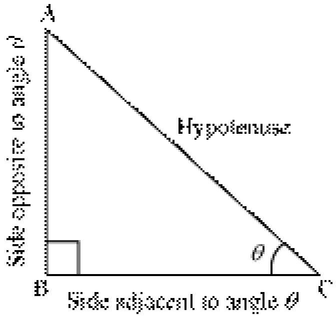
Introduction to Trigonometry

- **Trigonometric Ratio**



$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BC}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{AC}{AB}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{AC}{BC}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{BC}{AB}$$

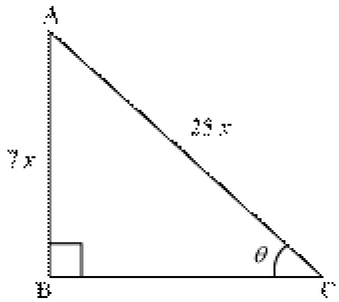
$$\text{Also, } \tan\theta = \frac{\sin\theta}{\cos\theta},\ \cot\theta = \frac{\cos\theta}{\sin\theta}$$

If one of the trigonometric ratios of an acute angle is known, then the remaining trigonometric ratios of the angle can be calculated.

Example:

If $\sin\theta = \frac{7}{25}$, then find the value of sec θ(1 + tan θ).

Solution:



It is given that $\sin\theta = \frac{7}{25}$

$\sin\theta = \frac{AB}{AC} = \frac{7}{25}$

⇒ AB = 7x and AC = 25x, where x is some positive integer

By applying Pythagoras theorem in ΔABC, we get:

$AB^2 + BC^2 = AC^2$

$\Rightarrow (7x)^2 + BC^2 = (25x)^2$

$\Rightarrow 49x^2 + BC^2 = 625x^2$

$\Rightarrow BC^2 = 625x^2 - 49x^2$

$\Rightarrow BC = \sqrt{576}\,x = 24x$

$\therefore \sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{25}{24}$

$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{7}{24}$

$\therefore \sec\theta(1+\tan\theta) = \frac{25}{24}\left(1+\frac{7}{24}\right) = \frac{25}{24}\times\frac{31}{24} = \frac{775}{576}$

- **Use trigonometric ratio in solving problem.**

Example:

If $\tan\theta = \frac{3}{5}$, then find the value of $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$

Solution:

$\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$

Take cosθ common from numerator and denominator both

$$= \frac{\frac{\sin\theta}{\cos\theta}+1}{\frac{\sin\theta}{\cos\theta}-1}$$

$$= \frac{\tan\theta+1}{\tan\theta-1}$$

$$= \frac{\frac{3}{5}+1}{\frac{3}{5}-1}$$

$$= \frac{\frac{3+5}{5}}{\frac{3-5}{5}}$$

$$= \frac{8}{-2}$$

$$= -4$$

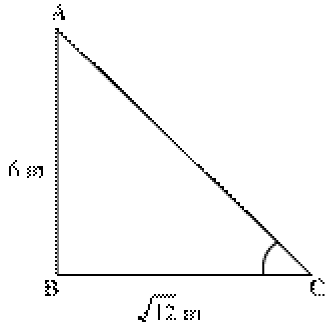
- **Trigonometric Ratios of some specific angles**

q	**0**	**30°**	**45°**	**60°**	**90°**
sin*q*	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos*q*	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan*q*	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec*q*	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec*q*	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot*q*	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Example 1:

ΔABC is right-angled at B and AB = 6 m, $\mathbf{BC}=\sqrt{\mathbf{12}}$ m. Find the measure of ∠A and ∠C.

Solution:



$AB = 6$ m,

$BC = \sqrt{12}$ m $= 2\sqrt{3}$ m

$$\tan C = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BC} = \frac{6}{2\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \tan C = \tan 60^\circ \qquad \left[\because \tan 60^\circ = \sqrt{3}\right]$$

$$\Rightarrow \angle C = 60^\circ$$

$$\therefore \angle A = 180^\circ - (90 + 60) = 30^\circ$$

Example 2:

Evaluate the expression

$$4\left(\cos^3 60^\circ - \sin^3 30^\circ\right) + 3\left(\sin 30^\circ - \cos 60^\circ\right)$$

Solution:

$$4\left(\cos^3 60^\circ - \sin^3 30^\circ\right) + 3\left(\sin 30^\circ - \cos 60^\circ\right)$$

$$= 4\left[\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^3\right] + 3\left(\frac{1}{2} - \frac{1}{2}\right)$$

$$= 4 \times 0 + 3 \times 0 = 0 + 0 = 0$$

- **Trigonometric Ratios of Complementary Angles**

$$\sin(90^\circ - \theta) = \cos\theta \qquad \cos(90^\circ - \theta) = \sin\theta$$

$$\tan(90^\circ - \theta) = \cot\theta \qquad \cot(90^\circ - \theta) = \tan\theta$$

$$\text{cosec}(90^\circ - \theta) = \sec\theta \qquad \sec(90^\circ - \theta) = \text{cosec}\,\theta$$

Where θ is an acute angle.

