32. Mean and Variance of a Random Variable

Exercise 32.1

1 A. Question

Which of the following distributions of probabilities of a random variable X are the probability distributions?

X:	3	2	1	0	-1
P(X):	0.3	0.2	0.4	0.1	0.05

Answer

The key point to solve the problem:

A given distribution of probabilities of a random variable X is said to be probability distribution if the sum of probabilities associated with each random variable is equal to 1

i.e. $\Sigma(p_i) = 1$

Given distribution is :

X:	3	2	1	0	-1
P(X):	0.3	0.2	0.4	0.1	0.05

Clearly,

Sum of probabilities = P(X = -1) + P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)

= 0.3 + 0.2 + 0.4 + 0.1 + 0.05

= 1.05 ≠ 1

∴ The given distribution is not a probability distribution.

1 B. Question

Which of the following distributions of probabilities of a random variable X are the probability distributions?

X:	0	1	2
P(X):	0.6	0.4	0.2

Answer

The key point to solve the problem:

A given distribution of probabilities of a random variable X is said to be probability distribution if the sum of probabilities associated with each random variable is equal to 1 i.e. $\Sigma(p_i) = 1$

Given distribution is :

X:	0	1	2
P(X):	0.6	0.4	0.2

Clearly,

Sum of probabilities = P(X = 0) + P(X = 1) + P(X = 2)

= 0.6 + 0.4 + 0.2

= 1.2 ≠ 1

∴ The given distribution is not a probability distribution.

1 C. Question

Which of the following distributions of probabilities of a random variable X are the probability distributions?

X:	0	1	2	3	4
P(X):	0.1	0.5	0.2	0.1	0.1

Answer

The key point to solve the problem:

A given distribution of probabilities of a random variable X is said to be probability distribution if the sum of probabilities associated with each random variable is equal to 1

i.e. $\Sigma(p_i) = 1$

Given distribution is :

X:	0	1	2	3	4
P(X):	0.1	0.5	0.2	0.1	0.1

Clearly,

Sum of probabilities = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)

= 0.1 + 0.5 + 0.2 + 0.1 + 0.1

= 1

∴ The given distribution is a probability distribution.

1 D. Question

Which of the following distributions of probabilities of a random variable X are the probability distributions?

X:	0	1	2	3
P(X):	0.3	0.2	0.4	0.1

Answer

The key point to solve the problem:

A given distribution of probabilities of a random variable X is said to be probability distribution if the sum of probabilities associated with each random variable is equal to 1

i.e. $\Sigma(p_i) = 1$

Given distribution is :

X:	0	1	2	3
P(X):	0.3	0.2	0.4	0.1

Clearly,

Sum of probabilities = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)

= 0.3 + 0.2 + 0.4 + 0.1

= 1

∴ The given distribution is a probability distribution.

2. Question

A random variable X has the following probability distribution:

Values of X:	-2	-1	0	1	2	3
P (X):	0.1	k	0.2	2k	0.3	k

Answer

The key point to solve the problem:

If a probability distribution is given then as per its definition, Sum of probabilities associated with each value of a random variable of given distribution is equal to 1

i.e. $\Sigma(p_i) = 1$

∴ P(X = -2) + P(X = -1) + P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1

0.1 + k + 0.2 + 2k + 0.3 + k = 1

0.6 + 4k = 1

4k = 0.4

K = 0.1

Value of k = 0.1

3. Question

A random variable X has the following probability distribution:

Values of X:	0	1	2	3	4	5	6	7	8
P (X):	a	3a	5a	7a	9a	11a	13a	15a	17a

Determine:

i. The value of a

ii. P (X<3), P (X≥3), P(0<X<5).

Answer

The key point to solve the problem:

If a probability distribution is given then as per its definition, Sum of probabilities associated with each value of a random variable of given distribution is equal to 1

i.e. $\Sigma(p_i) = 1$

∴ Sum of probabilities = 1

∴ a+3a+5a+7a+9a+11a+13a+15a+17a = 1

a(1+3+5+7+9+11+13+15+17) = 1

Thus, a = $\frac{1}{81}$ ....ans (i)

P(X<3) = P(X = 0) + P(X = 1) + P(X = 2)

= a+3a+5a

= 9a = $9 \times \frac{1}{81} = \frac{1}{9}$

P(X≥3) = 1 - P(X<3) {∵ sum of probabilities in distribution is 1}

= $1 - \frac{1}{9} = \frac{8}{9}$

P(0<X<5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)

= 3a + 5a + 7a + 9a

= 24a

= $\frac{24}{81} = \frac{8}{27}$ ....ans (ii)

4. Question

The probability distribution function of a random variable X is given by

X_i:	0	1	2
P_i	$3c^3$	$4c-10c^2$	$5c-1$

where c > 0

Find:

i. c ii. $P(X<2)$

iii. $P(1<X\leq 2)$

Answer

The key point to solve the problem:

If a probability distribution is given then as per its definition, Sum of probabilities associated with each value of a random variable of given distribution is equal to 1

i.e. $\Sigma(p_i) = 1$

Given probability distribution is:

X_i:	0	1	2
P_i	$3c^3$	$4c-10c^2$	$5c-1$

For the probability distribution:

$\Sigma(p_i) = 1$

$$\therefore 3c^3 + (4c - 10c^2) + (5c - 1) = 1$$

$$3c^3 - 10c^2 + 9c - 2 = 0$$

$$3c^3 - 3c^2 - 7c^2 + 7c + 2c - 2 = 0$$

Note: It's better to apply hit and trial method for solving cubic equation, just by putting 1 we get that it satisfies the equation. So (c - 1) will be its one factor. After that divide the polynomial with c-1 which will give a quadratic factor which will be factorized easily.

But we can also proceed as the equation is solved if you can think upto that extent.

$$3c^2(c - 1) - 7c(c - 1) + 2(c - 1) = 0$$

$$(c - 1)(3c^2 - 7c + 2) = 0$$

$$(c - 1)(3c^2 - 6c - c + 2) = 0$$

$(c - 1)\{3c(c - 2)-1(c - 2)\} = 0$

$(c - 1)(3c - 1)(c - 2) = 0$

$\therefore c = 1$ or $c = 2$ or $c = 1/3$

As we are given that $c > 0$

All three values satisfy the given condition, so they must be the values.

BUT BE CAUTIOUS :

As we know that the probability of any event lies between 0 and 1, and we are using c to represent probabilities. So We need to check whether for these values probabilities are defined are valid or not.

X_i : 0 1 2

P_i : $3c^3$ $4c-10c^2$ $5c-1$

As this distribution suggests c = 1 and c = 2 are going to make probabilities invalid.

So, c = 1/3 is the only solution for c … (i)

P(X<2) = 1 − P(X = 2)

= 1 − (5c − 1) = 2 − 5c

= 2 − 5/3 = 1/3

P(1 < X ≤ 3) = P(2)

= 5/3 − 1 = 2/3 ……(ii)

5. Question

Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$. Find the probability distribution of X.

Answer

Key point to solve the problem:

If a probability distribution is given then as per its definition, Sum of probabilities associated with each value of random variable of given distribution is equal to 1

i.e. $\Sigma(p_i) = 1$

Let,

$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4) = k$ (say)

$\therefore P(X = x_1) = k/2$

$P(X = x_2) = k/3$

$P(X = x_3) = k$

$P(X = x_4) = k/5$

$\because \Sigma(p_i) = 1$ {$\because$ it is given that it is a probability distribution }

$$\frac{k}{2} + k + \frac{k}{3} + \frac{k}{5} = 1$$

15k + 30k + 10k + 6k = 30 [by taace LCM]

61k = 30

k = 30/61

$\therefore$ the required probability distribution is :

X	P(X)
x_1	$\frac{k}{2} = \frac{30}{2 \times 61} = \frac{15}{61}$
x_2	$\frac{k}{3} = \frac{30}{3 \times 61} = \frac{10}{61}$
x_3	$k = \frac{30}{61}$
x_4	$\frac{k}{5} = \frac{30}{5 \times 61} = \frac{6}{61}$

6. Question

A random variable X takes the values 0, 1, 2 and 3 such that:

P(X = 0) = P(X>0) = P(X<0);

P(X = -3) = P(X = -2) = P(X = -1);

P (X = 1) = P(X = 2) = P(X = 3).

Obtain the probability distribution of X.

Answer

The key point to solve the problem:

If a probability distribution is given then as per its definition, Sum of probabilities associated with each value of a random variable of given distribution is equal to 1

i.e. $\Sigma(p_i) = 1$

Let, P(X = 0) = k

As sum of probabilities associated with each random variable is 1

∴ P(X<0) + P(X = 0) + P(X>0) = 1

k + k + k = 1 {∵ P(X = 0) = P(X>0) = P(X<0)}

3k = 1

∴ k = 1/3

Thus

P(X<0) = 1/3

P(X = -3) + P(X = -2) + P(X = -1) = 1/3

m+m+m = 1/3 {∵ P(X = -3) = P(X = -2) = P(X = -1) = m (say) }

m = 1/9

Similarly,

P(X>0) = 1/3

P (X = 1) + P(X = 2) + P(X = 3) = 1/3

n+n+n = 1/3 {∵ P(X = 3) = P(X = 2) = P(X = 1) = n (say) }

∴ n = 1/9

∴ the required probability distribution is :

X	P(X)
-3	$\frac{1}{9}$
-2	$\frac{1}{9}$
-1	$\frac{1}{9}$
0	$\frac{1}{3}$
1	$\frac{1}{9}$
2	$\frac{1}{9}$
3	$\frac{1}{9}$

7. Question

Two cards are drawn from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces.

Answer

In a deck of 52 cards, there are 4 aces each of one suit respectively.

Let X be the random variable denoting the number of aces for an event when two cards are drawn simultaneously.

∴ X can take values 0, 1 or 2.

P(X = 0) = $\frac{48C_2}{52C_2} = \frac{48\times47}{52\times51} = \frac{188}{221}$

[For selecting 0 aces, we removed all 4 aces from the deck and selected out of 48]

P(X = 1) = $\frac{4C_1\times48C_1}{52C_2} = \frac{48\times4\times2}{52\times51} = \frac{32}{221}$

[For selecting 1 ace, we need to select and 1 out of 4 and not any other]

P(X = 2) = $\frac{4C_2}{52C_2} = \frac{4\times3}{52\times51} = \frac{1}{221}$

[For selecting 2 aces, we need to select and 2 out of 4]

Now we have p_i and x_i.

As $\frac{1}{221}+\frac{32}{221}+\frac{188}{221}=1$

Thus, $\sum(p_i) = 1$

∴ Now we are ready to write the probability distribution for X:-

The following table gives probability distribution:

x_i	p_i
0	$\frac{188}{221}$
1	$\frac{32}{221}$
2	$\frac{1}{221}$

8. Question

Find the probability distribution of the number of heads, when three coins are tossed.

Answer

When we toss a coin three times we have the following possibilities:

{HHH,HHT,HTH,THH,HTT,THT,TTH,TTT}

Let X be a random variable representing some heads in 3 tosses of a coin.

∵ The probability of getting a head and probability of getting a tail are independent events and P(GETTING A TAIL) = P(GETTING A HEAD) = 1/2

∴ P(Head in the first toss) and P(Head in the second toss) and P(head in the third toss) can be given by their products.

Note: $P(A\cap B) = P(A)P(B)$ where A and B are independent events.

Thus,

P(X = 0) = P(TTT) = P(T)P(T)P(T) = 1/2 x 1/2 x 1/2 = 1/8

P(X = 1) = P(HTT or THT or TTH) = P(HTT)+P(THT)+P(TTH)

= P(H)P(T)P(T)+ P(T)P(H)P(T)+ P(T)P(T)P(H)

= 1/2 x 1/2 x 1/2 + 1/2 x 1/2 x 1/2 + 1/2 x 1/2 x 1/2

= 3/8

P(X = 2) = P(HHT or HTH or THH) = P(HHT)+P(HTH)+P(THH)

= P(H)P(H)P(T)+ P(H)P(T)P(H)+ P(T)P(H)P(H)

= 1/2 x 1/2 x 1/2 + 1/2 x 1/2 x 1/2 + 1/2 x 1/2 x 1/2

= 3/8

P(X = 3) = P(HHH) = P(H)P(H)P(H) = 1/2 x 1/2 x 1/2 = 1/8

Now we have p_i and x_i.

As $\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8} = 1$

Thus, $\Sigma(p_i) = 1$

∴ Now we are ready to write the probability distribution for X:-

The following table gives probability distribution:

x_i	p_i
0	$\frac{1}{8}$
1	$\frac{3}{8}$
2	$\frac{3}{8}$
3	$\frac{1}{8}$

9. Question

Four cards are drawn simultaneously from a well-shuffled pack of 52 playing cards. Find the probability distribution of the number of aces.

Answer

In a deck of 52 cards, there are 4 aces each of one suit respectively.

Let X be the random variable denoting the number of aces for an event when 4 cards are drawn simultaneously.

∴ X can take values 0 , 1 , 2 , 3 or 4

P(X = 0) = $\frac{48C_4}{52C_4}$

[For selecting 0 aces, we removed all 4 aces from the deck and selected out of 48]

P(X = 1) = $\frac{4C_1 \times 48C_3}{52C_4}$

[For selecting 1 ace, we selected and 1 out of 4aces and 3 cards from remaining 48]

P(X = 2) = $\frac{4C_2 \times 48C_2}{52C_4}$

[For selecting 2 aces, we selected and 2 out of 4aces and 2 cards from remaining 48]

P(X = 3) = $\frac{4C_3 \times 48C_1}{52C_4}$

[For selecting 3 aces, we selected and 3 out of 4aces and 1 card from remaining 48]

$P(X = 4) = \frac{4C_4}{52C_4}$

[For selecting 4 aces, we selected and 4 out of 4 aces]

Now we have p_i and $x_{i.}$

∴ Now we are ready to write the probability distribution for X:-

The following table gives probability distribution:

x_i	p_i
0	$\frac{48C_4}{52C_4}$
1	$\frac{4C_1 \times 48C_3}{52C_4}$
2	$\frac{4C_2 \times 48C_2}{52C_4}$
3	$\frac{4C_3 \times 48C_1}{52C_4}$
4	$\frac{4C_4}{52C_4}$

10. Question

A bag contains 4 red and 6 black balls. Three balls are drawn at random. Find the probability distribution of the number of red balls.

