Chapter

10

Surface Tension

Intermolecular Force

The force of attraction or repulsion acting between the molecules are known as intermolecular force. The nature of intermolecular force is electromagnetic.

The intermolecular forces of attraction may be classified into two types.

Cohesive force	Adhesive force
The force of attraction between molecules of same substance is called the force of cohesion. This force is lesser in liquids and least in gases.	The force of attraction between the molecules of the different substances is called the force of adhesion.
Ex. (i) Two drops of a liquid coalesce into one when brought in mutual contact. (ii) It is difficult to separate two sticky plates of glass welded with water. (iii) It is difficult to break a drop of mercury into small droplets because of large cohesive force between the mercury molecules.	Ex. (i) Adhesive force enables us to write on the blackboard with a chalk. (ii) A piece of paper sticks to another due to large force of adhesion between the paper and gum molecules. (iii) Water wets the glass surface due to force of adhesion.

Note : ❑ Cohesive or adhesive forces are inversely proportional to the eighth power of distance between the molecules.

Surface Tension

The property of a liquid due to which its free surface tries to have minimum surface area and behaves as if it were under tension somewhat like a stretched elastic membrane is called surface tension. A small liquid drop has spherical shape, as due to surface tension the liquid surface tries to have minimum surface area and for a given volume, the sphere has minimum surface area.

Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid, the direction of this force being perpendicular to the line and tangential to the free surface of liquid. So if F is the force acting on one side of imaginary line of length L, then $T = (F/L)$

(1) It depends only on the nature of liquid and is independent of the area of surface or length of line considered.

(2) It is a scalar as it has a unique direction which is not to be specified.

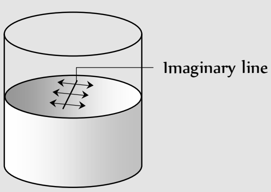
(3) Dimension : $[MT^{-2}]$. (Similar to force constant)

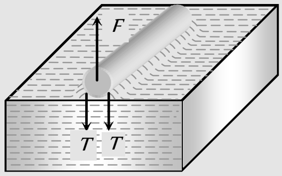
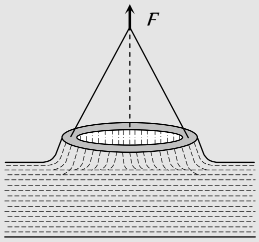
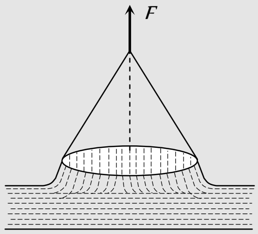
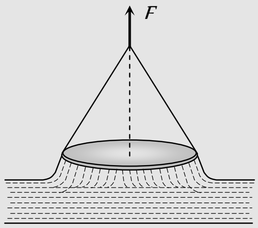
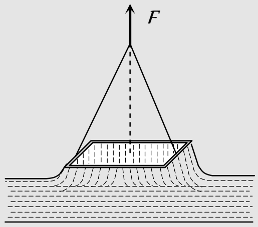
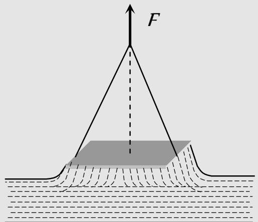
(4) Units : N/m (S.I.) and $Dyne/cm$ [C.G.S.]

(5) It is a molecular phenomenon and its root cause is the electromagnetic forces.

Force Due to Surface Tension

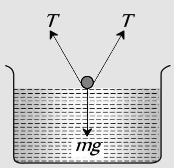
If a body of weight W is placed on the liquid surface, whose surface tension is T. If F is the minimum force required to pull it away from the water then value of F for different bodies can be calculated by the following table.

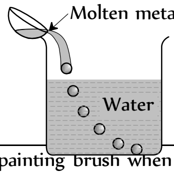
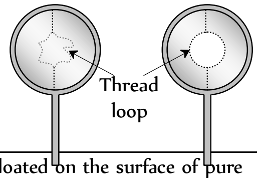
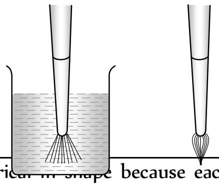
Body	Figure	Force
	 Fig. 10.1	

