms of energy

(i) Mechanical energy (Kinetic and Potential) (ii) Chemical energy (iii) Electrical energy

(iv) Magnetic energy (v) Nuclear energy (vi) Sound energy

(vii) Light energy (viii) Heat energy

(6) Transformation of energy : Conversion of energy from one form to another is possible through various devices and processes.

Mechanical → electrical	Light → Electrical	Chemical → electrical
Dynamo 	Photoelectri cell 	Primary cell 
Chemical → heat	**Sounds → Electrical**	**Heat → electrical**
Coal Burning 	Microphone 	Thermo-couple 
Heat → Mechanical	**Electrical → Mechanical**	**Electrical → Heat**
Engine 	Motor 	Heater 
Electrical → Sound	**Electrical → Chemical**	**Electrical → Light**
Speaker 	Voltameter 	Bulb 

Sample problems based on energy

***Problem* 15.** A particle of mass 'm' and charge 'q' is accelerated through a potential difference of 'V' *volt*. Its energy is

[UPSEAT 2001]

(a) qV (b) mqV (c) $\left(\frac{q}{m}\right)V$ (d) $\frac{q}{mV}$

Solution : (a) Energy of charged particle = charge × potential difference = qV

Problem **16.** An ice cream has a marked value of 700 *kcal*. How many kilowatt hour of energy will it deliver to the body as it is digested

(a) 0.81 *kWh* (b) 0.90 *kWh* (c) 1.11 *kWh* (d) 0.71 *kWh*

Solution : (a) 700 *k cal* = $700 \times 10^3 \times 4.2 J = \frac{700 \times 10^3 \times 4.2}{3.6 \times 10^6} = 0.81\, kWh$ [As $3.6 \times 10^6 J = 1 kWh$]

Problem **17.** A metallic wire of length *L metres* extends by *l metres* when stretched by suspending a weight *Mg* to it. The mechanical energy stored in the wire is

(a) $2Mgl$ (b) Mgl (c) $\frac{Mgl}{2}$ (d) $\frac{Mgl}{4}$

Solution : (c) Elastic potential energy stored in wire $U = \frac{1}{2}Fx = \frac{Mgl}{2}$.

6.10 Kinetic Energy

The energy possessed by a body by virtue of its motion is called kinetic energy.

Examples : (i) Flowing water possesses kinetic energy which is used to run the water mills.

(ii) Moving vehicle possesses kinetic energy.

(iii) Moving air (*i.e.* wind) possesses kinetic energy which is used to run wind mills.

(iv) The hammer possesses kinetic energy which is used to drive the nails in wood.

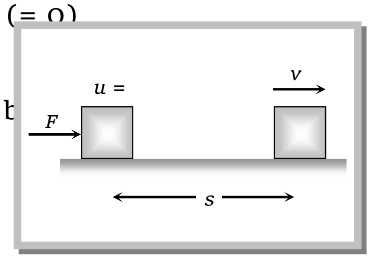
(v) A bullet fired from the gun has kinetic energy and due to this energy the bullet penetrates into a target.

(1) **Expression for kinetic energy :** Let

m = mass of the body, u = Initial velocity of the body (= 0)

F = Force acting on the body, a = Acceleration of the body

s = Distance travelled by the body, v = Final velocity of the b



From $v^2 = u^2 + 2as$

$$\Rightarrow \quad v^2 = 0 + 2as \quad \therefore s = \frac{v^2}{2a}$$

Since the displacement of the body is in the direction of the applied force, then work done by the force is

$$W = F \times s = ma \times \frac{v^2}{2a}$$

$$\Rightarrow \quad W = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy of the body $KE = W = \frac{1}{2}mv^2$

(2) **Calculus method :** Let a body is initially at rest and force $\vec{F}$ is applied on the body to displace it through $d\bar{s}$ along its own direction then small work done

$$dW = \vec{F}.d\vec{s} = F\,ds$$

$\Rightarrow$ $dW = m\,a\,ds$ [As $F = ma$]

$\Rightarrow$ $dW = m\dfrac{dv}{dt}ds$ $\left[\text{As } a = \dfrac{dv}{dt}\right]$

$\Rightarrow$ $dW = mdv.\dfrac{ds}{dt}$

$\Rightarrow$ $dW = m\,v\,dv$ ....(i) $\left[\text{As } \dfrac{ds}{dt} = v\right]$

Therefore work done on the body in order to increase its velocity from zero to v is given by

$$W = \int_0^v mv\,dv = m\int_0^v v\,dv = m\left[\frac{v^2}{2}\right]_0^v = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy of the body $KE = \frac{1}{2}mv^2$.

In vector form $KE = \frac{1}{2}m(\vec{v}.\vec{v})$

As m and $\vec{v}.\vec{v}$ are always positive, kinetic energy is always positive scalar *i.e.* kinetic energy can never be negative.

(3) **Kinetic energy depends on frame of reference :** The kinetic energy of a person of mass m, sitting in a train moving with speed v, is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of the earth.

(4) **Kinetic energy according to relativity :** As we know $E = \frac{1}{2}mv^2$.

But this formula is valid only for ($v << c$) If v is comparable to c (speed of light in free space = $3\times10^8\ m/s$) then according to Einstein theory of relativity

$$E = \frac{mc^2}{\sqrt{1-(v^2/c^2)}} - mc^2$$

(5) **Work-energy theorem:** From equation (i) $dW = mv\,dv$.

Work done on the body in order to increase its velocity from u to v is given by

$$W = \int_u^v mv\,dv = m\int_u^v v\,dv = m\left[\frac{v^2}{2}\right]_u^v$$

$\Rightarrow$ $W = \frac{1}{2}m[v^2 - u^2]$

Work done = change in kinetic energy

$$W = \Delta E$$

This is work energy theorem, it states that work done by a force acting on a body is equal to the change produced in the kinetic energy of the body.

This theorem is valid for a system in presence of all types of forces (external or internal, conservative or non-conservative).

If kinetic energy of the body increases, work is positive *i.e.* body moves in the direction of the force (or field) and if kinetic energy decreases work will be negative and object will move opposite to the force (or field).

Examples : (i) In case of vertical motion of body under gravity when the body is projected up, force of gravity is opposite to motion and so kinetic energy of the body decreases and when it falls down, force of gravity is in the direction of motion so kinetic energy increases.

(ii) When a body moves on a rough horizontal surface, as force of friction acts opposite to motion, kinetic energy will decrease and the decrease in kinetic energy is equal to the work done against friction.

(6) **Relation of kinetic energy with linear momentum:** As we know

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\left[\frac{P}{v}\right]v^2 \qquad [\text{As } P = mv]$$

$$\therefore \qquad E = \frac{1}{2}Pv$$

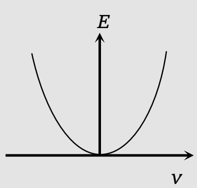
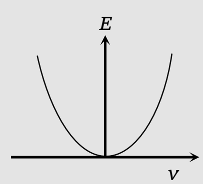
or $$E = \frac{P^2}{2m} \qquad \left[\text{As } v = \frac{P}{m}\right]$$

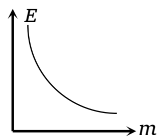
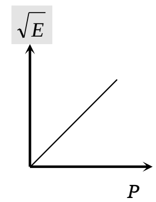
So we can say that kinetic energy $E = \frac{1}{2}mv^2 = \frac{1}{2}Pv = \frac{p^2}{2m}$

and Momentum $P = \frac{2E}{v} = \sqrt{2mE}$.

From above relation it is clear that a body can not have kinetic energy without having momentum and vice-versa.

(7) **Various graphs of kinetic energy**

	$E \propto v^2$ m = constant		$E \propto P^2$ m = constant

 $E \propto \frac{1}{m}$ P = constant	 $P \propto \sqrt{E}$ m = constant

Sample problem based on kinetic energy

Problem **18.** Consider the following two statements

1. Linear momentum of a system of particles is zero

2. Kinetic energy of a system of particles is zero

Then **[AIEEE 2003]**

(a) 1 implies 2 and 2 implies 1 (b) 1 does not imply 2 and 2 does not imply 1

(c) 1 implies 2 but 2 does not imply 1 (d) 1 does not imply 2 but 2 implies 1

Solution : (d) Momentum is a vector quantity whereas kinetic energy is a scalar quantity. If the kinetic energy of a system is zero then linear momentum definitely will be zero but if the momentum of a system is zero then kinetic energy may or may not be zero.

Problem **19.** A running man has half the kinetic energy of that of a boy of half of his mass. The man speeds up by 1 *m/s* so as to have same *K.E.* as that of boy. The original speed of the man will be **[Pb. PMT 2001]**

(a) $\sqrt{2}\, m/s$ (b) $(\sqrt{2}-1) m/s$ (c) $\frac{1}{(\sqrt{2}-1)} m/s$ (d) $\frac{1}{\sqrt{2}} m/s$

Solution : (c) Let m = mass of the boy, M = mass of the man, v = velocity of the boy and V = velocity of the man

Initial kinetic energy of man $= \frac{1}{2}MV^2 = \frac{1}{2}\left[\frac{1}{2}mv^2\right] = \frac{1}{2}\left[\frac{1}{2}\left(\frac{M}{2}\right)v^2\right]$ $\left[\text{As} m = \frac{M}{2} \text{ given}\right]$

$\Rightarrow \quad V^2 = \frac{v^2}{4} \Rightarrow V = \frac{v}{2}$ .....(i)

When the man speeds up by 1 *m/s* , $\frac{1}{2}M(V+1)^2 = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{M}{2}\right)v^2 \quad \Rightarrow (V+1)^2 = \frac{v^2}{2}$

$\Rightarrow V + 1 = \frac{v}{\sqrt{2}}$ .....(ii)

From (i) and (ii) we get speed of the man $V = \frac{1}{\sqrt{2}-1} m/s$.

Problem **20.** A body of mass 10 *kg* at rest is acted upon simultaneously by two forces 4*N* and 3*N* at right angles to each other. The kinetic energy of the body at the end of 10 sec is

(a) 100 J (b) 300 J (c) 50 J (d) 125 J

Solution : (d) As the forces are working at right angle to each other therefore net force on the body $F = \sqrt{4^2 + 3^2} = 5N$

Kinetic energy of the body = work done = $F \times s$ $= F \times \frac{1}{2}at^2 = F \times \frac{1}{2}\left(\frac{F}{m}\right)t^2 = 5 \times \frac{1}{2}\left(\frac{5}{10}\right)(10)^2 = 125\ J.$

Problem **21.** If the momentum of a body increases by 0.01%, its kinetic energy will increase by

(a) 0.01% (b) 0.02 % (c) 0.04 % (d) 0.08 %

Solution : (b) Kinetic energy $E = \frac{P^2}{2m}$ $\therefore\ E \propto P^2$

Percentage increase in kinetic energy = 2(% increase in momentum) [If change is very small]

= 2(0.01%) = 0.02%.

Problem **22.** If the momentum of a body is increased by 100 %, then the percentage increase in the kinetic energy is

[NCERT 1990; BHU 1999; Pb. PMT 1999; CPMT 1999, 2000; CBSE PMT 2001]

(a) 150 % (b) 200 % (c) 225 % (d) 300 %

Solution : (d) $E = \frac{P^2}{2m} \Rightarrow \frac{E_2}{E_1} = \left(\frac{P_2}{P_1}\right)^2 = \left(\frac{2P}{P}\right)^2 = 4$

$E_2 = 4\,E_1 = E_1 + 3E_1 = E_1 + 300$ % of E_1.

Problem **23.** A body of mass 5 *kg* is moving with a momentum of 10 *kg-m/s*. A force of 0.2 *N* acts on it in the direction of motion of the body for 10 seconds. The increase in its kinetic energy is

(a) 2.8 J (b) 3.2 J (c) 3.8 J (d) 4.4 J

Solution : (d) Change in momentum $= P_2 - P_1 = F \times t \Rightarrow P_2 = P_1 + F \times t = 10 + 0.2 \times 10 = 12\,kg\text{-}m/s$

Increase in kinetic energy $E = \frac{1}{2m}[P_2^2 - P_1^2] = \frac{1}{2m}[(12)^2 - (10)^2] = \frac{1}{2 \times 5}[144 - 100] = \frac{44}{10} = 4.4\,J.$

Problem **24.** Two masses of 1*g* and 9*g* are moving with equal kinetic energies. The ratio of the magnitudes of their respective linear momenta is

(a) 1 : 9 (b) 9 : 1 (c) 1 : 3 (d) 3 : 1

Solution : (c) $P = \sqrt{2mE}$ $\therefore\ P \propto \sqrt{m}$ if E = constant . So $\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$.

Problem **25.** A body of mass 2 *kg* is thrown upward with an energy 490 *J*. The height at which its kinetic energy would become half of its initial kinetic energy will be **[** $g = 9.8\,m/s^2$ **]**

(a) 35 m (b) 25 m (c) 12.5 m (d) 10 m

Solution : (c) If the kinetic energy would become half, then Potential energy = $\frac{1}{2}$(Initial kinetic energy)

$$\Rightarrow mgh = \frac{1}{2}[490] \Rightarrow 2 \times 9.8 \times h = \frac{1}{2}[490] \Rightarrow h = 12.5\,m$$

***Problem* 26.** A 300 g mass has a velocity of $(3\hat{i} + 4\hat{j})\, m/sec$ at a certain instant. What is its kinetic energy

(a) 1.35 J (b) 2.4 J (c) 3.75 J (d) 7.35 J

Solution : (c) $\vec{v} = (3\hat{i} + 4\hat{j})$ $\therefore v = \sqrt{3^2 + 4^2} = 5\,m/s$. So kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.3 \times (5)^2 = 3.75\ J$

6.11 Stopping of Vehicle by Retarding Force

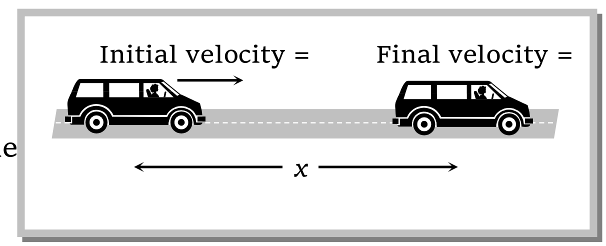
If a vehicle moves with some initial velocity and due to some retarding force it stops after covering some distance after some time.

(1) **Stopping distance :** Let m = Mass of vehicle, v = Velocity, P = Momentum, E = Kinetic energy

F = Stopping force, x = Stopping distance, t = Stopping time

Then, in this process stopping force does work on the vehicle and destroy the motion.

By the work- energy theorem



$$W = \Delta K = \frac{1}{2}mv^2$$

$\Rightarrow$ Stopping force (F) × Distance (x) = Kinetic ene

$\Rightarrow$ Stopping distance (x) $= \frac{\text{Kinetic energy } (E)}{\text{Stopping force } (F)}$

$$\Rightarrow \quad x = \frac{mv^2}{2F} \quad .....(i)$$

(2) **Stopping time :** By the impulse-momentum theorem

$$F \times t = \Delta P \Rightarrow F \times t = P$$

$$\therefore \quad t = \frac{P}{F}$$

or

$$t = \frac{mv}{F} \quad .....(ii)$$

(3) **Comparison of stopping distance and time for two vehicles :** Two vehicles of masses m_1 and m_2 are moving with velocities v_1 and v_2 respectively. When they are stopped by the same retarding force (F).

