3. Binary Operations

Exercise 3.1

1 A. Question

Determine whether each of the following operations define a binary operation on the given set or not:

‘*’ on N defined by a * b = a^b for all a,b N.

Answer

Given that ‘*’ is an operation that is valid in the Natural Numbers ‘N’ and it is defined as given:

⇒ a*b = a^b, where a,b∈N

Since a∈N and b∈N,

According to the problem it is given that on applying the operation ‘*’ for two given natural numbers it gives a natural number as a result of the operation,

⇒ a*b∈N ...... (1)

We also know that $p^q>0$ if p>0 and q>0.

So, we can state that,

⇒ $a^b>0$

⇒ $a^b\in N$ ...... (2)

From (1) and (2) we can see that both L.H.S and R.H.S gave only Natural numbers as a result.

Thus we can clearly state that ‘*’ is a Binary Operation on ‘N’.

1 B. Question

Determine whether each of the following operations define a binary operation on the given set or not:

‘O’ on Z defined by a O b = a^b for all a,b Z.

Answer

Given that ‘O’ is an operation that is valid in the Integers ‘Z’ and it is defined as given:

⇒ aOb = a^b, where a,b∈Z

Since a∈Z and b∈Z,

According to the problem it is given that on applying the operation ‘*’ for two given integers it gives Integers as a result of the operation,

⇒ aOb∈Z ...... (1)

Let us values of a = 2 and b = – 2 on substituting in the R.H.S side we get,

⇒ $a^b = 2^{-2}$

⇒ $a^b = \frac{1}{4}$

⇒ $a^b\notin Z$ ...... (2)

From (2), we can see that a^b doesn’t give only Integers as a result. So, this cannot be stated as a binary function.

∴ The operation ‘O’ does not define a binary function on Z.

1 C. Question

Determine whether each of the following operations define a binary operation on the given set or not:

'*' on N defined by a * b = a + b – 2 for all a,b N.

Answer

Given that '*' is an operation that is valid in the Natural Numbers 'N' and it is defined as given:

⇒ $a*b = a + b - 2$, where $a,b \in N$

Since $a \in N$ and $b \in N$,

According to the problem it is given that on applying the operation '*' for two given natural numbers it gives a natural number as a result of the operation,

⇒ $a*b \in N$ ...... (1)

Let us take the values of a = 1 and b = 1, substituting in the R.H.S side we get,

⇒ $a + b - 2 = 1 + 1 - 2$

⇒ $a + b - 2 = 0 \notin N$ ...... (2)

From (2), we can see that a + b – 2 doesn't give only Natural numbers as a result. So, this cannot be stated as a binary function.

∴ The operation '*' does not define a binary operation on N.

1 D. Question

Determine whether each of the following operations define a binary operation on the given set or not:

'x_6' on S={1,2,3,4,5} defined by a x_6 b = Remainder when ab is divided by 6.

Answer

Given that 'x_6' is an operation that is valid for the numbers in the Set S = {1,2,3,4,5} and it is defined as given:

⇒ ax_6b = Remainder when ab is divided by 6, where $a,b \in S$

Since $a \in S$ and $b \in S$,

According to the problem it is given that on applying the operation '*' for two given numbers in the set 'S' it gives one of the numbers in the set 'S' as a result of the operation,

⇒ $ax_6b \in S$ ...... (1)

Let us take the values of a = 3, b = 4,

⇒ $ab = 3\times4$

⇒ $ab = 12$

We know that 12 is a multiple of 6. So, on dividing 12 with 6 we get 0 as remainder which is not in the given set 'S'.

The operation 'x_6' does not define a binary operation on set S.

1 E. Question

Determine whether each of the following operations define a binary operation on the given set or not:

'$+_6$' on S = {0,1,2,3,4,5} defined by a $+_6$ b= $\begin{cases} a+b & ,\text{if } a+b<6 \\ a+b-6, & \text{if } a+b \geq 6 \end{cases}$

Answer

Given that '*' is an operation that is valid for the numbers in the set S = {0,1,2,3,4,5} and it is defined as given:

⇒ $a +_6 b = \begin{cases} a + b, \text{if } a + b < 6 \\ a + b - 6, \text{if } a + b \geq 6 \end{cases}$, where $a,b \in S$

Since a∈S and b∈S,

According to the problem it is given that on applying the operation '*' for two given numbers in the set 'S' it gives the number that present in the set 'S' as a result of the operation,

⇒ $a +_6 b \in S$ ...... (1)

Since the addition of any two whole numbers must give a whole number, we can say that,

⇒ $a + b \geq 0$

For $a + b < 6$, $a + b \in S$

⇒ $a + b \geq 6$ ⇒ $a + b - 6 \geq 0 \in W$

But according to the numbers given in the problem, the maximum value of the sum we can attain is 6.

So, we can say that $a + b - 6 \in S$.

∴ The operation ' $+_6$' defines a binary operation on S.

1 F. Question

Determine whether each of the following operations define a binary operation on the given set or not:

'O' on N defined by a O b = $a^b + b^a$ for all a,b N.

Answer

Given that 'O' is an operation that is valid in the Natural Numbers 'N' and it is defined as given:

⇒ aOb = $a^b + b^a$, where a,b∈N

Since a∈N and b∈N,

According to the problem it is given that on applying the operation 'O' for two given natural numbers it gives a natural number as a result of the operation,

⇒ aOb∈N ...... (1)

We know that $p^q > 0$ if $p > 0$ and $q > 0$.

⇒ $a^b > 0$ and $b^a > 0$.

We also know that the sum of two natural numbers is a natural number.

⇒ $a^b + b^a \in N$

∴ The operation 'O' defines the binary operation on N.

1 G. Question

Determine whether each of the following operations define a binary operation on the given set or not:

'*' on Q defined by $a * b = \frac{a-1}{b+1}$ for all a,b Q.

Answer

Given that '*' is an operation that is valid in the Rational Numbers 'Q' and it is defined as given:

⇒ $a * b = \frac{a-1}{b+1}$, where a,b∈Q

Since a∈Q and b∈Q,

According to the problem it is given that on applying the operation '*' for two given rational numbers it gives a rational number as a result of the operation,

⇒ a*b∈Q ...... (1)

Let the value of b = – 1 and a = 2

$\Rightarrow \frac{a-1}{b+1} = \frac{2-1}{-1+1}$

$\Rightarrow \frac{a-1}{b+1} = \frac{1}{0} = \text{undefined}$

$\Rightarrow \frac{a-1}{b+1} \notin Q$

∴ the operation ‘*’ does not define a binary operation on Q.

2 A. Question

Determine whether or not each definition * given below gives a binary operation. In the event that * is not a binary operation give justification of this.

On Z^+, defined * by a*b = a – b

Here, Z^+ denotes the set of all non – negative integers.

Answer

Given that ‘*’ is an operation that is valid in the Positive integers ‘Z^+’ and it is defined as given:

⇒ a*b = a – b, where a,b∈Z^+

Since a∈Z^+ and b∈Z^+,

According to the problem it is given that on applying the operation ‘*’ for two given positive integers it gives a positive integer as a result of the operation,

⇒ a*b∈Z^+ ...... (1)

Let us take the values a = 1 and b = 2

⇒ a – b = 1 – 2

⇒ a – b = – 1∉Z^+

∴ The operation * does not define a binary operation on Z^+

2 B. Question

Determine whether or not each definition * given below gives a binary operation. In the event that * is not a binary operation give justification of this.

On Z^+, defined * by a*b = ab

Here, Z^+ denotes the set of all non – negative integers.

Answer

Given that ‘*’ is an operation that is valid in the Positive integers ‘Z^+’ and it is defined as given:

⇒ a*b = ab, where a,b∈Z^+,

Since a∈Z^+ and b∈Z^+,

According to the problem it is given that on applying the operation ‘*’ for two given positive integers it gives a Positive integer as a result of the operation,

⇒ a*b∈$Z^{+ -}$ ...... (1)

We know that p q>0 if p>0 and q>0.

⇒ ab>0∈N

⇒ ab∈Z^+

∴ The operation * defines a binary operation on Z^+.

2 C. Question

Determine whether or not each definition * given below gives a binary operation. In the event that * is not a binary operation give justification of this.

On R, defined * by a*b = ab^2

Here, Z^+ denotes the set of all non – negative integers.

Answer

Given that '*' is an operation that is valid in the Real Numbers 'R' and it is defined as given:

⇒ a*b = ab^2, where a,b∈R

Since a∈R and b∈R,

According to the problem it is given that on applying the operation '*' for two given real numbers it gives a real number as a result of the operation,

⇒ a*b∈R ...... (1)

We know that ab∈R if a∈R and b∈R

∴ The operation * defines a binary operation on R.

2 D. Question

Determine whether or not each definition * given below gives a binary operation. In the event that * is not a binary operation give justification of this.

On Z^+, defined * by a*b = |a – b|

Here, Z^+ denotes the set of all non – negative integers.

Answer

Given that '*' is an operation that is valid in the Positive integers 'Z^+' and it is defined as given:

⇒ a*b = |a – b|, where a,b∈Z^+,

Since a∈Z^+ and b∈Z^+,

According to the problem it is given that on applying the operation '*' for two given positive integers it gives a Positive integer as a result of the operation,

⇒ a*b∈Z ...... (1)

Let us take a = 2 and b = 2,

⇒ |a – b| = |2 – 2|

⇒ |a – b| = |0|

⇒ |a – b| = 0∉Z^+

∴ The operation * does not define a binary function on Z^+.

2 E. Question

Determine whether or not each definition * given below gives a binary operation. In the event that * is not a binary operation give justification of this.

On Z^+, defined * by a*b = a

Here, Z^+ denotes the set of all non – negative integers.

Answer

Given that '*' is an operation that is valid in the Positive integers 'Z^+' and it is defined as given:

⇒ a*b = a, where a,b∈Z^+,

Since $a \in Z^+$ and $b \in Z^+$,

According to the problem it is given that on applying the operation '*' for two given positive integers it gives a Positive integer as a result of the operation,

$\Rightarrow a*b \in Z^{+-}$ ...... (1)

It is told from the problem $a \in Z^+$.

The operation '*' defines a binary operation on Z^+.

2 F. Question

Determine whether or not each definition * given below gives a binary operation. In the event that * is not a binary operation give justification of this.

On R, defined * by $a*b = a + 4b^2$

Here, Z^+ denotes the set of all non – negative integers.

Answer

Given that '*' is an operation that is valid in the Real Numbers 'R' and it is defined as given:

$\Rightarrow a*b = a + 4b^2$, where $a,b \in R$,

Since $a \in R$ and $b \in R$,

According to the problem it is given that on applying the operation '*' for two given real numbers it gives a Real number as a result of the operation,

$\Rightarrow a*b \in R$ ...... (1)

Since $b \in R$ then $b^2 \in R$,

We also know that the sum of two real numbers gives a real number. So,

$\Rightarrow a + 4b^2 \in R$ ...... (2)

From (1) and (2),

$\therefore$ The operation '*' defines a binary operation on R.

3. Question

Let * be a binary operation on the set I of integers, defined by $a*b = 2a + b - 3$. Find the value of $3*4$.

Answer

Given that * is a binary operation on the set I of integers.

The operation is defined by $a*b = 2a + b - 3$.

We need to find the value of $3*4$.

Since 3 and 4 belongs to the set of integers we can use the binary operation.

$\Rightarrow 3*4 = (2\times3) + 4-3$

$\Rightarrow 3*4 = 6 + 1$

$\Rightarrow 3*4 = 7$

$\therefore$ the value of $3*4$ is 7.

4. Question

Is * defined on the set {1,2,3,4,5} by a*b = LCM of a and b a binary operation? Justify your answer.

Answer

Given that * is an operation that is valid on the set S = {1,2,3,4,5} defined by a*b = LCM of a and b.

According to the problem it is given that on applying the operation * for two given numbers in the set ‘S’ it gives a number in the set ‘S’ as a result of the operation.

Let us take the values of a = 2 and b = 3.

We know that L.C.M of two prime numbers is given by the product of that two prime numbers.

⇒ L.C.M of a and b is $2\times3 = 6$.

⇒ $6 \notin S$

∴ The operation * does not define a binary operation on ‘S’.

5. Question

Let S = {a,b,c}. Find the total number of binary operations on S.

Answer

Given set S = {a,b,c}, we need to find the total number of binary operations possible for the set ‘S’.

We know that the total number of binary operations on a set ‘S’ with ‘n’ elements is given by n^{n^2}.

Here n = 3,

⇒ $n^{n^2} = 3^{3^2}$

⇒ $n^{n^2} = 3^9$

∴ The total number of binary operations possible on set ‘S’ is 3^9.

6. Question

Find the total number of binary operations on {a,b}

Answer

Given set S = {a,b}, we need to find the total number of binary operations possible for the set ‘S’.

We know that the total number of binary operations on a set ‘S’ with ‘n’ elements is given by n^{n^2}.

Here n = 2,

⇒ $n^{n^2} = 2^{2^2}$

⇒ $n^{n^2} = 2^4$

⇒ $n^{n^2} = 16$

∴ The total number of binary operations possible on set ‘S’ is 16.

7. Question

Prove that the operation * on the set

$M = \left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : b \in R - \{0\}\right\}$ defined by A*B=AB is a binary operation.

Answer

Given that * is an operation that is valid on the set $M = \left\{\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} : b \in R - \{0\}\right\}$ and it is defined as given:A*B = A B.

According to the problem it is given that on applying the operation * for two given numbers in the set ‘M’ it gives a number in the set ‘M’ as a result of the operation.

⇒ $A*B \in M$ ...... (1)

Let us take $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ and $B = \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix}$ here a∈R, b∈R, c∈R and d∈R then,

$\Rightarrow AB = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \times \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix}$

$\Rightarrow AB = \begin{pmatrix} ((a \times c) + (0 \times 0)) & ((a \times 0) + (0 \times d)) \\ ((0 \times c) + (b \times 0)) & ((0 \times 0) + (b \times d)) \end{pmatrix}$

$\Rightarrow AB = \begin{pmatrix} (ac + 0) & (0 + 0) \\ (0 + 0) & (0 + bd) \end{pmatrix}$

$\Rightarrow AB = \begin{pmatrix} ac & 0 \\ 0 & bd \end{pmatrix}$

Since a∈R and c∈R then ac∈R

And also b∈R and d∈R then bd∈R.