Needle (Length = l)		$F = 2l\,T + W$
Hollow disc (Inner radius = r_1 Outer radius = r_2)		$F = 2\pi\,(r_1 + r_2)T + W$
Thin ring (Radius = r)		$F = 2\pi\,(r + r)T + W$ $F = 4\pi rT + W$
Circular plate or disc (Radius = r)		$F = 2\pi rT + W$
Square frame (Side = l)		$F = 8l\,T + W$
Square plate		$F = 4l\,T + W$

Examples of Surface Tension

(1) When mercury is split on a clean glass plate, it forms globules. Tiny globules are spherical on the account of surface tension because force of gravity is negligible. The bigger globules get flattened from the middle but have round shape near the edges.	(2) When a greased iron needle is placed gently on the surface of water at rest, so that it does not prick the water surface, the needle floats on the surface of water despite it being heavier because the weight of needle is balanced by the vertical components of the forces of surface tension. If the water



	surface is pricked by one end of the needle, the needle sinks down.
(3) When a molten metal is poured into water from a suitable height, the falling stream of metal breaks up and the detached portion of the liquid in small quantity acquire the spherical shape. 	(4) Take a frame of wire and dip it in soap solution and take it out, a soap film will be formed in the frame. Place a loop of wet thread gently on the film. It will remain in the form, we place it on the film according to figure. Now, piercing the film with a pin at any point inside the loop, It immediately takes the circular form as shown in figure. 
(5) Hair of shaving brush/painting brush when dipped in water spread out, but as soon as it is taken out, its hair stick together. 	(6) If a small irregular piece of camphor is floated on the surface of pure water, it does not remain steady but dances about on the surface. This is because, irregular shaped camphor dissolves unequally and decreases the surface tension of the water locally. The unbalanced forces make it to move haphazardly in different directions.
(7) Rain drops are spherical in shape because each drop tends to acquire minimum surface area due to surface tension, and for a given volume, the surface area of sphere is minimum.	(8) Oil drop spreads on cold water. Whereas it may remain as a drop on hot water. This is due to the fact that the surface tension of oil is less than that of cold water and is more than that of hot water.

Factors Affecting Surface Tension

(1) **Temperature :** The surface tension of liquid decreases with rise of temperature. The surface tension of liquid is zero at its boiling point and it vanishes at critical temperature. At critical temperature, intermolecular forces for liquid and gases becomes equal and liquid can expand without any restriction. For small temperature differences, the variation in surface tension with temperature is linear and is given by the relation

$$T_t = T_0(1 - \alpha t)$$

where T_t, T_0 are the surface tensions at t^oC and 0^oC respectively and α is the temperature coefficient of surface tension.

Examples : (i) Hot soup tastes better than the cold soup.

(ii) Machinery parts get jammed in winter.

(2) **Impurities :** The presence of impurities either on the liquid surface or dissolved in it, considerably affect the surface tension, depending upon the degree of contamination. A highly soluble substance like sodium chloride when dissolved in water, increases the surface tension of water. But the sparingly soluble substances like phenol when dissolved in water, decreases the surface tension of water.

Applications of Surface Tension

(1) The oil and grease spots on clothes cannot be removed by pure water. On the other hand, when detergents (like soap) are added in water, the surface tension of water decreases. As a result of this, wetting power of soap solution increases. Also the force of adhesion between soap solution and oil or grease on the clothes increases. Thus, oil, grease and dirt particles get mixed with soap solution easily. Hence clothes are washed easily.

(2) The antiseptics have very low value of surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension, the antiseptics spreads properly over wound.

(3) Surface tension of all lubricating oils and paints is kept low so that they spread over a large area.

(4) Oil spreads over the surface of water because the surface tension of oil is less than the surface tension of cold water.

(5) A rough sea can be calmed by pouring oil on its surface.

(6) In soldering, addition of 'flux' reduces the surface tension of molten tin, hence, it spreads.

Molecular Theory of Surface Tension

The maximum distance upto which the force of attraction between two molecules is appreciable is called molecular range $(\approx 10^{-9} m)$. A sphere with a molecule as centre and radius equal to molecular range is called the sphere of influence. The liquid enclosed between free surface (PQ) of the liquid and an imaginary plane (RS) at a distance r (equal to molecular range) from the free surface of the liquid form a liquid film.

To understand the concept of tension acting on the free surface of a liquid, let us consider four liquid molecules like A, B, C and D. Their sphere of influence are shown in the figure.

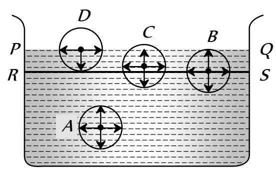


Fig. 10.2

(1) Molecule A is well within the liquid, so it is attracted equally in all directions. Hence the net force on this molecule is zero and it moves freely inside the liquid.