The ratio of their stopping distances $\frac{x_1}{x_2} = \frac{E_1}{E_2} = \frac{m_1 v_1^2}{m_2 v_2^2}$

and the ratio of their stopping time $\frac{t_1}{t_2} = \frac{P_1}{P_2} = \frac{m_1 v_1}{m_2 v_2}$

If vehicles possess same velocities		
$v_1 = v_2$	$\frac{x_1}{x_2} = \frac{m_1}{m_2}$	$\frac{t_1}{t_2} = \frac{m_1}{m_2}$
If vehicle possess same kinetic momentum		
$P_1 = P_2$	$\frac{x_1}{x_2} = \frac{E_1}{E_2} = \left(\frac{P_1^2}{2m_1}\right)\left(\frac{2m_2}{P_2^2}\right) = \frac{m_2}{m_1}$	$\frac{t_1}{t_2} = \frac{P_1}{P_2} = 1$
If vehicle possess same kinetic energy		
$E_1 = E_2$	$\frac{x_1}{x_2} = \frac{E_1}{E_2} = 1$	$\frac{t_1}{t_2} = \frac{P_1}{P_2} = \frac{\sqrt{2m_1 E_1}}{\sqrt{2m_2 E_2}} = \sqrt{\frac{m_1}{m_2}}$

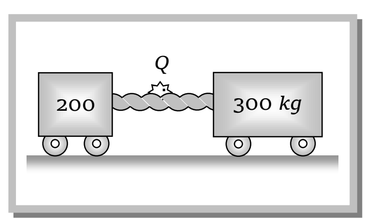
Note : ❑ If vehicle is stopped by friction then

Stopping distance $x = \frac{\frac{1}{2}mv^2}{F} = \frac{\frac{1}{2}mv^2}{ma} = \frac{v^2}{2\mu g}$ [As $a = \mu g$]

Stopping time $t = \frac{mv}{F} = \frac{mv}{m\mu g} = \frac{v}{\mu g}$

~~Sample problems based on stopping of vehicle~~

Problem 27. Two carts on horizontal straight rails are pushed apart by an explosion of a powder charge Q placed between the carts. Suppose the coefficients of friction between the carts and rails are identical. If the 200 *kg* cart travels a distance of 36 *metres* and stops, the distance covered by the cart weighing 300 *kg* is **[CPMT 1989]**



(a) 32 *metres*

(b) 24 *metres*

(c) 16 *metres*

(d) 12 *metres*

Solution : (c) Kinetic energy of cart will goes against friction. $\therefore\ E = \frac{P^2}{2m} = \mu mg \times s \Rightarrow s = \frac{P^2}{2\mu g m^2}$

As the two carts pushed apart by an explosion therefore they possess same linear momentum and coefficient of friction is same for both carts (given). Therefore the distance covered by the cart before coming to rest is given by

$$s \propto \frac{1}{m^2} \quad \therefore\ \frac{s_2}{s_1} = \left(\frac{m_1}{m_2}\right)^2 = \left(\frac{200}{300}\right)^2 = \frac{4}{9} \Rightarrow S_2 = \frac{4}{9} \times 36 = 16 \text{ metres}.$$

Problem **28.** An unloaded bus and a loaded bus are both moving with the same kinetic energy. The mass of the latter is twice that of the former. Brakes are applied to both, so as to exert equal retarding force. If x_1 and x_2 be the distance covered by the two buses respectively before coming to a stop, then

(a) $x_1 = x_2$ (b) $2x_1 = x_2$ (c) $4x_1 = x_2$ (d) $8x_1 = x_2$

Solution : (a) If the vehicle stops by retarding force then the ratio of stopping distance $\frac{x_1}{x_2} = \frac{E_1}{E_2}$.

But in the given problem kinetic energy of bus and car are given same *i.e.* $E_1 = E_2$. $\therefore x_1 = x_2$.

Problem **29.** A bus can be stopped by applying a retarding force F when it is moving with a speed v on a level road. The distance covered by it before coming to rest is s. If the load of the bus increases by 50 % because of passengers, for the same speed and same retarding force, the distance covered by the bus to come to rest shall be

(a) 1.5 s (b) 2 s (c) 1 s (d) 2.5 s

Solution : (a) Retarding force (F) × distance covered (x) = Kinetic energy $\left(\frac{1}{2}mv^2\right)$

If v and F are constants then $x \propto m$ $\therefore \frac{x_2}{x_1} = \frac{m_2}{m_1} = \frac{1.5\,m}{m} = 1.5 \Rightarrow x_2 = 1.5\,s$.

Problem **30.** A vehicle is moving on a rough horizontal road with velocity v. The stopping distance will be directly proportional to

(a) $\sqrt{v}$ (b) v (c) v^2 (d) v^3

Solution : (c) As $s = \frac{v^2}{2a}$ $\therefore s \propto v^2$.

6.12 Potential Energy

Potential energy is defined only for conservative forces. In the space occupied by conservative forces every point is associated with certain energy which is called the energy of position or potential energy. Potential energy generally are of three types : Elastic potential energy, Electric potential energy and Gravitational potential energy *etc.*

(1) **Change in potential energy :** Change in potential energy between any two points is defined in the terms of the work done by the associated conservative force in displacing the particle between these two points without any change in kinetic energy.

$$U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}.d\vec{r} = -W \qquad ......(i)$$

We can define a unique value of potential energy only by assigning some arbitrary value to a fixed point called the reference point. Whenever and wherever possible, we take the reference point at infinite and assume potential energy to be zero there, *i.e.* if take $r_1 = \infty$ and $r_2 = r$ then from equation (i)

$$U = -\int_{\infty}^{r} \vec{F}.d\vec{r} = -W$$

In case of conservative force (field) potential energy is equal to negative of work done in shifting the body from reference position to given position.

This is why in shifting a particle in a conservative field (say gravitational or electric), if the particle moves opposite to the field, work done by the field will be negative and so change in potential energy will be positive *i.e.* potential energy will increase. When the particle moves in the direction of field, work will be positive and change in potential energy will be negative *i.e.* potential energy will decrease.

(2) **Three dimensional formula for potential energy:** For only conservative fields $\vec{F}$ equals the negative gradient $(-\vec{\nabla})$ of the potential energy.

So $\vec{F} = -\vec{\nabla}U$ ($\vec{\nabla}$ read as Del operator or Nabla operator and $\vec{\nabla} = \frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}$)

$$\Rightarrow \quad \vec{F} = -\left[\frac{dU}{dx}\hat{i} + \frac{dU}{dy}\hat{j} + \frac{dU}{dz}\hat{k}\right]$$

where $\frac{dU}{dx}$ = Partial derivative of U *w.r.t.* x (keeping y and z constant)

$\frac{dU}{dy}$ = Partial derivative of U *w.r.t.* y (keeping x and z constant)

$\frac{dU}{dz}$ = Partial derivative of U *w.r.t.* z (keeping x and y constant)

(3) **Potential energy curve :** A graph plotted between the potential energy of a particle and its displacement from the centre of force is called potential energy curve.

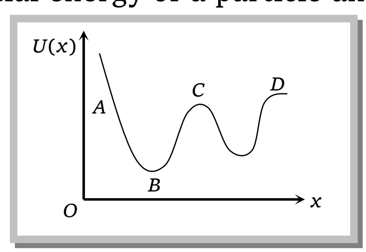


Figure shows a graph of potential energy function $U(x)$ for one dimensional motion.

As we know that negative gradient of the potential energy gives force.

$$\therefore \; -\frac{dU}{dx} = F$$

(4) **Nature of force :**

(i) Attractive force : On increasing x, if U increases $\frac{dU}{dx} = \text{positive}$

then F is negative in direction *i.e.* force is attractive in nature. In graph this is represented in region BC.

(ii) Repulsive force : On increasing x, if U decreases $\frac{dU}{dx} = \text{negative}$

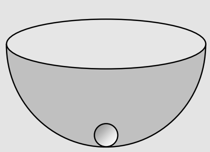
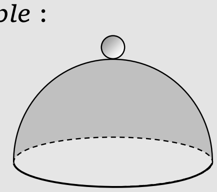
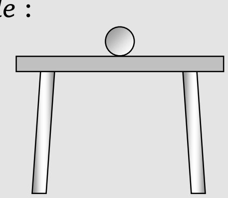
then F is positive in direction *i.e.* force is repulsive in nature. In graph this is represented in region AB.

(iii) Zero force : On increasing x, if U does not changes $\frac{dU}{dx} = 0$

then F is zero *i.e.* no force works on the particle. Point B, C and D represents the point of zero force or these points can be termed as position of equilibrium.

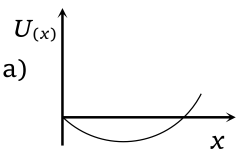
(5) **Types of equilibrium :** If net force acting on a particle is zero, it is said to be in equilibrium.

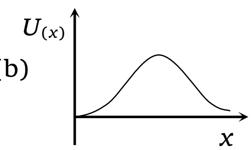
For equilibrium $\frac{dU}{dx} = 0$, but the equilibrium of particle can be of three types :

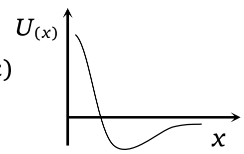
Stable	Unstable	Neutral
When a particle is displaced slightly from a position, then a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.	When a particle is displaced slightly from a position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.	When a particle is slightly displaced from a position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.
Potential energy is minimum.	Potential energy is maximum.	Potential energy is constant.
$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$
$\frac{d^2U}{dx^2} = \text{positive}$ *i.e.* rate of change of $\frac{dU}{dx}$ is positive.	$\frac{d^2U}{dx^2} = \text{negative}$ *i.e.* rate of change of $\frac{dU}{dx}$ is negative.	$\frac{d^2U}{dx^2} = 0$ *i.e.* rate of change of $\frac{dU}{dx}$ is zero.
Example : 	*Example* : 	*Example* : 
A marble placed at the bottom of a hemispherical bowl.	A marble balanced on top of a hemispherical bowl.	A marble placed on horizontal table.

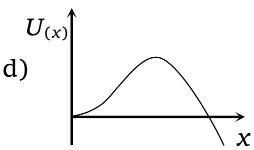
Sample problems based on potential energy

Problem **31.** A particle which is constrained to move along the *x*-axis, is subjected to a force in the same direction which varies with the distance *x* of the particle from the origin as $F(x) = -kx + ax^3$. Here *k* and *a* are positive constants. For $x \geq 0$, the functional from of the potential energy $U_{(x)}$ of the particle is **[IIT-JEE (Screening) 2002]**

(a) 

(b) 

(c) 

(d) 

Solution : (d) $F = -\frac{dU}{dx} \Rightarrow dU = -F.dx \Rightarrow U = -\int_0^x (-kx + ax^3)dx \Rightarrow U = \frac{kx^2}{2} - \frac{ax^4}{4}$

$\therefore$ We get $U = 0$ at *x* = 0 and $x = \sqrt{\frac{2k}{a}}$

Also we get $U =$ negative for $x > \sqrt{\frac{2k}{a}}$

From the given function we can see that *F* = 0 at *x* = 0 *i.e.* slope of *U*-*x* graph is zero at *x* = 0.

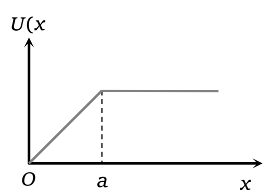
Problem **32.** The potential energy of a body is given by $A - Bx^2$ (where *x* is the displacement). The magnitude of force acting on the particle is

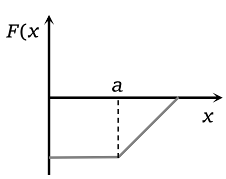
(a) Constant (b) Proportional to *x*

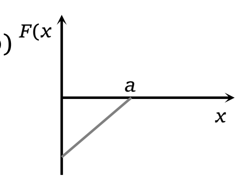
(c) Proportional to x^2 (d) Inversely proportional to *x*

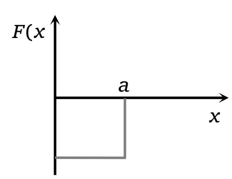
Solution : (b) $F = \frac{-dU}{dx} = -\frac{d}{dx}(A - Bx^2) = 2Bx \quad \therefore F \propto x$.

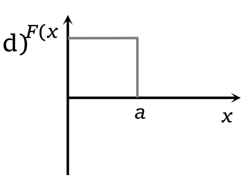
Problem **33.** The potential energy of a system is represented in the first figure. The force acting on the system will be represented by



(a) 

(b) 

(c) 

(d) 

Solution : (c) As slope of problem graph is positive and constant upto distance *a* then it becomes zero. Therefore from $F = -\frac{dU}{dx}$ we can say that upto distance *a* force will be constant (negative) and suddenly it becomes zero.

Problem **34.** A particle moves in a potential region given by $U = 8x^2 - 4x + 400$ *J*. Its state of equilibrium will be

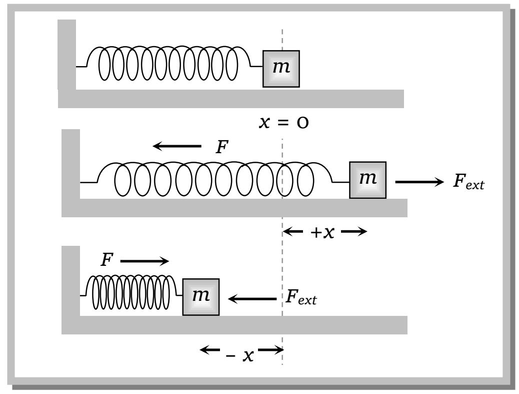
(a) $x = 25\ m$ (b) $x = 0.25\ m$ (c) $x = 0.025\ m$ (d) $x = 2.5\ m$

Solution : (b) $F = -\frac{dU}{dx} = -\frac{d}{dx}(8x^2 - 4x + 400)$

For the equilibrium condition $F = -\frac{dU}{dx} = 0 \Rightarrow 16x - 4 = 0 \Rightarrow x = 4/16 \therefore x = 0.25\ m$.

6.13 Elastic Potential Energy

(1) **Restoring force and spring constant :** When a spring is stretched or compressed from its normal position (x = 0) by a small distance x, then a restoring force is produced in the spring to bring it to the normal position.



According to Hooke's law this restoring force is proportional to the displacement x and its direction is always opposite to the displacement.

i.e. $\vec{F} \propto -\vec{x}$

or $\vec{F} = -k\,\vec{x}$ .....(i)

where k is called spring constant.

If x = 1, F = k (Numerically)

or $k = F$

Hence spring constant is numerically equal to force required to produce unit displacement (compression or extension) in the spring. If required force is more, then spring is said to be more stiff and vice-versa.

Actually k is a measure of the stiffness/softness of the spring.

Dimension : As $k = \frac{F}{x}$ $\therefore\ [k] = \frac{[F]}{[x]} = \frac{[MLT^{-2}]}{L} = [MT^{-2}]$

Units : S.I. unit *Newton/metre*, C.G.S unit *Dyne/cm*.

Note : ❑ Dimension of force constant is similar to surface tension.

(2) **Expression for elastic potential energy :** When a spring is stretched or compressed from its normal position (x = 0), work has to be done by external force against restoring force. $\vec{F}_{\text{ext}} = \vec{F}_{restoring} = k\vec{x}$

Let the spring is further stretched through the distance dx, then work done

$$dW = \vec{F}_{\text{ext}}.d\vec{x} = F_{\text{ext}}.dx\cos 0^o = kx\,dx \qquad [\text{As } \cos 0^o = 1]$$

Therefore total work done to stretch the spring through a distance x from its mean position is given by

$$W = \int_0^x dW = \int_0^x kx\,dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

This work done is stored as the potential energy of the stretched spring.

$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2$

$$U = \frac{1}{2}Fx \qquad \left[\text{As}\, k = \frac{F}{x}\right]$$

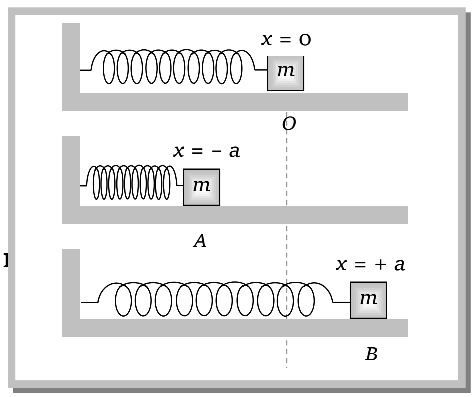
$$U = \frac{F^2}{2k} \qquad \left[\text{As } x = \frac{F}{k}\right]$$

$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$

Note : ❑ If spring is stretched from initial position x_1 to final position x_2 then work done

= Increment in elastic potential energy $= \frac{1}{2}k(x_2^2 - x_1^2)$

(3) **Energy graph for a spring :** If the mass attached with spring performs simple harmonic motion about its mean position then its potential energy at any position (x) can be given by



$$U = \frac{1}{2}kx^2 \qquad ....(i)$$

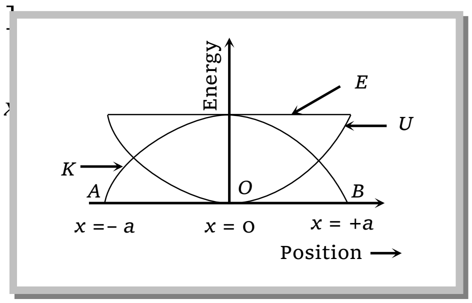
So for the extreme position

$$U = \frac{1}{2}ka^2 \qquad [\text{As } x = \pm a \text{ for extreme}]$$

This is maximum potential energy or the total energy of

$\therefore$ Total energy $E = \frac{1}{2}ka^2$ ....(ii)

[Because velocity of mass = 0 at extreme $\therefore$ $K = \frac{1}{2}mv^2 = 0$]



Now kinetic energy at any position $K = E - U = \frac{1}{2}ka^2 - \frac{1}{2}kx$

$$K = \frac{1}{2}k(a^2 - x^2) \qquad ....(iii)$$

From the above formula we can check that

$U_{max} = \frac{1}{2}ka^2$ [At extreme $x = \pm a$] and $U_{min} = 0$ [At mean $x = 0$]

$K_{max} = \frac{1}{2}ka^2$ [At mean $x = 0$] and $K_{min} = 0$ [At extreme $x = \pm a$]

$E = \frac{1}{2}ka^2 =$ constant (at all positions)

It mean kinetic energy changes parabolically *w.r.t.* position but total energy remain always constant irrespective to position of the mass

Sample problems based on elastic potential energy

Problem 35. A long spring is stretched by 2 *cm*, its potential energy is *U*. If the spring is stretched by 10 *cm*, the potential energy stored in it will be

(a) *U* / 25 (b) *U* / 5 (c) 5 *U* (d) 25 *U*

Solution : (d) Elastic potential energy of a spring $U = \frac{1}{2}kx^2$ $\therefore U \propto x^2$

So $\frac{U_2}{U_1} = \left(\frac{x_2}{x_1}\right)^2 \Rightarrow \frac{U_2}{U} = \left(\frac{10\ cm}{2\ cm}\right)^2 \Rightarrow U_2 = 25\ U$

Problem 36. A spring of spring constant $5 \times 10^3 N/m$ is stretched initially by 5 *cm* from the unstretched position. Then the work required to stretch it further by another 5 *cm* is

(a) 6.25 *N-m* (b) 12.50 *N-m* (c) 18.75 *N-m* (d) 25.00 *N-m*

Solution : (c) Work done to stretch the spring from x_1 to x_2

$$W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}5 \times 10^3[(10 \times 10^{-2})^2 - (5 \times 10^{-2})^2] = \frac{1}{2} \times 5 \times 10^3 \times 75 \times 10^{-4} = 18.75\ N.m\ .$$

Problem 37. Two springs of spring constants $1500\ N/m$ and $3000\ N/m$ respectively are stretched with the same force. They will have potential energy in the ratio

(a) 4 : 1 (b) 1 : 4 (c) 2 : 1 (d) 1 : 2

Solution : (c) Potential energy of spring $U = \frac{F^2}{2k}$ $\Rightarrow$ $\frac{U_1}{U_2} = \frac{k_2}{k_1} = \frac{3000}{1500} = 2:1$ [If *F* = constant]

Problem 38. A body is attached to the lower end of a vertical spiral spring and it is gradually lowered to its equilibrium position. This stretches the spring by a length *x*. If the same body attached to the same spring is allowed to fall suddenly, what would be the maximum stretching in this case

(a) *x* (b) 2*x* (c) 3*x* (d) *x*/2

Solution : (b) When spring is gradually lowered to it's equilibrium position

$kx = mg$ $\therefore$ $x = \frac{mg}{k}$.

