4. Inverse Trigonometric Functions

Exercise 4.1

1 A. Question

Find the principal value of each of the following:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

Answer

Let $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = y$

Then sin y = $\left(-\frac{\sqrt{3}}{2}\right) = -\sin\left(\frac{\pi}{3}\right) = \sin(-\frac{\pi}{3})$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin(-\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$

Therefore the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is $-\frac{\pi}{3}$

1 B. Question

Find the principal value of each of the following:

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

Answer

Let $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = y$

cos y = $-\frac{\sqrt{3}}{2}$

We need to find the value of y.

We know that the value of cos is negative for the second quadrant and hence the value lies in [0, π].

cos y = – cos $(\frac{\pi}{6})$

cos y = $\pi - \frac{\pi}{6}$

y = $\frac{5\pi}{6}$

1 C. Question

Find the principal value of each of the following:

$$\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$$

Answer

$\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)$

$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$

$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} - \frac{1}{\sqrt{2}} \times \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}\right)$

$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{\pi}{12}$

1 D. Question

Find the principal value of each of the following:

$\sin^{-1}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$

Answer

$\sin^{-1}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$

$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$

$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} + \frac{1}{\sqrt{2}} \times \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}\right)$

$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\pi}{3} + \frac{\pi}{4}$

$= \frac{7\pi}{12}$

1 E. Question

Find the principal value of each of the following:

$\sin^{-1}\left(\cos\frac{3\pi}{4}\right)$

Answer

Let $\sin^{-1}\left(\cos\frac{3\pi}{4}\right) = y$

Then $\sin y = \cos\frac{3\pi}{4} = -\sin\left(\pi - \frac{3\pi}{4}\right) = -\sin(\frac{\pi}{4})$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$-\sin\left(\frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

Therefore the principal value of $\sin^{-1}\left(\cos\frac{3\pi}{4}\right)$ is $-\frac{\pi}{4}$.

1 F. Question

Find the principal value of each of the following:

$\sin^{-1}\left(\tan\frac{5\pi}{4}\right)$

Answer

Let y = $\sin^{-1}\left(\tan\frac{5\pi}{4}\right)$

Therefore, sin y = $\left(\tan\frac{5\pi}{4}\right) = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1 = \sin\left(\frac{\pi}{2}\right)$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

And $\sin\left(\frac{\pi}{2}\right) = \tan\frac{5\pi}{4}$

Therefore the principal value of $\sin^{-1}\left(\tan\frac{5\pi}{4}\right)$ is $\frac{\pi}{2}$.

2 A. Question

Find the principal value of each of the following: $\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}}$

Answer

$\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}} = \sin^{-1}\frac{1}{2} - \sin^{-1}\left(2 \times \frac{1}{\sqrt{2}}\sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2}\right)$

$= \sin^{-1}\frac{1}{2} - \sin^{-1}(1)$

$= \frac{\pi}{6} - \frac{\pi}{2}$

$= -\frac{\pi}{3}$

2 B. Question

Find the principal value of each of the following: $\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right\}$

Answer

$\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right\}$

$= \sin^{-1}\left\{\cos\left(\frac{\pi}{3}\right)\right\}$

$= \sin^{-1}\left\{\frac{\sqrt{3}}{2}\right\}$

$= \frac{\pi}{6}$

3 A. Question

Find the domain of each of the following functions:

$f(x) = \sin^{-1}x^2$

Answer

Domain of $\sin^{-1}$ lies in the interval [–1, 1].

Therefore domain of $\sin^{-1}x^2$ lies in the interval [–1, 1].

$-1 \leq x^2 \leq 1$

But x^2 cannot take negative values,

So, $0 \leq x^2 \leq 1$

$-1 \leq x \leq 1$

Hence domain of $\sin^{-1}x^2$ is [–1, 1].

3 B. Question

Find the domain of each of the following functions:

$f(x) = \sin^{-1}x + \sin x$

Answer

Domain of $\sin^{-1}$ lies in the interval [–1, 1].

$-1 \leq x \leq 1$.