(2) Molecule B is little below the free surface of the liquid and it is also attracted equally in all directions. Hence the resultant force acts on it is also zero.

(3) Molecule C is just below the upper surface of the liquid film and the part of its sphere of influence is outside the free liquid surface. So the number of molecules in the upper half (attracting the molecules upward) is less than the number of molecule in the lower half (attracting the molecule downward). Thus the molecule C experiences a net downward force.

(4) Molecule D is just on the free surface of the liquid. The upper half of the sphere of influence has no liquid molecule. Hence the molecule D experiences a maximum downward force.

Thus all molecules lying on surface film experiences a net downward force. Therefore, free surface of the liquid behaves like a stretched membrane.

Surface Energy

The molecules on the liquid surface experience net downward force. So to bring a molecule from the interior of the liquid to the free surface, some work is required to be done against the intermolecular force of attraction, which will be stored as potential energy of the molecule on the surface. The potential energy of surface molecules per unit area of the surface is called surface energy.

Unit : *Joule/m* (S.I.) *erg/cm* (C.G.S.)

Dimension : $[MT]$

If a rectangular wire frame *ABCD*, equipped with a sliding wire *LM* dipped in soap solution, a film is formed over the frame. Due to the surface tension, the film will have a tendency to shrink and thereby, the sliding wire *LM* will be pulled in inward direction. However, the sliding wire can be held in this position under a force *F*, which is equal and opposite to the force acting on the sliding wire *LM* all along its length due to surface tension in the soap film.

If T is the force due to surface tension per unit length, then $F = T \times 2l$

Here l is length of the sliding wire *LM*. The length of the sliding wire has been taken as $2l$ for the reason that the film has got two free surfaces.

Suppose that the sliding wire *LM* is moved through a small distance *x*, so as to take the position $L'M'$. In this process, area of the film increases by $2l \times x$ (on the two sides) and to do so, the work done is given by

$W = F \times x = (T \times 2l) \times x = T \times (2lx) = T \times \Delta A$

$\therefore \quad W = T \times \Delta A \qquad [\Delta A$ = Total increase in area of the film]

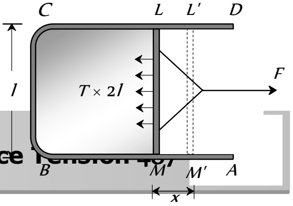


Fig. 10.3

If temperature of the film remains constant in this process, this work done is stored in the film as its surface energy.

From the above expression $T = \dfrac{W}{\Delta A}$ or $T = W \quad$ [If $\Delta A = 1$]

i.e. surface tension may be defined as the amount of work done in increasing the area of the liquid surface by unity against the force of surface tension at constant temperature.

Work Done in Blowing a Liquid Drop or Soap Bubble

(1) If the initial radius of liquid drop is r_1 and final radius of liquid drop is r_2 then

$W = T \times$ Increment in surface area

$W = T \times 4\pi[r_2^2 - r_1^2] \qquad$ [drop has only one free surface]

(2) In case of soap bubble

$W = T \times 8\pi[r_2^2 - r_1^2] \qquad$ [Bubble has two free surfaces]

Splitting of Bigger Drop

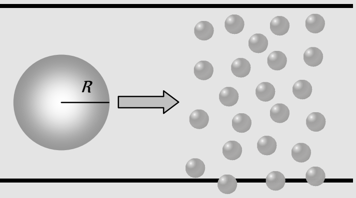
When a drop of radius *R* splits into *n* smaller drops, (each of radius *r*) then surface area of liquid increases. Hence the work is to be done against surface tension.

Since the volume of liquid remains constant therefore

$\dfrac{4}{3}\pi R^3 = n\dfrac{4}{3}\pi r^3 \quad \therefore \quad R^3 = nr^3$

Work done = $T \times \Delta A = T \times$ [Total final surface area of *n* drops – surface area of big drop] = $T[n4\pi r^2 - 4\pi R^2]$

Various formulae of work done			
$4\pi T[nr^2 - R^2]$	$4\pi R^2 T[n^{1/3} - 1]$	$4\pi T r^2 n^{2/3}[n^{1/3} - 1]$	$4\pi T R^3\left[\dfrac{1}{r} - \dfrac{1}{R}\right]$



If the work is not done by an external source then internal energy of liquid decreases, subsequently temperature decreases. This is the reason why spraying causes cooling.