When spring is allowed to fall suddenly it oscillates about it's mean position

Let *y* is the amplitude of vibration then at lower extreme, by the conservation of energy

$\Rightarrow \frac{1}{2}ky^2 = mgy \Rightarrow y = \frac{2mg}{k} = 2x.$

Problem 39. Two equal masses are attached to the two ends of a spring of spring constant *k*. The masses are pulled out symmetrically to stretch the spring by a length *x* over its natural length. The work done by the spring on each mass is

(a) $\frac{1}{2}kx^2$ (b) $-\frac{1}{2}kx^2$ (c) $\frac{1}{4}kx^2$ (d) $-\frac{1}{4}kx^2$

Solution : (d) If the spring is stretched by length *x*, then work done by two equal masses = $\frac{1}{2}kx^2$

So work done by each mass on the spring = $\frac{1}{4}kx^2$ $\therefore$ Work done by spring on each mass = $-\frac{1}{4}kx^2$.

6.14 Electrical Potential Energy

It is the energy associated with state of separation between charged particles that interact via electric force. For two point charge q_1 and q_2, separated by distance r.

$$U = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r}$$

While for a point charge q at a point in an electric field where the potential is V

$$U = qV$$

As charge can be positive or negative, electric potential energy can be positive or negative.

Sample problems based on electrical potential energy

***Problem* 40.** A proton has a positive charge. If two protons are brought near to one another, the potential energy of the system will

(a) Increase (b) Decrease

(c) Remain the same (d) Equal to the kinetic energy

Solution : (a) As the force is repulsive in nature between two protons. Therefore potential energy of the system increases.

***Problem* 41.** Two protons are situated at a distance of 100 *fermi* from each other. The potential energy of this system will be in *eV*

(a) 1.44 (b) 1.44×10^3 (c) 1.44×10^2 (d) 1.44×10^4

Solution : (d) $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{100 \times 10^{-15}} = 2.304 \times 10^{-15}\ J = \frac{2.304 \times 10^{-15}}{1.6 \times 10^{-19}}eV = 1.44 \times 10^4\ eV$

***Problem* 42.** $_{80}Hg^{208}$ nucleus is bombarded by α-particles with velocity $10^7\ m/s$. If the α-particle is approaching the *Hg* nucleus head-on then the distance of closest approach will be

(a) $1.115 \times 10^{-13}\ m$ (b) $11.15 \times 10^{-13}\ m$ (c) $111.5 \times 10^{-13}\ m$ (d) Zero

Solution : (a) When α particle moves towards the mercury nucleus its kinetic energy gets converted in potential energy of the system. At the distance of closest approach $\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

$$\Rightarrow \frac{1}{2} \times (1.6 \times 10^{-27})(10^7)^2 = 9 \times 10^9\frac{(2.e)(80\ e)}{r} \Rightarrow r = 1.115 \times 10^{-13}\ m.$$

***Problem* 43.** A charged particle A moves directly towards another charged particle B. For the $(A+B)$ system, the total momentum is P and the total energy is E

(a) P and E are conserved if both A and B are free to move

(b) (a) is true only if A and B have similar charges

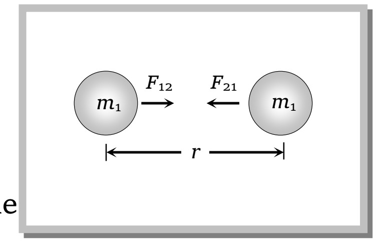
(c) If B is fixed, E is conserved but not P

(d) If B is fixed, neither E nor P is conserved

Solution : (a, c) If A and B are free to move, no external forces are acting and hence P and E both are conserved but when B is fixed (with the help of an external force) then E is conserved but P is not conserved.

6.15 Gravitational Potential Energy

It is the usual form of potential energy and is the energy associated with the state of separation between two bodies that interact via gravitational force.



For two particles of masses m_1 and m_2 separated by a distance r

Gravitational potential energy $U = -\frac{Gm_1m_2}{r}$

(1) If a body of mass m at height h relative to surface of earth the

Gravitational potential energy $U = \frac{mgh}{1+\frac{h}{R}}$

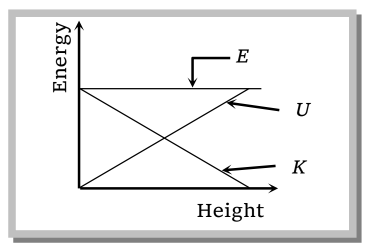
Where R = radius of earth, g = acceleration due to gravity at the surface of the earth.

(2) If $h << R$ then above formula reduces to $U = mgh$.

(3) If V is the gravitational potential at a point, the potential energy of a particle of mass m at that point will be

$$U = mV$$

(4) Energy height graph : When a body projected vertically upward from the ground level with some initial velocity then it possess kinetic energy but its potential energy is zero.



As the body moves upward its potential energy increases due to increase in height but kinetic energy decreases (due to decrease in velocity). At maximum height its kinetic energy becomes zero and potential energy maximum but through out the complete motion total energy remains constant as shown in the figure.

Sample problems based on gravitational potential energy

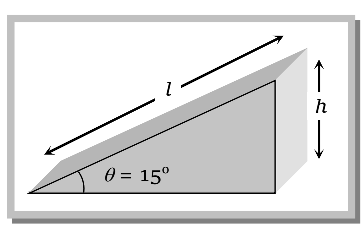
***Problem* 44.** The work done in pulling up a block of wood weighing 2kN for a length of 10 m on a smooth plane inclined at an angle of 15^o with the horizontal is (sin 15° = 0.259)

(a) 4.36 kJ (b) 5.17 kJ (c) 8.91 kJ (d) 9.82 kJ

Solution : (b) Work done = $mg \times h$

$= 2 \times 10^3 \times l \sin\theta$

$= 2 \times 10^3 \times 10 \times \sin 15^o = 5176\ J = 5.17\ kJ$

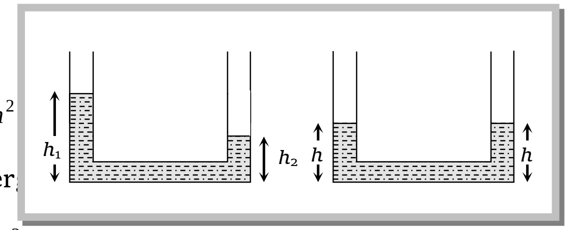


Problem 45. Two identical cylindrical vessels with their bases at same level each contains a liquid of density *d*. The height of the liquid in one vessel is h_1 and that in the other vessel is h_2. The area of either vases is *A*. The work done by gravity in equalizing the levels when the two vessels are connected, is **[SCRA 1996]**

(a) $(h_1 - h_2)gd$ (b) $(h_1 - h_2)gAd$ (c) $\frac{1}{2}(h_1 - h_2)^2 gAd$ (d) $\frac{1}{4}(h_1 - h_2)^2 gAd$

Solution : (d) Potential energy of liquid column is given by $mg\frac{h}{2} = Vdg\frac{h}{2} = Ahdg\frac{h}{2} = \frac{1}{2}Adgh^2$

Initial potential energy $= \frac{1}{2}Adgh_1^2 + \frac{1}{2}Adgh_2^2$



Final potential energy = $\frac{1}{2}Adgh^2 + \frac{1}{2}Adh^2g = Adgh^2$

Work done by gravity = change in potential energy

$$W = \left[\frac{1}{2}Adgh_1^2 + \frac{1}{2}Adgh_2^2\right] - Adgh^2$$

$$= Adg\left[\frac{h_1^2}{2} + \frac{h_2^2}{2}\right] - Adg\left(\frac{h_1 + h_2}{2}\right)^2 \quad \left[\text{As } h = \frac{h_1 + h_2}{2}\right]$$

$$= Adg\left[\frac{h_1^2}{2} + \frac{h_2^2}{2} - \left(\frac{h_1^2 + h_2^2 + 2h_1h_2}{4}\right)\right] = \frac{Adg}{4}(h_1 - h_2)^2$$

Problem 46. If *g* is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an abject of mass *m* raised from the surface of earth to a height equal to the radius of the earth *R*, is **[IIT-JEE1983]**

(a) $\frac{1}{2}mgR$ (b) $2mgR$ (c) mgR (d) $\frac{1}{4}mgR$

Solution : (a) Work done = gain in potential energy $= \frac{mgh}{1 + h/R} = \frac{mgR}{1 + R/R} = \frac{1}{2}mgR$ [As $h = R$ (given)]

Problem 47. The work done in raising a mass of 15 *gm* from the ground to a table of 1*m* height is

(a) 15 *J* (b) 152 *J* (c) 1500 *J* (d) 0.15 *J*

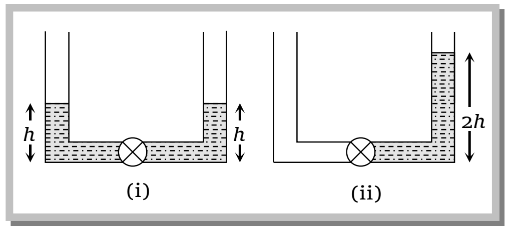
Solution : (d) $W = mgh = 15 \times 10^{-3} \times 10 \times 1 = 0.15\ J$.

Problem 48. A body is falling under gravity. When it loses a gravitational potential energy by *U*, its speed is *v*. The mass of the body shall be

(a) $\frac{2U}{v}$ (b) $\frac{U}{2v}$ (c) $\frac{2U}{v^2}$ (d) $\frac{U}{2v^2}$

Solution : (c) Loss in potential energy = gain in kinetic energy $\Rightarrow U = \frac{1}{2}mv^2 \therefore m = \frac{2U}{v^2}$.

Problem 49. A liquid of density *d* is pumped by a pump *P* from situation (i) to situation (ii) as shown in the diagram. If the cross-section of each of the vessels is a, then the work done in pumping (neglecting friction effects) is



(a) $2dgh$

(b) $dgha$

(c) $2dgh^2a$

(d) dgh^2a

Solution : (d) Potential energy of liquid column in first situation $= Vdg\frac{h}{2} + Vdg\frac{h}{2} = Vdgh = ahdgh = dgh^2a$

[As centre of mass of liquid column lies at height $\frac{h}{2}$]

Potential energy of the liquid column in second situation $= Vdg\left(\frac{2h}{2}\right) = (A \times 2h)dgh = 2dgh^2a$

Work done pumping = Change in potential energy = $2dgh^2a - dgh^2a = dgh^2a$.

***Problem* 50.** The mass of a bucket containing water is 10 *kg*. What is the work done in pulling up the bucket from a well of depth 10 *m* if water is pouring out at a uniform rate from a hole in it and there is loss of 2*kg* of water from it while it reaches the top $(g = 10\, m/\sec^2)$

(a) 1000 *J* (b) 800 *J* (c) 900 *J* (d) 500 *J*

Solution : (c) Gravitational force on bucket at starting position = *mg* = 10 × 10 = 100 *N*

Gravitational force on bucket at final position = 8 × 10 = 80 *N*

So the average force through out the vertical motion $= \frac{100 + 80}{2} = 90\ N$

∴ Work done = Force × displacement = 90 × 10 = 900 *J*.

***Problem* 51.** A rod of mass *m* and length *l* is lying on a horizontal table. The work done in making it stand on one end will be

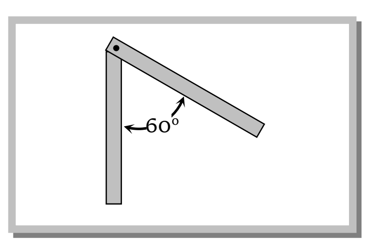
(a) *mgl* (b) $\frac{mgl}{2}$ (c) $\frac{mgl}{4}$ (d) 2*mgl*

Solution : (b) When the rod is lying on a horizontal table, its potential energy = 0

But when we make its stand vertical its centre of mass rises upto high $\frac{l}{2}$. So it's potential energy $= \frac{mgl}{2}$

∴ Work done = charge in potential energy $= mg\frac{l}{2} - 0 = \frac{mgl}{2}$.

***Problem* 52.** A *metre* stick, of mass 400 *g*, is pivoted at one end displaced through an angle 60^o. The increase in its potential energy is



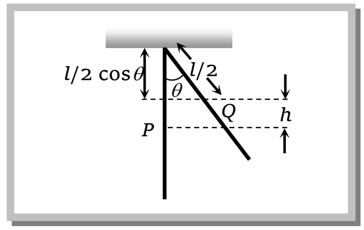
(a) 1 *J*

(b) 10 *J*

(c) 100 *J*

(d) 1000 J

Solution : (a) Centre of mass of a stick lies at the mid point and when the stick is displaced through an angle 60° it rises upto height 'h' from the initial position.



From the figure $h = \frac{l}{2} - \frac{l}{2}\cos\theta \;= \frac{l}{2}(1-\cos\theta)$

Hence the increment in potential energy of the stick = mgh

$$= mg\,\frac{l}{2}(1-\cos\theta) = 0.4 \times 10 \times \frac{1}{2}(1-\cos 60^{o}) = 1\,J$$

Problem **53.** Once a choice is made regarding zero potential energy reference state, the changes in potential energy

(a) Are same

(b) Are different

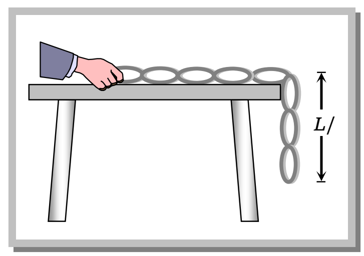
(c) Depend strictly on the choice of the zero of potential energy

(d) Become indeterminate

Solution : (a) Potential energy is a relative term but the difference in potential energy is absolute term. If reference level is fixed once then change in potential energy are same always.

6.16 Work Done in Pulling the Chain Against Gravity

A chain of length L and mass M is held on a frictionless table with $(1/n)^{th}$ of its length hanging over the edge.



Let $m = \frac{M}{L} =$ mass per unit length of the chain and y is the length of the chain hanging over the edge. So the mass of the chain of length y will be ym and the force acting on it due to gravity will be mgy.

The work done in pulling the dy length of the chain on the table.

$dW = F(-dy)$ [As y is decreasing]

i.e. $dW = mgy\,(-dy)$

So the work done in pulling the hanging portion on the table.