Example 1: Evaluate the expression

$\sin 28^\circ \sin 30^\circ \sin 54^\circ \sec 36^\circ \sec 62^\circ$

Solution:

$$\sin 28° \sin 30° \sin 54° \sec 36° \sec 62°$$
$$= (\sin 28° \sec 62°)(\sin 54° \sec 36°) \sin 30°$$
$$= \{\sin 28° \operatorname{cosec}(90° - 62°)\}\{\sin 54° \operatorname{cosec}(90° - 36°)\} \sin 30°$$
$$= (\sin 28° \operatorname{cosec} 28°)(\sin 54° \operatorname{cosec} 54°) \sin 30°$$
$$= \left(\sin 28° \frac{1}{\sin 28°}\right)\left(\sin 54° \frac{1}{\sin 54°}\right) \times \frac{1}{2}$$
$$= \frac{1}{2}$$

Example 2: Evaluate the expression

$$4\sqrt{3}(\sin 40° \sec 30° \sec 50°) + \frac{\sin^2 34° + \sin^2 56°}{\sec^2 31° - \cot^2 59°}$$

Solution:

$$4\sqrt{3}(\sin 40° \sec 30° \sec 50°) + \frac{\sin^2 34° + \sin^2 56°}{\sec^2 31° - \cot^2 59°}$$
$$= 4\sqrt{3}[\sec 30°(\sin 40° \sec 50°)] + \frac{\sin^2 34° + \sin^2 (90 - 56°)}{\sec^2 31° - \tan^2 (90 - 59°)}$$
$$[\because \cos(90° - \theta) = \sin\theta, \tan(90° - \theta) = \cot\theta]$$
$$= 4\sqrt{3}[\sec 30° \sin 40° \operatorname{cosec}(90 - 50°)] + \frac{\sin 34° + \cos^2 34°}{\sec^2 31° - \tan^2 31°}$$
$$= 4\sqrt{3}\left[\frac{2}{\sqrt{3}} \sin 40° \operatorname{cosec} 40°\right] + \frac{1}{1}$$
$$= 8 + 1 = 9$$

- **Trigonometric Identities**

1. 1. $\cos^2 A + \sin^2 A = 1$
2. 2. $1 + \tan^2 A = \sec^2 A$
3. 3. $1 + \cot^2 A = \operatorname{cosec}^2 A$

Example:

If $\cos\theta = \frac{5}{7}$, find the value of cot θ + cosec θ

Solution:

We have, $\cos\theta = \frac{5}{7}$

$$\text{Now,} \quad \sin^2\theta + \cos^2\theta = 1$$

$$\therefore \sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\left(\frac{5}{7}\right)^2}$$

$$= \sqrt{\frac{49-25}{49}} = \frac{2\sqrt{6}}{7}$$

$$\therefore \operatorname{cosec}\theta = \frac{7}{2\sqrt{6}}$$

$$\text{Also,} \ \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\frac{5}{7}}{\frac{2\sqrt{6}}{7}} = \frac{5}{2\sqrt{6}}$$

$$\therefore \cot\theta + \operatorname{cosec}\theta = \frac{5}{2\sqrt{6}} + \frac{7}{2\sqrt{6}}$$

$$= \frac{12}{2\sqrt{6}} = \frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \sqrt{6}$$

- **Use of trigonometric identities in proving relationships involving trigonometric ratio.**

Example: Prove the following identities
$\tan^2\theta + \cot^2\theta + 2 = \sec^2\theta \operatorname{cosec}^2\theta$
Solution:

We have

$$\text{LHS} = \tan^2\theta + \cot^2\theta + 2$$

$$= \tan^2\theta + \cot^2\theta + 2\cdot\tan\theta\cdot\cot\theta \qquad [\because \tan\theta\cdot\cot\theta = 1]$$

$$= \left(\tan\theta + \cot\theta\right)^2 \qquad \left[\because a^2 + b^2 + 2ab = (a+b)^2\right]$$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cdot\cos\theta}\right)^2$$

$$= \left(\frac{1}{\sin\theta\cdot\cos\theta}\right)^2$$

$$= \left(\sec\theta\cdot\text{cosec}\,\theta\right)^2$$

$$= \sec^2\theta\cdot\text{cosec}^2\theta$$

$$= \text{RHS}$$