By conservation of energy, Loss in thermal energy = work done against surface tension

$JQ = W$

$$\Rightarrow \qquad JmS\,\Delta\theta = 4\pi T R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$\Rightarrow J\frac{4}{3}\pi R^3 d\, S\Delta\theta = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

[As $m = V \times d = \dfrac{4}{3}\pi R^3 \times d$]

$\therefore$ Decrease in temperature $\Delta\theta = \dfrac{3T}{JSd}\left[\dfrac{1}{r} - \dfrac{1}{R}\right]$

where J = mechanical equivalent of heat, S = specific heat of liquid, d = density of liquid.

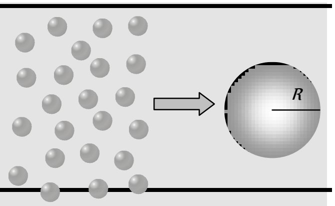
Formation of Bigger Drop

If *n* small drops of radius *r* coalesce to form a big drop of radius *R* then surface area of the liquid decreases.

Amount of surface energy released = Initial surface energy – final surface energy

$$E = n4\pi r^2 T - 4\pi R^2 T$$

Various formulae of released energy			
$4\pi T[nr^2 - R^2]$	$4\pi R^2 T(n^{1/3} - 1)$	$4\pi T r^2 n^{2/3}(n^{1/3} - 1)$	$4\pi T R^3\left[\dfrac{1}{r} - \dfrac{1}{R}\right]$



(i) If this released energy is absorbed by a big drop, its temperature increases and rise in temperature can be given by

$$\Delta\theta = \frac{3T}{JSd}\left[\frac{1}{r} - \frac{1}{R}\right]$$

(ii) If this released energy is converted into kinetic energy of a big drop without dissipation then by the law of conservation of energy.

$$\frac{1}{2}mv^2 = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{4}{3}\pi R^3 d\right]v^2 = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

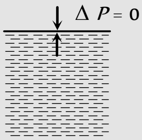
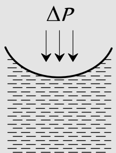
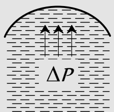
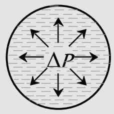
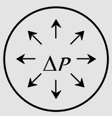
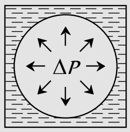
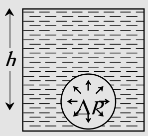
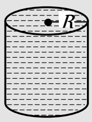
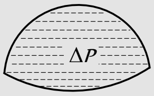
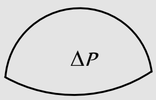
$$\Rightarrow v^2 = \frac{6T}{d}\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$\therefore\ v = \sqrt{\frac{6T}{d}\left(\frac{1}{r} - \frac{1}{R}\right)}$$

Excess Pressure

Due to the property of surface tension a drop or bubble tends to contract and so compresses the matter enclosed. This in turn increases the internal pressure which prevents further contraction and equilibrium is achieved. So in equilibrium the pressure inside a bubble or drop is greater than outside and the difference of pressure between two sides of the liquid surface is called excess pressure. In case of a drop, excess pressure is provided by hydrostatic pressure of the liquid within the drop while in case of bubble the gauge pressure of the gas confined in the bubble provides it.

Excess pressure in different cases is given in the following table :

Plane surface	Concave surface
 $\Delta P = 0$	 $\Delta P = \frac{2T}{R}$
Convex surface	**Drop**
 $\Delta P = \frac{2T}{R}$	 $\Delta P = \frac{2T}{R}$
Bubble in air	**Bubble in liquid**
 $\Delta P = \frac{4T}{R}$	 $\Delta P = \frac{2T}{R}$
Bubble at depth h below the free surface of liquid of density d	**Cylindrical liquid surface**
 $\Delta P = \frac{2T}{R} + hdg$	 $\Delta P = \frac{T}{R}$
Liquid surface of unequal radii	**Liquid film of unequal radii**
 $\Delta P = T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$	 $\Delta P = 2T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

Note : ❑ Excess pressure is inversely proportional to the radius of bubble (or drop), *i.e.*, pressure inside a smaller bubble (or drop) is higher than inside a larger bubble (or drop). That is why when two bubbles of different sizes are put in communication with each other, the air will rush from smaller to larger bubble, so that the smaller will shrink while the larger will expand till the smaller bubble reduces to droplet.