Answer

X represents the number of red balls drawn.

∴ X can take values 0,1,2 or 3

∵ there are total 10 balls

n(S) = total possible ways of selecting 2 balls = $10C_3$

P(X = 0) = P(selecting no red balls) = $\frac{6C_3}{10C_3} = \frac{6\times5\times4}{10\times9\times8} = \frac{1}{6}$

P(X = 1) = P(selecting 1 red ball and 2 black balls)

$= \frac{4C_1\times6C_2}{10C_3} = \frac{4\times6\times5\times3\times2}{10\times9\times8\times2} = \frac{1}{2}$

P(X = 2) = P(selecting 2 red balls and 1 black ball)

$= \frac{4C_2\times6C_1}{10C_3} = \frac{4\times3\times6\times3}{10\times9\times8} = \frac{3}{10}$

P(X = 3) = P(selecting 3 red balls and 0 black ball)

$= \frac{4C_3}{10C_3} = \frac{4\times3\times2\times1}{10\times9\times8} = \frac{1}{30}$

Now we have p_i and x_i.

Following table represents the probability distribution of X :

x_i	p_i
0	$\frac{1}{6}$
1	$\frac{1}{2}$
2	$\frac{3}{10}$
3	$\frac{1}{30}$

11. Question

Five defective mangoes are accidentally mixed with 15 goods ones. Four mangoes are drawn at random from this lot. Find the probability distribution of the number of defective mangoes.

Answer

X represents the number of defective mangoes drawn.

∴ X can take values 0,1,2 or 3

∵ there are total 20mangoes (15good+5defective) mangoes

n(S) = total possible ways of selecting 5 mangoes = $20C_4$

P(X = 0) = P(selecting no defective mango) = $\frac{15C_4}{20C_4} = \frac{15\times14\times13\times12}{20\times19\times18\times17} = \frac{91}{323}$

P(X = 1) = P(selecting 1 defective mango and 3 good mangoes)

$= \frac{5C_1\times15C_3}{20C_4} = \frac{455}{969}$

P(X = 2) = P(selecting 2 defective mangoes and 2 good mangoes)

$= \frac{5C_2\times15C_2}{20C_4} = \frac{70}{323}$

P(X = 3) = P(selecting 3 defective mangoes and 1 good mango)

$= \frac{5C_3\times15C_1}{20C_4} = \frac{10}{323}$

P(X = 4) = P(selecting 4 defective mangoes and no good mango)

$= \frac{5C_4}{20C_4} = \frac{1}{969}$

Now we have p_i and x_i.

Following table represents the probability distribution of X :

x_i	p_i
0	$\frac{91}{323}$
1	$\frac{455}{969}$
2	$\frac{70}{323}$
3	$\frac{10}{323}$
4	$\frac{1}{969}$

12. Question

Two dice are thrown together, and the number appearing on them noted. X denotes the sum of the two numbers. Assuming that all the 36 outcomes are equally likely, what is the probability distribution of X?

Answer

When two fair dice are thrown there are total 36 possible outcomes.

∵ X denotes the sum of 2 numbers appearing on dice.

∴ X can take values 2,3,4,5,6,7,8,9,10,11 and 12

As appearance of a number on a fair die is equally likely

i.e. P(appearing of 1) = P(appearing of 2) = P(appearing of 3) = P(appearing of 4) = P(appearing of 5) = P(appearing of 6) = 1/6

And also the appearance of numbers on two different dice is an independent event. So two find conditions like P(1 in the first dice and 2 in the second dice) can be given using multiplication rule of probability.

Note: P(A∩B) = P(A)P(B) where A and B are independent events.

$P(X = 2) = \frac{1}{36}$ {∵ (1,1) is the only combination resulting sum = 2}

$P(X = 3) = \frac{2}{36} = \frac{1}{18}$

{∵ (1,2) and (2,1) are the combinations resulting in sum = 3}

$P(X = 4) = \frac{3}{36} = \frac{1}{12}$

{∵ (1,3), (3,1) and (2,2) are the combinations resulting in sum = 4}

$P(X = 5) = \frac{4}{36} = \frac{1}{9}$

{∵ (3,2) (2,3) (1,4) and (4,1) are the combinations resulting in sum = 5}

$P(X = 6) = \frac{5}{36}$

{∵ (1,5) (5,1) (2,4) (4,2) (3,3) are the combinations resulting in sum = 6}

$P(X = 7) = \frac{6}{36} = \frac{1}{6}$

{∵ (1,6) (6,1) (2,5) (5,2) (3,4) (4,3) are the combinations resulting in sum = 7}

$P(X = 8) = \frac{5}{36}$

{∵ (3,5) (5,3) (2,6) (6,2) (4,4) are the combinations resulting in sum = 8}

$P(X = 9) = \frac{4}{36} = \frac{1}{9}$

{∵ (3,6) (6,3) (5,4) and (4,5) are the combinations resulting in sum = 9}

$P(X = 10) = \frac{3}{36} = \frac{1}{12}$

{∵ (6,4), (4,6) and (5,5) are the combinations resulting in sum = 10}

$P(X = 11) = \frac{2}{36} = \frac{1}{18}$

{∵ (5,6) and (6,5) are the combinations resulting in sum = 11}

$P(X = 12) = \frac{1}{36}$ {∵ (6,6) is the only combination resulting sum = 2}

Now we have p_i and x_i.

∴ Required probability distribution is:-

X:	2	3	4	5	6	7	8	9	10	11	12
P(X):	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

13. Question

A class has 15 students whose ages are 14, 17, 15, 14, 21, 19, 20, 16, 18, 17, 20, 17, 16, 19 and 20 years respectively. One student is selected in such a manner that each has the same chance of being selected and the age X of the selected student is recorded. What is the probability distribution of the random variable X?

Answer

Note Many of the times while solving such simple problems we make a mistake in counting. So at first, we should make a frequency table which tells us no of students in the class of the same age.

This makes our interpretation easier.

∴ frequency distribution table for age and number of students is:

Age	14	15	16	17	18	19	20	21
Number Of Students	2	1	2	3	1	2	3	1

X represents the age of a random student.

∴ X can take values 14,15,16,17,18,19,20 or 21

Total No. of students in class = 15

Using the above frequency table, we can easily see a total number of students of a particular age, and hence we can find probability easily.

P(X = 14) = $\frac{2}{15}$ {As probability = $\frac{\text{no of favourable outcomes}}{\text{total no of outcomes}}$ }

Similarly,

P(X = 15) = $\frac{1}{15}$; P(X = 16) = $\frac{2}{15}$; P(X = 17) = $\frac{3}{15} = \frac{1}{5}$

P(X = 18) = $\frac{1}{15}$; P(X = 19) = $\frac{2}{15}$; P(X = 20) = $\frac{3}{15} = \frac{1}{5}$

P(X = 21) = $\frac{1}{15}$

Now we have p_i and x_i.

∴ Required probability distribution is:-

X:	14	15	16	17	18	19	20	21
P(X):	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$

14. Question

Five defective bolts are accidentally mixed twenty good ones. If four bolts are drawn at random from this lot, find the probability distribution of the number of defective bolts.

Answer

X represents the number of defective bolts drawn.

∴ X can take values 0,1,2 or 3

∵ there are total 25 bolts (20 good + 5 defectives) bolts

n(S) = total possible ways of selecting 5 bolts = $25C_4$

P(X = 0) = P(selecting no defective bolt) = $\frac{20C_4}{25C_4} = \frac{20\times19\times18\times17}{25\times24\times23\times22} = \frac{969}{2530}$

P(X = 1) = P(selecting 1 defective bolt and 3 good bolts)

$= \frac{5C_1 \times 20C_3}{25C_4} = \frac{114}{253}$

P(X = 2) = P(selecting 2 defective bolts and 2 good bolts)

$= \frac{5C_2 \times 20C_2}{25C_4} = \frac{38}{253}$

P(X = 3) = P(selecting 3 defective bolts and 1 good bolt)

$= \frac{5C_3 \times 20C_1}{25C_4} = \frac{4}{253}$

P(X = 4) = P(selecting 4 defective bolts and no good bolt)

$= \frac{5C_4}{25C_4} = \frac{1}{2530}$

Now we have p_i and x_i.

Following table represents the probability distribution of X :

x_i	p_i
0	$\frac{969}{2530}$
1	$\frac{455}{969}$
2	$\frac{38}{253}$
3	$\frac{4}{253}$
4	$\frac{1}{2530}$

15. Question

Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of aces.

Answer

Note: While reading this question you might have observed that cards are being drawn successively and sometimes in other question you might have get cards are drawn simultaneously

You have to be careful regarding these two words while solving the question. Both have a different meaning.

E.g.:-

When you draw 3 cards out of 52 simultaneously, then total no of ways of drawing is simply $52C_3$ = 22100. You draw 3 cards at a time.

When you draw 3 cards out of 52 successively, then total no of ways of drawing is not simply $52C_3$, once you have drawn the first card, only 51 cards are remaining and so on. You are drawing only one card at a time

In such case total ways of drawing 3 cards = $52C_1 \times 51C_1 \times 50C_1 = 132600$

Hence both are entirely different. So Be cautious regarding it.

Let’s solve the problem now:

Note: In our problem, it is given that after every draw, we are replacing the card. So our sample space will not change

In a deck of 52 cards, there are 4 aces each of one suit respectively.

Let X be the random variable denoting the number of aces for an event when 2 cards are drawn successively.

∴ X can take values 0, 1 or 2

$P(X = 0) = \frac{48}{52} \times \frac{48}{52} = \frac{144}{169}$

{As we have to select 1 card at a time such that no ace is there so the first probability is 48/52 and as the drawn card is replaced, next time again probability is 48/52 }

Similarly, we proceed for other cases.

$P(X = 1) = \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} = \frac{24}{169}$

{we might get ace in the first card or second card, so both probabilities are added}

Similarly,

$P(X = 2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

Now we have p_i and x_i.

∴ Now we are ready to write the probability distribution for X:-

The following table gives probability distribution:

x_i	p_i
0	$\frac{144}{169}$
1	$\frac{24}{169}$
2	$\frac{1}{169}$

16. Question

Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of kings.

Answer

Note: While reading this question you might have observed that cards are being drawn successively and sometimes in other question you might have get cards are drawn simultaneously

You have to be careful regarding these two words while solving the question. Both have a different meaning.

E.g.:-

When you draw 3 cards out of 52 simultaneously, then total no of ways of drawing is simply $52C_3$ = 22100. You draw 3 cards at a time.

When you draw 3 cards out of 52 successively, then total no of ways of drawing is not simply $52C_3$, once you have drawn the first card, only 51 cards are remaining and so on. You are drawing only one card at a time

In such case total ways of drawing 3 cards = $52C_1 \times 51C_1 \times 50C_1 = 132600$

Hence both are entirely different. So Be cautious regarding it.

Let's solve the problem now:

Note: In our problem, it is given that after every draw, we are replacing the card. So our sample space will not change

In a deck of 52 cards, there are 4 kings each of one suit respectively.

Let X be the random variable denoting the number of kings for an event when 2 cards are drawn successively.

∴ X can take values 0, 1 or 2

$P(X = 0) = \frac{48}{52} \times \frac{48}{52} = \frac{144}{169}$

{As we have to select 1 card at a time such that no king is there so the first probability is 48/52 and as the drawn card is replaced, next time again probability is 48/52 }

Similarly, we proceed for other cases.

$P(X = 1) = \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} = \frac{24}{169}$

{we might get a king in the first card or second card, so both probabilities are added}

Similarly,

$P(X = 2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

Now we have p_i and x_i.

∴ Now we are ready to write the probability distribution for X:-

The following table gives probability distribution:

x_i	p_i
0	$\frac{144}{169}$
1	$\frac{24}{169}$
2	$\frac{1}{169}$

17. Question

Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces.

Answer

Note: While reading this question you might have observed that cards are being drawn successively and sometimes in other question you might have get cards are drawn simultaneously

You have to be careful regarding these two words while solving the question. Both have a different meaning.

E.g.:-

When you draw 3 cards out of 52 simultaneously, then total no of ways of drawing is simply $52C_3$ = 22100. You draw 3 cards at a time.

When you draw 3 cards out of 52 successively, then total no of ways of drawing is not simply $52C_3$, once you have drawn the first card, only 51 cards are remaining and so on. You are drawing only one card at a time.

In such case total ways of drawing 3 cards = $52C_1 \times 51C_1 \times 50C_1 = 132600$

Hence both are entirely different. So Be cautious regarding it.

Let's solve the problem now:

Note: In our problem, it is given that after every draw, we are not replacing the card. So our sample space will change in this case

In a deck of 52 cards, there are 4 aces each of one suit respectively.

Let X be the random variable denoting the number of aces for an event when 2 cards are drawn successively.

∴ X can take values 0, 1 or 2

P(X = 0) = $\frac{48}{52} \times \frac{47}{51} = \frac{188}{221}$

{As we have to select 1 card at a time such that no ace is there so first probability is 48/52 and as the drawn card is not replaced, next time probability is 47/51 because now one card is not in our sample space and that card is not from group of ace, So now ace cards are 47 }

Similarly, we proceed for other cases.

$P(X = 1) = \frac{4}{52} \times \frac{48}{51} + \frac{48}{52} \times \frac{4}{51} = \frac{32}{221}$

{we might get ace in the first card or in the second card, so both probabilities are added}

Similarly,

$P(X = 2) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

Now we have p_i and x_i.

∴ Now we are ready to write the probability distribution for X:-

The following table gives probability distribution:

x_i	p_i
0	$\frac{188}{221}$
1	$\frac{32}{221}$
2	$\frac{1}{221}$

18. Question

Find the probability distribution of the number of white balls drawn in a random draw of 3 balls without replacement, from a bag containing 4 white and 6 red balls.

Answer

Let X denote the number of white balls drawn in a random draw of 3 balls.

∴ X can take values 0,1,2 or 3.