⇒ AB∈R

∴ The operation ‘*’ defines a binary operation on ‘M’.

8. Question

Let S be the set of all rational numbers of the form m/n, where m∈Z and n = 1,2,3. Prove that * on S defined by a*b = ab is not a binary operation.

Answer

Given that * is an operation that is valid on the set S which consists of all rational numbers of the form $\frac{m}{n}$, here m∈Z and n = 1,2,3 and is defined by a*b = ab.

According to the problem it is given that on applying the operation * for two given numbers in the set ‘S’ it gives a number in the set ‘S’ as a result of the operation.

⇒ a*b∈S ...... (1)

Since a∈S and b∈S,

Let us take the values of $a = \frac{5}{3}$ and $b = \frac{8}{3}$ then,

$\Rightarrow ab = \frac{5}{3} \times \frac{8}{3}$

$\Rightarrow ab = \frac{40}{9} \notin S$ as 9∉n.

∴ The operation ‘*’ does not define a binary operation on ‘S’.

9. Question

The binary operation * defined on R×R → R is defined as a*b = 2a + b. Find (2*3)*4.

Answer

Given that * is an operation that is valid for the following Domain and Range R×R→R and is defined by a*b = 2ab.

We need to find the value of (2*3)*4

According to the problem the binary operation involving * is true for all real values of a and b.

⇒ (2*3)*4 = ((2×2) + 3)*4

⇒ (2*3)*4 = (4 + 3)*4

⇒ (2*3)*4 = 7*4

⇒ (2*3)*4 = (2×7) + 4

⇒ (2*3)*4 = 14 + 4

⇒ (2*3)*4 = 18

∴ The value of (2*3)*4 is 18.

10. Question

Let * be a binary operation on N given by a*b = LCM(a,b) for all a,b∈N. Find 5*7.

Answer

Given that * is an operation that is valid for the natural numbers ‘N’ and is defined by a*b = LCM(a,b).

We need to find the value of 5*7.

According to the Problem, Binary operation is assumed to be true for the values of a and b to be natural.

⇒ 5*7 = LCM(5,7)

We know that LCM of two prime numbers is the product of that given two prime numbers.

⇒ 5*7 = 5×7

⇒ 5*7 = 35

∴ The value of 5*7 is 35.

Exercise 3.2

1 A. Question

Let ‘*’ be a binary operation on N defined by a*b = L.C.M(a,b) for all a,b∈N.

Find 2*4, 3*5, 1*6.

Answer

Given that * is an operation that is valid on all natural numbers ‘N’ and is defined by a*b = L.C.M(a,b)

According to the problem, binary operation given is assumed to be true.

Let us find the values of 2*4,3*5,1*6

⇒ 2*4 = L.C.M(2,4) = 4

⇒ 3*5 = L.C.m(3,5)

⇒ 3*5 = 3×5 = 15

⇒ 1*6 = L.C.M(1,6)

⇒ 1*6 = 1×6 = 6

The values of 2*4 is 4 , 3*5 is 15 and 1*6 is 6

1 B. Question

Let ‘*’ be a binary operation on N defined by a*b = L.C.M(a,b) for all a,b∈N.

Check the commutativity and associativity of ‘*’ on N.

Answer

We know that commutative property is p*q = q*p, where * is a binary operation.

Let’s check the commutativity of given binary operation:

⇒ a*b = L.C.M(a,b)

⇒ b*a = L.C.M(b,a) = L.C.M(a,b)

⇒ b*a = a*b

∴ Commutative property holds for given binary operation ‘*’ on ‘N’.

We know that associative property is (p*q)*r = p*(q*r)

Let’s check the associativity of given binary operation:

⇒ (a*b)*c = (L.C.M(a,b))*c

⇒ (a*b)*c = L.C.M(a,b)*c

⇒ (a*b)*c = L.C.M(L.C.M(a,b),c)

⇒ (a*b)*c = L.C.M(a,b,c) ...... (1)

⇒ a*(b*c) = a*(L.C.M(b,c))

⇒ a*(b*c) = a*L.C.M(b,c)

⇒ a*(b*c) = L.C.M(a,L.C.M(b,c))

⇒ a*(b*c) = L.C.M(a,b,c) ...... (2)

From(1) and (2) we can say that associative property holds for binary function ‘*’ on ‘N’.

2 A. Question

Determine which of the following binary operations are associative and which are commutative:

* on N defined by a*b=1 for all a,b N

Answer

Given that * is a binary operation on N defined by a*b = 1 for all a,b∈N.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let’s check the commutativity of given binary operation:

⇒ a*b = 1

⇒ b*a = 1

⇒ b*a = a*b

∴ The commutative property holds for given binary operation ‘*’ on ‘N’.

We know that associative property is (p*q)*r = p*(q*r)

Let’s check the associativity of given binary operation:

⇒ (a*b)*c = (1)*c

⇒ (a*b)*c = 1*c

⇒ (a*b)*c = 1 ...... (1)

⇒ a*(b*c) = a*(1)

⇒ a*(b*c) = a*1

⇒ a*(b*c) = 1 ...... (2)

From (1) and (2) we can clearly say that,

Associative property holds for given binary operation ‘*’ on ‘N’.

2 B. Question

Determine which of the following binary operations are associative and which are commutative:

*on Q defined by $a * b = \frac{a+b}{2}$ for all a,b Q.

Answer

Given that * is a binary operation on N defined by $a * b = \frac{a+b}{2}$ for all a,b∈N.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let's check the commutativity of given binary operation:

⇒ $a * b = \frac{a+b}{2}$

⇒ $b * a = \frac{b+a}{2} = \frac{a+b}{2}$

⇒ b*a = a*b

∴ The commutative property holds for given binary operation '*' on 'N'.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

⇒ $(a * b) * c = \left(\frac{a+b}{2}\right) * c$

⇒ $(a * b) * c = \frac{\frac{a+b}{2} + c}{2}$

⇒ $(a * b) * c = \frac{a+b+2c}{4}$ ...... (1)

⇒ $a * (b * c) = a * \left(\frac{b+c}{2}\right)$

⇒ $a * (b * c) = \frac{a + \frac{b+c}{2}}{2}$

⇒ $a * (b * c) = \frac{2a+b+c}{4}$ ...... (2)

From (1) and (2) we can clearly say that associativity doesn't hold for the binary operation '*' on 'N'.

3. Question

Let A be any set containing more than one element. Let '*' be a binary operation on A defined by a*b = b for all a,b∈A. Is '*' commutative or associative on A?

Answer

Given that * is a binary operation on set A defined by a*b = b for all a,b∈A.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let's check the commutativity of given binary operation:

⇒ a*b = b

⇒ b*a = a

⇒ b*a≠a*b

∴ The commutative property does not hold for given binary operation '*' on 'A'.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

⇒ (a*b)*c = (b)*c

⇒ (a*b)*c = b*c

⇒ (a*b)*c = c ...... (1)

⇒ a*(b*c) = a*(c)

⇒ a*(b*c) = a*c

⇒ a*(b*c) = c ...... (2)

From (1) and (2) we can clearly say that associativity holds for the binary operation ‘*’ on ‘A’.

4 A. Question

Check the commutativity and associativity of each of the following binary operations:

‘*’ on Z defined by a*b = a + b + ab for all a,b∈Z

Answer

Given that * is a binary operation on Z defined by a*b = a + b + ab for all a,b∈Z.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let’s check the commutativity of given binary operation:

⇒ a*b = a + b + ab

⇒ b*a = b + a + ba

⇒ b*a = a + b + ab

⇒ b*a = a*b

∴ Commutative property holds for given binary operation ‘*’ on ‘Z’.

We know that associative property is (p*q)*r = p*(q*r)

Let’s check the associativity of given binary operation:

⇒ (a*b)*c = (a + b + ab)*c

⇒ (a*b)*c = (a + b + ab + c + ((a + b + ab)×c))

⇒ (a*b)*c = a + b + c + ab + ac + ac + abc ...... (1)

⇒ a*(b*c) = a*(b + c + bc)

⇒ a*(b*c) = (a + b + c + bc + (a×(b + c + bc)))

⇒ a*(b*c) = a + b + c + ab + bc + ac + abc ...... (2)

From (1) and (2) we can clearly say that associativity holds for the binary operation ‘*’ on ‘Z’.

4 B. Question

Check the commutativity and associativity of each of the following binary operations:

‘*’ on N defined by a*b = 2^{ab} for all a,b∈N

Answer

Given that * is a binary operation on N defined by a*b = 2^{ab} for all a,b∈N.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let’s check the commutativity of given binary operation:

⇒ a*b = 2^{ab}

⇒ b*a = 2^{ba} = 2^{ab}

⇒ b*a = a*b

∴ The commutative property holds for given binary operation ‘*’ on ‘N’.

We know that associative property is (p*q)*r = p*(q*r)

Let’s check the associativity of given binary operation:

⇒ (a*b)*c = (2^{ab})*c

⇒ $(a * b) * c = 2^{2^{ab}.c}$ ...... (1)

⇒ a*(b*c) = a*(2^{bc})

⇒ $a * (b * c) = 2^{a.2^{bc}}$ ...... (2)

From (1) and (2) we can clearly say that associativity doesn't hold for the binary operation '*' on 'N'.

4 C. Question

Check the commutativity and associativity of each of the following binary operations:

'*' on Q defined by a*b = a – b for all a,b∈Q

Answer

Given that * is a binary operation on Q defined by a*b = a – b for all a,b∈Q.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let's check the commutativity of given binary operation:

⇒ a*b = a – b

⇒ b*a = b = a

⇒ b*a≠a*b

∴ The commutative property doesn't hold for given binary operation '*' on 'Q'.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

⇒ (a*b)*c = (a – b)*c

⇒ (a*b)*c = a – b – c ...... (1)

⇒ a*(b*c) = a*(b – c)

⇒ a*(b*c) = a – (b – c)

⇒ a*(b*c) = a – b + c ...... (2)

From (1) and (2) we can clearly say that associativity doesn't hold for the binary operation '*' on 'Q'.

4 D. Question

Check the commutativity and associativity of each of the following binary operations:

'O' on Q defined by aOb = $a^2 + b^2$ for all a,b∈Q

Answer

Given that O is a binary operation on Q defined by aOb = $a^2 + b^2$ for all a,b∈Q.

We know that commutative property is pOq = qOp, where O is a binary operation.

Let's check the commutativity of given binary operation:

⇒ aOb = $a^2 + b^2$

⇒ bOa = $b^2 + a^2 = a^2 + b^2$

⇒ bOa = aOb

∴ Commutative property holds for given binary operation 'O' on 'Q'.

We know that associative property is (pOq)Or = pO(qOr)

Let's check the associativity of given binary operation:

⇒ (aOb)Oc = $(a^2 + b^2)$Oc

⇒ $(aOb)Oc = (a^2 + b^2)^2 + c^2$

⇒ (aOb)Oc = $a^4 + b^4 + 2a^2b^2 + c^2$ ...... (1)

⇒ aO(bOc) = aO$(b^2 + c^2)$

⇒ $aO(bOc) = a^2 + (b^2 + c^2)^2$

⇒ aO(bOc) = $a^2 + b^4 + c^4 + 2b^2c^2$ ...... (2)

From (1) and (2) we can clearly say that associativity doesn't hold for the binary operation '*' on 'Q'.

4 E. Question

Check the commutativity and associativity of each of the following binary operations:

'o' on Q defined by a o b = ab/2 for all a,b Q.

Answer

Given that o is a binary operation on Q defined by $aob = \frac{ab}{2}$ for all a,b∈Q.

We know that commutative property is poq = qop, where o is a binary operation.

Let's check the commutativity of given binary operation:

⇒ $aob = \frac{ab}{2}$

⇒ $boa = \frac{ba}{2} = \frac{ab}{2}$

⇒ b*a = a*b

∴ The commutative property holds for given binary operation 'o' on 'Q'.

We know that associative property is (poq)or = po(qor)

Let's check the associativity of given binary operation:

⇒ $(aob)oc = \left(\frac{ab}{2}\right)oc$

⇒ $(aob)oc = \frac{\left(\frac{ab}{2}\right)c}{2}$

⇒ $(aob)oc = \frac{abc}{4}$ ...... (1)

⇒ $ao(boc) = ao\left(\frac{bc}{2}\right)$

⇒ $ao(boc) = \frac{a\left(\frac{bc}{2}\right)}{2}$

⇒ $ao(boc) = \frac{abc}{4}$ ...... (2)

From (1) and (2) we can clearly say that associativity hold for the binary operation 'o' on 'Q'.

4 F. Question

Check the commutativity and associativity of each of the following binary operations:

'*' on Q defined by a*b = ab^2 for all a,b∈Q

Answer

Given that * is a binary operation on Q defined by $a*b = ab^2$ for all a,b∈Q.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let's check the commutativity of given binary operation:

⇒ $a*b = ab^2$

⇒ $b*a = ba^2$

⇒ b*a≠a*b

∴ Commutative property doesn't holds for given binary operation '*' on 'Q'.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

⇒ $(a*b)*c = (ab^2)*c$

⇒ $(a*b)*c = ab^2c^2$ ...... (1)

⇒ $a*(b*c) = a*(bc^2)$

⇒ $a*(b*c) = a(bc^2)^2$

⇒ $a*(b*c) = ab^2c^4$ ...... (2)

From (1) and (2) we can clearly say that associativity doesn't hold for the binary operation '*' on 'Q'.

4 G. Question

Check the commutativity and associativity of each of the following binary operations:

'*' on Q defined by a*b = a + ab for all a,b∈Q

Answer

Given that * is a binary operation on Q defined by a*b = a + ab for all a,b∈Q.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let's check the commutativity of given binary operation:

⇒ a*b = a + ab

⇒ b*a = b + ba = b + ab

⇒ b*a≠a*b

∴ Commutative property doesn't holds for given binary operation '*' on 'Q'.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

⇒ (a*b)*c = (a + ab)*c

⇒ (a*b)*c = a + ab + ((a + ab)×c)

⇒ (a*b)*c = a + ab + ac + abc ...... (1)

⇒ a*(b*c) = a*(b + bc)

⇒ a*(b*c) = a + (a×(b + bc))

⇒ a*(b*c) = a + ab + abc ...... (2)

From (1) and (2) we can clearly say that associativity doesn't hold for the binary operation '*' on 'Q'.