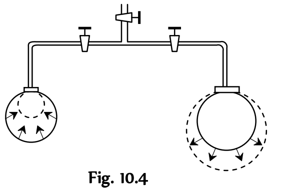


Fig. 10.4

Shape of Liquid Meniscus

We know that a liquid assumes the shape of the vessel in which it is contained *i.e.* it can not oppose permanently any force that tries to change its shape. As the effect of force is zero in a direction perpendicular to it, the free surface of liquid at rest adjusts itself at right angles to the resultant force.

When a capillary tube is dipped in a liquid, the liquid surface becomes curved near the point of contact. This curved surface is due to the resultant of two forces *i.e.* the force of cohesion and the force of adhesion. The curved surface of the liquid is called meniscus of the liquid.

If liquid molecule A is in contact with solid (*i.e.* wall of capillary tube) then forces acting on molecule A are

(i) Force of adhesion F_a (acts outwards at right angle to the wall of the tube).

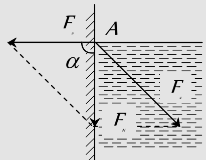
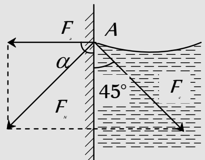
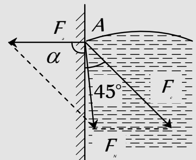
(ii) Force of cohesion F_c (acts at an angle 45^o to the vertical).

Resultant force F_R depends upon the value of F_a and F_c.

If resultant force F_R make an angle α with F_a.

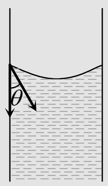
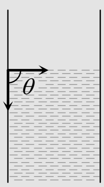
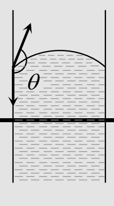
Then $\tan\alpha = \dfrac{F_c \sin 135^o}{F_a + F_c \cos 135^o} = \dfrac{F_c}{\sqrt{2}\,F_a - F_c}$

By knowing the direction of resultant force we can find out the shape of meniscus because the free surface of the liquid adjust itself at right angle to this resultant force.

If $F_c = \sqrt{2}Fa$ $\tan\alpha = \infty$ $\therefore$ $\alpha = 90^o$ *i.e.* the resultant force acts vertically downwards. Hence the liquid meniscus must be horizontal.	$F_c < \sqrt{2}Fa$ $\tan\alpha$ = positive $\therefore$ α is acute angle *i.e.* the resultant force directed outside the liquid. Hence the liquid meniscus must be concave upward.	$F_c > \sqrt{2}Fa$ $\tan\alpha$ = negative $\therefore$ α is obtuse angle *i.e.* the resultant force directed inside the liquid. Hence the liquid meniscus must be convex upward.
		
Example: Pure water in silver coated capillary tube.	Example: Water in glass capillary tube.	Example: Mercury in glass capillary tube.

Angle of Contact

Angle of contact between a liquid and a solid is defined as the angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid, both the tangents being drawn at the point of contact of the liquid with the solid.

$\theta < 90^o$ $F_a > \dfrac{F_c}{\sqrt{2}}$ concave meniscus. Liquid wets the solid surface	
$\theta = 90^o$ $F_a = \dfrac{F_c}{\sqrt{2}}$ plane meniscus. Liquid does not wet the solid surface.	
$\theta > 90^o$ $F_a < \dfrac{F_c}{\sqrt{2}}$ convex meniscus. Liquid does not wet the solid surface.	

(i) Its value lies between 0^o and 180^o

$\theta = 0^o$ for pure water and glass, $\theta = 8^o$ for tap water and glass, $\theta = 90^o$ for water and silver

$\theta = 138^o$ for mercury and glass, $\theta = 160^o$ for water and chromium

(ii) It is particular for a given pair of liquid and solid. Thus the angle of contact changes with the pair of solid and liquid.

(iii) It does not depends upon the inclination of the solid in the liquid.

(iv) On increasing the temperature, angle of contact decreases.

(v) Soluble impurities increases the angle of contact.

(vi) Partially soluble impurities decreases the angle of contact.

Capillarity

If a tube of very narrow bore (called capillary) is dipped in a liquid, it is found that the liquid in the capillary either ascends or descends relative to the surrounding liquid. This phenomenon is called capillarity.

The root cause of capillarity is the difference in pressures on two sides of (concave and convex) curved surface of liquid.

Examples of capillarity :

(i) Ink rises in the fine pores of blotting paper leaving the paper dry.