Since bag contains 6 red and 4 white ball, i.e. at a total of 10 balls

∴ total no. of ways of selecting 3 balls out of 10 = $10C_3$

For selecting 0 white balls, we will select all 3 balls from red

∴ P(X = 0) = P (not selecting any white ball) = $\frac{6C_3}{10C_3} = \frac{6\times5\times4}{10\times9\times8} = \frac{1}{6}$

For selecting 1 white ball, we will select all 2 balls from red and 1 from white

∴ $P(X = 1) = \frac{6C_2 \times 3C_1}{10C_3} = \frac{15}{30} = \frac{1}{2}$

For selecting 2 white balls, we will select all 1 ball from red and 2 from white

∴ $P(X = 1) = \frac{6C_1 \times 3C_2}{10C_3} = \frac{9}{30} = \frac{3}{10}$

For selecting 3 white balls, we will select all 0 balls from red and 2 from white

$\therefore$ P(X = 1) = $\frac{6C_0 \times 3C_3}{10C_3} = \frac{1}{30}$

Now we have p_i and x_i.

$\therefore$ Now we are ready to write the probability distribution for X:-

The following table gives probability distribution:

x_i	p_i
0	$\frac{1}{6}$
1	$\frac{1}{2}$
2	$\frac{3}{10}$
3	$\frac{1}{30}$

19. Question

Find the probability distribution of Y in two throws of two dice, where Y represents the number of times a total of 9 appears.

Answer

When two fair dice are thrown there are total 36 possible outcomes.

In our question, we are throwing two dice 2 times

$\because$ Y denotes the number of times, a sum of two numbers appearing on dice is equal to 9.

$\therefore$ Y can take values 0,1 or 2

Means In both the throw sum of 9 is not obtained, in one of throw of two dice 9 is obtained and in both the throws of two dice a sum of 9 is obtained.

$\because$ (3,6) (6,3) (5,4) and (4,5) are the combinations resulting in sum = 9

Let A denotes the event of getting a sum of 9 in a throw of 2 dice

$\therefore$ P(A) = probability of getting the sum of 9 in a throw of 2 dice is $\frac{4}{36} = \frac{1}{9}$

And P(A') = probability of not getting the sum of 9 in the throw of 2 dice = $\frac{32}{36} = \frac{8}{9}$

Note: P(A∩B) = P(A)P(B) where A and B are independent events.

P(Y = 0) = P(A')×P(A') = $\frac{8}{9} \times \frac{8}{9} = \frac{64}{81}$

P(Y = 1) = P(A)×P(A')+ P(A')×P(A)+ = $\frac{8}{9} \times \frac{1}{9} + \frac{1}{9} \times \frac{8}{9} = \frac{16}{81}$

P(Y = 2) = P(A)×P(A) = $\frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$

Now we have p_i and x_i.

∴ The required probability distribution is:-

X:	0	1	2
P(X):	$\frac{64}{81}$	$\frac{16}{81}$	$\frac{1}{81}$

20. Question

From a lot containing 25 items, 5 of which are defective, 4 are chosen at random. Let X be the number of defectives found. Obtain the probability distribution of X if the items are chosen without replacement.

Answer

X represents the number of defective items drawn.

∴ X can take values 0,1,2 or 3

∵ there are total 25 items (20 good+5 defectives) items

n(S) = total possible ways of selecting 5 items = $25C_4$

P(X = 0) = P(selecting no defective item) = $\frac{20C_4}{25C_4} = \frac{20\times19\times18\times17}{25\times24\times23\times22} = \frac{969}{2530}$

P(X = 1) = P(selecting 1 defective item and 3 good items)

$= \frac{5C_1\times20C_3}{25C_4} = \frac{114}{253}$

P(X = 2) = P(selecting 2 defective items and 2 good items)

$= \frac{5C_2\times20C_2}{25C_4} = \frac{38}{253}$

P(X = 3) = P(selecting 3 defective items and 1 good item)

$= \frac{5C_3\times20C_1}{25C_4} = \frac{4}{253}$

P(X = 4) = P(selecting 4 defective items and no good item)

$= \frac{5C_4}{25C_4} = \frac{1}{2530}$

Now we have p_i and x_i.

Following table represents probability distribution of X :

x_i	p_i
0	$\frac{969}{2530}$
1	$\frac{455}{969}$
2	$\frac{38}{253}$
3	$\frac{4}{253}$
4	$\frac{1}{2530}$

21. Question

Three cards are drawn successively with replacement from a well-shuffled deck of 52 cards. A random variable x denotes the number of hearts in the three cards drawn. Determine the probability distribution of X.

Answer

Note: While reading this question you might have observed that cards are being drawn successively and sometimes in other question you might have get cards are drawn simultaneously

You have to be careful regarding these two words while solving question. Both have a different meaning.

E.g.:-

When you draw 3 cards out of 52 simultaneously, then total no of ways of drawing is simply $52C_3$ = 22100. You draw 3 cards at a time.

When you draw 3 cards out of 52 successively, then total no of ways of drawing is not simply $52C_3$, once you have drawn the first card, only 51 cards are remaining and so on. You are drawing only one card at a time

In such case total ways of drawing 3 cards = $52C_1 \times 51C_1 \times 50C_1 = 132600$

Hence both are entirely different. So Be cautious regarding it.

Let's solve the problem now:

Note: In our problem, it is given that after every draw, we are replacing the card. So our sample space will not change

In a deck of 52 cards, there are 13 hearts.

Let X be the random variable denoting the number of hearts drawn for an event when 3 cards are drawn successively with replacement.

∴ X can take values 0 , 1, 2 or 3

$P(X = 0) = \frac{39}{52} \times \frac{39}{52} \times \frac{39}{52} = \frac{27}{64}$

{As we have to select 1 card at a time such that no heart is there so the first probability is 39/52 and as the drawn card is replaced, next time again probability is 39/52 and again the same thing }

Similarly, we proceed for other cases.

$$P(X = 1) = \frac{13}{52} \times \frac{39}{52} \times \frac{39}{52} + \frac{39}{52} \times \frac{13}{52} \times \frac{39}{52} + \frac{39}{52} \times \frac{39}{52} \times \frac{13}{52} = \frac{27}{64}$$

{we might get a heart in the first card or second card or third, so probabilities in all 3 cases are added as they are mutually exclusive events}

Similarly,

$$P(X = 2) = \frac{13}{52} \times \frac{13}{52} \times \frac{39}{52} + \frac{39}{52} \times \frac{13}{52} \times \frac{13}{52} + \frac{13}{52} \times \frac{39}{52} \times \frac{13}{52} = \frac{9}{64}$$

{First 2 cards are heart and 3^{rd} is non-heart, last 2 are hearts and so on cases. As these cases are mutually exclusive. Hence they are added }

Similarly,

$$P(X = 3) = \frac{13}{52} \times \frac{13}{52} \times \frac{13}{52} = \frac{1}{64}$$

Now we have p_i and x_i.

∴ Now we are ready to write the probability distribution for X:-

The following table gives probability distribution:

x_i	p_i
0	$\frac{27}{64}$
1	$\frac{27}{64}$
2	$\frac{9}{64}$
3	$\frac{1}{64}$

22. Question

An urn contains 4 red and 3 blue balls. Find the probability distribution of the number of blue balls in a random draw of 3 balls with replacement.

Answer

Let X denote the number of blue balls drawn in a random draw of 3 balls.

∴ X can take values 0,1,2 or 3.

Since bag contains 4 red and 3 blue balls i.e. at a total of 7 balls

∵ balls are drawn with replacement

For selecting 0 blue balls, we will select all 3 red balls

∴ P(X = 0) = P (not selecting any blue ball) = $\frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} = \frac{64}{343}$

For selecting 1 blue ball, we will select all 2 red balls and 1 blue

∴ P(X = 1) = $\frac{3}{7} \times \frac{4}{7} \times \frac{4}{7} + \frac{4}{7} \times \frac{3}{7} \times \frac{4}{7} + \frac{4}{7} \times \frac{4}{7} \times \frac{3}{7} = \frac{144}{343}$

For selecting 2 blue balls, we will select all 1 red ball and 2 blue

∴ P(X = 2) = $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{7} + \frac{4}{7} \times \frac{3}{7} \times \frac{3}{7} + \frac{3}{7} \times \frac{4}{7} \times \frac{3}{7} = \frac{108}{343}$

For selecting 3 blue balls, we will select all 3 blue and 0 red balls

∴ P(X = 3) = P (selecting all blue ball) = $\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} = \frac{27}{343}$

Now we have p_i and x_i.

∴ Now we are ready to write the probability distribution for X:-

The following table gives probability distribution:

x_i	p_i
0	$\frac{64}{343}$
1	$\frac{144}{343}$
2	$\frac{108}{343}$
3	$\frac{27}{343}$

23. Question

Two cards are drawn simultaneously from a well-shuffled deck of 52 cards. Find the probability distribution of the number of successes, when getting a spade is considered a success.

Answer

In a deck of 52 cards, there are 13 spades.

Let X be the random variable denoting the number of success and success here is getting a spade for an event when two cards are drawn simultaneously.

∴ We can say that the number of successes is equal to some spades obtained in each draw.

∴ X can take values 0, 1 or 2.

P(X = 0) = $\frac{39C_2}{52C_2} = \frac{39\times38}{52\times51} = \frac{19}{34}$

[For selecting 0 spades, we removed all 13 spades from the deck and selected out of 39]

P(X = 1) = $\frac{13C_1\times39C_1}{52C_2} = \frac{13\times39\times2}{52\times51} = \frac{13}{34}$

[For selecting 1 spade, we need to select and 1 out of 13 spade and not any other spade]

P(X = 2) = $\frac{13C_2}{52C_2} = \frac{13\times12}{52\times51} = \frac{2}{34}$

[For selecting 2 spades, we need to select and 2 out of 13 spades]

Now we have p_i and $x_{i.}$

∴ Now we are ready to write the probability distribution for X:-

The following table gives probability distribution:

X:	0	1	2
P(X):	$\frac{19}{34}$	$\frac{13}{34}$	$\frac{2}{34}$

24. Question

A fair die is tossed twice. If the number appearing on the top is less than 3, it is a success. Find the probability distribution of number of successes.

Answer

A fair dice is tossed twice.

Every time a throwing dice is an independent event.

Note: P(A∩B) = P(A)P(B) where A and B are independent events.

Let A denote the event of getting a number less than 3 on a single throw of dice.

∴ P(A) = $\frac{2}{6} = \frac{1}{3}$ {out of 6 outcomes 1 and 2 are favourable}

P(not A) = P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

As success is considered when number appearing on dice is less than 3.

Let X denotes the success.

As we are throwing two dice so that we can get success in both throws or either one of them, or we may even not get succeed.

∴ X can take values 0,1 or 2

P(X = 0) = P(not success)× P(not success) = P(A')× P(A') = $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

P(X = 1) = P(A) ×P(A')+ P(A')×P(A)

= $\frac{2}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} = \frac{4}{9}$

P(X = 2) = P(success)× P(success) = P(A)× P(A) = $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

Now we have p_i and $x_{i.}$

∴ Now we are ready to write the probability distribution for X:-

The following table gives probability distribution:

X:	0	1	2
P(X):	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{4}{9}$

25. Question

An urn contains 5 red and 2 black balls. Two balls are randomly selected. Let X represent the number of black balls. What are the possible values of X? Is X a random variable?

Answer

The key point to solve the problem:

A variable X is said to be a random variable if the sum of probabilities associated with each value of X gets equal to 1

i.e. $\Sigma(p_i) = 1$ where p_i is probability associated with x_i

X represents the number of black balls drawn.

∴ X can take values 0,1 or 2 as both balls drawn can be black which corresponds to X = 2, either of 2 balls can be black which corresponds to X = 1, and if neither of balls drawn is black it corresponds to X = 0

∵ there are a total of 7 balls

n(S) = total possible ways of selecting 2 balls = $7C_2$

P(X = 0) = P(selecting no black balls) = $\frac{5C_2}{7C_2} = \frac{5\times4}{7\times6} = \frac{10}{21}$

P(X = 1) = P(selecting 1 black ball and 1 red)

$= \frac{5C_1\times2C_1}{7C_2} = \frac{5\times2\times2}{7\times6} = \frac{10}{21}$

P(X = 0) = P(selecting all black balls) = $\frac{2C_2}{7C_2} = \frac{2}{7\times6} = \frac{1}{21}$

Now we have p_i and x_i.

x_i	p_i
0	$\frac{10}{21}$
1	$\frac{10}{21}$
2	$\frac{1}{21}$

Clearly, $\Sigma(p_i) = \frac{10}{21}+\frac{10}{21}+\frac{1}{21} = 1$

∴ X is a random variable, and the above table represents its probability distribution.

26. Question

Let X represent the difference between the number of heads and the number of tails when a coin is tossed 6 times. What are the possible values of X?

Answer

Since X represents the difference between some heads and number of tails when a coin is tossed 6 times.

As the coin is tossed 6 times following outcomes are possible:

Let H denotes heads and T denotes tails

Outcomes: 6H0T(6 heads 0 Tails), 5H1T, 4H2T, 3H3T, 2H4T and 1H5T

∴ differences can be 6,4,2,0,-2 and -4

∴ X can take values -4,-2,0,2,4 or 6 …

27. Question

From a lot of 10 bulbs, which includes 3 detectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of the number of defective bulbs.

Answer

X represents the number of defective bulbs drawn.

∴ X can take values 0,1 or 2 (as maximum bulbs drawn are 2)

∵ there are total 10 bulbs (7 good+3 defectives)

n(S) = total possible ways of selecting 2 items from sample = $10C_2$

P(X = 0) = P(selecting no defective bulb) = $\frac{7C_2}{10C_2} = \frac{7\times6}{10\times9} = \frac{7}{15}$

P(X = 1) = P(selecting 1 defective bulb and 1 good bulb)

$= \frac{7C_1\times3C_1}{10C_2} = \frac{7\times3\times2}{10\times9} = \frac{7}{15}$

P(X = 1) = P(selecting 0 defective bulb and 2 good bulb)

$= \frac{7C_0\times3C_2}{10C_2} = \frac{3\times2}{10\times9} = \frac{1}{15}$

Now we have p_i and x_i.

Following table represents the probability distribution of X :

x_i	p_i
0	$\frac{7}{15}$
1	$\frac{7}{15}$
2	$\frac{1}{15}$

28. Question

Four balls are to drawn without replacement from a box containing 8 red and 4 white balls. If X denotes the number of red balls drawn, find the probability distribution of X.