The domain of sin x lies in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$-1.57 \leq x \leq 1.57$

From the above we can see that the domain of $\sin^{-1}x + \sin x$ is the intersection of the domains of $\sin^{-1}x$ and sin x.

So domain of $\sin^{-1}x + \sin x$ is [–1, 1].

3 C. Question

Find the domain of each of the following functions:

$f(x) = \sin^{-1}\sqrt{x^2 - 1}$

Answer

Domain of $\sin^{-1}$ lies in the interval [–1, 1].

Therefore, Domain of $\sin^{-1}\sqrt{x^2 - 1}$ lies in the interval [–1, 1].

$-1 \leq \sqrt{x^2 - 1} \leq 1$

$0 \leq x^2 - 1 \leq 1$

$1 \leq x^2 \leq 2$

$\pm\sqrt{1} \leq x \leq \pm\sqrt{2}$

$\sqrt{2} \leq x \leq -1$ and $1 \leq x \leq \sqrt{2}$

Domain of $\sin^{-1}\sqrt{x^2 - 1}$ is $[-\sqrt{2}, 1] \cup [1, \sqrt{2}]$.

3 D. Question

Find the domain of each of the following functions:

$f(x) = \sin^{-1}x + \sin^{-1}2x$

Answer

Domain of $\sin^{-1}$ lies in the interval [–1, 1].

$-1 \leq x \leq 1$

Therefore, the domain of $\sin^{-1} 2x$ lies in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$-1 \leq 2x \leq 1$

$-\frac{1}{2} \leq x \leq \frac{1}{2}$

The domain of $\sin^{-1}x + \sin^{-1}2x$ is the intersection of the domains of $\sin^{-1}x$ and $\sin^{-1}2x$.

So, Domain of $\sin^{-1}x + \sin^{-1}2x$ is $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

4. Question

If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z + \sin^{-1}t = 2\pi$, then find the value of

$x^2 + y^2 + z^2 + t^2$.

Answer

Range of $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Give that $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z + \sin^{-1}t = 2\pi$

Each of $\sin^{-1}x$, $\sin^{-1}y$, $\sin^{-1}z$, $\sin^{-1}t$ takes value of $\frac{\pi}{2}$.

So,

x = 1, y = 1, z = 1 and t = 1.

Hence,

$= x^2 + y^2 + z^2 + t^2$

= 1 + 1 + 1 + 1

= 4

5. Question

If $(\sin^{-1}x)^2 + (\sin^{-1}y)^2 + (\sin^{-1}z)^2 = 3/4\ \pi^2$. Find $x^2 + y^2 + z^2$.

Answer

Range of $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Given that $(\sin^{-1}x)^2 + (\sin^{-1}y)^2 + (\sin^{-1}z)^2 = \frac{3}{4}\pi^2$

Each of $\sin^{-1}x$, $\sin^{-1}y$ and $\sin^{-1}z$ takes the value of $\frac{\pi}{2}$.

x = 1, y = 1, and z = 1.

Hence,

$= x^2 + y^2 + z^2$

= 1 + 1 + 1

= 3.

Exercise 4.2

1. Question

Find the domain of definition of $f(x) = \cos^{-1}(x^2 - 4)$.

Answer

Domain of $\cos^{-1}x$ lies in the interval [-1, 1].

Therefore, the domain of $\cos^{-1}(x^2 - 4)$ lies in the interval [-1, 1].

$-1 \leq x^2 - 4 \leq 1$

$3 \leq x^2 \leq 5$

$\pm\sqrt{3} \leq x \leq \pm\sqrt{5}$

$-\sqrt{5} \leq x \leq -\sqrt{3}$ and $\sqrt{3} \leq x \leq \sqrt{5}$

Domain of $\cos^{-1}(x^2 - 4)$ is $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$.

2. Question

Find the domain of $f(x) = \cos^{-1}2x + \sin^{-1}x$.

Answer

Domain of $\cos^{-1}x$ lies in the interval [-1, 1].

Therefore, the domain of $\cos^{-1}(2x)$ lies in the interval [-1, 1].