4 H. Question

Check the commutativity and associativity of each of the following binary operations:

'*' on R defined by a*b = a + b – 7 for all a,b∈Q

Answer

Given that * is a binary operation on R defined by a*b = a + b – 7 for all a,b∈R.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let's check the commutativity of given binary operation:

⇒ a*b = a + b – 7

⇒ b*a = b + a – 7 = a + b – 7

⇒ b*a = a*b

∴ Commutative property holds for given binary operation '*' on 'R'.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

⇒ (a*b)*c = (a + b – 7)*c

⇒ (a*b)*c = a + b – 7 + c – 7

⇒ (a*b)*c = a + b + c – 14 ...... (1)

⇒ a*(b*c) = a*(b + c – 7)

⇒ a*(b*c) = a + b + c – 7 – 7

⇒ a*(b*c) = a + b + c – 14 ...... (2)

From (1) and (2) we can clearly say that associativity holds for the binary operation '*' on 'R'.

4 I. Question

Check the commutativity and associativity of each of the following binary operations:

'*' on Q defined by a*b = $(a – b)^2$ for all a,b∈Q

Answer

Given that * is a binary operation on Q defined by a*b = $(a – b)^2$ for all a,b∈Q.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let's check the commutativity of given binary operation:

⇒ a*b = $(a – b)^2$

⇒ b*a = $(b – a)^2$ = $(a – b)^2$

⇒ b*a = a*b

∴ Commutative property holds for given binary operation '*' on 'Q'.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

⇒ (a*b)*c = $((a – b)^2)$*c

⇒ $(a * b) * c = ((a-b)^2 - c)^2$

⇒ (a*b)*c = $(a^2 + b^2 – 2ab – c)^2$ ...... (1)

⇒ a*(b*c) = a*$((b – c)^2)$

⇒ $a * (b * c) = (a^2 - (b-c)^2)^2$

$\Rightarrow a*(b*c) = (a^2 – b^2 – c^2 + 2bc)^2$ ...... (2)

From (1) and (2) we can clearly say that associativity doesn’t hold for the binary operation ‘*’ on ‘Q’.

4 J. Question

Check the commutativity and associativity of each of the following binary operations:

*’ on Q defined by a*b = ab + 1 for all a,b∈Q

Answer

Given that * is a binary operation on Q defined by a*b = ab + 1 for all a,b∈Q.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let’s check the commutativity of given binary operation:

⇒ a*b = ab + 1

⇒ b*a = ba + 1 = ab + 1

⇒ b*a = a*b

∴ Commutative property holds for given binary operation ‘*’ on ‘Q’.

We know that associative property is (p*q)*r = p*(q*r)

Let’s check the associativity of given binary operation:

⇒ (a*b)*c = (ab + 1)*c

⇒ (a*b)*c = ((ab + 1)×c) + 1

⇒ (a*b)*c = abc + c + 1 ...... (1)

⇒ a*(b*c) = a*(bc + 1)

⇒ a*(b*c) = (a×(bc + 1)) + 1

⇒ a*(b*c) = abc + a + 1 ...... (2)

From (1) and (2) we can clearly say that associativity doesn’t hold for the binary operation ‘*’ on ‘Q’.

4 K. Question

Check the commutativity and associativity of each of the following binary operations:

‘*’ on N defined by $a*b = a^b$ for all a,b∈N

Answer

Given that * is a binary operation on N defined by $a*b = a^b$ for all a,b∈N.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let’s check the commutativity of given binary operation:

$\Rightarrow a*b = a^b$

$\Rightarrow b*a = b^a$

⇒ b*a≠a*b

∴ Commutative property doen’t holds for given binary operation ‘*’ on ‘N’.

We know that associative property is (p*q)*r = p*(q*r)

Let’s check the associativity of given binary operation:

$\Rightarrow (a*b)*c = (a^b)*c$

$\Rightarrow (a*b)*c = (a^b)^c$

⇒ $(a*b)*c = a^{bc}$ ...... (1)

⇒ $a*(b*c) = a*(b^c)$

⇒ $a*(b*c) = a^{b^c}$ ...... (2)

From (1) and (2) we can clearly say that associativity doesn’t hold for the binary operation ‘*’ on ‘N’.

4 L. Question

Check the commutativity and associativity of each of the following binary operations:

‘*’ on Z defined by a*b = a – b for all a,b∈Z

Answer

Given that * is a binary operation on Z defined by a*b = a – b for all a,b∈Z.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let’s check the commutativity of given binary operation:

⇒ a*b = a – b

⇒ b*a = b – a

⇒ b*a≠a*b

∴ Commutative property doesn’t holds for given binary operation ‘*’ on ‘Z’.

We know that associative property is (p*q)*r = p*(q*r)

Let’s check the associativity of given binary operation:

⇒ (a*b)*c = (a – b)*c

⇒ (a*b)*c = (a – b) – c

⇒ (a*b)*c = a – b – c ...... (1)

⇒ a*(b*c) = a*(b – c)

⇒ a*(b*c) = a – (b – c)

⇒ a*(b*c) = a – b + c ...... (2)

From (1) and (2) we can clearly say that associativity doesn’t hold for the binary operation ‘*’ on ‘Z’.

4 M. Question

Check the commutativity and associativity of each of the following binary operations:

‘*’ on Q defined by a*b = ab/4 for all a,b∈Q

Answer

Given that * is a binary operation on Q defined by $a*b = \frac{ab}{4}$ for all a,b∈Q.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let’s check the commutativity of given binary operation:

⇒ $a*b = \frac{ab}{4}$

⇒ $b*a = \frac{ba}{4} = \frac{ab}{4}$

⇒ b*a = a*b

∴ Commutative property holds for given binary operation ‘*’ on ‘Q’.

We know that associative property is (p*q)*r = p*(q*r)

Let’s check the associativity of given binary operation:

⇒ $(a * b) * c = \left(\frac{ab}{4}\right) * c$

⇒ $(a * b) * c = \frac{\left(\frac{ab}{4}\right).c}{4}$

⇒ $(a * b) * c = \frac{abc}{16}$ ...... (1)

⇒ $a * (b * c) = a * \left(\frac{bc}{4}\right)$

⇒ $a * (b * c) = \frac{a.\left(\frac{bc}{4}\right)}{4}$

⇒ $a * (b * c) = \frac{abc}{16}$ ...... (2)

From (1) and (2) we can clearly say that associativity hold for the binary operation ‘*’ on ‘Q’.

4 N. Question

Check the commutativity and associativity of each of the following binary operations:

‘*’ on Z defined by a*b = a + b – ab for all a,b∈Z

Answer

Given that * is a binary operation on Z defined by a*b = a + b – ab for all a,b∈Z.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let’s check the commutativity of given binary operation:

⇒ a*b = a + b – ab

⇒ b*a = b + a – ba = a + b – ab

⇒ b*a = a*b

∴ Commutative property holds for given binary operation ‘*’ on ‘Z’.

We know that associative property is (p*q)*r = p*(q*r)

Let’s check the associativity of given binary operation:

⇒ (a*b)*c = (a + b – ab)*c

⇒ (a*b)*c = a + b – ab + c – ((a + b – ab)×c)

⇒ (a*b)*c = a + b + c – ab – ac – bc + abc ...... (1)

⇒ a*(b*c) = a*(b + c – bc)

⇒ a*(b*c) = a + b + c – bc – (a×(b + c – bc))

⇒ a*(b*c) = a + b + c – ab – ac – bc + abc ...... (2)

From (1) and (2) we can clearly say that associativity hold for the binary operation ‘*’ on ‘Z’.

4 O. Question

Check the commutativity and associativity of each of the following binary operations:

‘*’ on Q defined by a*b = gcd(a,b) for all a,b∈Q

Answer

Given that * is a binary operation on Q defined by a*b = g.c.d(a,b) for all a,b∈Q.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let’s check the commutativity of given binary operation:

⇒ a*b = g.c.d(a,b)

⇒ b*a = g.c.d(b,a) = g.c.d(a,b)

⇒ b*a = a*b

∴ Commutative property holds for given binary operation ‘*’ on ‘Q’.

We know that associative property is (p*q)*r = p*(q*r)

Let’s check the associativity of given binary operation:

⇒ (a*b)*c = (g.c.d(a,b))*c

⇒ (a*b)*c = g.c.d(g.c.d(a,b),c)

⇒ (a*b)*c = g.c.d(a,b,c) ...... (1)

⇒ a*(b*c) = a*(g.c.d(b,c))

⇒ a*(b*c) = g.c.d(a,g.c.d(b,c))

⇒ a*(b*c) = g.c.d(a,b,c) ...... (2)

From (1) and (2) we can clearly say that associativity hold for the binary operation ‘*’ on ‘Q’.

5. Question

If the binary operation o is defined by aob = a + b – ab on the set Q – { – 1} of all rational numbers other than – 1. Show that o is commutative on Q – { – 1}.

Answer

Given that o is a binary operation on Q – { – 1} defined by aob = a + b – ab for all a,b∈Q – { – 1}.

We know that commutative property is poq = qop, where o is a binary operation.

Let’s check the commutativity of given binary operation:

⇒ aob = a + b – ab

⇒ boa = b + a – ba = a + b – ab

⇒ b*a = a*b

∴ Commutative property holds for given binary operation ‘o’ on ‘Q – { – 1}’.

We know that associative property is (poq)or = po(qor)

Let’s check the associativity of given binary operation:

⇒ (aob)oc = (a + b – ab)oc

⇒ (aob)oc = a + b – ab + c – ((a + b – ab)×c)

⇒ (aob)oc = a + b + c – ab – ac – ab + abc ...... (1)

⇒ ao(boc) = ao(b + c – bc)

⇒ ao(boc) = a + b + c – bc – (a×(b + c – bc))

⇒ a*(b*c) = a + b + c – ab – bc – ac + abc ...... (2)

From (1) and (2) we can clearly say that associativity hold for the binary operation ‘*’ on ‘Q – { – 1}’.

6. Question

Show that the binary operation * on Z defined by a*b = 3a + 7b is not commutative.

Answer

Given that * is a binary operation on Z defined by a*b = 3a + 7b for all a,b∈Z.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let’s check the commutativity of given binary operation:

⇒ a*b = 3a + 7b

⇒ b*a = 3b + 7a

⇒ b*a≠a*b

∴ Commutative property doesn’t holds for given binary operation ‘*’ on ‘Z’.

7. Question

On the set Z of integers a binary operation * is defined by a*b = ab + 1 for all a,b∈Z. Prove that * is not associative on Z.

Answer

Given that * is a binary operation on Z defined by a*b = ab + 1 for all a,b∈Z.

We know that associative property is (p*q)*r = p*(q*r), where x is a binary operation.

Let’s check the associativity of given binary operation:

⇒ (a*b)*c = (ab + 1)*c

⇒ (a*b)*c = ((ab + 1)×c) + 1

⇒ (a*b)*c = 1 + c + abc ...... (1)

⇒ a*(b*c) = a*(bc + 1)

⇒ a*(b*c) = (a×(bc + 1)) + 1

⇒ a*(b*c) = abc + a + 1 ...... (2)

From (1) and (2) we can clearly say that associativity doesn’t hold for the binary operation ‘*’ on ‘Z’.

8. Question

Let S be the set of all real numbers except – 1 and let ‘*’ be an operation defined by a*b = a + b + ab for all ab∈S. Determine whether ‘*’ is a binary operation on ‘S’. if yes, Check its commutativity and associativity. Also, solve the equation (2*x)*3 = 7.

Answer

Given that ‘*’ is an operation that is valid on the set S which consists of all real numbers except – 1 i.e., R – {– 1} defined as a*b = a + b + ab

Let us assume a + b + ab = – 1

⇒ a + ab + b + 1 = 0

⇒ a(1 + b) + (1 + b) = 0

⇒ (a + 1)(b + 1) = 0

⇒ a = – 1 or b = – 1

But according to the problem, it is given that a≠ – 1 and b≠ – 1 so,

a + b + ab≠ – 1, so we can say that the operation ‘*’ defines a binary operation on set ‘S’.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let’s check the commutativity of given binary operation:

⇒ a*b = a + b + ab

⇒ b*a = b + a + ba = a + b + ab

⇒ b*a = a*b

∴ Commutative property holds for given binary operation '*' on 'S'.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

⇒ (a*b)*c = (a + b + ab)*c

⇒ (a*b)*c = a + b + ab + c + ((a + b + ab)×c)

⇒ (a*b)*c = a + b + c + ab + ac + bc + abc ...... (1)

⇒ a*(b*c) = a*(b + c + bc)

⇒ a*(b*c) = a + b + c + bc + (a×(b + c + bc))

⇒ a*(b*c) = a + b + c + ab + bc + ac + abc ...... (2)

From (1) and (2) we can clearly say that associativity holds for the binary operation '*' on 'N'.

We need to also solve for x in the given expression:

⇒ (2*x)*3 = 7

⇒ (2 + x + 2x)*3 = 7

⇒ (2 + 3x)*3 = 7

⇒ 2 + 3x + 3 + ((2 + 3x)×3) = 7

⇒ 5 + 3x + 6 + 9x = 7

⇒ 11 + 12x = 7

⇒ 12x = – 4

⇒ $x = \frac{-4}{12}$

⇒ $x = \frac{-1}{3}$

∴ the value of x is $\frac{-1}{3}$.

9. Question

On Q, the set of all rational numbers, * is defined by $a * b = \frac{a-b}{2}$, show that * is not associative.

Answer

Given that * is a binary operation on Q defined by $a * b = \frac{a-b}{2}$ for all a,b∈Q.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

⇒ $(a * b) * c = \left(\frac{a-b}{2}\right) * c$

⇒ $(a * b) * c = \frac{\left(\frac{a-b}{2}\right)-c}{2}$

⇒ $(a * b) * c = \frac{(a-b)-2c}{4}$

⇒ $(a * b) * c = \frac{a-b-2c}{4}$ ...... (1)

⇒ $a * (b * c) = a * \left(\frac{b-c}{2}\right)$

$\Rightarrow a * (b * c) = \frac{a - \left(\frac{b-c}{2}\right)}{2}$

$\Rightarrow a * (b * c) = \frac{2a-(b-c)}{4}$

$\Rightarrow a * (b * c) = \frac{2a-b+c}{4}$ ...... (2)

From (1) and (2) we can clearly say that associativity doesn't hold for the binary operation '*' on 'Q'.

