31. Probability

Exercise 31.1

1. Question

Ten cards numbered 1 through 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?

Answer

Ten cards are numbered, hence the sample space is

S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, n(S) = 10

Let event A be the even number card drawn, then

A = {2, 4, 6, 8, 10}, n(A)=5

Now probability that an even card is drawn is

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{10} = \frac{1}{2}$$

Let event B be the cards greater than 3, then

B = {4, 5, 6, 7, 8, 9, 10}, n(B) = 7

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{10}$$

So the probability of A∩B

So, A∩B = {4, 6, 8, 10}, n(A∩B)=4

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{4}{10}$$

So the required probability will become

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{10}}{\frac{7}{10}} = \frac{4}{7}$$

Hence the probability that an even card is drawn out of cards that are more than 3 is $\frac{4}{7}$

2 A. Question

Assume that each child born is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

the youngest is a girl,

Answer

Let b and g represents the boy and the girl child respectively.

Now if a family has two children, the sample space will be

S = {(b, b), (b, g), (g, b), (g, g)}, n(S)=4

Let A be the event that both children are girls, the

A= {(g, g)}, n(A)=1

So the probability that both children are girls is

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

If the youngest is a girl

Let B be the event that the youngest child is a girl

Then B = {(b, g), (g, g)}, n(B) = 2

And the corresponding probability becomes

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

The sample space for the both girls is girl child and the youngest is a girl will become

(A∩B) = {(g, g)}, n(A∩B)=1

And the corresponding probability becomes

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{4}$$

So the conditional probability that both are girls given that the youngest is a girl is

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

2 B. Question

Assume that each child born is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

at least one is a girl?

Answer

At least one is a girl

Let C be the event that at least one is a girl

Then C = {(b, g), (g, b), (g, g)}, n(C) = 3

And the corresponding probability becomes

$$P(C) = \frac{n(C)}{n(S)} = \frac{3}{4}$$

The sample space for the both girls is girl child and at least one is a girl will become

(A∩C) = {(g, g)}, n(A∩C)=1

And the corresponding probability becomes

$$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{1}{4}$$

So the conditional probability that both are girls given that the youngest is a girl is

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

3. Question

Given that the two numbers appearing on throwing two dice are different. Find the probability of the event ‘the sum of numbers on the dice is 4’.

Answer

Let S be the sample space of throwing two dice and the two numbers are different, then the sample space is

S = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5)}

n(S) = 30

Let B be the event that the sum if the numbers on the dice is 4, then the sample space becomes

B = {(1, 3), (3, 1)}, n(B) = 2

So the probability of the event 'the sum of numbers on the dice is 4' is

$$P(B|A) = \frac{n(B)}{n(A)} = \frac{2}{30} = \frac{1}{15}$$

4. Question

A coin is tossed three times, if head occurs on first two tosses, find the probability of getting head on third toss.

Answer

If h and t represents heads and tails respectively, then when a coin is tossed three times, the sample space becomes

S = {(HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)},n(S)=8

Let E be the event of head occurring on the third toss, then the favorable outcomes will be

E = {(HHH, HTH, THH, TTH)}, n(E) = 4

So the corresponding probability will be

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

Let F be the event that head occurs on first two toss, then the favorable outcomes will be

F = {(HHH,HHT)}, n(F) = 2

So the corresponding probability will be

$$P(F) = \frac{n(F)}{n(S)} = \frac{2}{8} = \frac{1}{4}$$

And the favorable outcome for getting head on all the three toss will be

(E∩F)={(HHH)}, n(E∩F)=1

And the corresponding probability becomes

$$P(E \cap F) = \frac{n(E \cap F)}{n(F)} = \frac{1}{8}$$

So if head occurs on first two tosses, the probability of getting head on third tosses

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{2}$$

5. Question

A die is thrown three times, find the probability that 4 appears on the third toss if it is given that 6 and 5 appear respectively on first two tosses.

Answer

When a die is thrown three times the number of all favorable outcomes are n(S) = 216

Let E be the event that 4 occurs on the third toss, then the favorable outcomes will be

E ={(1, 1, 4),(1, 2, 4), (1, 3, 4), (1, 4, 4), (1, 5, 4), (1, 6, 4)

(2, 1, 4),(2, 2, 4), (2, 3,4), (2, 4, 4), (2, 5, 4), (2, 6, 4)

(3, 1, 4), (3, 2, 4), (3, 3, 4), (3, 4, 4), (3, 5, 4), (3, 6,4)

(4, 1 ,4), (4, 2, 4), (4, 3, 4), (4, 4, 4), (4, 5, 4), (4, 6, 4)

(5, 1, 4), (5, 2,4), (5, 3, 4),(5,4, 4), (5, 5, 4), (5, 6, 4)

(6, 1, 4),(6, 2, 4), (6, 3, 4), (6,4, 4),(6, 5, 4), (6, 6, 4)}

n(E) = 36

So the probability that 4 appears on the third toss is

$$P(E) = \frac{n(E)}{n(S)} = \frac{36}{216}$$

Let F be the event that 6 and 5 appear on first two tosses respectively, and then the favorable outcomes will be

F = {(6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4), (6, 5, 6)}

n(F) = 6

So the probability that 6 and 5 appear on first two tosses respectively will be

$$P(F) = \frac{n(F)}{n(S)} = \frac{6}{216}$$

So the favorable outcomes that 6 and 5 appear respectively on first two tosses and 4 appears on the third toss will be

(E∩F) = {(6, 5, 4)}, n(E∩F) = 1

And the corresponding probability will be

$$P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{216}$$

So the conditional probability that if it is given that 6 and 5 appear respectively on first two tosses and 4 appears on the third toss will be

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$$

6. Question

Compute P (A/B), if P (B) = 0.5 and P (A∩B) = 0.32

Answer

Given: P(B) = 0.5, P(A∩B) = 0.32

To find: P(A|B)

We know that,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

By substituting the values we get

$$\Rightarrow P(A|B) = \frac{0.32}{0.5}$$

$$\Rightarrow P(A|B) = \frac{\frac{32}{100}}{\frac{50}{100}} = \frac{32}{50}$$

$\Rightarrow P(A|B) = \frac{16}{25} = 0.64$ is the required value

7. Question

If P (A) = 0.4, P (B) = 0.3 and P(B/A)=0.5, find P(A∩B) and P(A/B).

Answer

Given: P(A) = 0.4, P(B)=0.3, P(B|A) = 0.5

To find: P(A|B) and P(A∩B)

We know that,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

By substituting the values we get

$$\Rightarrow 0.5 = \frac{P(A \cap B)}{0.4}$$

$\Rightarrow P(A \cap B) = 0.5 \times 0.4 = 0.2$is the required value

Similarly,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

By substituting the values we get

$$P(A|B) = \frac{0.2}{0.3}$$

$\Rightarrow P(A|B) = \frac{2}{3} = 0.67$ is the required value

8. Question

If A and B are two events such that $P(A) = \frac{1}{3}, P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{11}{30}$, find P (A/B) and P (B/A).

Answer

Given: $P(A) = \frac{1}{3}, P(B) = \frac{1}{5}, P(A \cup B) = \frac{11}{30}$

To find: P(A|B) and P(B|A)

We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

By substituting the values we get

$$\Rightarrow \frac{11}{30} = \frac{1}{3} + \frac{1}{5} - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = \frac{1}{3} + \frac{1}{5} - \frac{11}{30} = \frac{10+6-11}{30}$$

$$\Rightarrow P(A \cap B) = \frac{5}{30} = \frac{1}{6}$$

We also know that,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

By substituting the values we get

$$P(A|B) = \frac{\frac{1}{6}}{\frac{1}{5}}$$

$\Rightarrow P(A|B) = \frac{5}{6}$ is the required value

Similarly,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

By substituting the values we get

$$P(B|A) = \frac{\frac{1}{6}}{\frac{1}{3}}$$

$\Rightarrow P(B|A) = \frac{3}{6} = \frac{1}{2}$ is the required valu

9. Question

A couple has two children. Find the probability that both the children are

(i) males, if it is known that at least one of the children is male.

(ii) females, if it is known that the elder child is a female.

Answer

Let b and g represents the boy and the girl child respectively.

Now if a family has two children, the sample space will be

S = {(b, b), (b, g), (g, b), (g, g)}, n(S)=4

(i) Let A be the event that both children are males, then

A= {(b, b)}, n(A)=1

So the probability that both children are males is

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

Let B be the event that at least one of the children is male

Then B = {(b, b), (b, g), (g, b)}, n(B) = 3

And the corresponding probability becomes

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$$

The sample space for the at least male and both being male will become

$(A \cap B) = \{(b, b)\}$, $n(A \cap B) = 1$

And the corresponding probability becomes

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{4}$$

So the conditional probability that both are males given that the at least one is male is

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

(ii) Let P be the event that both children are females, then

$P = \{(g, g)\}$, $n(P) = 1$

So the probability that both children are females is

$$P(P) = \frac{n(P)}{n(S)} = \frac{1}{4}$$

Let Q be the event that the elder child is a female

Then $Q = \{(g, b), (g, g)\}$, $n(Q) = 2$

And the corresponding probability becomes

$$P(Q) = \frac{n(Q)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

The sample space for the elder child being female and both child being female will become

$(P \cap Q) = \{(g, g)\}$, $n(P \cap Q) = 1$

And the corresponding probability becomes

$$P(P \cap Q) = \frac{n(P \cap Q)}{n(S)} = \frac{1}{4}$$

So the conditional probability that if the elder child is female then both children are females is

$$P(P|Q) = \frac{P(P \cap Q)}{P(Q)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Exercise 31.2

1. Question

From a pack of 52 cards, two are drawn one without replacement. Find the probability that both of them are kings.

Answer

Total number of all favorable cases is $n(S) = 52$

Let A be the event that first card drawn is a king. There are four kings in the pack. Hence, the probability of the first card is a king is

$P(A) = \frac{4}{52}$

Let B be the event that second card is also king without replacement. Then there are 3 kings left in the pack as the cards are not replaced. Therefore, the probability of the second card is also king is

$P(B|A) = \frac{3}{51}$

Then the probability of getting two kings without replacement is

$= P(A)P(B|A)$

$\Rightarrow = \frac{4}{52} \times \frac{3}{51}$

$\Rightarrow = \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$

The probability that both of them are kings is $\frac{1}{221}$

2. Question

From a pack of 52 cards, 4 are drawn one by one without replacement. Find the probability that all are aces (or, kings).

Answer

Total number of all favorable cases is n(S) = 52

Let A be the event that first card drawn is ace (or, kings). There are four aces (or, kings) in the pack. Hence, the probability of the first card is ace (or, kings) is

$P(A) = \frac{4}{52}$

Let B be the event that second card is also ace (or, kings) without replacement. Then there are 3 aces (or, kings) left in the pack as the cards are not replaced. Therefore, the probability of the second card is also ace (or, kings) is

$P(B|A) = \frac{3}{51}$

Let C be the event that third card is also ace (or, kings) without replacement. Then there are 2 aces (or, kings) left in the pack as the cards are not replaced. Therefore, the probability of the third card is also ace (or, kings) is

$P\left(\frac{C}{A \cap B}\right) = \frac{2}{50}$

Let D be the event that fourth card is also ace (or, kings) without replacement. Then there are 1 ace (or, kings) left in the pack as the cards are not replaced. Therefore, the probability of the fourth card is also ace (or, kings) is

$P\left(\frac{D}{A \cap B \cap C}\right) = \frac{1}{49}$

Then the probability all are aces (or, kings) is

$= P(A)P(B|A)P\left(\frac{C}{A \cap B}\right) P\left(\frac{D}{A \cap B \cap C}\right)$

$\Rightarrow = \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} = \frac{1}{270725}$

The probability that all are aces (or, kings) is $\frac{1}{270725}$

3. Question

Find the chance of drawing 2 white balls in succession from a bag containing 5 red and 7 white balls, the ball first drawn not being replaced.

Answer

Bag contains 5 red balls and 7 white balls. So the total number of all favorable cases is n(S) = 5+7=12

Let A represents first ball as white ball, and B be second ball as white ball.

Then the probability of drawing two white balls without replacement is

P(2 white balls without replacement)

$= P(A)P(B|A)$

$= \frac{7}{12} \times \frac{6}{11}$ (as there are 7 white balls in first draw out of 12 balls, and 6 white balls in second draw out of 11 balls as the balls are not replaced)

Hence the required probability is $\frac{7}{22}$

4. Question

A bag contains 25 tickets, numbered from 1 to 25. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show even numbers.

Answer

There are 25 tickets numbered from 1 to 25, so the sample space is

S = {1, 2, 3, ..., 25}, n(S) = 25

Number of even numbered tickets from 1 to 25 is

{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24} = 12

Let A represents first ticket with even number and B represents second ticket with even number

Then the probability of both tickets being even number without replacement is

P(both tickets showing even number without replacement)

$= P(A)P(B|A)$

$= \frac{12}{25} \times \frac{11}{24}$ (as there are 12 even numbered tickets out of 25 tickets in first draw, and 11 even numbered tickets out of 24 tickets in second draw as the tickets are not replaced)

Hence the required probability is $\frac{11}{50}$

5. Question

From a deck of cards, three cards are drawn on by one without replacement. Find the probability that each time it is a card of spade.

Answer

Total number of all favorable cases is n(S) = 52

Let A be the event that first card drawn is card of spade. There are 13 spade cards in the pack. Hence, the probability of the first card is spade is

$$P(A) = \frac{13}{52}$$

Let B be the event that second card is also card of spade without replacement. Then there are 12 spade cards left in the pack as the cards are not replaced. Therefore, the probability of the second card is also spade card is

$$P(B|A) = \frac{12}{51}$$

Let C be the event that third card is also spade card without replacement. Then there are 11 spade cards left in the pack as the cards are not replaced. Therefore, the probability of the third card is also a spade card is

$$P\left(\frac{C}{A \cap B}\right) = \frac{11}{50}$$

Then the probability all three are spade cards without replacement is

$$= P(A)P(B|A)P\left(\frac{C}{A \cap B}\right)$$

$\Rightarrow = \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} = \frac{11}{850}$ (as there are 13 spade cards in the pack of 52 in first draw, 12 spade cards in the pack of 51cards in the second draw as the cards are not replaced and 11spadecards in the pack of 50 cards in the third draw as the cards are not replaced.)

The required probability is $\frac{11}{850}$

6 A. Question

Two cards are drawn without replacement from a pack of 52 cards. Find the probability that

both are kings

Answer

Total number of all favorable cases is n(S) = 52

Let A be the event that first card drawn is a king. There are four kings in the pack. Hence, the probability of the first card is a king is

$$P(A) = \frac{4}{52}$$

Let B be the event that second card is also king without replacement. Then there are 3 kings left in the pack as the cards are not replaced. Therefore, the probability of the second card is also king is

$$P(B|A) = \frac{3}{51}$$

Then the probability of getting two kings without replacement is

$$= P(A)P(B|A)$$

$\Rightarrow = \frac{4}{52} \times \frac{3}{51}$ (as there are 4 kings out of 52 cards in first draw, and 3 kings out of 51 cards in the second draw as the cards are not replaced)

$$\Rightarrow = \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$$

The probability that both of them are kings is $\frac{1}{221}$

6 B. Question

Two cards are drawn without replacement from a pack of 52 cards. Find the probability that

the first is a king and the second is an ace

Answer

Total number of all favorable cases is n(S) = 52

Let A be the event that first card drawn is a king. There are four kings in the pack. Hence, the probability of the first card is a king is

$$P(A) = \frac{4}{52}$$

Let B be the event that second card is an ace without replacement. Then there are 4 aces in the pack as the cards are not replaced. Therefore, the probability of the second card is an ace is

$$P(B|A) = \frac{4}{51}$$

Then the probability of getting first is a king and the second is an ace without replacement is

$$= P(A)P(B|A)$$

$\Rightarrow = \frac{4}{52} \times \frac{4}{51}$ (as there are 4 kings out of 52 cards in first draw, and 4 aces out of 51 cards in the second draw as the cards are not replaced)

$$\Rightarrow = \frac{4}{663}$$

The probability that first is a king and the second is an ace without replacement is $\frac{4}{663}$

6 C. Question

Two cards are drawn without replacement from a pack of 52 cards. Find the probability that

the first is a heart and second is red.

Answer

Total number of all favorable cases is n(S) = 52

Let A be the event that first card drawn is a heart. There are 13 hearts in the pack. Hence, the probability of the first card is a heart is

$$P(A) = \frac{13}{52}$$

Let B be the event that second card is red without replacement. Then there are 26 red cards in the pack but as the cards are not replaced now there are 25 red cards as one heart which is red in color is already drawn out. Therefore, the probability of the second card is a red is

$$P(B|A) = \frac{25}{51}$$

Then the probability of getting first is a heart and the second is a red card without replacement is

$$= P(A)P(B|A)$$

$$\Rightarrow = \frac{13}{52} \times \frac{25}{51}$$

$$\Rightarrow = \frac{25}{204}$$

The probability that first is a heart and the second is red card without replacement is $\frac{25}{204}$

7. Question

A bag contains 20 tickets, numbered from 1 to 20. Two tickets are drawn without replacement. What is the probability that the first ticket has an even number and the second an odd number?

Answer

There are 20 tickets numbered from 1 to 25, so the sample space is

S = {1, 2, 3, …., 20}, n(S) = 20

Number of even numbered tickets from 1 to 20 is

{2, 4, 6, 8, 10, 12, 14, 16, 18, 20} = 10

Number of odd numbered tickets from 1 to 20 is

{1, 3, 5, 7, 9, 11, 13, 15, 17, 19} = 10

Let A represents first ticket with even number and B represents second ticket with odd number

Then the probability of first ticket having even number and second ticket having odd number without replacement is

$= P(A)P(B|A)$

$= \frac{10}{20} \times \frac{10}{19} = \frac{5}{19}$ (as there are 10 even numbered tickets out of 20 tickets in first draw, and 10 odd numbered tickets out of 19 tickets in second draw as the tickets are not replaced)

Hence the required probability is $\frac{5}{19}$

8. Question

An urn contains 3 white, 4 red and 5 black balls. Two balls are drawn one by one without replacement. What is the probability that at least one ball is black.

Answer

There are 3 white, 4 red and 5 black balls in the bag, so the number of all favorable outcomes in the sample space is

n(S) = 3+4+5=12

Let A be the event of not getting a black ball in the first draw, this means getting another color (red or white) ball out of 12 balls in the first draw. Hence the probability becomes

$P(A) = \frac{7}{12}$ (as there are 3 white + 4 red = 7 balls)

Let B represents the event of not getting a black ball in the second draw, this means getting another color (red or white) ball out of 11 balls in the second draw as the balls are not replaced. Hence the probability becomes

$P(B|A) = \frac{6}{11}$ (as there are 3 white + 4 red = 7 balls and one ball is already drawn in first draw so now there are total of 6 red and white balls)

Then the probability of at least one black ball without replacement

$= 1 - P(\text{none is black ball})$

$\Rightarrow = 1 - P(A \cap B)$

$\Rightarrow = 1 - P(A)P(B|A)$

$= 1 - \frac{7}{12} \times \frac{6}{11}$

$\Rightarrow = 1 - \frac{7}{22} = \frac{15}{22}$

Hence the required probability is $\frac{15}{22}$

9. Question

A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement, find the probability that none is red.

Answer

There are 5 white, 7 red and 3 black balls in the bag, so the number of all favorable outcomes in the sample space is n(S) = 5+7+3=15

Let A be the event of getting a red ball in the first draw. Hence the probability becomes

$P(A) = \frac{7}{15}$ (as there are 7 red balls out of 15 balls)

Let B represents the event of getting a red ball in the second draw. Hence the probability becomes

$P(B|A) = \frac{6}{14}$ (as there are 7 red balls and one red ball is already drawn in first draw so now there are total of 6 red balls)

Let C represents the event of getting a red ball in the third draw. Hence the probability becomes

$P\left(\frac{C}{A \cap B}\right) = \frac{5}{13}$ (as there are 7 red balls and two red balls are already drawn in first and second draw so now there are total of 5 red balls)

Then the probability of none being red balls without replacement

$= 1 - P(\text{all are red ball})$

$\Rightarrow = 1 - P(A \cap B)$

$\Rightarrow = 1 - P(A)P(B|A)P\left(\frac{C}{A \cap B}\right)$

$= 1 - \frac{7}{15} \times \frac{6}{14} \times \frac{5}{13}$

$\Rightarrow = 1 - \frac{1}{13} = \frac{12}{13}$

Hence the required probability is $\frac{12}{13}$

10. Question

A card is drawn from a well-shuffled deck of 52 cards and then a second card is drawn. Find the probability that the first card is a heart and the second card is diamond if the first card is not replaced.

Answer

There are 52 cards in a deck. Let A be the event that first card drawn is a heart. There are 13 hearts in the pack. Hence, the probability of the first card is a heart is

$P(A) = \frac{13}{52}$

Let B be the event that second card is a diamond without replacement. Then there are 13 diamond cards in the pack but as the cards are not replaced now there are 51 cards in total in the deck. Therefore, the probability of the second card is a red is

$P(B|A) = \frac{13}{51}$

Then the probability of getting first is a heart and the second is a diamond card without replacement is

$= P(A)P(B|A)$

$\Rightarrow = \frac{13}{52} \times \frac{13}{51}$

$\Rightarrow = \frac{13}{204}$

The probability that first is a heart and the second is diamond card without replacement is $\frac{13}{204}$

11. Question

An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?

Answer

There are 10 black and 5 white balls in the bag, so the number of all favorable outcomes in the sample space is

n(S) = 10+5=15

Let A be the event of getting a black ball in the first draw. Hence the probability becomes

$\mathrm{P(A)} = \frac{10}{15}$ (as there are 10 black balls out of 15 balls)

Let B represents the event of getting a black ball in the second draw. Hence the probability becomes

$\mathrm{P(B|A)} = \frac{9}{14}$ (as there are 10 black balls and one black ball is already drawn in first draw so now there are total of 9 black balls)

Then the probability of all being black balls without replacement

$\Rightarrow = \mathrm{P(A \cap B)}$

$\Rightarrow = \mathrm{P(A)P(B|A)}$

$= \frac{10}{15} \times \frac{9}{14}$

$\Rightarrow = \frac{3}{7}$

Hence the required probability is $\frac{3}{7}$

12. Question

Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability the first two cards are kings and third card drawn is an ace?

Answer

There are 52 cards in a deck. Let A be the event that first card drawn is a king. There are 4 hearts in the pack. Hence, the probability of the first card is a king is

$\mathrm{P(A)} = \frac{4}{52}$

Let B be the event that second card is also a king without replacement. Then there are 3 king cards out of 51 cards in the pack as the cards are not replaced. Therefore, the probability of the second card is a red is

$\mathrm{P(B|A)} = \frac{3}{51}$

Let C be the event that third card is an ace card without replacement. Then there are 4 ace cards out of 50 cards in the pack as the cards are not replaced. Therefore, the probability of the third card is an ace card is

$\mathrm{P}\left(\frac{\mathrm{C}}{\mathrm{A \cap B}}\right) = \frac{4}{50}$

Then the probabilities of getting first two cards are kings and third card drawn is an ace without replacement is

$= \mathrm{P(A)P(B|A)P}\left(\frac{\mathrm{C}}{\mathrm{A \cap B}}\right)$

$\Rightarrow = \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50}$

$\Rightarrow = \frac{2}{5525}$

The probability that first two cards are kings and third card drawn is an ace without replacement is $\frac{2}{5525}$

13. Question

A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale otherwise it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

Answer

Number of oranges the box contains = 15

Number of good oranges = 12

Number of bad oranges = 3

Probability that box is approved for sale

= Probability that first orange is good × probability that second orange is good, given first is good × probability that third orange is good, given first two are good

Let A represents a good orange

Then P(A)=P(getting first orange as good) $= \frac{12}{15}$

And

P(A│A)=P(getting second orange good, given first is good) $= \frac{11}{14}$

(as now there are 11 good oranges left out of 14 total oranges as one good orange is already drawn in first draw and are not replaced)

And also,

P(AA│A)=P(getting third orange good, given first two are good)= $\frac{10}{13}$

(as now there are 10 good oranges left out of 13 total oranges as two good orange is already drawn in first draw and are not replaced)

So the probability that box is approved for sale is

$= P(A)P(A|A)P(AA|A)$

$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{44}{91}$

Therefore probability that box is approved for sale is $\frac{44}{91}$

14. Question

A bag contains 4 white, 7 black 5 red balls. Three balls are drawn one after the other without replacement. Find the probability that the balls drawn are white, black and red respectively.

Answer

There are 4 white, 7 black and 5 red balls in the bag, so the number of all favorable outcomes in the sample space is

n(S) = 4+7+5=16

Let A be the event of getting a white ball in the first draw. Hence the probability becomes

$P(A) = \frac{4}{16}$ (as there are 4 white balls out of 16 balls)

Let B represents the event of getting a black ball in the second draw. Hence the probability becomes

$P(B|A) = \frac{7}{15}$ (as there are 7 black balls out of 15 balls as balls are not replaced back in the bag)

Let C represents the event of getting a red ball in the third draw. Hence the probability becomes

$P\left(\frac{C}{A\cap B}\right) = \frac{5}{14}$ (as there are 5 red balls out of 14 balls as balls are not replaced back in the bag)

Then the probability that the balls drawn are white, black and red respectively without replacement

$\Rightarrow = P(A \cap B \cap C)$

$\Rightarrow = P(A)P(B|A)P\left(\frac{C}{A \cap B}\right)$

$= \frac{4}{16} \times \frac{7}{15} \times \frac{5}{14}$

$\Rightarrow \frac{1}{24}$

Hence the required probability is $\frac{1}{24}$

Exercise 31.3

1. Question

If P (A) = 7/13, P (B) =9/13 and P (A ∩ B) = 4/13, find p (A/B).

Answer

We have, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

From the given data, P (A ∩ B) = $\frac{4}{13}$

And P (B) =$\frac{9}{13}$

Hence, $P\left(\frac{A}{B}\right) = \frac{\frac{4}{13}}{\frac{9}{13}}$

$= \frac{4}{9}$ (answer)

2. Question

If A and B are events such that P (A) =0.6, P (B) = 0.3 and P (A ∩ B) = 0.2, find P (A/B) and P(B/A).

Answer

We have, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

And $P(\frac{B}{A}) = \frac{P(A \cap B)}{P(A)}$

And also P (A ∩ B) = P (B ∩ A)

Hence $P\left(\frac{A}{B}\right) = \frac{0.2}{0.3} = \frac{2}{3}$ (answer)

And $P\left(\frac{B}{A}\right) = \frac{0.2}{0.6} = \frac{2}{6} = \frac{2}{3}$ (answer)

3. Question

If A and B are two events such that P (A ∩ B) = 0.32 and P (B) = 0.5, find P (A/B).

Answer

We have, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

Therefore $P\left(\frac{A}{B}\right) = \frac{0.32}{0.5}$

{multiply numerator and denominator by 2 to convert into the whole number}

= 0.64 (answer)

4. Question

If P (A) = 0.4, P (B) = 0.8, P (B/A) = 0.6. Find P (A/B) and P (A ∪ B).

Answer

We have, P (A ∪ B) = P (A)+ P (B)- P (A ∩ B)

From equation, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

P (B ∩ A) = $P\left(\frac{B}{A}\right)$ × P (A) = 0.6× 0.4

= 0.24

Hence, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{0.8}$

= 0.3 (answer)

And P (A ∪ B) = P (A)+ P (B)- P (A ∩ B)

= 0.4+0.8-0.24

= 0.96 (answer)

5 A. Question

If A and B are two events such that

P (A) = 1/3, P (B)=1/4 and P(A ∪ B) = 5/12, find P (A/B) and P (B/A).

Answer

We have, P (A) = $\frac{1}{3}$, P (B)=$\frac{1}{4}$ and P(A ∪ B) = $\frac{5}{12}$

We have, P (A ∪ B) = P (A)+ P (B)- P (A ∩ B)

Therefore, P (A ∩ B) = P (A)+ P (B) - P (A ∪ B)

$= \left(\frac{1}{3}\right) + \left(\frac{1}{4}\right) - \left(\frac{5}{12}\right)$

$= \frac{(4+3-5)}{12}$ {Make denominator same by taking LCM)

$= \frac{2}{12} = \frac{1}{6}$ (Answer)

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \left(\frac{\frac{1}{6}}{\frac{1}{4}}\right)$

$= \frac{4}{6} = \frac{2}{3}$ (answer)

And $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{3}{6} = \frac{1}{2}$ (answer)

5 B. Question

If A and B are two events such that

P (A) = 6/11, P (B) = 5/11 and P (A ∪ B) = 7/11, find P (A ∩ B), P (A/B), P (B/A)

Answer

We have P (A) = 6/11, P (B) = 5/11 and P (A ∪ B) = 7/11

We have, P (A ∪ B) = P (A)+ P (B)- P (A ∩ B)

Therefore, P (A ∩ B) = P (A)+ P (B) - P (A ∪ B)

$= \left(\frac{6}{11}\right) + \left(\frac{5}{11}\right) - \left(\frac{7}{11}\right)$

$= \frac{6+5-7}{11}$ { 11 is common in denominator }

$= \frac{4}{11}$ (answer)

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{11}}{\frac{5}{11}}$

$= \frac{4}{5}$ (answer)

And $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{4}{11}}{\frac{6}{11}}$

$= \frac{4}{6} = \frac{2}{3}$ (answer)

5 C. Question

If A and B are two events such that

P (A)= 7/13, P (B)=9/13 and P(A ∩ B)=4/13, find P (A/B).

Answer

We have, P (A)= 7/13, P (B)=9/13 and P(A ∩ B)=4/13

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{4}{13}}{\frac{9}{13}} = \frac{4}{9}$ (answer)1

5 D. Question

If A and B are two events such that

P(A)=1/2,P (B)=1/3 and P(A ∩ B) = 1/4, find P (A/B), P(B/A), P (A/B) and P (A/B).

Answer

We have, P(A)=1/2,P (B)=1/3 and P(A ∩ B) = 1/4

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{1}{4}}{\frac{1}{3}}$

$= \frac{3}{4}$ (answer)

And $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}}$

$= \frac{2}{4} = \frac{1}{2}$ (answer)

6. Question

If A and B are two events such that 2P (A) = P (B) = 5/13 and P (A/B) = 2/5, find P (A∪ B).

Answer

We have, 2P (A) = P (B) = $\frac{5}{13}$

2P (A) = $\frac{5}{13}$

P (A) = $\frac{5}{13\times2} = \frac{5}{26}$

Now, we need P (A ∩ B)

We have, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

P (A ∩ B) = $P\left(\frac{A}{B}\right) \times P(B)$

$= \left(\frac{2}{5}\right) \times \left(\frac{5}{13}\right)$

$= \frac{2}{13}$

Now, P (A ∪ B) = P (A)+ P (B)- P (A ∩ B)

$= \left(\frac{5}{26}\right) + \left(\frac{5}{13}\right) - \frac{2}{13}$

$= \frac{5+10-4}{26}$ (taking lcm as 26 is common factor)

$= \frac{11}{26}$ (answer)

7. Question

If P (A)=6/11,P (B)=5/11 and P (A ∪ B) = 7/11, find

i. P (A ∩ B)

ii. P (A /B)

iii. P (B/A)

Answer

We have, P (A ∪ B) = P (A)+ P (B)- P (A ∩ B)

i. From above formula,

P (A ∩ B)= P (A)+ P (B) - P (A ∪ B)

$=\left(\frac{6}{11}\right) + \left(\frac{5}{11}\right) - \left(\frac{7}{11}\right)$

$= \frac{6+5-7}{11} = \frac{4}{11}$ (answer)

ii. $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{4}{11}}{\frac{5}{11}} = \frac{4}{5}$ (answer)

iii. $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{4}{11}}{\frac{6}{11}} = \frac{4}{6} = \frac{2}{3}$ (answer)

8 A. Question

A coin is tossed three times. Find P (A/B) in each of the following:

A=Heads on third toss, B=Heads on first two tosses

Answer

When a coin is tossed three times, we have following outcomes

{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Total outcomes =8

From above outcomes,

A= Heads on third toss = {HHH, HTH, THH, TTH} =4

Therefore probability of occurrence of event A = P(A) $= \frac{4}{8} = \frac{1}{2}$

B= heads on first two tosses = {HHH, HHT} =2

Therefore probability of occurrence of event B = P(B) $= \frac{2}{8} = \frac{1}{4}$

Also we want P (A ∩ B) = probability of occurrence of both events A and B

=heads on first two tosses and heads on third toss

=heads on all tosses = {HHH} = 1(Occurrence Of both A and B events

Therefore, P (A ∩ B) = $\frac{1}{8}$

Hence, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{4}{8} = \frac{1}{2}$ (answer)

8 B. Question

A coin is tossed three times. Find P (A/B) in each of the following:

A=At least two heads, B=At most two heads

Answer

When a coin is tossed three times, we have following outcomes

{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Total outcomes =8

A= at least two heads ={HHH, HHT, THH, HTH} = 4

Therefore probability of occurrence of event A = P(A) $= \frac{4}{8} = \frac{1}{2}$

B= at most two heads ={HHT, HTH, HHT, THT, TTH, TTT, HTT} =7

Therefore probability of occurrence of event B = P(B) $= \frac{7}{8}$

Now Also we want P (A ∩ B) = probability of occurrence of both events A and B

= occurrence of atleast and atmost two heads ={HHT, HTH, THH} =3

Hence, probability of occurrence of both events A and B = P (A ∩ B)=$\frac{3}{8}$

Therefore, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{3/8}{7/8} = \frac{3}{7}$ (answer)

8 C. Question

A coin is tossed three times. Find P (A/B) in each of the following:

A=At most two tails, B=At least one tail.

Answer

When a coin is tossed three times, we have following outcomes

{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Total outcomes =8

A= atmost two tails ={HHH, HHT, HTH, THH, HTT, THT, TTH, } = 7

Therefore probability of occurrence of event A = P(A) =$\frac{7}{8}$

B= at least one tail ={HHT, HTH, HHT, THT, TTH, TTT, HTT} =7

Therefore probability of occurrence of event B = P(B) =$\frac{7}{8}$

Now Also we want P (A ∩ B) = probability of occurrence of both events A and B

= occurrence of atleast two tail and atleast one tail ={HHT, HTH, THH, HTT, THT, TTH,} =6

Hence, probability of occurrence of both events A and B = P (A ∩ B)=$\frac{6}{8}$

Therefore, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{6}{8}}{7/8} = \frac{6}{7}$ (answer)

9 A. Question

Two coins are tossed once. Find P(A/B) in each of the following:

A=Tail appears on one coin, B= One coin shows head.

Answer

When two coins are tossed, total outcomes are

{HH, HT, TH, TT} =4

A=Tail appears on one coin= {HT, TH} =2

Probability of A = P(A) =$\frac{2}{4} = \frac{1}{2}$

B= One coin shows head ={HT, TH} =2

Probability of B =P(B) =$\frac{2}{4} = \frac{1}{2}$

(A ∩ B) = one coin shows head and another shows tail = {HT, TH} =2

P (A ∩ B) =$\frac{2}{4} = \frac{1}{2}$

Hence, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{1/2}{1/2} = 1$ (answer)

9 B. Question

Two coins are tossed once. Find P(A/B) in each of the following:

A=No tail appears, B=No head appears.

Answer

When two coins are tossed, total outcomes are

{HH, HT, TH, TT} =4

A=No tail occurs = {HH} =1

Probability f A = P(A) = $\frac{1}{4}$

B= No head occurs={TT} =1

Probability of B =P(B) = $\frac{1}{4}$

(A ∩ B) = No tail and no head occurs = not possible =0

P (A ∩ B) = $\frac{0}{4} = 0$

Hence, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

= $\frac{0}{¼} = 0$ (answer)

10. Question

A die is thrown three times. Find P(A/B) and P (B/A), if A=4 appears on the third toss, B=6 and 5 appear respectively on first two tosses.

Answer

When a die is thrown 3 times, total possible outcomes are, 6^3 = 216

A= 4 appears on third toss =

{(1,1,4),(1,2,4),(1,3,4),(1,4,4),(1,5,4),(1,6,4)

(2,1,4),(2,2,4),(2,3,4),(2,4,4),(2,5,4),(2,6,4)

(3,1,4),(3,2,4),(3,3,4),(3,4,4),(3,5,4),(3,6,4)

(4,1,4), (4,2,4)(4,3,4),(4,4,4),(4,5,4),(4,6,4)

(5,1,4),(5,2,4),(5,3,4)(5,4,4),(5,5,4),(5,6,4)

(6,1,4),(6,2,4),(6,3,4),(6,4,4),(6,5,4),(6,6,4)}

=36

Hence P(A) = $\frac{36}{216} = \frac{1}{6}$

B= 6,5 appears respectively on first two tosses

{(6,5,1),(6,5,2),(6,5,3),(6,5,4),(6,5,5),(6,5,6)}

= 6

Hence P(B) = $\frac{6}{216} = \frac{1}{36}$

(A ∩ B) = 6,5 occurs on first two toss and 4 occur on third toss = (6,5,4) = 1

P (A ∩ B) = $\frac{1}{216}$

Hence $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{1}{216}}{\frac{1}{36}} = \frac{36}{216} = \frac{1}{6}$ (answer)

$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{216}}{⅙} = \frac{6}{216} = \frac{1}{36}$ (answer)

11. Question

Mother, father and son line up at random for a family picture. If A and B are two events given by A=son on one end, B= Father in the middle, find P (A/B) and P (B/A).

Answer

Total possible outcomes =

{FMS, FSM, MFS, SMF, SFM, MSF} =6

A= Son on end = {FMS, MFS, SMF, SFM} =4

Therefore P(A) = $\frac{4}{6} = \frac{2}{3}$

B= father in middle = {MFS, SFM} = 2

Therefore P(B) = $\frac{2}{6} = \frac{1}{3}$

(A ∩ B) = son on one end and father in middle

= {MFS, SFM} =2

P(A ∩ B) = $\frac{2}{6} = \frac{1}{3}$

Hence $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$ (answer)

. $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$ (answer)

12. Question

A dice is thrown twice, and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?

Answer

When a dice is thrown 2 times, total outcomes are

{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)

(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

= 36 possible outcomes

A= sum of numbers is 6

={(1,5),(2,4),(3,3),(4,2),(5,1)} = 5

P(A) = $\frac{5}{36}$

B= 4 has appeared ateast once

= {(1,4),(2,4),(3,4),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,4),(6,4)} =11

P(B)= $\frac{11}{36}$

(A ∩ B) = sum of two number is 6 and also 4 has occurred ateast once

(A ∩ B) ={ (2,4),(4,2)} = 2

P(A ∩ B) = $\frac{2}{36}$

Conditional probability = $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \left(\frac{2}{36}\right) / (\frac{5}{36} = \frac{2}{5}$ (answer)

13. Question

Two dice are thrown. Find the probability that the numbers appeared has the sum 8, if it is known that the second die always exhibits 4.

Answer

We know total possible outcomes when two dice are thrown = 36

A= sum is 8

{(2,6),(3,5),(4,4),(5,3),(6,2)} = 5

P(A) = $\frac{5}{36}$

B =second die exhibit 4

{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4)}= 6

P(B) = $\frac{6}{36}$

(A ∩ B) = sum of two number is 8 and also second die exhibit 4

=(4, 4) =1

P(A ∩ B) = $\frac{1}{36}$

Therefore, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = (\frac{1}{36})/(\frac{6}{36}) = \frac{1}{6}$ (answer)

14. Question

A pair of dice is thrown. Find the probability of getting 7 as the sum, if it is known that the second die always exhibits an odd number.

Answer

We know total possible outcomes when two dice are thrown = 36

A= sum is 7

{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1l} = 6

P(A) = $\frac{6}{36}$

B =second die exhibit odd number

{(1,1),(1,3),(1,5),(2,1),(2,3),(2,5),(3,1),(3,3),(3,5),(4,1),(4,3),(4,5),(5,1),(5,3),(5,5),(6,1),(6,3),(6,5)}= 18

P(B) = $\frac{18}{36}$

(A ∩ B) = sum of two number is 7 and also second die exhibit odd number

$=\{(2,5),(4,3),(6,1)\} = 3$

$P(A \cap B) = \frac{3}{36}$

Therefore, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= (\frac{3}{36})/(\frac{18}{3} = \frac{3}{18} = \frac{1}{6}$ (answer)

15. Question

A pair of dice is thrown. Find the probability of getting 7 as the sum if it is known that the second die always exhibits a prime number.

Answer

We know, when a pair of dice is thrown, total possible outcomes are = 36

A=No of outcomes for getting 7 as sum are {(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)} = 6

Therefore, P(A) = 6/36 $=\frac{1}{6}$

B= No of outcomes for second die exhibiting prime no =

{(1,2),(1,3),(1,5),(2,2),(2,3),(2,5),(3,2),(3,3),(3,5),(4,2),(4,3),(4,5),(5,2),(5,3),(5,5),(6,2),(6,3),

(6,5)} = 18 {since 2,3,5 are prime nos)

Hence P(B) $= \frac{18}{36} = \frac{1}{2}$

(A B) = outcomes of getting 7 as a sum with a second die showing prime no

= {(2,5), (5,2),(4,3)} = 3

Hence P(A B) = 3/36 $= \frac{1}{12}$

Therefore, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{2}{12} = \frac{1}{6}$ (answer)

16. Question

A die is rolled. If the outcome is an odd number, what is the probability that it is prime?

Answer

Total outcomes when a die is rolled = {1,2,3,4,5,6} = 6

A = outcome is odd number = {1,3,5} = 3

Hence P(A) $= \frac{3}{6} = \frac{1}{2}$

B= Outcome is prime number = { 2,3,5) = 3

Hence P(B) $= \frac{3}{6} = \frac{1}{2}$

(A B) = outcome is odd number and also prime number = {3,5} =2

Hence P(A B) $= \frac{2}{6} = \frac{1}{3}$

We require, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$ (answer)

17. Question

A pair of dice is thrown. Find the probability of getting the sum 8 or more, if 4 appears on the first die.

Answer

We know, when a pair of dice is thrown, total possible outcomes are = 36

A=No of outcomes for getting sum of 8 or more are

{(2,6),(3,5),(3,6),(4,4),(4,5),(4,6),(5,3),(5,4),(5,5),(5,6),(6,2),(6,3),(6,4),(6,5),(6,6)} = 15

Therefore, P(A) = $\frac{15}{36}$

B= No of outcomes for first die showing 4 =

{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)} = 6

Hence P(B) = $\frac{6}{36}$

(A B) = outcomes of getting sum as 8 or more with a first die showing 4

= {(4,4),(4,5),(4,6)} = 3

Hence P(A B) = $\frac{3}{36}$

Therefore, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{3}{36}}{\frac{6}{36}} = \frac{3}{6} = \frac{1}{2}$ (answer)

18. Question

Find the probability that the sum of the numbers showing on two dice is 8, given that least one die does not show five.

Answer

We know, when a pair of dice is thrown, total possible outcomes are = 36

A=No of outcomes for getting sum of 8 are

{(2,6),(3,5),(4,4),(5,3),(6,2)} = 5

Therefore, P(A) = $\frac{5}{36}$

B= No of outcomes for at least one die does not show 5

= {(1,1),(1,2),(1,3),(1,4),(1,6),(2,1),(2,2),(2,3),(2,4),(2,6),(3,1),(3,2),(3,3),(3,4),(3,6),(4,1),(4,2),

(4,3),(4,4),(4,6),(6,1),(6,2),(6,3),(6,4),(6,6) } = 25

Hence P(B) = $\frac{25}{36}$

(A B) = outcomes of getting sum as 8 with at least one die not showing 5

= {(2,6),(4,4),(6,2)= 3

Hence P(A B) = $\frac{3}{36}$

Therefore, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{3}{36}}{\frac{25}{36}} = \frac{3}{26}$ (answer)

19. Question

Two numbers are selected at random from integers 1 through 9 If the sum is even, find the probability that both the numbers are odd.

Answer

Given: Two numbers are selected at random from integers 1 to 9.

Let A be the event when both the numbers are odd.