$-1 \leq 2x \leq 1$

$$\frac{-1}{2} \leq x \leq \frac{1}{2}$$

Domain of $\cos^{-1}(2x)$ is $\left[\frac{-1}{2}, \frac{1}{2}\right]$.

Domain of $\sin^{-1}x$ lies in the interval [-1, 1].

∴ Domain of $\cos^{-1}(2x) + \sin^{-1}x$ lies in the interval $\left[\frac{-1}{2}, \frac{1}{2}\right]$.

3. Question

Find the domain of $f(x) = \cos^{-1}x + \cos x$.

Answer

Domain of $\cos^{-1}x$ lies in the interval [–1, 1].

Domain of cos x lies in the interval [0, π] = [0, 3.14]

∴ Domain of $\cos^{-1}x$ + cos x lies in the interval [–1, 1].

4 A. Question

Find the principal value of each of the following:

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

Answer

We know that for any x ∈ [–1, 1], $\cos^{-1}$ represents an angle in [0, π].

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ = an angle in [0, π] whose cosine is $\left(-\frac{\sqrt{3}}{2}\right)$.

$= \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

$$\therefore \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

4 B. Question

Find the principal value of each of the following:

$$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

Answer

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$.

Then, cos y = $-\frac{1}{\sqrt{2}}$

= -cos $\frac{\pi}{4}$

= cos $\left(\pi - \frac{\pi}{4}\right)$

= cos $\left(\frac{3\pi}{4}\right)$

We know that the range of the principal value branch of $\cos^{-1}$ is [0, π] and cos $\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

4 C. Question

Find the principal value of each of the following:

$$\cos^{-1}\left(\sin\frac{4\pi}{3}\right)$$

Answer

$\cos^{-1}\left(\sin\frac{4\pi}{3}\right)$

$= \cos^{-1}\left(\sin\left(\pi + \frac{\pi}{3}\right)\right)$

$= \cos^{-1}\frac{-\sqrt{3}}{2}$

For any x ∈ [-1,1], $\cos^{-1}x$ represents an angle in [0, π] whose cosine is x.

$$\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

∴ Principal value of $\cos^{-1}\left(\sin\frac{4\pi}{3}\right)$ is $\frac{5\pi}{6}$.

4 D. Question

Find the principal value of each of the following:

$$\cos^{-1}\left(\tan\frac{3\pi}{4}\right)$$

Answer

$\cos^{-1}\left(\tan\frac{3\pi}{4}\right)$

$= \cos^{-1}\left(\tan\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\right)$

$= \cos^{-1}(-1)$

For any x ∈ [-1, 1], $\cos^{-1}$x represents as an angle in [0, π] whose cosine is x.

$\cos^{-1}(-1) = \pi$

∴Principal value of $\cos^{-1}\left(\tan\frac{3\pi}{4}\right)$ is π.

5 A. Question

For the principal values, evaluate each of the following:

$$\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$$

Answer

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$.

Then, cos x = $\frac{1}{2}$ = cos $\left(\frac{\pi}{3}\right)$

∴ $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$.

Then, sin y = $\frac{1}{2}$ = sin $\left(\frac{\pi}{6}\right)$

∴ $\sin^{-1}\left(\frac{1}{2}\right) = \left(\frac{\pi}{6}\right)$

Hence, $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{3} + 2\left(\frac{\pi}{6}\right)$

$= \frac{\pi}{3} + \frac{\pi}{3}$

$= \frac{2\pi}{3}$

∴Principal value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

5 B. Question

For the principal values, evaluate each of the following:

$$\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$$

Answer

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$.

Then, cos x = $\frac{1}{2}$ = cos $\left(\frac{\pi}{3}\right)$

∴ $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = y$.