Answer

Let X denote the number of red balls drawn in a random draw of 4 balls.

∴ X can take values 0,1,2,3 or 4.

Since bag contains 8 red and 4 white balls i.e. at a total of 12 balls

∴ total no. of ways of selecting 4 balls out of 12 = $12C_4$

For selecting 0 red balls we will select all 3 balls from red

∴ P(X = 0) = P (not selecting any red ball) = $\frac{4C_4}{12C_4} = \frac{4\times3\times2\times1}{12\times11\times10\times9} = \frac{1}{495}$

For selecting 1 red ball, we will select 1 ball from 8 red and 3 balls from 4 white

∴ P(X = 1) = $\frac{4C_3\times8C_1}{12C_4} = \frac{4\times3\times2\times1}{12\times11\times10\times9}\times\frac{4\times8}{1} = \frac{32}{495}$

For selecting 2 red ball, we will select 2 balls from 8 red and 2 balls from 4 white

∴ P(X = 2) = $\frac{4C_2\times8C_2}{12C_4} = \frac{4\times3\times2\times1}{12\times11\times10\times9}\times\frac{4\times3\times8\times7}{2\times2} = \frac{168}{495}$

For selecting 3 red ball we will select 3 balls from 8 red and 1 ball from 4 white

∴ P(X = 3) = $\frac{4C_1\times8C_3}{12C_4} = \frac{4\times3\times2\times1}{12\times11\times10\times9}\times\frac{4\times8\times7\times6}{3\times2} = \frac{224}{495}$

∴ P(X = 4) = P (selecting all red ball) = $\frac{8C_4}{12C_4} = \frac{8\times7\times6\times5}{12\times11\times10\times9} = \frac{70}{495}$

Now we have p_i and x_i.

∴ Now we are ready to write the probability distribution for X :-

The following table gives probability distribution:

x_i	p_i
0	$\frac{1}{495}$
1	$\frac{32}{495}$
2	$\frac{168}{495}$
3	$\frac{224}{495}$
4	$\frac{70}{495}$

29. Question

The probability distribution of a random variable X is given below:

X:	0	1	2	3
P(X):	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

i. Determine the value of k

ii. Determine P $(X \leq 2)$ and $P(X>2)$

iii. Find P $(X \leq 2)$ + P $(X>2)$

Answer

The key point to solve the problem:

If a probability distribution is given then as per its definition, Sum of probabilities associated with each value of a random variable of given distribution is equal to 1

i.e. $\Sigma(p_i) = 1$

Given distribution is :

(i)

X:	0	1	2	3
P(X):	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

$\therefore k+\frac{k}{2}+\frac{k}{4}+\frac{k}{8} = 1$

$8k+4k+2k+k = 8$

$15k = 8$

$\therefore k = \frac{8}{15}$ (i)

(ii) $P(X>2) = P(X = 3) = \frac{k}{8}$

$\therefore$ P(X>2) = $\frac{1}{8} \times \frac{8}{15} = \frac{1}{15}$

P(X ≤ 2) = P(X = 0) + P(X = 1) + P(X = 2) = $k + \frac{k}{2} + \frac{k}{4} = \frac{7k}{4} = \frac{7}{4} \times \frac{8}{15} = \frac{14}{15}$ ...(ii)

(iii) $\therefore$ P(X>2) + P(X≤ 2) = $\frac{14}{15} + \frac{1}{15} = 1$

30. Question

Let X denote the number of colleges where you will apply after your results and P(X = x) denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x)=\begin{cases} kx & ,\text{If } x=0,1 \\ 2kx & ,\text{if } x=2 \\ k(5-x) & ,\text{if } x=3 \text{ or } 4 \\ 0 & ,\text{if } x>4 \end{cases}$$

Where k is a positive constant.

Find the value of k. Also, find the probability that you will get admission in (i) exactly one college (ii) at most 2 colleges (iii) at least 2 colleges.

Answer

Given,

$$P(X=x)=\begin{cases} kx & ,\text{If } x=0,1 \\ 2kx & ,\text{if } x=2 \\ k(5-x) & ,\text{if } x=3 \text{ or } 4 \\ 0 & ,\text{if } x>4 \end{cases}$$

Our variable is X and from equation we see that it is taking values X = 0,1,2,3,4 …… (any whole number)

And it represents the number of colleges in which you are going to get admission.

According to equation given we have :

$\therefore$ P(X = 0) = k×0 = 0

P(X = 1) = k×1 = k

P(X = 2) = 2k×2 = 4k

P(X = 3) = k(5-3) = 2k

P(X = 4) = k(5-4) = k

P(X>4) = 0

As in question, it is not given either X is random variable or the given distribution is a probability distribution, we can't apply any thing directly.

But we have a hint in the question,

As $P(X = 0) = 0$

It means the chance of not getting admission in any college 0

∴ admission is sure

Hence the sum of all probabilities must be equal to 1 as getting admission has become a sure event.

This makes the given distribution a probability distribution an X a random variable also.

∴ $k + 4k+2k+k = 1$

$8k = 1$

$k = 1/8$

i) P(getting admission in exactly one college) = $P(1) = k = 1/8$

ii) P(getting admission in atmost 2 colleges) = $P(X\leq 2)$

$= P(X = 1)+P(X = 2)$

$= k+4k = 5k = 5/8$

iii) P(getting admission in atleast 2 colleges) = $P(X\geq 2)$

$= P(X = 2)+P(X = 3)+P(X = 4)$

$= 4k+2k+k = 7k = 7/8$

Exercise 32.2

1 A. Question

Find the mean and standard deviation of each of the following probability distributions:

x_i :	2	3	4
p_i :	0.2	0.5	0.3

Answer

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Standard Deviation is given by SD = $\sqrt{\text{Variance}}$ where variance is given by:

Variance = $\Sigma x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

x_i	p_i	x_ip_i	$x_i^2p_i$
2	0.2	2 x 0.2 = 0.4	2^2 x 0.2 = 0.8
3	0.5	3 x 0.5 = 1.5	3^2 x 0.5 = 4.5
4	0.3	4 x 0.3 = 1.2	4^2 x 0.3 = 4.8

∴ mean = 0.4 + 1.5 + 1.2 = 3.1

And variance = 0.8 + 4.5 + 4.8 – $(3.1)^2$ = 0.49

∴ Standard deviation = √ 0.49 = 0.7

1 B. Question

Find the mean and standard deviation of each of the following probability distributions:

x_i :	1	3	4	5
p_i :	0.4	0.1	0.2	0.3

Answer

Mean of any probability distribution is given by Mean = Σx_ip_i

Standard Deviation is given by SD = √ Variance where variance is given by:

Variance = $\Sigma\ x_i^2p_i - (\Sigma x_ip_i)^2$

∴ first we need to find the products i.e. p_ix_i and $p_ix_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

x_i	p_i	x_ip_i	$x_i^2p_i$
1	0.4	1 x 0.4 = 0.4	1^2 x 0.4 = 0.4
3	0.1	3 x 0.1 = 0.3	3^2 x 0.1 = 0.9
4	0.2	4 x 0.2 = 0.8	4^2 x 0.2 = 3.2
5	0.3	5 x 0.3 = 1.5	5^2 x 0.3 = 7.5

∴ mean = 0.4 + 0.3+0.8+1.5= 3.0

And variance =0.4+0.9+3.2+7.5 – $(3.0)^2$ = 3

∴ Standard deviation = √ 3= 1.732

1 C. Question

Find the mean and standard deviation of each of the following probability distributions:

x_i :	-5	-4	1	2
p_i :	1/4	1/8	1/2	1/8

Answer

Mean of any probability distribution is given by Mean = Σx_ip_i

Standard Deviation is given by SD = √ Variance where variance is given by:

Variance = $\sum x_i^2p_i - (\sum x_ip_i)^2$

∴ first we need to find the products i.e. p_ix_i and $p_ix_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

x_i	p_i	x_ip_i	$x_i^2p_i$
-5	1/4 = 0.25	-5 x 0.25 = -1.25	$(-5)^2$ x 0.25 = 6.25
-4	1/8 = 0.125	-4 x 0.125 = -0.5	$(-4)^2$ x 0.125 = 2
1	½ = 0.5	1 x 0.5 = 0.5	1^2 x 0.5 = 0.5
2	1/8 = 0.125	2 x 0.125 = 0.25	2^2 x 0.125 = 0.5

∴ mean = -1.25-0.5+0.5+0.25 = -1

And variance = 6.25+2+0.5+0.5 – $(-1)^2$ = 8.25

∴ Standard deviation = √8.25= 2.9

1 D. Question

Find the mean and standard deviation of each of the following probability distributions:

x_i :	-1	0	1	2	3
p_i :	0.3	0.1	0.1	0.3	0.2

Answer

Mean of any probability distribution is given by Mean = $\sum x_ip_i$

Standard Deviation is given by SD = √ Variance where variance is given by:

Variance = $\sum x_i^2p_i - (\sum x_ip_i)^2$

∴ first we need to find the products i.e. p_ix_i and $p_ix_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

x_i	p_i	x_ip_i	$x_i^2p_i$
-1	0.3	-1 x 0.3 = -0.3	$(-1)^2$ x 0.3 = 0.3
0	0.1	-0 x 0.1 = 0	$(0)^2$ x 0.1 = 0
1	0.1	1 x 0.1 = 0.1	1^2 x 0.1 = 0.1
2	0.3	2 x 0.3 = 0.6	2^2 x 0.3 = 1.2
3	0.2	3 x 0.2 = 0.6	3^2 x 0.2 = 1.8

∴ mean = -0.3 + 0 + 0.1 + 0.6 + 0.6 = 1.0

And variance =0.3 + 0 + 0.1 + 1.2 + 1.8 – $(1)^2$ = 2.4

∴ Standard deviation = √2.4 = 1.5

1 E. Question

Find the mean and standard deviation of each of the following probability distributions:

x_i :	1	2	3	4
p_i :	0.4	0.3	0.2	0.1

Answer

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Standard Deviation is given by SD = $\sqrt{\text{Variance}}$ where variance is given by:

Variance = $\Sigma x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
1	0.4	1 x 0.4 = 0.4	$(1)^2$ x 0.4 = 0.4
2	0.3	2 x 0.3 = 0.6	2^2 x 0.3 = 1.2
3	0.2	3 x 0.2 = 0.6	3^2 x 0.2 = 1.8
4	0.1	4 x 0.1 = 0.4	4^2 x 0.1 = 1.6

∴ mean = 0.4+0.6+0.6+0.4 = 2.0

And variance = $0.4 + 1.2 + 1.8 + 1.6 - (2)^2 = 1.0$

∴ Standard deviation = $\sqrt{1} = 1$

1 F. Question

Find the mean and standard deviation of each of the following probability distributions:

x_i :	0	1	3	5
p_i :	0.2	0.5	0.2	0.1

Answer

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Standard Deviation is given by SD = $\sqrt{\text{Variance}}$ where variance is given by:

Variance = $\Sigma x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

x_i	p_i	x_ip_i	$x_i^2p_i$
0	0.2	0 x 0.2 = 0	$(0)^2$ x 0.2 = 0
1	0.5	1 x 0.5 = 0.5	1^2 x 0.5 = 0.5
3	0.2	3 x 0.2 = 0.6	3^2 x 0.2 = 1.8
5	0.1	5 x 0.1 = 0.5	5^2 x 0.1 = 2.5

∴ mean = 0+0.5+0.6+0.5 = 1.6

And variance = 0 +0.5 + 1.8 + 2.5– $(1.6)^2$ = 2.24

∴ Standard deviation = √2.24 = 1.497

1 G. Question

Find the mean and standard deviation of each of the following probability distributions:

x_i :	-2	-1	0	1	2
p_i :	0.1	0.2	0.4	0.2	0.1

Answer

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Standard Deviation is given by SD = √ Variance where variance is given by:

Variance = $\Sigma\, x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

x_i	p_i	x_ip_i	$x_i^2p_i$
-2	0.1	-2 x 0.1 = -0.2	$(-2)^2$ x 0.1 = 0.4
-1	0.2	-1 x 0.2 = -0.2	$(-1)^2$ x 0.2 = 0.2
0	0.4	0 x 0.4 =0	0 x 0.4 = 0
1	0.2	1 x 0.2 = 0.2	1^2 x 0.2 = 0.2
2	0.1	2 x 0.1 = 0.2	$(2)^2$ x 0.1 = 0.4

∴ mean = -0.2-0.2+0+0.2+0.2 = 0

And variance = 0 +0.4+0.2+0.2+0.4– $(0)^2$ = 1.2

∴ Standard deviation = √1.2 = 1.095

1 H. Question

Find the mean and standard deviation of each of the following probability distributions:

x_i :	-3	-1	0	1	3
p_i :	0.05	0.45	0.20	0.25	0.05

Answer

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Standard Deviation is given by SD = √ Variance where variance is given by:

Variance = $\Sigma x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
-3	0.05	-3 x 0.05 = -0.15	$(-3)^2$ x 0.05 = 0.45
-1	0.45	-1 x 0.45 = -0.45	$(-1)^2$ x 0.45 = 0.45
0	0.20	0 x 0.2 =0	0 x 0.4 = 0
1	0.25	1 x 0.25 = 0.25	1^2 x 0.25 = 0.25
3	0.05	3 x 0.05 = 0.15	$(3)^2$ x 0.05 = 0.45

∴ mean = -0.15-0.45+0+0.25+0.15 = -0.2

And variance = 0 +0.45+0.25+0.45+0.45− $(-0.2)^2$ = 1.56

∴ Standard deviation = √1.56 = 1.248

1 I. Question

Find the mean and standard deviation of each of the following probability distributions:

x_i :	0	1	2	3	4	5
p_i :	1/6	5/18	2/9	1/6	1/9	1/18

Answer

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Standard Deviation is given by SD = √ Variance where variance is given by:

Variance = $\Sigma x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

x_i	p_i	x_ip_i	$x_i^2p_i$
0	$\frac{1}{6}$	0	0
1	$\frac{5}{18}$	$1 \times \frac{5}{18} = \frac{5}{18}$	$1^2 \times \frac{5}{18} = \frac{5}{18}$
2	$\frac{2}{9}$	$2 \times \frac{2}{9} = \frac{4}{9}$	$2^2 \times \frac{2}{9} = \frac{8}{9}$
3	$\frac{1}{6}$	$3 \times \frac{1}{6} = \frac{1}{2}$	$3^2 \times \frac{1}{6} = \frac{3}{2}$
4	$\frac{1}{9}$	$4 \times \frac{1}{9} = \frac{4}{9}$	$4^2 \times \frac{1}{9} = \frac{16}{9}$
5	$\frac{1}{18}$	$5 \times \frac{1}{18} = \frac{5}{18}$	$5^2 \times \frac{1}{18} = \frac{25}{18}$

∴ Mean = $0 + \frac{5}{18} + \frac{4}{9} + \frac{1}{2} + \frac{4}{9} + \frac{5}{18} = \frac{35}{18}$

Variance = $0 + \frac{5}{18} + \frac{8}{9} + \frac{3}{2} + \frac{16}{9} + \frac{25}{18} - \left(\frac{35}{18}\right)^2 = \frac{35}{6} - \left(\frac{35}{18}\right)^2 = \frac{665}{324}$

∴ standard deviation = $\sqrt{\left(\frac{665}{324}\right)} = \frac{\sqrt{665}}{18}$

2. Question

A discrete random variable X has the probability distribution given below:

X:	0.5	1	1.5	2
P(X):	k	k^2	$2k^2$	k

(i) Find the value of k. (ii) Determine the mean of the distribution.