A = {(3, 1), (5, 1), (7, 1), (9, 1), (3, 5), (3, 7), (3, 9), (5, 3), (5, 7), (5, 9), (7, 3), (7, 5), (7, 9), (9, 3), (9, 5), (9, 7)}

Let B be the event when sum of both numbers is even.

B = {(1, 3(, (1, 5), (2, 4), (1, 7), (2, 6), (3, 5), (1, 9), (2, 8), (3, 7), (4, 6), (7, 5), (8, 4), (9, 3), (8, 6), (9, 5), (9, 7)}

A B = {(1, 3), (1, 5), (1, 7), (3, 5), (1, 9), (3, 7), (7, 5), (9, 3), (9, 5), (9, 7)}

$$P\left(\frac{A}{B}\right) = \left(\frac{n(A\cap B)}{n(B)}\right)$$

$$P\left(\frac{A}{B}\right) = \frac{5}{8}$$

20. Question

A die is thrown twice, and the sum of the numbers appearing is observed to be 8 What is the conditional probability that the number 5 has appeared at least once?

Answer

We know, when a pair of dice is thrown, total possible outcomes are = 36

A=No of outcomes for getting sum of 8 are

{(2,6),(3,5),(4,4),(5,3),(6,2)} = 5

Therefore, P(A) = $\frac{5}{36}$

B= No of outcomes for at least 5 has appeared once =

= {(1,5),(2,5),(3,5),(4,5),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,5)} =11

Hence P(B) = $\frac{11}{36}$

(A B) = outcomes of getting sum as 8 with at least one die showing 5

= {(3,5),(5,3)= 2

Hence P(A B) $= \frac{2}{36}$

Therefore, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}$ (answer)

21. Question

Two dice are thrownand it is known that the first die shows a 6 Find the probability that the sum of the numbers showing on two dice is 7

Answer

Given: Two dice are thrown.

Let A be the event when the sum of the numbers = 7

Therefore, A = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

Let B be the event when first dice shows 6

Therefore, B = {(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

A B = {(6, 1)}

$$P\left(\frac{A}{B}\right) = \left(\frac{n(A \cap B)}{n(B)}\right)$$

$$P\left(\frac{A}{B}\right) = \frac{1}{6}$$

22. Question

A pair of dice is thrown. Let E be the event that the sum is greater than or equal to 10 and F be the event”5 appears of the first-die”. Find P(E/F). If F is the event”5 appears on at least one die”, find P(E/F). (same as above question)

Answer

We know, when a pair of dice is thrown, total possible outcomes are = 36

E=No of outcomes for getting sum greater than or equal to 10

{(4,6),(5,5),(5,6),(6,4),(6,5),(6,6)} = 6

Therefore, P(E) = $\frac{6}{36}$

Case 1:

F= No of outcomes for 5 appears on first die =

= {(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)} =6

Hence P(B) = $\frac{6}{36}$

(E F) = outcomes of getting a sum greater than or equal to 10 and first die showing 5

= {(5,6),(5,5)= 2

Hence P(E F) = $\frac{2}{36}$

Therefore, $P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{2}{6} = \frac{1}{3}$ (answer)

Case 2:

F = 5 appears on atleast one die =

{(1,5),(2,5),(3,5),(4,5),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,5)} = 11

P(F) = $\frac{11}{36}$

(E F) = outcomes of getting a sum greater than or equal to 10 and at least die showing 5

= {(5,6),(5,5),(6,5)}= 3

Hence P(E F) = $\frac{3}{36}$

Therefore, $P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{3}{36}}{\frac{11}{36}} = \frac{3}{11}$ (answer)

23. Question

The probability that a student selected at random from a class will pass in Mathematics is 4/5, and probability that he/she passes in Mathematics and Computer Science is 1/2. What is the probability that he/she will pass in Computer Science if it is known that he/she has passed in Mathematics?

Answer

Let M = Mathematics , C= computer science

By given data,

P(M) = $\frac{4}{5}$ and P(M C) = $\frac{1}{2}$

We need to find, $P\left(\frac{C}{M}\right) = \frac{P(C \cap M)}{P(M)} = \frac{\frac{1}{2}}{\frac{4}{5}} = \frac{5}{4\times 2} = \frac{5}{8}$ (Answer)

24. Question

The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a trouser is 0.3, and the probability that he will buy a shirt given that he will buy a trouser is 0.4. Find the probability that he will buy both a shirt and a trouser. Find also the probability that he will buy a trouser given that he buys a shirt.

Answer

Let S= Shirt, T= Trouser

P(S) = 0.2 , P(T) = 0.3 and $P\left(\frac{S}{T}\right)$ = 0.4

We need to find P(S T) and $P\left(\frac{T}{S}\right)$

We know, $P\left(\frac{S}{T}\right) = \frac{P(S \cap T)}{P(T)}$ From given data, 0.4 = P (S T) / 0.3

P (S T) = 0.4×0.3 = 0.12 (Answer).

Now we want, $P\left(\frac{T}{S}\right)$

$P\left(\frac{T}{S}\right) = \frac{P(T \cap S)}{P(S)} = \frac{0.12}{0.2} = \frac{12}{20} = \frac{6}{10} = \frac{3}{5} = 0.6$ (answer)

25. Question

In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in XII. What is the probability that a student is chosen randomly studies in Class XII given that the chosen student is a girl?

Answer

Given: Total Number of students = 1000

Number of Girls = 430

% of girls in Class XII = 10%

Let A be the event of Student chosen studies in Class XII

And Let B be the event that the student chosen is a girl

Now, $P(B) = \frac{430}{1000}$

$$P(A \cap B) = \frac{43}{1000}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{43}{430} = \frac{1}{10}$$

26. Question

Ten cards numbered 1 through 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?

Answer

Given: Total Number of Cards = 10

Let A be the event of drawing a number more than 3.

Let B be the event of drawing an even number.

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{4}{10}}{\frac{7}{10}} = \frac{4}{7}$$

27. Question

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls? Given that

i. the youngest is girl

ii. at least one is a girl.

Answer

Let B= boy G= girl

And let us consider, in a sample space, the first child is elder and second child is younger.

Total possible outcome = {BB, BG, GB, GG} = 4

Let A= be the event that both the children are girls = 1

Therefore P(A) $=\frac{1}{4}$

Case 1.

Let B = event that youngest is girl = {BG, GG} =2

{Since we have considered second is younger in a sample space}

Therefore P(B) $=\frac{2}{4}$

And (A ∩ B) = both are girls and younger is also girl = (GG) = 1

Therefore , P (A ∩ B) = $\frac{1}{4}$

We require $P\left(\frac{A}{B}\right)$

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{1/4}{\frac{2}{4}} = \frac{1}{2}$ (answer)

Case 2.

Let B = event that at least one is girl = {BG,GB GG} =3

{Since we have considered second is younger in a sample space}

Therefore P(B) = $\frac{3}{4}$

And (A ∩ B) = both are girls and atlas one is girl = (GG) = 1

Therefore , P (A ∩ B) = $\frac{1}{4}$

We require $P\left(\frac{A}{B}\right)$

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{1/4}{\frac{3}{4}} = \frac{1}{3}$ (answer)

Exercise 31.4

1. Question

A coin is tossed thrice and all the eight outcomes are assumed equally likely. In which of the following cases are the following events A and B are independent?

i. A=the first throw results in head, B=the last throw results in tail

ii. A=the number of heads is odd, B=the number of tails is odd

iii. A=the number of heads is two, B=the last throw results in head

Answer

It is given that the coin is tossed thrice, so the sample space will be,

S={HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}

For independent event, P(A)*P(B)=P(A∩B)

i. A=the first throw results in head, B=the last throw results in tail

A={HHH,HHT,HTH,HTT}

$$P(A) = \frac{4}{8} = \frac{1}{2}$$

B={HHT,HTT,THT,TTT}

$$P(B) = \frac{4}{8} = \frac{1}{2}$$

A∩B={HHT,HTT}

$$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$$

$$P(A) * P(B) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$$

$$P(A) * P(B) = P(A \cap B)$$

Therefore A and B are independent events.

ii. A=the number of heads is odd, B=the number of tails is odd

A={HTT,THT,TTH}

$$P(A) = \frac{3}{8}$$

B={HTH,THH,HHT}

$$P(B) = \frac{3}{8}$$

A∩B={}=Ø

$$P(A) * P(B) != P(A \cap B)$$

A and B are not independent

iii. A=the number of heads is two, B=the last throw results in head

A={HHT,HTH,THT}

$$P(A) = \frac{3}{8}$$

B={HHH,HTH,THH,TTH}

$$P(B) = \frac{4}{8} = \frac{1}{2}$$

A∩B={HTH}

$$P(A \cap B) = \frac{1}{8}$$

$$P(A) * P(B) = \frac{3}{8} * \frac{1}{2} = \frac{3}{16}$$

$$P(A) * P(B) != P(A \cap B)$$

Therefore A and B are not independent events.

2. Question

Prove that is throwing a pair of dice, the occurrence of the number 4 on the first die is independent of the occurrence of 5 on the second die.

Answer

We are throwing two dice so,

Sample space S={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),

(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

Sample space contains 36 elements,

A= Number four appears on first die

A={(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)}

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

B=Number 5 on second die

B={(1,5), (2,5), (3,5),(4,5),(5,5),(6,5)}

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

P(A∩B)={(4,5)}

$$P(A \cap B) = \frac{1}{36}$$

$$P(A) * P(B) = \frac{1}{6} * \frac{1}{6} = \frac{1}{36}$$

$$P(A) * P(B) = P(A \cap B)$$

Therefore A and B are independent events

3. Question

A card is drawn form a pack of 52 cards so that each card is equally likely to be selected. In which of the following cases are the events A and N independent?

i. A=the card drawn is a king or queen, B=the card drawn is a queen or jack

ii. A=the card drawn is black, B=the card drawn is a king

iii. B=the card drawn is a spade, B=the card drawn in an ace

Answer

(i) In a pack of 52 cards there are 4 king,4 queen,4 jack

A=the card drawn is a king or queen

$$P(A) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

B=the card drawn is a queen or jack

$$P(A) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

A∩B=Card is a queen

$$P(A \cap B) = \frac{4}{52} = \frac{1}{13}$$

$$P(A) * P(B) = \frac{2}{13} * \frac{2}{13} = \frac{4}{139}$$

$$P(A) * P(B)! = \ P(A \cap B)$$

Therefore A and B are not independent events

(ii) In a pack of 52 cards there are 4 king,26 black

A=the card drawn is a black

$$P(A) = \frac{26}{52} = \frac{1}{2}$$

B=the card drawn is a king

$$P(A) = \frac{4}{52} = \frac{1}{13}$$

A∩B=Card is a black king

$$P(A \cap B) = \frac{2}{52} = \frac{1}{26}$$

$$P(A) * P(B) = \frac{1}{2} * \frac{1}{13} = \frac{1}{26}$$

$$P(A) * P(B) = \ P(A \cap B)$$

Therefore A and B are independent events

(iii) In a pack of 52 cards there are 13 spade,4 ace

A=the card drawn is a spade

$$P(A) = \frac{13}{52} = \frac{1}{4}$$

B=the card drawn is an ace

$$P(A) = \frac{4}{52} = \frac{1}{13}$$

A∩B=Card is an ace from spade

$$P(A \cap B) = \frac{1}{52}$$

$$P(A) * P(B) = \frac{1}{4} * \frac{1}{13} = \frac{1}{52}$$

$$P(A) * P(B) = P(A \cap B)$$

Therefore A and B are independent events

4. Question

A coin is tossed three times. Let the events A,B and C be defined as follows:

A=first toss is head, B=second toss is head, and C= exactly two heads are tossed in a row.

Check the independence of (i) A and B

(ii) B and C

(iii) C and A

Answer

Sample space S={(HHH),(HHT),(HTH),(HTT),(THH),(THT),(TTH),(TTT)}

A=first toss is head

A={HHH,HTH,HHT,HTT}

B=second toss is head

B={HHH,HHT,THT,THH}

$$P(A) = \frac{4}{8} = \frac{1}{2}$$

$$P(B) = \frac{4}{8} = \frac{1}{2}$$

A∩B={HHH,HHT}

$$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$$

$$P(A) * P(B) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$$

$$P(A) * P(B) = P(A \cap B)$$

Therefore A and B are independent events.

(ii) Sample space S={(HHH),(HHT),(HTH),(HTT),(THH),(THT),(TTH),(TTT)}

B=second toss is head

P(B)={HHH,HHT,THT,THH}

C= exactly two heads are tossed in a row.

P(C)={HHT,THH}

$$P(B) = \frac{4}{8} = \frac{1}{2}$$

$$P(C) = \frac{2}{8} = \frac{1}{4}$$

A∩B={HHT,THH}

$$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$$

$$P(A) * P(B) = \frac{1}{2} * \frac{1}{4} = \frac{1}{8}$$

$$P(A) * P(B) = !P(A \cap B)$$

Therefore B and C are not independent events.

(iii) Sample space S={(HHH),(HHT),(HTH),(HTT),(THH),(THT),(TTH),(TTT)}

C= exactly two heads are tossed in a row.

C={HHT,THH}

A=first toss is head

A={HHH,HTH,HHT,HTT}

$$P(C) = \frac{2}{8} = \frac{1}{4}$$

$$P(A) = \frac{4}{8} = \frac{1}{2}$$

A∩B={HHT}

$$P(A \cap B) = \frac{1}{8}$$

$$P(A) * P(B) = \frac{1}{4} * \frac{1}{2} = \frac{1}{8}$$

$$P(A) * P(B) = P(A \cap B)$$

Therefore A and C are independent events.

5. Question

If A and B be two events such that P(A) = 1/4, P (B) = 1/3 and P (A ∪ B) = 1/2, show that A and B are independent events.

Answer

P (A ∪ B)=P(A)+P(B)–P(A∩B)

$$\frac{1}{2} = \frac{1}{4} + \frac{1}{3} - P(A \cap B)$$

$$P(A \cap B) = \frac{1}{4} + \frac{1}{3} - \frac{1}{2}$$

$$= \frac{3 + 4 - 6}{12} = \frac{1}{12}$$

$$P(A) * P(B) = \frac{1}{4} * \frac{1}{3} = \frac{1}{12}$$

$$P(A) * P(B) = P(A \cap B)$$

Therefore A and B are independent events.

6. Question

Given two independent events A and B such that P (A)=0.3 and P(B) = 0.6 Find

i. P (A B)

ii. $P(A \cap \overline{B})$

iv. $P(\overline{A} \cap \overline{B})$

iv. P (A ∪ B)

v. P (A / B)

vi. P (B / A)

Answer

For independent event,

$P(A) * P(B) = P(A \cap B)$

P(A)*P(B)=0.3*0.6=0.18

ii. $P(A \cap \overline{B})$

$P(A \cap \overline{B}) = P(A) - P(A \cap B)$

$= 0.3 - 0.18 = 0.12$

iii. $P(\overline{A} \cap B)$

$P(\overline{A} \cap B) = P(B) - P(A \cap B)$

$= 0.6 - 0.18 = 0.42$

iv. $P(\overline{A} \cap \overline{B})$

$P(\overline{A} \cap \overline{B})$ =[(1–P(A)][1–P(B)] =[1–0.3][1–0.6]

=0.7*0.4=0.28

iv. P (A ∪ B)

P (A ∪ B)=P(A)+P(B)–P(A∩B)

=0.3+0.6–0.18=0.72

v. P (A / B)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.18}{0.6} = \frac{18}{60} = 0.3$$

vi. P (B / A)

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.18}{0.3} = \frac{18}{30} = 0.6$$

7. Question

If P (not B) = 0.65, P (A ∪ B) = 0.85, and A and B are independent events, then find p (A).

Answer

We know that,

P (A ∪ B)=P(A)+P(B)–P(A∩B)

P(not B)=1–P(B)

=1–0.65=0.35

Given that A and B are independent events,

$P(A) * P(B) = P(A \cap B)$

Therefore,

P (A ∪ B)=P(A)+P(B)–P(A∩B)

=P(A)+P(B)–[P(A)*P(B)]

P (A ∪ B)=P(A)[1–P(B)]+P(B)

0.85=P(A)*0.65+0.35

P(A)*0.65=0.50

$P(A) = \frac{0.5}{0.65} = 0.77$

8. Question

If A and B are two independent events such that $P(\overline{A} \cap B)$=2/15 and $P(A \cap \overline{B})$=1/6, then find P(B).

Answer

Let $\overline{A}$, $\overline{B}$ denote the complements of A, B respectively.

Given that, P(A∩$\overline{B}$))= $\frac{1}{6}$

P(A)P($\overline{B}$) $= \frac{1}{6}$

P(A)[1–P(B)] $= \frac{1}{6}$

P(A)=1/6[1–P(B)]

P(A'∩B)= $\frac{2}{15}$

P(A')P(B)= $\frac{2}{15}$

[1–P(A)]P(B))= $\frac{2}{15}$

[1–1/6{1–P(B)}]P(B))= $\frac{2}{15}$

[{6–6P(B)–1}/{6–6P(B)}]P(B))= $\frac{2}{15}$

15[5–6P(B)]P(B)=2[6–6P(B)]

15[5–6P(B)]P(B)=12[1–P(B)]

5[5–6P(B)]P(B)=4[1–P(B)]

25P(B)–30$[P(B)]^2$=4–4P(B)

–30$[P(B)]^2$+25P(B)+4P(B)–4=0

30$[P(B)]^2$–29P(B)+4=0

$30a^2$–29a+4=0 where P(B)=a

$30a^2$–24a–5a+4=0

6a(5a–4)–1(5a–4)=0

(6a–1)(5a–4)=0

6a–1=0

6a=1

a= $\frac{1}{6}$

$P(B) = \frac{1}{6}$

5a–4=0

5a=4

a== $\frac{4}{5}$

$P(B) = \frac{4}{5}$

Therefore, $P(B) = \frac{1}{6}, \frac{4}{5}$

9. Question

A and B are two independent events. The probability that A and B occur is 1/6 and the probability that neither of them occurs is 1/3. Find the probability of occurrence of two events.

Answer

P(A∩B)=$\frac{1}{6}$

$$P(\overline{A} \cap \overline{B}) = \frac{1}{3}$$

$$P(A \cap B) = P(A) * P(B) = \frac{1}{6}$$

$$P(B) = \frac{1}{6 * P(A)}$$

$$P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$$

$$\frac{1}{3} = 1 - P(A) - P(B) + \frac{1}{6}$$

$$P(A) + P(B) = 1 + \frac{1}{6} - \frac{1}{3} = \frac{5}{6}$$

$$P(A) + \frac{1}{6 * P(A)} = \frac{5}{6}$$

$6P(A)^2$–5P(A)+1=0

$6P(A)^2$–3P(A)–2P(A)+1=0

[2P(A)–1][3P(A)–1]=0

2P(A)–1=0

2P(A)=1

$$P(A) = \frac{1}{2}$$

3P(A)–1=0

3P(A)=1

$$P(A)=\frac{1}{3}$$

$$P(A)=\frac{1}{2}\text{ OR }\frac{1}{3}$$

10. Question

If A and B are two independent events such that P (A ∪ B) = 0.60 and P(A)= 0.2, find P(B).

Answer

Given that A and B are independent events,

$$P(A)*P(B)=\ P(A\cap B)$$

We know that,

P (A ∪ B)=P(A)+P(B)–P(A∩B)

Therefore,

P (A ∪ B)=P(A)+P(B)–P(A∩B)

=P(A)+P(B)–[P(A)*P(B)]

0.6=0.2+P(B)–0.2P(B)

0.6–0.2=P(B)[1–0.2]

0.4=P(B)*0.8

$$P(B)=\frac{0.4}{0.8}=\frac{1}{2}=0.5$$

11. Question

A die Is tossed twice. Find the probability of getting a number greater than 3 on each toss.

Answer

We are throwing two dice so,

Sample space S={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),

(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

Sample space contains 36 elements,

Let ,

A= getting a number greater than 3 on first toss

A={(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

$$P(A)=\frac{18}{36}=\frac{1}{2}$$

B= getting a number greater than 3 on second toss

B={(1,4),(1,5),(1,6), (2,4),(2,5),(2,6), (3,4),(3,5),(3,6), (4,4),(4,5),(4,6), (5,4),(5,5),(5,6), (6,4),(6,5),(6,6)}

$$P(B) = \frac{18}{36} = \frac{1}{2}$$

P(A∩B)=P(getting a number on each toss)

Given that A and B are independent events,

$$P(A) * P(B) = P(A \cap B)$$

$$P(A \cap B) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$$

12. Question

Given the probability that A can solve a problem is 2/3 and the probability that B can solve the same problem is 3/5. /find the probability that none of the two will be able to solve the problem.

Answer

Given that P(A can solve problem)=$\frac{2}{3}$

$$P(A) = \frac{2}{3}$$

$$P(\overline{A}) = 1 - P(A) = 1 - \frac{2}{3} = \frac{1}{3}$$

P(B can solve the problem)=$\frac{3}{5}$

$$P(B) = \frac{3}{5}$$

$$P(\overline{B}) = 1 - P(B) = 1 - \frac{3}{5} = \frac{2}{5}$$

P(none can solve the problem)=$P(\overline{A} \cap \overline{B}) = P(\overline{A}) * P(\overline{B}) = \frac{1}{3} * \frac{2}{5} = \frac{2}{15}$

Therefore, required probability=$\frac{2}{15}$

13. Question

An unbiased die is tossed twice. Find the probability of getting 4,5, or 6 on the first toss and 1,2,3 or 4 on the second toss.

Answer

Given an unbiased die is tossed twice so the sample space contain 36 elements.

Let A be the probability of getting 4,5,6 on first toss

A={(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

$$P(A) = \frac{18}{36} = \frac{1}{2}$$

Let B be the probability of getting 1,2,3,4 on the second toss

B={1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4), (3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4),(5,1),(5,2),(5,3),(5,4)

(6,1),(6,2),(6,3),(6,4)}

$$P(B) = \frac{24}{36} = \frac{2}{3}$$

P(getting 4,5,6 on the first toss and 1,2,3 or 4 on the second toss)

$$P(A \cap B) = P(A) * P(B)$$

$$= \frac{1}{2} * \frac{2}{3} = \frac{1}{3}$$

14. Question

A bag contains 3 red and 2 black balls. One ball Is drawn from it at random. Its colour is noted and then it is put back in the bag. A second draw is made and the same procedure is repeated. Find the probability of drawing (i) tow red balls, (ii) two black balls, (iii) first red and second black ball.

Answer

Given, bag contains 3 red and 2 black balls.

Probability of getting red ball=$P(A) = \frac{3}{5}$

Probability of getting black ball=$P(B) = \frac{2}{5}$

(i) two red balls

P(getting two getting red balls)=P(A)*P(A)

$$= \frac{3}{5} * \frac{3}{5} = \frac{9}{25}$$

(ii) two black balls

P(getting two black balls)=P(B)*P(B)

$$= \frac{2}{5} * \frac{2}{5} = \frac{4}{25}$$

(iii) first red and second black ball.

P(getting first red and second black ball)=P(A)*P(B)

$$= \frac{3}{5} * \frac{2}{5} = \frac{6}{25}$$

15. Question

Three cards are drawn with replacement from a well shuffled pack of cards. Find the probability that the cards drawn are king, queen and jack.

Answer

In a pack of 52 cards, there are 4 king,4 queen and 4 jack

Let A be drawn card is a king

$$P(A) = \frac{4}{52} = \frac{1}{13}$$

Let B be drawn card is a queen

$$P(B) = \frac{4}{52} = \frac{1}{13}$$

Let C be drawn card is a jack

$P(C) = \frac{4}{52} = \frac{1}{13}$

P(cards drawn are king, queen, jack)=
$P(A \cap B \cap C) + P(A \cap C \cap B) + P(B \cap A \cap C) + P(B \cap C \cap A) + P(C \cap B \cap A) + P(C \cap A \cap B)$

=P(A)P(B)P(C)+P(A)P(C)P(B)+P(B)P(A)P(C)+P(B)P(C)P(A)+P(C)P(A)P(B)+P(C)P(B)P(A)

=6*P(A)P(B)P(C)= $6 * \left(\frac{1}{13} * \frac{1}{13} * \frac{1}{13}\right) = \frac{6}{2197}$

16. Question

An article manufactured by a company consists of two parts X and Y. In the process of manufacture of the part X, 9 out of 100 parts may be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of part Y. Calculate the probability that the assembled product will not be defective.

Answer

Given,

An article manufactured by a company consists of two parts X and Y.

Part X has 9 out of 100 defective

That is, Part X has 91 out of 100 non defective

Part Y has 5 out of 100 defective

That is, Part Y has 95 out of 100 non defective

Consider,

X=a non defective part of X

Y=a non defective part of Y

$P(X) = \frac{91}{100}$ $P(Y) = \frac{95}{100}$

P(assembled product will not be defective)

P(neither X defective nor Y defective)

=P(X∩Y)

=P(X)P(Y)

$= \frac{91}{100} * \frac{95}{100} = 0.8645$

17. Question

The probability that A hits a target is 1/3 and the probability that B hits it, is 2/5. What is the probability that the target will be hit, if each one of A and B shoots at the target?

Answer

Given,

Probability that A hits a target is 1/3

P(A)=$\frac{1}{3}$

The probability that B hits it, is 2/5

P(b)=$\frac{2}{5}$

P(target will be hit)=1–P(target will not be hit)

=1–P(neither A nor B hit the target)

$= 1 - P(\bar{A} \cap \bar{B})$

$= 1 - P(\bar{A})P(\bar{B})$

$= 1 - [(1 - P(A))(1 - P(B))$

$= 1 - \left[1 - \frac{1}{3}\right]\left[1 - \frac{2}{5}\right]$

$= 1 - \frac{2}{3} * \frac{3}{5} = 1 - \frac{2}{5} = \frac{3}{5}$

18. Question

An anti–aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane?

Answer

Given that an anti–aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it.

Let

A=hitting the plane at first shot

P(A)=0.4

B=hitting the plane at second shot

P(B)=0.3

C=hitting the plane at third shot

P(C)=0.2

D=hitting the plane at fourth shot

P(D)=0.1

P(gun hits the plane)=1–P(gun does not hit the plane)

=1–P(none of the four shots hit the plane)

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})$

$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})P(\bar{D})$

$= 1 - [(1 - P(A))(1 - P(B))(1 - P(C))(1 - P(D))]$

=1–[(1–0.4)(1–0.3)(1–0.2)(1–0.1)]

=1–(0.6)(0.7)(0.8)(0.9)

=1–0.3024=0.6976

19. Question

The odds against a certain event are 5 to 2 and the odds in favour of another event, independent to the former are 6 to 5. Find the probability that (i) at least one of the events will occur, and (ii) none of the events will occur.

Answer

(i) The odds against certain event are 5 to 2

$P(\bar{A}) = \frac{5}{5 + 2} = \frac{5}{7}$

odds in favour of another event are 6 to5

$$P(B) = \frac{6}{6+5} = \frac{6}{11}$$

$$P(\overline{B}) = 1 - \frac{6}{11} = \frac{5}{11}$$

Find the probability that (i) at least one of the events will occur

=1–P(none of the events will occur)

$$= 1 - P(\overline{A} \cap \overline{B}) = 1 - P(\overline{A})P(\overline{B})$$

$$= 1 - \frac{5}{7} * \frac{5}{11} = 1 - \frac{25}{77} = \frac{52}{77}$$

(ii) none of the events will occur.

P(none of the events will occur)

$$= P(\overline{A} \cap \overline{B}) = P(\overline{A})P(\overline{B})$$

$$= \frac{5}{7} * \frac{5}{11} = \frac{25}{77}$$

20. Question

A die is thrown thrice. Find the probability of getting an odd number at least once.

Answer

The sample space has 216 outcomes.

A be getting an odd number in a throw of die

$$P(A) = \frac{3}{6} = \frac{1}{2} \; P(\overline{A}) = \frac{1}{2}$$

P(getting an odd number at least once)=1–P(getting no odd number)

$$= 1 - P(\overline{A} \cap \overline{A} \cap \overline{A})$$

$$= 1 - P(\overline{A})P(\overline{A})P(\overline{A})$$

$$= 1 - \frac{1}{2} * \frac{1}{2} * \frac{1}{2} = 1 - \frac{1}{8} = \frac{7}{8}$$

21. Question

Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that (i) both balls are red, (ii) first balls is black and second is red, (iii) one of them is black and other is red.

Answer

Given an urn contains 10 black balls and 8 red balls

Probability of getting red ball=$P(R) = \frac{8}{18}$

Probability of getting black ball=$P(B) = \frac{10}{18}$

(i) both balls are red

P(getting two getting red balls)=P(R)*P(R)

$$= \frac{8}{18} * \frac{8}{18} = \frac{64}{324} = \frac{16}{81}$$

(ii) first balls is black and second is red

P(first balls is black and second is red)=P(B)*P(R)= $\frac{10}{18} * \frac{8}{18} = \frac{80}{324} = \frac{20}{81}$

(iv) one of them is black and other is red.

P(one of them is black and other is red)=P(B)P(R)+P(R)P(B)

$$= \frac{10}{18} * \frac{8}{18} + \frac{8}{18} * \frac{10}{18} = 2 * \left(\frac{20}{81}\right) = \frac{40}{81}$$

22. Question

An urn contains 4 red and 7 black balls. Two balls are drawn at random with replacement. Find the probability of getting (i) 2 red balls, (ii) 2 black balls, (iii) one red and one black ball.

Answer

Given an urn contains 4 red and 7 black balls

Probability of getting red ball=$P(R) = \frac{4}{11}$

Probability of getting black ball=$P(B) = \frac{7}{11}$

(i) two red balls

P(getting two getting red balls)=P(R)*P(R)

$$= \frac{4}{11} * \frac{4}{11} = \frac{16}{121}$$

(ii) two black balls

P(getting two blue balls)=P(B)*P(B)

$$= \frac{7}{11} * \frac{7}{11} = \frac{49}{121}$$

(iii) first red and second black ball.

P(getting one red and one black ball)=P(R)*P(B)+P(B)*P(R)

$$= \frac{4}{11} * \frac{7}{11} + \frac{7}{11} * \frac{4}{11} = \frac{28}{121} + \frac{28}{121} = \frac{56}{121}$$

23. Question

The probabilities of two students A and B coming to the school in time are 3/7 and 5/7 respectively. Assuming that the events, ‘A coming in time’ and ‘B coming in time’ are independent, find the probability of only one of them coming to the school in time. Write at least one advantage of coming to school in time.

Answer

Given that A coming in time and B coming in time are independent.

Let A denote A coming in time

B denote B coming in time

$\overline{A}$ denotes complementary event of A

$$P\overline{(A)} = P(1 - A) = 1 - \frac{3}{7} = \frac{4}{7}$$

$\overline{B}$ denotes complementary event of B

$$P\overline{(B)} = P(1 - B) = 1 - \frac{2}{7} = \frac{5}{7}$$

P(only one coming in time)$= \mathrm{P}(\mathrm{A} \cap \overline{\mathrm{B}}) + \mathrm{P}(\overline{\mathrm{A}} \cap \mathrm{B})$

$= \mathrm{P}(\mathrm{A}) * \mathrm{P}(\overline{\mathrm{B}}) + \mathrm{P}(\overline{\mathrm{A}}) * \mathrm{P}(\mathrm{B})$

$= \frac{3}{7} * \frac{2}{7} + \frac{4}{7} * \frac{5}{7} = \frac{6}{49} + \frac{20}{49} = \frac{26}{49}$

24. Question

Two dice are thrown together and the total score is noted. The event E, F and G are “a total 4”, “a total of 9 or more”, and “a total divisible by 5”, respectively. Calculate P(E), P(F) and P(G) and decide which pairs of events, if any, are independent.

Answer

We are throwing two dice so,

Sample space S={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),

(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

Sample space contains 36 elements,

n(S)=36

let E be an event of getting sum 4

E={(1,3),(3,1),(2,2)}

n(E)=3

$\mathrm{P}(\mathrm{E}) = \frac{\mathrm{n}(\mathrm{E})}{\mathrm{n}(\mathrm{S})} = \frac{3}{36} = \frac{1}{12}$

F be the event of getting 9 or more

F ={(3,6),(6,3),(4,5),(5,4),(4,6),(6,4),(5,5),(5,6),(6,5),(6,6)}

n(F)=3

$\mathrm{P}(\mathrm{F}) = \frac{\mathrm{n}(\mathrm{F})}{\mathrm{n}(\mathrm{S})} = \frac{10}{36} = \frac{5}{18}$

G be the event getting a total divisible by 5

G= {(1,4),(4,1),(2,3),(3,2),(4,6),(6,4),(5,5)}

n(G)=7

$\mathrm{P}(\mathrm{G}) = \frac{\mathrm{n}(\mathrm{G})}{\mathrm{n}(\mathrm{S})} = \frac{7}{36}$

$\mathrm{P}(\mathrm{E}) \cap \mathrm{P}(\mathrm{F}) = \emptyset\ != \mathrm{P}(\mathrm{E}) * \mathrm{P}(\mathrm{F})$ So E and F are not independent

$\mathrm{P}(\mathrm{E}) \cap \mathrm{P}(\mathrm{G}) = \emptyset! = \mathrm{P}(\mathrm{E}) * \mathrm{P}(\mathrm{G})$ So E and G are not independent

$\mathrm{P}(\mathrm{F}) \cap \mathrm{P}(\mathrm{G}) = \frac{1}{36}$

$\mathrm{P}(\mathrm{F}) * \mathrm{P}(\mathrm{G}) = \frac{5}{18} * \frac{7}{36} = \frac{35}{648}$

$P(F) \cap P(G) != P(F) * P(G)$ So F and G are not independent

Therefore no pair is independent

25. Question

Let A and B be two independent events such that P(A) = p_1 and P(B) = p_2. Describe in words the events whose probabilities are:

i. p_1p_2

ii. $(1–p_1)\ p_2$

iii. 1–(1–p1)(1–p2)

iv. $p_1+p_2 = 2p_1p_2$

Answer

If the events are said to be independent, if the occurrence or non occurrence of one does not affect the probability of the occurrence or non occurrence of other.

i. p_1p_2

p_1p_2=P(A)P(B)

Both events A and B will occur

ii. $(1–p_1)\ p_2$

$(1–p_1)\ p_2$=[1–P(A)]P(B)

$= P(\overline{A})P(B)$

Event A does not occur but event B occur.

iii. 1–(1–p1)(1–p2)

1–(1–p1)(1–p2)=[1–(1–P(A))(1–P(B))]

$= 1 - P(\overline{A})P(\overline{B})$

At least one of the event will occur

iv. $p_1+p_2 = 2p_1p_2$

$p_1+p_2 = 2p_1p_2$

P(A)+P(B)=2P(A)P(B)

P(A)+P(B)–2P(A)P(B)=0

P(A)–P(A)P(B)+P(B)–P(A)P(B)=0

P(A)[1–P(B)]+P(B)[1–P(A)]=0

$P(A)P(\overline{B)} + P(B)P(\overline{A}) = 0$

$P(A)P(\overline{B)} = P(B)P(\overline{A})$

Exactly one of A and B occurs

Exercise 31.5

1. Question

A bag contains 6 black and 3 white balls. Another bag contains 5 black and 4 white balls. If one ball is drawn from each bag, find the probability that these two balls are of the same colour.

Answer

Given:

⇒ Bag A contains 6 black balls and 3 white balls

⇒ Bag B contains 5 black balls and 4 white balls

It is told that one ball is drawn is drawn from is each bag.

We need to find the probability that the balls are of the same colour.

Let us find the Probability of drawing each colour ball from the bag.

⇒ $P(B_1)$ = P(drawing black ball from bag A)

$$\Rightarrow P(B_1) = \frac{\text{no.of ways to draw a black ball from Bag A}}{\text{no.of ways to draw a ball from Bag A}}$$

$$\Rightarrow P(B_1) = \frac{6}{6+3}$$

$$\Rightarrow P(B_1) = \frac{6}{9} = \frac{2}{3}$$

⇒ $P(W_1)$ = P(drawing white ball from bag A)

$$\Rightarrow P(W_1) = \frac{\text{no.of ways to draw a white ball from Bag A}}{\text{no.of ways to draw a ball from Bag A}}$$

$$\Rightarrow P(W_1) = \frac{3}{6+3}$$

$$\Rightarrow P(W_1) = \frac{3}{9} = \frac{1}{3}$$

⇒ $P(B_2)$ = P(drawing black ball from bag B)

$$\Rightarrow P(B_2) = \frac{\text{no.of ways to draw a black ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$$

$$\Rightarrow P(B_2) = \frac{5}{5+4}$$

$$\Rightarrow P(B_2) = \frac{5}{9}$$

⇒ $P(W_2)$ = P(drawing white ball from bag B)

$$\Rightarrow P(W_2) = \frac{\text{no.of ways to draw a white ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$$

$$\Rightarrow P(W_2) = \frac{4}{5+4}$$

$$\Rightarrow P(W_2) = \frac{4}{9}$$

We need to find the probability of drawing the same colour balls from two bags

⇒ P(S) = P(drawing two balls of same colours) = P(drawing black balls from each bag) + (P(drawing white balls from each bag)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$$\Rightarrow P(S) = \left(P(B_1)P(B_2)\right) + \left(P(W_1)P(W_2)\right)$$

$$\Rightarrow P(S) = \left(\frac{2}{3} \times \frac{5}{9}\right) + \left(\frac{1}{3} \times \frac{4}{9}\right)$$

$$\Rightarrow P(S) = \frac{10}{27} + \frac{4}{27}$$

$$\Rightarrow P(S) = \frac{14}{27}.$$

∴ The required probability is $\frac{14}{27}$.

2. Question

A bag contains 3 red and 5 black balls and a second bag contains 6 red and 4 black balls. A ball is drawn from each bag. Find the probability that one is red and the other is black.

Answer

Given:

⇒ Bag A contains 3 red balls and 5 black balls

⇒ Bag B contains 6 red balls and 4 black balls

It is told that one ball is drawn is drawn from is each bag.

We need to find the probability that one ball is red and other is black.

Let us find the Probability of drawing each colour ball from the bag.

⇒ $P(B_1)$ = P(drawing black ball from bag A)

$\Rightarrow P(B_1) = \frac{\text{no.of ways to draw a black ball from Bag A}}{\text{no.of ways to draw a ball from Bag A}}$

$\Rightarrow P(B_1) = \frac{5}{5+3}$

$\Rightarrow P(B_1) = \frac{5}{8}$

⇒ $P(R_1)$ = P(drawing Red ball from bag A)

$\Rightarrow P(R_1) = \frac{\text{no.of ways to draw a Red ball from Bag A}}{\text{no.of ways to draw a ball from Bag A}}$

$\Rightarrow P(R_1) = \frac{3}{5+3}$

$\Rightarrow P(R_1) = \frac{3}{8}$

⇒ $P(B_2)$ = P(drawing black ball from bag B)

$\Rightarrow P(B_2) = \frac{\text{no.of ways to draw a black ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$

$\Rightarrow P(B_2) = \frac{4}{6+4}$

$\Rightarrow P(B_2) = \frac{4}{10} = \frac{2}{5}$

⇒ $P(R_2)$ = P(drawing Red ball from bag B)

$\Rightarrow P(R_2) = \frac{\text{no.of ways to draw a Red ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$

$\Rightarrow P(R_2) = \frac{6}{6+4}$

$\Rightarrow P(R_2) = \frac{6}{10} = \frac{3}{5}$

We need to find the probability of drawing the different colour balls from two bags

⇒ P(S) = P(drawing one red ball and one Black ball)

⇒ P(S) = P(drawing black balls from bag A and red ball from bag B) + P(drawing black balls from bag B and red ball from bag A)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(S) = \left(P(B_1)P(R_2)\right) + \left(P(R_1)P(B_2)\right)$

$\Rightarrow P(S) = \left(\frac{5}{8} \times \frac{3}{5}\right) + \left(\frac{3}{8} \times \frac{2}{5}\right)$

$\Rightarrow P(S) = \frac{15}{40} + \frac{6}{40}$

$\Rightarrow P(S) = \frac{21}{40}$.

∴ The required probability is $\frac{21}{40}$.

3. Question

Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that (i) both the balls are red. (ii) first ball is black and second is red. (iii) one of them is black and other is red.

Answer

Given:

⇒ Bag contains 10 black balls and 8 red balls

It is told that two balls are drawn from bag with replacement.

Let us find the Probability of drawing each colour ball from the bag.

⇒ $P(B_1)$ = P(drawing black ball from bag)

$\Rightarrow P(B_1) = \frac{\text{no.of ways to draw a black ball from Bag}}{\text{no.of ways to draw a ball from Bag}}$

$\Rightarrow P(B_1) = \frac{10}{10+8}$

$\Rightarrow P(B_1) = \frac{10}{18} = \frac{5}{9}$

⇒ $P(R_1)$ = P(drawing red ball from bag)

$\Rightarrow P(R_1) = \frac{\text{no.of ways to draw a red ball from Bag}}{\text{no.of ways to draw a ball from Bag}}$

$\Rightarrow P(R_1) = \frac{8}{10+8}$

$\Rightarrow P(R_1) = \frac{8}{18} = \frac{4}{9}$

We need to find:

(i) $P(D_{rr})$ = P(both balls drawn are red)

(ii) $P(D_{br})$ = P(first drawn is black and next is red)

(iii) $P(S_{rd})$ = P(one ball is red and other is black)

⇒ $P(D_{rr})$ = P(both balls drawn are red)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(D_{rr}) = \left(P(R_1)P(R_1)\right)$

$\Rightarrow P(D_{rr}) = \left(\frac{4}{9} \times \frac{4}{9}\right)$

$\Rightarrow P(D_{rr}) = \frac{16}{81}$

⇒ $P(D_{br})$ = P(first drawn is black and next is red)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(D_{br}) = (P(B_1)P(R_1))$

$\Rightarrow P(D_{br}) = \left(\frac{5}{9} \times \frac{4}{9}\right)$

$\Rightarrow P(D_{br}) = \frac{20}{81}$

$\Rightarrow$ P(S_{rb}) = P(one ball is red and other is black)

$\Rightarrow$ P(S_{rb}) = P(first drawn is red and next is black) + P(first drawn black and next is red)

$\Rightarrow$ P(S_{rb}) = P(D_{rb}) + P(D_{br})

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(S_{rb}) = (P(R_1)P(B_1)) + (P(B_1)P(R_1))$

$\Rightarrow P(S_{rb}) = \left(\frac{4}{9} \times \frac{5}{9}\right) + \left(\frac{5}{9} \times \frac{4}{9}\right)$

$\Rightarrow P(S_{rb}) = \frac{20}{81} + \frac{20}{81}$

$\Rightarrow P(S_{rb}) = \frac{40}{81}$

$\therefore$ The required probabilities are $\frac{16}{81}, \frac{20}{81}, \frac{40}{81}$.

4. Question

Two cards are drawn successively without replacement from a well-shuffled deck of 52 cards. Find the probability of exactly one ace.

Answer

Given that two cards are drawn from a well-shuffled deck of 52 cards.

We know that there will 4 aces present in a deck.

It is told that two cards successively without replacement.

Let us find the probability of drawing the cards.

$\Rightarrow$ P(A_1) = P(Drawing ace from 52 cards deck)

$\Rightarrow P(A_1) = \frac{\text{No.of ways of drawing a ace from deck}}{\text{No.of ways of drawing a card from deck}}$

$\Rightarrow P(A_1) = \frac{4}{52}$

$\Rightarrow$ P(O_1) = P(Drawing cards other than ace from 52 cards deck)

$\Rightarrow P(O_1) = \frac{\text{No.of ways of drawing card other than ace from deck}}{\text{No.of ways of drawing a card from deck}}$

$\Rightarrow P(O_1) = \frac{48}{52}$

$\Rightarrow$ P(A_2) = P(Drawing ace from remaining 51 cards deck)

$\Rightarrow P(A_2) = \frac{\text{No.of ways of drawing a ace from 51 cards deck}}{\text{No.of ways of drawing a card from 51 cards deck}}$

$\Rightarrow P(A_2) = \frac{4}{51}$

$\Rightarrow$ P(O_2) = P(Drawing a card other than ace from remaining 51 cards deck)

$\Rightarrow P(O_2) = \frac{\text{No.of ways of drawing a card other than ace from 51 cards deck}}{\text{No.of ways of drawing a card from 51 cards deck}}$

$\Rightarrow P(O_2) = \frac{48}{51}$

We need to find the probability of drawing exactly one ace

$\Rightarrow P(D_A)$ = P(Drawing exactly 1 ace in the drawn two cards)

$\Rightarrow P(D_A)$ = P(Drawing Ace first and others next) + (P(Drawing Other cards first and ace next)

Since drawing cards are independent their probabilities multiply each other,

$\Rightarrow P(D_A) = (P(A_1)P(O_2)) + (P(O_1)P(A_2))$

$\Rightarrow P(D_A) = \left(\frac{4}{52} \times \frac{48}{51}\right) + \left(\frac{48}{52} \times \frac{4}{51}\right)$

$\Rightarrow P(D_A) = \frac{96}{663}$

$\Rightarrow P(D_A) = \frac{32}{221}$

∴ The required probability is $\frac{32}{221}$.