(ii) A towel soaks water.

(iii) Oil rises in the long narrow spaces between the threads of a wick.

(iv) Wood swells in rainy season due to rise of moisture from air in the pores.

(v) Ploughing of fields is essential for preserving moisture in the soil.

(vi) Sand is drier soil than clay. This is because holes between the sand particles are not so fine as compared to that of clay, to draw up water by capillary action.

Ascent Formula

When one end of capillary tube of radius r is immersed into a liquid of density d which wets the sides of the capillary tube (water and capillary tube of glass), the shape of the liquid meniscus in the tube becomes concave upwards.

R = radius of curvature of liquid meniscus.

T = surface tension of liquid

P = atmospheric pressure

Pressure at point $A = P$, Pressure at point $B = P - \frac{2T}{R}$

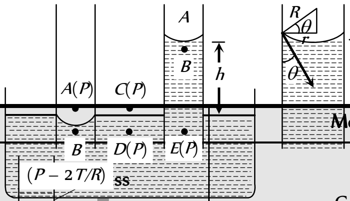


Fig. 10.5

Pressure at points C and D just above and below the plane surface of liquid in the vessel is also P (atmospheric pressure). The points B and D are in the same horizontal plane in the liquid but the pressure at these points is different.

In order to maintain the equilibrium the liquid level rises in the capillary tube upto height h.

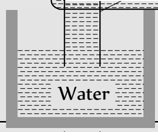
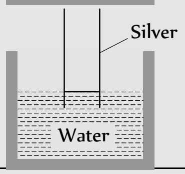
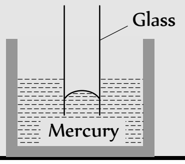
Pressure due to liquid column = pressure difference due to surface tension

$$\Rightarrow hdg = \frac{2T}{R}$$

$$\therefore\ h = \frac{2T}{Rdg} = \frac{2T\cos\theta}{rdg} \qquad \left[\text{As } R = \frac{r}{\cos\theta}\right]$$

(i) The capillary rise depends on the nature of liquid and solid both *i.e.* on T, d, θ and R.

(ii) Capillary action for various liquid-solid pair.

	Meniscus	Angle of contact	Level
	Concave	$\theta < 90^\circ$	Rises
	Plane	$\theta = 90^\circ$	No rise, no fall
	Convex	$\theta > 90^\circ$	Fall

(iii) For a given liquid and solid at a given place

$$h \propto \frac{1}{r} \qquad [\text{As } T, \theta, d \text{ and } g \text{ are constant}]$$

i.e. lesser the radius of capillary greater will be the rise and vice-versa. This is called **Jurin's law**.

(iv) If the weight of the liquid contained in the meniscus is taken into consideration then more accurate ascent formula is given by

$$h = \frac{2T\cos\theta}{rdg} - \frac{r}{3}$$

(v) In case of capillary of insufficient length *i.e.* $L < h$, the liquid will neither overflow from the upper end like a fountain nor will it tickle along the vertical sides of the tube. The liquid after reaching the upper end will increase the radius of its meniscus without changing nature such that :

$hr = Lr'$ $\quad \because\ L < h\ \therefore\ r' > r$

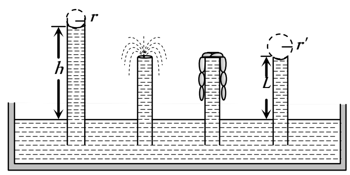


Fig. 10.6

(vi) If a capillary tube is dipped into a liquid and tilted at an angle α from vertical, then the vertical height of liquid column remains same whereas the length of liquid column (l) in the capillary tube increases.

$h = l\cos\alpha$ or $l = \frac{h}{\cos\alpha}$

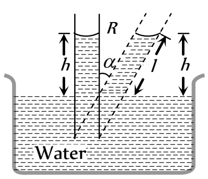


Fig. 10.7

(vii) It is important to note that in equilibrium, the height *h* is independent of the shape of capillary if the radius of meniscus remains the same. That is why the vertical height *h* of a liquid column in capillaries of different shapes and sizes will be same if the radius of meniscus remains the same.

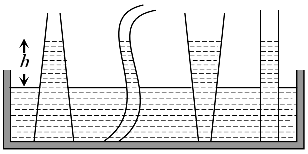


Fig. 10.8

Shape of Drops

Whether the liquid will be in equilibrium in the form of a drop or it will spread out; depends on the relative strength of the force due to surface tension at the three interfaces.