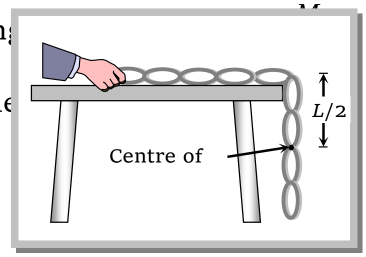
$$W = -\int_{L/n}^{0} mgy\,dy = mg\left[\frac{y^2}{2}\right]_{L/n}^{0} = \frac{mg\,L^2}{2n^2}$$

$\therefore$ $W = \frac{MgL}{2n^2}$ [As $m = M/L$]

Alternative method :

If point mass m is pulled through a height h then work done $W = mgh$

Similarly for a chain we can consider its centre of mass at the middle point of the hanging part *i.e.* at a height of $L/(2n)$ from the lower end and mass of the han

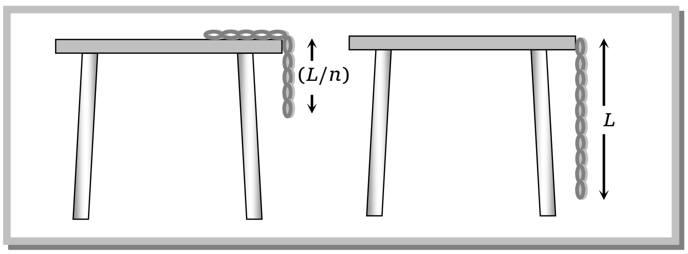


So work done to raise the centre of mass of the chain on the table

$$W = \frac{M}{n} \times g \times \frac{L}{2n} \qquad [\text{As } W = mgh]$$

or
$$W = \frac{MgL}{2n^2}$$

6.17 Velocity of Chain While Leaving the Table



Taking surface of table as a reference level (zero potential energy)

Potential energy of chain when $1/n^{\text{th}}$ length hanging from the edge $= \frac{-MgL}{2n^2}$

Potential energy of chain when it leaves the table $= -\frac{MgL}{2}$

Kinetic energy of chain = loss in potential energy

$$\Rightarrow \qquad \frac{1}{2}Mv^2 = \frac{MgL}{2} - \frac{MgL}{2n^2}$$

$$\Rightarrow \qquad \frac{1}{2}Mv^2 = \frac{MgL}{2}\left[1 - \frac{1}{n^2}\right]$$

$\therefore$ Velocity of chain $v = \sqrt{gL\left(1 - \frac{1}{n^2}\right)}$

Sample problem based on chain

Problem 54. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is

(a) MgL (b) $\frac{MgL}{3}$ (c) $\frac{MgL}{9}$ (d) $\frac{MgL}{18}$

Solution : (d) As 1/3 part of the chain is hanging from the edge of the table. So by substituting $n = 3$ in standard expression

$$W = \frac{MgL}{2n^2} = \frac{MgL}{2(3)^2} = \frac{MgL}{18}.$$

***Problem* 55.** A chain is placed on a frictionless table with one fourth of it hanging over the edge. If the length of the chain is 2*m* and its mass is 4*kg*, the energy need to be spent to pull it back to the table is

(a) 32 *J* (b) 16 *J* (c) 10 *J* (d) 2.5 *J*

Solution : (d) $W = \frac{MgL}{2n^2} = \frac{4 \times 10 \times 2}{2 \times (4)^2} = 2.5\ J.$

***Problem* 56.** A uniform chain of length 2*m* is held on a smooth horizontal table so that half of it hangs over the edge. If it is released from rest, the velocity with which it leaves the table will be nearest to

(a) 2 *m/s* (b) 4 *m/s* (c) 6 *m/s* (d) 8 *m/s*

Solution : (b) $v = \sqrt{gL\left(1 - \frac{1}{n^2}\right)} = \sqrt{10 \times 2\left(1 - \frac{1}{(2)^2}\right)} = \sqrt{15} = 3.87 \simeq 4\ m/s$ (approx.)

6.18 Law of Conservation of Energy

(1) **Law of conservation of energy**

For a body or an isolated system by work-energy theorem we have $K_2 - K_1 = \int \vec{F}.d\vec{r}$ .....(i)

But according to definition of potential energy in a conservative field $U_2 - U_1 = -\int \vec{F}.d\vec{r}$ .....(ii)

So from equation (i) and (ii) we have

$$K_2 - K_1 = -(U_2 - U_1)$$

or $K_2 + U_2 = K_1 + U_1$

i.e. $K + U$ = constant.

For an isolated system or body in presence of conservative forces the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depends upon time. This is known as the law of conservation of mechanical energy.

$\Delta(K + U) = \Delta E = 0$ [As E is constant in a conservative field]

$\therefore$ $\Delta K + \Delta U = 0$

i.e. if the kinetic energy of the body increases its potential energy will decrease by an equal amount and vice-versa.

(2) **Law of conservation of total energy :** If some non-conservative force like friction is also acting on the particle, the mechanical energy is no more constant. It changes by the amount of work done by the frictional force.

$$\Delta(K+U) = \Delta E = W_f \qquad \text{[where } W_f \text{ is the work done against friction]}$$

The lost energy is transformed into heat and the heat energy developed is exactly equal to loss in mechanical energy.

We can, therefore, write $\Delta E + Q = 0$ [where Q is the heat produced]

This shows that if the forces are conservative and non-conservative both, it is not the mechanical energy alone which is conserved, but it is the total energy, may be heat, light, sound or mechanical *etc*., which is conserved.

In other words : "*Energy may be transformed from one kind to another but it cannot be created or destroyed. The total energy in an isolated system is constant*". This is the law of conservation of energy.

Sample problems based on conservation of energy

***Problem* 57.** Two stones each of mass 5*kg* fall on a wheel from a height of 10*m*. The wheel stirs 2*kg* water. The rise in temperature of water would be

(a) 2.6° *C* (b) 1.2° *C* (c) 0.32° *C* (d) 0.12° *C*

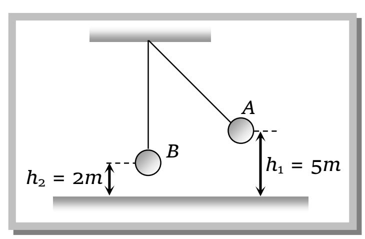
Solution : (d) For the given condition potential energy of the two masses will convert into heat and temperature of water will increase $W = JQ \Rightarrow 2m \times g \times h = J(m_w\, S\, \Delta t) \Rightarrow 2\times 5\times 10\times 10 = 4.2(2\times 10^3 \times 1 \times \Delta t)$

$\therefore \Delta t = \dfrac{1000}{8.4\times 10^3} = 0.119\,^oC = 0.12\,^oC$.

***Problem* 58.** A boy is sitting on a swing at a maximum height of 5*m* above the ground. When the swing passes through the mean position which is 2*m* above the ground its velocity is approximately **[MP PET 1990]**

(a) 7.6 *m/s* (b) 9.8 *m/s* (c) 6.26 *m/s* (d) None of these

Solution : (a) By the conservation of energy Total energy at point *A* = Total energy at point *B*

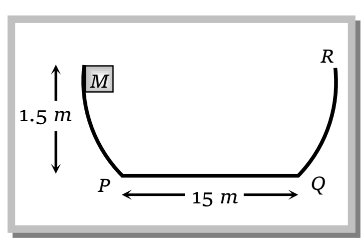


$\Rightarrow mgh_1 = mgh_2 + \frac{1}{2}mv^2$

$\Rightarrow 9.8\times 5 = 9.8\times 2 + \frac{1}{2}v^2$

$\Rightarrow v^2 = 58.8 \quad \therefore v = 7.6\, m/s$

***Problem* 59.** A block of mass *M* slides along the sides of a bowl as shown in the figure. The walls of the bowl are frictionless and the base has coefficient of friction 0.2. If the block is released from the top of the side, which is 1.5 *m* high, where will the block come to rest ? Given that the length of the base is 15 *m*



(a) 1 *m* from *P*

(b) Mid point

(c) 2 m from P

(d) At Q

Solution : (b) Potential energy of block at starting point = Kinetic energy at point P = Work done against friction in traveling a distance s from point P.

$\therefore \quad mgh = \mu mgs \Rightarrow s = \frac{h}{\mu} = \frac{1.5}{0.2} = 7.5\, m$

i.e. block come to rest at the mid point between P and Q.

Problem 60. If we throw a body upwards with velocity of $4\, ms^{-1}$ at what height its kinetic energy reduces to half of the initial value ? Take $g = 10\, m/s^2$

(a) 4m (b) 2 m (c) 1 m (d) None of these

Solution : (d) We know kinetic energy $K = \frac{1}{2}mv^2 \quad \therefore v \propto \sqrt{K}$

When kinetic energy of the body reduces to half its velocity becomes $v = \frac{u}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\, m/s$

From the equation $v^2 = u^2 - 2gh \Rightarrow (2\sqrt{2})^2 = (4)^2 - 2 \times 10\, h \quad \therefore h = \frac{16-8}{20} = 0.4\, m$.

Problem 61. A 2kg block is dropped from a height of 0.4 m on a spring of force constant $K = 1960\, Nm^{-1}$. The maximum compression of the spring is

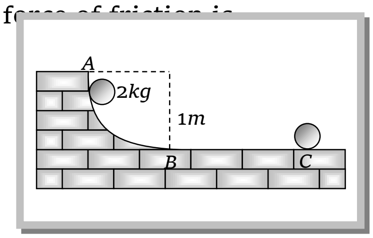
(a) 0.1 m (b) 0.2 m (c) 0.3 m (d) 0.4 m

Solution : (a) When a block is dropped from a height, its potential energy gets converted into kinetic energy and finally spring get compressed due to this energy.

$\therefore$ Gravitational potential energy of block = Elastic potential energy of spring

$\Rightarrow mgh = \frac{1}{2}Kx^2 \Rightarrow x = \sqrt{\frac{2\, mgh}{K}} = \sqrt{\frac{2 \times 2 \times 10 \times 0.4}{1960}} = 0.09\, m \simeq 0.1\, m$.

Problem 62. A block of mass 2kg is released from A on the track that is one quadrant of a circle of radius 1m. It slides down the track and reaches B with a speed of $4\, ms^{-1}$ and finally stops at C at a distance of 3m from B. The work done against the force of friction is



(a) 10 J

(b) 20 J

(c) 2 J

(d) 6 J

Solution : (b) Block possess potential energy at point A = $mgh = 2 \times 10 \times 1 = 20\, J$

Finally block stops at point C. So its total energy goes against friction *i.e.* work done against friction is 20 J.

Problem 63. A stone projected vertically upwards from the ground reaches a maximum height h. When it is at a height $\frac{3h}{4}$, the ratio of its kinetic and potential energies is

(a) 3 : 4 (b) 1 : 3 (c) 4 : 3 (d) 3 : 1

Solution : (b) At the maximum height, Total energy = Potential energy = mgh

At the height $\frac{3h}{4}$, Potential energy = $mg\frac{3h}{4}=\frac{3}{4}mgh$

and Kinetic energy = Total energy – Potential energy $=mgh-3\frac{mgh}{4}=\frac{1}{4}mgh$

$\therefore \quad \frac{\text{Kinetic energy}}{\text{Potential energy}}=\frac{1}{3}$.

6.19 Power

Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{\text{av.}})=\frac{\Delta W}{\Delta t}=\frac{W}{t}$

Instantaneous power $(P_{\text{inst.}})=\frac{dW}{dt}=\frac{\vec{F}.d\vec{s}}{dt}$ [As $dW=\vec{F}.d\vec{s}$]

$P_{\text{inst}}=\vec{F}.\vec{v}$ [As $\vec{v}=\frac{d\vec{s}}{dt}$]

i.e. power is equal to the scalar product of force with velocity.

Important points

(1) Dimension : $[P]=[F][v]=[MLT^{-2}][LT^{-1}]$

$\therefore \quad [P]=[ML^2T^{-3}]$

(2) Units : *Watt* or *Joule/sec* [S.I.]

Erg/sec [C.G.S.]

Practical units : *Kilowatt* (*kW*), *Mega watt* (*MW*) and Horse power (*hp*)

Relations between different units : $1\,watt=1\,Joule/\text{sec}=10^7\,erg/\text{sec}$

$1hp=746\ Watt$

$1\,MW=10^6\ Watt$

$1\,kW=10^3\ Watt$

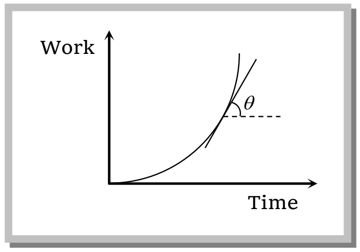
(3) If work done by the two bodies is same then power $\propto\frac{1}{\text{time}}$

i.e. the body which perform the given work in lesser time possess more power and vice-versa.

(4) As power = work/time, any unit of power multiplied by a unit of time gives unit of work (or energy) and not power, *i.e.* Kilowatt-hour or watt-day are units of work or energy.

$$1\,KWh = 10^3 \frac{J}{sec} \times (60 \times 60\,sec) = 3.6 \times 10^6 \; Joule$$

(5) The slope of work time curve gives the instantaneous power. As $P = dW/dt = \tan\theta$



(6) Area under power time curve gives the work done as $P = \frac{dW}{dt}$

$$\therefore \; W = \int P\,dt$$

$\therefore$ W = Area under P-t curve

6.20 Position and Velocity of an Automobile *w.r.t* Time

An automobile of mass m accelerates, starting from rest, while the engine supplies constant power P, its position and velocity changes *w.r.t* time.

(1) **Velocity :** As $Fv = P$ = constant

i.e. $\quad m\frac{dv}{dt}v = P \qquad \left[\text{As } F = \frac{mdv}{dt}\right]$

or $\quad \int v\,dv = \int \frac{P}{m}dt$

By integrating both sides we get $\frac{v^2}{2} = \frac{P}{m}t + C_1$

As initially the body is at rest *i.e.* $v = 0$ at $t = 0$, so $C_1 = 0$

$$\therefore \qquad v = \left(\frac{2Pt}{m}\right)^{1/2}$$

(2) **Position :** From the above expression $v = \left(\frac{2Pt}{m}\right)^{1/2}$

or $\quad \frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2} \qquad \left[\text{As } v = \frac{ds}{dt}\right]$

i.e. $\quad \int ds = \int \left(\frac{2Pt}{m}\right)^{1/2} dt$

By integrating both sides we get $s = \left(\frac{2P}{m}\right)^{1/2} . \frac{2}{3} t^{3/2} + C_2$

Now as at t = 0, s = 0, so $C_2 = 0$

$\therefore$ $$s = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$$

Sample problems based on power

Problem **64.** A car of mass 'm' is driven with acceleration 'a' along a straight level road against a constant external resistive force 'R'. When the velocity of the car is 'v', the rate at which the engine of the car is doing work will be

[MP PMT/PET 1998; JIMPER 2000]

(a) Rv (b) mav (c) $(R + ma)v$ (d) $(ma - R)v$

Solution : (c) The engine has to do work against resistive force R as well as car is moving with acceleration a.

Power = Force × velocity = $(R+ma)v$.

Problem **65.** A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to

(a) v (b) v^2 (c) v^3 (d) v^4

Solution : (c) Force $= v\frac{dm}{dt} = v\frac{d}{dt}(V \times \rho) = v\rho\frac{d}{dt}[A \times l] = v\rho A\frac{dl}{dt} = \rho A v^2$

Power = $F \times v = \rho A v^2 \times v = \rho A v^3$ $\therefore P \propto v^3$.

Problem **66.** A pump motor is used to deliver water at a certain rate from a given pipe. To obtain twice as much water from the same pipe in the same time, power of the motor has to be increased to **[JIPMER 2002]**

(a) 16 times (b) 4 times (c) 8 times (d) 2 times

Solution : (d) $P = \frac{\text{work done}}{\text{time}} = \frac{mgh}{t}$ $\therefore P \propto m$

i.e. To obtain twice water from the same pipe in the same time, the power of motor has to be increased to 2 times.