5. Question

A speaks truth in 75% and B in 80% of the cases. In what percentage of cases are they likely to contradict each other in narrating the same incident?

Answer

Given:

⇒ A speaks truth in 75% of cases

⇒ B speaks truth in 80% of cases

Now,

$\Rightarrow P(T_A)$ = P(A speaking truth) = 0.75

$\Rightarrow P(N_A)$ = P(A not speaking truth) = 1-0.75

$\Rightarrow P(N_A) = 0.25$

$\Rightarrow P(T_B)$ = P(B speaking truth) = 0.80

$\Rightarrow P(N_B)$ = P(B not speaking truth) = 1-0.80

$\Rightarrow P(N_B) = 0.20$

We need to find the probability for case in which A and B contradict each other for narrating an incident.

This happens only when A not telling truth while B is telling truth and vice-versa.

$\Rightarrow P(C_{AB})$ = P(A and B contradict each other)

$\Rightarrow P(C_{AB})$ = P(A tells truth and B doesn't) + P(B tells truth and A doesn't)

Since the speaking of A and B are independent events their probabilities will multiply each other.

$\Rightarrow P(C_{AB}) = (P(T_A)P(N_B)) + (P(N_A)P(T_B))$

$\Rightarrow P(C_{AB}) = (0.75 \times 0.20) + (0.25 \times 0.80)$

$\Rightarrow P(C_{AB}) = 0.15 + 0.20$

$\Rightarrow P(C_{AB}) = 0.35$

∴ The required probability is 0.35.

6. Question

Kamal and Monica appeared for an interview for two vacancies. The probability of Kamal’s selection is 1/3 and that of Monika’s selection is 1/5. Find the probability that

i. Both of them will be selected

ii. None of them will be selected

iii. At least one of them will be selected

iv. Only one of them will be selected.

Answer

Given that,

⇒ $P(K_S)$ = P(Kamal’s selection)

⇒ $P(K_S) = \frac{1}{3}$

⇒ $P(K_N)$ = P(Not selecting Kamal)

⇒ $P(K_N) = 1 - \frac{1}{3}$

⇒ $P(K_N) = \frac{2}{3}$

⇒ $P(V_S)$ = P(Monika’s selection)

⇒ $P(V_S) = \frac{1}{5}$

⇒ $P(V_N)$ = P(Not selecting Monika)

⇒ $P(V_N) = 1 - \frac{1}{5}$

⇒ $P(V_N) = \frac{4}{5}$

We need to find:

i. Both of them will be selected

ii. None of them will be selected

iii. At least one of them will be selected

iv. Only one of them will be selected.

⇒ $P(S_{both})$ = P(Both of them are selected)

Since selection of each person is an independent event their probabilities multiply each other

⇒ $P(S_{both}) = P(K_S)P(V_S)$

⇒ $P(S_{both}) = \frac{1}{3} \times \frac{1}{5}$

⇒ $P(S_{both}) = \frac{1}{15}$

⇒ $P(S_{none})$ = P(None of them are selected)

Since selection of each person is an independent event their probabilities multiply each other

⇒ $P(S_{none}) = P(K_N)P(V_N)$

⇒ $P(S_{none}) = \frac{2}{3} \times \frac{4}{5}$

⇒ $P(S_{none}) = \frac{8}{15}$

⇒ P(S_{atone}) = P(Selecting at least one of them)

⇒ P(S_{atone}) = P(selecting only Kamal) + P(selecting only Monika) + P(Selecting both)

Since selection of each person is an independent event their probabilities multiply each other

⇒ $P(S_{atone}) = \left(P(K_S)P(V_N)\right) + \left(P(K_N)P(V_S)\right) + \left(P(K_S)P(V_S)\right)$

⇒ $P(S_{atone}) = \left(\frac{1}{3}\times\frac{4}{5}\right) + \left(\frac{2}{3}\times\frac{1}{5}\right) + \left(\frac{1}{3}\times\frac{1}{5}\right)$

⇒ $P(S_{atone}) = \frac{4}{15} + \frac{2}{15} + \frac{1}{15}$

⇒ $P(S_{atone}) = \frac{7}{15}$

⇒ P(S_{one}) = P(Only one of them is selected)

⇒ P(S_{one}) = P(only Kamal is selected) + P(only Monika is selected)

Since selection of each person is an independent event their probabilities multiply each other

⇒ $P(S_{one}) = \left(P(K_S)P(V_N)\right) + \left(P(K_N)P(V_S)\right)$

⇒ $P(S_{one}) = \left(\frac{1}{3}\times\frac{4}{5}\right) + \left(\frac{2}{3}\times\frac{1}{5}\right)$

⇒ $P(S_{one}) = \frac{6}{15} = \frac{2}{5}$

∴ The required probabilities are $\frac{1}{15}, \frac{8}{15}, \frac{7}{15}, \frac{2}{5}$.

7. Question

A bag contains 3 white, 4 red and 5 black balls. Two balls are drawn one after the other, without replacement. What is the probability that one is white and the other is black?

Answer

Given:

⇒ Bag contains 3 white, 4 red and 5 black balls.

It is told that two balls are drawn from bag without replacement.

Let us find the Probability of drawing each colour ball from the bag.

⇒ P(B_1) = P(drawing black ball from bag on first draw)

⇒ $P(B_1) = \frac{\text{no.of ways to draw a black ball from Bag}}{\text{no.of ways to draw a ball from Bag}}$

⇒ $P(B_1) = \frac{5}{3+4+5}$

⇒ $P(B_1) = \frac{5}{12}$

⇒ P(W_1) = P(drawing white ball from bag on first draw)

⇒ $P(W_1) = \frac{\text{no.of ways to draw a white ball from Bag}}{\text{no.of ways to draw a ball from Bag}}$

⇒ $P(W_1) = \frac{3}{3+4+5}$

⇒ $P(W_1) = \frac{3}{12}$

⇒ P(B_2) = P(drawing black ball from bag on second draw)

⇒ $P(B_2) = \frac{\text{no.of ways to draw a black ball from Bag after drawing a ball}}{\text{no.of ways to draw a ball from Bag after drawing a ball}}$

$\Rightarrow P(B_2) = \frac{5}{11}$

$\Rightarrow P(W_2)$ = P(drawing white ball from bag on second draw)

$\Rightarrow P(W_2) = \frac{\text{no.of ways to draw a white ball from Bag after drawing a ball}}{\text{no.of ways to draw a ball from Bag after drawing a ball}}$

$\Rightarrow P(W_2) = \frac{3}{11}$

We need to find:

$\Rightarrow P(S_{WB})$ = P(one ball is White and other is black)

$\Rightarrow P(S_{WB})$ = P(first drawn is White and next is black) + P(first drawn black and next is white)

$\Rightarrow P(S_{WB}) = P(D_{WB}) + P(D_{BW})$

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(S_{WB}) = \left(P(W_1)P(B_2)\right) + \left(P(B_1)P(W_2)\right)$

$\Rightarrow P(S_{WB}) = \left(\frac{3}{12} \times \frac{5}{11}\right) + \left(\frac{5}{12} \times \frac{3}{11}\right)$

$\Rightarrow P(S_{WB}) = \frac{15}{132} + \frac{15}{132}$

$\Rightarrow P(S_{WB}) = \frac{30}{132}$

$\Rightarrow P(S_{WB}) = \frac{5}{22}$

∴ The required probability is $\frac{5}{22}$.

8. Question

A bag contains 8 red and 6 green balls. Three balls are drawn one after another without replacement. Find the probability that at least two balls drawn are green.

Answer

Given:

Bag contains 8 red and 6 green balls

It is told that three balls are drawn without replacement.

We need to find the probability that at least two balls drawn are green.

The possible cases of drawing balls are as shown below:

i. Green,Green,Red

ii. Green,Red,Green

iii. Red,Green,Green

iv. Green,Green,Green

$\Rightarrow P(D_{at2greens})$ = P(GGR) + P(GRG) + P(RGG) + P(GGG)

Since drawing balls are independent events so, the probabilities multiply each other.

$\Rightarrow P(D_{at2greens}) = \left(P(G)P(G)P(R)\right) + \left(P(G)P(R)P(G)\right) + \left(P(R)P(G)P(G)\right) + \left(P(G)P(G)P(G)\right)$

$\Rightarrow P(D_{at2greens}) = \left(\frac{6}{14} \times \frac{5}{13} \times \frac{8}{12}\right) + \left(\frac{6}{14} \times \frac{8}{13} \times \frac{5}{12}\right) + \left(\frac{8}{14} \times \frac{6}{13} \times \frac{5}{12}\right) + \left(\frac{6}{14} \times \frac{5}{13} \times \frac{4}{12}\right)$

$\Rightarrow P\left(D_{at2greens}\right) = \frac{(240+240+240+120)}{2184}$

$\Rightarrow P\left(D_{at2greens}\right) = \frac{840}{2184}$

$\Rightarrow P\left(D_{at2greens}\right) = \frac{5}{13}$

∴ The required probability is $\frac{5}{13}$.

9. Question

Arun and Tarun appeared for an interview for two vacancies. The probability of Arun's selections is 1/4 and that of Tarun's rejection is 2/3. Find the probability that at least one of them will be selected.

Answer

Given that,

$\Rightarrow P(A_S)$ = P(Arun's selection)

$\Rightarrow P(A_S) = \frac{1}{4}$

$\Rightarrow P(A_N)$ = P(Arun's rejection)

$\Rightarrow P(A_N) = 1 - \frac{1}{4}$

$\Rightarrow P(A_N) = \frac{3}{4}$

$\Rightarrow P(T_N)$ = P(Tarun's rejection)

$\Rightarrow P(T_N) = \frac{2}{3}$

$\Rightarrow P(T_S)$ = P(Tarun's selection)

$\Rightarrow P(T_S) = 1 - \frac{2}{3}$

$\Rightarrow P(T_S) = \frac{1}{3}$

We need to find the probability that at least one of them will be selected

$\Rightarrow P(S_{atone})$ = P(Selecting at least one of them)

$\Rightarrow P(S_{atone})$ = P(selecting only Arun) + P(selecting only Tarun) + P(Selecting both)

Since selection of each person is an independent event their probabilities multiply each other

$\Rightarrow P(S_{atone}) = \left(P(A_S)P(T_N)\right) + \left(P(A_N)P(T_S)\right) + \left(P(A_S)P(T_S)\right)$

$\Rightarrow P(S_{atone}) = \left(\frac{1}{4} \times \frac{2}{3}\right) + \left(\frac{3}{4} \times \frac{1}{3}\right) + \left(\frac{1}{4} \times \frac{1}{3}\right)$

$\Rightarrow P(S_{atone}) = \frac{2}{12} + \frac{3}{12} + \frac{1}{12}$

$\Rightarrow P(S_{atone}) = \frac{6}{12} = \frac{1}{2}$

∴ The required probability is $\frac{1}{2}$.

10. Question

A and B toss a coin alternately till one of them gets a head and wins the game. If A starts the game, find the probability that B will win the game.

Answer

Given that A and B toss a coin until one of them gets a head to win the game.

Let us find the probability of getting the head.

⇒ $P(A_H)$ = P(A getting a head on tossing a coin)

$\Rightarrow P(A_H) = \frac{1}{2}$

⇒ $P(A_N)$ = P(A not getting head on tossing a coin)

$\Rightarrow P(A_N) = \frac{1}{2}$

⇒ $P(B_H)$ = P(B getting a head on tossing a coin)

$\Rightarrow P(B_H) = \frac{1}{2}$

⇒ $P(B_N)$ = P(B not getting head on tossing a coin)

$\Rightarrow P(B_N) = \frac{1}{2}$

It is told that A starts the game.

We need to find the probability that B wins the games.

B wins the game only when A losses in$1^{st}, 3^{rd}, 5^{th}$,...... tosses.

This can be shown as follows:

⇒ $P(W_B)$ = P(B wins the game)

$\Rightarrow P(W_B) = P(A_N B_H) + P(A_N B_N A_N B_H) + P(A_N B_N A_N B_N A_N B_H) + \ldots\ldots\ldots\ldots\ldots\ldots$

Since tossing a coin by each person is an independent event, the probabilities multiply each other.

$\Rightarrow P(W_B) = (P(A_N)P(B_H)) + (P(A_N)P(B_N)P(A_N)P(B_H)) + \ldots\ldots\ldots\ldots\ldots$

$\Rightarrow P(W_B) = \left(\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) + \ldots\ldots\ldots\ldots\ldots\ldots$

$\Rightarrow P(W_B) = \left(\frac{1}{2}\right)^2 \times \left(1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \ldots\ldots\ldots\ldots\ldots\right)$

The terms in the bracket resembles the infinite geometric series sequence:

We know that the sum of a Infinite geometric series with first term ‘a’ and common ratio ‘r’ is $S_\infty = \frac{a}{1-r}$

$\Rightarrow P(W_B) = \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{1-\left(\frac{1}{2}\right)^2}\right)$

$\Rightarrow P(W_B) = \frac{1}{4} \times \left(\frac{1}{1-\frac{1}{4}}\right)$

$\Rightarrow P(W_B) = \frac{1}{4} \times \left(\frac{1}{\frac{3}{4}}\right)$

$\Rightarrow P(W_B) = \frac{1}{3}$

∴ The required probability is $\frac{1}{3}$.

11. Question

Two cards are drawn from a well shuffled pack of 52 cards, one after another without replacement. Find the probability that one of these is red card and the other a black card?

Answer

Given that two cards are drawn from a well-shuffled deck of 52 cards.

We know that there will 26 Red and 26 Black cards present in a deck.

It is told that two cards successively without replacement.

Let us find the probability of drawing the cards.

⇒ $P(R_1)$ = P(Drawing Red card from 52 cards deck)

$\Rightarrow P(R_1) = \frac{\text{No.of ways of drawing a red card from deck}}{\text{No.of ways of drawing a card from deck}}$

$\Rightarrow P(R_1) = \frac{26}{52}$

⇒ $P(B_1)$ = P(Drawing Black card from 52 cards deck)

$\Rightarrow P(B_1) = \frac{\text{No.of ways of drawing black card from deck}}{\text{No.of ways of drawing a card from deck}}$

$\Rightarrow P(B_1) = \frac{26}{52}$

⇒ $P(R_2)$ = P(Drawing Red card from remaining 51 cards deck)

$\Rightarrow P(R_2) = \frac{\text{No.of ways of drawing a red card from 51 cards deck}}{\text{No.of ways of drawing a card from 51 cards deck}}$

$\Rightarrow P(R_2) = \frac{26}{51}$

⇒ $P(B_2)$ = P(Drawing Black card from remaining 51 cards deck)

$\Rightarrow P(B_2) = \frac{\text{No.of ways of drawing Black card from 51 cards deck}}{\text{No.of ways of drawing a card from 51 cards deck}}$

$\Rightarrow P(B_2) = \frac{26}{51}$

We need to find the probability of drawing exactly one Red and one black card

⇒ $P(D_{RB})$ = P(Drawing exactly one red and one black card)

⇒ $P(D_{RB})$ = P(Drawing Red first and Black next) + (P(Drawing Black first and red next)

Since drawing cards are independent their probabilities multiply each other,

⇒ $P(D_{RB}) = (P(R_1)P(B_2)) + (P(B_1)P(R_2))$

$\Rightarrow P(D_{RB}) = \left(\frac{26}{52} \times \frac{26}{51}\right) + \left(\frac{26}{52} \times \frac{26}{51}\right)$

$\Rightarrow P(D_{RB}) = \frac{676}{2652}$

$\Rightarrow P(D_{RB}) = \frac{13}{51}$

∴ The required probability is $\frac{13}{51}$.

12. Question

Tickets are numbered from 1 to 10. Two tickets are drawn one after the other at random. Find the probability that the number on one of the tickets is a multiple of 5 and on the other a multiple of 4.

Answer

Given that tickets are numbered from 1 to 10.

It is told that two tickets are drawn one after other in random.

We need to find the probability that number on ticket is a multiple of 5 and other number is multiple of 4.

Let us find the individual probabilities first

⇒ $P(T_{51})$ = P(Number on ticket is multiple of 5 in 1^{st} draw)

⇒ $P(T_{51}) = \frac{\text{No.of ways of picking a ticket whose number is muliple of 5 in } 1^{st} \text{ draw}}{\text{No.of ways of picking a ticket in } 1^{st} \text{ draw}}$

⇒ $P(T_{51}) = \frac{2}{10}$

⇒ $P(T_{51}) = \frac{1}{5}$

⇒ $P(T_{41})$ = P(Number on ticket is multiple of 4 in 1^{st} draw)

⇒ $P(T_{41}) = \frac{\text{No.of ways of picking a ticket whose number is muliple of 4 in } 1^{st} \text{ draw}}{\text{No.of ways of picking a ticket in } 1^{st} \text{ draw}}$

⇒ $P(T_{41}) = \frac{2}{10}$

⇒ $P(T_{41}) = \frac{1}{5}$

⇒ $P(T_{52})$ = P(Number on ticket is multiple of 5 in 2^{nd} draw)

⇒ $P(T_{52}) = \frac{\text{No.of ways of picking a ticket whose number is muliple of 5 in } 2^{nd} \text{ draw}}{\text{No.of ways of picking a ticket in } 2^{nd} \text{ draw}}$

⇒ $P(T_{52}) = \frac{2}{9}$

⇒ $P(T_{42})$ = P(Number on ticket is multiple of 4 in 2^{nd} draw)

⇒ $P(T_{42}) = \frac{\text{No.of ways of picking a ticket whose number is muliple of 5 in } 2^{nd} \text{ draw}}{\text{No.of ways of picking a ticket in } 2^{nd} \text{ draw}}$

⇒ $P(T_{42}) = \frac{2}{9}$

⇒ $P(D_{54})$ = P(Drawing one ticket which is multiple of 5 and other is multiple of 4)

⇒ $P(D_{54})$ = P(Drawing 5 multiple ticket first and 4 multiple ticket next) + (P(Drawing 4 multiple ticket first and 5 multiple ticket next)

Since drawing tickets are independent their probabilities multiply each other,

⇒ $P(D_{54}) = (P(T_{51})P(T_{42})) + (P(T_{41})P(T_{52}))$

⇒ $P(D_{54}) = \left(\frac{2}{10} \times \frac{2}{9}\right) + \left(\frac{2}{10} \times \frac{2}{9}\right)$

⇒ $P(D_{54}) = \frac{8}{90}$

⇒ $P(D_{54}) = \frac{4}{45}$

∴ The required probability is $\frac{4}{45}$.

13. Question

In a family, the husband tells a lie in 30% cases and the wife in 35% cases. Find the probability that both contradict each other on the same fact.

Answer

Given:

⇒ Husband tells lies in 30% of cases

⇒ Wife tells lies in 35% of cases

Now,

⇒ $P(N_H)$ = P(Husband telling lies) = 0.30

⇒ $P(T_H)$ = P(Husband telling truth) = 1-0.30

⇒ $P(T_H) = 0.70$

⇒ $P(N_W)$ = P(Wife telling lies) = 0.35

⇒ $P(T_W)$ = P(Wife telling truth) = 1-0.35

⇒ $P(T_W) = 0.65$

We need to find the probability for case in which husband and wife contradict each other for narrating a fact.

This happens only when husband not telling truth while Wife is telling truth and vice-versa.

⇒ $P(C_{HW})$ = P(Husband and Wife contradict each other)

⇒ $P(C_{HW})$ = P(Husband tells truth and Wife doesn't) + P(Wife tells truth and Husband doesn't)

Since the speaking of husband and Wife are independent events their probabilities will multiply each other.

⇒ $P(C_{HW}) = (P(T_H)P(N_W)) + (P(N_H)P(T_W))$

⇒ $P(C_{HW}) = (0.70\times0.35) + (0.30\times0.65)$

⇒ $P(C_{HW}) = 0.245 + 0.195$

⇒ $P(C_{HW}) = 0.44$

∴ The required probability is 0.44.

14. Question

A husband and wife appear in an interview for two vacancies for the same post. The probability of husband's selection is 1/7 and that of wife's selection is 1/5. What is the probability that

i. Both of them will be selected

ii. Only one them will be selected

iii. None of them will be selected

Answer

Given that,

⇒ $P(H_S)$ = P(Husband's selection)

⇒ $P(H_S) = \frac{1}{7}$

⇒ $P(H_N)$ = P(Not selecting Husband)

⇒ $P(H_N) = 1 - \frac{1}{7}$

⇒ $P(H_N) = \frac{6}{7}$

⇒ $P(W_S)$ = P(wife's selection)

⇒ $P(W_S) = \frac{1}{5}$

⇒ $P(W_N)$ = P(Not selecting Wife)

⇒ $P(W_N) = 1 - \frac{1}{5}$

$\Rightarrow P(W_N) = \frac{4}{5}$

We need to find:

i. Both of them will be selected

ii. Only one of them will be selected

iii. None of them will be selected

$\Rightarrow P(S_{both}) = P(\text{Both of them are selected})$

Since selection of each person is an independent event their probabilities multiply each other

$\Rightarrow P(S_{both}) = P(H_S)P(W_S)$

$\Rightarrow P(S_{both}) = \frac{1}{7} \times \frac{1}{5}$

$\Rightarrow P(S_{both}) = \frac{1}{35}$

$\Rightarrow P(S_{one}) = P(\text{Only one of them is selected})$

$\Rightarrow P(S_{one}) = P(\text{only Husband is selected}) + P(\text{only Wife is selected})$

Since selection of each person is an independent event their probabilities multiply each other

$\Rightarrow P(S_{one}) = \left(P(H_S)P(W_N)\right) + \left(P(H_N)P(W_S)\right)$

$\Rightarrow P(S_{one}) = \left(\frac{1}{7} \times \frac{4}{5}\right) + \left(\frac{6}{7} \times \frac{1}{5}\right)$

$\Rightarrow P(S_{one}) = \frac{10}{35} = \frac{2}{7}$

$\Rightarrow P(S_{none}) = P(\text{None of them are selected})$

Since selection of each person is an independent event their probabilities multiply each other

$\Rightarrow P(S_{none}) = P(H_N)P(W_N)$

$\Rightarrow P(S_{none}) = \frac{6}{7} \times \frac{4}{5}$

$\Rightarrow P(S_{none}) = \frac{24}{35}$

∴ The required probabilities are $\frac{1}{35}, \frac{2}{7}, \frac{24}{35}$.

15. Question

A bag contains 7 white, 5 black and 4 red balls. Four balls are drawn without replacement. Find the probability that at least three balls are black.

Answer

Given:

Bag contains 7 white, 5 black and 4 red balls

It is told that four balls are drawn without replacement.

We need to find the probability that at least three balls drawn are Black.

The possible cases of drawing balls are as shown below:

i. Black, Black, Black, Others

ii. Black, Black, Others, Black

iii. Black, Others, Black, Black

iv. Others, Black, Black, Black

v. Black, Black, Black, Black

⇒ $P(D_{at3blacks}) = P(BBBO) + P(BBOB) + P(BOBB) + P(OBBB) + P(BBBB)$

Since drawing balls are independent events so, the probabilities multiply each other.

⇒ $P(D_{at3blacks}) = \left(P(B)P(B)P(B)P(O)\right) + \left(P(B)P(B)P(O)P(B)\right) + \left(P(B)P(O)P(B)P(B)\right) + \left(P(O)P(B)P(B)P(B)\right) + \left(P(B)P(B)P(B)P(B)\right)$

⇒ $P(D_{at3blacks}) = \left(\frac{5}{16} \times \frac{4}{15} \times \frac{3}{14} \times \frac{11}{13}\right) + \left(\frac{5}{16} \times \frac{4}{15} \times \frac{11}{14} \times \frac{3}{13}\right) + \left(\frac{5}{16} \times \frac{11}{15} \times \frac{4}{14} \times \frac{3}{13}\right) + \left(\frac{5}{16} \times \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13}\right)$

⇒ $P(D_{at3blacks}) = \frac{(1980 + 120)}{43680}$

⇒ $P(D_{at3blacks}) = \frac{2100}{43680}$

⇒ $P(D_{at3blacks}) = \frac{5}{108}$

∴ The required probability is $\frac{5}{108}$.

16. Question

A, B, and C are independent witness of an event which is known to have occurred. A speaks the truth three times out of four, B four times out of five and C five times out of six. What is the probability that the occurrence will be reported truthfully by majority of three witnesses?

Answer

Given:

A speaks truth three times out of four

B speaks truth four times out of five

C speaks truth five times out of six

⇒ $P(T_A)$ = P(A speaks truth)

⇒ $P(T_A) = \frac{3}{4}$

⇒ $P(T_B)$ = P(B speaks truth)

⇒ $P(T_B) = \frac{4}{5}$

⇒ $P(T_C)$ = P(C speaks truth)

⇒ $P(T_C) = \frac{5}{6}$

⇒ $P(N_A)$ = P(A speaks lies)

⇒ $P(N_A) = 1 - \frac{3}{4}$

⇒ $P(N_A) = \frac{1}{4}$

⇒ $P(N_B)$ = P(B speaks lies)

⇒ $P(N_B) = 1 - \frac{4}{5}$

⇒ $P(N_B) = \frac{1}{5}$

⇒ $P(N_C)$ = P(C speaks lies)

⇒ $P(N_C) = 1 - \frac{5}{6}$

⇒ $P(N_C) = \frac{1}{6}$.

It is told that the occurrence is will be reported by majority of witness. This is only possible when at least two of the witnesses speaks truth.

⇒ $P(M) = P(T_AT_BN_C) + P(T_AN_BT_C) + P(N_AT_BT_c) + P(T_AT_BT_C)$

Since speaking by a person is independent the probabilities will multiply each other.

⇒ $P(M) = (P(T_A)P(T_B)P(N_C)) + (P(T_A)P(N_B)P(T_C)) + (P(N_A)P(T_B)P(T_c)) + (P(T_A)P(T_B)P(T_C))$

⇒ $P(M) = \left(\frac{3}{4} \times \frac{4}{5} \times \frac{1}{6}\right) + \left(\frac{3}{4} \times \frac{1}{5} \times \frac{5}{6}\right) + \left(\frac{1}{4} \times \frac{4}{5} \times \frac{5}{6}\right) + \left(\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}\right)$

⇒ $P(M) = \frac{12 + 15 + 20 + 60}{120}$

⇒ $P(M) = \frac{107}{120}$.

∴ The required probability is $\frac{107}{120}$.

17. Question

A bag contains 4 white balls and 2 black balls. Another contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find the probability that

i. Both are white

ii. Both are black

iii. One is white and one is black

Answer

Given:

⇒ Bag A contains 4 white balls and 2 black balls

⇒ Bag B contains 3 white balls and 5 black balls

It is told that one ball is drawn is drawn from is each bag.

Let us find the Probability of drawing each colour ball from the bag.

⇒ $P(B_1)$ = P(drawing black ball from bag A)

⇒ $P(B_1) = \frac{\text{no.of ways to draw a black ball from Bag A}}{\text{no.of ways to draw a ball from Bag A}}$

⇒ $P(B_1) = \frac{2}{2+4}$

⇒ $P(B_1) = \frac{2}{6} = \frac{1}{3}$

⇒ $P(W_1)$ = P(drawing white ball from bag A)

⇒ $P(W_1) = \frac{\text{no.of ways to draw a white ball from Bag A}}{\text{no.of ways to draw a ball from Bag A}}$

⇒ $P(W_1) = \frac{4}{2+4}$

⇒ $P(W_1) = \frac{4}{6} = \frac{2}{3}$

⇒ $P(B_2)$ = P(drawing black ball from bag B)

$\Rightarrow P(B_2) = \frac{\text{no.of ways to draw a black ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$

$\Rightarrow P(B_2) = \frac{5}{5+3}$

$\Rightarrow P(B_2) = \frac{5}{8}$

$\Rightarrow P(W_2)$ = P(drawing white ball from bag B)

$\Rightarrow P(W_2) = \frac{\text{no.of ways to draw a white ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$

$\Rightarrow P(W_2) = \frac{3}{5+3}$

$\Rightarrow P(W_2) = \frac{3}{8}$

We need to find:

i. $P(D_{WW})$ = P(Both are white)

ii. $P(D_{BB})$ = P(Both are black)

iii. $P(D_{WB})$ = P(One drawn is white and other is black)

$\Rightarrow P(D_{WW})$ = P(Both are White)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(D_{WW}) = \left(P(W_1)P(W_2)\right)$

$\Rightarrow P(D_{WW}) = \left(\frac{2}{3} \times \frac{3}{8}\right)$

$\Rightarrow P(D_{WW}) = \frac{1}{4}$

$P(D_{BB})$ = P(Both balls are black)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(D_{BB}) = \left(P(B_1)P(B_2)\right)$

$\Rightarrow P(D_{BB}) = \left(\frac{1}{3} \times \frac{5}{8}\right)$

$\Rightarrow P(D_{WW}) = \frac{5}{24}$

$P(D_{WB})$ = P(One ball drawn is white and other is black)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(D_{WB}) = \left(P(W_1)P(B_2)\right) + \left(P(B_1)P(W_2)\right)$

$\Rightarrow P(D_{WB}) = \left(\frac{2}{3} \times \frac{5}{8}\right) + \left(\frac{1}{3} \times \frac{3}{8}\right)$

$\Rightarrow P(D_{WB}) = \frac{10}{24} + \frac{3}{24}$

$\Rightarrow P(D_{WB}) = \frac{13}{24}$.

∴ The required probabilities are $\frac{1}{4}, \frac{5}{24}, \frac{13}{24}$.

18. Question

A bag contains 4 white, 7 black and 5 red balls. 4 balls are drawn with replacement. What is the probability that at least two are white?

Answer

Given:

Bag contains 4 white, 7 black and 5 red balls

It is told that four balls are drawn with replacement.

We need to find the probability that at least two balls drawn are white.

Let us find the individual probabilities to draw a ball

⇒ P(W) = P(drawing a white ball)

$$\Rightarrow P(W) = \frac{\text{no.of ways of drawing a white ball from bag}}{\text{no.of ways of drawing a ball from bag}}$$

$$\Rightarrow P(W) = \frac{4}{16} = \frac{1}{4}$$

⇒ P(O) = P(drawing balls other than white)

$$\Rightarrow P(W) = 1 - \frac{1}{4}$$

$$\Rightarrow P(W) = \frac{3}{4}$$

The possible cases of drawing balls are as shown below:

i. Two whites and two others – 6 cases to draw

ii. Three whites and 1 others – 4 cases to draw

iii. Four whites – 1 case to draw

Since drawing balls are independent events so, the probabilities multiply each other.

$$\Rightarrow P(D_{at2\,whites}) = (P(2\text{ whitesand }2\text{ others})) + (P(3\text{ whites and }1\text{ others})) + (P(4\text{ whites}))$$

$$\Rightarrow P(D_{at2\,whites}) = 6 \times \left(\frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4}\right) + 4 \times \left(\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right)$$

$$\Rightarrow P(D_{at2\,whites}) = \frac{(54 + 12 + 1)}{256}$$

$$\Rightarrow P(D_{at2\,whites}) = \frac{67}{256}$$

∴ The required probability is $\frac{67}{256}$.

19. Question

Three cards are drawn with replacement from a well shuffled pack of 52 cards. Find the probability that the cards are a king, a queen and a jack.

Answer

Given that three cards are drawn from a well-shuffled deck of 52 cards with replacement.

We know that there will 4 kings, 4 queens, 4 jacks present in a deck.

Let us find the probability of drawing the cards.

⇒ P(K) = P(Drawing King from 52 cards deck)

$$\Rightarrow P(K) = \frac{\text{No.of ways of drawing a king from deck}}{\text{No.of ways of drawing a card from deck}}$$

$$\Rightarrow P(K) = \frac{4}{52} = \frac{1}{13}$$

⇒ P(Q) = P(Drawing Queen from 52 cards deck)

$\Rightarrow P(Q) = \frac{\text{No.of ways of drawing a queen from deck}}{\text{No.of ways of drawing a card from deck}}$

$\Rightarrow P(Q) = \frac{4}{52} = \frac{1}{13}$

⇒ P(J) = P(Drawing Jack from 52 cards deck)

$\Rightarrow P(J) = \frac{\text{No.of ways of drawing a Jack from deck}}{\text{No.of ways of drawing a card from deck}}$

$\Rightarrow P(J) = \frac{4}{52} = \frac{1}{13}$

We need to find the probability of drawing one king, one queen, one jack

There will be 6 cases or sequences for drawing these three cards

⇒ $P(D_{KQJ})$ = P(Drawing King, queen and jack)

Since drawing cards are independent their probabilities multiply each other,

$\Rightarrow P(D_{KQJ}) = 6 \times \left(\frac{1}{13} \times \frac{1}{13} \times \frac{1}{13}\right)$

$\Rightarrow P(D_{KQJ}) = \frac{6}{2197}$

∴ The required probability is $\frac{6}{2197}$.

20. Question

A bag contains 4 red and 5 black balls, a second bag contains 3 red and 7 black balls. One ball is drawn at random from each bag, find the probability that the (i) balls are of different colours (ii) balls are of the same colour.

Answer

Given:

⇒ Bag A contains 4 red balls and 5 black balls

⇒ Bag B contains 3 red balls and 7 black balls

It is told that one ball is drawn is drawn from is each bag.

We need to find the probability that the balls are of same colour.

Let us find the Probability of drawing each colour ball from the bag.

⇒ $P(B_1)$ = P(drawing black ball from bag A)

$\Rightarrow P(B_1) = \frac{\text{no.of ways to draw a black ball from Bag A}}{\text{no.of ways to draw a ball from Bag A}}$

$\Rightarrow P(B_1) = \frac{5}{5+4}$

$\Rightarrow P(B_1) = \frac{5}{9}$

⇒ $P(R_1)$ = P(drawing red ball from bag A)

$\Rightarrow P(R_1) = \frac{\text{no.of ways to draw a red ball from Bag A}}{\text{no.of ways to draw a ball from Bag A}}$

$\Rightarrow P(R_1) = \frac{4}{5+4}$

$\Rightarrow P(R_1) = \frac{4}{9}$

⇒ $P(B_2)$ = P(drawing black ball from bag B)

$\Rightarrow P(B_2) = \frac{\text{no.of ways to draw a black ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$

$\Rightarrow P(B_2) = \frac{7}{7+3}$

$\Rightarrow P(B_2) = \frac{7}{10}$

$\Rightarrow P(R_2)$ = P(drawing red ball from bag B)

$\Rightarrow P(R_2) = \frac{\text{no.of ways to draw a red ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$

$\Rightarrow P(R_2) = \frac{3}{7+3}$

$\Rightarrow P(R_2) = \frac{3}{10}$

We need to find the probability of drawing the different colour balls from two bags

$\Rightarrow P(D_{RB})$ = P(drawing one red ball and one Black ball)

$\Rightarrow P(D_{RB})$ = P(drawing black balls from bag A and red ball from bag B) + P(drawing black balls from bag B and red ball from bag A)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(D_{RB}) = \left(P(B_1)P(R_2)\right) + \left(P(R_1)P(B_2)\right)$

$\Rightarrow P(D_{RB}) = \left(\frac{5}{9} \times \frac{3}{10}\right) + \left(\frac{7}{10} \times \frac{4}{9}\right)$

$\Rightarrow P(D_{RB}) = \frac{15}{90} + \frac{28}{90}$

$\Rightarrow P(D_{RB}) = \frac{43}{90}.$

We need to find the probability of drawing the same colour balls from two bags

$\Rightarrow$ P(S) = P(drawing two balls of same colours) = P(drawing black balls from each bag) + (P(drawing white balls from each bag)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(S) = \left(P(B_1)P(B_2)\right) + \left(P(W_1)P(W_2)\right)$

$\Rightarrow P(S) = \left(\frac{5}{9} \times \frac{7}{10}\right) + \left(\frac{4}{9} \times \frac{3}{10}\right)$

$\Rightarrow P(S) = \frac{35}{90} + \frac{12}{90}$

$\Rightarrow P(S) = \frac{47}{90}.$

$\therefore$ The required probabilities are $\frac{43}{90}, \frac{47}{90}.$

21. Question

A can hit a target 3 times in 6 shots, B:2 times in 6 shots and C:4 times in 4 shots. They fix a volley. What is the probability that at least 2 shots hit?

Answer

Given:

A hits a target 3 times out of 6 shots

B hits a target 2 times out of 6 shots

C hits a target 4 times out of 4 shots

⇒ $P(T_A)$ = P(A hits target)

⇒ $P(T_A) = \frac{3}{6} = \frac{1}{2}$

⇒ $P(T_B)$ = P(B hits target)

⇒ $P(T_B) = \frac{2}{6} = \frac{1}{3}$

⇒ $P(T_C)$ = P(C hits target)

⇒ $P(T_C) = \frac{4}{4} = 1$

⇒ $P(N_A)$ = P(A doesn't hits target)

⇒ $P(N_A) = 1 - \frac{1}{2}$

⇒ $P(N_A) = \frac{1}{2}$

⇒ $P(N_B)$ = P(B doesn't hits target)

⇒ $P(N_B) = 1 - \frac{1}{3}$

⇒ $P(N_B) = \frac{2}{3}$

⇒ $P(N_C)$ = P(C doesn't hits target)

⇒ $P(N_C) = 1 - 1$

⇒ $P(N_C) = 0.$

It is told that the target is to be hit by at least two shots. This is only possible when at least two of the Persons hits the target.

⇒ $P(M) = P(T_AT_BN_C) + P(T_AN_BT_C) + P(N_AT_BT_c) + P(T_AT_BT_C)$

Since hitting by a person is independent the probabilities will multiply each other.

⇒ $P(M) = (P(T_A)P(T_B)P(N_C)) + (P(T_A)P(N_B)P(T_C)) + (P(N_A)P(T_B)P(T_c)) + (P(T_A)P(T_B)P(T_C))$

⇒ $P(M) = \left(\frac{1}{2}\times\frac{1}{3}\times 0\right) + \left(\frac{1}{2}\times\frac{2}{3}\times 1\right) + \left(\frac{1}{2}\times\frac{1}{3}\times 1\right) + \left(\frac{1}{2}\times\frac{1}{3}\times 1\right)$

⇒ $P(M) = \frac{0+2+1+1}{6}$

⇒ $P(M) = \frac{4}{6}.$

⇒ $P(M) = \frac{2}{3}$

∴ The required probability is $\frac{2}{3}$.

22. Question

The probability of student A passing an examination is 2/9 and of student B passing is 5/9. Assuming the two events: 'A passes', 'B passes' as independent, find the probability of : (i) only A passing the examination (ii) only one of them passing the examination.

Answer

Given that,

⇒ $P(A_P)$ = P(A passing an examination)

⇒ $P(A_P) = \frac{2}{9}$

⇒ $P(A_N)$ = P(A Not passing an examination)

$\Rightarrow P(A_N) = 1 - \frac{2}{9}$

$\Rightarrow P(A_N) = \frac{7}{9}$

⇒ $P(B_P)$ = P(B passing an examination)

$\Rightarrow P(B_P) = \frac{5}{9}$

⇒ $P(B_N)$ = P(B Not passing an examination)

$\Rightarrow P(B_N) = 1 - \frac{5}{9}$

$\Rightarrow P(B_N) = \frac{4}{9}$

We need to find probability that:

i. Only A passing the examination

ii. Only one of them passing the examination

⇒ $P(S_A)$ = P(Only A passing the examination)

This happens only in the case B must fail

Since passing examination is an independent event their probabilities multiply each other

$\Rightarrow P(S_A) = P(A_P)P(B_N)$

$\Rightarrow P(S_A) = \frac{2}{9} \times \frac{4}{9}$

$\Rightarrow P(S_A) = \frac{8}{81}$

⇒ $P(S_{one})$ = P(Only one of them passed the examination)

⇒ $P(S_{one})$ = P(only A passed the examination) + P(only B passed the examination)

Since passing examination is an independent event their probabilities multiply each other

$\Rightarrow P(S_{one}) = \left(P(A_P)P(B_N)\right) + \left(P(A_N)P(B_P)\right)$

$\Rightarrow P(S_{one}) = \left(\frac{2}{9} \times \frac{4}{9}\right) + \left(\frac{7}{9} \times \frac{5}{9}\right)$

$\Rightarrow P(S_{one}) = \frac{8}{81} + \frac{35}{81}$

$\Rightarrow P(S_{one}) = \frac{43}{81}$

∴ The required probabilities are $\frac{8}{81}, \frac{43}{81}$.

23. Question

There are three urns A, B and C. Urn A contains 4 red balls and 3 black balls. Urn B contains 5 red balls and 4 black balls. Urn C contains 4 red and 4 black balls. One ball is drawn from each of these urns. What is the probability that 3 balls drawn consist of 2 red balls and a black ball?

Answer

Given:

⇒ Urn A contains 4 red balls and 3 black balls

⇒ Urn B contains 5 red balls and 4 black balls

⇒ Urn C contains 4 red balls and 4 black balls

It is told that one ball is drawn is drawn from is each urn.

We need to find the probability that two balls are red and other ball is black.

Let us find the Probability of drawing each colour ball from the Urn.

⇒ $P(B_1)$ = P(drawing black ball from Urn A)

⇒ $P(B_1) = \frac{\text{no.of ways to draw a black ball from Urn A}}{\text{no.of ways to draw a ball from Urn A}}$

⇒ $P(B_1) = \frac{3}{4+3}$

⇒ $P(B_1) = \frac{3}{7}$

⇒ $P(R_1)$ = P(drawing Red ball from urn A)

⇒ $P(R_1) = \frac{\text{no.of ways to draw a Red ball from Urn A}}{\text{no.of ways to draw a ball from Urn A}}$

⇒ $P(R_1) = \frac{4}{4+3}$

⇒ $P(R_1) = \frac{4}{7}$

⇒ $P(B_2)$ = P(drawing black ball from Urn B)

⇒ $P(B_2) = \frac{\text{no.of ways to draw a black ball from Urn B}}{\text{no.of ways to draw a ball from Urn B}}$

⇒ $P(B_2) = \frac{4}{5+4}$

⇒ $P(B_2) = \frac{4}{9}$

⇒ $P(R_2)$ = P(drawing Red ball from urn B)

⇒ $P(R_2) = \frac{\text{no.of ways to draw a Red ball from Urn B}}{\text{no.of ways to draw a ball from Urn B}}$

⇒ $P(R_2) = \frac{5}{5+4}$

⇒ $P(R_2) = \frac{5}{9}$

⇒ $P(B_3)$ = P(drawing black ball from Urn C)

⇒ $P(B_3) = \frac{\text{no.of ways to draw a black ball from Urn C}}{\text{no.of ways to draw a ball from Urn C}}$

⇒ $P(B_3) = \frac{4}{4+4}$

⇒ $P(B_3) = \frac{4}{8} = \frac{1}{2}$

⇒ $P(R_3)$ = P(drawing Red ball from urn C)

⇒ $P(R_3) = \frac{\text{no.of ways to draw a Red ball from Urn C}}{\text{no.of ways to draw a ball from Urn C}}$

⇒ $P(R_3) = \frac{4}{4+4}$

⇒ $P(R_3) = \frac{4}{8} = \frac{1}{2}$

We need to find the probability of 2 red balls and 1 black ball from three bags

⇒ P(S) = P(drawing two red ball and one Black ball)

⇒ P(S) = P(drawing black balls from bag A red ball from bag B and Red ball from bag C) + P(drawing black balls from bag B red ball from bag A and Red ball from bag C) + P(drawing black balls from bag C red ball from bag A and Red ball from bag B)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

⇒ $P(S) = (P(B_1)P(R_2)P(R_3)) + (P(R_1)P(B_2)P(R_3)) + \left(P(R_1)P(R_2)P(B_3)\right)$

⇒ $P(S) = \left(\frac{3}{7}\times\frac{5}{9}\times\frac{1}{2}\right) + \left(\frac{4}{7}\times\frac{4}{9}\times\frac{1}{2}\right) + \left(\frac{4}{7}\times\frac{5}{9}\times\frac{1}{2}\right)$

⇒ $P(S) = \frac{15}{126} + \frac{16}{126} + \frac{20}{126}$

⇒ $P(S) = \frac{51}{126}$

⇒ $P(S) = \frac{17}{42}$.