Then, sin y = $-\frac{1}{2}$ = -sin $\left(\frac{\pi}{6}\right)$ = sin $\left(-\frac{\pi}{6}\right)$

∴ $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Hence, $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3} - 2\left(-\frac{\pi}{6}\right)$

$= \frac{\pi}{3} + \frac{\pi}{3}$

$= \frac{2\pi}{3}$

∴ Principal value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

5 C. Question

For the principal values, evaluate each of the following:

$$\sin^{-1}\left(-\frac{1}{2}\right) + -2\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

Answer

Let Let $\sin^{-1}\left(-\frac{1}{2}\right) = x$

Then, sin x = $-\frac{1}{2}$ = -sin $\left(\frac{\pi}{6}\right)$ = sin $\left(-\frac{\pi}{6}\right)$

$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Let $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ = y

Then, cos y = $\frac{-\sqrt{3}}{2}$ = cos $\left(\pi - \frac{\pi}{6}\right)$

$$\therefore \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

Hence, $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -\frac{\pi}{6} + 2\left(\frac{5\pi}{6}\right)$

$$= -\frac{\pi}{6} + \frac{10\pi}{6}$$

$$= \frac{-\pi + 10\pi}{6}$$

$$= \frac{9\pi}{6}$$

$$= \frac{3\pi}{2}$$

∴ Principal value of $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is $\frac{3\pi}{2}$.

5 D. Question

For the principal values, evaluate each of the following:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Answer

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= -\frac{\pi}{3} + \frac{\pi}{6}$ {Since $\sin^{-1}x$ = An angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is x,

Similarly, $\cos^{-1}$ = An angle in [0, π] whose cosine is x}

$$= -\frac{\pi}{6}$$

Hence,

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{6}$$

∴ Principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $-\frac{\pi}{6}$

Exercise 4.3

1 A. Question

Find the principal value of each of the following:

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Answer

We know that, for any x ∈ R, $\tan^{-1}$ represent an angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x.

So, $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ = An angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is $\frac{1}{\sqrt{3}}$

$= \frac{\pi}{6}$

$\therefore \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

Hence, the Principal value of $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

1 B. Question

Find the principal value of each of the following:

$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

Answer

We know that, for any x ∈ R, $\tan^{-1}$ represent an angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x.

So, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ = An angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is $-\frac{1}{\sqrt{3}}$

$= -\frac{\pi}{6}$

$\therefore \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

Hence, Principal value of $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is $-\frac{\pi}{6}$.

1 C. Question

Find the principal value of each of the following:

$\tan^{-1}\left(\cos\frac{\pi}{2}\right)$

Answer

$\tan^{-1}\left(\cos\frac{\pi}{2}\right) = \tan^{-1}(0)$ [∵ $\cos\frac{\pi}{2} = 0$]

We know that, for any x ∈ R, $\tan^{-1}$ represent an angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x.

$\therefore \tan^{-1}(0) = 0$

Hence,

Principle value of $\tan^{-1}\left(\cos\frac{\pi}{2}\right)$ is 0.

1 D. Question

Find the principal value of each of the following:

$$\tan^{-1}\left(2\cos\frac{2\pi}{3}\right)$$

Answer

$\tan^{-1}\left(2\cos\frac{2\pi}{3}\right) = \tan^{-1}\left(2 \times \frac{-1}{2}\right)$

$= \tan^{-1}(-1)$

We know that, for any x ∈ R, $\tan^{-1}$ represent an angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x.

$\therefore \tan^{-1}(-1) = -\frac{\pi}{4}$

Hence, Principle value of $\tan^{-1}\left(2\cos\frac{2\pi}{3}\right)$ is $-\frac{\pi}{4}$.

2 A. Question

For the principal values, evaluate each of the following:

$$\tan^{-1}(-1) + \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

Answer

Let $\tan^{-1}(-1) = x$.

Then tan x = -1

$= -\tan\frac{\pi}{4}$

$= \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$

$\therefore \tan^{-1}(-1) = \frac{3\pi}{4}$

Let $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = y$.