Problem **67.** A force applied by an engine of a train of mass $2.05 \times 10^6\,kg$ changes its velocity from 5 m/s to 25 m/s in 5 minutes. The power of the engine is

(a) 1.025 MW (b) 2.05 MW (c) 5MW (d) 5 MW

Solution : (b) Power $= \frac{\text{Work done}}{\text{time}} = \frac{\text{Increase in kinetic energy}}{\text{time}} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{t} = \frac{\frac{1}{2} \times 2.05 \times 10^6 \times [25^2 - 5^2]}{5 \times 60}$

$= 2.05 \times 10^6\ watt = 2.05\ MW$

Problem **68.** From a water fall, water is falling at the rate of 100 kg/s on the blades of turbine. If the height of the fall is 100m then the power delivered to the turbine is approximately equal to

(a) 100kW (b) 10 kW (c) 1kW (d) 1000 kW

Solution : (a) Power $= \frac{\text{Work done}}{t} = \frac{mgh}{t} = 100 \times 10 \times 100 = 10^5\, watt = 100\, kW$ $\left[\text{As} \frac{m}{t} = 100 \frac{kg}{\text{sec}} \text{(given)}\right]$

Problem **69.** A particle moves with a velocity $\vec{v} = 5\hat{i} - 3\hat{j} + 6\hat{k}\, ms^{-1}$ under the influence of a constant force $\vec{F} = 10\hat{i} + 10\hat{j} + 20\hat{k}\, N$. The instantaneous power applied to the particle is

(a) 200 $J\text{-}s^{-1}$ (b) 40 $J\text{-}s^{-1}$ (c) 140 $J\text{-}s^{-1}$ (d) 170 $J\text{-}s^{-1}$

Solution : (c) $P = \vec{F}.\vec{v} = (10\hat{i} + 10\hat{j} + 20\hat{k}).(5\hat{i} - 3\hat{j} + 6\hat{k}) = 50 - 30 + 120 = 140\ J\text{-}s^{-1}$

Problem **70.** A car of mass 1250 *kg* experience a resistance of 750 *N* when it moves at 30ms^{-1}. If the engine can develop 30*kW* at this speed, the maximum acceleration that the engine can produce is

(a) $0.8ms^{-2}$ (b) $0.2ms^{-2}$ (c) $0.4ms^{-1}$ (d) $0.5ms^{-2}$

Solution : (b) Power = Force × velocity = (Resistive force + Accelerating force) × velocity

$\Rightarrow\ 30 \times 10^3 = (750 + ma) \times 30 \Rightarrow ma = 1000 - 750 \Rightarrow a = \frac{250}{1250} = 0.2\, ms^{-2}$.

Problem **71.** A bus weighing 100 quintals moves on a rough road with a constant speed of 72*km/h*. The friction of the road is 9% of its weight and that of air is 1% of its weight. What is the power of the engine. Take $g = 10 m/s^2$

(a) 50 *kW* (b) 100 *kW* (c) 150 *kW* (d) 200 *kW*

Solution : (d) Weight of a bus = mass × g $= 100 \times 100\, kg \times 10\, m/s^2 = 10^5\, N$

Total friction force = 10% of weight = 10^4 *N*

∴ Power = Force × velocity $= 10^4\, N \times 72\, km/h = 10^4 \times 20\, watt = 2 \times 10^5\, watt = 200\, kW$.

Problem **72.** Two men with weights in the ratio 5 : 3 run up a staircase in times in the ratio 11 : 9. The ratio of power of first to that of second is

(a) $\frac{15}{11}$ (b) $\frac{11}{15}$ (c) $\frac{11}{9}$ (d) $\frac{9}{11}$

Solution : (a) Power (P) = $\frac{mgh}{t}$ or $P \propto \frac{m}{t} \Rightarrow \frac{P_1}{P_2} = \frac{m_1}{m_2}\frac{t_2}{t_1} = \left(\frac{5}{3}\right)\left(\frac{9}{11}\right) = \frac{45}{33} = \frac{15}{11}$ (g and h are constants)

Problem **73.** A dam is situated at a height of 550 *metre* above sea level and supplies water to a power house which is at a height of 50 *metre* above sea level. 2000 *kg* of water passes through the turbines per second. The maximum electrical power output of the power house if the whole system were 80% efficient is

(a) 8 *MW* (b) 10 *MW* (c) 12.5 *MW* (d) 16 *MW*

Solution : (a) Power $= \frac{\text{work done}}{\text{time}} = \frac{mg\Delta h}{t} = \frac{2000 \times 10 \times (550 - 50)}{1} = 10\ MW$

But the system is 80% efficient ∴ Power output = 10 × 80% = 8 *MW*.

Problem **74.** A constant force *F* is applied on a body. The power (*P*) generated is related to the time elapsed (*t*) as

(a) $P \propto t^2$ (b) $P \propto t$ (c) $P \propto \sqrt{t}$ (d) $P \propto t^{3/2}$

Solution : (b) $F = \frac{mdv}{dt}$ ∴ $F\,dt = mdv \Rightarrow v = \frac{F}{m}t$

Now $P = F \times v = F \times \frac{F}{m} t = \frac{F^2 t}{m}$

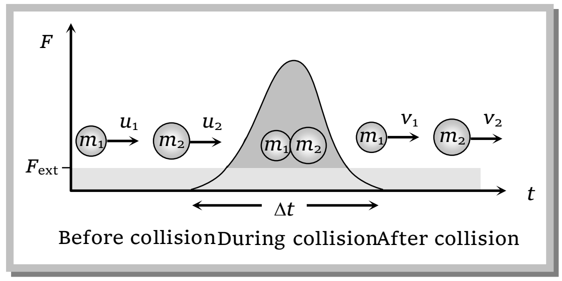
If force and mass are constants then $P \propto t$.

6.21 Collision

Collision is an isolated event in which a strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particle change.

In collision particles may or may not come in real touch e.g. in collision between two billiard balls or a ball and bat there is physical contact while in collision of alpha particle by a nucleus (*i.e.* Rutherford scattering experiment) there is no physical contact.

(1) **Stages of collision :** There are three distinct identifiable stages in collision, namely, before, during and after. In the before and after stage the interaction forces are zero. Between these two stages, the interaction forces are very large and often the dominating forces governing the motion of bodies. The magnitude of the interacting force is often unknown, therefore, Newton's second law cannot be used, the law of conservation of momentum is useful in relating the initial and final velocities.



(2) **Momentum and energy conservation in collision :**

(i) Momentum conservation : In a collision the effect of external forces such as gravity or friction are not taken into account as due to small duration of collision (Δt) average impulsive force responsible for collision is much larger than external force acting on the system and since this impulsive force is 'Internal' therefore the total momentum of system always remains conserved.

(ii) Energy conservation : In a collision 'total energy' is also always conserved. Here total energy includes all forms of energy such as mechanical energy, internal energy, excitation energy, radiant energy or even mass energy.

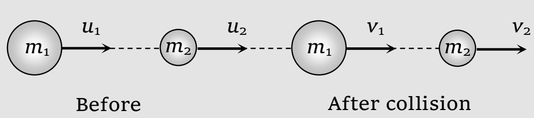
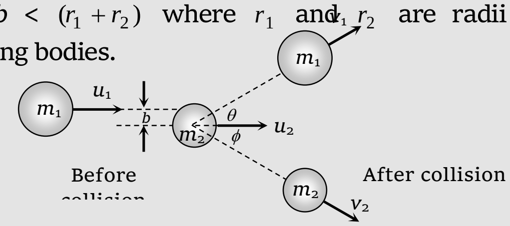
These laws are the fundamental laws of physics and applicable for any type of collision but this is not true for conservation of kinetic energy.

(3) **Types of collision :** (i) On the basis of conservation of kinetic energy.

Perfectly elastic collision	Inelastic collision	Perfectly inelastic collision
If in a collision, kinetic energy after collision is equal to kinetic energy before collision, the collision is said to be perfectly elastic.	If in a collision kinetic energy after collision is not equal to kinetic energy before collision, the collision is said to inelastic.	If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic.
Coefficient of restitution $e = 1$	Coefficient of restitution $0 < e < 1$	Coefficient of restitution $e = 0$
	Here kinetic energy appears in other forms. In some cases	The term 'perfectly inelastic' does not necessarily mean that

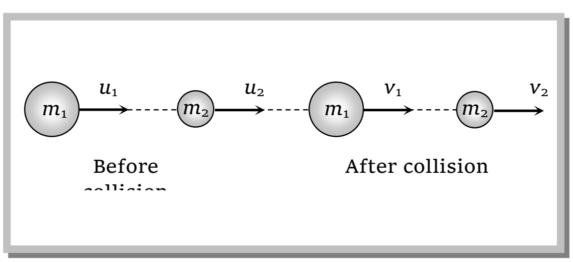
$(KE)_{final} = (KE)_{initial}$	$(KE)_{final} < (KE)_{initial}$ such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation) while in other cases $(KE)_{final} > (KE)_{initial}$ such as when internal energy stored in the colliding particles is released	all the initial kinetic energy is lost, it implies that the loss in kinetic energy is as large as it can be. (Consistent with momentum conservation).
Examples : (1) Collision between atomic particles (2) Bouncing of ball with same velocity after the collision with earth.	*Examples* : (1) Collision between two billiard balls. (2) Collision between two automobile on a road. In fact all majority of collision belong to this category.	*Example* : Collision between a bullet and a block of wood into which it is fired. When the bullet remains embeded in the block.

(ii) On the basis of the direction of colliding bodies

Head on or one dimensional collision	**Oblique collision**
In a collision if the motion of colliding particles before and after the collision is along the same line the collision is said to be head on or one dimensional.	If two particle collision is 'glancing' *i.e.* such that their directions of motion after collision are not along the initial line of motion, the collision is called oblique. If in oblique collision the particles before and after collision are in same plane, the collision is called 2-dimensional otherwise 3-dimensional.
Impact parameter b is zero for this type of collision. 	Impact parameter b lies between 0 and $(r_1 + r_2)$ *i.e.* $0 < b < (r_1 + r_2)$ where r_1 and r_2 are radii of colliding bodies. 
Example : collision of two gliders on an air track.	*Example* : Collision of billiard balls.

6.22 Perfectly Elastic Head on Collision

Let two bodies of masses m_1 and m_2 moving with initial velocities u_1 and u_2 in the same direction and they collide such that after collision their final velocities are v_1 and v_2 respectively.



According to law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad ......(i)$$

$\Rightarrow$
$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad ......(ii)$$

According to law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad .....(iii)$$

$\Rightarrow$
$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \qquad .....(iv)$$

Dividing equation (iv) by equation (ii)

$$v_1 + u_1 = v_2 + u_2 \qquad .....(v)$$

$\Rightarrow$
$$u_1 - u_2 = v_2 - v_1 \qquad .....(vi)$$

Relative velocity of approach = Relative velocity of separation

Note : ❑ The ratio of relative velocity of separation and relative velocity of approach is defined as coefficient of restitution. $e = \frac{v_2 - v_1}{u_1 - u_2}$ or $v_2 - v_1 = e(u_1 - u_2)$

❑ For perfectly elastic collision $e = 1$ $\therefore v_2 - v_1 = u_1 - u_2$ (As shown in eq. (vi)

❑ For perfectly inelastic collision $e = 0$ $\therefore v_2 - v_1 = 0$ or $v_2 = v_1$

It means that two body stick together and move with same velocity.

❑ For inelastic collision $0 < e < 1$ $\therefore v_2 - v_1 = e(u_1 - u_2)$

In short we can say that e is the degree of elasticity of collision and it is dimension less quantity.

Further from equation (v) we get $v_2 = v_1 + u_1 - u_2$

Substituting this value of v_2 in equation (i) and rearranging we get

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} \qquad ......(vii)$$

Similarly we get $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$ ......(viii)

(1) **Special cases of head on elastic collision**

(i) **If projectile and target are of same mass *i.e.* $m_1 = m_2$**

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2}{m_1 + m_2}u_2$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$

Substituting $m_1 = m_2$ we get

$v_1 = u_2$ and $v_2 = u_1$

It means when two bodies of equal masses undergo head on elastic collision, their velocities get interchanged.

Example : Collision of two billiard balls

Sub case : $u_2 = 0$ *i.e.* target is at rest
$v_1 = 0$ and $v_2 = u_1$

(ii) If massive projectile collides with a light target *i.e.* $m_1 >> m_2$

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2 u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$

Substituting $m_2 = 0$, we get

$v_1 = u_1$ and $v_2 = 2u_1 - u_2$

Sub case : $u_2 = 0$ i.*e.* target is at rest

Example : Collision of a truck with a cyclist

Before collision After collision

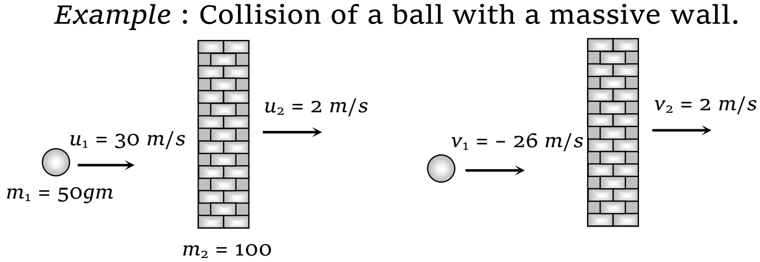
(iii) If light projectile collides with a very heavy target *i.e.* $m_1 << m_2$

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2 u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$

Substituting $m_1 = 0$, we get

$v_1 = -u_1 + 2u_2$ and $v_2 = u_2$

Example : Collision of a ball with a massive wall.



Before collision After collision

Sub case : $u_2 = 0$ i.e. target is at rest

$v_1 = -u_1$ and $v_2 = 0$

i.e. the ball rebounds with same speed in opposite direction when

(2) **Kinetic energy transfer during head on elastic collision**

Kinetic energy of projectile before collision $K_i = \frac{1}{2}m_1 u_1^2$

Kinetic energy of projectile after collision $K_f = \frac{1}{2}m_1 v_1^2$

Kinetic energy transferred from projectile to target ΔK = decrease in kinetic energy in projectile

$$\Delta K = \frac{1}{2}m_1 u_1^2 - \frac{1}{2}m_1 v_1^2 \quad = \frac{1}{2}m_1(u_1^2 - v_1^2)$$

Fractional decrease in kinetic energy $\frac{\Delta K}{K} = \frac{\frac{1}{2}m_1(u_1^2 - v_1^2)}{\frac{1}{2}m_1 u_1^2} = 1 - \left(\frac{v_1}{u_1}\right)^2$ .....(i)

We can substitute the value of v_1 from the equation $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2 u_2}{m_1 + m_2}$

If the target is at rest *i.e.* $u_2 = 0$ then $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$

From equation (i) $\frac{\Delta K}{K} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$ .....(ii)

or $\frac{\Delta K}{K} = \frac{4m_1 m_2}{(m_1 + m_2)^2}$ .....(iii)

or $\frac{\Delta K}{K} = \frac{4m_1 m_2}{(m_1 - m_2)^2 + 4m_1 m_2}$ .....(iv)

Note : ❑ Greater the difference in masses less will be transfer of kinetic energy and vice versa

❑ Transfer of kinetic energy will be maximum when the difference in masses is minimum

i.e. $m_1 - m_2 = 0$ or $m_1 = m_2$ then $\frac{\Delta K}{K} = 1 = 100\%$

So the transfer of kinetic energy in head on elastic collision (when target is at rest) is maximum when the masses of particles are equal i.e. mass ratio is 1 and the transfer of kinetic energy is 100%.

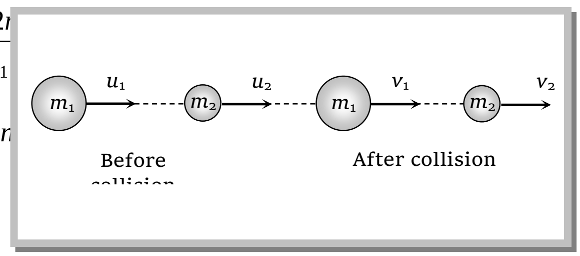
❑ If $m_2 = n m_1$ then from equation (iii) we get $\frac{\Delta K}{K} = \frac{4n}{(1+n)^2}$

- Kinetic energy retained by the projectile $\left(\frac{\Delta K}{K}\right)_{\text{Retained}} = 1 -$ kinetic energy transferred by projectile

$$\Rightarrow \qquad \left(\frac{\Delta K}{K}\right)_{\text{Retained}} = 1 - \left[1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2\right] = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

(3) **Velocity, momentum and kinetic energy of stationary target after head on elastic collision**

(i) Velocity of target : We know $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m}{m_1}$



$\Rightarrow \quad v_2 = \frac{2m_1 u_1}{m_1 + m_2} = \frac{2u_1}{1 + m_2/m_1}$ [As $u_2 = 0$ and Let $\frac{m_2}{m_1} = n$

$\therefore \qquad v_2 = \frac{2u_1}{1+n}$

(ii) Momentum of target : $P_2 = m_2 v_2 = \frac{2nm_1u_1}{1+n}$ $\qquad \left[\text{As } m_2 = m_1 n \text{ and } v_2 = \frac{2u_1}{1+n}\right]$

$\therefore \qquad P_2 = \frac{2m_1u_1}{1+(1/n)}$

(iii) Kinetic energy of target : $K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}nm_1\left(\frac{2u_1}{1+n}\right)^2 = \frac{2\,m_1u_1^2 n}{(1+n)^2}$

$$= \frac{4(K_1)n}{(1-n)^2 + 4n} \qquad \left[\text{As } K_1 = \frac{1}{2}m_1u_1^2\right]$$

(iv) Relation between masses for maximum velocity, momentum and kinetic energy

Velocity	$v_2 = \frac{2u_1}{1+n}$	For v_2 to be maximum n must be minimum *i.e.* $n = \frac{m_2}{m_1} \to 0 \therefore m_2 << m_1$	Target should be very light.
Momentum	$P_2 = \frac{2m_1u_1}{(1+1/n)}$	For P_2 to be maximum, $(1/n)$ must be minimum or n must be maximum. *i.e.* $n = \frac{m_2}{m_1} \to \infty \therefore m_2 >> m_1$	Target should be massive.
Kinetic energy	$K_2 = \frac{4K_1 n}{(1-n)^2 + 4n}$	For K_2 to be maximum $(1-n)^2$ must be minimum. *i.e.* $1 - n = 0 \Rightarrow n = 1 = \frac{m_2}{m_1} \therefore m_2 = m_1$	Target and projectile should be of equal mass.