∴ The required probability is $\frac{17}{42}$.

24. Question

X is taking up subjects – Mathematics, Physics and Chemistry in the examination. His probabilities of getting grade A in these subjects are 0.2, 0.3 and 0.5 respectively. Find the probability that he gets

i. Grade A in all subjects

ii. Grade A in no subjects

iii. Grade A in two subjects

Answer

Given:

⇒ $P(M_A)$ = P(getting A in mathematics)

⇒ $P(M_A) = 0.2$

⇒ $P(M_N)$ = P(not getting A in mathematics)

⇒ $P(M_N) = 1-0.2$

⇒ $P(M_N) = 0.8$

⇒ $P(P_A)$ = P(getting A in physics)

⇒ $P(P_A) = 0.3$

⇒ $P(P_N)$ = P(not getting A in physics)

⇒ $P(P_N) = 1-0.7$

⇒ $P(P_N) = 0.3$

⇒ $P(C_A)$ = P(getting A in Chemistry)

⇒ $P(C_A) = 0.5$

⇒ $P(C_N)$ = P(not getting A in chemistry)

⇒ $P(C_N) = 1-0.5$

⇒ $P(C_N) = 0.5$

We need to find the probability that:

i. X gets A in all subjects

ii. X gets A in no subjects

iii. X gets A in two subjects

⇒ $P(X_{all})$ = P(getting A in all subjects)

Since getting A in different subjects is an independent event, their probabilities multiply each other

⇒ $P(X_{all}) = (P(M_A)P(P_A)P(C_A))$

⇒ $P(X_{all}) = 0.2\times0.3\times0.5$

⇒ $P(X_{all}) = 0.03$

⇒ $P(X_{none})$ = P(getting A in no subjects)

Since getting A in different subjects is an independent event, their probabilities multiply each other

⇒ $P(X_{none}) = (P(M_N)P(P_N)P(C_N))$

⇒ $P(X_{none}) = 0.8\times0.7\times0.5$

⇒ $P(X_{none}) = 0.28$

⇒ $P(X_{two})$ = P(getting A in any two subjects)

Since getting A in different subjects is an independent event, their probabilities multiply each other

⇒ $P(X_{two}) = (P(M_A)P(P_A)P(C_N)) + (P(M_A)P(P_N)P(C_A)) + (P(M_N)P(P_A)P(C_A))$

⇒ $P(X_{two}) = (0.2\times0.3\times0.5) + (0.2\times0.7\times0.5) + (0.8\times0.3\times0.5)$

⇒ $P(X_{two}) = 0.03 + 0.07 + 0.12$

⇒ $P(X_{two}) = 0.22$

∴ The required probabilities are 0.03, 0.28, 0.22.

25. Question

A and B take turns in throwing two dice, the first to throw 9 being awarded the prize. Show that their chance of winning are in the ration 9:8.

Answer

Given that A and B throws two dice.

The first who throw 9 awarded a prize.

The possibilities of getting 9 on throwing two dice are:

$\{(3,6),(4,5),(5,4),(6,3)\}$

⇒ $P(S_9)$ = P(getting sum 9)

⇒ $P(S_9) = \frac{4}{36}$

⇒ $P(S_9) = \frac{1}{9}$

⇒ $P(S_N)$ = P(not getting sum 9)

⇒ $P(S_N) = 1-\frac{1}{9}$

⇒ $P(S_N) = \frac{8}{9}$

Let us assume A starts the game, A wins the game only when he gets 9 while throwing dice in 1^{st},3^{rd},5^{th},......

times

Here the probability of getting sum 9 on throwing a dice is same for both the players A and B

Since throwing a dice is an independent event, their probabilities multiply each other

$\Rightarrow P(A_{wins}) = P(S_9) + P(S_N)P(S_N)P(S_9) + P(S_N)P(S_N)P(S_N)P(S_N)P(S_9) + \ldots\ldots\ldots\ldots$

$\Rightarrow P(A_{wins}) = \frac{1}{9} + \left(\frac{8}{9}\times\frac{8}{9}\times\frac{1}{9}\right) + \left(\frac{8}{9}\times\frac{8}{9}\times\frac{8}{9}\times\frac{8}{9}\times\frac{1}{9}\right) + \ldots\ldots\ldots\ldots\ldots\ldots$

$\Rightarrow P(A_{wins}) = \frac{1}{9}\times\left(1 + \left(\frac{8}{9}\right)^2 + \left(\frac{8}{9}\right)^4 + \ldots\ldots\ldots\ldots\right)$

The series in the brackets resembles the Infinite geometric series.

We know that sum of a infinite geometric series with first term 'a' and common ratio 'o' is $S_\infty = \frac{a}{1-r}$.

$\Rightarrow P(A_{wins}) = \frac{1}{9}\times\left(\frac{1}{1-\left(\frac{8}{9}\right)^2}\right)$

$\Rightarrow P(A_{wins}) = \frac{1}{9}\times\left(\frac{1}{1-\frac{64}{81}}\right)$

$\Rightarrow P(A_{wins}) = \frac{1}{9}\times\left(\frac{1}{\frac{17}{81}}\right)$

$\Rightarrow P(A_{wins}) = \frac{9}{17}$

$\Rightarrow P(B_{wins}) = 1-P(A_{wins})$

$\Rightarrow P(B_{wins}) = 1-\frac{9}{17}$

$\Rightarrow P(B_{wins}) = \frac{8}{17}$

$\Rightarrow P(A_{wins}):P(B_{wins}) = \frac{9}{17}:\frac{8}{17}$

$\Rightarrow P(A_{wins}):P(B_{wins}) = 9:8$

∴ Thus proved

26. Question

A, B and C in order toss a coin. The one to throw a head wins. What are their respective chances of winning assuming that the game may continue indefinitely?

Answer

Given that A, B and C toss a coin until one of them gets a head to win the game.

Let us find the probability of getting the head.

$\Rightarrow P(A_H)$ = P(A getting a head on tossing a coin)

$\Rightarrow P(A_H) = \frac{1}{2}$

$\Rightarrow P(A_N)$ = P(A not getting head on tossing a coin)

$\Rightarrow P(A_N) = \frac{1}{2}$

$\Rightarrow P(B_H)$ = P(B getting a head on tossing a coin)

$\Rightarrow P(B_H) = \frac{1}{2}$

$\Rightarrow P(B_N) = P(B \text{ not getting head on tossing a coin})$

$\Rightarrow P(B_N) = \frac{1}{2}$

$\Rightarrow P(C_H) = P(C \text{ getting a head on tossing a coin})$

$\Rightarrow P(C_H) = \frac{1}{2}$

$\Rightarrow P(C_N) = P(C \text{ not getting head on tossing a coin})$

$\Rightarrow P(C_N) = \frac{1}{2}$

It is told that A starts the game.

A tosses in $1^{st}, 4^{th}, 7^{th}, \ldots\ldots$ tosses.

This can be shown as follows:

$\Rightarrow P(W_A) = P(A \text{ wins the game})$

$\Rightarrow P(W_A) = P(A_H) + P(A_N B_N C_N A_H) + P(A_N B_N C_N A_N B_N C_N A_H) + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Since tossing a coin by each person is an independent event, the probabilities multiply each other.

$\Rightarrow P(W_A) = (P(A_H)) + (P(A_N)P(B_N)P(C_N)P(A_H)) + \ldots\ldots\ldots\ldots\ldots\ldots$

$$\Rightarrow P(W_A) = \left(\frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Rightarrow P(W_A) = \left(\frac{1}{2}\right) \times \left(1 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^6 + \ldots\ldots\ldots\ldots\ldots\right)$$

The terms in the bracket resembles the infinite geometric series sequence:

We know that the sum of a Infinite geometric series with first term ‘a’ and common ratio ‘r’ is $S_\infty = \frac{a}{1-r}$

$$\Rightarrow P(W_A) = \left(\frac{1}{2}\right) \times \left(\frac{1}{1-\left(\frac{1}{2}\right)^3}\right)$$

$$\Rightarrow P(W_A) = \frac{1}{2} \times \left(\frac{1}{1-\frac{1}{8}}\right)$$

$$\Rightarrow P(W_A) = \frac{1}{2} \times \left(\frac{1}{\frac{7}{8}}\right)$$

$$\Rightarrow P(W_A) = \frac{4}{7}$$

B tosses in $2^{nd}, 5^{th}, 8^{th}, \ldots\ldots$ tosses.

This can be shown as follows:

$\Rightarrow P(W_B) = P(B \text{ wins the game})$

$\Rightarrow P(W_B) = P(A_N B_H) + P(A_N B_N C_N A_N B_H) + P(A_N B_N C_N A_N B_N C_N A_N B_H) + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Since tossing a coin by each person is an independent event, the probabilities multiply each other.

$\Rightarrow P(W_B) = (P(A_N)P(B_H)) + (P(A_N)P(B_N)P(C_N)P(A_N)P(B_H)) + \ldots\ldots\ldots\ldots\ldots\ldots$

$$\Rightarrow P(W_B) = \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Rightarrow P(W_B) = \left(\frac{1}{2}\right)^2 \times \left(1 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^6 + \ldots\ldots\ldots\ldots\ldots\right)$$

The terms in the bracket resembles the infinite geometric series sequence:

We know that the sum of a Infinite geometric series with first term 'a' and common ratio 'r' is $s_{\infty} = \frac{a}{1-r}$

$\Rightarrow P(W_B) = \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{1-\left(\frac{1}{2}\right)^3}\right)$

$\Rightarrow P(W_B) = \frac{1}{4} \times \left(\frac{1}{1-\frac{1}{8}}\right)$

$\Rightarrow P(W_B) = \frac{1}{4} \times \left(\frac{1}{\frac{7}{8}}\right)$

$\Rightarrow P(W_B) = \frac{2}{7}$

$\Rightarrow P(W_C) = P(\text{winning of C})$

$\Rightarrow P(W_C) = 1-P(W_A)-P(W_B)$

$\Rightarrow P(W_C) = 1 - \frac{4}{7} - \frac{2}{7}$

$\Rightarrow P(W_C) = \frac{1}{7}$

∴ The chances of winning of A,B and C are $\frac{4}{7}, \frac{2}{7}, \frac{1}{7}$.

27. Question

Three persons A, B, C throw a die in succession till one gets a 'six' and wins the game. Find their respective probabilities of winning.

Answer

Given that A,B and C throws a die.

The first who throw 6 wins the game.

$\Rightarrow P(S_6) = P(\text{getting } 6)$

$\Rightarrow P(S_6) = \frac{1}{6}$

$\Rightarrow P(S_N) = P(\text{not getting } 6)$

$\Rightarrow P(S_N) = 1 - \frac{1}{6}$

$\Rightarrow P(S_N) = \frac{5}{6}$

Let us assume A starts the game, A wins the game only when he gets 6 while throwing dice in $1^{st}, 4^{th}, 7^{th}$,...... times

Here the probability of getting sum 6 on throwing a dice is same for the players A, B and C

Since throwing a dice is an independent event, their probabilities multiply each other

$\Rightarrow P(A_{wins}) = P(S_9) + P(S_N)P(S_N)P(S_N)P(S_9) + P(S_N)P(S_N)P(S_N)P(S_N)P(S_N)P(S_N)P(S_9) + \ldots\ldots\ldots\ldots\ldots$

$\Rightarrow P(A_{wins}) = \frac{1}{6} + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{9}\right) + \ldots\ldots\ldots\ldots\ldots\ldots$

$\Rightarrow P(A_{wins}) = \frac{1}{6} \times \left(1 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^6 + \ldots\ldots\ldots\ldots\ldots\right)$

The series in the brackets resembles the Infinite geometric series.

We know that sum of a infinite geometric series with first term ‘a’ and common ratio ‘o’ is $s_{\infty} = \frac{a}{1-r}$.

$$\Rightarrow P(A_{wins}) = \frac{1}{6}\times\left(\frac{1}{1-\left(\frac{5}{6}\right)^3}\right)$$

$$\Rightarrow P(A_{wins}) = \frac{1}{6}\times\left(\frac{1}{1-\frac{125}{216}}\right)$$

$$\Rightarrow P(A_{wins}) = \frac{1}{6}\times\left(\frac{1}{\frac{91}{216}}\right)$$

$$\Rightarrow P(A_{wins}) = \frac{36}{91}$$

B wins the game only when he gets 6 while throwing dice in 2^{nd},5^{th},8^{th},...... times and others doesn’t get 6.

Since throwing a dice is an independent event, their probabilities multiply each other

$\Rightarrow P(B_{wins}) = (P(S_N)P(S_9)) + (P(S_N)P(S_N)P(S_N)P(S_N)P(S_9)) + (P(S_N)P(S_N)P(S_N)P(S_N)P(S_N)P(S_N)P(S_N)P(S_9)) + \ldots\ldots\ldots\ldots\ldots$

$$\Rightarrow P(B_{wins}) = \left(\frac{5}{6}\times\frac{1}{6}\right) + \left(\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}\right) + \left(\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{9}\right) + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Rightarrow P(B_{wins}) = \frac{5}{36}\times\left(1 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^6 + \ldots\ldots\ldots\ldots\ldots\right)$$

The series in the brackets resembles the Infinite geometric series.

We know that sum of a infinite geometric series with first term ‘a’ and common ratio ‘o’ is $s_{\infty} = \frac{a}{1-r}$.

$$\Rightarrow P(B_{wins}) = \frac{5}{36}\times\left(\frac{1}{1-\left(\frac{5}{6}\right)^3}\right)$$

$$\Rightarrow P(B_{wins}) = \frac{5}{36}\times\left(\frac{1}{1-\frac{125}{216}}\right)$$

$$\Rightarrow P(B_{wins}) = \frac{5}{36}\times\left(\frac{1}{\frac{91}{216}}\right)$$

$$\Rightarrow P(B_{wins}) = \frac{30}{91}$$

$\Rightarrow P(C_{wins}) = 1\text{-}P(A_{wins})\text{-}P(B_{wins})$

$$\Rightarrow P(C_{wins}) = 1 - \frac{36}{91} - \frac{30}{91}$$

$$\Rightarrow P(C_{wins}) = \frac{25}{91}$$

∴ The probabilities of winning of A, B and C is $\frac{36}{91}, \frac{30}{91}, \frac{25}{91}$.

28. Question

A and B take turns in throwing two dice, the first to throw 10 being awarded the prize, show that if A has the first throw, their chance of winning are in the ratio 12:11.

Answer

Given that A and B throws two dice.

The first who throw 10 awarded a prize.

The possibilities of getting 10 on throwing two dice are:

$\{(4,6),(5,5),(6,4)\}$

$\Rightarrow P(S_{10}) = P(\text{getting sum } 10)$

$\Rightarrow P(S_{10}) = \frac{3}{36}$

$\Rightarrow P(S_{10}) = \frac{1}{12}$

$\Rightarrow P(S_N) = P(\text{not getting sum } 10)$

$\Rightarrow P(S_N) = 1 - \frac{1}{12}$

$\Rightarrow P(S_N) = \frac{11}{12}$

Let us assume A starts the game, A wins the game only when he gets 10 while throwing dice in 1^{st},3^{rd},5^{th}, ... times

Here the probability of getting sum 10 on throwing a dice is same for both the players A and B

Since throwing a dice is an independent event, their probabilities multiply each other

$\Rightarrow P(A_{wins}) = P(S_{10}) + P(S_N)P(S_N)P(S_{10}) + P(S_N)P(S_N)P(S_N)P(S_N)P(S_{10}) + \ldots\ldots\ldots\ldots\ldots$

$\Rightarrow P(A_{wins}) = \frac{1}{12} + \left(\frac{11}{12}\times\frac{11}{12}\times\frac{1}{12}\right) + \left(\frac{11}{12}\times\frac{11}{12}\times\frac{11}{12}\times\frac{11}{12}\times\frac{1}{12}\right) + \ldots\ldots\ldots\ldots\ldots\ldots$

$\Rightarrow P(A_{wins}) = \frac{1}{12}\times\left(1 + \left(\frac{11}{12}\right)^2 + \left(\frac{11}{12}\right)^4 + \ldots\ldots\ldots\ldots\ldots\right)$

The series in the brackets resembles the Infinite geometric series.

We know that sum of a infinite geometric series with first term 'a' and common ratio 'o' is $S_\infty = \frac{a}{1-r}$.

$\Rightarrow P(A_{wins}) = \frac{1}{12}\times\left(\frac{1}{1-\left(\frac{11}{12}\right)^2}\right)$

$\Rightarrow P(A_{wins}) = \frac{1}{12}\times\left(\frac{1}{1-\frac{121}{144}}\right)$

$\Rightarrow P(A_{wins}) = \frac{1}{12}\times\left(\frac{1}{\frac{23}{144}}\right)$

$\Rightarrow P(A_{wins}) = \frac{12}{23}$

$\Rightarrow P(B_{wins}) = 1 - P(A_{wins})$

$\Rightarrow P(B_{wins}) = 1 - \frac{12}{23}$

$\Rightarrow P(B_{wins}) = \frac{11}{23}$

$\Rightarrow P(A_{wins}):P(B_{wins}) = \frac{12}{23}:\frac{11}{23}$

$\Rightarrow P(A_{wins}):P(B_{wins}) = 12:11$

∴ Thus proved

29. Question

There are 3 red and 5 black balls in bag 'A'; and 2 red and 3 black balls in bag 'B'. One ball is drawn from bag 'A' and two from bag 'B'. Find the probability that out of the 3 balls drawn one is red and 2 are black.

Answer

Given:

⇒ Bag A contains 3 red balls and 5 black balls

⇒ Bag B contains 2 red balls and 3 black balls

It is told that one ball is drawn from bag A and two balls from bag B.

We need to find the probability that one ball is red and other two are black.

Let us find the Probability of drawing each colour ball from the bag.

⇒ $P(B_1)$ = P(drawing black ball from bag A)

$\Rightarrow P(B_1) = \frac{\text{no.of ways to draw a black ball from Bag A}}{\text{no.of ways to draw a ball from Bag A}}$

$\Rightarrow P(B_1) = \frac{5}{5+3}$

$\Rightarrow P(B_1) = \frac{5}{8}$

⇒ $P(R_1)$ = P(drawing Red ball from bag A)

$\Rightarrow P(R_1) = \frac{\text{no.of ways to draw a Red ball from Bag A}}{\text{no.of ways to draw a ball from Bag A}}$

$\Rightarrow P(R_1) = \frac{3}{5+3}$

$\Rightarrow P(R_1) = \frac{3}{8}$

⇒ $P(B_{21})$ = P(drawing black ball from bag B in 1^{st} draw)

$\Rightarrow P(B_{21}) = \frac{\text{no.of ways to draw a black ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$

$\Rightarrow P(B_{21}) = \frac{3}{3+2}$

$\Rightarrow P(B_{21}) = \frac{3}{5}$

⇒ $P(R_{21})$ = P(drawing Red ball from bag B in 1^{st} draw)

$\Rightarrow P(R_{21}) = \frac{\text{no.of ways to draw a Red ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$

$\Rightarrow P(R_{21}) = \frac{2}{3+2}$

$\Rightarrow P(R_{21}) = \frac{2}{5}$

⇒ $P(B_{22})$ = P(drawing black ball from bag B in 2^{nd} draw after drawing red ball)

$\Rightarrow P(B_{22}) = \frac{\text{no.of ways to draw a black ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$

$\Rightarrow P(B_{22}) = \frac{3}{3+1}$

$\Rightarrow P(B_{22}) = \frac{3}{4}$

⇒ $P(R_{22})$ = P(drawing Red ball from bag B in 2^{nd} draw after drawing Black ball)

$\Rightarrow P(R_{22}) = \frac{\text{no.of ways to draw a Red ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$

$\Rightarrow P(R_{22}) = \frac{2}{2+2}$

$\Rightarrow P(R_{22}) = \frac{2}{4}$

⇒ P(B_{221}) = P(drawing black ball from bag B in 2nd draw after drawing black ball)

$\Rightarrow P(B_{221}) = \frac{\text{no.of ways to draw a black ball from Bag B}}{\text{no.of ways to draw a ball from Bag B}}$

$\Rightarrow P(B_{221}) = \frac{2}{2+2}$

$\Rightarrow P(B_{22}) = \frac{2}{4}$

We need to find the probability of drawing a red and two black balls from two bags

⇒ P(S) = P(drawing one red ball and two Black balls)

⇒ P(S) = P(drawing red ball from bag A and black balls from bag B) + P(drawing black ball from bag A and red and black balls from bag B)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(S) = (P(R_1)P(B_{21})P(B_{221})) + (P(B_1)P(R_{21})P(B_{22})) + (P(B_1)P(B_{21})P(R_{22}))$

$\Rightarrow P(S) = \left(\frac{3}{8}\times\frac{3}{5}\times\frac{2}{4}\right) + \left(\frac{5}{8}\times\frac{2}{5}\times\frac{3}{4}\right) + \left(\frac{5}{8}\times\frac{3}{5}\times\frac{2}{4}\right)$

$\Rightarrow P(S) = \frac{18}{160} + \frac{30}{160} + \frac{30}{160}$

$\Rightarrow P(S) = \frac{78}{160} = \frac{39}{80}.$

∴ The required probability is $\frac{39}{80}$.

30. Question

Fatima and John appear in an interview for two vacancies for the same post. The probability of Fatima’s selection is 1/7 and that of John’s selection is 1/5. What is the probability that

i. Both of them are selected

ii. Only one of them will be selected

iii. None of them will be selected

Answer

Given that,

⇒ P(F_S) = P(Fatima’s selection)

$\Rightarrow P(F_S) = \frac{1}{7}$

⇒ P(F_N) = P(Not selecting Fatima)

$\Rightarrow P(F_N) = 1 - \frac{1}{7}$

$\Rightarrow P(F_N) = \frac{6}{7}$

⇒ P(J_S) = P(John’s selection)

$\Rightarrow P(J_S) = \frac{1}{5}$

⇒ P(J_N) = P(Not selecting John)

$\Rightarrow P(J_N) = 1 - \frac{1}{5}$

$\Rightarrow P(J_N) = \frac{4}{5}$

We need to find:

i. Both of them will be selected

ii. Only one of them will be selected

iii. None of them will be selected

⇒ $P(S_{both})$ = P(Both of them are selected)

Since selection of each person is an independent event their probabilities multiply each other

⇒ $P(S_{both}) = P(F_S)P(J_S)$

⇒ $P(S_{both}) = \frac{1}{7} \times \frac{1}{5}$

⇒ $P(S_{both}) = \frac{1}{35}$

⇒ $P(S_{one})$ = P(Only one of them is selected)

⇒ $P(S_{one})$ = P(only Fatima is selected) + P(only John is selected)

Since selection of each person is an independent event their probabilities multiply each other

⇒ $P(S_{one}) = \left(P(F_S)P(J_N)\right) + \left(P(F_N)P(J_S)\right)$

⇒ $P(S_{one}) = \left(\frac{1}{7} \times \frac{4}{5}\right) + \left(\frac{6}{7} \times \frac{1}{5}\right)$

⇒ $P(S_{one}) = \frac{10}{35} = \frac{2}{7}$

⇒ $P(S_{none})$ = P(None of them are selected)

Since selection of each person is an independent event their probabilities multiply each other

⇒ $P(S_{none}) = P(F_N)P(J_N)$

⇒ $P(S_{none}) = \frac{6}{7} \times \frac{4}{5}$

⇒ $P(S_{none}) = \frac{24}{35}$

∴ The required probabilities are $\frac{1}{35}, \frac{2}{7}, \frac{24}{35}$.

31. Question

A bag contains 8 marbles of which 3 are blue and 5 are red. One marble is drawn at random, its colour is noted and the marble is replaced in the bag. A marble is again drawn from the bag and its colour is noted. Find the probability that the marble will be

i. Blue followed by red

ii. Blue and red in any order

iii. Of the same colour

Answer

Given:

Bag contains 3 blue and 5 red marbles

It is told that two marbles are drawn with replacement

Let us find the probability of drawing each marble from bag

⇒ P(B) = P(Drawing a Blue Marble)

$\Rightarrow P(B) = \frac{(\text{No.of ways of drawing a blue marble from bag})}{\text{No.of ways of drawing a marble from bag}}$

$\Rightarrow P(B) = \frac{3}{8}$

$\Rightarrow$ P(R) = P(Drawing a Red Marble)

$\Rightarrow P(R) = \frac{(\text{No.of ways of drawing a red marble from bag})}{\text{No.of ways of drawing a marble from bag}}$

$\Rightarrow P(R) = \frac{5}{8}$

We need to find the probability that the marbles drawn:

i. Blue followed by red

ii. Blue and red in any order

iii. Of the same colour

$\Rightarrow$ $P(S_{BR})$ = P(drawing Blue marble followed by Red)

Since drawing a marble is an independent event, the probabilities multiply each other.

$\Rightarrow P(S_{BR}) = \left(P(B)P(R)\right)$

$\Rightarrow P(S_{BR}) = \left(\frac{3}{8} \times \frac{5}{8}\right)$

$\Rightarrow P(S_{BR}) = \frac{15}{64}$

$\Rightarrow$ $P(S_{any})$ = P(drawing Blue and red marble in any order)

$\Rightarrow$ $P(S_{any})$ = P(drawing Blue marble followed by red) + P(drawing Red marble followed by Blue)

Since drawing a marble is an independent event, the probabilities multiply each other.

$\Rightarrow P(S_{any}) = \left(P(B)P(R)\right) + \left(P(R)P(B)\right)$

$\Rightarrow P(S_{any}) = \left(\frac{3}{8} \times \frac{5}{8}\right) + \left(\frac{5}{8} \times \frac{3}{8}\right)$

$\Rightarrow P(S_{any}) = \frac{15}{64} + \frac{15}{64}$

$\Rightarrow P(S_{any}) = \frac{30}{64} = \frac{15}{32}.$

$\Rightarrow$ P(S) = P(drawing two marbles of same colour)

$\Rightarrow$ P(S) = P(drawing black balls from each bag) + (P(drawing white balls from each bag)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(S) = \left(P(B)P(B)\right) + \left(P(R)P(R)\right)$

$\Rightarrow P(S) = \left(\frac{3}{8} \times \frac{3}{8}\right) + \left(\frac{5}{8} \times \frac{5}{8}\right)$

$\Rightarrow P(S) = \frac{9}{64} + \frac{25}{64}$

$\Rightarrow P(S) = \frac{34}{64} = \frac{17}{32}.$

$\therefore$ The required probabilities are $\frac{15}{64}, \frac{15}{32}, \frac{17}{32}$.

32. Question

An urn contains 7 red and 4 blue balls. Two balls are drawn at random with replacement. Find the probability of getting.

i. 2 red balls

ii. 2 blue balls

iii. One red and one blue ball

Answer

Given:

Urn contains 7 red and 4 blue balls

It is told that two balls are drawn with replacement

Let us find the probability of drawing each marble from bag

⇒ P(B) = P(Drawing a Blue Ball)

$\Rightarrow P(B) = \frac{(\text{No.of ways of drawing a blue ball from urn})}{\text{No.of ways of drawing a ball from urn}}$

$\Rightarrow P(B) = \frac{4}{7+4}$

$\Rightarrow P(B) = \frac{4}{11}$

⇒ P(R) = P(Drawing a Red Ball)

$\Rightarrow P(R) = \frac{(\text{No.of ways of drawing a red ball from urn})}{\text{No.of ways of drawing a ball from urn}}$

$\Rightarrow P(R) = \frac{7}{7+4}$

$\Rightarrow P(R) = \frac{7}{11}$

We need to find the probability that the balls drawn:

i. Both in red colour

ii. Both in blue colour

iii. One red and one blue

⇒ $P(S_{RR})$ = P(drawing Two red colour balls)

Since drawing a ball is an independent event, the probabilities multiply each other.

$\Rightarrow P(S_{RR}) = \big(P(R)P(R)\big)$

$\Rightarrow P(S_{RR}) = \left(\frac{7}{11} \times \frac{7}{11}\right)$

$\Rightarrow P(S_{RR}) = \frac{49}{121}$

⇒ $P(S_{BB})$ = P(drawing two blue colour balls)

Since drawing a ball is independent for each bag, the probabilities multiply each other.

$\Rightarrow P(S_{BB}) = \big(P(B)P(B)\big)$

$\Rightarrow P(S_{BB}) = \left(\frac{4}{11} \times \frac{4}{11}\right)$

$\Rightarrow P(S_{BB}) = \frac{16}{121}$

⇒ $P(S_{any})$ = P(drawing Blue and red ball in any order)

⇒ $P(S_{any})$ = P(drawing Blue ball followed by red) + P(drawing Red ball followed by Blue)

Since drawing a ball is an independent event, the probabilities multiply each other.

$\Rightarrow P(S_{any}) = (P(B)P(R)) + (P(R)P(B))$

$\Rightarrow P(S_{any}) = \left(\frac{7}{11} \times \frac{4}{11}\right) + \left(\frac{4}{11} \times \frac{7}{11}\right)$

$\Rightarrow P(S_{any}) = \frac{28}{121} + \frac{28}{121}$

$\Rightarrow P(S_{any}) = \frac{56}{121}.$

∴ The required probabilities are $\frac{49}{121}, \frac{16}{121}, \frac{56}{121}.$

33. Question

A card is drawn from a deck of 52 cards. The outcome is noted, the card is replaced and the deck reshuffled. Another card is then drawn from the deck.

i. What is the probability that both the cards are of the same suit?

ii. What is the probability that the first card is an ace and the second card is a red queen?

Answer

Given that two cards are drawn from the deck with replacement.

We know that there will four suits in a deck and each suit contains 13 cards namely Spades, Hearts, Diamonds, Clubs.

Also, there will total of 4 aces and 2 red queens and 2 black queens

Let us find the probability required:

⇒ $P(D_{same})$ = P(For selecting a card from a single suit out of 52 cards)

$\Rightarrow P(D_{same}) = \frac{13}{52}$

$\Rightarrow P(D_{same}) = \frac{1}{4}$

This probability will be same for all the suits.

$\Rightarrow P(D_S) = P(D_H) = P(D_D) = P(D_C) = \frac{1}{4}$

⇒ $P(D_{ace})$ = P(Drawing an ace)

$\Rightarrow P(D_{ace}) = \frac{4}{52}$

$\Rightarrow P(D_{ace}) = \frac{1}{13}$

⇒ $P(D_{redQ})$ = P(Drawing a red queen)

$\Rightarrow P(D_{redQ}) = \frac{2}{52}$

$\Rightarrow P(D_{redQ}) = \frac{1}{26}$

We need to find the probability of getting:

i. Both cards from same deck

ii. First an ace and second a red queen

⇒ $P(S_{same})$ = P(getting both cards from same deck)

We may get two cards any of the four decks. So, each deck’s probability is taken into consideration.

⇒ $P(S_{same})$ = P(both cards from spade) + P(both cards from hearts) + P(both cards from diamond) + P(both cards from club)

Since drawing a card is an independent event, their probabilities multiply each other.

⇒ $P(S_{same}) = (P(D_S)P(D_S)) + (P(D_H)P(D_H)) + (P(D_D)P(D_D)) + (P(D_C)P(D_C))$

⇒ $P(S_{same}) = \left(\frac{1}{4}\times\frac{1}{4}\right) + \left(\frac{1}{4}\times\frac{1}{4}\right) + \left(\frac{1}{4}\times\frac{1}{4}\right) + \left(\frac{1}{4}\times\frac{1}{4}\right)$

⇒ $P(S_{same}) = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$

⇒ $P(S_{same}) = \frac{4}{16}$

⇒ $P(S_{same}) = \frac{1}{4}$

⇒ $P(S_{AR})$ = P(getting an ace first and followed red queen)

Since drawing a card is an independent event, their probabilities multiply each other.

⇒ $P(S_{AR}) = (P(D_A)P(D_{redQ}))$

⇒ $P(S_{AR}) = \left(\frac{1}{13}\times\frac{1}{26}\right)$

⇒ $P(S_{AR}) = \frac{1}{338}$

∴ The required probabilities are $\frac{1}{4}, \frac{1}{338}$.

34. Question

Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among 100 students, what is the probability that: (i) you both enter the same section? (ii) you both enter the different section?

Answer

Given:

Section A contains 40 students

Section B contains 60 students

It is told that you and your friend are among 100 students

We need to find:

i. You both enter the same section

ii. You both enter the different section

⇒ $P(S_{same})$ = P(You and your friend enter same section)

This happens only in the case if you and your friend belongs to either section A or section B.

⇒ $P(S_{same})$ = P(Both belongs to section A) + P(Both belongs to section B)

⇒ $P(S_{same}) = \frac{\text{No.of ways to select 2 students in section A}}{\text{No.of ways to select 2 students out of 100 students}} + \frac{\text{No.of ways to select 2 students in section B}}{\text{No.of ways to select 2 students out of 100 students}}$

⇒ $P(S_{same}) = \left(\frac{{}^{40}C_2}{{}^{100}C_2}\right) + \left(\frac{{}^{60}C_2}{{}^{100}C_2}\right)$

⇒ $P(S_{same}) = \left(\frac{40\times39}{100\times99}\right) + \left(\frac{60\times59}{100\times99}\right)$

⇒ $P(S_{Same}) = \left(\frac{1560 + 3540}{9900}\right)$

⇒ $P(S_{same}) = \frac{5100}{9900}$

$\Rightarrow P(S_{same}) = \frac{51}{99} = \frac{17}{33}$

$\Rightarrow P(S_{diff})$ = P(You and your friend enter different section)

This happens only in the case if you belongs to A and your friend belongs to section B.

$\Rightarrow P(S_{diff})$ = 1-P(Both belongs to same section)

$\Rightarrow P(S_{diff}) = 1\text{-}P(S_{same})$

$\Rightarrow P(S_{diff}) = 1 - \frac{17}{33}$

$\Rightarrow P(S_{diff}) = \frac{16}{33}$

∴ The required probabilities are $\frac{17}{33}, \frac{16}{33}$.

35. Question

In a hockey match, both teams A and B scored same number of goals up to the end of the game, so to decide the winner, the referee asked both the captains throw a die alternately and decide that the team, whose captain gets a first six, will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match and state whether the decision of the referee was fair or not.

Answer

Given that teams A and B scored same number of goals.

It is asked captains of A and B to throw a die.

The first who throw 6 awarded a prize.

$\Rightarrow P(S_6)$ = P(getting 6)

$\Rightarrow P(S_6) = \frac{1}{6}$

$\Rightarrow P(S_N)$ = P(not getting 6)

$\Rightarrow P(S_N) = 1 - \frac{1}{6}$

$\Rightarrow P(S_N) = \frac{5}{6}$

It is given A starts the game, A wins the game only when he gets 6 while throwing die in $1^{st}, 3^{rd}, 5^{th}$,...... times

Here the probability of getting 6 on throwing a die is same for both the players A and B

Since throwing a die is an independent event, their probabilities multiply each other

$\Rightarrow P(A_{wins}) = P(S_6) + P(S_N)P(S_N)P(S_6) + P(S_N)P(S_N)P(S_N)P(S_N)P(S_6) + \ldots\ldots\ldots\ldots\ldots$

$\Rightarrow P(A_{wins}) = \frac{1}{6} + \left(\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}\right) + \left(\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}\right) + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$\Rightarrow P(A_{wins}) = \frac{1}{6}\times\left(1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \ldots\ldots\ldots\ldots\ldots\right)$

The series in the brackets resembles the Infinite geometric series.

We know that sum of a infinite geometric series with first term ‘a’ and common ratio ‘o’ is $s_\infty = \frac{a}{1-r}$.

$\Rightarrow P(A_{wins}) = \frac{1}{6}\times\left(\frac{1}{1-\left(\frac{5}{6}\right)^2}\right)$

$\Rightarrow P(A_{wins}) = \frac{1}{6} \times \left(\frac{1}{1-\frac{25}{36}}\right)$

$\Rightarrow P(A_{wins}) = \frac{1}{6} \times \left(\frac{1}{\frac{11}{36}}\right)$

$\Rightarrow P(A_{wins}) = \frac{6}{11}$

$\Rightarrow P(B_{wins}) = 1-P(A_{wins})$

$\Rightarrow P(B_{wins}) = 1 - \frac{6}{11}$

$\Rightarrow P(B_{wins}) = \frac{5}{11}$

Since the probabilities if winnings of A and B are not equal, the decision of the referee is not fair

36. Question

A and B throw a pair of die alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins.

Answer

Given that A and B throw a pair of die alternatively.

It is told that A wins if he gets sum of 7 and B wins if he gets sum of 10.

Let us find the probability of getting sum 7 and 10.

The possibilities of getting 7 on throwing a pair of die:

{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)} = 6 cases

The possibilities of getting 10 on throwing a pair of die:

{(4,6),(5,5),(6,4)} = 3 cases

$\Rightarrow P(S_7) = P(\text{getting sum 7})$

$\Rightarrow P(S_7) = \frac{\text{No.of ways of getting sum 7 on throwing a pair of die}}{\text{No.of ways of getting sum on throwing a pair of die}}$

$\Rightarrow P(S_7) = \frac{6}{36}$

$\Rightarrow P(S_7) = \frac{1}{6}$

$\Rightarrow P(S_{N7}) = P(\text{not getting sum 7})$

$\Rightarrow P(S_{N7}) = 1-P(S_7)$

$\Rightarrow P(S_{N7}) = 1 - \frac{1}{6}$

$\Rightarrow P(S_{N7}) = \frac{5}{6}$

$\Rightarrow P(S_{10}) = P(\text{getting sum 10})$

$\Rightarrow P(S_{10}) = \frac{\text{No.of ways of getting sum 10 on throwing a pair of die}}{\text{No.of ways of getting sum on throwing a pair of die}}$

$\Rightarrow P(S_{10}) = \frac{3}{36}$

$\Rightarrow P(S_{10}) = \frac{1}{12}$

$\Rightarrow P(S_{N10}) = P(\text{not getting sum 10})$

⇒ $P(S_{N10}) = 1 - P(S_{10})$

⇒ $P(S_{N10}) = 1 - \frac{1}{12}$

⇒ $P(S_{N10}) = \frac{11}{12}$

It is told that A starts the game.

We need to find the probability that B wins the games.

B wins the game only when A losses in $1^{st}, 3^{rd}, 5^{th}$,...... throws.

This can be shown as follows:

⇒ $P(B_{WINS})$ = P(B wins the game)

⇒ $P(B_{Wins}) = P(S_{N7}S_{10}) + P(S_{N7}S_{N10}S_{N7}S_{10}) + P(S_{N7}S_{N10}S_{N7}S_{N10}S_{N7}S_{10}) + \ldots\ldots\ldots\ldots$

Since throwing a pair of die by each person is an independent event, the probabilities multiply each other.

⇒ $P(B_{wins}) = (P(S_{N7})P(S_{10})) + (P(S_{N7})P(S_{N10})P(S_{N7})P(S_{10})) + \ldots\ldots\ldots\ldots$

⇒ $P(B_{WINS}) = \left(\frac{5}{6} \times \frac{1}{12}\right) + \left(\frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{1}{12}\right) + \left(\frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{1}{12}\right) + \ldots\ldots\ldots\ldots\ldots\ldots$

⇒ $P(B_{WINS}) = \left(\frac{5}{72}\right) \times \left(1 + \left(\frac{55}{72}\right) + \left(\frac{55}{72}\right)^2 + \ldots\ldots\ldots\ldots\ldots\right)$

The terms in the bracket resembles the infinite geometric series sequence:

We know that the sum of a Infinite geometric series with first term 'a' and common ratio 'r' is $s_\infty = \frac{a}{1-r}$

⇒ $P(B_{WINS}) = \left(\frac{5}{72}\right) \times \left(\frac{1}{1-\frac{55}{72}}\right)$

⇒ $P(B_{WINS}) = \frac{5}{72} \times \left(\frac{1}{\frac{17}{72}}\right)$

⇒ $P(B_{WINS}) = \frac{5}{17}$

∴ The required probability is $\frac{5}{17}$.

Exercise 31.6

1. Question

A bag A contains 5 white and 6 black balls. Another bag B contains 4 white and 3 black balls. A ball is transferred from bag A to the bag B, and then a ball is taken out of the second bag. Find the probability of this ball being black.

Answer

Given:

Bag A contains 5 white and 6 black balls.

Bag B contains 4 white and 3 black balls.

A ball is transferred from bag A to bag B, and then a ball is drawn from bag B.

There are two mutually exclusive ways to draw a black ball from bag B –

a. A white ball is transferred from bag A to bag B, and then, a black ball is drawn from bag B

b. A black ball is transferred from bag A to bag B, and then, a black ball is drawn from bag B

Let E_1 be the event that white ball is drawn from bag A and E_2 be the event that black ball is drawn from bag A.

Now, we have

$$P(E_1) = \frac{\text{Number of white balls in bag A}}{\text{Total number of balls in bag A}}$$

$$\Rightarrow P(E_1) = \frac{5}{5+6}$$

$$\therefore P(E_1) = \frac{5}{11}$$

We also have

$$P(E_2) = \frac{\text{Number of black balls in bag A}}{\text{Total number of balls in bag A}}$$

$$\Rightarrow P(E_2) = \frac{6}{5+6}$$

$$\therefore P(E_2) = \frac{6}{11}$$

Let E_3 denote the event that black ball is drawn from bag B.

Hence, we have

$$P(E_3|E_1) = \frac{\text{Number of black balls in bag B after adding a white ball from bag A}}{\text{Total number of balls in bag B after adding a white ball from bag A}}$$

$$\Rightarrow P(E_3|E_1) = \frac{3}{3+5}$$

$$\therefore P(E_3|E_1) = \frac{3}{8}$$

We also have

$$P(E_3|E_2) = \frac{\text{Number of black balls in bag B after adding a black ball from bag A}}{\text{Total number of balls in bag B after adding a black ball from bag A}}$$

$$\Rightarrow P(E_3|E_2) = \frac{4}{4+4}$$

$$\therefore P(E_3|E_2) = \frac{4}{8}$$

Using the theorem of total probability, we get

$P(E_3) = P(E_1)P(E_3|E_1) + P(E_2)P(E_3|E_2)$

$$\Rightarrow P(E_3) = \frac{5}{11} \times \frac{3}{8} + \frac{6}{11} \times \frac{4}{8}$$

$$\Rightarrow P(E_3) = \frac{15}{88} + \frac{24}{88}$$

$$\therefore P(E_3) = \frac{39}{88}$$

Thus, the probability of the drawn ball being black is $\frac{39}{88}$.

2. Question

A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled at random from one of the two purses, what is the probability that it is a silver coin?

Answer

Given:

First purse contains 2 silver and 4 copper coins.

Second purse contains 4 silver and 3 copper coins.

A coin is pulled a random from one of the two purses.

There are two mutually exclusive ways to pull a silver coin from one of the two purses –

a. The first purse is selected, and then, a silver coin is pulled from the first purse

b. The second purse is selected, and then, a silver coin is pulled from the second purse

Let E_1 be the event that the first purse is selected and E_2 be the event that the second purse is selected.

Since there are only two purses and each purse has an equal probability of being selected, we have

$$P(E_1) = P(E_2) = \frac{1}{2}$$

Let E_3 denote the event that a silver coin is pulled.