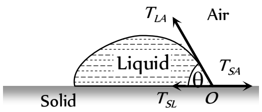


Fig. 10.9

T_{LA} = surface tension at liquid-air interface, T_{SA} = surface tension at solid-air interface.

T_{SL} = surface tension at solid-liquid interface, θ = angle of contact between the liquid and solid.

For the equilibrium of molecule

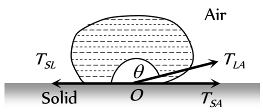


Fig. 10.10

$T_{SL} + T_{LA}\cos\theta = T_{SA}$ or $\cos\theta = \dfrac{T_{SA} - T_{SL}}{T_{LA}}$

Special Cases

$T_{SA} > T_{SL}$, $\cos\theta$ is positive *i.e.* $0^o < \theta < 90^o$.

This condition is fulfilled when the molecules of liquid are strongly attracted to that of solid.

Example : (i) Water on glass.

(ii) Kerosene oil on any surface.

$T_{SA} < T_{SL}$, $\cos\theta$ is negative *i.e.* $90^o < \theta < 180^o$.

This condition is fulfilled when the molecules of the liquid are strongly attracted *to* themselves and weakly *w.r.t.* that of solid.

Example : (i) Mercury on glass surface.

(ii) Water on lotus leaf (or a waxy or oily surface)

$(T_{SL} + T_{LA}\cos\theta) > T_{SA}$

In this condition, the molecule of liquid will not be in equilibrium and experience a net force at the interface. As a result, the liquid spreads.

Example : (i) Water on a clean glass plate.

Useful Facts and Formulae

(1) **Formation of double bubble** : If r_1 and r_2 are the radii of smaller and larger bubble and P_0 is the atmospheric pressure, then the pressure inside them will be $P_1 = P_0 + \dfrac{4T}{r_1}$ and $P_2 = P_0 + \dfrac{4T}{r_2}$.

Now as $r_1 < r_2$ $\therefore$ $P_1 > P_2$

So for interface

$$\Delta P = P_1 - P_2 = 4T\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \quad \text{...(i)}$$

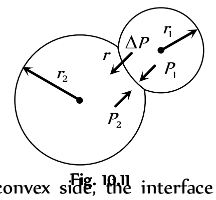


Fig. 10.11

As excess pressure acts from concave to convex side, the interface will be concave towards the smaller bubble and convex towards larger bubble and if *r* is the radius of interface.

$$\Delta P = \frac{4T}{r} \quad \text{...(ii)}$$

From (i) and (ii) $\dfrac{1}{r} = \dfrac{1}{r_1} - \dfrac{1}{r_2}$

$\therefore$ Radius of the interface $r = \dfrac{r_1 r_2}{r_2 - r_1}$

(2) **Formation of a single bubble**

(i) Under isothermal condition two soap bubble of radii '*a*' and '*b*' coalesce to form a single bubble of radius '*c*'.

If the external pressure is P_0 then pressure inside bubbles

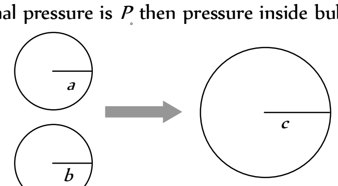


Fig. 10.12

$P_a = \left(P_0 + \dfrac{4T}{a}\right)$, $P_b = \left(P_0 + \dfrac{4T}{b}\right)$ and $P_c = \left(P_0 + \dfrac{4T}{c}\right)$

and volume of the bubbles

$V_a = \dfrac{4}{3}\pi a^3$, $V_b = \dfrac{4}{3}\pi b^3$, $V_c = \dfrac{4}{3}\pi c^3$

Now as mass is conserved $\mu_a + \mu_b = \mu_c$

$$\Rightarrow \frac{P_aV_a}{RT_a} + \frac{P_bV_b}{RT_b} = \frac{P_cV_c}{RT_c}$$

$$\left[\text{As } PV = \mu RT, \ i.e., \mu = \frac{PV}{RT}\right]$$

$$\Rightarrow P_aV_a + P_bV_b = P_cV_c \quad ...(i)$$

[As temperature is constant, *i.e.*, $T_a = T_b = T_c$]