Sample problem based on head on elastic collision

***Problem* 75.** n small balls each of mass m impinge elastically each second on a surface with velocity u. The force experienced by the surface will be

(a) mnu (b) $2\ mnu$ (c) $4\ mnu$ (d) $\frac{1}{2}mnu$

Solution : (b) As the ball rebounds with same velocity therefore change in velocity = $2u$ and the mass colliding with the surface per second = nm

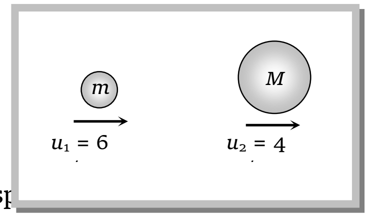
Force experienced by the surface $F = m\frac{dv}{dt}$ $\therefore F = 2\ mnu$.

Problem 76. A particle of mass m moving with horizontal speed 6 m/sec. If $m<<M$ then for one dimensional elastic collision, the speed of lighter particle after collision will be

(a) 2 m/sec in original direction (b) 2 m/sec opposite to the original direction

(c) 4 m/sec opposite to the original direction (d) 4 m/sec in original direction

Solution : (a) $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$



Substituting $m_1 = 0$, $v_1 = -u_1 + 2u_2$

$\Rightarrow v_1 = -6 + 2(4) = 2m/s$

i.e. the lighter particle will move in original direction with the sp

Problem 77. A body of mass m moving with velocity v makes a head-on collision with another body of mass $2m$ which is initially at rest. The loss of kinetic energy of the colliding body (mass m) is**[MP PMT 1996;**

(a) $\frac{1}{2}$ of its initial kinetic energy (b) $\frac{1}{9}$ of its initial kinetic energy

(c) $\frac{8}{9}$ of its initial kinetic energy (d) $\frac{1}{4}$ of its initial kinetic energy

Solution : (c) Loss of kinetic energy of the colliding body $\frac{\Delta K}{K} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 = 1 - \left(\frac{m - 2m}{m + 2m}\right)^2 = 1 - \left(\frac{1}{3}\right)^2$

$\Delta K = \left(1 - \frac{1}{9}\right)K = \frac{8}{9}K$ $\therefore$ Loss of kinetic energy is $\frac{8}{9}$ of its initial kinetic energy.

Problem 78. A ball of mass m moving with velocity V, makes a head on elastic collision with a ball of the same mass moving with velocity $2V$ towards it. Taking direction of V as positive velocities of the two balls after collision are

(a) $-V$ and $2V$ (b) $2V$ and $-V$ (c) V and $-2V$ (d) $-2V$ and V

Solution : (d) Initial velocities of balls are $+V$ and $-2V$ respectively and we know that for given condition velocities get interchanged after collision. So the velocities of two balls after collision are $-2V$ and V respectively.

Problem 79. Consider the following statements

Assertion (A) : In an elastic collision of two billiard balls, the total kinetic energy is conserved during the short time of collision of the balls (*i.e.*, when they are in contact)

Reason (R) : Energy spent against friction does not follow the law of conservation of energy of these statements

(a) Both A and R are true and the R is a correct explanation of A

(b) Both A and R are true but the R is not a correct explanation of the A

(c) A is true but the R is false

(d) Both A and R are false

Solution : (d) (i) When they are in contact some part of kinetic energy may convert in potential energy so it is not conserved during the short time of collision. (ii) Law of conservation of energy is always true.

***Problem* 80.** A big ball of mass M, moving with velocity u strikes a small ball of mass m, which is at rest. Finally small ball attains velocity u and big ball v. Then what is the value of v

(a) $\frac{M-m}{M+m}u$ (b) $\frac{m}{M+m}u$ (c) $\frac{2m}{M+m}u$ (d) $\frac{M}{M+m}u$

Solution : (a) From the standard equation $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 = \left(\frac{M-m}{M+m}\right)u$.

***Problem* 81.** A car of mass $400kg$ and travelling at 72 *kmph* crashes into a truck of mass $4000kg$ and travelling at 9 *kmph*, in the same direction. The car bounces back at a speed of 18 *kmph*. The speed of the truck after the impact is

(a) 9 *kmph* (b) 18 *kmph* (c) 27 *kmph* (d) 36 *kmph*

Solution : (b) By the law of conservation of linear momentum $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow 400 \times 72 + 4000 \times 9 = 400 \times (-18) + 4000 \times v_2 \Rightarrow v_2 = 18\ km/h$.

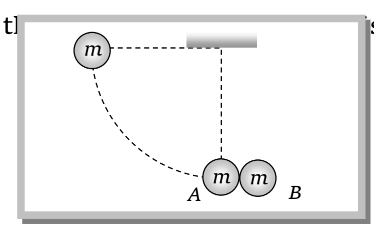
***Problem* 82.** A smooth sphere of mass M moving with velocity u directly collides elastically with another sphere of mass m at rest. After collision their final velocities are V and v respectively. The value of v is **[MP PET 1995]**

(a) $\frac{2uM}{m}$ (b) $\frac{2um}{M}$ (c) $\frac{2u}{1+\frac{m}{M}}$ (d) $\frac{2u}{1+\frac{M}{m}}$

Solution : (c) Final velocity of the target $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$

As initially target is at rest so by substituting $u_2 = 0$ we get $v_2 = \frac{2Mu}{M+m} = \frac{2u}{1+\frac{m}{M}}$.

***Problem* 83.** A sphere of mass 0.1 *kg* is attached to a cord of 1*m* length. Starting from the height of its point of suspension this sphere hits a block of same mass at rest on a frictionless table, If the impact is elastic, then the kinetic energy of t[…]s



(a) 1 *J*

(b) 10 *J*

(c) 0.1 *J*

(d) 0.5 *J*

Solution : (a) As two blocks are of same mass and the collision is perfectly elastic therefore their velocities gets interchanged *i.e.* the block A comes into rest and complete kinetic energy transferred to block B.

Now kinetic energy of block B after collision = Kinetic energy of block A before collision

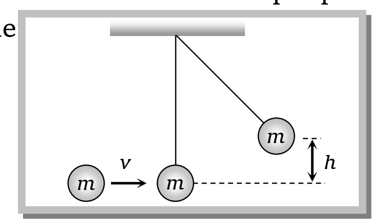
= Potential energy of block A at the original height

= mgh = 0.1 × 10 × 1 = 1 J.

***Problem* 84.** A ball moving horizontally with speed v strikes the bob of a simple pendulum at rest. The mass of the bob is equal to that of the ball. If the collision is elastic the bob will rise to a height

(a) $\frac{v^2}{g}$ (b) $\frac{v^2}{2g}$ (c) $\frac{v^2}{4g}$ (d) $\frac{v^2}{8g}$

Solution : (b) Total kinetic energy of the ball will transfer to the bob of simple pendulum. Let it rises to height 'h' by the law of conservation of ene



$$\frac{1}{2}mv^2 = mgh$$

$$\therefore\ h = \frac{v^2}{2g}$$

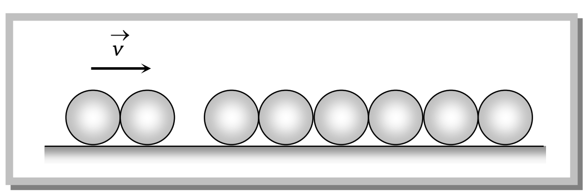
***Problem* 85.** A moving body with a mass m_1 strikes a stationary body of mass m_2. The masses m_1 and m_2 should be in the ratio $\frac{m_1}{m_2}$ so as to decrease the velocity of the first body 1.5 times assuming a perfectly elastic impact. Then the ratio $\frac{m_1}{m_2}$ is

(a) 1/ 25 (b) 1/5 (c) 5 (d) 25

Solution : (c) $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$ [As u_2 = 0 and $\left(v_1 = \frac{u_1}{1.5}\right)$ given]

$$\Rightarrow \frac{u_1}{1.5} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 \Rightarrow m_1 + m_2 = 1.5(m_1 - m_2) \Rightarrow \frac{m_1}{m_2} = 5.$$

***Problem* 86.** Six identical balls are lined in a straight groove made on a horizontal frictionless surface as shown. Two similar balls each moving with a velocity v collide with the row of 6 balls from left. What will happen



(a) One ball from the right rolls out with a speed $2v$ and the remaining balls will remain at rest

(b) Two balls from the right roll out with speed v each and the remaining balls will remain stationary

(c) All the six balls in the row will roll out with speed $v/6$ each and the two colliding balls will come to rest

(d) The colliding balls will come to rest and no ball rolls out from right

Solution : (b) Only this condition satisfies the law of conservation of linear momentum.

Problem **87.** A moving mass of 8 *kg* collides elastically with a stationary mass of 2 *kg*. If *E* be the initial kinetic energy of the mass, the kinetic energy left with it after collision will be

(a) 0.80 *E* (b) 0.64 *E* (c) 0.36 *E* (d) 0.08 *E*

Solution : (c) Kinetic energy retained by projectile $\frac{\Delta K}{K} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \Rightarrow \Delta K = \left(\frac{8-2}{8+2}\right)^2 E = = \frac{9}{25} E = 0.36 E$.

Problem **88.** A neutron travelling with a velocity *v* and K.E. *E* collides perfectly elastically head on with the nucleus of an atom of mass number *A* at rest. The fraction of total energy retained by neutron is

(a) $\left(\frac{A-1}{A+1}\right)^2$ (b) $\left(\frac{A+1}{A-1}\right)^2$ (c) $\left(\frac{A-1}{A}\right)^2$ (d) $\left(\frac{A+1}{A}\right)^2$

Solution : (a) Fraction of kinetic energy retained by projectile $\frac{\Delta K}{K} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$

Mass of neutron (m_1) = 1 and Mass of atom (m_2) = *A* $\therefore \frac{\Delta K}{K} = \left(\frac{1-A}{1+A}\right)^2$ or $\left(\frac{A-1}{A+1}\right)^2$.

Problem **89.** A neutron with 0.6MeV kinetic energy directly collides with a stationary carbon nucleus (mass number 12). The kinetic energy of carbon nucleus after the collision is

(a) 1.7 *MeV* (b) 0.17 *MeV* (c) 17 *MeV* (d) Zero

Solution : (b) Kinetic energy transferred to stationary target (carbon nucleus) $\frac{\Delta K}{K} = \left[1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2\right]$

$\frac{\Delta K}{K} = \left[1 - \left(\frac{1-12}{1+12}\right)^2\right] = \left[1 - \frac{121}{169}\right] = \frac{48}{169}$ $\therefore \Delta K = \frac{48}{169} \times (0.6 MeV) = 0.17 MeV$.

Problem **90.** A body of mass *m* moving along a straight line collides with a body of mass *nm* which is also moving with a velocity *kv* in the same direction. If the first body comes to rest after the collision, then the velocity of second body after the collision would be

(a) $\frac{nv}{(1+nk)}$ (b) $\frac{nv}{(1-nk)}$ (c) $\frac{(1-nk)v}{n}$ (d) $\frac{(1+nk)v}{n}$

Solution : (d) Initial momentum = $mv + nm(kv)$ and final momentum $= 0 + nm\,V$

By the conservation of momentum, $mv + nm(kv) = 0 + nm\,V$

$\Rightarrow v + nkv = nV \Rightarrow nV = (1+nk)v \Rightarrow V = \frac{(1+nk)v}{n}$

Problem **91.** Which one of the following statement does not hold good when two balls of masses m_1 and m_2 undergo elastic collision

(a) When $m_1 < m_2$ and m_2 at rest, there will be maximum transfer of momentum

(b) When $m_1 > m_2$ and m_2 at rest, after collision the ball of mass m_2 moves with four times the velocity of m_1

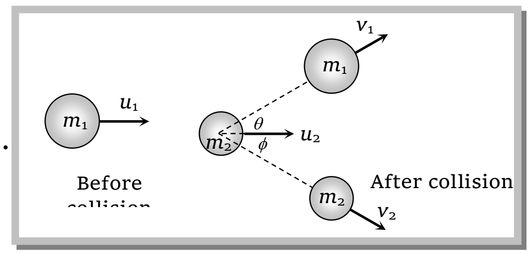
(c) When $m_1 = m_2$ and m_2 at rest, there will be maximum transfer of kinetic energy

(d) When collision is oblique and m_2 at rest with $m_1 = m_2$, after collision the balls move in opposite directions

Solution : (b, d) We know that transfer of momentum will be maximum when target is massive and transfer of kinetic energy will be maximum when target and projectile are having same mass. It means statement (a) and (c) are correct, but statement (b) and (d) are incorrect because when target is very light, then after collision it will move with double the velocity of projectile and when collision is oblique and m_2 at rest with $m_1 = m_2$, after collision the ball move perpendicular to each other.

6.23 Perfectly Elastic Oblique Collision

Let two bodies moving as shown in figure.



By law of conservation of momentum

Along x-axis, $m_1u_1 + m_2u_2 = m_1v_1 \cos\theta + m_2v_2 \cos\phi$ .....(i)

Along y-axis, $0 = m_1v_1 \sin\theta - m_2v_2 \sin\phi$ ...

By law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad .....(iii)$$

In case of oblique collision it becomes difficult to solve problem when some experimental data are provided as in these situations more unknown variables are involved than equations formed.

Special condition : If $m_1 = m_2$ and $u_2 = 0$ substituting these values in equation (i), (ii) and (iii) we get

$$u_1 = v_1 \cos\theta + v_2 \cos\phi \quad .....(iv)$$

$$0 = v_1 \sin\theta - v_2 \sin\phi \quad .....(v)$$

and $$u_1^2 = v_1^2 + v_2^2 \quad .....(vi)$$

Squaring (iv) and (v) and adding we get

$$u_1^2 = v_1^2 + v_2^2 + 2v_1u_2 \cos(\theta + \phi) \quad .....(vii)$$

Using (vi) and (vii) we get $\cos(\theta + \phi) = 0$

$\therefore$ $\theta + \phi = \pi / 2$

i.e. after perfectly elastic oblique collision of two bodies of equal masses (if the second body is at rest), the scattering angle $\theta + \phi$ would be 90^o.

Sample problems based on oblique elastic collision

Problem **92.** A ball moving with velocity of $9 m/s$ collides with another similar stationary ball. After the collision both the balls move in directions making an angle of 30^o with the initial direction. After the collision their speed will be

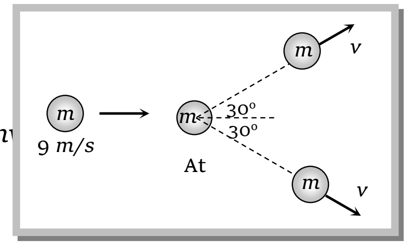
(a) $2.6 m/s$ (b) $5.2 m/s$ (c) $0.52 m/s$ (d) $52 m/s$

Solution : (b) Initial horizontal momentum of the system = $m \times 9$

Final horizontal momentum of the system = $2mv \cos 30^\circ$

According to law of conservation of momentum, $m \times 9 = 2m$

$\Rightarrow$ $v = 5.2\ m/s$



Problem 93. A ball of mass $1kg$, moving with a velocity of $0.4 m/s$ collides with another stationary ball. After the collision, the first ball moves with a velocity of $0.3 m/s$ in a direction making an angle of 90^o with its initial direction. The momentum of second ball after collision will be (in *kg-m/s*)

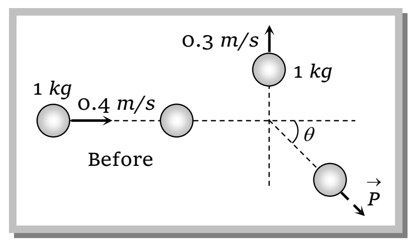
(a) 0.1 (b) 0.3 (c) 0.5 (d) 0.7

Solution : (c) Let second ball moves with momentum P making an angle θ from the horizontal (as shown in the figure).