Hence, we have

$$P(E_3|E_1) = \frac{\text{Number of silver coins in the first purse}}{\text{Total number of coins in the first purse}}$$

$$\Rightarrow P(E_3|E_1) = \frac{2}{2+4}$$

$$\Rightarrow P(E_3|E_1) = \frac{2}{6}$$

$$\therefore P(E_3|E_1) = \frac{1}{3}$$

We also have

$$P(E_3|E_2) = \frac{\text{Number of silver coins in the second purse}}{\text{Total number of coins in the second purse}}$$

$$\Rightarrow P(E_3|E_2) = \frac{4}{4+3}$$

$$\therefore P(E_3|E_2) = \frac{4}{7}$$

Using the theorem of total probability, we get

$P(E_3) = P(E_1)P(E_3|E_1) + P(E_2)P(E_3|E_2)$

$$\Rightarrow P(E_3) = \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{4}{7}$$

$$\Rightarrow P(E_3) = \frac{1}{6} + \frac{2}{7}$$

$$\Rightarrow P(E_3) = \frac{7+12}{42}$$

$$\therefore P(E_3) = \frac{19}{42}$$

Thus, the probability of pulling a silver coin is $\frac{19}{42}$.

3. Question

One bag contains 4 yellow and 5 red balls. Another bag contains 6 yellow and 3 red balls. A ball is transferred from the first bag to the second bag, and then a ball is drawn from the second bag. Find the probability that ball drawn is yellow.

Answer

Given:

The bag I contains 4 yellow and 5 red balls.

Bag II contains 6 yellow and 3 red balls.

A ball is transferred from bag I to bag II and then a ball is drawn from bag II.

There are two mutually exclusive ways to draw a yellow ball from bag II –

a. A yellow ball is transferred from the bag I to bag II, and then, a yellow ball is drawn from bag II

b. A red ball is transferred from the bag I to bag II, and then, a yellow ball is drawn from bag II

Let E_1 be the event that yellow ball is drawn from the bag I and E_2 be the event that red ball is drawn from the bag I.

Now, we have

$$P(E_1) = \frac{\text{Number of yellow balls in the bag I}}{\text{Total number of balls in the bag I}}$$

$$\Rightarrow P(E_1) = \frac{4}{4+5}$$

$$\therefore P(E_1) = \frac{4}{9}$$

We also have

$$P(E_2) = \frac{\text{Number of red balls in the bag I}}{\text{Total number of balls in the bag I}}$$

$$\Rightarrow P(E_2) = \frac{5}{4+5}$$

$$\therefore P(E_2) = \frac{5}{9}$$

Let E_3 denote the event that yellow ball is drawn from bag II.

Hence, we have

$$P(E_3|E_1) = \frac{\text{Number of yellow balls in bag II after adding a yellow ball from bag I}}{\text{Total number of balls in bag II after adding a yellow ball from bag I}}$$

$$\Rightarrow P(E_3|E_1) = \frac{7}{7+3}$$

$$\therefore P(E_3|E_1) = \frac{7}{10}$$

We also have

$$P(E_3|E_2) = \frac{\text{Number of yellow balls in bag II after adding a red ball from bag I}}{\text{Total number of balls in bag II after adding a red ball from bag I}}$$

$$\Rightarrow P(E_3|E_2) = \frac{6}{6+4}$$

$\therefore P(E_3|E_2) = \frac{6}{10}$

Using the theorem of total probability, we get

$P(E_3) = P(E_1)P(E_3|E_1) + P(E_2)P(E_3|E_2)$

$\Rightarrow P(E_3) = \frac{4}{9} \times \frac{7}{10} + \frac{5}{9} \times \frac{6}{10}$

$\Rightarrow P(E_3) = \frac{28}{90} + \frac{30}{90}$

$\Rightarrow P(E_3) = \frac{58}{90}$

$\therefore P(E_3) = \frac{29}{45}$

Thus, the probability of the drawn ball being yellow is $\frac{29}{45}$.

4. Question

A bag contains 3 white and 2 black balls, and another bag contains 2 white and 4 black balls. One bag is chosen at random. From the selected bag, one ball is drawn. Find the probability that the ball drawn is white.

Answer

Given:

The bag I contains 3 white and 2 black balls.

Bag II contains 2 white and 4 black balls.

A bag is chosen, and a ball is drawn from it.

There are two mutually exclusive ways to draw a white ball from one of the two bags –

a. The bag I is selected, and then, a white ball is drawn from the bag I

b. Bag II is selected, and then, a white ball is drawn from bag II

Let E_1 be the event that bag I is selected and E_2 be the event that bag II is selected.

Since there are only two bags and each bag has an equal probability of being selected, we have

$P(E_1) = P(E_2) = \frac{1}{2}$

Let E_3 denote the event that a white ball is drawn.

Hence, we have

$P(E_3|E_1) = \frac{\text{Number of white balls in the bag I}}{\text{Total number of balls in the bag I}}$

$\Rightarrow P(E_3|E_1) = \frac{3}{3+2}$

$\therefore P(E_3|E_1) = \frac{3}{5}$

We also have

$P(E_3|E_2) = \frac{\text{Number of white balls in bag II}}{\text{Total number of balls in bag II}}$

$\Rightarrow P(E_3|E_2) = \frac{2}{2+4}$

$$\Rightarrow P(E_3|E_2) = \frac{2}{6}$$

$$\therefore P(E_3|E_2) = \frac{1}{3}$$

Using the theorem of total probability, we get

$P(E_3) = P(E_1)P(E_3|E_1) + P(E_2)P(E_3|E_2)$

$$\Rightarrow P(E_3) = \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{1}{3}$$

$$\Rightarrow P(E_3) = \frac{3}{10} + \frac{1}{6}$$

$$\Rightarrow P(E_3) = \frac{9+5}{30}$$

$$\Rightarrow P(E_3) = \frac{14}{30}$$

$$\therefore P(E_3) = \frac{7}{15}$$

Thus, the probability of the drawn ball being white is $\frac{7}{15}$.

5. Question

The contents of three bags I, II and III are as follows:

Bag I: 1 white, 2 black and 3 red balls

Bag II: 2 white, 1 black and 1 red ball

Bag III: 4 white, 5 black and 3 red balls

A bag is chosen at random and two balls are drawn. What is the probability that the balls drawn are white and red?

Answer

Given:

Bag I contains 1 white, 2 black and 3 red balls

Bag II contains 2 white, 1 black and 1 red ball

Bag III contains 4 white, 5 black and 3 red balls

A bag is chosen and two balls are drawn from it.

There are three mutually exclusive ways to draw a white and a red ball from one of the three bags –

a. Bag I is selected, and then, a white and a red ball are drawn from bag I

b. Bag II is selected, and then, a white and a red ball are drawn from bag II

c. Bag III is selected, and then, a white and a red ball are drawn from bag III

Let E_1 be the event that bag I is selected, E_2 be the event that bag II is selected and E_3 be the event that bag III is selected.

Since there are only three bags and each bag has an equal probability of being selected, we have

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let E_4 denote the event that a white and a red ball are drawn.

Hence, we have

$$P(E_4|E_1) = \frac{\text{Number of ways of selecting a white and a red ball from bag I}}{\text{Number of ways of selecting two balls from bag I}}$$

$$\Rightarrow P(E_4|E_1) = \frac{\binom{1}{1} \times \binom{3}{1}}{\binom{6}{2}}$$

$$\Rightarrow P(E_4|E_1) = \frac{1 \times 3}{\frac{6 \times 5}{2}}$$

$$\Rightarrow P(E_4|E_1) = \frac{6}{6 \times 5}$$

$$\therefore P(E_4|E_1) = \frac{1}{5}$$

We also have

$$P(E_4|E_2) = \frac{\text{Number of ways of selecting a white and a red ball from bag II}}{\text{Number of ways of selecting two balls from bag II}}$$

$$\Rightarrow P(E_4|E_2) = \frac{\binom{2}{1} \times \binom{1}{1}}{\binom{4}{2}}$$

$$\Rightarrow P(E_4|E_2) = \frac{2 \times 1}{\frac{4 \times 3}{2}}$$

$$\Rightarrow P(E_4|E_2) = \frac{4}{4 \times 3}$$

$$\therefore P(E_4|E_2) = \frac{1}{3}$$

Similarly, we also have

$$P(E_4|E_3) = \frac{\text{Number of ways of selecting a white and a red ball from bag III}}{\text{Number of ways of selecting two balls from bag III}}$$

$$\Rightarrow P(E_4|E_3) = \frac{\binom{4}{1} \times \binom{3}{1}}{\binom{12}{2}}$$

$$\Rightarrow P(E_4|E_3) = \frac{4 \times 3}{\frac{12 \times 11}{2}}$$

$$\Rightarrow P(E_4|E_3) = \frac{24}{12 \times 11}$$

$$\therefore P(E_4|E_3) = \frac{2}{11}$$

Using the theorem of total probability, we get

$P(E_4) = P(E_1)P(E_4|E_1) + P(E_2)P(E_4|E_2) + P(E_3)P(E_4|E_3)$

$$\Rightarrow P(E_4) = \frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{11}$$

$$\Rightarrow P(E_4) = \frac{1}{15} + \frac{1}{9} + \frac{2}{33}$$

$$\Rightarrow P(E_4) = \frac{3+5}{45} + \frac{2}{33}$$

$$\Rightarrow P(E_4) = \frac{8}{45} + \frac{2}{33}$$

$$\Rightarrow P(E_4) = \frac{88+30}{495}$$

$$\therefore P(E_4) = \frac{118}{495}$$

Thus, the probability of the drawn balls being white and red is $\frac{118}{495}$.

6. Question

An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the sum of the numbers obtained is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2,3,4,...,12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8?

Answer

Given:

An unbiased coin is tossed.

If heads occurs, a pair of dice is rolled and sum is noted.

If tails occurs, a number from 2, 3, 4,..., 12 is noted.

There are two mutually exclusive ways to note a number, which is either 7 or 8 –

a. Toss result is a head, and then, sum of the obtained numbers on rolling two dice is 7 or 8

b. Toss result is a tail, and then, a card numbered 7 or 8 is picked from the pack

Let E_1 be the event that the result of the toss is a head and E_2 be the event that the result of the toss is a tail.

Since there are only two outcomes for a coin toss and each outcome has an equal probability of occurring, we have

$$P(E_1) = P(E_2) = \frac{1}{2}$$

Let E_3 denote the event that the noted number is 7 or 8.

Hence, we have

$$P(E_3|E_1) = \frac{\text{Number of ways of obtaining the sum 7 or 8 when two dice are rolled}}{\text{Total number of possible outcomes of sum when two dice are rolled}}$$

When two dice are rolled, we obtain a sum of 7 when the outcomes are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1).

We obtain a sum of 8 when the outcomes are (2, 6), (3, 5), (4, 4), (5, 3) and (6, 2).

Hence, 11 ways out of 36 give the sum 7 or 8.

$$\therefore P(E_3|E_1) = \frac{11}{36}$$

We also have

$$P(E_3|E_2) = \frac{\text{Number of ways of selecting a card with number 7 or 8}}{\text{Total number of ways of selecting a card}}$$

$\therefore P(E_3|E_2) = \frac{2}{11}$

Using the theorem of total probability, we get

$P(E_3) = P(E_1)P(E_3|E_1) + P(E_2)P(E_3|E_2)$

$\Rightarrow P(E_3) = \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{11}$

$\Rightarrow P(E_3) = \frac{11}{72} + \frac{1}{11}$

$\Rightarrow P(E_3) = \frac{121 + 72}{792}$

$\therefore P(E_3) = \frac{193}{792}$

Thus, the probability of the noted number being 7 or 8 is $\frac{193}{792}$.

7. Question

A factory has two machines A and B. Past records show that the machine A produced 60% of the items of output and machine B produce 40% of the items. Further 2% of the items produced by machine A were defective and 1% produced by machine B were defective. If an item is drawn at random, what is the probability that it is defective?

Answer

Given:

Machine A produced 60% of the total items.

Machine B produced 40% of the total items.

2% of the items produced by machine A were defective.

1% of the items produced by machine B were defective.

There are two mutually exclusive ways to draw a defective item produced by one of the two machines –

a. Item was produced by machine A, and then, the item is defective

b. Item was produced by machine B, and then, the item is defective

Let E_1 be the event that the item was produced by machine A and E_2 be the event that the item was produced by machine B.

As 60% of the total items were produced by machine A, we have

$P(E_1) = \frac{60}{100}$

$\therefore P(E_1) = \frac{3}{5}$

Similarly, as 40% of the total items were produced by machine B, we have

$P(E_2) = \frac{40}{100}$

$\therefore P(E_2) = \frac{2}{5}$

Let E_3 denote the event that the item is defective.

Hence, we have

$P(E_3|E_1) = P(\text{item produced by machine A is defective})$

2% of the items produced by machine A are defective.

$$\therefore P(E_3|E_1) = \frac{2}{100}$$

We also have

$P(E_3|E_2) = P(\text{item produced by machine B is defective})$

1% of the items produced by machine A are defective.

$$\therefore P(E_3|E_2) = \frac{1}{100}$$

Using the theorem of total probability, we get

$P(E_3) = P(E_1)P(E_3|E_1) + P(E_2)P(E_3|E_2)$

$$\Rightarrow P(E_3) = \frac{3}{5} \times \frac{2}{100} + \frac{2}{5} \times \frac{1}{100}$$

$$\Rightarrow P(E_3) = \frac{6}{500} + \frac{2}{500}$$

$$\Rightarrow P(E_3) = \frac{8}{500}$$

$$\therefore P(E_3) = \frac{2}{125}$$

Thus, the probability of the item being defective is $\frac{2}{125}$.

8. Question

The bag A contains 8 white and 7 black balls while the bag B contains 5 white and 4 black balls. One balls is randomly picked up from the bag A and mixed up with the balls in bag B. Then a ball is randomly drawn out from it. Find the probability that ball drawn is white.

Answer

Given:

Bag A contains 8 white and 7 black balls.

Bag B contains 5 white and 4 black balls.

A ball is transferred from bag A to bag B and then a ball is drawn from bag B.

There are two mutually exclusive ways to draw a white ball from bag B –

a. A white ball is transferred from bag A to bag B, and then, a white ball is drawn from bag B

b. A black ball is transferred from bag A to bag B, and then, a white ball is drawn from bag B

Let E_1 be the event that white ball is drawn from bag A and E_2 be the event that black ball is drawn from bag A.

Now, we have

$$P(E_1) = \frac{\text{Number of white balls in bag A}}{\text{Total number of balls in bag A}}$$

$$\Rightarrow P(E_1) = \frac{8}{8+7}$$

$$\therefore P(E_1) = \frac{8}{15}$$

We also have

$$P(E_2) = \frac{\text{Number of black balls in bag A}}{\text{Total number of balls in bag A}}$$

$$\Rightarrow P(E_2) = \frac{7}{8+7}$$

$$\therefore P(E_2) = \frac{7}{15}$$

Let E_3 denote the event that white ball is drawn from bag B.

Hence, we have

$$P(E_3|E_1) = \frac{\text{Number of white balls in bag B after adding a white ball from bag A}}{\text{Total number of balls in bag B after adding a white ball from bag A}}$$

$$\Rightarrow P(E_3|E_1) = \frac{6}{6+4}$$

$$\therefore P(E_3|E_1) = \frac{6}{10}$$

We also have

$$P(E_3|E_2) = \frac{\text{Number of white balls in bag B after adding a black ball from bag A}}{\text{Total number of balls in bag B after adding a black ball from bag A}}$$

$$\Rightarrow P(E_3|E_2) = \frac{5}{5+5}$$

$$\therefore P(E_3|E_2) = \frac{5}{10}$$

Using the theorem of total probability, we get

$P(E_3) = P(E_1)P(E_3|E_1) + P(E_2)P(E_3|E_2)$

$$\Rightarrow P(E_3) = \frac{8}{15} \times \frac{6}{10} + \frac{7}{15} \times \frac{5}{10}$$

$$\Rightarrow P(E_3) = \frac{48}{150} + \frac{35}{150}$$

$$\therefore P(E_3) = \frac{83}{150}$$

Thus, the probability of the drawn ball being white is $\frac{83}{150}$.

9. Question

A bag contains 4 white and 5 black balls and another bag contains 3 white and 4 black balls. A ball is taken out from the first bag and without seeing its colour is put in the second bag. A ball is taken out form the latter. Find the probability that the ball drawn is white.

Answer

Given:

Bag I contains 4 white and 5 black balls.

Bag II contains 3 white and 4 black balls.

A ball is transferred from bag I to bag II and then a ball is drawn from bag II.

There are two mutually exclusive ways to draw a white ball from bag II –

a. A white ball is transferred from bag I to bag II, and then, a white ball is drawn from bag II

b. A black ball is transferred from bag I to bag II, and then, a white ball is drawn from bag II

Let E_1 be the event that white ball is drawn from bag I and E_2 be the event that black ball is drawn from bag I.

Now, we have

$$P(E_1) = \frac{\text{Number of white balls in bag I}}{\text{Total number of balls in bag I}}$$

$$\Rightarrow P(E_1) = \frac{4}{4+5}$$

$$\therefore P(E_1) = \frac{4}{9}$$

We also have

$$P(E_2) = \frac{\text{Number of black balls in bag I}}{\text{Total number of balls in bag I}}$$

$$\Rightarrow P(E_2) = \frac{5}{4+5}$$

$$\therefore P(E_2) = \frac{5}{9}$$

Let E_3 denote the event that white ball is drawn from bag II.

Hence, we have

$$P(E_3|E_1) = \frac{\text{Number of white balls in bag II after adding a white ball from bag I}}{\text{Total number of balls in bag II after adding a white ball from bag I}}$$

$$\Rightarrow P(E_3|E_1) = \frac{4}{4+4}$$

$$\therefore P(E_3|E_1) = \frac{4}{8}$$

We also have

$$P(E_3|E_2) = \frac{\text{Number of white balls in bag II after adding a black ball from bag I}}{\text{Total number of balls in bag II after adding a black ball from bag I}}$$

$$\Rightarrow P(E_3|E_2) = \frac{3}{3+5}$$

$$\therefore P(E_3|E_2) = \frac{3}{8}$$

Using the theorem of total probability, we get

$P(E_3) = P(E_1)P(E_3|E_1) + P(E_2)P(E_3|E_2)$

$$\Rightarrow P(E_3) = \frac{4}{9} \times \frac{4}{8} + \frac{5}{9} \times \frac{3}{8}$$

$$\Rightarrow P(E_3) = \frac{16}{72} + \frac{15}{72}$$

$$\therefore P(E_3) = \frac{31}{72}$$

Thus, the probability of the drawn ball being white is $\frac{83}{150}$.

10. Question

One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is white.

Answer

Given:

Bag I contains 4 white and 5 black balls.

Bag II contains 6 white and 7 black balls.

A ball is transferred from bag I to bag II and then a ball is drawn from bag II.

There are two mutually exclusive ways to draw a white ball from bag II –

a. A white ball is transferred from bag I to bag II, and then, a white ball is drawn from bag II

b. A black ball is transferred from bag I to bag II, and then, a white ball is drawn from bag II

Let E_1 be the event that white ball is drawn from bag I and E_2 be the event that black ball is drawn from bag I.

Now, we have

$$P(E_1) = \frac{\text{Number of white balls in bag I}}{\text{Total number of balls in bag I}}$$

$$\Rightarrow P(E_1) = \frac{4}{4+5}$$

$$\therefore P(E_1) = \frac{4}{9}$$

We also have

$$P(E_2) = \frac{\text{Number of black balls in bag I}}{\text{Total number of balls in bag I}}$$

$$\Rightarrow P(E_2) = \frac{5}{4+5}$$

$$\therefore P(E_2) = \frac{5}{9}$$

Let E_3 denote the event that white ball is drawn from bag II.

Hence, we have

$$P(E_3|E_1) = \frac{\text{Number of white balls in bag II after adding a white ball from bag I}}{\text{Total number of balls in bag II after adding a white ball from bag I}}$$

$$\Rightarrow P(E_3|E_1) = \frac{7}{7+7}$$

$$\therefore P(E_3|E_1) = \frac{7}{14}$$

We also have

$$P(E_3|E_2) = \frac{\text{Number of white balls in bag II after adding a black ball from bag I}}{\text{Total number of balls in bag II after adding a black ball from bag I}}$$

$$\Rightarrow P(E_3|E_2) = \frac{6}{6+8}$$

$$\therefore P(E_3|E_2) = \frac{6}{14}$$

Using the theorem of total probability, we get

$P(E_3) = P(E_1)P(E_3|E_1) + P(E_2)P(E_3|E_2)$

$$\Rightarrow P(E_3) = \frac{4}{9} \times \frac{7}{14} + \frac{5}{9} \times \frac{6}{14}$$

$$\Rightarrow P(E_3) = \frac{28}{126} + \frac{30}{126}$$

$$\Rightarrow P(E_3) = \frac{58}{126}$$

$$\therefore P(E_3) = \frac{29}{63}$$

Thus, the probability of the drawn ball being white is $\frac{29}{63}$.

11. Question

An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn from first urn and put into the second urn and then a ball is drawn from the latter. Find the probability that it is a white ball.

Answer

Given:

Urn I contains 10 white and 3 black balls

Urn II contains 3 white and 5 black balls

Two balls are transferred from urn I to urn II and then a ball is drawn from urn II.

There are three mutually exclusive ways to draw a white ball from urn II –

a. Two white balls are transferred from urn I to urn II, and then, a white ball is drawn from urn II

b. Two black balls are transferred from urn I to urn II, and then, a white ball is drawn from urn II

c. A white and a black ball are transferred from urn I to urn II, and then, a white ball is drawn from urn II

Let E_1 be the event that two white balls are drawn from urn I, E_2 be the event that two black balls are drawn from urn I and E_3 be the event that a white and a black ball are drawn from urn I.

Now, we have

$$P(E_1) = \frac{\text{Number of ways of drawing 2 white balls from urn I}}{\text{Number of ways of drawing 2 balls from urn I}}$$

$$\Rightarrow P(E_1) = \frac{\binom{10}{2}}{\binom{13}{2}}$$

$$\Rightarrow P(E_1) = \frac{\frac{10 \times 9}{2}}{\frac{13 \times 12}{2}}$$

$$\Rightarrow P(E_1) = \frac{90}{156}$$

$$\therefore P(E_1) = \frac{15}{26}$$

We also have

$$P(E_2) = \frac{\text{Number of ways of drawing 2 black balls from urn I}}{\text{Number of ways of drawing 2 balls from urn I}}$$

$$\Rightarrow P(E_2) = \frac{\binom{3}{2}}{\binom{13}{2}}$$

$$\Rightarrow P(E_2) = \frac{\frac{3 \times 2}{2}}{\frac{13 \times 12}{2}}$$

$$\Rightarrow P(E_2) = \frac{6}{156}$$

$$\therefore P(E_2) = \frac{1}{26}$$

Similarly, we have

$$P(E_3) = \frac{\text{Number of ways of drawing 1 white and 1 black ball from urn I}}{\text{Number of ways of drawing 2 balls from urn I}}$$

$$\Rightarrow P(E_3) = \frac{\binom{10}{1} \times \binom{3}{1}}{\binom{13}{2}}$$

$$\Rightarrow P(E_3) = \frac{10 \times 3}{\frac{13 \times 12}{2}}$$

$$\Rightarrow P(E_3) = \frac{60}{156}$$

$$\therefore P(E_3) = \frac{10}{26}$$

Let E_4 denote the event that a white is drawn.

Hence, we have

$$P(E_4|E_1) = \frac{\text{Number of white balls in urn II after adding two white balls from urn I}}{\text{Total number of balls in urn II}}$$

$$\Rightarrow P(E_4|E_1) = \frac{5}{5+5}$$

$$\therefore P(E_4|E_1) = \frac{5}{10}$$

We also have

$$P(E_4|E_2) = \frac{\text{Number of white balls in urn II after adding two black balls from urn I}}{\text{Total number of balls in urn II}}$$

$$\Rightarrow P(E_4|E_2) = \frac{3}{3+7}$$

$$\therefore P(E_4|E_2) = \frac{3}{10}$$

Similarly, we also have

$$P(E_4|E_3) = \frac{\text{Number of white balls in urn II after adding a white, a black ball from urn I}}{\text{Total number of balls in urn II}}$$

$$\Rightarrow P(E_4|E_3) = \frac{4}{4+6}$$

$$\therefore P(E_4|E_3) = \frac{4}{10}$$

Using the theorem of total probability, we get

$P(E_4) = P(E_1)P(E_4|E_1) + P(E_2)P(E_4|E_2) + P(E_3)P(E_4|E_3)$

$$\Rightarrow P(E_4) = \frac{15}{26} \times \frac{5}{10} + \frac{1}{26} \times \frac{3}{10} + \frac{10}{26} \times \frac{4}{10}$$

$$\Rightarrow P(E_4) = \frac{75}{260} + \frac{3}{260} + \frac{40}{260}$$

$$\Rightarrow P(E_4) = \frac{118}{260}$$

$$\therefore P(E_4) = \frac{59}{130}$$

Thus, the probability of the drawn ball being white is $\frac{59}{130}$.

12. Question

A bag contains 6 red and 8 black balls and another bag contains 8 red and 6 black balls. A ball is drawn from the first bag and without noticing its colour is put in the second bag. A ball is drawn from the second bag. Find the probability that the ball drawn is red in colour.

Answer

Given:

Bag I contains 6 red and 8 black balls.

Bag II contains 8 red and 6 black balls.

A ball is transferred from bag I to bag II and then a ball is drawn from bag II.

There are two mutually exclusive ways to draw a red ball from bag II –

a. A red ball is transferred from bag I to bag II, and then, a red ball is drawn from bag II

b. A black ball is transferred from bag I to bag II, and then, a red ball is drawn from bag II

Let E_1 be the event that red ball is drawn from bag I and E_2 be the event that black ball is drawn from bag I.

Now, we have

$$P(E_1) = \frac{\text{Number of red balls in bag I}}{\text{Total number of balls in bag I}}$$

$$\Rightarrow P(E_1) = \frac{6}{6+8}$$

$$\Rightarrow P(E_1) = \frac{6}{14}$$

$$\therefore P(E_1) = \frac{3}{7}$$

We also have

$$P(E_2) = \frac{\text{Number of black balls in bag I}}{\text{Total number of balls in bag I}}$$

$$\Rightarrow P(E_2) = \frac{8}{6+8}$$

$\Rightarrow P(E_2) = \frac{8}{14}$

$\therefore P(E_2) = \frac{4}{7}$

Let E_3 denote the event that red ball is drawn from bag II.

Hence, we have

$P(E_3|E_1) = \frac{\text{Number of red balls in bag II after adding a red ball from bag I}}{\text{Total number of balls in bag II after adding a red ball from bag I}}$

$\Rightarrow P(E_3|E_1) = \frac{9}{9+6}$

$\therefore P(E_3|E_1) = \frac{9}{15}$

We also have

$P(E_3|E_2) = \frac{\text{Number of red balls in bag II after adding a black ball from bag I}}{\text{Total number of balls in bag II after adding a black ball from bag I}}$

$\Rightarrow P(E_3|E_2) = \frac{8}{8+7}$

$\therefore P(E_3|E_2) = \frac{8}{15}$

Using the theorem of total probability, we get

$P(E_3) = P(E_1)P(E_3|E_1) + P(E_2)P(E_3|E_2)$

$\Rightarrow P(E_3) = \frac{3}{7} \times \frac{9}{15} + \frac{4}{7} \times \frac{8}{15}$

$\Rightarrow P(E_3) = \frac{27}{105} + \frac{32}{105}$

$\therefore P(E_3) = \frac{59}{105}$

Thus, the probability of the drawn ball being red is $\frac{59}{105}$.

13. Question

Three machines E_1,E_2,E_3 in a certain factory produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the tubes produced one each of machines E_1 and E_2 are defective, and that 5% of those produced on E_3 are defective. If one tube is picked up at random from a day’s production, calculate the probability that it is defective.

Answer

Given:

Machine E_1 produces 50% of the total output.

Machine E_2 produces 25% of the total output.

Machine E_3 produces 25% of the total output.

4% of the tubes produced by machine E_1 are defective.

4% of the tubes produced by machine E_2 are defective.

5% of the tubes produced by machine E_3 are defective.

There are three mutually exclusive ways to pick up a defective tube produced by one of the three machines –

a. Tube was produced by machine E_1, and then, the tube is defective

b. Tube was produced by machine E_2, and then, the tube is defective

c. Tube was produced by machine E_3, and then, the tube is defective

Let X_1 be the event that the tube is produced by machine E_1, X_2 be the event that the tube is produced by machine E_2 and X_3 be the event that the tube is produced by machine E_3.

As 50% of the total output is produced by machine E_1, we have

$$P(X_1) = \frac{50}{100}$$

$$\therefore P(X_1) = \frac{1}{2}$$

Similarly, as each of machines E_2 and E_3 produces 25% of the total tubes, we have

$$P(X_2) = P(X_3) = \frac{25}{100}$$

$$\therefore P(X_2) = P(X_3) = \frac{1}{4}$$

Let X_4 denote the event that the tube is defective.

Hence, we have

$$P(X_4|X_1) = P(\text{tube produced by machine } E_1 \text{ is defective})$$

4% of the tubes produced by machine E_1 are defective. $\Rightarrow P(X_4|X_1) = \frac{4}{100}$

$$\therefore P(X_4|X_1) = \frac{1}{25}$$

Similarly, $P(X_4|X_2) = \frac{1}{25}$

We also have

$$P(X_4|X_1) = P(\text{tube produced by machine } E_3 \text{ is defective})$$

5% of the tubes produced by machine E_3 are defective. $\Rightarrow P(X_4|X_3) = \frac{5}{100}$

$$\therefore P(X_4|X_3) = \frac{1}{20}$$

Using the theorem of total probability, we get

$P(X_4) = P(X_1)P(X_4|X_1) + P(X_2)P(X_4|X_2) + P(X_3)P(X_4|X_3)$

$$\Rightarrow P(X_4) = \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20}$$

$$\Rightarrow P(X_4) = \frac{1}{50} + \frac{1}{100} + \frac{1}{80}$$

$$\Rightarrow P(X_4) = \frac{2+1}{100} + \frac{1}{80}$$

$$\Rightarrow P(X_4) = \frac{3}{100} + \frac{1}{80}$$

$$\Rightarrow P(X_4) = \frac{12+5}{400}$$

$$\therefore P(X_4) = \frac{17}{400}$$

Thus, the probability of the tube being defective is $\frac{17}{400}$.

Exercise 31.7

1. Question

The contents of the urns I,II,III are as follows:

Urn I: 1 white, 2 black and 3 red balls

Urn II: 2 white, 1 black and 1 red balls

Urn III: 4 white, 5 black and 3 red balls.

One urn is chosen at random and two balls are drawn. They happen to be white and red. What is the probability that they come from Urns I,II,III?

Answer

Given:

Urn I has 1 white, 2 black and 3 red balls

Urn II has 2 white, 1 black and 1 red balls

Urn III has 4 white, 5 black and 3 red balls

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing Urn I

U_2 = choosing Urn II

U_3 = choosing Urn III

A = choosing 1 white and 1 red ball from urn

We know that each urn is most likely to choose. So, probability of choosing a urn will be same for every Urn.

$$\Rightarrow P(U_1) = \frac{1}{3}$$

$$\Rightarrow P(U_2) = \frac{1}{3}$$

$$\Rightarrow P(U_3) = \frac{1}{3}$$

The Probability of choosing balls from each Urn differs from Urn to Urn and the probabilities are as follows:

$\Rightarrow P(A|U_1)$ = P(Choosing required balls from Urn 1)

$$\Rightarrow P(A|U_1) = \frac{\text{Number of way of choosing 1 white and 1 red ball from Urn1}}{\text{Number of ways of choosing 2 balls from Urn1}}$$

$$\Rightarrow P(A|U_1) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$$

$$\Rightarrow P(A|U_1) = \frac{1\times 3}{\frac{6\times 5}{1\times 2}}$$

$$\Rightarrow P(A|U_1) = \frac{1}{5}$$

$\Rightarrow P(A|U_2)$ = P(Choosing required balls from Urn 2)

$\Rightarrow P(A|U_2) = \frac{\text{Number of way of choosing 1 white and 1red ball from Urn2}}{\text{Number of ways of choosing 2 balls from Urn2}}$

$\Rightarrow P(A|U_2) = \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2}$

$\Rightarrow P(A|U_2) = \frac{2\times1}{\frac{4\times3}{1\times2}}$

$\Rightarrow P(A|U_2) = \frac{1}{3}$

$\Rightarrow$ P(A|U_3) = P(Choosing required balls from Urn 3)

$\Rightarrow P(A|U_3) = \frac{\text{Number of way of choosing 1 white and 1red ball from Urn3}}{\text{Number of ways of choosing 2 balls from Urn3}}$

$\Rightarrow P(A|U_3) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2}$

$\Rightarrow P(A|U_3) = \frac{4\times3}{\frac{12\times11}{1\times2}}$

$\Rightarrow P(A|U_3) = \frac{2}{11}$

Now we find

P(U_1|A) = P(The chosen balls are from Urn1)

P(U_2|A) = P(The chosen balls are from Urn2)

P(U_3|A) = P(The chosen balls are from Urn3)

Using Baye’s theorem:

$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$

$\Rightarrow P(U_1|A) = \frac{\frac{1}{3}\times\frac{1}{5}}{\left(\frac{1}{3}\times\frac{1}{5}\right) + \left(\frac{1}{3}\times\frac{1}{3}\right) + \left(\frac{1}{3}\times\frac{2}{11}\right)}$

$\Rightarrow P(U_1|A) = \frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$

$\Rightarrow P(U_1|A) = \frac{\frac{1}{5}}{\frac{118}{165}}$

$\Rightarrow P(U_1|A) = \frac{33}{118}$

$\Rightarrow P(U_2|A) = \frac{P(U_2)P(A|U_2)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$

$\Rightarrow P(U_2|A) = \frac{\frac{1}{3}\times\frac{1}{3}}{\left(\frac{1}{3}\times\frac{1}{5}\right) + \left(\frac{1}{3}\times\frac{1}{3}\right) + \left(\frac{1}{3}\times\frac{2}{11}\right)}$

$\Rightarrow P(U_2|A) = \frac{\frac{1}{3}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$

$\Rightarrow P(U_2|A) = \frac{\frac{1}{3}}{\frac{118}{165}}$

$\Rightarrow P(U_2|A) = \frac{55}{118}$

$\Rightarrow P(U_3|A) = \frac{P(U_3)P(A|U_3)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$

$\Rightarrow P(U_3|A) = \frac{\frac{1}{3}\times\frac{2}{11}}{\left(\frac{1}{3}\times\frac{1}{5}\right)+\left(\frac{1}{3}\times\frac{1}{3}\right)+\left(\frac{1}{3}\times\frac{2}{11}\right)}$

$\Rightarrow P(U_3|A) = \frac{\frac{2}{11}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$

$\Rightarrow P(U_3|A) = \frac{\frac{2}{11}}{\frac{118}{165}}$

$\Rightarrow P(U_3|A) = \frac{30}{118}$

∴ The required probabilities are $\frac{33}{118}, \frac{55}{118}, \frac{30}{118}$.

2. Question

A bag A contains 2 white and 3 red balls and a bag B contains 4 white and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag B.

Answer

Given:

Bag A has 2 white and 3 red balls

Bag B has 4 white and 5 red balls

Let us assume B_1, B_2, B_3 and A be the events as follows:

B_1 = choosing Bag I

B_2 = choosing Bag II

A = choosing red ball from Bag

We know that each Bag is most likely to choose. So, probability of choosing a bag will be same for every bag.

$\Rightarrow P(B_1) = \frac{1}{2}$

$\Rightarrow P(B_2) = \frac{1}{2}$

The Probability of choosing balls from each Bag differs from bag to bag and the probabilities are as follows:

⇒ $P(A|B_1)$ = P(Choosing red ball from Bag 1)

$\Rightarrow P(A|B_1) = \frac{\text{Number of way of choosing red ball from bag1}}{\text{Number of ways of choosing a ball from bag1}}$

$\Rightarrow P(A|B_1) = \frac{{}^3C_1}{{}^5C_1}$

$\Rightarrow P(A|B_1) = \frac{3}{5}$

⇒ $P(A|B_2)$ = P(Choosing red ball from Bag 2)

$\Rightarrow P(A|B_2) = \frac{\text{Number of way of choosing a red ball from bag2}}{\text{Number of ways of choosing a ball from bag2}}$

$\Rightarrow P(A|B_2) = \frac{{}^5C_1}{{}^9C_1}$

$\Rightarrow P(A|B_2) = \frac{5}{9}$

Now we find

$P(B_2|A)$ = P(The chosen ball is from bag2)

Using Baye's theorem:

$\Rightarrow P(B_2|A) = \frac{P(B_1)P(A|B_1)}{P(B_1)P(A|B_1)+P(B_2)P(A|B_2)}$

$\Rightarrow P(B_2|A) = \frac{\frac{1}{2}\times\frac{5}{9}}{\left(\frac{1}{2}\times\frac{3}{5}\right)+\left(\frac{1}{2}\times\frac{5}{9}\right)}$

$\Rightarrow P(B_2|A) = \frac{\frac{5}{9}}{\frac{3}{5}+\frac{5}{9}}$

$\Rightarrow P(B_2|A) = \frac{\frac{5}{9}}{\frac{52}{45}}$

$\Rightarrow P(B_2|A) = \frac{25}{52}$

∴ The required probabilities are $\frac{25}{52}$.

3. Question

Three urns contains 2 white and 3 black balls; 3 white and 2 black balls; 4 white and 1 black balls respectively. One ball is drawn from an urn chosen at random and it was found to be white. Find the probability that it was drawn from the first urn.

Answer

Given:

Urn I has 2 white and 3 black balls

Urn II has 3 white and 2 black balls

Urn III has 4 white, 1 black red balls

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing Urn I

U_2 = choosing Urn II

U_3 = choosing Urn III

A = choosing 1 white ball from an urn

We know that each urn is most likely to choose. So, probability of choosing a urn will be same for every Urn.

$\Rightarrow P(U_1) = \frac{1}{3}$

$\Rightarrow P(U_2) = \frac{1}{3}$

$\Rightarrow P(U_3) = \frac{1}{3}$

The Probability of choosing balls from each Urn differs from Urn to Urn and the probabilities are as follows:

$\Rightarrow P(A|U_1)$ = P(Choosing white ball from Urn 1)

$\Rightarrow P(A|U_1) = \frac{\text{Number of way of choosing 1 white ball from Urn1}}{\text{Number of ways of choosing 1 ball from Urn1}}$

$\Rightarrow P(A|U_1) = \frac{{}^2C_1}{{}^5C_1}$

$\Rightarrow P(A|U_1) = \frac{2}{5}$

$\Rightarrow P(A|U_2)$ = P(Choosing white ball from Urn 2)

$\Rightarrow P(A|U_2) = \frac{\text{Number of way of choosing 1 white ball from Urn2}}{\text{Number of ways of choosing 1 balls from Urn2}}$

$\Rightarrow P(A|U_2) = \frac{^3C_1}{^5C_1}$

$\Rightarrow P(A|U_2) = \frac{3}{5}$

$\Rightarrow$ P(A|U_3) = P(Choosing required balls from Urn 3)

$\Rightarrow P(A|U_3) = \frac{\text{Number of way of choosing 1 white ball from Urn3}}{\text{Number of ways of choosing 1 ball from Urn3}}$

$\Rightarrow P(A|U_3) = \frac{^4C_1}{^5C_1}$

$\Rightarrow P(A|U_3) = \frac{4}{5}$

Now we find

P(U_1|A) = P(The chosen balls are from Urn1)

Using Baye’s theorem:

$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$

$\Rightarrow P(U_1|A) = \frac{\frac{1}{3}\times\frac{2}{5}}{\left(\frac{1}{3}\times\frac{2}{5}\right) + \left(\frac{1}{3}\times\frac{3}{5}\right) + \left(\frac{1}{3}\times\frac{4}{5}\right)}$

$\Rightarrow P(U_1|A) = \frac{\frac{2}{5}}{\frac{2}{5}+\frac{3}{5}+\frac{4}{5}}$

$\Rightarrow P(U_1|A) = \frac{\frac{2}{5}}{\frac{9}{5}}$

$\Rightarrow P(U_1|A) = \frac{2}{9}$

∴ The required probabilities is $\frac{2}{9}$.

4. Question

The contents of the three urns are as follows:

Urn I contains 7 white and 3 black balls

Urn II contains 4 white and 6 black balls

Urn III contains 2 white and 8 black balls.

One of these urns are chosen at random probabilities 0.20,0.60 and 0.20 respectively. From the chosen urn two balls are drawn without replacement. If both these balls are white, what is the probability that these came from urn 3?

Answer

Given:

Urn I has 7 white and 3 black balls

Urn II has 4 white and 6 black balls

Urn III has 2 white and 8 black balls

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing Urn I

U_2 = choosing Urn II

U_3 = choosing Urn III

A = choosing 1 white and 1 red ball from urn

From the problem,

$\Rightarrow P(U_1) = 0.20$

$\Rightarrow P(U_2) = 0.60$

$\Rightarrow P(U_3) = 0.20$

The Probability of choosing balls from each Urn differs from Urn to Urn and the probabilities are as follows:

$\Rightarrow$ $P(A|U_1)$ = P(Choosing required balls from Urn 1)

$\Rightarrow P(A|U_1) = \frac{\text{Number of way of choosing 2 white balls from Urn1}}{\text{Number of ways of choosing 2 balls from Urn1}}$

$\Rightarrow P(A|U_1) = \frac{^7C_2}{^{10}C_2}$

$\Rightarrow P(A|U_1) = \frac{\frac{7\times6}{1\times2}}{\frac{10\times9}{1\times2}}$

$\Rightarrow P(A|U_1) = \frac{7}{15}$

$\Rightarrow$ $P(A|U_2)$ = P(Choosing required balls from Urn 2)

$\Rightarrow P(A|U_2) = \frac{\text{Number of way of choosing 2 white balls from Urn2}}{\text{Number of ways of choosing 2 balls from Urn2}}$

$\Rightarrow P(A|U_2) = \frac{^4C_2}{^{10}C_2}$

$\Rightarrow P(A|U_2) = \frac{\frac{(4\times3)}{1\times2}}{\frac{10\times9}{1\times2}}$

$\Rightarrow P(A|U_2) = \frac{2}{15}$

$\Rightarrow$ $P(A|U_3)$ = P(Choosing required balls from Urn 3)

$\Rightarrow P(A|U_3) = \frac{\text{Number of way of choosing 2 white balls from Urn3}}{\text{Number of ways of choosing 2 balls from Urn3}}$

$\Rightarrow P(A|U_3) = \frac{^2C_2}{^{10}C_2}$

$\Rightarrow P(A|U_3) = \frac{1}{\frac{10\times9}{1\times2}}$

$\Rightarrow P(A|U_3) = \frac{1}{45}$

Now we find

$P(U_3|A)$ = P(The chosen balls are from Urn3)

Using Baye's theorem:

$\Rightarrow P(U_3|A) = \frac{P(U_3)P(A|U_3)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$

$\Rightarrow P(U_3|A) = \frac{0.2\times\frac{1}{45}}{\left(0.2\times\frac{7}{15}\right) + \left(0.6\times\frac{2}{15}\right) + \left(0.2\times\frac{1}{45}\right)}$

$\Rightarrow P(U_3|A) = \frac{\frac{1}{225}}{\frac{7}{75}+\frac{2}{25}+\frac{1}{225}}$

$\Rightarrow P(U_3|A) = \frac{\frac{1}{225}}{\frac{8}{45}}$

$\Rightarrow P(U_3|A) = \frac{1}{40}$

∴ The required probabilities is $\frac{1}{40}$.

5. Question

Suppose a girl thrown a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3,4,5 or 6, she tosses a coin once and notes whether a ‘head’ or ‘tail’ is obtained. If she obtained exactly one ‘tail’, what is the probability that she threw 3,4,5 or 6 with the die?