Answer

To find the value of k we will be using the very basic idea of probability.

Note: We know that the sum of the probabilities of all random variables taken from a given sample space is equal to 1.

∴ P(X=0.5) + P(X=1) + P(X=1.5) + P(X=2) = 1

∴ $k + k^2 + 2k^2 + k = 1$

⇒ $3k^2 + 2k - 1 = 0$

⇒ $3k^2 + 3k - k - 1 = 0$

⇒ $3k(k+1) - (k+1) = 0$

⇒ $(3k-1)(k+1) = 0$

∴ k = 1/3 or k = -1

∵ k represents probability of an event. Hence $0 \le P(X) \le 1$

$\therefore$ k = 1/3

Mean of any probability distribution is given by- Mean = $\Sigma x_i p_i$

Now we have,

X: 0.5 1 1.5 2

P(X): 1/3 1/9 2/9 1/3

$\therefore$ first we need to find the product i.e. $p_i x_i$ and add them to get mean.

$\therefore$ Mean = 0.5 x (1/3) + 1 x (1/9) + 1.5 x (2/9) +2 x (1/3)

$$= \frac{1}{6} + \frac{1}{9} + \frac{1}{3} + \frac{2}{3} = \frac{5}{18} + 1 = \frac{23}{18}$$

3. Question

Find the mean variance and standard deviation of the following probability distribution

X_i: a b

P_i: p q

Where p+q=1.

Answer

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Standard Deviation is given by SD = √ Variance where variance is given by:

Variance = $\Sigma x_i^2 p_i - (\Sigma x_i p_i)^2$

$\therefore$ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

$\therefore$ $p_1 x_1$ = ap and $p_2 x_2$ = bq Similarly $p_1 x_1^2 = a^2 p$ and $p_2 x_2^2 = b^2 q$

$\therefore$ Mean = ap + bq

Variance = $a^2p + b^2q - (ap + bq)^2$

$$= a^2p + b^2q - (a^2p^2 + b^2q^2 + 2abpq)$$

$$= a^2p(1 - p) + b^2q(1 - q) - 2abpq$$

$= a^2pq + b^2pq - 2abpq$ [∵ p+q = 1 ……given]

$$= pq(a^2 + b^2 - 2ab) = pq(a - b)^2$$

$\therefore$ SD = $\sqrt{\{pq(a-b)^2\}} = |a-b|\sqrt{pq}$

4. Question

Find the mean and variance of the number of tails in three tosses of a coin.

Answer

When we toss a coin three times we have the following possibilities:

{HHH,HHT,HTH,THH,HTT,THT,TTH,TTT}

Let X be a random variable representing number of tails in 3 tosses of a coin.

∵ probability of getting a head or probability of getting a tail are independent events and P(GETTING A HEAD) = P(GETTING A TAIL) = 1/2

∴ P(Head in first toss) and P(Head in second toss) and P(head in third toss) can be given by their individual products.

Note: $P(A \cap B) = P(A)P(B)$ where A and B are independent events.

Thus,

P(X=0) = P(HHH) = P(H)P(H)P(H) = 1/2 x 1/2 x 1/2 = 1/8

P(X=1) = P(HHT or HTH or THH) = P(HHT)+P(HTH)+P(THH)

= P(H)P(H)P(T)+ P(H)P(T)P(H)+ P(T)P(H)P(H)

= 1/2 x 1/2 x 1/2 + 1/2 x 1/2 x 1/2 + 1/2 x 1/2 x 1/2

= 3/8

P(X=2) = P(HTT or THT or TTH) = P(HTT)+P(THT)+P(TTH)

= P(H)P(T)P(T)+ P(T)P(H)P(T)+ P(T)P(T)P(H)

= 1/2 x 1/2 x 1/2 + 1/2 x 1/2 x 1/2 + 1/2 x 1/2 x 1/2

= 3/8

P(X=3) = P(TTT) = P(T)P(T)P(T) = 1/2 x 1/2 x 1/2 = 1/8

Now we have p_i and x_i.

Let's proceed to find mean and variance.

Mean of any probability distribution is given by Mean = $\sum x_i p_i$

Variance is given by:

Variance = $\sum x_i^2 p_i - (\sum x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

x_i	p_i	x_ip_i	$x_i^2p_i$
0	1/8	0 x (1/8) = 0	0
1	3/8	1 x (3/8) = 3/8	$(1)^2$ x (3/8) = 3/8
2	3/8	2 x (3/8) = 3/4	2^2 x (3/8) = 3/2
3	1/8	3 x (1/8) = 3/8	3^2 x (1/8) = 9/8

∴ Mean = $0+\frac{3}{8}+\frac{3}{4}+\frac{3}{8}=\frac{18}{8}=\frac{3}{2}$

Variance = $0+\frac{3}{8}+\frac{3}{2}+\frac{9}{8}-\left(\frac{3}{2}\right)^2=\left(3-\frac{9}{4}\right)=\frac{3}{4}$

5. Question

Two cards are drawn simultaneously from a pack of 52 cards. Compute the mean and standard deviation of the number of kings.

Answer

In a deck of 52 cards there are 4 kings each of one suit respectively.

Let X be the random variable denoting the number of kings for an event when two cards are drawn simultaneously.

∴ X can take values 0 , 1 or 2.

P(X=0) = $\frac{48C_2}{52C_2}=\frac{48\times47}{52\times51}=\frac{188}{221}$

[For selecting 0 kings, we removed all 4 kings from deck and selected out of 48]

P(X=1) = $\frac{4C_1\times48C_1}{52C_2}=\frac{48\times4\times2}{52\times51}=\frac{32}{221}$

[For selecting 1 king, we need to select and 1 out of 4 and not any other]

P(X=2) = $\frac{4C_2}{52C_2}=\frac{4\times3}{52\times51}=\frac{1}{221}$

[For selecting 2 king, we need to select and 2 out of 4]

Now we have p_i and x_i.

Let's proceed to find mean and standard deviation.

Mean of any probability distribution is given by Mean = $\sum x_ip_i$

Standard Deviation is given by SD = √ Variance where variance is given by:

Variance = $\sum x_i^2p_i - (\sum x_ip_i)^2$

∴ first we need to find the products i.e. p_ix_i and $p_ix_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

x_i	p_i	x_ip_i	$x_i^2p_i$
0	$\frac{188}{221}$	0	0
1	$\frac{32}{221}$	$1 \times \frac{32}{221} = \frac{32}{221}$	$1^2 \times \frac{32}{221} = \frac{32}{221}$
2	$\frac{1}{221}$	$2 \times \frac{1}{221} = \frac{2}{221}$	$2^2 \times \frac{1}{221} = \frac{4}{221}$

∴ mean = $0 + \frac{32}{221} + \frac{2}{221} = \frac{34}{221}$

Variance = $\frac{32}{221} + \frac{4}{221} - \left(\frac{34}{221}\right)^2 = \frac{400}{2873}$

∴ Standard deviation = √variance = √(400/2873)

$= \frac{20}{\sqrt{2873}}$

6. Question

Find the mean, variance and standard deviation of the number of tails in three tosses of a coin.

Answer

When we toss a coin three times we have the following possibilities:

{HHH,HHT,HTH,THH,HTT,THT,TTH,TTT}

Let X be a random variable representing number of tails in 3 tosses of a coin.

∵ probability of getting a head or probability of getting a tail are independent events and P(GETTING A HEAD) = P(GETTING A TAIL) = 1/2

∴ P(Head in first toss) and P(Head in second toss) and P(head in third toss) can be given by their individual products.

Note: P(A∩B) = P(A)P(B) where A and B are independent events.

Thus,

P(X=0) = P(HHH) = P(H)P(H)P(H) = 1/2 x 1/2 x 1/2 = 1/8

P(X=1) = P(HHT or HTH or THH) = P(HHT)+P(HTH)+P(THH)

= P(H)P(H)P(T)+ P(H)P(T)P(H)+ P(T)P(H)P(H)

= 1/2 x 1/2 x 1/2 + 1/2 x 1/2 x 1/2 + 1/2 x 1/2 x 1/2

= 3/8

P(X=2) = P(HTT or THT or TTH) = P(HTT)+P(THT)+P(TTH)

= P(H)P(T)P(T)+ P(T)P(H)P(T)+ P(T)P(T)P(H)

= 1/2 x 1/2 x 1/2 + 1/2 x 1/2 x 1/2 + 1/2 x 1/2 x 1/2

= 3/8

P(X=3) = P(TTT) = P(T)P(T)P(T) = 1/2 x 1/2 x 1/2 = 1/8

Now we have p_i and x_i.

Let's proceed to find mean and variance.

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Variance is given by:

Variance = $\Sigma\, x_i^2 p_i - (\Sigma x_i p_i)^2$

Standard Deviation is given by SD = √ Variance

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	1/8	0 x (1/8) = 0	0
1	3/8	1 x (3/8) = 3/8	$(1)^2$ x (3/8) = 3/8
2	3/8	2 x (3/8) = 3/4	2^2 x (3/8) = 3/2
3	1/8	3 x (1/8) = 3/8	3^2 x (1/8) = 9/8

∴ Mean = $0+\frac{3}{8}+\frac{3}{4}+\frac{3}{8}=\frac{18}{8}=\frac{3}{2}$

Variance = $0+\frac{3}{8}+\frac{3}{2}+\frac{9}{8}-\left(\frac{3}{2}\right)^2=(3-\frac{9}{4})=\frac{3}{4}$

Standard Deviation = √(3/4) = $\frac{\sqrt{3}}{2}$ = 0.87

7. Question

Two bad eggs are accidently mixed up with ten good ones. Three eggs are drawn at random with replacement from this lot. Compute the mean for the number of bad eggs drawn.

Answer

As there are total of two bad eggs. Therefore while drawing 3 eggs we can draw 1 bad egg or 2 or 0 bad eggs.

Let X be the random variable denoting number of bad eggs that can be drawn in each draw.

Clearly X can take values 0,1 or 2

P(X=0) = P(all 3 are good eggs) = $\frac{2C_0\times 10C_3}{12C_3}=\frac{10\times 9\times 8}{12\times 11\times 10}=\frac{120}{220}=\frac{6}{11}$

[Since there are 10 good eggs so for selecting all good we took all three from 10 and 0 eggs from 2 bad ones. Total sample points are no of ways of selecting 3 eggs from total of 12 eggs]

Similarly,

P(X=1) = P(1 bad and 2 good eggs) = $\frac{2C_1 \times 10C_2}{12C_3} = \frac{9}{22}$

P(X=2) = P(2 Bad eggs and 1 good egg) = $\frac{2C_2 \times 10C_1}{12C_3} = \frac{1}{22}$

Now we have p_i and x_i.

Let's proceed to find mean

Mean of any probability distribution is given by Mean = $\sum x_i p_i$

∴ first we need to find the products i.e. $p_i x_i$ and add them to get mean.

Following table gives the required products :

x_i	p_i	$x_i p_i$
0	6/11	0
1	9/22	1 x (9/22) = 9/22
2	1/22	2 x (1/22) = 1/11

∴ mean = $0 + \frac{9}{22} + \frac{1}{11} = \frac{11}{22} = \frac{1}{2}$

8. Question

A pair of fair dice is thrown. Let X be the random variable which denotes the minimum of the two numbers which appear. Find the probability distribution, mean and variance of X.

Answer

When a pair of fair dice is thrown there are total 36 possible outcomes.

X denotes the minimum of two numbers which appear

∴ X can take values 1,2,3,4,5 and 6

P(X=1) = 11/36

[Possible Pairs: (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)]

P(X=2) = 9/36

[Possible Pairs: (2,2),(3,2),(4,2),(5,2),(6,2),(2,6),(2,5),(2,4),(2,3)]

P(X=3) = 7/36

[Possible Pairs: (3,3),(3,4),(4,3),(5,3),(3,5),(3,6),(6,3)]

P(X=4) = 5/36

[Possible Pairs: (4,4),(5,4),(4,5),(4,6),(6,4)]

P(X=5) = 3/36

[Possible Pairs (5,5),(5,6),(6,5)]

P(X=6) = 1/36

[Possible Pairs: (6,6)]

Now we have p_i and x_i.

Let's proceed to find mean and variance.

Mean of any probability distribution is given by Mean = $\sum x_i p_i$

Variance is given by:

Variance = $\sum x_i^2 p_i - (\sum x_i p_i)^2$

Standard Deviation is given by SD = $\sqrt{\text{Variance}}$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

Required Probability distribution table:-

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
1	11/36	1 x (11/36) = 11/36	$(1)^2$ x (11/36) = 11/36
2	9/36	2 x (9/36) = 18/36	2^2 x (9/36) = 1
3	7/36	3 x (7/36) = 21/36	3^2 x (7/36) = 63/36
4	5/36	4 x (5/36) =20/36	4^2 x (5/36) = 80/36
5	3/36	5 x (3/36) = 15/36	5^2 x (3/36) = 75/36
6	1/36	6 x (1/36) = 6/36	6^2 x (1/36) = 1

∴ Mean = $\frac{11}{36}+\frac{18}{36}+\frac{21}{36}+\frac{20}{36}+\frac{15}{36}+\frac{6}{36}=\frac{91}{36}$

Variance = $\frac{11}{36}+1+\frac{63}{36}+\frac{80}{36}+\frac{75}{36}+1-\left(\frac{91}{36}\right)^2=\frac{301}{36}-\left(\frac{91}{36}\right)^2=\frac{2555}{1296}$

Standard deviation = √variance = $\frac{\sqrt{2555}}{\sqrt{1296}}=1.403$

9. Question

A fair coin is tossed four times. Let X denote the number of heads occurring. Find the probability distribution, mean and variance of X.