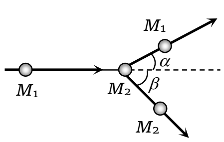
By the conservation of horizontal momentum $1 \times 0.4 = P \cos \theta$

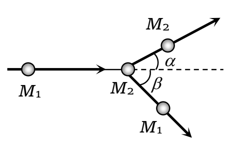
By the conservation of vertical momentum $0.3 = P \sin \theta$

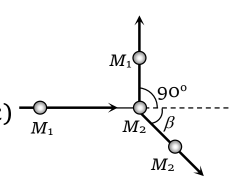
From (i) and (ii) we get $P = 0.5$ *kg-m/s*

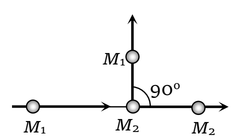


Problem 94. Keeping the principle of conservation of momentum in mind which of the following collision diagram is not correct

(a) 

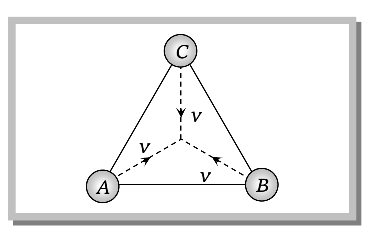
(b) 

(c) 

(d) 

Solution : (d) In this condition the final resultant momentum makes some angle with *x*-axis. Which is not possible because initial momentum is along the *x*-axis and according to law of conservation of momentum initial and final momentum should be equal in magnitude and direction both.

Problem 95. Three particles *A*, *B* and *C* of equal mass are moving with the same velocity *v* along the medians of an equilateral triangle. These particle collide at the centre *G* of triangle. After collision *A* becomes stationary, *B* retraces its path with velocity *v* then the magnitude and direction of velocity of *C* will be



(a) *v* and opposite to *B*

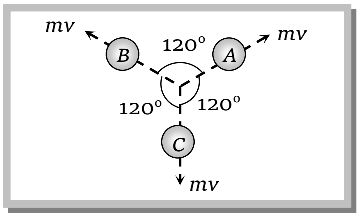
(b) *v* and in the direction of *A*

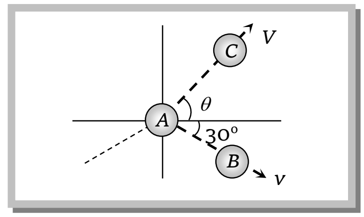
(c) *v* and in the direction of *C*

(d) v and in the direction of B

Solution : (d) From the figure (I) it is clear that before collision initial momentum of the system = 0

After the collision, A becomes stationary, B retraces its path with velocity v. Let C moves with velocity V making an angle θ from the horizontal. As the initial momentum of the system is zero, therefore horizontal and vertical momentum after the collision should also be equal to zero.





From figure (II) Horizontal momentum $v\cos\theta + v\cos 30^o = 0$ .....(i)

Vertical momentum $v\sin\theta - v\sin 30^o = 0$ .....(ii)

By solving (i) and (ii) we get $\theta = -30^o$ and $V = v$ *i.e.* the C will move with velocity v in the direction of B.

Problem **96.** A ball B_1 of mass M moving northwards with velocity v collides elastically with another ball B_2 of same mass but moving eastwards with the same velocity v. Which of the following statements will be true

(a) B_1 comes to rest but B_2 moves with velocity $\sqrt{2}v$

(b) B_1 moves with velocity $\sqrt{2}v$ but B_2 comes to rest

(c) Both move with velocity $v/\sqrt{2}$ in north east direction

(d) B_1 moves eastwards and B_2 moves north wards

Solution : (d) Horizontal momentum and vertical momentum both should remain conserve before and after collision. This is possible only for the (d) option.

6.24 Head on Inelastic Collision

(1) **Velocity after collision :** Let two bodies A and B collide inelastically and coefficient of restitution is e.

Where $$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

$\Rightarrow$ $v_2 - v_1 = e(u_1 - u_2)$

$\therefore$ $v_2 = v_1 + e(u_1 - u_2)$ ......(i)

From the law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad ......(ii)$$

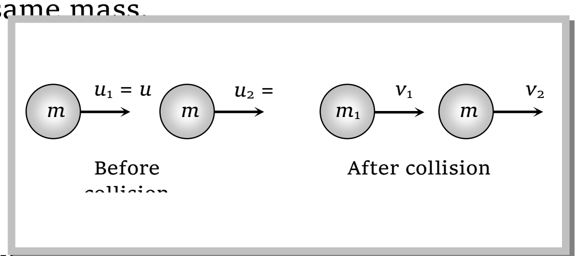
By solving (i) and (ii) we get

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2$$

Similarly $$v_2 = \left[\frac{(1+e)m_1}{m_1 + m_2}\right]u_1 + \left(\frac{m_2 - e\,m_1}{m_1 + m_2}\right)u_2$$

By substituting e = 1, we get the value of v_1 and u_2 for perfectly elastic head on collision.

(2) **Ratio of velocities after inelastic collision :** A sphere of mass m moving with velocity u hits inelastically with another stationary sphere of same mass.



$\therefore$ $$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{u - 0}$$

$\Rightarrow$ $$v_2 - v_1 = eu \qquad ......(i)$$

By conservation of momentum :

Momentum before collision = Momentum after collision

$$mu = mv_1 + mv_2$$

$\Rightarrow$ $$v_1 + v_2 = u \qquad ......(ii)$$

Solving equation (i) and (ii) we get $v_1 = \frac{u}{2}(1-e)$ and $v_2 = \frac{u}{2}(1+e)$

$\therefore$ $$\frac{v_1}{v_2} = \frac{1-e}{1+e}$$

(3) **Loss in kinetic energy**

Loss (ΔK) = Total initial kinetic energy – Total final kinetic energy

$$= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

Substituting the value of v_1 and v_2 from the above expression

$$\text{Loss } (\Delta K) = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(1 - e^2)(u_1 - u_2)^2$$

By substituting e = 1 we get ΔK = 0 *i.e.* for perfectly elastic collision loss of kinetic energy will be zero or kinetic energy remains constant before and after the collision.

***S**ample problems based on inelastic collision*

***Problem* 97.** A body of mass $40kg$ having velocity $4m/s$ collides with another body of mass $60kg$ having velocity $2m/s$. If the collision is inelastic, then loss in kinetic energy will be **[CPMT 1996; UP PMT 19**

(a) 440 *J* (b) 392 *J* (c) 48 *J* (d) 144 *J*

Solution : (c) Loss of K.E. in inelastic collision

$$\Delta K = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2 = \frac{1}{2}\frac{40 \times 60}{(40 + 60)}(4-2)^2 = \frac{1}{2}\frac{2400}{100}4 = 48\,J.$$

Problem 98. One sphere collides with another sphere of same mass at rest inelastically. If the value of coefficient of restitution is $\frac{1}{2}$, the ratio of their speeds after collision shall be

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

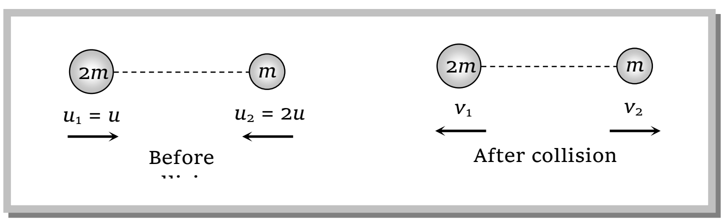
Solution : (c) $\frac{v_1}{v_2} = \frac{1-e}{1+e} = \frac{1-1/2}{1+1/2} = \frac{1/2}{3/2} = \frac{1}{3}$.

Problem 99. The ratio of masses of two balls is 2 : 1 and before collision the ratio of their velocities is 1 : 2 in mutually opposite direction. After collision each ball moves in an opposite direction to its initial direction. If $e = (5/6)$, the ratio of speed of each ball before and after collision would be

(a) (5/6) times (b) Equal

(c) Not related (d) Double for the first ball and half for the second ball

Solution : (a) Let masses of the two ball are $2m$ and m, and their speeds are u and $2u$ respectively.



By conservation of momentum $m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2 \Rightarrow 2mu - 2mu = mv_2 - 2mv_1 \Rightarrow v_2 = 2v_1$

Coefficient of restitution $= -\frac{(\vec{v}_2 - \vec{v}_1)}{(\vec{u}_2 - \vec{u}_1)} = -\frac{(2v_1 + v_1)}{(-2u - u)} = \frac{-3v_1}{-3u} = \frac{v_1}{u} = \frac{5}{6}$ [As $e = \frac{5}{6}$ given]

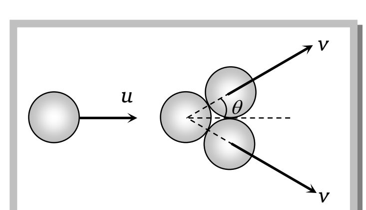
$\Rightarrow \frac{v_1}{u_1} = \frac{5}{6}$ = ratio of the speed of first ball before and after collision.

Similarly we can calculate the ratio of second ball before and after collision, $\frac{v_2}{u_2} = \frac{2v_1}{2u} = \frac{v_1}{u} = \frac{5}{6}$.

Problem 100. Two identical billiard balls are in contact on a table. A third identical ball strikes them symmetrically and come to rest after impact. The coefficient of restitution is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{\sqrt{3}}{2}$

Solution : (a) $\sin\theta = \frac{r}{2r} = \frac{1}{2} \Rightarrow \theta = 30°$



From conservation of linear momentum $mu = 2mv\cos 30^o$ or $v = \frac{u}{\sqrt{3}}$

Now $e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$ in common normal direction.

Hence, $e = \frac{v}{u\cos 30^o} = \frac{u/\sqrt{3}}{u\sqrt{3}/2} = \frac{2}{3}$

Problem **101.** A body of mass $3kg$, moving with a speed of $4ms^{-1}$, collides head on with a stationary body of mass $2kg$. Their relative velocity of separation after the collision is $2ms^{-1}$. Then

(a) The coefficient of restitution is 0.5 (b) The impulse of the collision is 7.2 *N-s*

(c) The loss of kinetic energy due to collision is 3.6 *J* (d) The loss of kinetic energy

Solution: (a,b,c) $m_1 = 3kg$, $m_2 = 2kg$, $u_1 = 4m/s$, $u_2 = 0$

Relative velocity of approach $u_1 - u_2 = 4m/s$

Relative velocity of separation $v_2 - v_1 = 2m/s$ (given)

Coefficient of restitution $e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}} = \frac{2}{4} = \frac{1}{2} = 0.5$

Loss in kinetic energy $= \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(1-e)^2(u_1 - u_2)^2 = \frac{1}{2}\frac{3\times 2}{3+2}\left[1 - \left(\frac{1}{2}\right)^2\right](4)^2 = 7.2J$

Final velocity of m_1 mass, $v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left[\frac{(1+e)m_2}{m_1 + m_2}\right]u_2 = \frac{(3 - 0.5\times 2)}{3+2}\times 4 + 0 = \frac{8}{5}m/s$

Impulse of collision = change in momentum of mass m_1 (or m_2) = $m_1v_1 - m_1u_1$

$$= 3\times\frac{8}{5} - 3\times 4 = \frac{24}{5} - 12 = 4.8 - 12 = -7.2\ N\text{-}s\ .$$

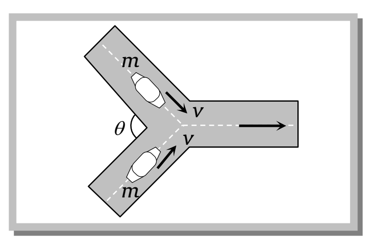
Problem **102.** Two cars of same mass are moving with same speed *v* on two different roads inclined at an angle θ with each other, as shown in the figure. At the junction of these roads the two cars collide inelastically and move simultaneously with the same speed. The speed of these cars would be

(a) $v\cos\frac{\theta}{2}$

(b) $\frac{v}{2}\cos\theta$

(c) $\frac{v}{2}\cos\frac{\theta}{2}$

(d) $2v\cos\theta$



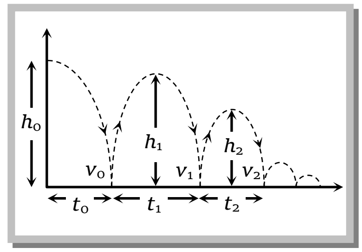
Solution : (a) Initial horizontal momentum of the system $= mv\cos\frac{\theta}{2} + mv\cos\frac{\theta}{2}$.

If after the collision cars move with common velocity *V* then final horizontal momentum of the system = $2mV$.

By the law of conservation of momentum, $2mV = mv\cos\frac{\theta}{2} + mv\cos\frac{\theta}{2} \Rightarrow V = v\cos\frac{\theta}{2}$.

6.25 Rebounding of Ball After Collision With Ground

If a ball is dropped from a height h on a horizontal floor, then it strikes with the floor with a speed.



$$v_0 = \sqrt{2gh_0} \qquad \text{[From } v^2 = u^2 + 2gh]$$

and it rebounds from the floor with a speed

$$v_1 = e\,v_0 = e\sqrt{2gh_0} \qquad \left[\text{As } e = \frac{\text{velocity after collision}}{\text{velocity before collision}}\right]$$

(1) **First height of rebound :** $h_1 = \dfrac{v_1^2}{2g} = e^2 h_0$

$\therefore \qquad h_1 = e^2 h_0$

(2) **Height of the ball after n^{th} rebound :** Obviously, the velocity of ball after n^{th} rebound will be

$$v_n = e^n v_0$$

Therefore the height after n^{th} rebound will be $h_n = \dfrac{v_n^2}{2g} = e^{2n} h_0$

$\therefore \qquad h_n = e^{2n} h_0$

(3) **Total distance travelled by the ball before it stops bouncing**

$$H = h_0 + 2h_1 + 2h_2 + 2h_3 + ... = h_0 + 2e^2 h_0 + 2e^4 h_0 + 2e^6 h_0 + ...$$

$$H = h_0[1 + 2e^2(1 + e^2 + e^4 + e^6 ....)] = h_0\left[1 + 2e^2\left(\frac{1}{1-e^2}\right)\right] \qquad \left[\text{As } 1 + e^2 + e^4 + ... = \frac{1}{1-e^2}\right]$$

$$\therefore \qquad H = h_0\left[\frac{1+e^2}{1-e^2}\right]$$

(4) **Total time taken by the ball to stop bouncing**

$$T = t_0 + 2t_1 + 2t_2 + 2t_3 + ... = \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + ...$$

$$= \sqrt{\frac{2h_0}{g}}\,[1 + 2e + 2e^2 + ...] \qquad [\text{As } h_1 = e^2 h_0 ;\ h_2 = e^4 h_0]$$

$$= \sqrt{\frac{2h_0}{g}}\,[1 + 2e(1 + e + e^2 + e^3 + ...)] = \sqrt{\frac{2h_0}{g}}\left[1 + 2e\left(\frac{1}{1-e}\right)\right] = \sqrt{\frac{2h_0}{g}}\left(\frac{1+e}{1-e}\right)$$

$\therefore \qquad T = \left(\frac{1+e}{1-e}\right)\sqrt{\frac{2h_0}{g}}$

Sample problems based on rebound of ball after collision with ground

Problem **103.** The change of momentum in each ball of mass $60gm$, moving in opposite directions with speeds $4m/s$ collide and rebound with the same speed, is

(a) $0.98\ kg\text{-}m/s$ (b) $0.73\ kg\text{-}m/s$ (c) $0.48\ kg\text{-}m/s$ (d) $0.22\ kg\text{-}m/s$

Solution : (c) Momentum before collision = mv, Momentum after collision = – mv

$\therefore$ Change in momentum $= 2mv = 2\times 60\times 10^{-3}\times 4 = 480\times 10^{-3}\ kg\text{-}m/s = 0.48\ kg\text{-}m/s$

Problem **104.** A body falling from a height of 20m rebounds from hard floor. If it loses 20% energy in the impact, then coefficient of restitution is

(a) 0.89 (b) 0.56 (c) 0.23 (d) 0.18

Solution : (a) It loses 20% energy in impact and only 80% energy remains with the ball

So ball will rise upto height $h_2 = 80\%$ of $h_1 = \frac{80}{100}\times 20 = 16\ m$

Now coefficient of restitution $e = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{16}{20}} = \sqrt{0.8} = 0.89$.

Problem **105.** A rubber ball is dropped from a height of $5m$ on a planet where the acceleration due to gravity is not known. On bouncing, it rises to $1.8m$. The ball loses its velocity on bouncing by a factor of **[CBSE PMT 1998]**

(a) 16/25 (b) 2/5 (c) 3/5 (d) 9/25

Solution : (c) If ball falls from height h_1, then it collides with ground with speed $v_1 = \sqrt{2gh_1}$ .....(i)

and if it rebound with velocity v_2, then it goes upto height h_2 from ground, $v_2 = \sqrt{2gh_2}$ .....(ii)

From (i) and (ii) $\frac{v_2}{v_1} = \sqrt{\frac{2gh_2}{2gh_1}} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{1.8}{5}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$.

6.26 Perfectly Inelastic Collision

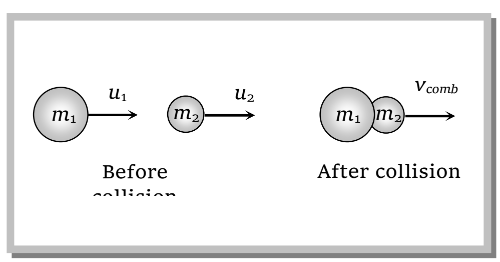
In such types of collisions the bodies move independently before collision but after collision as a one single body.