10. Question

On Z, the set of all integers, a binary operation * is defined by a*b = a + 3b – 4. Prove that * is neither commutative nor associative on Z.

Answer

Given that * is a binary operation on Z defined by a*b = a + 3b – 4 for all a,b∈Z.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let's check the commutativity of given binary operation:

⇒ a*b = a + 3b – 4

⇒ b*a = b + 3a – 4

⇒ b*a ≠ a*b

∴ The commutative property doesn't hold for given binary operation '*' on 'Z'.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

⇒ (a*b)*c = (a + 3b – 4)*c

⇒ (a*b)*c = a + 3b – 4 + 3c – 4

⇒ (a*b)*c = a + 3b + 3c – 8 ...... (1)

⇒ a*(b*c) = a*(b + 3c – 4)

⇒ a*(b*c) = a + (3×(b + 3c – 4)) – 4

⇒ a*(b*c) = a + 3b + 9c – 12 – 4

⇒ a*(b*c) = a + 3b + 9c – 16 ...... (2)

From (1) and (2) we can clearly say that associativity doesn't hold for the binary operation '*' on 'Z'.

11. Question

On the set Q of all rational numbers if a binary operation * is defined by $a * b = \frac{ab}{5}$, prove that * is associative on Q.

Answer

Given that * is a binary operation on Q defined by $a * b = \frac{ab}{5}$ for all a,b∈Q.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

$\Rightarrow (a * b) * c = \left(\frac{ab}{5}\right) * c$

$\Rightarrow (a * b) * c = \frac{\left(\frac{ab}{5}\right).c}{5}$

$\Rightarrow (a * b) * c = \frac{abc}{25}$ ...... (1)

$\Rightarrow a * (b * c) = a * \left(\frac{bc}{5}\right)$

$\Rightarrow a * (b * c) = \frac{a.\left(\frac{bc}{5}\right)}{5}$

$\Rightarrow a * (b * c) = \frac{abc}{25}$ ...... (2)

From (1) and (2) we can clearly say that associativity hold for the binary operation '*' on 'Q'.

12. Question

The binary operation * is defined by $a * b = \frac{ab}{7}$ on the set Q if all rational numbers. Show that * is associative.

Answer

Given that * is a binary operation on Q defined by $a * b = \frac{ab}{7}$ for all a,b∈Q.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

$\Rightarrow (a * b) * c = \left(\frac{ab}{7}\right) * c$

$\Rightarrow (a * b) * c = \frac{\left(\frac{ab}{7}\right).c}{7}$

$\Rightarrow (a * b) * c = \frac{abc}{49}$ ...... (1)

$\Rightarrow a * (b * c) = a * \left(\frac{bc}{7}\right)$

$\Rightarrow a * (b * c) = \frac{a.\left(\frac{bc}{7}\right)}{7}$

$\Rightarrow a * (b * c) = \frac{abc}{49}$ ...... (2)

From (1) and (2) we can clearly say that associativity hold for the binary operation '*' on 'Q'.

13. Question

On Q, the set of all rational numbers a binary operation * is defined by $a * b = \frac{a+b}{2}$. Show that * is not associative on Q.

Answer

Given that * is a binary operation on Q defined by $a * b = \frac{a+b}{2}$ for all a,b∈Q.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

$\Rightarrow (a * b) * c = \left(\frac{a+b}{2}\right) * c$

$\Rightarrow (a * b) * c = \frac{\left(\frac{a+b}{2}\right) + c}{2}$

$\Rightarrow (a * b) * c = \frac{a + b + 2c}{4}$ ...... (1)

$\Rightarrow a * (b * c) = a * \left(\frac{b+c}{2}\right)$

$\Rightarrow a * (b * c) = \frac{a + \left(\frac{b+c}{2}\right)}{2}$

$\Rightarrow a * (b * c) = \frac{2a+b+c}{4}$

$\Rightarrow a * (b * c) = \frac{2a+b+c}{4}$ ...... (2)

From (1) and (2) we can clearly say that associativity doesn't hold for the binary operation '*' on 'N'.

14. Question

Let S be the set of all rational numbers except 1 and * be defined on S by a*b = a + b – ab, for all a,b∈S.

Prove that:

i. * is a binary operation on S

ii. * is commutative as well as associative.

Answer

Given that '*' is an operation that is valid on the set S which consists of all real numbers except 1 i.e., R – {1} defined as a*b = a + b – ab

Let us assume a + b – ab = 1

⇒ a – ab + b – 1 = 0

⇒ a(1 – b) – 1(1 – b) = 0

⇒ (1 – a)(1 – b) = 0

⇒ a = 1 or b = 1

But according to the problem, it is given that a≠1 and b≠1 so,

a + b + ab≠1, so we can say that the operation '*' defines a binary operation on set 'S'.

We know that commutative property is p*q = q*p, where * is a binary operation.

Let's check the commutativity of given binary operation:

⇒ a*b = a + b – ab

⇒ b*a = b + a – ba = a + b – ab

⇒ b*a = a*b

∴ Commutative property holds for given binary operation '*' on 'S'.

We know that associative property is (p*q)*r = p*(q*r)

Let's check the associativity of given binary operation:

⇒ (a*b)*c = (a + b – ab)*c

⇒ (a*b)*c = a + b – ab + c – ((a + b – ab)×c)

⇒ (a*b)*c = a + b + c – ab – ac – bc + abc ...... (1)

⇒ a*(b*c) = a*(b + c – bc)

⇒ a*(b*c) = a + b + c – bc – (a×(b + c + bc))

⇒ a*(b*c) = a + b + c – ab – bc – ac + abc ...... (2)

From (1) and (2) we can clearly say that associativity holds for the binary operation '*' on 'S'.

Exercise 3.3

1. Question

Find the identity element in the set I^+ of all positive integers defined by a*b = a + b for all a,b∈I^+.

Answer

Given that binary operation ‘*’ is valid for set ‘I^+’ of all positive integers defined by a*b = a + b for all a,b∈I^+.

Let us assume a∈I^+ and the identity element that we need to compute be e∈I^+.

We know that he Identity property is defined as follows:

⇒ a*e = e*a = a

⇒ a + e = a

⇒ e = a – a

⇒ e = 0

∴ The required Identity element w.r.t * is 0.

2. Question

Find the identity element in the set of all rational numbers except – 1 with respect to * defined by a*b = a + b + ab

Answer

Given that binary operation ‘*’ is valid for set of all rational numbers Q defined by a*b = a + b + ab for all a,b∈R.

Let us assume a∈R and the identity element that we need to compute be e∈R.

We know that he Identity property is defined as follows:

⇒ a*e = e*a = a

⇒ a + e + ea = a

⇒ e + ae = a – a

⇒ e(1 + a) = 0

⇒ e = 0

∴ The required Identity element w.r.t * is 0.

3. Question

If the binary operation * on the set Z is defined by a*b = a + b – 5, then find the identity element with respect to *.

Answer

Given that binary operation ‘*’ is valid for set ‘Z’ defined by a*b = a + b – 5 for all a,b∈Z.

Let us assume a∈Z and the identity element that we need to compute be e∈Z.

We know that he Identity property is defined as follows:

⇒ a*e = e*a = a

⇒ a + e – 5 = a

⇒ e – 5 = a – a

⇒ e = 5

∴ The required Identity element w.r.t * is 5.

4. Question

On the set Z of integers, if the binary operation * is defined by a*b = a + b + 2, then find the identity element.

Answer

Given that binary operation ‘*’ is valid for set ‘Z’ of integers defined by a*b = a + b for all a,b∈Z.

Let us assume a∈Z and the identity element that we need to compute be e∈Z.

We know that he Identity property is defined as follows:

⇒ a*e = e*a = a

⇒ a + e + 2 = a

⇒ e + 2 = a – a

⇒ e = – 2

∴ The required Identity element w.r.t * is – 2.

Exercise 3.4

1. Question

Let * be a binary operation on Z defined by a * b = a + b – 4 for all a, b ∈ Z.

i. Show that ‘ * ’ is both commutative and associative.

ii. Find the identity element in Z.

iii. Find the invertible elements in Z

Answer

i. We are given with the set Z which is the set of integers.

A general binary operation is nothing but association of any pair of elements a, b from an arbitrary set X to another element of X. This gives rise to a general definition as follows:

A binary operation * on a set is a function * : A X A → A. We denote * (a, b) as a * b.

Here the function *: Z X Z → Z is given by a * b = a + b – 4For the ‘ * ’ to be commutative, a * b = b * a must be true for all a, b belong to Z. Let’s check.

1. a * b = a + b – 4

2. b * a = b + a – 4

a * b = b * a (as shown by 1 and 2)

Hence ‘ * ’ is commutative.

For the ‘ * ’ to be associative, a * (b * c) = (a * b) * c must hold for every a, b, c∈ Z.

3. a * (b * c) = a * (b + c – 4) = a + (b + c – 4) – 4 = a + b + c – 8

4. (a * b) * c = (a + b – 4) * c = (a + b – 4) + c – 4 = a + b + c – 8

⇒ a * (b * c) = (a * b) * c

Hence ‘ * ’ is associative.

ii. Identity Element: Given a binary operation *: Z X Z → Z, an element e ∈Z, if it exists, is called an identity of the operation * , if a * e = a = e * a ∀ a ∈Z.

Let e be the identity element of Z.

Therefore, a * e = a⇒ a + e – 4 = a⇒ e – 4 = 0⇒ e = 4

iii. Given a binary operation $* : Z X Z \rightarrow Z$ with the identity element e in A, an element a $\in$ Z is said to be invertible with respect to the operation, if there exists an element b in Z such that a * b = e = b * a and b is called the inverse of a and is denoted by a^{-1}.

Let us proceed with the solution.

Let b ∈ Z be the invertible elements in Z of a, here a ∈ Z.

a * b = e (We know the identity element from previous)⇒ a + b – 4 = 4⇒ b = 8 – a

2. Question

Let * be a binary operation on Q_0 (Set of non – zero rational numbers) defined by a * b = 3ab/5 for all a, b ∈ Q_0.

Show that * is commutative as well as associative. Also, find its identity element, if it exists.

Answer

We are given with the set Q_0 which is the set of non - zero rational numbers.

A general binary operation is nothing but association of any pair of elements a, b from an arbitrary set X to another element of X. This gives rise to a general definition as follows:

A binary operation * on a set is a function * : A X A → A. We denote * (a, b) as a * b.

Here the function*: $Q_0 \text{ X } Q_0 \rightarrow Q_0 \text{ is given by } a * b = \frac{3ab}{5}, a, b \in Q_0$For the ‘ * ’ to be commutative, a * b = b * a must be true for all a, b ∈ Q_0. Let’s check.

$1. a * b = \frac{3ab}{5}$$2. b * a = \frac{3ba}{5} = \frac{3ab}{5}$⇒ a * b = b * a (as shown by 1 and 2)

Hence ‘ * ’ is commutative on Q_0

For the ‘ * ’ to be associative, a * (b * c) = (a * b) * c must hold for every a, b, c∈ Q_0.

$$3. a * (b * c) = a * \left(\frac{3bc}{5}\right)$$

$$= \frac{3}{5}.\left[\frac{3abc}{5}\right] = \frac{9abc}{25}$$

$$4. (a * b) * c = \left[\frac{3ab}{5}\right] * c$$

$$= \frac{3}{5}.\left[\frac{3abc}{5}\right] = \left[\frac{9abc}{25}\right]$$

⇒ 3. = 4.

Hence ‘ * ’ is associative on Q_0

Identity Element: Given a binary operation*: A X A → A, an element e ∈A, if it exists, is called an identity of the operation*, if a*e = a = e*a ∀ a ∈A**.**

Let e be the identity element of Q_0.

Therefore, a * e = a (a ∈Q_0)

$$\Rightarrow \frac{3ae}{5} = a$$

$$\Rightarrow \frac{3e}{5} = 1$$

$$\Rightarrow e = \frac{3}{5}$$

iii. Given a binary operation $* : A \text{ X } A \rightarrow A$ with the identity element e in A, an element a ∈ A is said to be invertible with respect to the operation, if there exists an element b in A such that a * b = e = b * a and b is called the inverse of a and is denoted by a^{-1}.

Let us proceed with the solution.

Here the function*: $Q_0 \, X \, Q_0 \rightarrow Q_0$ is given by $a * b = \frac{3ab}{5}, a, b \in Q_0$

Let $b \in Q_0$ be the invertible elements in Q_0 of a, where $a \in Q_0$.

∴a * b = e (We know the identity element from previous)

$$\Rightarrow \frac{3ab}{5} = \frac{5}{3}$$

$$\Rightarrow b = \frac{25}{9a} \text{ (Required inverse of a)}$$

3. Question

Let * be a binary operation on Q – {– 1} defined by a * b = a + b + ab for all, a, b ∈ Q – {– 1}. Then,

i. Show that ‘ * ’ is both commutative and associative on Q – {– 1}.

ii. Find the identity element in Q – {– 1}.

iii. Show that every element of Q – {– 1}. Is invertible. Also, find the inverse of an arbitrary element.

Answer

i. We are given with the set Q – {– 1}.

A general binary operation is nothing but association of any pair of elements a, b from an arbitrary set X to another element of X. This gives rise to a general definition as follows:

A binary operation * on a set is a function * : A X A → A. We denote * (a, b) as a * b.

Here the function *: Q – {– 1}X Q – {– 1} → Q – {– 1} is given by a * b = a + b + ab

For the ‘ * ’ to be commutative, a * b = b * a must be true for all a, b belong to Q – {– 1}. Let’s check.

1. a * b = a + b + ab2. b * a = b + a + ab = a + b + ab⇒ a * b = b * a (as shown by 1 and 2)

Hence ‘ * ’ is commutative on Q – {– 1}

For the ‘ * ’ to be associative, a * (b * c) = (a * b) * c must hold for every a, b, c∈ Q – {– 1}.

3. a * (b * c) = a * (b + c + bc)

= a + (b + c + bc) + a(b + c + bc)

= a + b + c + ab + bc + ac + abc

4. (a * b) * c = (a + b + ab) * c

= a + b + ab + c + (a + b + ab)c

= a + b + c + ab + bc + ac + abc⇒ 3. = 4.