Answer

Say, H represents event of getting a head and T represents getting a tail.

When we toss a coin 4 times we have the following possibilities:

{HHHH,HHHT,HHTH,THHH,HTHH,THHT,TTHH,HHTT,THTH…………,TTTT}

A total of 2^4 = 16 possibilities.

Let X be a random variable representing number of heads occuring in 4 tosses of a coin.

∵ probability of getting a head or probability of getting a tail are independent events and P(GETTING A HEAD) = P(GETTING A TAIL) = 1/2

∴ P(Head in first toss) and P(Head in second toss) and P(head in third toss) and P(tail in 4^{th} toss) can be given by their individual products.

Note: P(A∩B) = P(A)P(B) where A and B are independent events.

Thus,

P(X=0) = P(TTTT) = P(T)P(T)P(T)P(T) = 1/2 x 1/2 x 1/2 x 1/2 = 1/16

Selecting a coin out of 4 which will show head rest all showing tail

∴ $4C_1 \times P(HHHT) = 4C_1 \times \{P(H)P(H)P(H)P(T)\} = 4C_1 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 4C_1 \left(\frac{1}{2}\right)^4$

P(X=1) = $4C_1 \times \left(\frac{1}{2}\right)^4 = 4 \times \frac{1}{16} = \frac{1}{4}$

Similarly,

P(X=2) = $4C_2 \times \left(\frac{1}{2}\right)^4 = \frac{6}{16} = \frac{3}{8}$

P(X=3) = $4C_3 \times \left(\frac{1}{2}\right)^4 = \frac{4}{16} = \frac{1}{4}$

P(X=4) = P(HHHH) = 1/16

Now we have p_i and x_i.

Let's proceed to find mean and variance.

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Variance is given by:

Variance = $\Sigma x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table representing probability distribution gives the required products :

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	1/16	0 x (1/16) = 0	0
1	1/4	1 x (1/4) = 1/4	$(1)^2$ x (1/4) = 1/4
2	3/8	2 x (3/8) = 3/4	2^2 x (3/8) = 3/2
3	1/4	3 x (1/4) = 3/4	3^2 x (1/4) = 9/4
4	1/16	4 x (1/16) = 1/4	4^2 x (1/16) = 1

∴ Mean = $0 + \frac{1}{4} + \frac{3}{4} + \frac{3}{4} + \frac{1}{4} = \frac{8}{4} = 2$

Variance $= \frac{1}{4} + \frac{3}{2} + \frac{9}{4} + 1 - 2^2 = 5 - 4 = 1$ .....ans

10. Question

A fair die is tossed. Let X denote twice the number appearing. Find probability distribution, mean and variance of X.

Answer

When a fair dice is thrown there are total 6 possible outcomes.

∵ X denote twice the number appearing on die

∴ X can take values 2,4,6,8,10 and 12

As appearance of a number on a fair die is equally likely

i.e. P(appearing of 1) = P(appearing of 2) = P(appearing of 3) = P(appearing of 4) = P(appearing of 5) = P(appearing of 6) = 1/6

∴ appearance of twice of the number is also equally likely with a probability of 1/6.

P(X=2)=P(X=4)=P(X=6)=P(X=8)=P(X=10)=P(X=12)=1/6

Now we have p_i and x_i.

Let's proceed to find mean and variance.

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Variance is given by:

Variance = $\Sigma\, x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

Required Probability distribution table:-

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
2	1/6	2 x (1/6) = 2/6	$(2)^2$ x (1/6) = 4/6
4	1/6	4 x (1/6) = 4/6	4^2 x (1/6) = 16/6
6	1/6	6 x (1/6) = 6/6	6^2 x (1/6) = 36/6
8	1/6	8 x (1/6) = 8/6	8^2 x (1/6) = 64/6
10	1/6	10 x (1/6) = 10/6	10^2 x (1/6) = 100/6
12	1/6	12 x (1/6) = 12/6	12^2 x (1/6) = 144/6

∴ Mean $= \frac{2}{6} + \frac{4}{6} + \frac{6}{6} + \frac{8}{6} + \frac{10}{6} + \frac{12}{6} = 7$

Variance $= \frac{4}{6} + \frac{16}{6} + \frac{36}{6} + \frac{64}{6} + \frac{100}{6} + \frac{144}{6} - (7)^2 = \frac{364}{6} - (7)^2 = \frac{70}{6}$

11. Question

A fair die is tossed. Let X denote 1 or 3 according as an odd or an even number appears. Find the probability distribution, mean and variance of X.

Answer

When a fair dice is thrown there are total 6 possible outcomes.

∵ X denote 1 or 3 according as an odd or an even number appears.

P(appearing of even number on a die) = 3/6 [favourable outcomes {2,4,6}]

P(appearing of an odd number on a die) = 3/6 [favourable outcomes {1,4,3}]

P(X=1) = 3/6 = 1/2

P(X=3) = 3/6 = 1/2

Now we have p_i and x_i.

Let's proceed to find mean and variance.

Mean of any probability distribution is given by Mean = $\sum x_i p_i$

Variance is given by:

Variance = $\sum x_i^2 p_i - (\sum x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table gives the required products :

Required Probability distribution table:-

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
1	1/2	1 x (1/2) = 1/2	$(1)^2$ x (1/2) =1/2
3	1/2	3 x (1/2) = 3/2	3^2 x (1/2) = 9/2

∴ Mean = $\frac{1}{2}+\frac{3}{2}= 2$

Variance = $\frac{1}{2}+\frac{9}{2}-2^2=5-4=1 \ \ldots\ldots \mathrm{ANS}$

12. Question

A fair coin is tossed four times. Let X denote the longest string of heads occurring. Find the probability distribution, mean and variance of X.

Answer

Say, H represents event of getting a head and T represents getting a tail.

When we toss a coin 4 times we have the following possibilities:

{HHHH,HHHT,HHTH,THHH,HTHH,THHT,TTHH,HHTT,THTH…………,TTTT}

A total of 2^4 = 16 possibilities.

∵ probability of getting a head or probability of getting a tail are independent events and P(GETTING A HEAD) = P(GETTING A TAIL) = 1/2

∴ P(Head in first toss) and P(Head in second toss) and P(head in third toss) and P(tail in 4^{th} toss) can be given by their individual products.

Note: $P(A \cap B) = P(A)P(B)$ where A and B are independent events.

As X is a random variable representing longest string of head occurring in 4 tosses.

∴ X can take following values:

X = 0 [all tails (TTTT)]

X = 1 [Longest string contains only 1 head e.g. (HTTT),(TTTH),(HTHT)..]

X = 2 [Longest string contain only 2 head e.g. (HHTT),(HHTH),(THHT)…]

X = 3 [Longest string contain only 3 head e.g. (HHHT) And (THHH)]

X = 4 [Longest string contain 4 heads i.e. (HHHH)]

Thus,

P(X=0) = 1/16

P(X=1) = 7/16 [by counting number of favourable outcomes as explained]

P(X=2) = 5/16

P(X=3) = 2/16

P(X=4) = 1/16

Now we have p_i and x_i.

Let's proceed to find mean and variance.

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Variance is given by:

Variance = $\Sigma\, x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table representing probability distribution gives the required products :

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	1/16	0 x (1/16) = 0	0
1	7/16	1 x (7/16) = 7/16	$(1)^2$ x (7/16) = 7/16
2	5/16	2 x (5/16) = 10/16	2^2 x (5/16) = 20/16
3	2/16	3 x (2/16) = 6/16	3^2 x (2/16) = 18/16
4	1/16	4 x (1/16) = 1/4	4^2 x (1/16) = 1

$\therefore$ Mean = $0 + \frac{7}{16} + \frac{10}{16} + \frac{6}{16} + \frac{1}{4} = \frac{27}{16} = 1.7$

Variance = $\frac{7}{16} + \frac{20}{16} + \frac{18}{16} + 1 - 1.7^2 = \frac{61}{16} - 1.7^2 = 0.935$ …..ans

13. Question

Two cards are selected at random from a box which contains five cards numbered 1,1,2,2, and 3. Let X denote the sum and Y the maximum of the two numbers drawn. Find the probability distribution, mean and variance of X and Y.

Answer

As box contains cards numbered as 1,1,2,2 and 3

$\therefore$ possible sums of card numbers are 2,3,4 and 5

Hence, X can take values 2,3,4 and 5

X=2 [when drawn cards are (1,1)]

X=3 [when drawn cards are (1,2) or (2,1)]

X=4 [when drawn cards are (2,2) or (3,1) or (1,3)]

X=5 [when drawn cards are (2,3) or (3,2)]

As Y is a random variable representing maximum of the two numbers drawn

$\therefore$ Y can take values 1,2 and 3.

Y=1 [when drawn cards are 1 and 1]

Y=2 [when drawn cards are (1,2) or (2,2) or (2,1)]

Y=3 [when drawn cards are (1,3) or (3,1) or (2,3) or (3,2)]

Note : P(1) represents probability of drawing card numbered as 1, similarly P(2) and P(3)

$\therefore$ P(X=2) = P(1)P(1) = $\frac{2}{5} \times \frac{1}{4} = 0.1$

[For drawing first card we had 2 favourable outcomes as 1,1 out of total 5 ,in second time of drawing ,as we drew a card numbered as 1 we are having 1 favourable outcome out of total remaining of 4]

Similarly,

P(X=3) = P(2)P(1) + P(1)P(2) = $\frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{4} = 0.4$

P(X=4) = P(2)P(2)+P(3)P(1)+P(1)P(3) = $\frac{2}{5} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4} = 0.3$

P(X=5) = P(2)P(3)+P(3)P(2) = $\frac{2}{5} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} = 0.2$

Similarly,

P(Y=1) = P(1)P(1) = $\frac{2}{5} \times \frac{1}{4} = 0.1$

$P(Y=2) = P(1)P(2)+P(2)P(1)+P(2)P(2) = \frac{2}{5}\times\frac{2}{4}+\frac{2}{5}\times\frac{2}{4}+\frac{2}{5}\times\frac{1}{4} = 0.5$

$P(Y=3) = P(2)P(3)+P(3)P(2)+ P(1)P(3)+P(3)P(1)$

$= \frac{2}{5}\times\frac{1}{4}+\frac{2}{5}\times\frac{1}{4}+\frac{2}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{2}{4} = 0.4$

Now we have p_i and x_i.

Let's proceed to find mean and variance.

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Variance is given by:

Variance = $\Sigma\, x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table representing probability distribution gives the required products :

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	1/16	0 x (1/16) = 0	0
1	7/16	1 x (7/16) = 7/16	$(1)^2$ x (7/16) = 7/16
2	5/16	2 x (5/16) = 10/16	2^2 x (5/16) = 20/16
3	2/16	3 x (2/16) = 6/16	3^2 x (2/16) = 18/16

∴ Mean for(X) = 0.2+1.2+1.2+1 = 3.6

Variance for(X) = $0.4+3.6+4.8+5.0-3.6^2 = 13.8-3.6^2 = 0.84$

Similarly probability distribution for Y is given below:

y_i	p_i	$y_i p_i$	$y_i^2 p_i$
1	0.1	1 x (0.1) = 0.1	$(1)^2$ x (0.1) = 0.1
2	0.5	2 x (0.5) = 1.0	2^2 x (0.5) = 2.0
3	0.4	3 x (0.4) = 1.2	3^2 x (0.4) = 3.6

∴ Mean for(Y) = 0.1+1.0+1.2 = 2.3

Variance for(Y) = $0.1+2.0+3.6-2.3^2 = 5.7-2.3^2 = 0.41$

14. Question

A die is tossed twice. A 'success' is getting an odd number on a toss. Find the variance of the number of successes.

Answer

As success is considered when we get an odd number when we roll a die.

As die is rolled twice , so we can get no success or a single success or we can get odd both the times an odd number.

If X is the random variable denoting the success then X can take value 0,1 or 2

∵ P(getting an odd number in a single rolling of die) = 3/6 = 1/2

As rolling a die is an independent event:

∴ P(getting an odd on first roll and probability of getting odd on second roll)=P(getting an odd on first roll) x P(getting an odd on second roll)

Note: P(A∩B) = P(A)P(B) where A and B are independent events.

∴ P(X=0) = P(even number on first throw) x P(even on second throw)

$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

P(X=1) = P(even number on first throw) x P(odd on second throw) +

P(odd number on first throw) x P(even on second throw)

$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$

P(X=2) = P(odd number on first throw) x P(odd on second throw)

$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Now we have p_i and x_i.

Let's proceed to find mean and variance.

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Variance is given by:

Variance = $\Sigma\, x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table representing probability distribution gives the required products :

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	1/4	0 x (1/4) = 0	0
1	1/2	1 x (1/2) = 1/2	$(1)^2$ x (1/2) = 1/2
2	1/4	2 x (1/4) = 1/2	2^2 x (1/4) = 1

∵ Variance = $\Sigma\, x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ Variance = $0 + \frac{1}{2} + 1 - \left(0 + \frac{1}{2} + \frac{1}{2}\right)^2 = \frac{3}{2} - 1 = \frac{1}{2} = 0.5$

15. Question

A box contains 14 bulbs, out of which 5 are defective. 3 bulbs are randomly drawn, one by one without replacement, from the box. Find the probability distribution of the number of defective bulbs.

Answer

Let X be the random variable denoting the number of defective bulbs drawn in each draw. Since we are drawing a maximum of 3 bulbs at a time, So we can get at max 3 defective bulbs as total defective bulbs are 5.