Answer

Given:

Let us assume D_1, D_2 and A be the events as follows:

D_1 = Throwing die and getting 1 or 2

D_2 = Throwing die and getting 3,4,5 or 6

A = getting exactly one tail

$\Rightarrow P(D_1) = \frac{2}{6}$

$\Rightarrow p(D_1) = \frac{1}{3}$

$\Rightarrow P(D_2) = \frac{4}{6}$

$\Rightarrow P(D_2) = \frac{2}{3}$

⇒ $P(A|D_1)$ = P(getting 1 tail on getting 1 or 2 from die)

$\Rightarrow P(A|D_1) = \frac{\text{Number of way of getting 1 tail from tossing coin 3 times}}{\text{Number of ways of getting tails and heads from tossing coin 3 times}}$

$\Rightarrow P(A|D_1) = \frac{{}^3C_1}{2^3}$

$\Rightarrow P(A|D_1) = \frac{3}{8}$

⇒ $P(A|D_2)$ = P(Getting 1 tail on getting 3,4,5 or 6 from die)

$\Rightarrow P(A|D_2) = \frac{\text{Number of way of getting 1 tail from tossing coin 1 time}}{\text{Number of ways of getting head and tail from tossing coin once}}$

$\Rightarrow P(A|D_2) = \frac{{}^1C_1}{2^1}$

$\Rightarrow P(A|D_2) = \frac{1}{2}$

Now we find

$P(D_2|A)$ = P(The tail we get here come after getting 3,4,5 or 6 from die)

Using Baye’s theorem:

$\Rightarrow P(D_2|A) = \frac{P(D_2)P(A|D_2)}{P(D_1)P(A|D_1)+P(D_2)P(A|D_2)}$

$\Rightarrow P(D_2|A) = \frac{\frac{2}{3}\times\frac{1}{2}}{\left(\frac{1}{3}\times\frac{3}{8}\right)+\left(\frac{2}{3}\times\frac{1}{2}\right)}$

$\Rightarrow P(D_2|A) = \frac{\frac{1}{3}}{\frac{1}{8}+\frac{1}{3}}$

$\Rightarrow P(D_2|A) = \frac{\frac{1}{3}}{\frac{11}{24}}$

$\Rightarrow P(D_2|A) = \frac{8}{11}$

∴ The required probability is $\frac{8}{11}$.

6. Question

Two groups are competing for the positions of the Board of Directors of corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

Answer

Let us assume U_1, U2 and A be the events as follows:

U_1 = First team wins

U_2 = Second team wins

A = Introduction of project

From the problem ir is given that,

$\Rightarrow P(U_1) = 0.6$

$\Rightarrow P(U_2) = 0.4$

⇒ P(A|U_1) = P(introduction of project by team 1) = 0.7

⇒ P(A|U_2) = P(introduction of project by team 2) = 0.3

Now we find

P(U_2|A) = P(Project introduced by team 2)

Using Baye’s theorem:

$\Rightarrow P(U_2|A) = \frac{P(U_2)P(A|U_2)}{P(U_1)P(A|U_1)+P(U_2)P(A|U_2)}$

$\Rightarrow P(U_2|A) = \frac{0.4\times0.3}{(0.6\times0.7)+(0.4\times0.3)}$

$\Rightarrow P(U_2|A) = \frac{0.12}{0.42+0.12}$

$\Rightarrow P(U_2|A) = \frac{0.12}{0.54}$

$\Rightarrow P(U_2|A) = \frac{2}{9}$

∴ The required probability is $\frac{2}{9}$.

7. Question

Suppose 5 men out of 100 and 25 women out of 1000 are good orators. An orator is chosen at random. Find the probability that a male person is selected. Assume that there are equal number of men and women.

Answer

Let us assume U_1, U_2 and A be the events as follows:

U_1 = choosing men

U_2 = choosing women

A = choosing orator

From the problems it is clear that men and women are equal in number. So, probability of choosing a men and women will be same.

$\Rightarrow P(U_1) = \frac{1}{2}$

$\Rightarrow P(U_2) = \frac{1}{2}$

From the problem:

$\Rightarrow P(A|U_1)$ = P(Choosing orator from men) = $\frac{5}{100}$

$\Rightarrow P(A|U_2)$ = P(Choosing orator from women) = $\frac{25}{1000}$

Now we find

$P(U_1|A)$ = P(The chosen orator is men)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{2}\times\frac{5}{100}}{\left(\frac{1}{2}\times\frac{5}{100}\right)+\left(\frac{1}{2}\times\frac{25}{1000}\right)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{40}}{\frac{1}{40}+\frac{1}{80}}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{40}}{\frac{3}{80}}$$

$$\Rightarrow P(U_1|A) = \frac{2}{3}$$

$\therefore$ The required probability is $\frac{2}{3}$.

8. Question

A letter is known to have come either from LONDON or CLIFTON. On the envelope just two consecutive letters are visible. What is the probability that the letter has come from

i. LONDON

ii. CLIFTON

Answer

Let us assume U_1, U_2 and A be the events as follows:

U_1 = Letter comes from LONDON

U_2 = Letter comes from CLIFTON

A = Event of getting two visible consecutive letters are ON

We know that letter comes from each place is most likely to occur. So, probability of getting letter will be same from each place.

$\Rightarrow P(U_1) = \frac{1}{2}$

$\Rightarrow P(U_2) = \frac{1}{2}$

Now,

$\Rightarrow P(A|U_1) = P(\text{visible two letters ON from london })$

$\Rightarrow P(A|U_1) = \frac{\text{Number of way of getting consecutive letters ON from LONDON}}{\text{Number of ways of getting consecutive letters from LONDON}}$

$\Rightarrow P(A|U_1) = \frac{2}{5}$

$\Rightarrow P(A|U_2) = P(\text{visible two letters ON from CLIFTON})$

$\Rightarrow P(A|U_2) = \frac{\text{Number of way of gettingconsecutive ON from CLIFTON}}{\text{Number of ways of getting consecutive letters from CLIFTON}}$

$\Rightarrow P(A|U_2) = \frac{1}{6}$

Now we find

$P(U_1|A) = P(\text{The envelope comes from LONDON})$

$P(U_2|A) = P(\text{The envelope comes from CLIFTON})$

Using Baye's theorem:

$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$

$\Rightarrow P(U_1|A) = \frac{\frac{1}{2}\times\frac{2}{5}}{\left(\frac{1}{2}\times\frac{2}{5}\right) + \left(\frac{1}{2}\times\frac{1}{6}\right)}$

$\Rightarrow P(U_1|A) = \frac{\frac{1}{5}}{\frac{1}{5} + \frac{1}{12}}$

$\Rightarrow P(U_1|A) = \frac{\frac{1}{5}}{\frac{17}{60}}$

$\Rightarrow P(U_1|A) = \frac{12}{17}$

$\Rightarrow P(U_2|A) = \frac{P(U_2)P(A|U_2)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$

$\Rightarrow P(U_2|A) = \frac{\frac{1}{2}\times\frac{1}{6}}{\left(\frac{1}{2}\times\frac{2}{5}\right) + \left(\frac{1}{2}\times\frac{1}{6}\right)}$

$\Rightarrow P(U_2|A) = \frac{\frac{1}{12}}{\frac{1}{5} + \frac{1}{12}}$

$\Rightarrow P(U_2|A) = \frac{\frac{1}{12}}{\frac{17}{60}}$

$\Rightarrow P(U_2|A) = \frac{5}{17}$

The required probabilities are $\frac{12}{17}, \frac{5}{17}$.

9. Question

In a class 5% of the boys and 10% of the girls have an IQ of more than 150. In this class, 60% of the students are boys. If a student is selected at random and found to have an IQ of more than 150, find the probability that the student is a boy.

Answer

Let us assume U_1, U_2 and A be the events as follows:

U_1 = choosing boy

U_2 = choosing girl

A = choosing a student with IQ more than 150

From the problem:

$\Rightarrow P(U_1) = 0.6$

$\Rightarrow P(U_2) = 0.4$

$\Rightarrow P(A|U_1)$ = P(Boy whose IQ is more than 150)

$\Rightarrow P(A|U_1) = 0.05$

$\Rightarrow P(A|U_2)$ = P(Girl whose IQ is more than 150)

$\Rightarrow P(A|U_2) = 0.1$

Now we find

$P(U_1|A)$ = P(The choosen student whose IQ is more than 150 is a boy)

Using Baye's theorem:

$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$

$\Rightarrow P(U_1|A) = \frac{0.6\times 0.05}{(0.6\times 0.05) + (0.4\times 0.1)}$

$\Rightarrow P(U_1|A) = \frac{0.03}{0.03 + 0.04}$

$\Rightarrow P(U_1|A) = \frac{0.03}{0.07}$

$\Rightarrow P(U_1|A) = \frac{3}{7}$

$\therefore$ The required probabilities is $\frac{3}{7}$.

10. Question

A factory has three machines X,Y and Z producing 1000, 2000 and 3000 bolts per day respectively. The machine X produces 1% defectives bolts, Y produces 1.5% and Z produces 2% defective bolts. At the end of a day, a bolt is drawn at random and is found to be defective. What is the probability that this defective bolt has been produced by machine X?

Answer

Given:

Factory X has produced 1000 bolts per day

Factory Y has produced 2000 bolts per day

Factory Z has produced 3000 bolts per day.

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing factory X

U_2 = choosing factory Y

U_3 = choosing factory Z

A = choosing a defective bolt

$$\Rightarrow P(U_1) = \frac{1000}{1000+2000+3000} = \frac{1}{6}$$

$$\Rightarrow P(U_2) = \frac{2000}{1000+2000+3000} = \frac{2}{6}$$

$$\Rightarrow P(U_3) = \frac{3000}{1000+2000+3000} = \frac{3}{6}$$

From the problem:

⇒ $P(A|U_1)$ = P(Choosing defective bolt from factory X)

$$\Rightarrow P(A|U_1) = 0.01$$

⇒ $P(A|U_2)$ = P(Choosing defective bolt from factory Y)

$$\Rightarrow P(A|U_2) = 0.015$$

⇒ $P(A|U_3)$ = P(Choosing defective bolt from factory Z)

$$\Rightarrow P(A|U_3) = 0.02$$

Now we find

$P(U_1|A)$ = P(The chosen bolt is from factory X)

Using Baye’s theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{6}\times 0.01}{\left(\frac{1}{6}\times 0.01\right) + \left(\frac{2}{6}\times 0.015\right) + \left(\frac{3}{6}\times 0.02\right)}$$

$$\Rightarrow P(U_1|A) = \frac{0.01}{0.01 + 0.03 + 0.06}$$

$$\Rightarrow P(U_1|A) = \frac{0.01}{0.1}$$

$$\Rightarrow P(U_1|A) = \frac{1}{10}$$

∴ The required probability is $\frac{1}{10}$.

11. Question

An insurance company insured 3000 scooters, 4000 cars and 5000 trucks. The probabilities of the accident involving a scooter, a car and a truck are 0.02, 0.03 and 0.04 respectively. One of the insured vehicles meet with and accident. Find the probability that it is a (i) scooter (ii) car (iii) truck.

Answer

Given:

Company has 3000 scooters, 4000 cars and 5000 trucks.

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing scooter

U_2 = choosing car

U_3 = choosing truck

A = Involving in accident

$$\Rightarrow P(U_1) = \frac{3000}{3000+4000+5000} = \frac{3}{12}$$

$\Rightarrow P(U_2) = \frac{4000}{3000+4000+5000} = \frac{4}{12}$

$\Rightarrow P(U_3) = \frac{5000}{3000+4000+5000} = \frac{5}{12}$

From the problem:

⇒ $P(A|U_1)$ = P(scooter involving in accident)

$\Rightarrow P(A|U_1) = 0.02$

⇒ $P(A|U_2)$ = P(car involving in a accident)

$\Rightarrow P(A|U_2) = 0.03$

⇒ $P(A|U_3)$ = P(truck involving in accident)

$\Rightarrow P(A|U_3) = 0.04$

Now we find

$P(U_1|A)$ = P(The vehicle which met with accident is a scooter)

$P(U_2|A)$ = P(The vehicle which met with accident is a car)

$P(U_3|A)$ = P(The vehicle which met with accident is a truck)

Using Baye's theorem:

$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1)+P(U_2)P(A|U_2)+P(U_3)P(A|U_3)}$

$\Rightarrow P(U_1|A) = \frac{\frac{3}{12}\times 0.02}{\left(\frac{3}{12}\times 0.02\right)+\left(\frac{4}{12}\times 0.03\right)+\left(\frac{5}{12}\times 0.04\right)}$

$\Rightarrow P(U_1|A) = \frac{0.06}{0.06+0.12+0.20}$

$\Rightarrow P(U_1|A) = \frac{0.06}{0.38}$

$\Rightarrow P(U_1|A) = \frac{3}{19}$

$\Rightarrow P(U_2|A) = \frac{P(U_2)P(A|U_2)}{P(U_1)P(A|U_1)+P(U_2)P(A|U_2)+P(U_3)P(A|U_3)}$

$\Rightarrow P(U_2|A) = \frac{\frac{4}{12}\times 0.03}{\left(\frac{3}{12}\times 0.02\right)+\left(\frac{4}{12}\times 0.03\right)+\left(\frac{5}{12}\times 0.04\right)}$

$\Rightarrow P(U_2|A) = \frac{0.12}{0.06+0.12+0.20}$

$\Rightarrow P(U_2|A) = \frac{0.12}{0.38}$

$\Rightarrow P(U_2|A) = \frac{6}{19}$

$\Rightarrow P(U_3|A) = \frac{P(U_3)P(A|U_3)}{P(U_1)P(A|U_1)+P(U_2)P(A|U_2)+P(U_3)P(A|U_3)}$

$\Rightarrow P(U_3|A) = \frac{\frac{5}{12}\times 0.04}{\left(\frac{3}{12}\times 0.02\right)+\left(\frac{4}{12}\times 0.03\right)+\left(\frac{5}{12}\times 0.04\right)}$

$\Rightarrow P(U_3|A) = \frac{0.20}{0.06+0.12+0.20}$

$\Rightarrow P(U_3|A) = \frac{0.20}{0.38}$

$\Rightarrow P(U_3|A) = \frac{10}{19}$

∴ The required probabilities are $\frac{3}{19}, \frac{6}{19}, \frac{10}{19}$.

12. Question

Suppose we have four boxes A,B,C,D containing coloured marbles as given below:

Box	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn form box A? box B? box C?

Answer

Let us assume U_1, U_2, U_3, U_4 and A be the events as follows:

U_1 = choosing Box A

U_2 = choosing Box B

U_3 = choosing Box C

U_4 = choosing Box D

A = choosing red marble from box

We know that each box is most likely to choose. So, probability of choosing a box will be same for every Urn.

$\Rightarrow P(U_1) = \frac{1}{4}$

$\Rightarrow P(U_2) = \frac{1}{4}$

$\Rightarrow P(U_3) = \frac{1}{4}$

$\Rightarrow P(U_4) = \frac{1}{4}$

The Probability of choosing marble from each box differs from box to box and the probabilities are as follows:

$\Rightarrow P(A|U_1)$ = P(Choosing red marble from Box A)

$\Rightarrow P(A|U_1) = \frac{\text{Number of way of choosing red marble from BoxA}}{\text{Number of ways of choosing marble from BoxA}}$

$\Rightarrow P(A|U_1) = \frac{{}^{1}C_1}{{}^{10}C_1}$

$\Rightarrow P(A|U_1) = \frac{1}{10}$

$\Rightarrow P(A|U_2)$ = P(Choosing red marble from Box B)

$\Rightarrow P(A|U_2) = \frac{\text{Number of way of choosing red marble from Box B}}{\text{Number of ways of choosing marble from Box B}}$

$\Rightarrow P(A|U_2) = \frac{{}^{6}C_1}{10C_1}$

$\Rightarrow P(A|U_2) = \frac{6}{10}$

$\Rightarrow P(A|U_3)$ = P(Choosing red marble from Box C)

$$\Rightarrow P(A|U_3) = \frac{\text{Number of way of choosing red marble from Box C}}{\text{Number of ways of choosing Marble from BoxC}}$$

$$\Rightarrow P(A|U_3) = \frac{{}^{8}C_1}{{}^{10}C_1}$$

$$\Rightarrow P(A|U_3) = \frac{8}{10}$$

Since there is no red marbles in Box D, the probability of choosing red marble is 0.

i.e, $P(A|U_4) = 0$

Now we find

$P(U_1|A)$ = P(The chosen red marble is from Box A)

$P(U_2|A)$ = P(The chosen red marble is from Box B)

$P(U_3|A)$ = P(The chosen red marble is from Box C)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3) + P(U_4)P(A|U_4)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{4}\times\frac{1}{10}}{\left(\frac{1}{4}\times\frac{1}{10}\right) + \left(\frac{1}{4}\times\frac{6}{10}\right) + \left(\frac{1}{4}\times\frac{8}{10}\right) + \left(\frac{1}{4}\times 0\right)}$$

$$\Rightarrow P(U_1|A) = \frac{1}{1+6+8+0}$$

$$\Rightarrow P(U_1|A) = \frac{1}{15}$$

$$\Rightarrow P(U_2|A) = \frac{P(U_2)P(A|U_2)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3) + P(U_4)P(A|U_4)}$$

$$\Rightarrow P(U_2|A) = \frac{\frac{1}{4}\times\frac{6}{10}}{\left(\frac{1}{4}\times\frac{1}{10}\right) + \left(\frac{1}{4}\times\frac{6}{10}\right) + \left(\frac{1}{4}\times\frac{8}{10}\right) + \left(\frac{1}{4}\times 0\right)}$$

$$\Rightarrow P(U_2|A) = \frac{6}{1+6+8}$$

$$\Rightarrow P(U_2|A) = \frac{6}{15}$$

$$\Rightarrow P(U_3|A) = \frac{P(U_3)P(A|U_3)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3) + P(U_4)P(A|U_4)}$$

$$\Rightarrow P(U_3|A) = \frac{\frac{1}{4}\times\frac{8}{10}}{\left(\frac{1}{4}\times\frac{1}{10}\right) + \left(\frac{1}{4}\times\frac{6}{10}\right) + \left(\frac{1}{4}\times\frac{8}{10}\right) + \left(\frac{1}{4}\times 0\right)}$$

$$\Rightarrow P(U_3|A) = \frac{8}{1+6+8+0}$$

$$\Rightarrow P(U_3|A) = \frac{8}{15}$$

∴ The required probabilities are $\frac{1}{15}, \frac{6}{15}, \frac{8}{15}$.

13. Question

A manufacturer has three machine operators A,B and C. The first operator A produces 1% defective items, whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job for 30% of the time and C on the job for 20% of the time. A defective item is produced. What is the probability that it was produced by A?

Answer

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing Operator A

U_2 = choosing Operator B

U_3 = choosing Operator C

A = Producing defective item

From the problem:

⇒ $P(U_1) = 0.5$

⇒ $P(U_2) = 0.3$

⇒ $P(U_3) = 0.2$

⇒ $P(A|U_1)$ = P(Producing defective item by operator A)

⇒ $P(A|U_1) = 0.01$

⇒ $P(A|U_2)$ = P(Producing defective item by operator B)

⇒ $P(A|U_2) = 0.05$

⇒ $P(A|U_3)$ = P(Producing defective item by operator C)

⇒ $P(A|U_3) = 0.07$

Now we find

$P(U_1|A)$ = P(The defective item is produced by operator A)

Using Baye’s theorem:

⇒ $P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$

⇒ $P(U_1|A) = \frac{0.5 \times 0.01}{(0.5 \times 0.01) + (0.3 \times 0.05) + (0.2 \times 0.07)}$

⇒ $P(U_1|A) = \frac{5}{5 + 15 + 14}$

⇒ $P(U_1|A) = \frac{5}{34}$

∴ The required probability is $\frac{5}{34}$.

14. Question

An item is manufactured by three machines A,B and C. Out of the total number of items manufactured during a specified period, 50% are manufacture on machine A, 30% on B and 20% on C. 2% of the items produced on A and 2% of items produced on B are defective and 3% of these produced on C are defective. All the items stored at one godown. One item is drawn at random and is found to be defective. What is the probability that it was manufactured on machine A?

Answer

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing Machine A

U_2 = choosing Machine B

U_3 = choosing Machine C

A = manufacturing defective item

⇒ $P(U_1) = 0.5$

$\Rightarrow P(U_2) = 0.3$

$\Rightarrow P(U_3) = 0.2$

$\Rightarrow P(A|U_1) = P(\text{Manufacturing defective item from machine A})$

$\Rightarrow P(A|U_1) = 0.02$

$\Rightarrow P(A|U_2) = P(\text{Manufacturing defective item from machine B})$

$\Rightarrow P(A|U_2) = 0.02$

$\Rightarrow P(A|U_3) = P(\text{Manufacturing defective item from machine C})$

$\Rightarrow P(A|U_3) = 0.03$

Now we find

$P(U_1|A) = P(\text{The defective item is manufactured from machine A})$

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_1|A) = \frac{0.5\times0.02}{(0.5\times0.02) + (0.3\times0.02) + (0.2\times0.03)}$$

$$\Rightarrow P(U_1|A) = \frac{0.01}{0.01 + 0.006 + 0.006}$$

$$\Rightarrow P(U_1|A) = \frac{0.01}{0.022}$$

$$\Rightarrow P(U_1|A) = \frac{10}{22} = \frac{5}{11}$$

∴ The required probability is $\frac{5}{11}$.

15. Question

There are three coins. One is two - headed coin (having head on both faces), another is biased coin that comes up heads 75% of the times the third is also a biased coin that comes up tail 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two - headed coin?

Answer

Given:

Coin 1 has heads on both sides

Coin 2 and 3 are biased coins

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing coin 1

U_2 = choosing coin 2

U_3 = choosing coin 3

A = getting head on tossing the coin

We know that each coin is most likely to choose. So, probability of choosing a coin will be same for every coin.

$\Rightarrow P(U_1) = \frac{1}{3}$

$\Rightarrow P(U_2) = \frac{1}{3}$

$\Rightarrow P(U_3) = \frac{1}{3}$

From the problem:

$\Rightarrow P(A|U_1)$ = P(getting head on tossing coin 1)

$\Rightarrow P(A|U_1) = 1$

$\Rightarrow P(A|U_2)$ = P(getting head on tossing coin 2)

$\Rightarrow P(A|U_2) = 0.75$

$\Rightarrow P(A|U_3)$ = P(getting head on tossing coin 3)

$\Rightarrow P(A|U_3) = 0.40$

Now we find

$P(U_1|A)$ = P(The head we get after is from tossing coin 1)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{3}\times 1}{\left(\frac{1}{3}\times 1\right) + \left(\frac{1}{3}\times 0.75\right) + \left(\frac{1}{3}\times 0.40\right)}$$

$$\Rightarrow P(U_1|A) = \frac{1}{1 + 0.75 + 0.40}$$

$$\Rightarrow P(U_1|A) = \frac{1}{2.25}$$

$$\Rightarrow P(U_1|A) = \frac{4}{9}$$

∴ The required probability is $\frac{4}{9}$.

16. Question

In a factory, machine A produces 30% of the total output, machine B produces 25% and the machine C produces the remaining output. If defective items produced by machines A, B, C are 1%,1.2%, 2% respectively. Three machines working together produce 10000 items in a day. An item is drawn at random form a day's output and found to be defective. Find the probability that it was produced by machine B?

Answer

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing Machine A

U_2 = choosing Machine B

U_3 = choosing Machine C

A = Producing a defective output

From the problem:

$\Rightarrow P(U_1) = 0.3$

$\Rightarrow P(U_2) = 0.25$

$\Rightarrow P(U_3) = 0.45$

$\Rightarrow P(A|U_1)$ = P(Producing defective output from Machine A)

$\Rightarrow P(A|U_1) = 0.01$

⇒ $P(A|U_2)$ = P(Producing defective output from Machine B)

$$\Rightarrow P(A|U_2) = 0.012$$

⇒ $P(A|U_3)$ = P(Producing defective output from Machine C)

$$\Rightarrow P(A|U_3) = 0.02$$

Now we find

$P(U_2|A)$ = P(The found defective item is produced by Machine B)

Using Baye's theorem:

$$\Rightarrow P(U_2|A) = \frac{P(U_2)P(A|U_2)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_2|A) = \frac{0.25 \times 0.012}{(0.3 \times 0.01) + (0.25 \times 0.012) + (0.45 \times 0.02)}$$

$$\Rightarrow P(U_2|A) = \frac{300}{300 + 300 + 900}$$

$$\Rightarrow P(U_2|A) = \frac{300}{1500}$$

$$\Rightarrow P(U_2|A) = \frac{1}{5}$$

∴ The required probability is $\frac{1}{5}$.

17. Question

A company has two plants to manufacture bicycles. The first plant manufactures 60% of the bicycles and the second plant 40%. Out of that 80% of the bicycles are rated of standard quality at the first plant and 90% of standard quality at the second plant. A bicycle is picked up at random and found to be standard quality. Find the probability that it comes from the second plant.

Answer

Let us assume U_1, U_2 and A be the events as follows:

U_1 = Choosing first plant to manufacture bicycles

U_2 = choosing second plant to manufacture bicycles

A = Picking standard quality cycle

From the Problem

$$\Rightarrow P(U_1) = 0.6$$

$$\Rightarrow P(U_2) = 0.4$$

⇒ $P(A|U_1)$ = P(Picking standard quality cycle from first plant)

$$\Rightarrow P(A|U_1) = 0.8$$

⇒ $P(A|U_2)$ = P(Picking standard quality cycle from second plant)

$$\Rightarrow P(A|U_2) = 0.9$$

Now we find

$P(U_2|A)$ = P(The chosen standard quality cycle is from second plant)

Using Baye's theorem:

$$\Rightarrow P(U_2|A) = \frac{P(U_2)P(A|U_2)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$$

$$\Rightarrow P(U_2|A) = \frac{0.4\times0.9}{(0.6\times0.8)+(0.4\times0.9)}$$

$$\Rightarrow P(U_2|A) = \frac{0.36}{0.48+0.36}$$

$$\Rightarrow P(U_2|A) = \frac{0.36}{0.84}$$

$$\Rightarrow P(U_2|A) = \frac{3}{7}$$

∴ The required probability is $\frac{3}{7}$.

18. Question

Three urns A,B and C contain 6 red and 4 white; 2 red and 6 white; and 1 red and 5 white balls respectively. An urn is chosen at random and a ball is drawn. If the ball drawn is found to be red, find the probability that the ball was drawn from urn A.

Answer

Given:

Urn A has 6 red and 4 white balls

Urn B has 2 red and 6 white balls

Urn C has 1 red and 5 white balls

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing Urn A

U_2 = choosing Urn B

U_3 = choosing Urn C

A = choosing red ball from urn

We know that each urn is most likely to choose. So, probability of choosing a urn will be same for every Urn.

$$\Rightarrow P(U_1) = \frac{1}{3}$$

$$\Rightarrow P(U_2) = \frac{1}{3}$$

$$\Rightarrow P(U_3) = \frac{1}{3}$$

The Probability of choosing balls from each Urn differs from Urn to Urn and the probabilities are as follows:

⇒ $P(A|U_1)$ = P(Choosing red ball from Urn A)

$$\Rightarrow P(A|U_1) = \frac{\text{Number of way of choosing red ball from UrnA}}{\text{Number of ways of choosing a balls from UrnA}}$$

$$\Rightarrow P(A|U_1) = \frac{{}^{6}C_1}{{}^{10}C_1}$$

$$\Rightarrow P(A|U_1) = \frac{6}{10}$$

⇒ $P(A|U_2)$ = P(Choosing red ball from Urn B)

$$\Rightarrow P(A|U_2) = \frac{\text{Number of way of choosing red ball from UrnB}}{\text{Number of ways of choosing a balls from UrnB}}$$

$$\Rightarrow P(A|U_2) = \frac{{}^{2}C_1}{{}^{8}C_1}$$

$$\Rightarrow P(A|U_2) = \frac{2}{8}$$

$\Rightarrow$ P(A|U_3) = P(Choosing red ball from Urn C)

$$\Rightarrow P(A|U_3) = \frac{\text{Number of way of choosing red ball from UrnC}}{\text{Number of ways of choosing a balls from UrnC}}$$

$$\Rightarrow P(A|U_3) = \frac{{}^1C_1}{{}^6C_1}$$

$$\Rightarrow P(A|U_3) = \frac{1}{6}$$

Now we find

P(U_1|A) = P(The red ball is from Urn A)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{3}\times\frac{6}{10}}{\left(\frac{1}{3}\times\frac{6}{10}\right) + \left(\frac{1}{3}\times\frac{2}{8}\right) + \left(\frac{1}{3}\times\frac{1}{6}\right)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{6}{10}}{\frac{6}{10}+\frac{2}{8}+\frac{1}{6}}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{6}{10}}{\frac{61}{60}}$$

$$\Rightarrow P(U_1|A) = \frac{36}{61}$$

$\therefore$ The required probability is $\frac{36}{61}$.

19. Question

In a group of 400 people, 160 are smokers and non - vegetarian, 100 are smokers and vegetarian and the remaining are non - smokers and vegetarian. The probabilities of getting a special chest disease are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the disease. What is the probability that the selected person is a smoker and non - vegetarian?

Answer

Given:

From 400 people

Smokers and non - vegetarian are 160

Smokers and vegetarian are 100

Non - smokers and vegetarian are 140

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing Smokers and non - vegetarian

U_2 = choosing Smokers and vegetarian

U_3 = choosing Non - smokers and vegetarian

A = getting special chest disease

From the problem

$$\Rightarrow P(U_1) = \frac{160}{400} = \frac{2}{5}$$

$$\Rightarrow P(U_2) = \frac{100}{400} = \frac{1}{4}$$

$\Rightarrow P(U_3) = \frac{140}{400} = \frac{7}{20}$

$\Rightarrow P(A|U_1)$ = P(Smoker and non - vegetarian getting Chest disease)

$\Rightarrow P(A|U_1) = 0.35$

$\Rightarrow P(A|U_2)$ = P(Smoker and vegetarian getting chest disease)

$\Rightarrow P(A|U_2) = 0.20$

$\Rightarrow P(A|U_3)$ = P(Non - smoker and vegetarian getting chest disease)

$\Rightarrow P(A|U_3) = 0.10$

Now we find

$P(U_1|A)$ = P(The selected chest diseased person is a smoker and non - vegetarian)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{2}{5}\times 0.35}{\left(\frac{2}{5}\times 0.35\right) + \left(\frac{1}{4}\times 0.20\right) + \left(\frac{7}{20}\times 0.10\right)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{7}{50}}{\frac{7}{50} + \frac{1}{20} + \frac{7}{200}}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{7}{50}}{\frac{9}{40}}$$

$$\Rightarrow P(U_1|A) = \frac{28}{45}$$

∴ The required probability is $\frac{28}{45}$.

20. Question

A factory has three machines A,B and C, which produce 100,200 and 300 items of a particular type daily. The machines produce 2%, 3% and 5% defective items respectively. One day when the production was over, and item was picked up randomly and it was found to be defective. Find the probability that it was produced by machine A.

Answer

Given:

Machine A produce 100 items

Machine B produce 200 items

Machine C produce 300 items

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing Machine A

U_2 = choosing Machine B

U_3 = choosing Machine C

A = Producing defective item by machine

$\Rightarrow P(U_1) = \frac{100}{100+200+300} = \frac{1}{6}$

$\Rightarrow P(U_2) = \frac{200}{100+200+300} = \frac{2}{6}$

$\Rightarrow P(U_3) = \frac{300}{100+200+300} = \frac{3}{6}$

From the problem

$\Rightarrow$ P(A|U_1) = P(Producing defective item by Machine A)

$\Rightarrow P(A|U_1) = 0.02$

$\Rightarrow$ P(A|U_2) = P(Producing defective item by Machine B)

$\Rightarrow P(A|U_2) = 0.03$

$\Rightarrow$ P(A|U_3) = P(Producing defective item by Machine C)

$\Rightarrow P(A|U_3) = 0.05$

Now we find

P(U_1|A) = P(The chosen defective item is produced by machine A)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{6}\times 0.02}{\left(\frac{1}{6}\times 0.02\right) + \left(\frac{2}{6}\times 0.03\right) + \left(\frac{3}{6}\times 0.05\right)}$$

$$\Rightarrow P(U_1|A) = \frac{0.02}{0.02+0.06+0.15}$$

$$\Rightarrow P(U_1|A) = \frac{0.02}{0.23}$$

$$\Rightarrow P(U_1|A) = \frac{2}{23}$$

∴ The required probability is $\frac{2}{23}$.

21. Question

A bag contains 1 white and 6 red balls, and a second bag contains 4 white and 3 red balls. One of the bags is picked up at random and a ball is randomly drawn from it, and is found to be white in colour. Find the probability that the drawn ball was from the first bag.

Answer

Given:

Bag I has 1 white and 6 red balls

Bag II has 4 white and 3 red balls

Let us assume U_1, U_2 and A be the events as follows:

U_1 = choosing Bag I

U_2 = choosing Bag II

A = choosing white ball from urn

We know that each bag is most likely to choose. So, probability of choosing a bag will be same for every bag.

$\Rightarrow P(U_1) = \frac{1}{2}$

$\Rightarrow P(U_2) = \frac{1}{2}$

The Probability of choosing ball from each Bag differs from Bag to Bag and the probabilities are as follows:

⇒ $P(A|U_1)$ = P(Choosing white ball from Bag I)

$$\Rightarrow P(A|U_1) = \frac{\text{Number of way of choosing white ball from UrnI}}{\text{Number of ways of choosing a balls from UrnI}}$$

$$\Rightarrow P(A|U_1) = \frac{{}^1C_1}{{}^7C_1}$$

$$\Rightarrow P(A|U_1) = \frac{1}{7}$$

⇒ $P(A|U_2)$ = P(Choosing white ball from Bag II)

$$\Rightarrow P(A|U_2) = \frac{\text{Number of way of choosing whiteball from Bag II}}{\text{Number of ways of choosing a balls from Bag II}}$$

$$\Rightarrow P(A|U_2) = \frac{{}^4C_1}{{}^7C_1}$$

$$\Rightarrow P(A|U_2) = \frac{4}{7}$$

Now we find

$P(U_1|A)$ = P(The chosen ball is from Bag I)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{2}\times\frac{1}{7}}{\left(\frac{1}{2}\times\frac{1}{7}\right) + \left(\frac{1}{2}\times\frac{4}{7}\right)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{7}}{\frac{1}{7}+\frac{4}{7}}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{7}}{\frac{5}{7}}$$

$$\Rightarrow P(U_1|A) = \frac{1}{5}$$

∴ The required probabilities are $\frac{1}{5}$.

22. Question

In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Further more, 60% of the students in the college are girls. A student is found to be taller than 1.75 metres. Find the probability that the selected students is girl.

Answer

Let us assume U_1, U_2 and A be the events as follows:

U_1 = choosing Boy

U_2 = choosing Girl

A = choosing student who is taller than 1.75 metres.

From the problem

$$\Rightarrow P(U_1) = 0.4$$

$$\Rightarrow P(U_2) = 0.6$$

⇒ $P(A|U_1)$ = P(Choosing boy who is taller than 1.75 metres)

$$\Rightarrow P(A|U_1) = 0.04$$

⇒ $P(A|U_2)$ = P(Choosing girl taller than 1.75 metres)

⇒ $P(A|U_2) = 0.01$

Now we find

$P(U_2|A)$ = P(The chosen student is a girl taller than 1.75 metres)

Using Baye's theorem:

⇒ $P(U_2|A) = \frac{P(U_2)P(A|U_2)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$

⇒ $P(U_2|A) = \frac{0.6\times0.01}{(0.4\times0.04) + (0.6\times0.01)}$

⇒ $P(U_2|A) = \frac{0.006}{0.016 + 0.006}$

⇒ $P(U_2|A) = \frac{0.006}{0.022}$

⇒ $P(U_2|A) = \frac{3}{11}$

∴ The required probability is $\frac{3}{11}$.

23. Question

For A,B and C the chances of being selected as the manager of a firm are in the ration 4:1:2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8 and 0.5. If the change does take place, find the probability that it is due to the appointment of B or C.

Answer

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing Firm A

U_2 = choosing Firm B

U_3 = choosing Firm C

A = A change takes place

From the problem

⇒ $P(U_1) = \frac{4}{4+1+2} = \frac{4}{7}$

⇒ $P(U_2) = \frac{1}{4+1+2} = \frac{1}{7}$

⇒ $P(U_3) = \frac{2}{4+1+2} = \frac{2}{7}$

⇒ $P(A|U_1)$ = P(Change occurs due to firm A)

⇒ $P(A|U_1) = 0.3$

⇒ $P(A|U_2)$ = P(Change occurs due to Firm B)

⇒ $P(A|U_2) = 0.8$

⇒ $P(A|U_3)$ = P(Change occurs due to firm C)

⇒ $P(A|U_3) = 0.5$

Now we find

$P(U_2|A) + P(U_3|A)$ = P(The change occurred due to appointing B or C)

Using Baye's theorem:

$$\Rightarrow P(U_2|A)\ +\ P(U_3|A)\ =\ \frac{P(U_2)P(A|U_2)+P(U_3)P(A|U_3)}{P(U_1)P(A|U_1)+P(U_2)P(A|U_2)+P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_2|A)\ +\ P(U_3|A)\ =\ \frac{\left(\frac{1}{7}\times 0.8\right)+\left(\frac{2}{7}\times 0.5\right)}{\left(\frac{4}{7}\times 0.3\right)+\left(\frac{1}{7}\times 0.8\right)+\left(\frac{2}{7}\times 0.5\right)}$$

$$\Rightarrow P(U_2|A)\ +\ P(U_3|A)\ =\ \frac{0.8+1.0}{1.2+0.8+1.0}$$

$$\Rightarrow P(U_2|A)\ +\ P(U_3|A)\ =\ \frac{1.8}{3.0}$$

$$\Rightarrow P(U_2|A)\ +\ P(U_3|A)\ =\ \frac{3}{5}$$

∴ The required probability is $\frac{3}{5}$.

24. Question

Three persons A,B and C apply for a job of Manager in a private company. Chances of their selection (A,B and C) are in the ratio 1:2:4. The probabilities that A,B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the changes do not take place, find the probability that it is due to the appointment of C.

Answer

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing Person A

U_2 = choosing Person B

U_3 = choosing Person C

A = Improve in profit occurs

From the problem

$$\Rightarrow P(U_1)\ =\ \frac{1}{1+2+4}\ =\ \frac{1}{7}$$

$$\Rightarrow P(U_2)\ =\ \frac{2}{1+2+4}\ =\ \frac{2}{7}$$

$$\Rightarrow P(U_3)\ =\ \frac{4}{1+2+4}\ =\ \frac{4}{7}$$

⇒ $P(A|U_1)$ = P(Change occurs due to person A)

$$\Rightarrow P(A|U_1)\ =\ 0.8$$

⇒ $P(A|U_2)$ = P(Change occurs due to person B)

$$\Rightarrow P(A|U_2)\ =\ 0.5$$

⇒ $P(A|U_3)$ = P(Change occurs due to person C)

$$\Rightarrow P(A|U_3)\ =\ 0.3$$

Now we find

1 - $P(U_3|A)$ = P(The change does not occurred due to appointing C)

Using Baye's theorem:

$$\Rightarrow 1-P(U_3|A)\ =\ 1-\frac{P(U_3)P(A|U_3)}{P(U_1)P(A|U_1)+P(U_2)P(A|U_2)+P(U_3)P(A|U_3)}$$

$$\Rightarrow 1 - P(U_3|A) = 1 - \frac{\left(\frac{4}{7} \times 0.3\right)}{\left(\frac{4}{7} \times 0.3\right) + \left(\frac{1}{7} \times 0.8\right) + \left(\frac{2}{7} \times 0.5\right)}$$

$$\Rightarrow 1 - P(U_3|A) = 1 - \frac{1.2}{1.2 + 0.8 + 1.0}$$

$$\Rightarrow 1 - P(U_3|A) = 1 - \frac{1.8}{3.0}$$

$$\Rightarrow 1 - P(U_3|A) = \frac{2}{5}$$

∴ The required probability is $\frac{2}{5}$.

25. Question

An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0.01 and that of a motorcycle is 0.02. An insured vehicle met with an accident. Find the probability that the accidented vehicle was a motorcycle.

Answer

Given:

Company insured 2000 scooters and 3000 motorcycles

Let us assume U_1, U_2 and A be the events as follows:

U_1 = choosing Scooter

U_2 = choosing Motorcycle

A = accident involving vehicle

From the problem

$$\Rightarrow P(U_1) = \frac{2000}{2000 + 3000} = \frac{2}{5}$$

$$\Rightarrow P(U_2) = \frac{3000}{2000 + 3000} = \frac{3}{5}$$

⇒ $P(A|U_1)$ = P(accident involving scooter)

$$\Rightarrow P(A|U_1) = 0.01$$

⇒ $P(A|U_2)$ = P(accident involving motorcycles)

$$\Rightarrow P(A|U_2) = 0.02$$

Now we find

$P(U_2|A)$ = P(The vehicle involved in an accident is motorcycle)

Using Baye’s theorem:

$$\Rightarrow P(U_2|A) = \frac{P(U_2)P(A|U_2)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_2|A) = \frac{\frac{3}{5} \times 0.02}{\left(\frac{2}{5} \times 0.01\right) + \left(\frac{3}{5} \times 0.02\right)}$$

$$\Rightarrow P(U_2|A) = \frac{0.06}{0.02 + 0.06}$$

$$\Rightarrow P(U_2|A) = \frac{0.06}{0.08}$$

$$\Rightarrow P(U_2|A) = \frac{3}{4}$$

∴ The required probability is $\frac{3}{4}$.

26. Question

Of the students in a college, it is known that 60% reside in a hostel and 40% do not reseide in hostel. Previous year results report that 30% of students residing in hostel attain A grade and 20% of ones not residing in hostel attain A grade in their annual examination. At the end of the year, one students is chosen at random from the college and he has an A grade. What is the probability that the selected student is a hosteler?

Answer

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = Student residing in Hostel

U_2 = Student not residing in hostel

A = Attaining A grade

From the problem

$\Rightarrow P(U_1) = 0.6$

$\Rightarrow P(U_2) = 0.4$

$\Rightarrow P(A|U_1)$ = P(student in hostel attains A grade)

$\Rightarrow P(A|U_1) = 0.3$

$\Rightarrow P(A|U_2)$ = P(Student not in hostel attains A grade)

$\Rightarrow P(A|U_2) = 0.2$

Now we find

$P(U_1|A)$ = P(The student who got A grade is a hosteller)

Using Baye's theorem:

$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$

$\Rightarrow P(U_1|A) = \frac{0.6\times0.3}{(0.6\times0.3) + (0.4\times0.2)}$

$\Rightarrow P(U_1|A) = \frac{0.18}{0.18 + 0.08}$

$\Rightarrow P(U_1|A) = \frac{0.18}{0.26}$

$\Rightarrow P(U_1|A) = \frac{9}{13}$

$\therefore$ The required probabilities are $\frac{9}{13}$.

27. Question

There are three coins. One is two headed coin, another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is choosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

Answer

Given:

Coin 1 is two heads, coin2 is biased and coin 3 is unbiased

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = choosing coin 1

U_2 = choosing coin 2

U_3 = choosing coin 3

A = getting heads

We know that each coin is most likely to choose. So, probability of choosing a coin will be same for every coin.

$\Rightarrow P(U_1) = \frac{1}{3}$

$\Rightarrow P(U_2) = \frac{1}{3}$

$\Rightarrow P(U_3) = \frac{1}{3}$

From the problem

$\Rightarrow P(A|U_1)$ = P(getting heads on tossing coin 1)

$\Rightarrow P(A|U_1) = 1$

$\Rightarrow P(A|U_2)$ = P(getting heads on tossing coin 2)

$\Rightarrow P(A|U_2) = 0.75$

$\Rightarrow P(A|U_3)$ = P(getting heads on tossing coin 3)

$\Rightarrow P(A|U_3) = 0.5$

Now we find

$P(U_1|A)$ = P(The coin tossed to get head is Coin 1)

Using Baye's theorem:

$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$

$\Rightarrow P(U_1|A) = \frac{\frac{1}{3}\times 1}{\left(\frac{1}{3}\times 1\right) + \left(\frac{1}{3}\times 0.75\right) + \left(\frac{1}{3}\times 0.5\right)}$

$\Rightarrow P(U_1|A) = \frac{1}{1 + 0.75 + 0.5}$

$\Rightarrow P(U_1|A) = \frac{1}{2.25}$

$\Rightarrow P(U_1|A) = \frac{4}{9}$

$\therefore$ The required probability is $\frac{4}{9}$.