Hence ‘ * ’ is associative on Q – {– 1}

ii. Identity Element: Given a binary operation*: A X A → A, an element e ∈A, if it exists, is called an identity of the operation*, if a*e = a = e*a ∀ a ∈A.

Let e be the identity element of Q – {– 1}, and a be an element of Q – {– 1}.

Therefore, a * e = a⇒ a + e + ae = a⇒ e + ea = 0⇒ e(1 + a) = 0⇒ e = 0.

(1 + a≠0 as the a is not equal to 1 as given in the question)

iii. Given a binary operation $* : A \, X \, A \rightarrow A$ with the identity element e in A, an element a ∈ A is said to be invertible with respect to the operation, if there exists an element b in A such that a * b = e = b * a and b is called the inverse of a and is denoted by a^{-1}.

Let us proceed with the solution.

Let b ∈ Q – {– 1} be the invertible element/s in Q – {– 1} of a, here a ∈ Q – {– 1}.

∴a * b = e (We know the identity element from previous)⇒ a + b + ab = 0⇒ b + ab = – a⇒ b(1 + a) = – a
$\Rightarrow\ b = \frac{-a}{1 + a}$ (Required invertible elements, a≠ – 1, b≠ – 1)

4. Question

Let A = R_0 x R, where R_0 denote the set of all non – zero real numbers. A binary operation ‘O’ is defined on A as follows: (a, b) ^{O} (c, d) = (ac, bc + d) for all (a, b), (c, d)∈ R_0 x R.

i. Show that ‘O’ is commutative and associative on A

ii. Find the identity element in A

iii. Find the invertible elements in A.

Answer

In this question, we do not have individuals as elements but ordered pairs as the elements of the Cartesian product R_0 x R. Hence this is the same problem as the above problems but in the form of ordered pairs belonging to the Cartesian product.

A = R_0 x R

i. We are given with the set A which is the Cartesian product of R_0 and R.

Here the function *: A X A → A is given by (a, b)°(c, d) = (ac, bc + d)

So here the operation on two ordered pairs gives us a ordered pair. So the first element of the first ordered pair (a) multiplies with the first element of the second ordered pair (c) to make (ac) in the resulting pair. To make the second element of the final pair, second element of the first ordered pair (b) multiplies with the first element of the second ordered pair (c) to form (bc) and then this is added to the second element of the second ordered pair (d) to make (bc + d). Hence the final ordered pair is (ac, bc + d). So this is the operation basically.

For the ‘^{0}’ to be commutative, a * b = b * a must be true for all a, b belong to A. Let’s check.Note: Here a, b represent the ordered pairs (a, b) and (c, d) respectively.

1. (a, b)°(c, d) = (ac, bc + d)2. (c, d)°(a, b) = (ca, da + b)⇒ a * b≠b * a (as shown by 1 and 2)

Hence ‘^{0}’ is not commutative on A.

For the ‘ * ’ to be associative, a * (b * c) = (a * b) * c must hold for every a, b, c∈ A. Here c = (e, f)

3. (a, b)°[(c, d)°(e, f)] = (a, b)°(ce, de + f)

= (ace, bce + de + f)

4. [(a, b)°(c, d)]°(e, f) = (ac, bc + d)°(e, f)

= (ace, [bc + d]e + f)= (ace, bce + de + f)⇒ 3. = 4.

Hence ‘^{0}’ is associative on A.

ii. Identity Element: Given a binary operation *: A X A → A, an element e ∈A, if it exists, is called an identity of the operation * , if p * e = p = e * p ∀ p ∈A.

Here in this case, p and e are an ordered pairs.

Let e = (a, b) be the identity element of A and p = (x, y) where p∈ A.

∴, (x, y)°(a, b) = (a, b)⇒ (xa, ya + b) = (a, b)⇒ ∴xa = a, ya + b = b

(Since, ordered pairs are only equal when both the first and second terms are equal)⇒ x = 1 (x gets cancelled out)⇒ y(1) + b = b

⇒ b = 0

The ordered pair (1, 0) is the identity of the operation ‘^{0}’ on A.

iii. Given a binary operation $*: A \times A \rightarrow A$ with the identity element e in A, an element a $\in$ A is said to be invertible with respect to the operation, if there exists an element b in A such that a * b = e = b * a and b is called the inverse of a and is denoted by a^{-1}.

For this example, a, b, e are ordered pairs.

Let us proceed with the solution.

Let (c, d) $\in$ A be the invertible elements in A of (a, b) here (a, b) $\in$ A.

∴(a, b)°(c, d) = (1, 0) (We know the identity element from previous)⇒ (ac, bc + d) = (1, 0)⇒ ac = 1, bc + d = 0

(Since, ordered pairs are only equal when both the first and second terms are equal)$\Rightarrow\ c = \frac{1}{a}$

$$\Rightarrow\ d = -bc = \frac{-b}{a}$$

$$(c, d) = \left(\frac{1}{a}, \frac{-b}{a}\right) \text{ are the inverse of } (a, b)$$

(Required invertible element)

5. Question

Let ‘o’ be a binary operation on the set Q_0 if all non - zero rational numbers defined by a o b = ab/2, for all a, b $\in Q_0$.

i. Show that ‘o’ is both commutative and associate.

ii. Find the identity element in Q_0.

iii. Find the invertible elements of Q_0.

Answer

We are given with the set Q_0 which is the set of non - zero rational numbers.

A general binary operation is nothing but association of any pair of elements a, b from an arbitrary set X to another element of X. This gives rise to a general definition as follows:

i. A binary operation * on a set is a function*: $A \times A \rightarrow A$. We denote $*(a, b)$ as $a * b$.

Here the function o: $Q_0 \times Q_0 \rightarrow Q_0$ is given by $aob = \frac{ab}{2}, a, b \in Q_0$

For the ‘o’ to be commutative, aob = boa must be true for all a, b $\in Q_0$. Let’s check.

$1.\ aob = \frac{ab}{2} 2.\ b * a = \frac{ba}{2} = \frac{ab}{2}$⇒ a * b = b * a (as shown by 1 and 2)

Hence ‘o’ is commutative on Q_0

For the ‘o’ to be associative, ao(boc) = (aob)oc must hold for every a, b, c $\in Q_0$.

$$3.\ ao(boc) = ao\left(\frac{bc}{2}\right)$$

$$= \frac{1}{2}.\left[\frac{abc}{2}\right] = \frac{abc}{4} 4.\ (a * b) * c = \left[\frac{ab}{2}\right] * c$$

$$= \frac{1}{2}.\left[\frac{abc}{2}\right] = \left[\frac{abc}{4}\right]$$

$$\Rightarrow\ 3. = \ 4.$$

Hence ‘o’ is associative on Q_0

ii. Identity Element: Given a binary operation*: A X A → A, an element e ∈A, if it exists, is called an identity of the operation*, if a*e = a = e*a ∀ a ∈A**.**

Let e be the identity element of Q_0.

Therefore, aoe = a (a ∈Q_0)

$$\Rightarrow \frac{ae}{2} = a$$

$$\Rightarrow \frac{e}{2} = 1$$

$$\Rightarrow e = 2 \text{ (The identity element of } Q_0\text{)}$$

iii. Given a binary operation $* : A X A \rightarrow A$ with the identity element e in A, an element a $\in$ A is said to be invertible with respect to the operation, if there exists an element b in A such that a * b = e = b * a and b is called the inverse of a and is denoted by a^{-1}.

Let us proceed with the solution.

Let b $\in$ Q_0 be the invertible elements in Q_0 of a, where a $\in$ Q_0.

∴a * b = e (We know the identity element from previous)

$$\Rightarrow \frac{ab}{2} = 2$$

$$\Rightarrow b = \frac{4}{a} \text{ (Required inverse of a)}$$

6. Question

On R – {1}, a binary operation * is defined by a * b = a + b – ab. Prove that * is commutative and associative. Find the identity element for * on R – {1}. Also, prove that every element of R – {1} is invertible.

Answer

i. We are given with the set R – {– 1}.

A general binary operation is nothing but an association of any pair of elements a, b from an arbitrary set X to another element of X. This gives rise to a general definition as follows:

A binary operation * on a set is a function * : A X A → A. We denote * (a, b) as a * b.

Here the function *: R – {1}X R – {1} → R – {1} is given by a * b = a + b – ab

For the ‘ * ’ to be commutative, a * b = b * a must be true for all a, b belong to R – {1}. Let’s check.

1. a * b = a + b – ab2. b * a = b + a – ba = a + b – ab⇒ a * b = b * a (as shown by 1 and 2)

Hence ‘ * ’ is commutative on R – {1}

For the ‘ * ’ to be associative, a * (b * c) = (a * b) * c must hold for every a, b, c∈ R – {1}.

3. a * (b * c) = a * (b + c – bc)

= a + (b + c – bc) – a(b + c + bc)

= a + b + c – ab – bc – ac + abc

4. (a * b) * c = (a + b – ab) * c

= a + b – ab + c – (a + b – ab)c

= a + b + c – ab – bc – ac + abc⇒ 3. = 4.

Hence ‘ * ’ is associative on R – {1}

ii. Identity Element: Given a binary operation*: A X A → A, an element e ∈A, if it exists, is called an identity of the operation*, if a*e = a = e*a ∀ a ∈A.

Let e be the identity element of R – {1} and a be an element of R – {1}.

Therefore, a * e = a⇒ a + e – ae = a⇒ e + ea = 0⇒ e(1 – a) = 0⇒ e = 0.

(1 – a≠0 as the a cannot be equal to 1 as the operation is valid in R – {1})

iii. Given a binary operation $* : A X A \rightarrow A$ with the identity element e in A, an element a $\in$ A is said to be invertible with respect to the operation, if there exists an element b in A such that a * b = e = b * a and b is called the inverse of a and is denoted by a^{-1}.

Let us proceed with the solution.

Let b $\in$ R – {1} be the invertible element/s in R – {1} of a, here a $\in$ R – {1}.

∴a * b = e (We know the identity element from previous)⇒ a + b – ab = 0⇒ b – ab = – a⇒ b(1 – a) = – a

$\Rightarrow\ b\ =\ \frac{-a}{1-a}$ (Here $a \neq 1, b \neq 1$)

7. Question

Let R_0 denote the set of all non - zero real numbers and let A = R_0 x R_0. If '0' is a binary operation on A defined by (a, b)0(c, d) = (ac, bd), (c, d)∈A.

i. Show that '0' is both commutative and associative on A

ii. Find the identity element in A

iii. Find the invertible element in A.

Answer

In this question, we do not have individuals as elements but ordered pairs as the elements of the Cartesian product R_0 x R. Hence this is the same problem as the above problems but in the form of ordered pairs belonging to the Cartesian product.

A = R_0 x R_0

i. We are given with the set A which is the Cartesian product of R_0 and R_0.

Here the function *: A X A → A is given by (a, b)°(c, d) = (ac, bd)

So here the operation on two ordered pairs gives us a ordered pair. So the first element of the first ordered pair (a) multiplies with the first element of the second ordered pair (c) to make (ac) in the resulting pair. To make the second element of the final pair, second element of the first ordered pair (b) multiplies with the second element of the second ordered pair (c) to form (bd). Hence the final ordered pair is (ac, bd). So this is the operation basically.

For the '^{0}' to be commutative, p * q = p * q must be true for all p, q belong to A. Let's check.Note: Here p, q represent the ordered pairs (a, b) and (c, d) respectively.

1. (a, b)°(c, d) = (ac, bd)2. (c, d)°(a, b) = (ca, db) = (ac, bd)⇒ p * q = q * p (as shown by 1 and 2)

Hence '^{0}' is commutative on A.

For the ' * ' to be associative, a * (b * c) = (a * b) * c must hold for every a, b, c∈ A. Here r = (e, f)

3. (a, b)°[(c, d)°(e, f)] = (a, b)°(ce, df)

= (ace, bdf)

4. [(a, b)°(c, d)]°(e, f) = (ac, bd)°(e, f)

= (ace, bdf)⇒ 3. = 4.

Hence '^{0}' is associative on A.

ii. Identity Element: Given a binary operation *: A X A → A, an element e ∈A, if it exists, is called an identity of the operation * , if p * e = p = e * p ∀ p ∈A.

Here in this case, p and e are an ordered pairs.

Let e = (a, b) be the identity element of A and p = (x, y) where p $\in$ A.

$\therefore$, (x, y)°(a, b) = (x, y)$\Rightarrow$ (xa, yb) = (x, y)

(Since, ordered pairs are only equal when both the first and second terms are equal)$\Rightarrow$ $\therefore$xa = x, yb = y

$\Rightarrow$ x(a – 1) = 0;y(b – 1) = 0

Since x, y$\neq$0, we have, a – 1 = 0 and b – 1 = 0.

a = 1, b = 1

The ordered pair (1, 1) is the identity of the operation ‘o’ on A.

iii. Given a binary operation $* : A \times A \rightarrow A$ with the identity element e in A, an element a $\in$ A is said to be invertible with respect to the operation, if there exists an element b in A such that a * b = e = b * a and b is called the inverse of a and is denoted by a^{-1}.

For this example, a, b, e are ordered pairs.

Let us proceed with the solution.

Let (c, d) $\in$ A be the invertible elements in A of (a, b). Here (a, b) $\in$ A.

$\therefore$(a, b)°(c, d) = (1, 1) (We know the identity element from previous)$\Rightarrow$ (ac, bd) = (1, 1)$\Rightarrow$ ac = 1, bd = 1

(Since, ordered pairs are only equal when both the first and second terms are equal)$\Rightarrow$ $c = \frac{1}{a}$

$\Rightarrow d = \frac{1}{b}$

$(c, d) = \left(\frac{1}{a}, \frac{1}{b}\right)$ are the inverse of (a, b)

(Required invertible element)

8. Question

Let * be the binary operation on N defined by a * b = HCF of a and b. Does there exist identity for this binary operation on N?

Answer

The binary operation * on N defined as:

a * b = H.C.F. of a and b, a, b $\in$ N.

Therefore,

b * a = H.C.F. of b and a = H.C.F

Hence, * is commutative on N.

