

# Coordinate Geometry

---

- **Distance formula**

The distance between the points P  $(x_1, y_1)$  and Q  $(x_2, y_2)$  is given by  
 $PQ = \sqrt{x_2 - x_1^2 + y_2 - y_1^2}$

**Example 1:**

Find the values of  $l$ , if the distance between the points  $(-5, 3)$  and  $(l, 6)$  is 5 units.

**Solution:**

The given points are A  $(-5, 3)$  and B  $(l, 6)$ .

It is given that  $AB = 5$  units

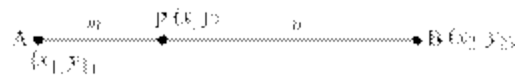
By distance formula we have

$$\sqrt{\lambda - 5^2 + 6 - 3^2} = 5 \Rightarrow \lambda + 5^2 + 9 = 25 \Rightarrow \lambda^2 + 25 + 10\lambda + 9 = 25 \Rightarrow \lambda^2 + 10\lambda + 9 = 0 \Rightarrow \lambda + 9 \lambda + 1 = 0 \Rightarrow \lambda = -1, \text{ or } \lambda = -9$$

Required values of  $l$  are  $-1$  or  $-9$ .

- The distance of a point  $(x, y)$  from the origin O  $(0, 0)$  is given by  $OP = \sqrt{x^2 + y^2}$ .

- **Section formula:**



The co-ordinates of the point P  $(x, y)$ , which divides the line segment joining the points A  $(x_1, y_1)$  and B  $(x_2, y_2)$  internally in the ratio  $m:n$ , are given by:

$$P \ x, y = \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}$$

**Example:** In what ratio does the point  $(-4, 7)$  divide the line segment joining the points P  $(-1, 1)$  and Q  $(-6, 11)$ .

**Solution:** Let the point  $(-4, 7)$  divide the line segment joining the points P  $(-1, 1)$  and Q  $(-6, 11)$  in the ratio  $\lambda : 1$ .

Thus, by section formula, we have:

$$-6\lambda + -1\lambda + 1, 11\lambda + 1\lambda + 1 = -4, 7 \Rightarrow -6\lambda - 1\lambda + 1 = -4, 11\lambda + 1\lambda + 1 = 7 \Rightarrow -6\lambda - 1 = -4\lambda - 4 \Rightarrow 2\lambda = 3 \Rightarrow \lambda = \frac{3}{2}$$

Therefore, the required ratio is  $3:2$ .

- The **mid-point** of the line segment joining the points A  $(x_1, y_1)$  and B  $(x_2, y_2)$  is  $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$ .

[Note: Here,  $m = n = 1$ ]

- If A  $(x_1, y_1)$ , B  $(x_2, y_2)$  and C  $(x_3, y_3)$  are the vertices of  $\Delta ABC$ , then the coordinates of its **centroid** are given by the point  $\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}$ .

- **Area of a triangle**

The area of a triangle whose vertices are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  is given by the numerical value of the expression

$$\frac{1}{2}(x_1y_2 - y_1x_2 + x_2y_3 - y_2x_3 + x_3y_1 - y_3x_1)$$

**Example 1:**

Find the area of the triangle whose vertices are P  $(-2, 2)$ , Q  $(2, 0)$  and R  $(8, 5)$ .

**Solution:**

We have P  $(-2, 2)$ , Q  $(2, 0)$  and R  $(8, 5)$  as the given points.

Let  $(x_1, y_1) = (-2, 2)$ ;  $(x_2, y_2) = (2, 0)$ ;  $(x_3, y_3) = (8, 5)$

$$\text{area of } \Delta PQR = \frac{1}{2}(x_1y_2 - y_1x_2 + x_2y_3 - y_2x_3 + x_3y_1 - y_3x_1) \Rightarrow \text{area of } \Delta PQR = \frac{1}{2}(-2 \cdot 0 - 2 \cdot 2 + 2 \cdot 5 - 0 \cdot 8 + 8 \cdot 2 - 5 \cdot (-2))$$

$$= \frac{1}{2}(0 - 4 + 10 - 0 + 16 - (-10)) = \frac{1}{2}(22) = 11$$

are units

**Example 2:**

If the points  $(-4, 1)$ ,  $(2, 4)$  and  $(p, 6)$  are collinear, then find the value of  $p$ .

**Solution:**

Since  $(-4, 1)$ ,  $(2, 4)$ ,  $(p, 6)$  are collinear, the area of the triangle formed by these points is zero.

$$\therefore \frac{1}{2}(-4 \cdot 4 - 1 \cdot 2 + 2 \cdot 6 - 4 \cdot p + p \cdot 1 - 6 \cdot (-4)) = 0 \Rightarrow -16 - 2 + 12 - 4p + p + 24 = 0 \Rightarrow -2 + 10 - 3p = 0 \Rightarrow 8 - 3p = 0 \Rightarrow 3p = 8 \Rightarrow p = \frac{8}{3}$$