28. Question

Assume that the chances of a patient having a heart attack is 40%. It is also assumed that meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options and patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

Answer

Let us assume U_1, U_2 and A be the events as follows:

U_1 = Selecting meditation or yoga

U_2 = selecting Drugs

A = Getting heart attack

From the problem,

$\Rightarrow P(U_1) = \frac{1}{2}$

$\Rightarrow P(U_2) = \frac{1}{2}$

$\Rightarrow$ P(A) = 0.4

$\Rightarrow P(A|U_1)$ = P(getting heart attack even after practicing yoga or meditation)

$\Rightarrow P(A|U_1) = 0.4 \times 0.7$

$\Rightarrow P(A|U_1) = 0.28$

$\Rightarrow P(A|U_2)$ = P(getting heart attack even after using drugs)

$\Rightarrow P(A|U_2) = 0.4 \times 0.65$

$\Rightarrow P(A|U_2) = 0.26$

Now we find

$P(U_1|A)$ = P(The patient got heart attack even after practicing yoga or meditation)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{2}\times 0.28}{\left(\frac{1}{2}\times 0.28\right) + \left(\frac{1}{2}\times 0.26\right)}$$

$$\Rightarrow P(U_1|A) = \frac{0.28}{0.28 + 0.26}$$

$$\Rightarrow P(U_1|A) = \frac{0.28}{0.54}$$

$$\Rightarrow P(U_1|A) = \frac{14}{27}$$

$\therefore$ The required probability is $\frac{14}{27}$.

29. Question

Coloured balls are distributed in four boxes as shown in the following table:

Box	Colour			
	Black	White	Red	Blue
I	3	4	5	6
II	2	2	2	2
III	1	2	3	1
IV	4	3	1	5

A box is selected at random and then a ball is randomly drawn from the selected box. The colour of the ball is black, what is the probability that ball drawn is from the box III.

Answer

Given:

Box I has 3 Black, 4 White, 5 Red, 6 Blue balls

Box II has 2 Black, 2 White, 2 Red, 2 Blue balls

Box III has 1 Black, 2 White, 3 Red, 1 Blue balls

Box IV has 4 Black, 3 White, 1 Red, 5 Blue balls

Let us assume U_1, U_2, U_3, U_4 and A be the events as follows:

U_1 = choosing Box I

U_2 = choosing Box II

U_3 = choosing Box III

U_4 = choosing Box IV

A = choosing Black ball from box

We know that each urn is most likely to choose. So, probability of choosing a urn will be same for every Urn.

$\Rightarrow P(U_1) = \frac{1}{4}$

$\Rightarrow P(U_2) = \frac{1}{4}$

$\Rightarrow P(U_3) = \frac{1}{4}$

$\Rightarrow P(U_4) = \frac{1}{4}$

The Probability of choosing balls from each box differs from box to box and the probabilities are as follows:

$\Rightarrow$ P(A|U_1) = P(Choosing black ball from Box I)

$\Rightarrow P(A|U_1) = \frac{\text{Number of way of choosing black ball from Box I}}{\text{Number of ways of choosing a balls from BoxI}}$

$\Rightarrow P(A|U_1) = \frac{{}^{3}C_1}{{}^{18}C_1}$

$\Rightarrow P(A|U_1) = \frac{3}{18}$

$\Rightarrow$ P(A|U_2) = P(Choosing black ball from Box II)

$\Rightarrow P(A|U_2) = \frac{\text{Number of way of choosing black ball from BoxII}}{\text{Number of ways of choosing a ball from BoxII}}$

$\Rightarrow P(A|U_2) = \frac{{}^{2}C_1}{{}^{8}C_1}$

$\Rightarrow P(A|U_2) = \frac{2}{8}$

$\Rightarrow P(A|U_2) = \frac{1}{4}$

$\Rightarrow$ P(A|U_3) = P(Choosing black ball from Box III)

$\Rightarrow P(A|U_3) = \frac{\text{Number of way of choosing black ball from BoxIII}}{\text{Number of ways of choosing a balls from BoxIII}}$

$\Rightarrow P(A|U_3) = \frac{{}^{1}C_1}{{}^{7}C_1}$

$\Rightarrow P(A|U_3) = \frac{1}{7}$

$\Rightarrow$ P(A|U_3) = P(Choosing black ball from Box III)

$\Rightarrow P(A|U_4) = \frac{\text{Number of way of choosing black ball from BoxIV}}{\text{Number of ways of choosing a balls from BoxIV}}$

$\Rightarrow P(A|U_4) = \frac{{}^{4}C_1}{{}^{13}C_1}$

$\Rightarrow P(A|U_4) = \frac{4}{13}$

Now we find

$P(U_3|A)$ = P(The black ball is from BallIII)

Using Baye's theorem:

$$\Rightarrow P(U_3|A) = \frac{P(U_3)P(A|U_3)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3) + P(U_4)P(A|U_4)}$$

$$\Rightarrow P(U_3|A) = \frac{\frac{1}{4}\times\frac{1}{7}}{\left(\frac{1}{4}\times\frac{1}{6}\right) + \left(\frac{1}{4}\times\frac{1}{4}\right) + \left(\frac{1}{4}\times\frac{1}{7}\right) + \left(\frac{1}{4}\times\frac{4}{13}\right)}$$

$$\Rightarrow P(U_3|A) = \frac{\frac{1}{7}}{\frac{1}{6} + \frac{1}{4} + \frac{1}{7} + \frac{4}{13}}$$

$$\Rightarrow P(U_3|A) = \frac{\frac{1}{7}}{\frac{947}{1092}}$$

$$\Rightarrow P(U_3|A) = \frac{156}{947}$$

∴ The required probability is $\frac{156}{947}$.

30. Question

If a machine is correctly set up it produces 90% acceptable items. If it is incorrectly set up it produces only 40% acceptable items. Past experience shows that 80% of the setups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly set up.

Answer

Let us assume U_1, U_2 and A be the events as follows:

U_1 = Machine is correctly set up

U_2 = Machine is incorrectly set up

A = produce two acceptable items

From the problem

$$\Rightarrow P(U_1) = 0.8$$

$$\Rightarrow P(U_2) = 0.2$$

⇒ $P(A|U_1)$ = P(producing 2 acceptable items if machine is correctly set up)

$$\Rightarrow P(A|U_1) = 0.9 \times 0.9$$

$$\Rightarrow P(A|U_1) = 0.81$$

⇒ $P(A|U_2)$ = P(producing 2 acceptable items if machine is not correctly set up)

$$\Rightarrow P(A|U_2) = 0.4 \times 0.4$$

$$\Rightarrow P(A|U_2) = 0.16$$

Now we find

$P(U_1|A)$ = P(Machine is correctly set up for producing 2 acceptable items)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_1|A) = \frac{0.8\times0.81}{(0.8\times0.81) + (0.2\times0.16)}$$

$$\Rightarrow P(U_1|A) = \frac{0.648}{0.648 + 0.032}$$

$\Rightarrow P(U_1|A) = \frac{0.648}{0.68}$

$\Rightarrow P(U_1|A) = \frac{81}{85}$

∴ The required probability is $\frac{81}{85}$.

31. Question

Bag A contains 3 red and 5 black balls, white bag B contains 4 red and 4 black balls. Two balls are transferred at random from bag A to bag B and then a ball is drawn from bag B at random. If the ball drawn from bag B is found to be red, find the probability that two red balls were transferred from bag A to bag B.

Answer

Given:

Bag A contains 3 red and 5 black balls

Bag B contains 4 red and 4 black balls

It is told that two balls are transferred from bag A to bag B, the possible cases (events) will be as follows:

(i) U_1 = Transferring 2 red balls from bag A to bag B

(ii) U_2 = Transferring 1 red ball and 1 black ball from bag A to bag B

(iii) U_3 = Transferring 2 black balls from bag A to bag B

Let us assume the event A as follows:

⇒ A = Drawing red ball from bag B

Now,

⇒ $P(U_1)$ = P(transferring 2 red balls from bag A to bag B)

$\Rightarrow P(U_1) = \frac{{}^3C_2}{{}^8C_2}$

$\Rightarrow P(U_1) = \frac{3\times2}{8\times7}$

$\Rightarrow P(U_1) = \frac{3}{28}$

⇒ $P(U_2)$ = P(transferring 1 red and 1 black ball from bag A to bag B)

$\Rightarrow P(U_2) = \frac{{}^3C_1\times{}^5C_1}{{}^8C_2}$

$\Rightarrow P(U_2) = \frac{3\times5}{\frac{8\times7}{1\times2}}$

$\Rightarrow P(U_2) = \frac{15}{28}$

⇒ $P(U_3)$ = P(transferring 2 black balls from bag A to bag B)

$\Rightarrow P(U_3) = \frac{{}^5C_2}{{}^8C_2}$

$\Rightarrow P(U_3) = \frac{5\times4}{8\times7}$

$\Rightarrow P(U_3) = \frac{10}{28}$

⇒ $P(A|U_1)$ = P(drawing red ball after transferring 2 red balls from bag A to bag B)

$\Rightarrow P(A|U_1) = \frac{{}^6C_1}{{}^{10}C_1}$

$\Rightarrow P(A|U_1) = \frac{6}{10}$

$\Rightarrow$ P(A|U_2) = P(drawing red ball after transferring 1 red and 1 black ball from bag A to bag B)

$\Rightarrow P(A|U_2) = \frac{^5C_1}{^{10}C_1}$

$\Rightarrow P(A|U_2) = \frac{5}{10}$

$\Rightarrow$ P(A|U_3) = P(drawing red ball after transferring 2 black balls from bag A to bag B)

$\Rightarrow P(A|U_3) = \frac{^4C_1}{^{10}C_1}$

$\Rightarrow P(A|U_3) = \frac{4}{10}$

We need to find

$\Rightarrow$ P(U_1|A) = P(red ball is drawn after transferring two red balls from bag A to bag B)

Using Baye's theorem,

$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$

$\Rightarrow P(U_1|A) = \frac{\frac{3}{28}\times\frac{6}{10}}{\left(\frac{3}{28}\times\frac{6}{10}\right) + \left(\frac{15}{28}\times\frac{5}{10}\right) + \left(\frac{10}{28}\times\frac{4}{10}\right)}$

$\Rightarrow P(U_1|A) = \frac{18}{18 + 75 + 40}$

$\Rightarrow P(U_1|A) = \frac{18}{133}$

$\therefore$ The required probability is $\frac{18}{133}$.

32. Question

By examining the chest X - ray, probability that T.B is detected when a person is actually suffering is 0.99. The probability that the doctor diagnoses incorrectly that a person has T.B. on the basis of X - ray is 0.001. In a certain city 1 in 1000 persons suffers from T.B. A person is selected at random is diagnosed to have T.B. What is the chance that he actually has T.B.?

Answer

Let us assume U_1, U_2 and A be the events as follows:

U_1 = Person having T.B

U_2 = Person not having T.B

A = Diagnosing T.B disease

From the problem:

$\Rightarrow P(U_1) = \frac{1}{1000}$

$\Rightarrow P(U_2) = \frac{999}{1000}$

$\Rightarrow$ P(A|U_1) = P(diagnosing T.B disease for the person who actually having T.B)

$P(A|U_1) = 0.99$

$\Rightarrow$ P(A|U_2) = P(diagnosing T.B disease for the person who don't actually have T.B)

$\Rightarrow P(A|U_2) = 0.001$

Now we find

$P(U_1|A)$ = P(The person has T.B and diagnosed T.B)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{1000}\times 0.99}{\left(\frac{1}{1000}\times 0.99\right) + \left(\frac{999}{1000}\times 0.001\right)}$$

$$\Rightarrow P(U_1|A) = \frac{0.99}{0.99 + 0.999}$$

$$\Rightarrow P(U_1|A) = \frac{0.99}{1.989}$$

$$\Rightarrow P(U_1|A) = \frac{11}{221}$$

∴ The required probability is $\frac{11}{221}$.

33. Question

A test for detection of a particular disease is not fool proof. The test will correctly detect the disease 90% of the time, but will incorrectly detect the disease 1% of the time. For a large population of which an estimated 0.2% have the disease, a person is selected at random, given the test, and told that he has the disease. What are the chances that the person actually have the disease?

Answer

Let us assume U_1, U_2 and A be the events as follows:

U_1 = Person actually has a disease

U_2 = Person doesn't has a disease

A = detection of disease

From the problem

$$\Rightarrow P(U_1) = \frac{0.2}{100} = \frac{2}{1000}$$

$$\Rightarrow P(U_2) = \frac{998}{1000}$$

⇒ $P(A|U_1)$ = P(Test correctly detected)

$$\Rightarrow P(A|U_1) = \frac{90}{100}$$

⇒ $P(A|U_2)$ = P(Test incorrectly detected)

$$\Rightarrow P(A|U_2) = \frac{1}{100}$$

Now we find

$P(U_1|A)$ = P(The person actually has the disease and correctly detected)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{2}{1000}\times\frac{90}{100}}{\left(\frac{2}{1000}\times\frac{90}{100}\right) + \left(\frac{998}{1000}\times\frac{1}{100}\right)}$$

$$\Rightarrow P(U_1|A) = \frac{180}{180 + 998}$$

$\Rightarrow P(U_1|A) = \frac{180}{1178}$

$\Rightarrow P(U_1|A) = \frac{90}{589}$

∴ The required probability is $\frac{90}{589}$.

34. Question

Let d_1, d_2, d_3 be three mutually exclusive diseases. Let S be the set of observable symptoms of these diseases. A doctor has the following information from a random sample of 5000 patients: 1800 had disease d_1, 2100 has disease d_2 and the others had disease d_3. 1500 patients with disease d_1, 1200 patients with disease d_2 and 900 patients with disease d_3 showed the symptom. Which of the diseases is the patient most likely to have?

Answer

Given:

1800 patients had disease d_1

2100 patients had disease d_2

1100 patients had disease d_3

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = Person with disease d_1

U_2 = Person with disease d_2

U_3 = Person with disease d_3

A = Showing the symptom S

Now,

$\Rightarrow P(U_1) = \frac{1800}{5000} = \frac{18}{50}$

$\Rightarrow P(U_2) = \frac{2100}{5000} = \frac{21}{50}$

$\Rightarrow P(U_3) = \frac{1100}{5000} = \frac{11}{50}$

$\Rightarrow P(A|U_1)$ = P(patient who has disease d_1 showed the symptom S)

$\Rightarrow P(A|U_1) = \frac{1500}{1800}$

$\Rightarrow P(A|U_1) = \frac{5}{6}$

$\Rightarrow P(A|U_2)$ = P(patient who has disease d_2 showed the symptom S)

$\Rightarrow P(A|U_2) = \frac{1200}{2100}$

$\Rightarrow P(A|U_2) = \frac{4}{7}$

$\Rightarrow P(A|U_3)$ = P(patient who has disease d_3 showed the symptom S)

$\Rightarrow P(A|U_3) = \frac{900}{1100}$

$\Rightarrow P(A|U_3) = \frac{9}{11}$

Now we find

$P(U_1|A)$ = P(The patient who showed symptom S has disease d_1)

P(U_2|A) = P(The patient who showed symptom S has disease d_2)

P(U_3|A) = P(The patient who showed symptom S has disease d_3)

Using Baye’s theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1)+P(U_2)P(A|U_2)+P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{18}{50}\times\frac{5}{6}}{\left(\frac{18}{50}\times\frac{5}{6}\right)+\left(\frac{21}{50}\times\frac{4}{7}\right)+\left(\frac{11}{50}\times\frac{9}{11}\right)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{3}{10}}{\frac{3}{10}+\frac{6}{25}+\frac{9}{50}}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{3}{10}}{\frac{36}{50}}$$

$$\Rightarrow P(U_1|A) = \frac{5}{12}$$

$$\Rightarrow P(U_2|A) = \frac{P(U_2)P(A|U_2)}{P(U_1)P(A|U_1)+P(U_2)P(A|U_2)+P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_2|A) = \frac{\frac{21}{50}\times\frac{4}{7}}{\left(\frac{18}{50}\times\frac{5}{6}\right)+\left(\frac{21}{50}\times\frac{4}{7}\right)+\left(\frac{11}{50}\times\frac{9}{11}\right)}$$

$$\Rightarrow P(U_2|A) = \frac{\frac{6}{25}}{\frac{3}{10}+\frac{6}{25}+\frac{9}{50}}$$

$$\Rightarrow P(U_2|A) = \frac{\frac{6}{25}}{\frac{36}{50}}$$

$$\Rightarrow P(U_2|A) = \frac{1}{3}$$

$$\Rightarrow P(U_3|A) = \frac{P(U_3)P(A|U_3)}{P(U_1)P(A|U_1)+P(U_2)P(A|U_2)+P(U_3)P(A|U_3)}$$

$$\Rightarrow P(U_3|A) = \frac{\frac{11}{50}\times\frac{9}{11}}{\left(\frac{18}{50}\times\frac{5}{6}\right)+\left(\frac{21}{50}\times\frac{4}{7}\right)+\left(\frac{11}{50}\times\frac{9}{11}\right)}$$

$$\Rightarrow P(U_3|A) = \frac{\frac{9}{50}}{\frac{3}{10}+\frac{6}{25}+\frac{9}{50}}$$

$$\Rightarrow P(U_3|A) = \frac{\frac{9}{50}}{\frac{36}{50}}$$

$$\Rightarrow P(U_3|A) = \frac{1}{4}$$

Since the probability of P(U_1|A) is larger than other disease’s probability. The patient mostly likely to have the disease d_1.

35. Question

A is known to speak truth 3 times out of 5 times. He throws a die and reports that it is one. Find the probability that it is actually one.

Answer

Let us assume U_1, U_2 and A be the events as follows:

U_1 = Getting 1 on throwing a die

U_2 = Getting other than 1 on throwing a die

A = Reporting 1 after throwing the die

From the problem,

$\Rightarrow P(U_1) = \frac{1}{6}$

$\Rightarrow P(U_2) = \frac{5}{6}$

$\Rightarrow$ P(A|U_1) = P(Reporting 1 on actually getting 1 on throwing a die)

$\Rightarrow$ P(A|U_1) = P(Telling the truth)

$\Rightarrow P(A|U_1) = \frac{3}{5}$

$\Rightarrow$ P(A|U_2) = P(Reporting 1 but not getting 1 on throwing a die)

$\Rightarrow$ P(A|U_2) = P(Not telling the truth)

$P(A|U_2) = \frac{2}{5}$

Now we find

P(U_1|A) = P(the die actually shows 1 given that man reports 1)

Using Baye’s theorem:

$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$

$\Rightarrow P(U_1|A) = \frac{\frac{1}{6}\times\frac{3}{5}}{\left(\frac{1}{6}\times\frac{3}{5}\right) + \left(\frac{5}{6}\times\frac{2}{5}\right)}$

$\Rightarrow P(U_1|A) = \frac{3}{3+10}$

$\Rightarrow P(U_1|A) = \frac{3}{13}$

∴ The required probability is $\frac{3}{13}$.

36. Question

A speaks the truth 8 times out of 10 times. A die is tossed. He reports that it was 5. What is the probability that it was actually 5?

Answer

Let us assume U_1, U_2 and A be the events as follows:

U_1 = Getting 5 on throwing a die

U_2 = Getting other than 5 on throwing a die

A = Reporting 5 after throwing the die

From the problem,

$\Rightarrow P(U_1) = \frac{1}{6}$

$\Rightarrow P(U_2) = \frac{5}{6}$

$\Rightarrow$ P(A|U_1) = P(Reporting 5 on actually getting 5 on throwing a die)

$\Rightarrow$ P(A|U_1) = P(Telling the truth)

$\Rightarrow P(A|U_1) = \frac{8}{10}$

⇒ $P(A|U_2)$ = P(Reporting 5 but not getting 5 on throwing a die)

⇒ $P(A|U_2)$ = P(Not telling the truth)

$$P(A|U_2) = \frac{2}{10}$$

Now we find

$P(U_1|A)$ = P(the die actually shows 5 given that man reports 5)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$$

$$\Rightarrow P(U_1|A) = \frac{\frac{1}{6}\times\frac{8}{10}}{\left(\frac{1}{6}\times\frac{8}{10}\right) + \left(\frac{5}{6}\times\frac{2}{10}\right)}$$

$$\Rightarrow P(U_1|A) = \frac{8}{8+10}$$

$$\Rightarrow P(U_1|A) = \frac{8}{18}$$

$$\Rightarrow P(U_1|A) = \frac{4}{9}$$

∴ The required probability is $\frac{4}{9}$.

37. Question

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/4. What is the probability that a student knows the answer given that he answered it correctly?

Answer

Let us assume U_1, U_2, and A be the events as follows:

U_1 = Knowing the answer

U_2 = Guessing the answer

A = Answering the question correctly

From the problem

$$\Rightarrow P(U_1) = \frac{3}{4}$$

$$\Rightarrow P(U_2) = \frac{1}{4}$$

⇒ $P(A|U_1)$ = P(Answering the question correctly by knowing the answer)

⇒ $P(A|U_1) = 1$

⇒ $P(A|U_2)$ = P(Answering the question correctly by guessing the answer)

$$\Rightarrow P(A|U_2) = \frac{1}{4}$$

Now we find

$P(U_1|A)$ = P(The answer is known given that he answerd the question correctly)

Using Baye's theorem:

$$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2)}$$

$\Rightarrow P(U_1|A) = \frac{\frac{3}{4}\times 1}{\left(\frac{3}{4}\times 1\right)+\left(\frac{1}{4}\times\frac{1}{4}\right)}$

$\Rightarrow P(U_1|A) = \frac{3}{3+\frac{1}{4}}$

$\Rightarrow P(U_1|A) = \frac{3}{\frac{13}{4}}$

$\Rightarrow P(U_1|A) = \frac{12}{13}$

∴ The required probability is $\frac{12}{13}$.

38. Question

A laboratory blood test is 99% effective in detecting a certain disease when its infection is present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1% of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

Answer

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = Person has disease

U_2 = Person doesn't has disease

A = Blood test result is positive

From the problem,

$\Rightarrow P(U_1) = \frac{0.1}{100} = 0.001$

$\Rightarrow P(U_2) = 0.999$

⇒ $P(A|U_1)$ = P(Blood test results shows positive for the person with disease)

$\Rightarrow P(A|U_1) = 0.99$

⇒ $P(A|U_2)$ = P(Blood test results shows positive for the person without disease)

$\Rightarrow P(A|U_2) = 0.005$

Now we find

$P(U_1|A)$ = P(The Person has a disease given that the blood test results shows positive)

Using Baye's theorem:

$\Rightarrow P(U_1|A) = \frac{P(U_1)P(A|U_1)}{P(U_1)P(A|U_1)+P(U_2)P(A|U_2)}$

$\Rightarrow P(U_1|A) = \frac{0.001\times 0.99}{(0.001\times 0.99)+(0.999\times 0.005)}$

$\Rightarrow P(U_1|A) = \frac{0.00099}{0.00099+0.004995}$

$\Rightarrow P(U_1|A) = \frac{0.00099}{0.005985}$

$\Rightarrow P(U_1|A) = \frac{990}{5985}$

$\Rightarrow P(U_1|A) = \frac{22}{133}$

∴ The required probability is $\frac{22}{133}$.

39. Question

There are three categories of students in a class of 60 students: A: Very hardworking; B: Regular but not so hardworking; C: Careless and irregular 10 students are in category A, 30 in category B and rest in category C. It is found that the probability of students of category A, unable to get good marks in the final year examination is 0.002, of category B it is 0.02 and of category C, this probability is 0.20. A student selected at random was found to be one who could not get good marks in the examination. Find the probability that this student is of category C.

Answer

Given:

10 students are in category A

30 students are in category B

20 students are in category C

Let us assume U_1, U_2, U_3 and A be the events as follows:

U_1 = Choosing student from category A

U_2 = choosing student from category B

U_3 = choosing student from category C

A = Not getting good marks in final examination

Now,

$\Rightarrow P(U_1) = \frac{10}{60} = \frac{1}{6}$

$\Rightarrow P(U_2) = \frac{30}{60} = \frac{3}{6}$

$\Rightarrow P(U_3) = \frac{20}{60} = \frac{2}{6}$

$\Rightarrow$ $P(A|U_1)$ = P(student not getting good marks from category A)

$\Rightarrow P(A|U_1) = 0.002$

$\Rightarrow$ $P(A|U_2)$ = P(student not getting good marks from category B)

$\Rightarrow P(A|U_2) = 0.02$

$\Rightarrow$ $P(A|U_3)$ = P(student not getting good marks from category C)

$\Rightarrow P(A|U_3) = 0.2$

Now we find

$P(U_3|A)$ = P(The student is from category C given that he didn’t get good marks in final examination)

Using Baye’s theorem:

$\Rightarrow P(U_3|A) = \frac{P(U_3)P(A|U_3)}{P(U_1)P(A|U_1) + P(U_2)P(A|U_2) + P(U_3)P(A|U_3)}$

$\Rightarrow P(U_3|A) = \frac{\frac{2}{6}\times 0.2}{\left(\frac{1}{6}\times 0.002\right) + \left(\frac{3}{6}\times 0.02\right) + \left(\frac{2}{6}\times 0.2\right)}$

$\Rightarrow P(U_3|A) = \frac{0.4}{0.002 + 0.06 + 0.4}$

$\Rightarrow P(U_3|A) = \frac{0.4}{0.462}$

$\Rightarrow P(U_3|A) = \frac{200}{231}$

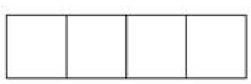
∴ The required probability is $\frac{200}{231}$.

Very short answer

1. Question

A four digit number is formed using the digits 1, 2, 3, 5 with no repetitions. Write the probability that the number is divisible by 5.

Answer



Given, a four-digit number is formed with the digits 1, 2, 3, 5 with no repetitions. There are as many numbers as there are ways of filling in 4 vacant places.

The no. of ways in which the 4 places can be filled, by multiplication principle, is $4 \times 3 \times 2 \times 1 = 24$ ways

For the Numbers divisible by 5,

the 4th place can be only filled by 1 way i.e., by 5, the 1st place can be filled by 3 ways (since repetition is not allowed), the second place by 2 ways and the third by 1 way.

∴ total numbers divisible by $5 = 3 \times 2 \times 1 \times 1 = 6$

P (no. divisible by 5)$= \frac{6}{24} = \frac{1}{4} = 0.25$

2. Question

When three dice are thrown, write the probability of getting 4 or 5 on each of the dice simultaneously.

Answer

When three dice are thrown, no. of events that can happen$= 6^3 = 6 \times 6 \times 6 = 216$

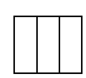
∴P(getting 4 or 5 simultaneously)$\frac{2}{6} \times \frac{2}{6} \times \frac{2}{6} = \frac{8}{216} = \frac{1}{27} = 0.0370$

3. Question

Three digit numbers are formed with the digits 0, 2, 4, 6 and 8. Write the probability of forming a three digit number with the same digits.

Answer

The probability of forming 3-digit no. with digits 0,2,4,6,8 is as follows,



Since it is a 3 digit no., first place cannot have 0 as it will make it a 2 digit no.

∴ 1st place can be filled in 4 ways, 2nd place in 5 ways and 3rd in 5 ways.

∴ total number$= 4 \times 5 \times 5 = 100$

The three digit numbers with same digits are 222,444,666,888

∴P (forming three-digit no.)$= \frac{4}{100} = \frac{1}{25} = 0.04$

4. Question

A ordinary cube has four plane faces, one face marked 2 and another face marked 3, find the probability of getting a total of 7 in 5 throws.

Answer

Total no. of ways= 6^5

A total of 7 in 5 throws can be obtained by getting two 2's ,one 3 and two blanks.

$$\text{Probability} = \left(\frac{1}{6}\right)^3 \times \left(\frac{4}{6}\right)^2 \times 5!$$

$$= \frac{120 \times 16}{6^5}$$

5. Question

Three numbers are chosen from 1 to 20. Find the probability that they are consecutive.

Answer

Given, no from 1 to 20

Sample space=20!

Three no consecutively=18!×3!

Now, considering three numbers as a single digit, we get 18 nos and these can be arranged in $3!$ ways

$$\therefore \text{ probability} = \frac{18! \times 3!}{20!}$$

$$= \frac{3}{190}$$

6. Question

6 boys and 6 girls sit in a row at random. Find the probability that all the girls sit together.

Answer

Given there are 6 boys and 6 girls

∴No of ways in which 6 boys and 6 girls sitting together in a row=7!

6 girls sitting arrangement=$6!$

∴ required probability=$\frac{7!\times 6!}{12!}$

$$= \frac{1}{132}$$

7. Question

If A and B are two independent events such that P(A) = 0.3 and $P(A \cup \overline{B}) = 0.8$. Find P(B).

Answer

Given A and B are independent events => $P(A \cap B) = P(A) \times P(B)$

Since A and B are independent, A and $\overline{B}$ are also independent

$P(A \cap \overline{B}) = P(A) \times P(\overline{B})$

By theorem,

$P(A \cup \overline{B}) = P(A) + P(\overline{B}) - P(A \cap \overline{B})$

0.8= 0.3+(1-P(B))-0.3(1-P(B))

0.8= 0.3+1-P(B)-0.3+0.3P(B)

0.8-1= -0.7P(B)

-0.2= -0.7P(B)

$\Rightarrow\ P(B)=\frac{0.2}{0.7}$

$=\frac{2}{7}$

8. Question

An unbiased die with face marked 1, 2, 3, 4, 5, 6 is rolled four times. Out of 4 face values obtained, find the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5.

Answer

Favourable points are 2,3,4,5

Therefore, Probability$\frac{4}{6}=\frac{2}{3}$

Since the die is rolled 4 times

$\therefore$ Required probability $=\left(\frac{2}{3}\right)^4$

$=\frac{16}{81}$

9. Question

If A and B are two events write the expression for the probability of occurrence of exactly one of two events.

Answer

If A and B are independent, then

P (exactly one of them occurs) = P(A) + P(B) — 2P(A)× P(B)

10. Question

Write the probability that a number selected at random from the set of first 100 natural numbers is a cube.

Answer

Let S = (1,2,3,..........,100)

Let A be the set of cubes from the set of first 100 natural numbers,

A = (1,8,27,64)

P (selected no. being cube)= $\frac{n(A)}{n(S)}$

$=\frac{4}{100}$

$=\frac{1}{25}$

=0.04

11. Question

In a competition A, B and C are participating. The probability that A wins is twice that of B, the probability that B wins is twice that of C . Find the probability that A losses.

Answer

Given P(A)=2 P(B)

P(B)=2 P(C)

P(A)=2(2P(C))

=4P(C)

By definition we know,

P(A) +P(B)+ P(C)=1

⇒ 4P(C) + 2P(C) +P(C) =1

⇒ 7P(C)=1

⇒ P(C)=$\frac{1}{7}$

Therefore P(A) = 4 $\times$P(C)

$= 4 \times \frac{1}{7}$

$= \frac{4}{7}$

∴ P(A loses)= $1 - \frac{4}{7}$

$= \frac{3}{7}$

12. Question

If A, B, C are mutually exclusive and exhaustive events associated to a random experiment, then write the value of P(A) + P(B) + P(C).

Answer

Given $P(A \cap B) = P(B \cap C) = P(A \cap C) = 0$

⇒ $P(A \cap B \cap C) = 0$

Also given $P(A \cup B \cup C) = 1$

By formula we know, $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Substituting the values we get

1 = P(A)+P(B)+P(C)

⇒ P(A)+P(B)+P(C) = 1

13. Question

If two events A and B are such that $P(\overline{A}) = 0.3$, $P(B) = 0.4$ and $P(A \cap \overline{B}) = 0.5$, find $P(\frac{B}{\overline{A \cap B}})$

Answer

Given $P(\overline{A}) = 0.3, P(B) = 0.4 \text{ and } P(A \cap \overline{B}) = 0.5$

We know that $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

But, P(A)=1-0.3=0.7

Therefore, $P(A \cap B) = 0.7 - 0.5 = 0.2$

Now, P(A∪B)=P(A)+P(B)-P(A∩B)

= 0.7 + 0.4-0.4

= 0.9

Therefore, $P(\overline{A \cup B}) = 1 - P(A \cup B)$

=1-0.9

=0.1

$\Rightarrow P\left(\frac{B}{\overline{\overline{A} \cap \overline{B}}}\right) = \frac{1}{4}$

14. Question

If A and B are two independent events, then write $P(A \cap \overline{B})$ in terms of P(A) and P(B).

Answer

Given, A and B are independent.

$\therefore$ A and $\overline{B}$ are also independent.

$P(A \cap \overline{B}) = P(A)P(\overline{B})$

=P(A)(1-P(B))

=P(A)-P(A)P(B)

15. Question

If P(A) = 0.3, P(B) = 0.6, P(B/A) = 0.5, find $P(A \cup B)$.

Answer

Given, P(A)=0.3,P(B)=0.6,P$(\frac{B}{A}) = 0.5$

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

$P(A \cap B) = P\left(\frac{B}{A}\right)P(A)$

=0.5 X 0.3

=0.15

P(A∪B)=P(A)+P(B)-P(A∩B)

=0.3+0.6-0.15

=0.75

16. Question

If A, B and C are independent events such that P(A) = P(B) = P(C) = p, then find the probability of occurrence of at least two of A, B and C.

Answer

Given, P(A)=P(B)=P(C)=p

P(occurrence of at least two of A,B,C)=P(A)P(B)P($\overline{C}$)+P(A)P($\overline{B}$)P(C)+P($\overline{A}$)P(B)P(C)+P(A)P(B)P(C)

=p×p(1-p)+p×(1-p)p+(1-p)p × p + p × p × p

$=3p^2 (1-p)+p^3$

$=3p^2 -3p^3 +p^3$

$=3p^2 -2p^3$

17. Question

If A and B are independent events then write expression for P (exactly one of A, B occurs).

Answer

Given A and B are independent events, then

P(exactly one of A,B occurs)= $P(A)P(\overline{B}) + P(B)P(\overline{A})$

18. Question

If A and B are independent events such that P(A) = p, p(B) = 2p and P (Exactly one of A and B occurs) = 5/9, find the value of p.

Answer

Given P(A)=p and P(B)=2p

∴P(exactly one of A and B occurs)= P(A)P($\overline{B}$)+P($\overline{A}$)P(B)

$\Rightarrow \frac{5}{9} = p(1-2p) + (1-2p)2p$

$\Rightarrow p - 2p^2 + 2p - 2p^2 = \frac{5}{9}$

$\Rightarrow 3p - 4p^2 - \frac{5}{9} = 0$

⇒27p-36p^2 -5=0

⇒36p^2 -27p+5=0

⇒36p^2 -12p-15p+5=0

⇒12p(3p-1)-5(3p-1)=0

⇒ (12p-5)(3p-1)=0

⇒12p-5=0

$\Rightarrow p = \frac{5}{12}$

also, 3p-1=0

$\Rightarrow p = \frac{1}{3}$

∴value of $p = \frac{5}{12}, \frac{1}{3}$

MCQ

1. Question

Mark the correct alternative in each of the following:

If one ball is drawn at random from each of three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, then the probability that 2 white and 1 black balls will be drawn is

A. 13/32

B. 1/4

C. 1/32

D. 3/16

Answer

There are 3 ways of drawing 2 white balls and 1 black ball from the three boxes, i.e.,

WWB,WBW,BWW

For box 1,

$(W)=\frac{3}{4}, P(B)=\frac{1}{4}$

For box 2,

$P(W)=\frac{2}{4}, P(B)=\frac{2}{4}$

For box 3,

$P(W)=\frac{1}{4}, P(B)=\frac{3}{4}$

$\therefore P(WWB)=\frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}=\frac{9}{32}$

$P(WBW)=\frac{3}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{3}{32}$

$P(BWW)=\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{1}{32}$

$P(2W \text{ and } 1B)=\frac{9}{32}+\frac{3}{32}+\frac{1}{32}=\frac{13}{32}$

2. Question

Mark the correct alternative in each of the following:

A and B draw two cards each, one after another, from a pack of well-shuffled pack of 52 cards. The probability that all the four cards drawn are of the same suit is

A. $\frac{44}{85\times 49}$

B. $\frac{11}{85\times 49}$

C. $\frac{13\times 24}{17\times 25\times 49}$

D. none of these

Answer

Total No. Of cards =52

No. Of cards for each suit = 13

Total types Of French suit = 4

$\text{Prob.}=\frac{\text{conditional case}}{\text{total case}}$

For this conditional case of selecting all four same suit,it can be either hearts or spades or clubs or diamonds

So Total no. Of conditional case= no. of case of selecting all four cards of hearts suit +spades suit + clubs suit + diamonds suit.

$=13C_4+13C_4+13C_4+13C_4$

$=4\times 13C_4$

2860

Total no. of case$= \; 52C_4$

=270725

So, probability of getting all four cards of the same suit$= \frac{2860}{270725}$

$= \frac{44}{85 \times 49}$

3. Question

Mark the correct alternative in each of the following:

A and B are two events such that P(A) = 0.25 and P(B) = 0.50. The probability of both happening together is 0.14. The probability of both A and B not happening is

A. 0.39

B. 0.25

C. 0.11

D. none of these

Answer

Given P(A∪B)=0.14,P(A)=0.25,P(B)=0.50

By addition theorem,

P(A∪B)=P(A)+P(B)-P(A∩B)

P(A∩B)=0.75-0.14=0.61

∴P(both A and B not happening)=1-P(A∩B)=1-0.61=0.39

4. Question

Mark the correct alternative in each of the following:

The probabilities of a student getting I, II and III division in an examination are $\frac{1}{10}, \frac{3}{5}$ and $\frac{1}{4}$ respectively. The probability that the student fails in the examination is

A. $\frac{197}{200}$

B. $\frac{27}{100}$

C. $\frac{83}{100}$

D. none of these

Answer

P(student fails)=P(student not getting 1^{st} division)XP(student not getting 2^{nd} division)XP(student not getting 3^{rd} division)

$= \left(1 - \frac{1}{10}\right)\left(1 - \frac{3}{5}\right)(1 - \frac{1}{4})$

$= \frac{27}{100}$

5. Question

Mark the correct alternative in each of the following:

India play two matches each with West Indies and Australia. In any match the probabilities of India getting 0, 1 and 2 points are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is

A. 0.0875

B. 1/16

C. 0.1125

D. none of these

Answer

India wins = W = 2 pt

India loses = L = 0 pt

India draws = D = 1 pt

Total matches played by India = 4

This means that the total points needs to be 7 at least

Either India wins all 4 matches or India wins 3 and draws 1

the probability of WWWW = (0.5)*(0.5)*(0.5)*(0.5) = 0.0625

India wins 3 matches and draws 1: probability in that case = (0.5)*(0.5)*(0.5)*(0.05) = 0.00625

The 4 possible outcomes will be:

WWWD, WWDW, WDWW, DWWW

Total probability of India wins 3 matches and draws 1 = 4*(0.00625) = 0.025

P(total points have to be at least 7) = 0.0625 + 0.025 = 0.0875

6. Question

Mark the correct alternative in each of the following:

Three faces of an ordinary dice are yellow, two faces are red and one face is blue. The dice is rolled 3 times. The probability that yellow red and blue face appear in the first second and third throws respectively, is

A. 1/36

B. 1/6

C. 1/30

D. none of these

Answer

Probability of getting blue face is $\frac{1}{6}$.

Probability of getting yellow face is $\frac{3}{6}$.

Probability of getting red face is $\frac{2}{6}$.

probability that yellow red and blue face appear in the first second and third throws is $P = \frac{3}{6} \times \frac{2}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

7. Question

Mark the correct alternative in each of the following:

The probability that a leap year will have 53 Friday 53 Saturdays is

A. 2/7

B. 3/7

C. 4/7

D. 1/7

Answer

There are 366 days in a leap year.52×7

=364 days.So, there are 7 possible combinations for the last 2 days (Sunday, Monday) (Monday, Tuesday) ... (Saturday, Sunday). Of these 7, 3 give you either 53 Saturdays or Fridays.So, ans is 3/7.

8. Question

Mark the correct alternative in each of the following:

A person writes 4 letters and addresses 4 envelopes. If the letters are placed in the envelopes at random, then the probability that all letters are not placed in the right envelopes, is

A. 1/4

B. 11/24

C. 15/24

D. 23/24

Answer

let P=all letters are dispatched in right letters

Therefore, $\overline{\mathrm{P}} = 1 - \mathrm{P}$

$= 1 - \frac{1}{4!}$

$= \frac{4! - 1}{4!}$

$= \frac{23}{24}$

9. Question

Mark the correct alternative in each of the following:

A speaks truth in 75% cases and B speaks truth in 80% cases. Probability that they contradict each other in a statement, is

A. 7/20

B. 13/20

C. 3/5

D. 2/5

Answer

Let A = Event that A speaks the truthB = Event that B speaks the truthThen $\mathrm{P(A)} = \frac{75}{100}$

$= \frac{3}{4}\mathrm{P(B)} = \frac{80}{100}$

$= \frac{4}{5}$P(A-lie) $= 1 - \frac{3}{4}$

$= \frac{1}{4}$P(B-lie) $= 1 - \frac{4}{5}$

$= \frac{1}{5}$Now,A and B contradict each other =[A lies and B true] or [B true and B lies]= P(A).P(B-lie) + P(A-lie).P(B)

$$= \frac{3}{5} \times \frac{1}{5} + \frac{1}{4} \times \frac{4}{5}$$

$$= \frac{7}{20}$$

10. Question

Mark the correct alternative in each of the following:

Three integers are chosen at random from the first 20 integers. The probability that their product is even is

A. 2/19

B. 3/29

C. 17/19

D. 4/19

Answer

the product of 3 randomly chosen integers are even only if there are even nos.

P(even) = 1-P(odd)

P(odd) $= \frac{10}{20} \times \frac{9}{19} \times \frac{8}{18}$

$$= \frac{2}{19}$$

P(even) $= \frac{17}{19}$

11. Question

Mark the correct alternative in each of the following:

Out of 30 consecutive integers, 2 are chosen at random. The probability that their sum is odd, is

A. 14/29

B. 16/29

C. 15/29

D. 10/29

Answer

. There are 15 odd numbers and 15 even numbers.Sum is odd if you choose 1 odd and 1 even.So, the probability of sum being odd is $\frac{15C_1 \times 15C_1}{30C_2}$

$$= \frac{15 \times 15}{435}$$

$$= \frac{15}{29}$$

12. Question

Mark the correct alternative in each of the following:

A bag contains 5 black balls, 4 white balls and 3 red balls. If a ball is selected random wise, the probability that it is black or red ball is

A. 1/3

B. 1/4

C. 5/12

D. 2/3

Answer

P(black or red)= $\frac{5C_1+3C_1}{12C_1}$

$= \frac{5+3}{12}$

$= \frac{8}{12}$

$= \frac{2}{3}$

13. Question

Mark the correct alternative in each of the following:

Two dice are thrown simultaneously. The probability of getting a pair of aces is

A. 1/36

B. 1/3

C. 1/6

D. none of these

Answer

for this experiment, out of the 36 sample point, the favorable one is (1,1)

$\therefore \mathrm{P(pair\ ofaces)} = \frac{1}{36}$

14. Question

Mark the correct alternative in each of the following:

An urn contains 9 balls two of which are red, three blue and four black. Three balls are drawn at random. The probability that they are of the same colour is

A. 5/84

B. 3/9

C. 3/7

D. 7/17

Answer

P(same colour)= $\frac{2C_3}{9C_3} + \frac{3C_3}{9C_3} + \frac{4C_3}{9C_3}$

$= \frac{0}{84} + \frac{1}{84} + \frac{4}{84}$

$= \frac{5}{84}$

15. Question

Mark the correct alternative in each of the following:

A coin is tossed three times. If events A and B are defined as A = Two heads come, B = Last should be head. Then, A and B are

A. independent

B. dependent

C. both

D. mutually exclusive

Answer

since event A is two heads come and event B is last should be head, both are dependent.