∴ X can take values 0,1,2 and 3

P(X=0) = P(drawing no defective bulbs)

As we are finding probability for 0 defective bulbs ,so we will select all 3 bulbs

from 9 good bulbs.

n(s) = total possible ways = $14C_3$

∴ P(X=0) = $\frac{9C_3}{14C_3} = \frac{9\times8\times7}{14\times13\times12} = \frac{3}{13}$

P(X=1) = P(drawing 1 defective bulbs and 2 good bulbs)

As we are finding probability for 1 defective bulbs ,so we will select 2 bulbs

from 9 good bulbs and 1 from 5 defective ones

∴ P(X=1) = $\frac{9C_2\times5C_1}{14C_3} = \frac{3\times9\times8\times5}{14\times13\times12} = \frac{45}{91}$

Similarly,

P(X=2) = $\frac{9C_1\times5C_2}{14C_3} = \frac{3\times9\times4\times5}{14\times13\times12} = \frac{45}{182}$

P(X=3) = $\frac{5C_3}{14C_3} = \frac{5\times4\times3}{14\times13\times12} = \frac{5}{182}$

So, Probability distribution is given below:

x_i	p_i
0	$\frac{3}{13}$
1	$\frac{45}{91}$
2	$\frac{45}{182}$
3	$\frac{5}{182}$

16. Question

In roulette, Fig. 32.2, the wheel has 13 numbers 0,1,2,….,12 marked on equally spaced slots. A player sets ₹10 on a given number. He receives ₹100 from the organizer of the game if the ball comes to rest in this slot; otherwise he gets nothing. If X denotes the player’s net gain/loss, find E (X).

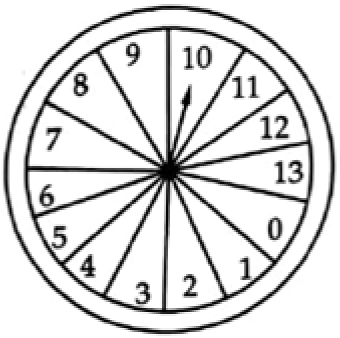

Fig. 32.2

Answer

As player sets Rs 10 on a number ,if he wins he get Rs 100

∴ his profit is Rs 90.

If he loses, he suffers a loss of Rs 10

He gets a profit when ball comes to rest in his selected slot.

Total possible outcome = 13

Favourable outcomes = 1

∴ probability of getting profit = 1/13

And probability of loss = 12/13

If X is the random variable denoting gain and loss of player

∴ X can take values 90 and -10

P(X=90) = 1/13

And P(X=-10) = 12/13

Now we have p_i and x_i.

Let's proceed to find mean

Mean of any probability distribution is given by Mean = $\sum x_i p_i$

∴ first we need to find the products i.e. $p_i x_i$ and add them to get mean

Following table representing probability distribution gives the required products :

x_i	p_i	$x_i p_i$
90	$\frac{1}{13}$	$\frac{90}{13}$
-10	$\frac{12}{13}$	$-\frac{120}{13}$

E(X) = Mean = $\frac{90}{13} + \left(-\frac{120}{13}\right) = -\frac{30}{13}$ …… ans

17. Question

Three cards are drawn at random (without replacement) from a well shuffled pack of 52 cards. Find the probability distribution of number of red cards. Hence find the mean of the distribution.

Answer

We have total 26 red cards in a deck of 52 cards.

As we are drawing maximum 3 cards at a time so we can get maximum 3 red cards.

If X denotes the number of red cards ,then X can take values from 0,1,2 and 3

P(X=0) = probability of drawing no red cards

We need to select all 3 cards from remaining 26 cards

Total possible ways of selecting 3 cards = $52C_3$

∴ P(X=0) = $\frac{26C_3}{52C_3} = \frac{2600}{22100} = \frac{2}{17}$

P(X=1) = P(selecting one red and 2 black cards) = $\frac{26C_1 \times 26C_2}{52C_3} = \frac{8450}{22100} = \frac{13}{34}$

P(X=2) = P(selecting 2 red and 1 black cards) = $\frac{26C_1 \times 26C_2}{52C_3} = \frac{8450}{22100} = \frac{13}{34}$

P(X=3) = $\frac{26C_3}{52C_3} = \frac{2600}{22100} = \frac{2}{17}$

So, Probability distribution is given below:

x_i	p_i	p_ix_i
0	$\frac{2}{17}$	0
1	$\frac{13}{34}$	$\frac{13}{34}$
2	$\frac{13}{34}$	$\frac{26}{34}$
3	$\frac{2}{17}$	$\frac{6}{17}$

Mean = $0 + \frac{13}{34} + \frac{26}{34} + \frac{6}{17} = \frac{51}{34} = \frac{3}{2} = 1.5 \ldots \ldots \text{ans}$

18. Question

An urn contains 5 red 2 black balls. Two balls are randomly drawn, without replacement. Let X represent the number of black balls drawn. What are the possible values of X? Is X a random variable? If yes, find the mean and variance of X.

Answer

X represents the number of black balls drawn.

∴ X can take values 0,1 and 2

∵ there are total 7 balls

n(S) = total possible ways of selecting 2 balls = $7C_2$

P(X=0) = P(selecting no black balls) = $\frac{5C_2}{7C_2} = \frac{5\times4}{7\times6} = \frac{20}{42} = \frac{10}{21}$

P(X=1) = P(selecting 1 black ball and 1 red ball)

$= \frac{5C_1\times2C_1}{7C_2} = \frac{5\times2\times2}{7\times6} = \frac{20}{42} = \frac{10}{21}$

P(X=2) = P(selecting 2 black ball and 0 red ball) = $\frac{5C_0\times2C_2}{7C_2} = \frac{2}{7\times6} = \frac{2}{42} = \frac{1}{21}$

X is said to be a random variable if some of the probabilities associated with each value of X is 1

Here,

P(X=0) + P(X=1) + P(X=2) = $\frac{20}{42} + \frac{20}{42} + \frac{2}{42} = \frac{42}{42} = 1$

∴ X is a random variable.

Now we have p_i and x_i.

Let's proceed to find mean and variance.

Mean of any probability distribution is given by Mean = $\Sigma x_i p_i$

Variance is given by:

Variance = $\Sigma\, x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table representing probability distribution gives the required products :

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	$\frac{10}{21}$	0	0
1	$\frac{10}{21}$	$\frac{10}{21}$	$\frac{10}{21}$
2	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{4}{21}$

∴ Mean = $\frac{10}{21} + \frac{2}{21} = \frac{12}{21} = \frac{4}{7}$

∵ Variance = $\Sigma\, x_i^2 p_i - (\Sigma x_i p_i)^2$

∴ Variance = $0 + \frac{4}{21} + \frac{10}{21} - \left(\frac{4}{7}\right)^2 = \frac{14}{21} - \frac{16}{49} = \frac{50}{147}$

19. Question

Two numbers are selected at random (without replacement) from positive integers 2,3,4,5,6 and 7. Let X denote the larger of the two numbers obtained. Find the mean and variance of the probability distribution of X.

Answer

∵ two numbers are selected at random like {(2,3) or (5,4) or (4,5)..etc}

Total ways of selecting two numbers without replacement = 6 x 5 =30

As X denote the larger of two numbers selected

∴ X can take values 3,4,5,6 and 7

P(X=3) = P(larger number is 3) = $\frac{2}{30}$ [{2,3},{3,2}]

P(X=4) = P(larger number is 4) = $\frac{4}{30}$ [{2,4},{4,2},{3,4},{4,3}]

P(X=5) = P(larger number is 5) = $\frac{6}{30}$ [{2,5},{3,5},{4,5} and their reverse order]

P(X=6) = P(larger number is 6) = $\frac{8}{30}$ [{2,6},{3,6},{4,6},{5,6} and their reverse order]

P(X=7) = P(larger number is 7) = $\frac{10}{30}$ [{2,7},{3,7},{4,7},{5,7},{6,7} and their reverse order]

Now we have p_i and x_i.

Let's proceed to find mean and variance.

Mean of any probability distribution is given by Mean = $\sum x_i p_i$

Variance is given by:

Variance = $\sum x_i^2 p_i - (\sum x_i p_i)^2$

∴ first we need to find the products i.e. $p_i x_i$ and $p_i x_i^2$ and add them to get mean and apply the above formula to get the variance.

Following table representing probability distribution gives the required products :

x_i	p_i	x_ip_i	$x_i^2p_i$
3	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{18}{30}$
4	$\frac{4}{30}$	$\frac{16}{30}$	$\frac{64}{30}$
5	$\frac{6}{30}$	$\frac{30}{30}$	$\frac{150}{30}$
6	$\frac{8}{30}$	$\frac{48}{30}$	$\frac{288}{30}$
7	$\frac{10}{30}$	$\frac{70}{30}$	$\frac{490}{30}$

∴ Mean = $\frac{6}{30}+\frac{16}{30}+\frac{30}{30}+\frac{48}{30}+\frac{70}{30}=\frac{17}{3}$

∵ Variance = $\sum x_i^2p_i - (\sum x_ip_i)^2$

∴ Variance = $\frac{18}{30}+\frac{64}{30}+\frac{150}{30}+\frac{288}{30}+\frac{490}{30}-\left(\frac{17}{3}\right)^2=\frac{101}{3}-\frac{289}{9}=\frac{14}{9}$

20. Question

In a game, a man wins ₹5 for getting a number greater than 4 and loses ₹1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quits as and when he gets a number than 4. Find the expected value of amount he wins/lose.

Answer

We are asked to find the expected amount he wins or lose i.e we have to find the mean of probability distribution of random variable X denoting the win/loss.

As he decided to throw the dice thrice but to quit at the instant he loses

∴ if he wins in all throw he can make earning of Rs 15

If he wins in first two throw and lose in last, he earns Rs (10-1) = Rs 9

If he wins in first throw and then loses ,he earns = Rs 4

If he loses in first throw itself, he earns Rs = -1

Thus X can take values -1,4,9 and 15

P(getting a number greater than 4 in a throw of die) = 2/6 = 1/3

P(getting a number not greater than 4 in a throw of die) = 4/6 = 2/3

P(X=-1) = P(getting number less than or equal to 4) = 2/3

P(X=4) = P(getting > 4) x P(getting ≤ 4) = $\frac{1}{3}\times\frac{2}{3}=\frac{2}{9}$

P(X=9) = P(getting > 4) x P(getting > 4) x P(getting ≤ 4) = $\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3}=\frac{2}{27}$

P(X=15) = P(getting > 4) x P(getting > 4) x P(getting > 4) = $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$

So, Probability distribution is given below:

x_i	p_i	p_ix_i
-1	$\frac{2}{3}$	$-\frac{2}{3}$
4	$\frac{2}{9}$	$\frac{8}{9}$
9	$\frac{2}{27}$	$\frac{18}{27}$
15	$\frac{1}{27}$	$\frac{15}{27}$

∵ Mean = $\sum x_i p_i$

Mean = $-\frac{2}{3} + \frac{8}{9} + \frac{18}{27} + \frac{15}{27} = \frac{39}{27} = 1.44$

He can win around Rs 1.45

Very Short Answer

1. Question

Write the values of 'a' for which the following distribution of probabilities becomes a probability distribution:

$X = x_i$:	-2	-1	0	1
$P(X = x_i)$	$\frac{1-a}{4}$	$\frac{1+2a}{4}$	$\frac{1-2a}{4}$	$\frac{1+a}{4}$

Answer

The probability distribution of X is:

$X = x_i$:	-2	-1	0	1
$P(X = x_i)$	$\frac{1-a}{4}$	$\frac{1+2a}{4}$	$\frac{1-2a}{4}$	$\frac{1+a}{4}$

We Know that

$$\sum_{i=1}^{n} pi = 1$$

Therefore

$$\frac{1-a}{4} + \frac{1+2a}{4} + \frac{1-2a}{4} + \frac{1+a}{4} = 1$$

$$\Rightarrow \frac{1-a+1+2a+1-2a+1+a}{4} = 1$$

$$\Rightarrow 1-a+1+2a+1-2a+1+a = 4$$

4=4

a = 0

2. Question

For what value of k the following distribution is a probability distribution?

$X=x_i$:	0	1	2	3
$P(X=x_i)$	$2k^4$	$3k^2-5k^3$	$2k-3k^2$	$3k-1$

Answer

The probability distribution of X is:

$X=x_i$:	0	1	2	3
$P(X=x_i)$	$2k^4$	$3k^2-5k^3$	$2k-3k^2$	$3k-1$

We Know that

$$\sum_{i=1}^{n} pi = 1$$

Therefore,

$2k^4+3k^2-5k^3+2k-3k^2+3k-1=1$

$2k^4-5k^3+5k-2=0$

$2(k^4-1)-5k^2-1)=0$

$2(k^2-1)(k^2+1)-5k(k-1)(k+1)=0$

$2(k-1)(k+1)(k^2+1)-5k(k-1)(k+1)=0$

$(k-1)(k+1)(2(k^2+1)-5k=0$

$(k-1)(k+1)(2k^2-5k+2)=0$

$(k-1)(k+1)(2k^2-k-4k+2)=0$

$(k-1)(k+1)(k(2k-1)-2(2k-1))=0$

$(k-1)(K+1)(2k-1)(k-2)$

$k-1=0$

$\boxed{k=1}$

$K+1=0$

$$\boxed{k=-1}$$

2k-1=0

$$\boxed{k = \frac{1}{2}}$$

k-2=0

$$\boxed{k=2}$$

3. Question

If X denotes the number on the upper face of a cubical die when it is thrown, find the mean of X

Answer

.

The sample space of the experiment consist of 6 elementary events where, x_i=1, 2, 3,4,5,6

The random variable X i.e. number on the upper face of a cubical die when it is thrown is-1,2,3,4,5,6

P(X=1) =P (1) =$\frac{1}{6}$

P(X=2) =P (2) =$\frac{1}{6}$

P(X=3) =P (3) =$\frac{1}{6}$

P(X=4) =P (4) =$\frac{1}{6}$

P(X=5) =P (5) =$\frac{1}{6}$

P(X=6) =P (6) =$\frac{1}{6}$

The probability distribution of X is

X_i	1	2	3	4	5	6
P_i	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$Mean\ of\ X = E(X) = \sum_{i=1}^{n} x_i p x_i$$

$$\Rightarrow \left(1\times\frac{1}{6}\right)+\left(2\times\frac{1}{6}\right)+\left(3\times\frac{1}{6}\right)+\left(4\times\frac{1}{6}\right)+\left(5\times\frac{1}{6}\right)+\left(6\times\frac{1}{6}\right)$$

$$\Rightarrow \left(\frac{1}{6}\right)+\left(\frac{2}{6}\right)+\left(\frac{3}{6}\right)+\left(\frac{4}{6}\right)+\left(\frac{5}{6}\right)+\left(\frac{6}{6}\right)$$

$$\Rightarrow \frac{21}{6}$$

E(X) = 3.5

4. Question

If the probability distribution of a random variable X is given by

$X=x_i$:	1	2	3	4
$P(X=x_i)$:	2k	4k	3k	K

Write the value of k.