Let a, b, c $\in$ N.

a * (b * c) = a * (HCF of b and c) = HCF of a, b and c

(a * b) * c = (HCF of a and b) * c = HCF of a, b and c

For example, take the numbers 2, 4, 7 $\in$ N.

Factors of 2 = 1, 2
Factors of 4 = 1, 2, 4
HCF = 1

Factors of 7 = 1, 7
HCF = 1

Factors of 2 = 1, 2

Factors of 4 = 2,2, 1

Factors of 7 = 1, 7

HCF = 1

HCF = 1

Here, if the operation is applied to the above numbers as follows:

(2 * 4) * 7 = 1 * 7 = 1 (1 is the HCF of 2, 4 and HCF of 1, 7 is 1). Also,

2 * (4 * 7) = 2 * 1 = 1 (Similar as the above reason)

Therefore, * is associative.

Now, Identity Element:

Given a binary operation *: N X N → N, an element e ∈A, if it exists, is called an identity of the operation * , if a * e = a = e * a ∀ a ∈A.

But there does not exist any value of e ∈N such that a * e = a = e * a

Therefore, this operation does not have any identity.

9. Question

Let A = RxR and * be a binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Find the binary element for * on A, if any.

Answer

A = R X R

For the ‘ * ’ to be commutative, p * q = p * q must be true for all p, q belong to A. Let’s check.Note: Here p, q ∈ A represent the ordered pairs (a, b) and (c, d) respectively.

p * q = (a, b) * (c, d) = (a + c, b + d)q * p = (c, d) * (a, b) = (c + a, d + b) = (a + c, b + d)⇒ p * q = q * p (binary operation * is commutative)

For the ‘ * ’ to be associative, a * (b * c) = (a * b) * c must hold for every a, b, c∈ A. Here r = (e, f)

p * (q * r) = (a, b) * ((c, d) * (e, f)) = (a, b) * (c + e, d + f) = (a + c + e, b + d + f)(p * q) * r = ((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)⇒ p * (q * r) = (p * q) * r (Associative)

Binary elements:

Identity Element: Given a binary operation*: A X A → A, an element e ∈A, if it exists, is called an identity of the operation*, if p*e = p = e*p ∀ p ∈A.

Here p = (a, b) and e = (x, y). For the element e to exist,

(a, b) * (x, y) = (a, b)⇒ (a + x, b + y) = (a, b)⇒ a + x = a, b + y = bSince, ordered pairs are only equal when both the first and second terms are equal⇒ x = 0, y = 0Hence the identity element e = (x, y) = (0, 0)

Given a binary operation $* : A X A \rightarrow A$ with the identity element e in A, an element a ∈ A is said to be invertible with respect to the operation, if there exists an element b in A such that a * b = e = b * a and b is called the inverse of a and is denoted by a^{-1}.

Let i = (r, s) be the inverse of p = (a, b) in A.

Therefore, (r, s) * (a, b) = e = (0, 0)⇒ (r + a, s + b) = (0, 0)∵r + a = 0, r + b = 0r = – a, s = – b

Therefore, (r, s) = (– a, – b) is the inverse pair.

Exercise 3.5

1. Question

Construct the composition table for x_4 on set S = {0, 1, 2, 3}.

Answer

A composition table consists of elements which are a result of operation on the set elements.

Here we have the operation, a x_4b = remainder of ab divided by 4 where a, b $\in$ S.

X_4	0	1	2	3
0	$0 \times_4 0 = 0$	$1 \times_4 0 = 0$	$2 \times_4 0 = 0$	$3 \times_4 0 = 0$
1	$0 \times_4 1 = 0$	$1 \times_4 1 = 1$	$2 \times_4 1 = 2$	$3 \times_4 1 = 3$
2	$0 \times_4 2 = 0$	$1 \times_4 2 = 2$	$2 \times_4 2 = 0$	$3 \times_4 2 = 2$
3	$0 \times_4 3 = 0$	$1 \times_4 3 = 3$	$2 \times_4 3 = 2$	$3 \times_4 3 = 1$

This is the composition table of x_4 on S = {0, 1, 2, 3}

For example, take 3 x_4 2 = remainder of (3 x 2) divided by 4

$$\text{Remainder of } \frac{6}{4} = 2$$

We also see that the operation x_4 is valid because all the output elements belong to the set S.

2. Question

Construct the composition table for $+_5$ on set S = {0, 1, 2, 3, 4}

Answer

A composition table consists of elements which are a result of operation on the set elements.

Here we have the operation, a $+_5$ b = remainder of a + b divided by 5 where a, b $\in$ S.

$+_5$	0	1	2	3	4
0	$0 +_5 0 = 0$	$1 +_5 0 = 0$	$2 +_5 0 = 2$	$3 +_5 0 = 3$	$4 +_5 0 = 4$
1	$0 +_5 1 = 1$	$1 +_5 1 = 2$	$2 +_5 1 = 3$	$3 +_5 1 = 4$	$4 +_5 1 = 0$
2	$0 +_5 2 = 1$	$1 +_5 2 = 3$	$2 +_5 2 = 4$	$3 +_5 2 = 0$	$4 +_5 2 = 1$
3	$0 +_5 3 = 3$	$1 +_5 3 = 4$	$2 +_5 3 = 0$	$3 +_5 3 = 1$	$4 +_5 3 = 2$
4	$0 +_5 4 = 4$	$1 +_5 4 = 0$	$2 +_5 4 = 1$	$3 +_5 4 = 2$	$4 +_5 4 = 3$

This is the composition table of x_4 on S = {0, 1, 2, 3, 4}

For example, take 3 $+_5$ 2 = remainder of (3 + 2) divided by 5

$$\text{Remainder of } \frac{5}{5} = 0$$

We also see that the operation $+_5$ is valid because all the output elements belong to the set S.

3. Question

Construct the composition table for x_6 on set S = {0, 1, 2, 3, 4, 5}.

Answer

A composition table consists of elements which are a result of operation on the set elements.

Here we have the operation, a x_6 b = remainder of ab divided by 6 where a, b $\in$ S.

x_6	0	1	2	3	4	5
0	$0 \times_6 0 = 0$	0	0	0	0	0
1	$0 \times_6 1 = 0$	1	2	3	4	5
2	$0 \times_6 2 = 0$	2	4	0	2	4
3	$0 \times_6 3 = 0$	3	0	3	0	3
4	$0 \times_6 4 = 0$	4	2	0	4	2
5	$0 \times_6 5 = 0$	5	4	3	2	1

This is the composition table of x_4 on S = {0, 1, 2, 3, 4, 5}

For example, take 3 x_6 2 = remainder of (3 x 2) divided by 6

$$\text{Remainder of } \frac{6}{6} = 0$$

4. Question

Construct the composition table for x_5 on Z_5 = {0, 1, 2, 3, 4}

Answer

A composition table consists of elements which are a result of operation on the set elements.

Here we have the operation, a x_5b = remainder of ab divided by 5 where a, b $\in$ S. Similar as problem 1.

x_5	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

5. Question

For the binary operation x_{10} on set S = {1, 3, 7, 9}, find the inverse of e3.

Answer

x_{10}	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

A composition table consists of elements which are a result of operation on the set elements.

Here we have the operation, a x_{10}b = remainder of ab divided by 10 where a, b $\in$ S.

For b$\in$S to be an inverse of a$\in$S, a x_{10}b = e, where e is the identity element.

We know for multiplication operation we have the identity element as 1.

So e = 1.

For a = 3,

3 x_{10} (inverse of 3) = 1

$$\text{Remainder of } \frac{3(\mathrm{i})}{10} = 1, \mathrm{i} \text{ is the inverse of } 3.$$

From the table above, 3 x_{10} 7 = 1

Hence we can conclude that 'inverse of 3' must be 7.

6. Question

For the binary operation x_7 on the set of S = {1, 2, 3, 4, 5, 6} compute $3^{-1} x_7 4$.

Answer

A composition table consists of elements which are a result of an operation on the set elements.

Here we have the operation, a x_7b = remainder of ab divided by 7 where a, b ∈ S.

x_7	1	2	3	4	5	6
1	1 x_7 1 = 1	2	3	4	5	6
2	1 x_7 2 = 2	4	6	1	3	5
3	1 x_7 2 = 3	6	2	5	1	4
4	1 x_7 4 = 4	1	5	2	6	3
5	1 x_7 5 = 5	3	1	6	4	2
6	1 x_7 6 = 6	5	4	3	2	1

For b∈S to be an inverse of a∈S, a x_7b = e, where e is the identity element.

We know for multiplication operation we have the identity element as 1.

So e = 1.

For a = 3,

3 x_7 (inverse of 3) = 1

$$\text{Remainder of } \frac{3(\mathrm{i})}{7} = 1, \mathrm{i} \text{ is the inverse of } 3.$$

From the table above, 3 x_7 5 = 1

Hence we can conclude that 'inverse of 3' must be 5.

Therefore the expression:

$3^{-1} x_7 4 = 5 x_7 4 = 6$. (From the table above)

7. Question

Find the inverse of 5 under multiplication modulo 11 on Z_{11}

Answer

A composition table consists of elements which are a result of an operation on the set elements.

Here we have the operation, a x_{11} b = remainder of ab divided by 11 where a, b ∈ S

x_{11}	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	1	3	5	7	9
3	3	6	9	1	4	7	10	3	5	8
4	4	8	1	5	9	2	6	10	3	7
5	5	10	4	9	3	8	2	7	1	6
6	6	1	7	2	8	3	9	4	10	5
7	7	3	10	6	2	9	5	1	8	4
8	8	5	3	10	7	4	1	9	6	3
9	9	7	5	3	1	10	8	6	4	2
10	10	9	8	7	6	5	4	3	2	1

Example, 4 x_{11} 9 = Remainder of (4 x 9) divided by 11

$$\text{Remainder of } \frac{(4)(9)}{11} = \text{Remainder of } \frac{36}{11} = 3$$

For b∈S to be an inverse of a∈S, a x_7b = e, where e is the identity element.

We know for multiplication operation we have the identity element as 1.

So e = 1.

For a = 5,

5 x_{11} (inverse of 3) = 1

$$\text{Remainder of } \frac{5(\mathrm{i})}{11} = 1, \text{i is the inverse of } 5.$$

From the table above, 5 x_{11} 9 = 1.

Hence, i = 9.

8. Question

Write the multiplication table for the set of integers modulo 5.

Answer

A composition table consists of elements which are a result of operation on the set elements.

Here we have the operation, a x_5b = remainder of ab divided by 5 where a, b ∈ S.

X_4	0	1	2	3	4
0	0 $x_4$0 = 0	1 x_4 0 = 0	2 x_4 0 = 0	3 x_4 0 = 0	4 x_4 0 = 0
1	0 x_4 1 = 0	1 x_4 1 = 1	2 x_4 1 = 2	3 x_4 1 = 3	4 x_4 1 = 4
2	0 x_4 2 = 0	1 x_4 2 = 2	2 x_4 2 = 0	3 x_4 2 = 1	4 x_4 2 = 3
3	0 x_4 3 = 0	1 x_4 3 = 3	2 x_4 3 = 1	3 x_4 3 = 4	4 $x_4$3 = 2
4	0 x_4 4 = 0	1 x_4 4 = 4	2 x_4 4 = 3	3 x_4 4 = 2	4 x_4 4 = 1

This is the composition table of x_4 on S = {0, 1, 2, 3, 5}

For example, take 3 x_4 2 = remainder of (3 x 2) divided by 4

$$\text{Remainder of } \frac{6}{4} = 2$$

We also see that the operation x_4 is valid because all the output elements belong to the set S.

9 A. Question

Consider the binary operation * and 0 defined by the following tables on set S = {a, b, c, d}.

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

Answer

We observe the following:

a * b = b * a = b

c * a = a * c = c

a * d = d * a = d

b * c = c * b = d

b * d = d * b = c

c * d = d * c = b

There ‘ * ’ is commutative.

Also,

a * (b * c) = a * (d) = d (From above)

(a * b) * c = (b) * c = d (Also from above)

Hence, ‘ * ’ is associative too.

Therefore, to find the identity element, e for e belong to S, we need:

a * e = e * a = a, a belong to S.

Therefore, a * e = a

⇒ e = a (since, a * a = a, from the given table)

To find out the inverse, a * x = e = b * x, x belongs to S

⇒ a * x = e

⇒ x = a (From the given table)

Therefore, the inverse of a is a, b is b, c is c and d is d.

9 B. Question

Consider the binary operation * and 0 defined by the following tables on set S = {a, b, c, d}.

0	a	b	c	d
a	a	a	a	a
b	a	b	c	d
c	a	c	d	b
d	a	d	b	c

Answer

We observe the following:

0	a	b	c	d
a	a	a	a	a
b	a	b	c	d
c	a	c	d	b
d	a	d	b	c

a * b = b * a = a a * a = a

c * a = a * c = a b * b = b

a * d = d * a = a c * c = d

b * c = c * b = c d * d = c

b * d = d * b = d

c * d = d * c = b

There ‘ * ’ is commutative.

Also,

a * (b * c) = a * (c) = a (From above)

(a * b) * c = (a) * c = a (Also from above)

Hence, ‘ * ’ is associative too.

Therefore, to find the identity element, e for e belong to S, we need:

a * e = e * a = a, a belong to S.

Therefore, a * e = a

We find that there is no unique element e which satisfies the condition.

e = a or b or c or d for a.

Since the identity is not unique, the inverse will also be not unique.

Very short answer

1. Question

Write the identity element for the binary operation * on the set R_0 of all non-zero real numbers by the rule $a * b = \frac{ab}{2}$ for all a, b ∈ R_0.

Answer

The given binary operation is $a*b = \frac{ab}{2}$

And from the definition of identity element e, a*e = a ....(i)

We have,

a*e = $\frac{ae}{2}$ ....(ii)

Thus from (i) & (ii) ,

$\frac{ae}{2}$ = a

or, $\frac{e}{2}$ = 1

∴ e = 2

Hence the identity element for this binary operation is 2.

2. Question

On the set Z of all integers a binary operation * is defined by a * b = a + b + 2 for all a, b ∈ Z. Write the inverse of 4.

Answer

The given binary operation is a*b = a+b+2

In order to find the inverse of the relation, we have to find the identity element first.