Substituting the value of pressure and volume

$$\Rightarrow \left[P_0 + \frac{4T}{a}\right]\left[\frac{4}{3}\pi a^3\right] + \left[P_0 + \frac{4T}{b}\right]\left[\frac{4}{3}\pi b^3\right]$$

$$= \left[P_0 + \frac{4T}{c}\right]\left[\frac{4}{3}\pi c^3\right]$$

$$\Rightarrow 4T(a^2 + b^2 - c^2) = P_0(c^3 - a^3 - b^3)$$

$\therefore$ Surface tension of the liquid $T = \dfrac{P_0(c^3 - a^3 - b^3)}{4(a^2 + b^2 - c^2)}$

(ii) If two bubble coalesce in vacuum then by substituting $P_0 = 0$ in the above expression we get

$$a^2 + b^2 - c^2 = 0 \quad \therefore \ c^2 = a^2 + b^2$$

Radius of new bubble $= c = \sqrt{a^2 + b^2}$ or can be expressed as $r = \sqrt{r_1^2 + r_2^2}$.

(3) The difference of levels of liquid column in two limbs of *U*-tube of unequal radii r_1 and r_2 is

$$h = h_1 - h_2 = \frac{2T\cos\theta}{dg}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

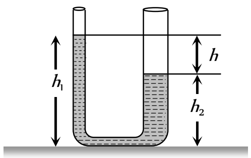


Fig. 10.13

(4) A large force (F) is required to draw apart two glass plate normally enclosing a thin water film because the thin water film formed between the two glass plates will have concave surface all around. Since on the concave side of a liquid surface, pressure is more, work will have to be done in drawing the plates apart.

$F = \dfrac{2AT}{t}$ where T= surface tension of water film, t= thickness of film, A = area of film.

(5) When a soap bubble is charged, then its size increases due to outward force on the bubble.

(6) The materials, which when coated on a surface and water does not enter through that surface are known as water proofing agents. For example wax *etc*. Water proofing agent increases the angle of contact.

(7) Values of surface tension of some liquids.

Liquid	Surface tension *Newton/metre*
Mercury	0.465
Water	0.075
Soap solution	0.030
Glycerine	0.063
Carbon tetrachloride	0.027
Ethyl alcohol	0.022

Tips & Tricks

✍ Surface tension does not depend on the area of the surface.

✍ When there is no external force, the shape of a liquid drop is determined by the surface tension of the liquid.

✍ Soap helps in better cleaning of clothes because it reduces the surface tension of the liquid.

✍ If a beaker is filled with liquid of density ρ upto a height *h*, then the mean pressure on the walls of the beaker is $h\rho g/2$.

✍ The pressure on the concave side of a curved surface is always greater than that on its convex side.

✍ Molecular forces do not obey the inverse square law of distance.

✍ The molecular forces are of electrical origin.

✍ Work done in forming a soap bubble of radius *R* is $8\pi R^2T$, where $T =$ surface tension.

✍ Energy is always required to split a drop of liquid into a number of small drops. It is because, the surface area of the small drops formed is greater than the surface area of the original single drop.

✍ Work done in breaking a drop of radius *R* into *n* drops of equal size $= 4\pi R^2T(n^{1/3} - 1)$.

Same amount of energy is liberated in combining *n* drops into a single drop.

✍ When the liquid drops merge into each other to form a larger drop, energy is released.

✍ Surface tension of molten cadmium increases with the increases in temperature.

✍ Detergents decrease both the angle of contact as well as surface tension.

✍ Angle of contact is independent of the angle of inclination of the walls.

✍ The materials used for water proofing increases the angle of contact as well as surface tension.

✍ A liquid does not wet the containing vessel if its angle of contact is obtuse.

✍ In case of liquids which do not wet the walls of the containing vessel, the force of adhesion is less than $1/\sqrt{2}$ times the force of cohesion.

✍ The liquid rises in a capillary tube, when the angle of contact is acute.

✍ The height of the liquid column in a capillary tube on the moon is six times that on the earth.

✍ Angle of contact between a liquid and a solid surface. Increases with increase in temperature of the liquid and decreases on adding impurity to the liquid.

✍ For a liquid – solid interface, if the angle of contact is acute, then

(i) The liquid will wet the solid.

(ii) The liquid will rise in the capillary tube made of such a solid and

(iii) Meniscus of the liquid will be concave.

✍ In case the angle of contact is obtuse, then

(i) The liquid will not wet the solid.

(ii) The liquid will get depressed in the tube and

(iii) Meniscus of the liquid will be convex.

✍ When the capillary tube is of insufficient length, the liquid will not overflow. It rises upto the top end of the tube and then adjusts the radius of curvature of its meniscus.