Answer

The probability distribution of X is:

$X=x_i$:	1	2	3	4
$P(X=x_i)$:	2k	4k	3k	k

We Know that

$$\sum_{i=1}^{n} pi = 1$$

Therefore

⇒ 2k+4k+3k+k=1

⇒ 10k=1

$$k = \frac{1}{10}$$

5. Question

Find the mean of the following probability distribution:

$X=x_i$:	1	2	3
$P(X=x_i)$:	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{5}{8}$

Answer

The probability distribution of X is:

$X=x_i$:	1	2	3
$P(X=x_i)$:	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{5}{8}$

We Know that

$$\text{Mean of } X = E(X) = \sum_{i=1}^{n} x_i\, px_i$$

Therefore

$$\Rightarrow \left(1 \times \frac{1}{4}\right) + \left(2 \times \frac{1}{8}\right) + \left(3 \times \frac{5}{8}\right)$$

$$\Rightarrow \left(\frac{1}{4}\right) + \left(\frac{2}{8}\right) + \left(\frac{15}{8}\right)$$

$$= \frac{19}{8}$$

6. Question

If the probability distribution of a random variable X is as given below:

$X=x_i$:	1	2	3	4
$P(X=x_i)$:	c	2c	3c	4c

Write the value of P (X ≤ 2).

Answer

The probability distribution of X is:

$X=x_i$:	1	2	3	4
$P(X=x_i)$:	c	2c	3c	4c

We Know that

$$\sum_{i=1}^{n} pi = 1$$

Therefore

⇒ c+2c+3c+4c=1

$$\Rightarrow\ 10c = 1$$

$$\Rightarrow\ c = \frac{1}{10}$$

P(X≤2)=P(X=1)+P(X=2)

⇒ c+2c

⇒ 3c

$$\Rightarrow\ 3 \times \frac{1}{10}$$

$$= \frac{3}{10}$$

7. Question

A random variable has the following probability distribution:

$X=x_i$:	1	2	3	4
$P(X=x_i)$:	K	2k	3k	4k

Write the value of P (X ≥3).

Answer

The probability distribution of X is:

$X=x_i$:	1	2	3	4
$P(X=x_i)$:	K	2k	3k	4k

We Know that

$$\sum_{i=1}^{n} pi = 1$$

Therefore

⇒ k+2k+3k+4k=1

⇒ 10k=1

$$k = \frac{1}{10}$$

P(X≥3)=P(X=3)+P(X=4)

⇒ 3k+4k

$$\Rightarrow \ 7k$$

$$\Rightarrow \ 7 \times \frac{1}{10}$$

$$= \frac{7}{10}$$

MCQ

1. Question

If a random variable X has the following probability distribution:

X:	0	1	2	3	4	5	6	7	8
P(X):	a	3a	5a	7a	9a	11a	13a	15a	17a

Then the value of a is

A. $\frac{7}{81}$

B. $\frac{5}{81}$

C. $\frac{2}{81}$

D. $\frac{1}{81}$

Answer

The probability distribution of X is:

X:	0	1	2	3	4	5	6	7	8
P(X):	a	3a	5a	7a	9a	11a	13a	15a	17a

We Know that

$$\sum_{i=1}^{n} pi = 1$$

Therefore,

⇒ a+3a+5a+7a+9a+11a+13a+15a+17a=1

⇒ 81a=1

$a = \frac{1}{81}$

Option (A) is incorrect because by putting $a = \frac{7}{81}$, the sum total of pxi is not equal to 1

Option (B) is incorrect because by putting $a = \frac{5}{81}$, the sum total of pxi is not equal to 1

Option () is incorrect because by putting $a = \frac{2}{81}$, the sum total of pxi is not equal to 1

2. Question

A random variable X has the following probability distribution:

X:	1	2	3	4	5	6	7	8
P(X):	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X: X is a prime number}, F = {X: X < 4}, the probability P (E ∪ F) is

A. 0.50

B. 0.77

C. 0.35

D. 0.87

Answer

E=(X:X is a prime number)=(2,3,5,7)

P(E)=P(X=2)+(X=3)+(X=5)+(X=7)

P(E)=0.23+0.12+0.20+0.07=0.62

F=(X:X<4)=(1,2,3)

P(F)=P(X=1)+(X=2)+(X=3)

P(F)=0.15+0.23+0.12=0.5

E∩F=(X is a prime number as well as <4)=(2,3)

P(E∩F)=P(X=2)+P(X=3)

⇒ 0.23+0.12=0.35

P(E∪F)=P(E)+P(F)-P(E∩F)

⇒ 0.62+0.50-0.35

$= 0.77$

3. Question

A random variable X takes the values 0, 1, 2, 3 and its mean is 1.3. If P (X =3) = 2P (X = 1) and P(X = 2) = 0.3, then P(X = 0) is

A. 0.1

B. 0.2

C. 0.3

D. 0.4

Answer

Let the value P(X=0) be x and the value of P(X=1) be A

GIVEN- P(X=3) =2P(X=1)

=2(A) =2A

P(X=2) =0.3

So, The Probability distribution of X is

X_i	0	1	2	3
P_i	x	A	0.3	2A

Mean of E(X) =1.3

$$E(X) = \sum_{i=1}^{n} x_i \, px_i$$

⇒ (0×x)+(1×A)+(2×0.3)+(3×2A)=1.3

⇒ A+0.6+6A=1.3

⇒ 7A+0.6=1.3

⇒ 7A=0.7

$$\Rightarrow \; A = \frac{0.7}{7}$$

⇒ A=0.1

By putting the value of A in probability distribution, we get

X_i	0	1	2	3
P_i	x	0.1	0.3	0.2

We Know that

$$\sum_{i=1}^{n} pi = 1$$

Therefore,

⇒ x+0.1+0.3+0.2=1

x=1-0.6

x=0.4

4. Question

A random variable has the following probability distribution:

$X=x_i$	0	1	2	3	4	5	6	7
$P(X=x_i)$	0	2p	2p	3p	P^2	$2p^2$	$7p^2$	2p

The value of p is

A. 1/10

B. −1

C. – 1/10

D. 1/5

Answer

The probability distribution of X is:

$X=x_i$	0	1	2	3	4	5	6	7
$P(X=x_i)$	0	2p	2p	3p	p^2	$2p^2$	$7p^2$	2p

We Know that

$$\sum_{i=1}^{n} \mathrm{pi} = 1$$

Therefore,

⇒ $2p+2p+3p+p^2+2p^2+7p^2+2p=1$

⇒ $10p^2+9p-1=0$

⇒ $10p^2+10p-p-1=0$

⇒ $10p(p+1)-1(p+1)=0$

⇒ $(10p-1)(p+1)$

$10p-1=0$ or $p+1=0$

$$p = \frac{1}{10} \text{ or } p = -1$$

We considered $p = \frac{1}{10}$ because p is a probability and probability cannot be in negative

Option (B) is incorrect because p is a probability and probability cannot be in negative

Option (C) is incorrect because p is a probability and probability cannot be in negative

Option (D) is incorrect because by putting $p = \frac{1}{5}$, the sum total of pxi is not equal to 1

5. Question

If X is a random-variable with probability distribution as given below:

$X=x_i$:	0	1	2	3
$P(X=x_i)$:	K	3k	3k	K

The value of k and its variance are

A. $\frac{1}{8}, \frac{22}{27}$

B. $\frac{1}{8}, \frac{23}{27}$

C. $\frac{1}{8}, \frac{24}{27}$

D. $\frac{1}{8}, \frac{3}{4}$

Answer

The probability distribution of X is:

$X=x_i$:	1	2	3	4
$P(X=x_i)$:	K	3k	3k	K

We Know that

$$\sum_{i=1}^{n} pi = 1$$

Therefore

⇒ k+3k+3k+k=1

$\Rightarrow$ 8k=1

$$k=\frac{1}{8}$$

$X=x_i$:	1	2	3	4
$P(X=x_i)$:	K	3k	3k	K

$$E(X)=\sum_{i=1}^{n} x_i\, px_i$$

$\Rightarrow$ (1×k)+(2×3k)+(3×3k)+(4×k)

$\Rightarrow$ k+6k+9k+4k=20k

$$\Rightarrow\ 20\times\frac{1}{8}=\frac{5}{2}$$

$$E(X^2)=\sum_{i=1}^{n} x_i^2\, px_i$$

$\Rightarrow$ (1^2 ×k)+(2^2 ×3k)+(3^2 ×3k)+(4^2 ×k)

$\Rightarrow$ k+12k+27k+16k=56k

$$\Rightarrow\ 56\times\frac{1}{8}=7$$

Var(X)=E(X^2) - $(E(X))^2$

$$\Rightarrow\ 7-\left(\frac{5}{2}\right)^2$$

$$\Rightarrow\ 7-\frac{25}{4}$$

$$=\frac{28-25}{4}$$

$$=\frac{3}{4}$$

6. Question

The probability distribution of a discrete random variable X is given below:

X:	2	3	4	5
P(X):	5/k	7/k	9/k	11/k

The value of E(X) is

A. 8

B. 16

C. 32

D. 48

Answer

The probability distribution of X is:

$X=x_i$:	2	3	4	5
$P(X=x_i)$:	5/K	7/k	9/k	11/K

We Know that

$$\sum_{i=1}^{n} pi = 1$$

Therefore

$$\frac{5}{k}+\frac{7}{k}+\frac{9}{k}+\frac{11}{k}=1$$

$$\Rightarrow \frac{32}{k}=1$$

⇒ k=32

$$\text{Mean of } X = E(X) = \sum_{i=1}^{n} x_i \, px_i$$

$$\Rightarrow \left(2 \times \frac{5}{k}\right) + \left(3 \times \frac{7}{k}\right) + \left(4 \times \frac{9}{k}\right) + \left(5 \times \frac{11}{k}\right)$$

$$\Rightarrow \left(\frac{10}{k}\right) + \left(\frac{21}{k}\right) + \left(\frac{36}{k}\right) + \left(\frac{55}{k}\right)$$

$$\Rightarrow \frac{122}{k}$$

$$= \frac{122}{32}$$

=3.8125

7. Question

For the following probability distribution:

X:	-4	-3	-2	-1	0
P(X):	0.1	0.2	0.3	0.2	0.2

The value of E(X) is

A. 0

B. −1

C. −2

D. −1.8

Answer

The probability distribution of X is:

X:	-4	-3	-2	-1	0
P(X):	0.1	0.2	0.3	0.2	0.2

We Know that

$$\text{Mean of X} = E(X) = \sum_{i=1}^{n} x_i \, px_i$$

Therefore,

(-4×0.1)+(-3×0.2)+(-2×0.3)+(-1×0.2)+(0×0.2)

⇒ -0.4-0.6-0.6-0.2

$= -1.8$

8. Question

For the following probability distribution:

X:	1	2	3	4
P(X):	1/10	1/5	3/10	2/5

The value of $E(X^2)$ is

A. 3

B. 5

C. 7

D. 10

Answer

The probability distribution of X is:

X:	1	2	3	4
P(X):	1/10	1/5	3/10	2/5

$$E(X) = \sum_{i=1}^{n} x_i \, px_i$$

$$\Rightarrow \left(1 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{5}\right) + \left(3 \times \frac{3}{10}\right) + \left(4 \times \frac{2}{5}\right)$$

$$\Rightarrow \frac{1}{10} + \frac{2}{5} + \frac{9}{10} + \frac{8}{5}$$

$$\Rightarrow \frac{30}{10} = 3$$

$$E(X^2) = \sum_{i=1}^{n} x_i^2 \, px_i$$

$$\Rightarrow \left(1^2 \times \frac{1}{10}\right) + \left(2^2 \times \frac{1}{5}\right) + \left(3^2 \times \frac{3}{10}\right) + \left(4^2 \times \frac{2}{5}\right)$$

$$\Rightarrow \frac{1}{10} + \frac{4}{5} + \frac{27}{10} + \frac{32}{5}$$

$$= \frac{100}{10} = 10$$

9. Question

Let X be a discrete random variable. Then the variance of X is

A. $E(X^2)$

B. $E(X^2) + (E(X))^2$

C. $E(X^2) - (E(X))^2$

D. $\sqrt{E(X^2) - (E(X))^2}$

Answer

Let X be a random variable whose possible values $x_1, x_2, \ldots\ldots\ldots x_n$

Possibilities $p(x_1)$, $p(x_2)$,…………$p(x_n)$respectively.

Let μ = E (X) be the mean of X. The variance of X, denoted by Var (X) σ_x^2 or defined as

$$\mathrm{Var(X)} = \sum_{i=1}^{n} (x-\mu)^2 p(x_i)$$

$$\Rightarrow \sum_{i=1}^{n} (x_i^2 + \mu^2 - 2\mu x_i) p(x_i)$$

$$\Rightarrow \sum_{i=1}^{n} x_i^2 p(x_i) + \sum_{i=1}^{n} \mu^2 p(x_i) - \sum_{i=1}^{n} 2\mu x_i p(x_i)$$

$$\Rightarrow \sum_{i=1}^{n} x_i^2 p(x_i) + \mu^2 \sum_{i=1}^{n} p(x_i) - 2\mu \sum_{i=1}^{n} x_i p(x_i)$$

$$\Rightarrow \sum_{i=1}^{n} x_i^2 p(x_i) + \mu^2 - 2\mu^2$$

$$\Rightarrow \sum_{i=1}^{n} x_i^2 p(x_i) - \mu^2$$

$$\Rightarrow \sum_{i=1}^{n} x_i^2 p(x_i) - \left(\sum_{i=1}^{n} x_i \, p x_i \right)^2$$

$$= E(X^2) - (E(X))^2$$