16. Question

Mark the correct alternative in each of the following:

Five persons entered the lift cabin on the ground floor of an 8 floor house. Suppose that each of them independently and with equal probability can leave the cabin at any floor beginning with the first, then the probability of all 5 persons leaving at different floors is

A. $\frac{^7P_5}{7^5}$ B. $\frac{7^2}{^7P_5}$

C. $\frac{6}{^6P_5}$ D. $\frac{^5P_5}{5^5}$

Answer

P(all 5 persons leaving at diff. floors)=$\frac{^7P_5}{7^5}$

17. Question

Mark the correct alternative in each of the following:

A box contains 10 good articles and 6 with defects. One item is drawn at random. The probability that it is either good or has a defect is

A. 64/64

B. 49/64

C. 40/64

D. 24/64

Answer

P(good articles)$= \frac{^{10}C_1}{^{16}C_1}$

P(defective)$= \frac{^6C_1}{^{16}C_1}$

$\therefore \text{P(good or defective)} = \frac{10}{16} + \frac{6}{16}$

$= \frac{16}{16}$

$= \frac{64}{64}$

18. Question

Mark the correct alternative in each of the following:

A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, the probability that it is rusted or is a nail is

A. 3/16

B. 5/16

C. 11/16

D. 14/16

Answer

Given half of nails and nuts are rusted.

Rusted nails=3

Rusted nuts=5

P(item rusted or nail)= $\frac{8C_1}{16C_1} + \frac{6C_1}{16C_1}$

$= \frac{8}{16} + \frac{6}{16}$

$= \frac{14}{16}$

19. Question

Mark the correct alternative in each of the following:

A bag contains 5 brown and 4 white socks. A man pulls out two socks. The probability that these are of the same colour is

A. 5/108

B. 18/108

C. 30/108

D. 48/108

Answer

2 brown socks from 5 can be chosen in $5C_2$ ways, similarly 2 white socks can be selected in 4C_2 ways.

P(same colour)= $\frac{5C_2}{9C_2} + \frac{4C_2}{9C_2}$

$= \frac{10}{36} + \frac{6}{36}$

$= \frac{16}{36}$

$= \frac{48}{108}$

20. Question

Mark the correct alternative in each of the following:

If S is the sample space and $P(A) = \frac{1}{3}P(B)$ and S = A∪B, where A and B are two mutually exclusive events, then P(A) =

A. 1/4

B. 1/2

C. 3/4

D. 3/8

Answer

P(A) = 1/3 P(B) → Given

3 P(A) = P(B) → (1)

Now, since it's given that A and B are mutually exclusive:

$P(A \cap B) = 0$ → (2)

Now then,

$P(S) = P(A \cup B) = 1$ → (3)

$P(A \cup B) = P(A) + P(B) + P(A \cap B)$

P(A) + P(B) = 1 → From (2) & (3)

P(A) + 3 P(A) = 1 → From (1)

4 P(A) = 1

P(A) = 1/4

21. Question

Mark the correct alternative in each of the following:

If A and B are two events, then $P(\bar{A} \cap B) =$

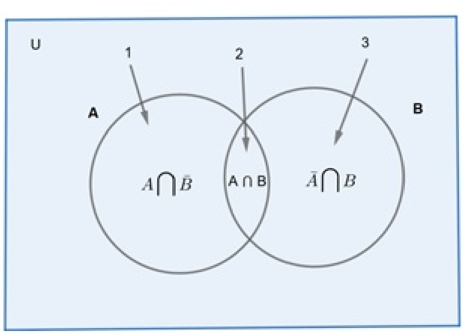
A. $P(\bar{A})P(\bar{B})$

B. 1 – P (A) – P(B)

C. $P(A) + P(B) - P(A \cap B)$

D. $P(B) - P(A \cap B)$

Answer



In the give picture we have,

$A \cap \bar{B}$= (1)

$A \cap B$ = (2)

$\bar{A} \cap B$ = (3)

Now, according to the addition theorem of probability we get the equation:

$P(B) = P(A \cap B) + P\bar{A} \cap B$

$P\bar{A} \cap B = P(B) - P(A \cap B)$

22. Question

Mark the correct alternative in each of the following:

If P (A ∪ B) = 0.8 and P (A ∩ B) = 0.3, then $P(\bar{A}) + P(\bar{B}) =$

A. 0.3

B. 0.5

C. 0.7

D. 0.9

Answer

For events A and B, addition theorem of probability →

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ → (1)

Also,

$P(A) = 1 - P(\bar{A})$ → (2)

$P(B) = 1 - P(\bar{B})$ → (3)

Using equation (1)

0.8 = P(A) + P(B) – 0.3

P(A) + P(B) = 1.1

$1 - P(\bar{A}) + 1 - P(\bar{B}) = 1.1$ → From (2) & (3)

$2 - \{P(\bar{A}) + P(\bar{B})\} = 1.1$

$P(\bar{A}) + P(\bar{B}) = 2 - 1.1$

$P(\bar{A}) + P(\bar{B}) = 0.9$

23. Question

Mark the correct alternative in each of the following:

A bag X contains 2 white and 3 black balls and another bag Y contains 4 white and 2 black balls. One bag is selected at random and a ball is drawn from it. Then, the probability chosen to be white is

A. 2/15

B. 7/15

C. 8/15

D. 14/15

Answer

Let A be the event of drawing a white ball.

Event M_1 = Selecting bag X

Event M_2 = Selecting bag Y

Let’s see what all probabilities we can find out from the given problem:

→ Selecting bag X = $P(M_1) = \frac{1}{2}$

→ Selecting bag Y = $P(M_2) = \frac{1}{2}$

→ Drawing white ball from bag X = $P\left(\frac{A}{M_1}\right) = \frac{2}{5}$

→ Drawing white ball from bag Y = $P\left(\frac{A}{M_2}\right) = \frac{4}{6} = \frac{2}{3}$

Now, according to law of total probability →

$$P(A) = P(M_1) \times P(A|M_1) + P(M_2) \times P(A|M_2)$$

$$= \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{2}{3}$$

$$= \frac{1}{5} + \frac{1}{3}$$

$$= \frac{8}{15}$$

24. Question

Mark the correct alternative in each of the following:

Two persons A and B take turns in throwing a pair of dice. The first person to throw 9 from both dice will be awarded the prize. If A throws first, then the probability that B wins the game is

A. 9/17

B. 8/17

C. 8/9

D. 1/9

Answer

Let’s find out the probability of getting ‘9’ from throw of a pair of dice first:

9 can be obtained in the given 4 cases →

[(3,6), (4,5), (5,4), (6,3)]

Getting 9 → $\frac{4}{36} = \frac{1}{9}$

Not getting 9 → $\frac{32}{36} = \frac{8}{9}$

B will win when A won’t get 9 in his/her throw and B will get 9 in his/her throw, i.e. B wins after 1 round of throws, P = $\frac{8}{9} \times \frac{1}{9}$

Now then,

P(B wins) = P(Getting 9 in 2^{nd} throw) + P(Getting 9 in 4^{th} throw) + P(Getting 9 in 6^{th} throw) + ...

$$= \frac{8}{9} \times \frac{1}{9} + \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{1}{9} + \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{1}{9} + \ldots..$$

$= \frac{8}{81}\left[1 + \frac{64}{81} + \left(\frac{64}{81}\right) + \ \ldots.\right]$ → GP (Geometric Progression)

$$= \frac{8}{81} \times \frac{1}{1 - \frac{64}{81}}$$

$= \frac{8}{81} \times \frac{81}{17}$

$= \frac{8}{17}$

25. Question

Mark the correct alternative in each of the following:

The probability that in a year of 22^{nd} century chosen at random, there will be 53 Sundays, is

A. 3/28

B. 2/28

C. 7/28

D. 5/28

Answer

P(53 Sundays) = P(Leap Years) $\times$P(53 Sundays in a Leap Year) + P(Non-Leap Years)$\times$P(53 Sundays in a Non-Leap Year) → (1)

Now,

P(53 Sundays in a Leap Year) $= \frac{2}{7}$ → (52 Weeks + 2 days → Saturday + Sunday or Sunday + Monday)

P(53 Sundays in a non-Leap Year) $= \frac{1}{7}$ → (52 Weeks + 1 day)

Next step in solving this problem requires the total number of leap years in 22^{nd} Century.

Leap Years = 24

Non Leap Years = 76

∴ P(Leap Year) $= \frac{25}{100}$

P(Non-Leap Year) $= \frac{75}{100}$

We now have all the required information, using equation (1) we get →

$= \frac{25}{100} \times \frac{2}{7} + \frac{75}{100} \times \frac{1}{7}$

$= \frac{1}{4} \times \frac{2}{7} + \frac{3}{4} \times \frac{1}{7}$

$= \frac{2}{28} + \frac{3}{28}$

$= \frac{5}{28}$

26. Question

Mark the correct alternative in each of the following:

From a set of 100 cards numbered 1 to 100, one card is drawn at random. The probability that the number obtained on the card is divisible by 6 or 8 but not by 24 is

A. 6/25

B. 1/5

C. 1/6

D. 2/5

Answer

No. divisible by 6 between 1-100 (A) = 16

No. divisible by 8 between 1-100 (B) = 12

No. divisible by 6 and 8 between 1-100 (C) = 4

No. divisible by 24 between 1-100 (D) = 4

P(No. divisible by 6 or 8) = P(A) + P(B) – P(C)

$$= \frac{16}{100} + \frac{12}{100} - \frac{4}{100}$$

$$= \frac{24}{100}$$

$$= \frac{6}{25}$$

P(divisible by 6 or 8 but not by 24) = P(divisible by 6 or 8) – P(divisible by 24)

$$= \frac{6}{25} - \frac{4}{100}$$

$$= \frac{5}{25}$$

$$= \frac{1}{5}$$

27. Question

Mark the correct alternative in each of the following:

If A and B are two events such that $P(A) = \frac{4}{5}$, and $P(A \cap B) = \frac{7}{10}$, then P(B/A) =

A. 1/10

B. 1/8

C. 7/8

D. 17/20

Answer

P(A) = $\frac{4}{5}$, P(A⋂B) = $\frac{7}{10}$ → (Given)

Now we know,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{7}{10}}{\frac{4}{5}}$$

$$= \frac{7}{10} \times \frac{5}{4}$$

$$= \frac{7}{8}$$

28. Question

Mark the correct alternative in each of the following:

If A and B are two events associated to a random experiment such that $P(A \cap B) = \frac{7}{10}$ and P(B) = 17/20, then P(A/B) =

A. 14/17

B. 17/20

C. 7/8

D. 1/8

Answer

$P(B) = \frac{17}{20}, P(A \cap B) = \frac{7}{10}$

Now we know,

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{7}{10}}{\frac{17}{20}}$

$= \frac{7}{10} \times \frac{20}{17}$

$= \frac{14}{17}$

29. Question

Mark the correct alternative in each of the following:

Associated to a random experiment two events A and B are such that P(B) = 3/5, P(A/B) = 1/2 and P(A∪B) = 4/5. The value of P(A) is

A. 3/10

B. 1/2

C. 1/10

D. 3/5

Answer

According to the addition theorem of probability for two events A and B we have,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) + P(B) - P(A \cap B) = \frac{4}{5} \rightarrow (1)$

Now,

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(A \cap B) = \frac{1}{2} \times P(B) \rightarrow$ (Given)

$P(A \cap B) = \frac{1}{2} \times \frac{3}{5} \rightarrow$ (Given)

$P(A \cap B) = \frac{3}{10}$

Now using Equation (1),

$P(A) + P(B) - P(A \cap B) = \frac{4}{5}$

$P(A) + \frac{3}{5} - \frac{3}{10} = \frac{4}{5}$

$P(A) + \frac{3}{10} = \frac{4}{5}$

$P(A) = \frac{4}{5} - \frac{3}{10}$

$P(A) = \frac{5}{10}$

$= \frac{1}{2}$

30. Question

Mark the correct alternative in each of the following:

If P(A) = 3/10, P(B) = 2/5 and P(A ∪B) = 3/5, then P(A/B) + P(B/A) equals

A. 1/4

B. 7/12

C. 5/12

D. 1/3

Answer

$P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$, $P(A \cup B) = \frac{3}{5} \rightarrow$ (Given)

According to the addition theorem of probability for two events A and B we have,

$P(A) + P(B) - P(A \cap B) = P(A \cup B)$

$\frac{3}{10} + \frac{2}{5} - P(A \cap B) = \frac{3}{5}$

$\frac{7}{10} - P(A \cap B) = \frac{3}{5}$

$P(A \cap B) = \frac{7}{10} - \frac{3}{5}$

$P(A \cap B) = \frac{1}{10}$

Now then,

$P(A|B) + P(B|A) = \frac{P(A \cap B)}{P(B)} + \frac{P(A \cap B)}{P(A)}$

$= \frac{\frac{1}{10}}{\frac{2}{5}} + \frac{\frac{1}{10}}{\frac{3}{10}}$

$= \frac{1}{10} \times \frac{5}{2} + \frac{1}{10} \times \frac{10}{3}$

$= \frac{1}{4} + \frac{1}{3}$

$= \frac{7}{12}$

31. Question

Mark the correct alternative in each of the following:

Let P(A) = 7/13, P(B) = 9/13 and P(A ∩ B) = 4/13. Then $P\left(\overline{A}/B\right) =$

A. 5/9

B. 4/9

C. 4/13

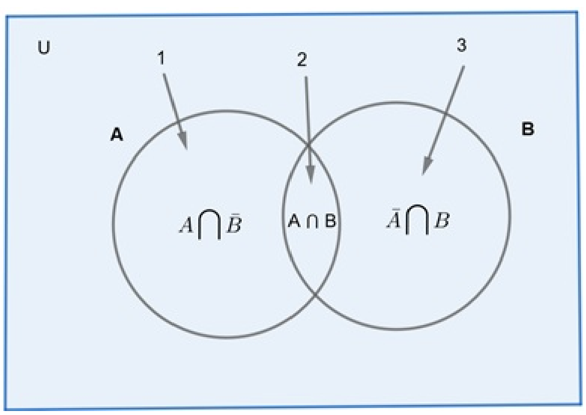
D. 6/13

Answer

$P(A) = \frac{7}{13}, P(B) = \frac{9}{13}, P(A \cap B) = \frac{4}{13}$

Now,

$P(\bar{A}|B) = \frac{P(\bar{A} \cap B)}{P(B)} \rightarrow (1)$



$P(B) = P(A \cap B) + P(\overline{A} \cap B) \rightarrow$ Refer to the Picture

$P\bar{A} \cap B = P(B) - P(A \cap B)$

$= \frac{9}{13} - \frac{4}{13}$

$= \frac{5}{13}$

Now using equation (1),

$P(\bar{A}|B) = \frac{P(\bar{A} \cap B)}{P(B)}$

$= \frac{\frac{5}{13}}{\frac{9}{13}}$

$= \frac{5}{13} \times \frac{13}{9}$

$= \frac{5}{9}$

32. Question

Mark the correct alternative in each of the following:

If $P(A) = \frac{2}{5}, P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P(\overline{A} / \overline{B}) P(\overline{B} / \overline{A})$ is equal to

A. 5/6

B. 5/7

C. 25/42

D. 1

Answer

$P(A) = \frac{2}{5}, P(B) = \frac{3}{10}, P(A \cap B) = \frac{1}{5}$ → (Given)

To find →

$P\left(\frac{\overline{A}}{\overline{B}}\right) \times P\left(\frac{\overline{B}}{\overline{A}}\right) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} \times \frac{P(\overline{A} \cap \overline{B})}{P(\overline{A})}$ → (1)

Let’s find the required values first and then we can put them into equation (1) to find the answer.

P(A) = 1 – P($\overline{A}$)

P($\overline{A}$) = 1 – P(A)

$= 1 - \frac{2}{5}$

$= \frac{3}{5}$

P(B) = 1 – P($\overline{B}$)

= 1 – P(B)

$= 1 - \frac{3}{10}$

$= \frac{7}{10}$

Also, according to the addition theorem of probability,

P(A⋃B) = P(A) + P(B) – P(A⋂B)

$= \frac{2}{5} + \frac{3}{10} - \frac{1}{5}$

$= \frac{4 + 3 - 2}{10}$

$= \frac{5}{10}$

$= \frac{1}{2}$

Also,

——$P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B})$

= 1 – P(A⋃B)

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

Now let’s put all the values in Equation (1)

$P\left(\frac{\bar{A}}{\bar{B}}\right) \times P\left(\frac{\bar{B}}{\bar{A}}\right) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} \times \frac{P(\bar{A}\cap\bar{B})}{P(\bar{A})}$

$= \frac{\frac{1}{2}}{\frac{7}{10}} \times \frac{\frac{1}{2}}{\frac{3}{5}}$

$= \frac{1}{2} \times \frac{10}{7} \times \frac{1}{2} \times \frac{5}{3}$

$= \frac{5}{7} \times \frac{5}{6}$

$= \frac{25}{42}$

33. Question

Mark the correct alternative in each of the following:

If A and B are two events such that $P(A) = \frac{3}{8}, P(B) = \frac{1}{3}, P(A/B) = \frac{1}{4}$, then $P(\bar{A}\cap\bar{B})$ equals

A. 1/12

B. 3/4

C. 1/4

D. 3/16

Answer

$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P\left(\frac{A}{B}\right) = \frac{1}{4}$ → (Given)

——$P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B})$

= 1 – P(A⋃B) → (1)

Now,

$P\left(\frac{A}{B}\right) = \frac{1}{4}$

$\frac{P(A\cap B)}{P(B)} = \frac{1}{4}$

$P(A\cap B) = \frac{1}{4} \times P(B)$

$P(A\cap B) = \frac{1}{4} \times \frac{1}{3}$

$P(A \cap B) = \frac{1}{12}$

Also, according to the addition theorem of probability,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{12}$

$= \frac{6 + 4 - 1}{12}$

$= \frac{9}{12}$

$= \frac{3}{4}$

Now using the value of $P(A \cup B)$ in Equation (1) we get,

$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$= 1 - \frac{3}{4}$

$= \frac{1}{4}$

34. Question

Mark the correct alternative in each of the following:

Let A and B be two events such that $P(A) = \frac{3}{8}, P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$. Then P(A/B) $P(\bar{A} / B)$ is equals to

A. 2/5

B. 3/8

C. 3/20

D. 6/25

Answer

$P(A) = \frac{3}{8}, P(B) = \frac{5}{8}, P(A \cup B) = \frac{3}{4} \rightarrow$ (Given)

Now, according to the addition theorem of probability,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

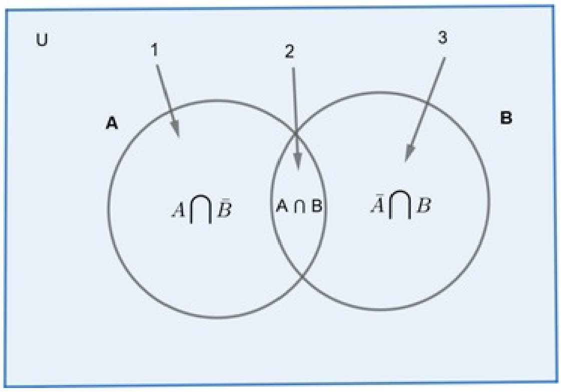
$P(A) + P(B) - P(A \cap B) = \frac{3}{4}$

$\frac{3}{8} + \frac{5}{8} - P(A \cap B) = \frac{3}{4}$

$1 - P(A \cap B) = \frac{3}{4}$

$P(A \cap B) = 1 - \frac{3}{4}$

$= \frac{1}{4}$



According to the figure we get,

$$P(B) = P(A\cap B) + P(\overline{A}\cap B)$$

$$P(\overline{A}\cap B) = P(B) - P(A\cap B)$$

$$= \frac{5}{8} - \frac{1}{4}$$

$$= \frac{3}{8}$$

Now then,

$$P\left(\frac{A}{B}\right) \times P\left(\frac{\overline{A}}{B}\right) = \frac{P(A\cap B)}{P(B)} \times \frac{P(\overline{A}\cap B)}{P(B)}$$

$$= \frac{\frac{1}{4}}{\frac{5}{8}} \times \frac{\frac{3}{8}}{\frac{5}{8}}$$

$$= \frac{1}{4} \times \frac{8}{5} \times \frac{3}{8} \times \frac{8}{5}$$

$$= \frac{2}{5} \times \frac{3}{5}$$

$$= \frac{6}{25}$$

35. Question

Mark the correct alternative in each of the following:

If $P(B) = \frac{3}{5}, P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then $P\left(\overline{A \cup B}\right) + P\left(\bar{A} \cup B\right) =$

A. 1/5

B. 4/5

C. 1/2

D. 1

Answer

$P(B) = \frac{3}{5}, P\left(\frac{A}{B}\right) = \frac{1}{2}, P(AUB) = \frac{4}{5} \rightarrow$ (Given)

——$P(\overline{A}\cap\overline{B}) = P(\overline{AUB})$

$= 1 - P(A \cup B) \rightarrow (1)$

$P(\overline{A} \cup B) = [1 - P(\text{only } A)]$

$= 1 - [P(A) - P(A \cap B)] \rightarrow (2)$

Now,

$P\left(\frac{A}{B}\right) = \frac{1}{2}$

$\frac{P(A \cap B)}{P(B)} = \mathbf{\frac{1}{2}}$

$P(A \cap B) = P(B) \times 1/2$

$P(A \cap B) = \frac{3}{5} \times \frac{1}{2}$

$= \frac{3}{10} \rightarrow (3)$

According to the addition theorem of probability,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) + \frac{3}{5} - \frac{3}{10} = \frac{4}{5} \rightarrow$ From (3) and (Given)

$P(A) + \frac{3}{10} = \frac{4}{5}$

$P(A) = \frac{4}{5} - \frac{3}{10}$

$P(A) = \frac{5}{10} = \frac{1}{2} \rightarrow (4)$

———Now using Equation (1) and (2)

$P(A \cup B) + P(\overline{A} \cup B) = 1 - P(A \cup B) + 1 - [P(A) - P(A \cap B)]$

$= 1 - \frac{4}{5} + 1 - \left(\frac{1}{2} - \frac{3}{10}\right)$

$= \frac{1}{5} + 1 - \frac{2}{10}$

$= \frac{6}{5} - \frac{2}{10}$

$= \frac{10}{10}$

$= 1$

36. Question

Mark the correct alternative in each of the following:

If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$, then $P(A \cup B) =$

A. 0.24

B. 0.3

C. 0.48

D. 0.96

Answer

P(A) =0.4 , $P\left(\frac{B}{A}\right)$ =0.6 , P(B) = 0.8 → (Given)

$P\left(\frac{B}{A}\right)$ =0.6

$$\frac{P(A \cap B)}{P(A)} = 0.6$$

P(A⋂B) = P(A) × 0.6

P(A⋂B) = 0.4 × 0.6

P(A⋂B) = 0.24 → (1)

Now, according to the addition theorem of probability,

P(A⋃B) = P(A) + P(B) – P(A⋂B)

= 0.4 + 0.8 – 0.24 → From (1)

= 1.2 - 0.24

= 0.96

37. Question

Mark the correct alternative in each of the following:

If $P(B)=\frac{3}{5}, P(A/B)=\frac{1}{2}$ and $P(A \cup B)=\frac{4}{5}$, then $P(B/\bar{A})=$

A. 1/5

B. 3/10

C. 1/2

D. 3/5

Answer

P(B) = $\frac{3}{5}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$, P(A⋃B) = $\frac{4}{5}$ → (Given)

Now as we know,

$$P\left(\frac{A}{B}\right) = \frac{1}{2}$$

$$\frac{P(A \cap B)}{P(B)} = \mathbf{\frac{1}{2}}$$

P(A⋂B) = P(B) × 1/2

P(A⋂B) = $\frac{3}{5} \times \frac{1}{2}$

= $\frac{3}{10}$ → (1)

Now, according to the addition theorem of probability,

P(A⋃B) = P(A) + P(B) – P(A⋂B)

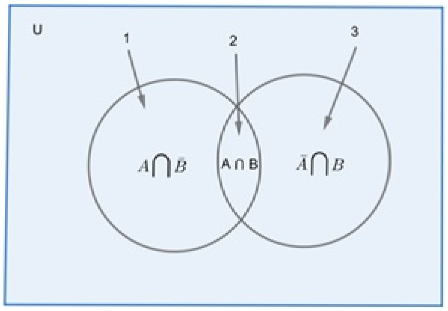
$$\frac{4}{5} = P(A) + \frac{3}{5} - \frac{3}{10}$$

$P(A) + \frac{3}{10} = \frac{4}{5}$

$P(A) = \frac{4}{5} - \frac{3}{10}$

$P(A) = \frac{5}{10} = \frac{1}{2} \rightarrow (2)$

Now according to the image:



$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= \frac{3}{5} - \frac{3}{10}$

$= \frac{3}{10} \rightarrow (3)$

Also, $P(\bar{A}) = 1 - P(A)$

$= 1 - \frac{1}{2}$

$= \frac{1}{2} \rightarrow (4)$

Now,

$P\left(\frac{B}{\bar{A}}\right) = \frac{P(B \cap \bar{A})}{P(\bar{A})}$

$= \frac{\frac{3}{10}}{\frac{1}{2}}$

$= \frac{3}{10} \times \frac{2}{1}$

$= \frac{3}{5}$

38. Question

Mark the correct alternative in each of the following:

If A and B are two events such that P(A) = 0.4, P(B) = 0.3 and P(A ∪ B) = 0.5, then $P(\overline{B} \cap A)$ equals.

A. 2/3

B. 1/2

C. 3/10

D. 1/5

Answer

P(A) =0.4 , P(B) =0.3 , P(A⋃B) = 0.5 → (Given)

Now, according to the addition theorem of probability,

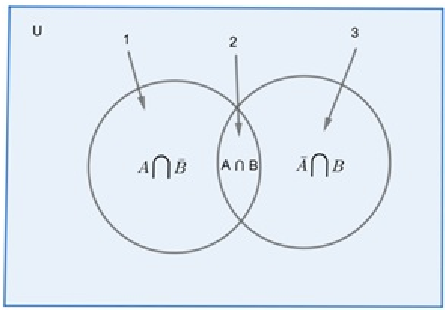
P(A⋃B) = P(A) + P(B) – P(A⋂B)

0.5 = 0.4 + 0.3 – P(A∩B)

P(A∩B) = 0.7 – 0.5

P(A∩B) = 0.2 → (1)

Now according to the image:



$P(\overline{B} \bigcap A)$ = P(A) – P(A⋂B)

= 0.4 – 0.2

= 0.2

$= \frac{2}{10}$

$= \frac{1}{5}$

39. Question

Mark the correct alternative in each of the following:

If A and B are two events such that A ≠ ϕ, B = ϕ, then

A. $P(A/B) = \frac{P(A \cap B)}{P(B)}$

B. P(A/B) = P(A) P(B)

C. P(A/B) = P(B/A) = 1

D. $P(A/B) = \frac{P(A)}{P(B)}$

Answer

According to Conditional Probability :

If A and B are two events associated with a random experiment such that A ≠ ϕ and B ≠ ϕ then,

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ and $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

So, option (A) is correct.

40. Question

Mark the correct alternative in each of the following:

If A and B are two events such that P(A) ≠ 0 and P(B) ≠ 1, then $P\left(\bar{A}/\bar{B}\right)$

A. 1 – P(A/B)

B. $1-P\left(\bar{A}/B\right)$

C. $\dfrac{1-P(A\cup B)}{P\left(\bar{B}\right)}$

D. $\dfrac{P\left(\bar{A}\right)}{P\left(\bar{B}\right)}$

Answer

—— $P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} = \frac{P\,(\overline{A\cup B})}{P(\bar{B})}$ → (1)

Also,

$P(\bar{A}\cap\bar{B}) = P\,(\overline{A\cup B})$

= 1 – P(A⋃B) → (2)

Using (2) in Equation (1) we get,

$$P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{1-P(A\cup B)}{P(\bar{B})}$$

41. Question

Mark the correct alternative in each of the following:

If the events A and B are independent, then P(A ∩ B) is equal to

A. P(A) + P(B)

B. P(A) – P(B)

C. P(A) P(B)

D. $\dfrac{P(A)}{P(B)}$

Answer

As we know,

$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$ → (1)

Now as events A and B are independent events,

$P\left(\frac{A}{B}\right) = P(A)$ and $P\left(\frac{B}{A}\right) = P(B)$ → (2)

Using the value obtained in (2) in (1) we get,

$$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$$

P(A∩B) = P(B) × $P\left(\frac{A}{B}\right)$

$P(A \cap B) = P(B)\ P(A)$

42. Question

Mark the correct alternative in each of the following:

If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(\bar{A} \cap \bar{B})$ equals.

A. 4/15

B. 8/45

C. 1/3

D. 2/9

Answer

$P(A) = \frac{3}{5}, P(B) = \frac{4}{9}$ → (Given)

Now as we know,

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$ → (1)

Since, A and B are two independent events,

$P(A \cap B) = P(A)\ P(B)$

$P(A \cap B) = \frac{3}{5} \times \frac{4}{9}$

$= \frac{12}{45}$

$= \frac{4}{15}$ → (2)

Now, according to the addition theorem of probability,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{3}{5} + \frac{4}{9} - \frac{4}{15}$ → From (2)

$P(A \cup B) = \frac{27 + 20 - 12}{45}$

$= \frac{35}{45}$

$= \frac{7}{9}$ → (3)

Using (3) in equation (1) we get,

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - \frac{7}{9}$

$= \frac{2}{9}$

43. Question

Mark the correct alternative in each of the following:

If A and B are two independent events such that P(A) = 0.3, P(A ∪ B) = 0.5, then P(A/B) – P(B/A) =

A. 2/7

B. 3/35

C. 1/70

D. 1/7

Answer

$P(A) = 0.3$, $P(A\cup B) = 0.5 \rightarrow$ (Given)

Since, A and B are two independent events,

$P(A\cap B) = P(A)\, P(B)$

$P(A\cap B) = 0.3 \times P(B) \rightarrow (1)$

Also, according to the addition theorem of probability,

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$0.5 = 0.3 + P(B) - 0.3P(B) \rightarrow$ From (Given) & (1)

$0.7P(B) = 0.2$

$P(B) = \frac{0.2}{0.7} = \frac{2}{7} \rightarrow (2)$

Putting value of P(B) in equation (1) we get,

$P(A\cap B) = 0.3 \times \frac{2}{7} = \frac{3}{10} \times \frac{2}{7}$

$P(A\cap B) = \frac{6}{70} \rightarrow (3)$

Now,

$$P\left(\frac{A}{B}\right) - P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(B)} - \frac{P(A \cap B)}{P(A)}$$

$= \frac{\frac{6}{70}}{\frac{2}{7}} - \frac{\frac{6}{70}}{\frac{3}{10}} \rightarrow$ From (3) & (2) and (Given)

$$= \frac{6}{70} \times \frac{7}{2} - \frac{6}{70} \times \frac{10}{3}$$

$$= \frac{3}{10} - \frac{2}{7}$$

$$= \frac{1}{70}$$

44. Question

Mark the correct alternative in each of the following:

A flash light has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, the probability that both are dead is

A. 3/28

B. 1/14

C. 9/64

D. 33/56

Answer

Total Number of Batteries = 8

Total Number of Dead Batteries = 3

Now Let,

M = Event of selecting the first dead battery

$$P(M) = \frac{3}{8}$$

N= Event of selecting the second dead battery

$P(N) = \frac{2}{7}$ → (Without replacement)

P(Both dead batteries are selected) = P(M) × P(N)

$$= \frac{3}{8} \times \frac{2}{7}$$

$$= \frac{3}{28}$$

45. Question

Mark the correct alternative in each of the following:

A bag contains 5 red and 3 blue balls. If 3 balls are selected drawn at random without replacement, then the probability of getting exactly one red ball is

A. 15/29

B. 15/56

C. 45/196

D. 135/392

Answer

Total Red Balls = 5

Total Blue Balls = 3

M = Event getting Red ball

$$P(M) = \frac{5}{8}$$

N = Event getting Blue ball

$$P(N) = \frac{3}{8}$$

Now,

P(Getting Exactly one red ball) = P(RBB) + P(BRB) + P(BBR)

$$= \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6}$$

$$= \frac{5}{56} + \frac{5}{56} + \frac{5}{56}$$

$$= \frac{15}{56}$$

46. Question

Mark the correct alternative in each of the following:

A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then the probability that exactly two of the three balls were red, the first ball being red, is

C. 15/28 D. 5/28

Answer

Total Red Balls = 5

Total Blue Balls = 3

M = Event getting Red ball

N = Event getting Blue ball

Now,

P(Getting exactly 2 red balls of 3 balls, the first ball being red) →

$= \frac{4}{7} \times \frac{3}{6} + \frac{3}{7} \times \frac{4}{6}$ → (Since 1st ball is already red, P(red ball) = $\frac{4}{7}$ and so on..)

$$= \frac{2}{7} + \frac{2}{7}$$

$$= \frac{4}{7}$$

47. Question

Mark the correct alternative in each of the following:

In a college 30% students fail in Physics, 25% fail in Mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in Physics if she has failed in Mathematics is

A. 1/10

B. 1/3

C. 2/5

D. 9/20

Answer

A = Failing in Physics

B = Failing in Mathematics

P(A) = 30%

$$= \frac{30}{100}$$

$$= \frac{3}{10}$$

P(B) = 25%

$$= \frac{25}{100}$$

$= \frac{1}{4}$ → (Given)

P(A∩B) = 10%

$$= \frac{10}{100}$$

$= \frac{1}{10}$

Now we know,

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{1}{10}}{\frac{1}{4}}$

$= \frac{1}{10} \times \frac{4}{1}$

$= \frac{4}{10}$

$= \frac{2}{5}$

48. Question

Mark the correct alternative in each of the following:

Three person, A, B and C fire a target in turn starting with A. Their probabilities of hitting the target are 0.4, 0.3 and 0.2 respectively. The probability of two hits is

A. 0.024

B. 0.452

C. 0.336

D. 0.188

Answer

A = Event of Person A hitting the target.

P(A) = 0.4

$P(\overline{A})$ = 1 – P(A) = 1 – 0.4

$P(\overline{A})$ = 0.6 → (1)

B = Event of Person B hitting the target.

P(B) = 0.3

$P(\overline{B})$ = 1 – P(B) = 1 – 0.3

$P(\overline{B})$ = 0.7 → (2)

C = Event of Person C hitting the target.

P(C) = 0.2

$P(\overline{C})$ = 1 – P(C) = 1 – 0.2

$P(\overline{C})$ = 0.8 → (3)

Now then,

$P(\text{Two Hits}) = P(A\,B\,\overline{C}) + P(A\,\overline{B}\,C) + P(\overline{A}\,B\,C)$

= 0.4 × 0.3 × 0.8 + 0.4 × 0.7 × 0.2 + 0.6 × 0.3 × 0.2

= 0.096 + 0.056 + 0.036

= 0.188

49. Question

Mark the correct alternative in each of the following:

A and B are two students. Their chances of solving a problem correctly are 1/3 and 1/4 respectively. If the probability of their making common error is 1/20 and they obtain the same answer, then the probability of their answer to be current is

A. 10/13

B. 13/120

C. 1/40

D. 1/12

Answer

E_1 = Both Solve correctly

$$P(E_1) = \frac{1}{4} \times \frac{1}{3}$$

$$= \frac{1}{12}$$

E_2 = Both Solve incorrectly

$$P(E_2) = \frac{3}{4} \times \frac{2}{3}$$

$$= \frac{6}{12}$$

A = They get the same result

Using Baye's Theorem →

$$P\left(\frac{E1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{12} \times 1}{\frac{1}{12} \times 1 + \frac{6}{12} \times \frac{1}{20}}$$

$$= \frac{20}{26}$$

$$= \frac{10}{13}$$

50. Question

Mark the correct alternative in each of the following:

Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. The probability that both cards are queen is

A. $\frac{1}{13} \times \frac{1}{13}$

B. $\frac{1}{13}+\frac{1}{13}$

C. $\frac{1}{13}\times\frac{1}{17}$

D. $\frac{1}{13}\times\frac{4}{5}$

Answer

M = Event that Queen is draw in in first draw.

P(M) = $\frac{4}{52}$

N = Event that Queen is drawn in second draw.

P(N) = $\frac{4}{52}$

Now, Probability that both the cards drawn are queen:

P = P(M) $\times$ P(N)

$$P = \frac{4}{52}\times\frac{4}{52}$$

$$= \frac{1}{13}\times\frac{1}{13}$$

51. Question

Mark the correct alternative in each of the following:

A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue balls is

A. 167/168

B. 1/28

C. 2/21

D. 3/28

Answer

P(G) = Probability of drawing a green ball = $\frac{3}{8}$

P(B) = Probability of drawing a blue ball = $\frac{2}{8}$

P(O) = Probability of drawing an orange ball = $\frac{3}{8}$

Important → When taking the probability of green/ball in 2^{nd} draw, remember to reduce the total number of balls and number of green/blue ball

While finding the probability. Example – P(GGB) = $\frac{3}{8}\times\frac{2}{7}\times\frac{2}{6}$

Now Probability of drawing 2 green balls and one blue ball :

P = P(GGB) + P(GBG) + P(BGG)

$$= \frac{3}{8}\times\frac{2}{7}\times\frac{2}{6}+\frac{3}{8}\times\frac{2}{7}\times\frac{2}{6}+\frac{2}{8}\times\frac{3}{7}\times\frac{2}{6}$$

$= \frac{1}{28} + \frac{1}{28} + \frac{1}{28}$

$= \frac{3}{28}$

52. Question

Mark the correct alternative in each of the following:

If two events are independent, then

A. they must be mutually exclusive

B. the sum of their probabilities must be equal to 1

C. (a) and (b) both are correct

D. none of the above is correct

Answer

Let A and B be two independent events, then,

$P(A \cap B) = P(A) \times P(B)$

$\therefore P(A \cap B) \neq 0$ → Not mutually exclusive

Or,

$P(A) + P(B) \neq 1$ → Their sum of probabilities is not 1

∴ None of the above is correct.

53. Question

Mark the correct alternative in each of the following:

Two dice are thrown. If it is known that the sum of the numbers on the dice was less than 6, the probability of getting a sum 3, is

A. 1/18

B. 5/18

C. 1/5

D. 2/5

Answer

A = Event of getting a sum of 3.

A = { (1,2) , (2,1) }

B = Event of sum of the numbers less than 6.

B = { (1,1) , (1,2) , (1,3) , (1,4) , (2,1) , (2,2) , (2,3) , (3,1) , (3,2) , (4,1) }

$(A \cap B)$ = { (1,2) , (2,1) } → (1)

Now, we have →

n(A) = 2

n(B) = 10

$n(A \cap B) = n(A) = 2$ → (2)

Now,

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{2}{10}$$

$$= \frac{1}{5}$$

54. Question

Mark the correct alternative in each of the following:

If A and B are such that $P(A \cup B) = \frac{5}{9}$ and $P(\bar{A} \cup \bar{B}) = \frac{2}{3}$, then $P(\bar{A}) + P(\bar{B}) =$

A. 9/10

B. 10/9

C. 8/9

D. 9/8

Answer

$P(A \cup B) = \frac{5}{9}$, $P(\bar{A} \cup \bar{B}) = \frac{2}{3}$ → (Given)

———Now,

$P(\bar{A} \cup \bar{B}) = P(A \cap B)$ → (1)

———$P(A \cap B) = 1 - P(A \cap B)$

$P(A \cap B) = 1 - \frac{2}{3}$ → From (Given) & (1)

$$P(A \cap B) = \frac{1}{3}$$

Also,

$P(\bar{A}) = 1 - P(A)$ → (2)

$P(\bar{B}) = 1 - P(B)$ → (3)

Now, according to the addition theorem of probability:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{5}{9} = P(A) + P(B) - \frac{1}{3}$$

$$P(A) + P(B) = \frac{5}{9} + \frac{1}{3}$$

$1 - P(\bar{A}) + 1 - P(\bar{B}) = \frac{8}{9}$ → From (2) & (3)

$$P(\bar{A}) + P(\bar{B}) = 2 - \frac{8}{9}$$

$$= \frac{10}{9}$$

55. Question

Mark the correct alternative in each of the following:

If A and B are two events such that P(A/B) = p, P(A) = p, P(B) = 1/3 and $P\left(A\cup B\right)=\frac{5}{9}$, then p =

A. 2/3

B. 3/5

C. 1/3

D. 3/4

Answer

$P\left(\frac{A}{B}\right)$ = p , P(A) = p , P(B) = $\frac{1}{3}$, P(A∪B) = $\frac{5}{9}$ → (Given)

Now,

$$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$$

$p = \frac{P(A\cap B)}{\frac{1}{3}}$ → From (Given)

P(A∩B) = $\frac{p}{3}$ → (1)

Now, according to the addition theorem of probability:

P(A⋃B) = P(A) + P(B) – P(A⋂B)

$$\frac{5}{9} = p + \frac{1}{3} - \frac{p}{3}$$

$$p - \frac{p}{3} = \frac{5}{9} - \frac{1}{3}$$

$$\frac{2p}{3} = \frac{2}{9}$$

$$p = \frac{2}{9} \times \frac{3}{2}$$

$$= \frac{1}{3}$$

56. Question

Mark the correct alternative in each of the following:

A die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number of the die and a spade card is

A. 1/2

B. 1/4

C. 1/8

D. 3/4

Answer

M = Event of getting an even number.

P(M) = 1/2 → (1)

N = Event of getting a spade card.

P(N) = 1/4 → (2)

P(Even number of the die and spade card) = P(MN)

$= \frac{1}{2} \times \frac{1}{4} \rightarrow$ From (1) & (2)

$= \frac{1}{8}$

57. Question

Mark the correct alternative in each of the following:

Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family ahs at least one girl is

A. 1/2

B. 1/3

C. 2/3

D. 4/7

Answer

The sample set we have, where the first letter represents the eldest child of the family→

S = { GGG, GBG, GGB, GBB , BBB, BBG, BGB, BGG }

Now let,

M = Event of choosing a family with a girl as the eldest child.

= { GGG, GBG, GGB, GBB }

N = Event of choosing a family with at-least one girl child.

= { GGG, GBG, GGB, GBB, BBG, BGB, BGG }

We have,

n(M) = 4

n(N) = 7

n(M∩N) = 4

Now,

$$P\left(\frac{M}{N}\right) = \frac{n(M \cap N)}{n(N)}$$

$= \frac{4}{7}$

58. Question

Mark the correct alternative in each of the following:

Let A and B be two events. If P(A) = 0.2, P(B) = 0.4, P(A ∪ B) = 0.6, then P(A/B) is equal to

A. 0.8

B. 0.5

C. 0.3

D. 0

Answer

P(A) = 0.2 , P(B) = 0.4 , P(A∪B) = 0.6 → (Given)

According to the addition theorem of probability,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = 0.2 + 0.4 - P(A \cap B)$

$0.6 = 0.6 - P(A \cap B)$

$P(A \cap B) = 0 \rightarrow (1)$

Now,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0}{0.4}$$

$= 0$

59. Question

Mark the correct alternative in each of the following:

Let A and B be two events such that P(A) = 0.6, P(B) = 0.2 and P(A/B) = 0.5. Then $P\left(\overline{A} / \overline{B}\right)$ equals.

A. 1/10

B. 3/10

C. 3/8

D. 6/7

Answer

$P(A) = 0.6$, $P(B) = 0.2$, $P\left(\frac{A}{B}\right) = 0.5 \rightarrow$ (Given)

$$P\left(\frac{\overline{A}}{\overline{B}}\right) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})}$$

$$= \frac{P\,(\overline{A \cup B})}{P(\overline{B})} \rightarrow (1)$$

$P(\overline{B}) = 1 - P(B)$

$= 1 - 0.2$

$= 0.8$

Also,

$P(\overline{A} \cap \overline{B}) = P\,(\overline{A \cup B})$

$= 1 - P(A \cup B) \rightarrow (3)$

Using (2) & (3) in Equation (1) we get,

$$P\left(\frac{\overline{A}}{\overline{B}}\right) = \frac{1 - P(A \cup B)}{0.8} \rightarrow (3)$$

Now,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$P(A \cap B) = 0.5 \times P(B) \rightarrow$ From (Given)

$P(A \cap B) = 0.5 \times 0.2$

$P(A \cap B) = 0.1$

According to the addition theorem of probability,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.6 + 0.2 - 0.1$

$= 0.8 - 0.1$

$= 0.7 \rightarrow (4)$

Putting (4) in equation (3) we get,

$$P\left(\frac{\overline{A}}{\overline{B}}\right) = \frac{1 - P(A \cup B)}{0.8}$$

$$= \frac{1 - 0.7}{0.8}$$

$$= \frac{3}{8}$$