Let that identity element be e then

a*e = a

From que.

a*e = a+e+2

So, from the above two relations we have

a+e+2 = a

or, e+2 = 0

∴ e = -2

Hence the identity element is -2 for this binary operation.

Now let a’ be the inverse of this relation

Then as per the definition of the inverse element

a*a’ = e

∴ a+a’+2 = -2

∴ a’ = -4 -a

And for 4 ,i.e. a = 4

a’ = -4 – 4

∴ a’ = -8

Thus the inverse element of 4 is -8 for the given binary operation.

3. Question

Define a binary operation on a set.

Answer

Binary operations on a set are calculations that combine two elements of the set (called operands) to produce another element of the same set.

The binary operations * on a non-empty set A are functions from A × A to A. The binary operation, *: A × A → A. It is an operation of two elements of the set whose domains and co-domain are in the same set.

4. Question

Define a commutative binary operation on a set.

Answer

A binary operation * on a set A is commutative if a * b = b * a, for all (a, b) ∈ A (non-empty set).

Let addition be the operating binary operation for a = 8 and b = 9, a + b = 17 = b + a.

5. Question

Define an associative binary operation on a set.

Answer

The associative property of binary operations hold if, for a non-empty set A, we can write (a * b) *c = a*(b * c).

Suppose **N** be the set of natural numbers and multiplication be the binary operation. Let a = 4, b = 5 c = 6. We can write (a × b) × c = 120 = a × (b × c).

6. Question

Write the total number of binary operations on a set consisting of two elements.

Answer

Let A be a non-empty set having n elements. Then, the total number of binary operations on this set, i.e., from A×A is n^{n^2}.

Here n = 2

Thus, total number of possible binary operations are 2^4 = 16.

7. Question

Write the identity element for the binary operation * defined on the set R of all real numbers by the rule $a * b = \frac{3ab}{7}$ for all a, b ∈ R.

Answer

The given binary operation is a*b = $\frac{3ab}{7}$

Let e be the identity element

Thus, as per the definition of an identity element a*e = a

Applying this in the relation given we have

$\frac{3ae}{7}$ = a

Or, $\frac{3e}{7}$=1

∴ e = $\frac{7}{3}$

Thus, the identity element for the given binary operation is $\frac{7}{3}$.

8. Question

Let * be a binary operation, on the set of all non-zero real numbers, given by $a * b = \frac{ab}{5}$ for all a, b ∈ R – {0}. Write the value of x given by 2 * (x * 5) = 10.

Answer

The given binary operation is a*b = $\frac{ab}{5}$

We have to find the value of x so that 2*(x*5) = 10

Firstly, x*5 = $\frac{5x}{5}$ = x

Now 2*(x*5) = 2*x = $\frac{2x}{5}$

As per the question, it is equal to 10

∴ $\frac{2x}{5}$ = 10

∴ x = 25

9. Question

Write the inverse of 5 under multiplication modulo 11 on the set {1, 2, ..., 10}.

Answer

From the definition of multiplication modulo

a*b = $\begin{Bmatrix} a \times b, if\ it\ is\ < m \\ a \times b - m, if\ it\ is\ \geq m \end{Bmatrix}$ (m is the base of the modulo)

Here the base is 11 and the set is {1,2,...,10}

So, we can make the composition table as :-

*	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	1	3	5	7	9
3	3	6	9	1	4	7	10	2	5	7
4	4	8	1	5	9	2	6	10	3	7
5	5	10	4	9	3	8	2	7	1	6
6	6	1	7	2	8	3	9	4	10	5
7	7	3	10	6	2	9	5	1	8	4
8	8	5	2	10	7	4	1	9	6	3
9	9	7	5	3	1	10	8	6	4	2
10	10	9	8	7	6	5	4	3	2	1

Thus, the identity element is 1.

So, the inverse of 5 is 9.

10. Question

Define identity element for a binary operation defined on a set.

Answer

If A be the non-empty set and * be the binary operation on A. An element e is the identity element of a ∈ A, if a * e = a = e * a.

If the binary operation is addition(+), e = 0 and for * is multiplication(×), e = 1.

11. Question

Write the composition table for the binary operation multiplication modulo 10($\times_{10}$) on the set S = {2, 4, 6, 8}.

Answer

From the definition of multiplication modulo

a*b = $\begin{Bmatrix} a \times b, if\ it\ is\ < m \\ a \times b - m, if\ it\ is\ \geq m \end{Bmatrix}$ (m is the base of the modulo)

And here the base is 3 and the set is S = {2,4,6,8}.

So, the composition table can be formed as :-

*	2	4	6	8
2	4	8	2	6
4	8	6	4	2
6	2	4	6	8
8	6	2	8	4

12. Question

For the binary operation multiplication modulo 10($\times_{10}$) defined on the set S = {1, 3, 7, 9}, write the inverse of 3.

Answer

From the definition of multiplication modulo

a*b = $\begin{Bmatrix} a \times b, if\ it\ is\ < m \\ a \times b - m, if\ it\ is\ \geq m \end{Bmatrix}$ (m is the base of the modulo)

Here the base is 10 and the set is S = {1,3,7,9}

So, We can make the composition table as :-

*	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

The identity element is 1.

So, the inverse element of 3 is 7.

13. Question

For the binary operation multiplication modulo 5($\times_5$) defined on the set S = {1, 2, 3, 4}. Write the value of (3 $\times_5$ $4^{-1})^{-1}$.

Answer

From the definition of multiplication modulo

a*b = $\begin{Bmatrix} a \times b, if\ it\ is\ < \ m \\ a \times b - m\ , if\ it\ is\ \geq m \end{Bmatrix}$ (m is the base of the modulo)

Here the base is 5 and the set is S = {1,2,3,4}

So, we can make a composition table as :-

*	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

The identity element is 1.

The inverse element of 4 is 4 itself.

$\therefore (3 \times_5 4^{-1})^{-1} = (3 \times_5 1)^{-1} = 3^{-1} = 2.$

14. Question

Write the composition table for the binary operation $\times_5$ (multiplication modulo 5) on the set S = {0, 1, 2, 3, 4}.

Answer

From the definition of multiplication modulo

a*b = $\begin{Bmatrix} a \times b, if\ it\ is\ <\ m \\ a \times b - m\ , if\ it\ is\ \geq m \end{Bmatrix}$ (m is the base of the modulo)

Here the base is 5 and the set is S = {0,1,2,3,4}

As the multiplication of 0 with any number will yield zero so we don’t consider it in the composition table.

So, we can make a composition table as :-

*	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

15. Question

A binary operation * is defined on the set R of all real numbers by the rule $a * b = \sqrt{a^2 + b^2}$ for all a, b ∈ R. Write the identity element for * on R.

Answer

The given binary operation is a*b = $\sqrt{(a^2 + b^2)}$

We have to find the identity element for the above relation.

Let that element be e.

∴ From the definition of identity element ,

a*e = a

∴ $\sqrt{(a^2 + e^2)}$ = a

Squaring both sides, we get,

$a^2 + e^2 = a^2$

∴ e = 0

Thus, the identity element for this operation is 0.

16. Question

Let $+_6$ (addition modulo 6) be a binary operation on S = {0, 1, 2, 3, 4, 5}. Write the value of $2 +_6 4^{-1} +_6 3^{-1}$.

Answer

From the definition of addition modulo :-

a*b = $\begin{Bmatrix} a+b\,, if\ it\ is < m \\ a+b-m, if\ it\ is\ \geq m \end{Bmatrix}$ (m is the base of the modulo)

Here the base is 6 and the set is S = {0,1,2,3,4,5}

So, we can make the composition table as :-

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

It is clear that 1 is the identity element of this binary operation.

The inverse of 4 is 3 while the inverse of 3 is 4.

$\therefore 2 +_6 4^{-1} +_6 3^{-1} = 2 +_6 3 +_6 4 = 3$

17. Question

Let * be a binary operation defined by a * b = 3a + 4b – 2. Find 4 * 5.

Answer

The given binary operation is a*b = 3a + 4b – 2

We have to find the value of 4*5

Applying the above relation we have

4*5 = 3(4) + 4(5) -2 = 12 + 20 – 2 = 30.

Thus the required solution is 30.

18. Question

If the binary operation * on the set Z of integers is defined by a * b = a + $3b^2$, find the value of 2 * 4.

Answer

The given binary operation is a*b = a + $3b^2$

We have to find the value of 2*4

Applying the relation, we have

$2*4 = 2 + 3(4^2) = 2 + 3(16) = 2 + 48 = 50.$

Thus, the required solution is 50.

19. Question

Let * be a binary operation on N given by a * b = HCF (a, b), a, b $\in$ N. Write the value of 22 * 4.

Answer

The given binary operation is a*b = HCF(a,b)

We have to find the value of 22*4

Applying the relation we have

22*4 = HCF(22,4) = 2

Hence the required solution is 2.

20. Question

Let * be a binary operation on set of integers I, defined by a * b = 2a + b –3. Find the value of 3 * 4.

Answer

The given binary operation is a*b = 2a+b-3

We have to find the value of 3*4

$\therefore$ 3*4 = 2(3) + 4 -3

= 6+ 1

= 7

Thus, the required value is 7.

MCQ

1. Question

Mark the correct alternative in each of the following:

If $a * b = a^2 + b^2$, then the value of (4 * 5) * 3 is

A. $(4^2 + 5^2) + 3^2$

B. $(4 + 5)^2 + 3^2$

C. $41^2 + 3^2$

D. $(4 + 5 + 3)^2$

Answer

Given $a * b = a^2 + b^2$

$\Rightarrow (4 * 5) * 3 = [(4^2 + 5^2)]*3$

= 41 * 3

$= 41^2 + 3^2$

2. Question

Mark the correct alternative in each of the following:

If a * b denote the bigger among a and b and if a · b = (a * b) + 3, then 4.7 =

A. 14

B. 31

C. 10

D. 8

Answer

Given that a * b denote the bigger among a and b

and a · b = (a * b) + 3

⇒ 4 · 7 = (4 * 7) + 3

Now as 7 > 4

⇒ (4 * 7) + 3 = 7 + 3

= 10

3. Question

Mark the correct alternative in each of the following:

On the power set p of a non-empty set A, we define an operation Δ by

$$X \Delta Y = \left(\bar{X} \cap Y\right) \cup \left(X \cap \bar{Y}\right)$$

Then which are of the following statements is true about Δ

A. commutative and associative without an identity

B. commutative but not associative with an identity

C. associative but not commutative without an identity

D. associative and commutative with an identity

Answer

∵ $X \Delta Y = (\bar{X} \cap Y) \cup (X \cap \bar{Y})$

⇒ $Y \Delta X = (\bar{Y} \cap X) \cup (Y \cap \bar{X})$

⇒ $X \Delta Y = Y \Delta X$

4. Question

Mark the correct alternative in each of the following:

If the binary operation * on Z is defined by a * b = $a^2 - b^2$ + ab + 4, then value of (2 * 3) * 4 is

A. 233

B. 33

C. 55

D.–55

Answer

Given that a * b = $a^2 - b^2$ + ab + 4

To calculate (2 * 3) * 4

⇒ 2 * 3 = $2^2 - 3^2$ + 2.3 + 4 = 5

⇒ (2 * 3) * 4 = 5 * 4

$\Rightarrow 5 * 4 = 5^2 – 4^2 + 5.4 + 4$

$= 33$

5. Question

Mark the correct alternative in each of the following:

For the binary operation * on Z defined by a * b = a + b + 1 the identity element is

A. 0

B. –1

C. 1

D. 2

Answer

For Identity element a * e = a = e * a , with ‘e’ being the identity element.

Given a * b = a + b + 1

$\Rightarrow$ a * e = a + e + 1 = a ....(1)

$\Rightarrow$ e * a = e + a + 1 = a .... (2)

$\Rightarrow$ e = -1 is the required identity element.

6. Question

Mark the correct alternative in each of the following:

If a binary operation * is defined on the set Z of integers as a * b = 3a – b, then the value of (2 * 3) *4 is

A. 2

B. 3

C. 4

D. 5

Answer

Given that a * b = 3a – b

To calculate (2 * 3) * 4

$\Rightarrow$ 2 * 3 = 3.2 – 3 = 3

$\Rightarrow$ (2 * 3) * 4 = 3 * 4

$\Rightarrow$ 3 * 4 = 3.3 – 4

= 5

7. Question

Mark the correct alternative in each of the following:

Q^+ denote the set of all positive rational numbers. If the binary operation $\odot$ on Q^+ is defined as a $a \odot b = \frac{ab}{2}$, then the inverse of 3 is

A. $\frac{4}{3}$

B. 2

C. $\frac{1}{3}$

D. $\frac{2}{3}$

Answer

For Identity element a * e = a = e * a, with ‘e’ being the identity element.

Given $a * b = \frac{ab}{2}$

⇒ $a * e = \frac{ae}{2} = a$

⇒ $e = 2$

For Inverse element a * b = e = b * a , with ‘b’ being the inverse element ‘a’.

Given $a * b = \frac{ab}{2}$

Now for a = 3

⇒ $3 * b = \frac{3b}{2} = 2$ ....(1)

⇒ $b * 3 = \frac{3b}{2} = 2$ ....(2)

⇒ $b = \frac{4}{3}$ is the required inverse element of a=3.

8. Question

Mark the correct alternative in each of the following:

If G is the set of all matrices of the form $\begin{bmatrix} x & x \\ x & x \end{bmatrix}$, where x ∈ R – {0}, then the identity element with respect to the multiplication of matrices as binary operation, is

A. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

B. $\begin{bmatrix} -1/2 & -1/2 \\ -1/2 & -1/2 \end{bmatrix}$

C. $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$

D. $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$

Answer

For Identity element a * e = a = e * a, with ‘e’ being the identity element.

Given $a * b = \begin{bmatrix} a & a \\ a & a \end{bmatrix} \times \begin{bmatrix} b & b \\ b & b \end{bmatrix}$

⇒ $a * e = \begin{bmatrix} a & a \\ a & a \end{bmatrix} \times \begin{bmatrix} e & e \\ e & e \end{bmatrix} = \begin{bmatrix} a & a \\ a & a \end{bmatrix}$

⇒ $\begin{bmatrix} a & a \\ a & a \end{bmatrix} \times \begin{bmatrix} e & e \\ e & e \end{bmatrix} = \begin{bmatrix} 2ae & 2ae \\ 2ae & 2ae \end{bmatrix}$

⇒ 2ae=a

$\Rightarrow e = \frac{1}{2}$

$\Rightarrow e = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

9. Question

Mark the correct alternative in each of the following:

Q^+ is the set of all positive rational numbers with the binary operation * defined by $a * b = \frac{ab}{2}$ for all a, b, ∈ Q^+. The inverse of an element a ∈ Q^+ is

A. a

B. $\frac{1}{a}$

C. $\frac{2}{a}$

D. $\frac{4}{a}$

Answer

For Identity element a * e = a = e * a , with 'e' being the identity element.