(1) **When the colliding bodies are moving in the same direction**

By the law of conservation of momentum

$$m_1u_1 + m_2u_2 = (m_1+m_2)v_{comb}$$

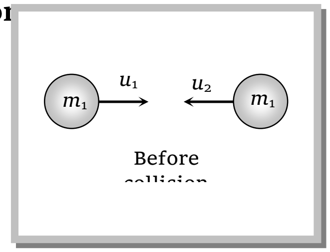
$$\Rightarrow \qquad v_{comb} = \frac{m_1u_1 + m_2u_2}{m_1+m_2}$$



Loss in kinetic energy $\Delta K = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1 + m_2)v_{comb}^2$

$\Delta K = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$ [By substituting the value of v_{comb}]

(2) **When the colliding bodies are moving in the opposite directions**



By the law of conservation of momentum

$m_1u_1 + m_2(-u_2) = (m_1 + m_2)v_{\text{comb}}$ (Taking left to right as positive)

$\therefore v_{\text{comb}} = \frac{m_1u_1 - m_2u_2}{m_1 + m_2}$

when $m_1u_1 > m_2u_2$ then $v_{\text{comb}} > 0$ (positive)

i.e. the combined body will move along the direction of motion of mass m_1.

when $m_1u_1 < m_2u_2$ then $v_{\text{comb}} < 0$ (negative)

i.e. the combined body will move in a direction opposite to the motion of mass m_1.

(3) **Loss in kinetic energy**

ΔK = Initial kinetic energy – Final kinetic energy

$= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}(m_1 + m_2)v_{\text{comb}}^2\right)$

$= \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 + u_2)^2$

Sample problems based on perfectly inelastic collision

***Problem* 106.** Which of the following is not a perfectly inelastic collision

(a) Striking of two glass balls (b) A bullet striking a bag of sand

(c) An electron captured by a proton (d) A man jumping onto a moving cart

Solution : (a) For perfectly elastic collision relative velocity of separation should be zero *i.e.* the colliding body should move together with common velocity.

***Problem* 107.** A metal ball of mass $2kg$ moving with a velocity of $36 km/h$ has an head-on collision with a stationary ball of mass $3kg$. If after the collision, the two balls move together, the loss in kinetic energy due to collision is

[CBSE 1997; AIIMS 2001]

(a) $40\ J$ (b) $60\ J$ (c) $100\ J$ (d) $140\ J$

Solution : (b) Loss in kinetic energy $\Delta K = \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 - u_2)^2 = \frac{1}{2}\frac{2 \times 3}{2 + 3}(10 - 0)^2 = 60\ J$.

Problem **108.** A mass of $20kg$ moving with a speed of $10\,m/s$ collides with another stationary mass of 5 *kg*. As a result of the collision, the two masses stick together. The kinetic energy of the composite mass will be **[MP PMT 2000]**

(a) 600 *J* (b) 800 *J* (c) 1000 *J* (d) 1200 *J*

Solution : (b) By conservation of momentum $m_1u_1 + m_2u_2 = (m_1 + m_2)V$

Velocity of composite mass $V = \dfrac{m_1u_1 + m_2u_2}{m_1 + m_2} = \dfrac{20 \times 10 + 5 \times 0}{20 + 5} = 8\,m/s$

$\therefore$ Kinetic energy of composite mass $= \dfrac{1}{2}(m_1 + m_2)V^2 = \dfrac{1}{2}(20 + 5) \times 8^2 = 800\ J.$

Problem **109.** A neutron having mass of $1.67 \times 10^{-27}\,kg$ and moving at $10^8\,m/s$ collides with a deutron at rest and sticks to it. If the mass of the deutron is $3.34 \times 10^{-27}\,kg$; the speed of the combination is **[CBSE PMT 2000]**

(a) $2.56 \times 10^3\,m/s$ (b) $2.98 \times 10^5\,m/s$ (c) $3.33 \times 10^7\,m/s$ (d) $5.01 \times 10^9\,m/s$

Solution : (c) $m_1 = 1.67 \times 10^{-27}\,kg$, $u_1 = 10^8\,m/s$, $m_2 = 3.34 \times 10^{-27}\,kg$ and $u_2 = 0$

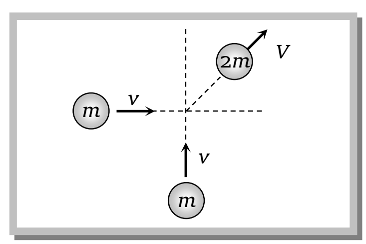
Speed of the combination $V = \dfrac{m_1u_1 + m_2u_2}{m_1 + m_2} = \dfrac{1.67 \times 10^{-27} \times 10^8 + 0}{1.67 \times 10^{-27} + 3.34 \times 10^{-27}} = 3.33 \times 10^7\,m/s.$

Problem **110.** A particle of mass m moving eastward with a speed v collides with another particle of the same mass moving northward with the same speed v. The two particles coalesce on collision. The new particle of mass $2m$ will move in the north-easterly direction with a velocity **[NCERT 1980; CPMT 1991; MP PET 1999; DPMT 1999]**

(a) $v/2$ (b) $2v$ (c) $v/\sqrt{2}$ (d) v

Solution : (c) Initially both the particles are moving perpendicular to each other with momentum *mv*.

So the net initial momentum $= \sqrt{(mv)^2 + (mv)^2} = \sqrt{2}\,mv$.



After the inelastic collision both the particles (system) moves with velocity *V*, so linear momentum = 2*mV*

By the law of conservation of momentum $\sqrt{2}\,mv = 2mV$

$\therefore\ V = v/\sqrt{2}.$

Problem **111.** A particle of mass 'm' moving with velocity 'v' collides inelastically with a stationary particle of mass '$2m$'. The speed of the system after collision will be

(a) $\dfrac{v}{2}$ (b) $2v$ (c) $\dfrac{v}{3}$ (d) $3v$

Solution : (c) By the conservation of momentum $mv + 2m \times 0 = 3mV \ \therefore\ V = \dfrac{v}{3}.$

Problem **112.** A ball moving with speed *v* hits another identical ball at rest. The two balls stick together after collision. If specific heat of the material of the balls is *S*, the temperature rise resulting from the collision is **[Roorkee 1999]**

(a) $\frac{v^2}{8S}$ (b) $\frac{v^2}{4S}$ (c) $\frac{v^2}{2S}$ (d) $\frac{v^2}{S}$

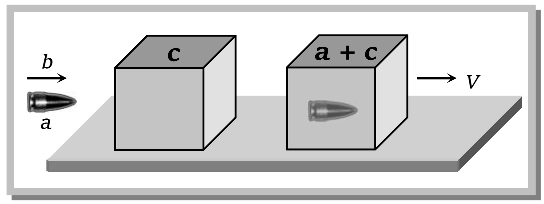
Solution : (b) Kinetic energy of ball will raise the temperature of the system $\frac{1}{2}mv^2 = (2m)S\,\Delta t \Rightarrow \Delta t = \frac{v^2}{4S}$.

Problem **113.** A bullet of mass a is fired with velocity b in a large block of mass c. The final velocity of the system will be

(a) $\frac{c}{a+c}$ (b) $\frac{ab}{a+c}$ (c) $\frac{(a+b)}{c}$ (d) $\frac{(a+c)}{a}b$

Solution : (b) Initially bullet moves with velocity b and after collision bullet get embedded in block and both move together with common velocity.

By the conservation of momentum $a \times b + 0 = (a$



$\therefore\ V = \frac{ab}{a+c}$

Problem **114.** A particle of mass 1g having velocity $3\hat{i} - 2\hat{j}$ has a glued impact with another particle of mass 2g and velocity as $4\hat{j} - 6\hat{k}$. Velocity of the formed particle is

(a) $5.6ms^{-1}$ (b) 0 (c) $6.4ms^{-1}$ (d) $4.6ms^{-1}$

Solution : (d) By conservation of momentum $m\vec{u}_1 + m_2\vec{u}_2 = (m_1 + m_2)\vec{V}$

$$\therefore\ \vec{V} = \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1 + m_2} = \frac{1(3i - 2\hat{j}) + 2(4j - 6\hat{k})}{m_1 + m_2} = \frac{3i + 6j - 12\hat{k}}{(1+2)} = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$|\vec{V}| = \sqrt{(1)^2 + (2)^2 + (-4)^2} = \sqrt{1 + 4 + 16} = 4.6ms^{-1}.$$

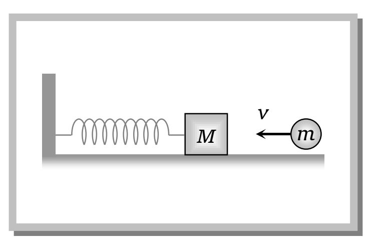
Problem **115.** A body of mass 2kg is placed on a horizontal frictionless surface. It is connected to one end of a spring whose force constant is $250\ N/m$. The other end of the spring is joined with the wall. A particle of mass 0.15kg moving horizontally with speed v sticks to the body after collision. If it compresses the spring by $10\,cm$, the velocity of the particle is

(a) $3m/s$ (b) $5m/s$ (c) $10\,m/s$ (d) $15\,m/s$

Solution : (d) By the conservation of momentum

Initial momentum of particle = Final momentum of system ⇒ $m \times v = (m + M)\ V$

$\therefore$ velocity of system $V = \frac{mv}{(m+M)}$



Now the spring compresses due to kinetic energy of the system so by the conservation of energy

$$\frac{1}{2}kx^2 = \frac{1}{2}(m+M)V^2 = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2$$

$$\Rightarrow kx^2 = \frac{m^2v^2}{m+M} \Rightarrow v = \sqrt{\frac{kx^2(m+M)}{m^2}} = \frac{x}{m}\sqrt{k(m+M)}$$

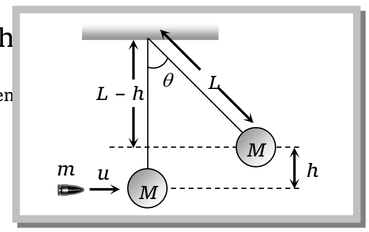
Putting m = 0.15 kg, M = 2 kg, k = 250 N/m, x = 0.1 m we get v = 15 m/s.

6.27 Collision Between Bullet and Vertically Suspended Block

A bullet of mass m is fired horizontally with velocity u in block of mass M suspended by vertical thread.

After the collision bullet gets embedded in block. Let the combined system raised upto height h and the string makes an angle θ with the vertical.

(1) **Velocity of system**



Let v be the velocity of the system (block + bullet) just after th

Momentum$_{\text{bullet}}$ + Momentum$_{\text{block}}$ = Momentum$_{\text{bullet and block systen}}$

$$mu + 0 = (m+M)v$$

$\therefore$ $$v = \frac{mu}{(m+M)} \qquad ......(i)$$

(2) **Velocity of bullet :** Due to energy which remains in the bullet block system, just after the collision, the system (bullet + block) rises upto height h.

By the conservation of mechanical energy $\frac{1}{2}(m+M)v^2 = (m+M)gh \Rightarrow v = \sqrt{2gh}$

Now substituting this value in the equation (i) we get $\sqrt{2gh} = \frac{mu}{m+M}$

$\therefore$ $$u = \left[\frac{(m+M)\sqrt{2gh}}{m}\right]$$

(3) **Loss in kinetic energy :** We know the formula for loss of kinetic energy in perfectly inelastic collision

$$\Delta K = \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}(u_1+u_2)^2$$

$\therefore$ $$\Delta K = \frac{1}{2}\frac{mM}{m+M}u^2$$ [As $u_1 = u$, $u_2 = 0$, $m_1 = m$ and $m_2 = M$]

(4) **Angle of string from the vertical**

From the expression of velocity of bullet $u = \left[\frac{(m+M)\sqrt{2gh}}{m}\right]$ we can get $h = \frac{u^2}{2g}\left(\frac{m}{m+M}\right)^2$

From the figure $\cos\theta = \frac{L-h}{L} = 1 - \frac{h}{L} = 1 - \frac{u^2}{2gL}\left(\frac{m}{m+M}\right)^2$

or $$\theta = \cos^{-1}\left[1 - \frac{1}{2gL}\left(\frac{mu}{m+M}\right)^2\right]$$

Problems based on collision between bullet and block

Problem 116. A bullet of mass m moving with velocity v strikes a block of mass M at rest and gets embeded into it. The kinetic energy of the composite block will be

(a) $\frac{1}{2}mv^2 \times \frac{m}{(m+M)}$ (b) $\frac{1}{2}mv^2 \times \frac{M}{(m+M)}$ (c) $\frac{1}{2}mv^2 \times \frac{(M+m)}{M}$ (d) $\frac{1}{2}Mv^2 \times \frac{m}{(m+M)}$

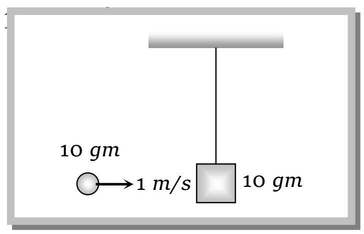
Solution : (a) By conservation of momentum,

Momentum of the bullet (mv) = momentum of the composite block $(m + M)V$

$\Rightarrow$ Velocity of composite block $V = \frac{mv}{m+M}$

$\therefore$ Kinetic energy $= \frac{1}{2}(m+M)V^2 = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2}\frac{m^2v^2}{m+M} = \frac{1}{2}mv^2\left(\frac{m}{m+M}\right)$.

Problem 117. A mass of $10 gm$, moving horizontally with a velocity of $100\, cm/\sec$, strikes the bob of a pendulum and strikes to it. The mass of the bob is also $10 gm$ (see fig.) The maximum height to which the system can be raised is ($g =$



(a) Zero

(b) $5 cm$

(c) $2.5 cm$

(d) $1.25 cm$

Solution : (d) By the conservation of momentum,

Momentum of the bullet = Momentum of system $\Rightarrow 10 \times 1 = (10+10) \times v \Rightarrow v = \frac{1}{2} m/s$

Now maximum height reached by system $H_{max} = \frac{v^2}{2g} = \frac{(1/2)^2}{2 \times 10} m = 1.25\, cm$.

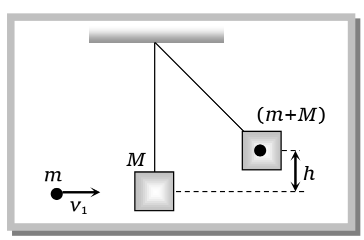
Problem 118. A bullet of mass m moving with a velocity v strikes a suspended wooden block of mass M as shown in the figure and sticks to it. If the block rises to a height h the initial velocity of the bullet is **[MP PMT 1997]**

(a) $\frac{m+M}{m}\sqrt{2gh}$

(b) $\sqrt{2gh}$

(c) $\frac{M+m}{M}\sqrt{2gh}$

(d) $\frac{m}{M+m}\sqrt{2gh}$



Solution : (a) By the conservation of momentum $mv = (m+M)V$

and if the system goes upto height h then $V = \sqrt{2gh}$

$\therefore mv = (m+M)\sqrt{2gh} \Rightarrow v = \frac{m+M}{m}\sqrt{2gh}$.

Problem 119. A bag P (mass M) hangs by a long thread and a bullet (mass m) comes horizontally with velocity v and gets caught in the bag. Then for the combined (beg + bullet) system the

(a) Momentum is $\frac{mvM}{M+m}$ (b) Kinetic energy $\frac{mV^2}{2}$

(c) Momentum is $\frac{mv\,(M+m)}{M}$ (d) Kinetic energy is $\frac{m^2V^2}{2(M+m)}$

Solution : (d) Velocity of combined system $V = \frac{mv}{m+M}$

Momentum for combined system $= (m+M)V = (m+M)\frac{mv}{m+M}$

Kinetic energy for combined system $= \frac{1}{2}(m+M)V^2 = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2}(m+M)\frac{m^2v^2}{(m+M)^2} = \frac{m^2v^2}{2(m+M)}$.

Problem 120. A wooden block of mass M is suspended by a cord and is at rest. A bullet of mass m, moving with a velocity v pierces through the block and comes out with a velocity $v/2$ in the same direction. If there is no loss in kinetic energy, then upto what height the block will rise

(a) $m^2v^2/2M^2g$ (b) $m^2v^2/8M^2g$ (c) $m^2v^2/4Mg$ (d) $m^2v^2/2Mg$

Solution : (b) By the conservation of momentum

Initial momentum = Final momentum

$$mv + M\times 0 = m\frac{v}{2} + M\times V \Rightarrow V = \frac{m}{2M}v$$

If block rises upto height h then $h = \frac{V^2}{2g} = \frac{(mv/2M)^2}{2g} = \frac{m^2v^2}{8M^2g}$.