Given $a * b = \frac{ab}{2}$

$\Rightarrow a * e = \frac{ae}{2} = a$

⇒ e=2

For Inverse element a * b = e = b * a , with 'b' being the inverse element 'a'.

Given $a * b = \frac{ab}{2}$

$\Rightarrow a * b = \frac{ab}{2} = 2$ ....(1)

$\Rightarrow b * a = \frac{ba}{2} = 2$ ....(2)

$\Rightarrow b = \frac{4}{a}$ is the required inverse element of a.

10. Question

Mark the correct alternative in each of the following:

If the binary operation $\odot$ is defined on the set Q^+ of all positive rational numbers by $a \odot b = \frac{ab}{4}$. Then,

$3 \odot \left(\frac{1}{5} \odot \frac{1}{2}\right)$ is equal to

A. $\frac{3}{160}$

B. $\frac{5}{160}$

C. $\frac{3}{10}$

D. $\frac{3}{40}$

Answer

Given that $a * b = \frac{ab}{4}$

To calculate $3 * \left(\frac{1}{5} * \frac{1}{2}\right)$

$\Rightarrow \frac{1}{5} * \frac{1}{2} = \frac{\frac{1}{5} \cdot \frac{1}{2}}{4} = \frac{1}{40}$

$\Rightarrow 3 * \left(\frac{1}{5} * \frac{1}{2}\right) = 3 * \frac{1}{40}$

$\Rightarrow 3 * \frac{1}{40} = \frac{3.\frac{1}{40}}{4}$

$= \frac{3}{160}$

11. Question

Mark the correct alternative in each of the following:

Let * be a binary operation defined on set Q – {1} by the rule a * b = a + b – ab. Then, the identity element for * is

A. 1

B. $\frac{a-1}{a}$

C. $\frac{a}{a-1}$

D. 0

Answer

For Identity element a * e = a = e * a , with ‘e’ being the identity element.

Given $a * b = a + b - ab$

⇒ a * e =a + e – ae

=a

⇒ a + e - ae=a

⇒ e(1-a) =0

⇒ e=0

12. Question

Mark the correct alternative in each of the following:

Which of the following is true?

A. * defined by $a * b = \frac{a+b}{2}$ is a binary operation on Z

B. * defined by $a * b = \frac{a+b}{2}$ is a binary operation on Q

C. all binary commutative operations are associative

D. subtraction is a binary operation on N

Answer

Option C and D are self-explanatory and option A is false because The o/p will result in a rational quantity whereas according to the definition it belongs to an integer value.

13. Question

The binary operation * defined on N by a * b = a + b + ab for all a, b ∈ N is

A. commutative only

B. associative only

C. commutative and associative both

D. none of these

Answer

1) <u>Commutative</u>:

⇒ a * b = a + b + ab ...(1)

⇒ b * a = b + a + ba ...(2)

⇒ a * b= b * a

2) <u>Associative</u>:

⇒ (a * b)* c = (a + b + ab) * c

(a + b + ab) * c = a + b + ab + c +(a + b + ab)c

(a + b + ab) * c = a + b + c + ab +ac + bc + abc

⇒ a * (b * c) = a * (b + c + bc)

a * (b + c + bc) = a + b + c + bc +(b + c + bc)a

a * (b + c + bc) = a + b + c + ab +ac + bc + abc

⇒ (a * b)* c = a * (b * c)

14. Question

Mark the correct alternative in each of the following:

If a binary operation * is defined by a * b = $a^2 + b^2$ + ab + 1, then (2 * 3) * 2 is equal to

A. 20

B. 40

C. 400

D. 445

Answer

Given a * b = $a^2 + b^2$ + ab + 1

⇒ (2 * 3) * 2 = [($2^2 + 3^2$ + 2.3 + 1)]*2

$\Rightarrow [(2^2 + 3^2 + 2.3 + 1)]*2 = 20 * 2$

$\Rightarrow 20 * 2 = 20^2 + 2^2 + 20.2 + 1 = 445$

15. Question

Mark the correct alternative in each of the following:

Let * be a binary operation on R defined by a * b = ab + 1. Then, * is

A. commutative but not associative

B. associative but not commutative

C. neither commutative nor associative

D. both commutative and associative

Answer

1) <u>Commutative</u>:

$\Rightarrow a * b = ab + 1$ ...(1)

$\Rightarrow b * a = ba + 1$ ...(2)

$\Rightarrow a * b = b * a$

2) <u>NOT Associative</u>:

$\Rightarrow (a * b) * c = (ab + 1) * c$

$(ab + 1) * c = (ab + 1)c + 1$

$(ab + 1) * c = abc + c + 1$

$\Rightarrow a * (b * c) = a * (bc + 1)$

$a * (bc + 1) = a(bc + 1) + 1$

$a * (bc + 1) = abc + a$

$\Rightarrow (a * b) * c \neq a * (b * c)$

16. Question

Mark the correct alternative in each of the following:

Subtraction of integers is

A. commutative but not associative

B. commutative and associative

C. associative but not commutative

D. neither commutative nor associative

Answer

1) <u>NOT Commutative</u>:

$\Rightarrow a * b = a - b$ ...(1)

$\Rightarrow b * a = b - a$ ...(2)

$\Rightarrow a * b \neq b * a$

2) <u>NOT Associative</u>:

$\Rightarrow (a * b) * c = (a - b) * c$

$(a - b) * c = (a - b) - c$

(a - b) * c = a – b - c

⇒ a * (b * c) = a * (b - c)

a * (b - c) = a – (b - c)

a * (b - c) = a – b + c

⇒ (a * b)* c ≠ a * (b * c)

17. Question

Mark the correct alternative in each of the following:

The law a + b = b + a is called

A. closure law

B. associative law

C. commutative law

D. distributive law

Answer

The law a + b = b + a is called is called commutative law.

Eg. 2+3=3+2

18. Question

Mark the correct alternative in each of the following:

An operation * is defined on the set Z of non-zero integers by $a * b = \frac{a}{b}$ for all a, b ∈ Z. Then the property satisfied is

A. closure

B. commutative

C. associative

D. none of these

Answer

1) NOT Commutative:

$\Rightarrow\ a * b = \frac{a}{b} \ldots(1)$

$\Rightarrow\ b * a = \frac{b}{a} \ldots(2)$

$\Rightarrow\ a * b \neq b * a$

2) NOT Associative:

$\Rightarrow\ (a * b) * c = \left(\frac{a}{b}\right) * c$

$= \frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$

$\Rightarrow\ a * (b * c) = a * \left(\frac{b}{c}\right)$

$$a * (b - c) = \frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\Rightarrow (a * b) * c \neq a * (b * c)$$

19. Question

Mark the correct alternative in each of the following:

On Z an operation * is defined by $a * b = a^2 + b^2$ for all a, b ∈ Z. The operation * on Z is

A. commutative and associative

B. associative but not commutative

C. not associative

D. not a binary operation

Answer

1) <u>Commutative</u>:

⇒ $a * b = a^2 + b^2$ ...(1)

⇒ $b * a = b^2 + a^2$ ...(2)

⇒ a * b= b * a

2) <u>NOT Associative</u>:

⇒ $(a * b)* c = (a^2 + b^2) * c$

$= (a^2 + b^2)^2 + c^2$

⇒ $a * (b * c) = a * (b^2 + a^2)$

$= a^2 + (b^2 + c^2)^2$

⇒ (a * b)* c ≠ a * (b * c)

20. Question

Mark the correct alternative in each of the following:

A binary operation * on Z defined by a * b = 3a + b for all a, b ∈ Z, is

A. commutative

B. associative

C. not commutative

D. commutative and associative

Answer

<u>Commutative</u>:

⇒ a * b = 3a + b ...(1)

⇒ b * a = 3b + a ...(2)

⇒ a * b ≠ b * a

21. Question

Mark the correct alternative in each of the following:

Let * be a binary operation on Q^+ defined by $a * b = \frac{ab}{100}$ for all a, b ∈ Q^+. The inverse of 0.1 is

A. 10^5

B. 10^4

C. 10^6

D. none of these

Answer

For Identity element a * e = a = e * a , with ‘e’ being the identity element.

Given $a * b = \frac{ab}{100}$

$$\Rightarrow a * e = \frac{ae}{100} = a$$

$$\Rightarrow e = 100$$

For Inverse element a * b = e = b * a , with ‘b’ being the inverse element ‘a’.

Given $a * b = \frac{ab}{100}$

Now for a = 0.1

$$\Rightarrow 0.1 * b = \frac{(0.1)b}{100} = 100 \ \ldots.(1)$$

$$\Rightarrow b * 0.1 = \frac{(0.1)b}{100} = 100 \ \ldots.(2)$$

⇒ b $=10^5$ is the required inverse element of a=0.1.

22. Question

Mark the correct alternative in each of the following:

Let * be a binary operation on N defined by a * b = a + b + 10 for all a, b ∈ N. The identity element for * in N is

A. -10

B. 0

C. 10

D. non-existent

Answer

For Identity element a * e = a = e * a, with ‘e’ being the identity element.

Given a * b = a + b + 10

⇒ a * e =a+e+10=a

⇒ e=-10 (Not a Natural number)

23. Question

Mark the correct alternative in each of the following:

Consider the binary operation * defined on Q - {1} by the rule a * b = a + b – ab for all a, b ∈ Q – {1}. The identity element in Q – {1} is

A. 0

B. 1

C. $\frac{1}{2}$

D. -1

Answer

For Identity element a * e = a = e * a, with ‘e’ being the identity element.

Given a * b =a + b - ab

⇒ a * e =a + e – ae

=a

⇒ e(1-a) =0

⇒ e=0

24. Question

Mark the correct alternative in each of the following:

For the binary operation * defined on R – {1} by the rule a * b = a + b + ab for all a, b ∈ R – {1}, the inverse of a is

A. -a

B. $-\frac{a}{a+1}$

C. $\frac{1}{a}$

D. a^2

Answer

For Identity element a * e = a = e * a , with ‘e’ being the identity element.

Given a * b =a + b + ab

⇒ a * e =a + e + ea

=a

⇒ e=0

For Inverse element a * b = e = b * a , with ‘b’ being the inverse element ‘a’.

Given a * b =a + b + ab

Now for a = 0.1

⇒ a * b =a + b + ab

=0 ….(1)

⇒ b * a =a + b + ab

=0 ….(2)

⇒ $b = -\frac{a}{1+a}$ is the required inverse element of a .

25. Question

Mark the correct alternative in each of the following:

For the multiplication of matrices as a binary operation on the set of all matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, a, b ∈ R the inverse of $\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$ is

A. $\begin{bmatrix} -2 & 3 \\ -3 & -2 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 2/13 & -3/13 \\ 3/13 & 2/13 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Answer

For Identity element a * e = a = e * a , with 'e' being the identity element.

The identity element of matrix multiplication is the identity matrix.

The identity element e = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Therefore, the inverse element of $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ is the inverse of the matrix.

Given: A = $\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{13}\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 2/13 & -3/13 \\ 3/13 & 2/13 \end{bmatrix}$

Hence, C is the correct answer.

26. Question

Mark the correct alternative in each of the following:

On the set Q^+ of all positive rational numbers a binary operation * is defined by $a * b = \frac{ab}{2}$ for all a, b ∈ Q^+.

The inverse of 8 is

A. $\frac{1}{8}$

B. $\frac{1}{2}$

C. 2

D. 4

Answer

For Identity element a * e = a = e * a , with ‘e’ being the identity element.

Given $a * b = \frac{ab}{2}$

⇒ $a * e = \frac{ae}{2} = a$

⇒ e=2

For Inverse element a * b = e = b * a , with ‘b’ being the inverse element a=8 .

Given $a * b = \frac{ab}{2}$

⇒ $8 * b = \frac{8b}{2} = 2$ ....(1)

⇒ $b * 8 = \frac{8b}{2} = 2$ ....(2)

⇒ $b = \frac{1}{2}$ is the required inverse element of a=8.

27. Question

Mark the correct alternative in each of the following:

Let * be a binary operation defined on Q^+ by the rule $a * b = \frac{ab}{3}$ for all a, b ∈ Q^+. The inverse of 4 * 6 is

A. $\frac{9}{8}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. none of these

Answer

As $4 * 6 = \frac{4 \times 6}{3}$

=8

For Identity element a * e = a = e * a , with ‘e’ being the identity element.

Given $a * b = \frac{ab}{3}$

⇒ $a * e = \frac{ae}{3} = a$

⇒ e=3

For Inverse element a * b = e = b * a , with ‘b’ being the inverse element a=8 .

Given $a * b = \frac{ab}{3}$

⇒ $8 * b = \frac{8b}{3} = 3$ ....(1)

⇒ $b * 8 = \frac{8b}{3} = 3$ ....(2)

⇒ $b = \frac{9}{8}$ is the required inverse element of $a = (4 * 6) = 8$.

28. Question

Mark the correct alternative in each of the following:

The number of binary operation that can be defined on a set of 2 elements is

A. 8

B. 4

C. 16

D. 64

Answer

As Total number of binary operations that can be defined on a set on N elements are N^{N^2}

⇒ For N=2 ⇒ $N^{N^2} = 2^4 = 16$

Hence the number of binary operations that can be defined on a set of 2 elements are 16.

29. Question

Mark the correct alternative in each of the following:

The number of commutative binary operations that can be defined on a set of 2 elements is

A. 8

B. 6

C. 4

D. 2

Answer

As Total number of commutative binary operations that can be defined on a set on N elements are $N^{\frac{N(N-1)}{2}}$

⇒ For N=2

$N^{\frac{N(N-1)}{2}} = 2^1$

=2

Hence the number of binary operations that can be defined on a set of 2 elements are 16.