Then cos y = $\frac{-1}{\sqrt{2}}$

$= -\cos\frac{\pi}{4}$

$= \cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

$\therefore \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4}$

Hence, $\tan^{-1}(-1) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4} + \frac{3\pi}{4} = \frac{6\pi}{4} = \frac{3\pi}{2}$

2 B. Question

For the principal values, evaluate each of the following:

$$\tan^{-1}\left\{2\sin\left(4\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\}$$

Answer

Let $\cos^{-1}\dfrac{\sqrt{3}}{2} = x$

cos x = cos $\left(\frac{\pi}{6}\right)$

x = $\left(\frac{\pi}{6}\right)$

So now,

$$\tan^{-1}\left\{2\sin\left(4\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\} = \tan^{-1}\left\{2\sin\left(4\frac{\pi}{6}\right)\right\}$$

$$\tan^{-1}\left\{2\sin\left(4\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\} = \tan^{-1}\left\{2\sin\left(2\frac{\pi}{3}\right)\right\}$$

$$\tan^{-1}\left\{2\sin\left(4\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\} = \tan^{-1}\left(2\frac{\sqrt{3}}{2}\right)$$

$$\tan^{-1}\left\{2\sin\left(4\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\} = \tan^{-1}(\sqrt{3})$$

$$\tan^{-1}\left\{2\sin\left(4\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\} = \frac{\pi}{3}$$

3 A. Question

Evaluate each of the following:

$$\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$$

Answer

Let $\tan^{-1}(1) = x.$

Then tan x = 1 = $\frac{\pi}{4}$

∴ $\tan^{-1}(1) = \frac{\pi}{4}$ ......(i)

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y.$

Then cos y = $-\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi-\frac{\pi}{3}\right) = \cos\frac{2\pi}{3}.$

∴ $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ ......(ii)

Again,

Let $\sin^{-1}\left(-\frac{1}{2}\right) = z.$

Then sin z = $-\frac{1}{2} = -\sin\frac{\pi}{6} = \sin\left(-\frac{\pi}{6}\right)$

∴ $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ ......(iii)

Now,

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} + \left(-\frac{\pi}{6}\right)$ [from (i), (ii), (iii)]

$= \frac{3\pi + 8\pi - 2\pi}{12}$

$= \frac{9\pi}{12}$

$= \frac{3\pi}{4}$

3 B. Question

Evaluate each of the following:

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \tan^{-1}(-\sqrt{3}) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$$

Answer

$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \tan^{-1}(-\sqrt{3}) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$

We know that, for any x ∈ R, $\tan^{-1}$ represent an angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x.

$\therefore \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \frac{-\pi}{6}$,

$\tan^{-1}(-\sqrt{3}) = \frac{-\pi}{3}$ and,

$\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) = \tan(-1)$ $[\because \sin\left(-\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right) = -1]$

$= -\frac{\pi}{4}$

Now, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \tan^{-1}(-\sqrt{3}) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$ becomes,

$= \left(\frac{-\pi}{6}\right) + \left(\frac{-\pi}{3}\right) + \left(\frac{-\pi}{4}\right)$

$= \frac{-2\pi - 4\pi - 3\pi}{12}$

$= \frac{-9\pi}{12}$

$= \frac{-3\pi}{4}$

Therefore the principle value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \tan^{-1}(-\sqrt{3}) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$ is $\frac{-3\pi}{4}$.

3 C. Question

Evaluate each of the following:

$$\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left\{\cos\left(\frac{13\pi}{6}\right)\right\}$$

Answer

$\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left\{\cos\left(\frac{13\pi}{6}\right)\right\}$

Firstly, $\tan\frac{5\pi}{6} = \tan\left(\pi - \frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$ ......(i)

Also, $\cos\left(\frac{13\pi}{6}\right) = \cos\left(2\pi + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ ......(ii)

From (i) and (ii),

$\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left\{\cos\left(\frac{13\pi}{6}\right)\right\}$ becomes,

$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Now,

We know that, for any x ∈ R, $\tan^{-1}$ represent an angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x.

$\therefore \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

We know that, for any x ∈ [-1, 1], $\tan^{-1}$ represent an angle in $(0, \pi)$ whose cosine is x.

$\therefore \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

Hence, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{6} + \frac{\pi}{6} = 0$

Therefore, Principal Value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left\{\cos\left(\frac{13\pi}{6}\right)\right\}$ is 0.

Exercise 4.4

1 A. Question

Find the principal values of each of the following:

$\sec^{-1}(-\sqrt{2})$

Answer

Let $\sec^{-1}(-\sqrt{2}) = y$

$\Rightarrow \sec y = -\sqrt{2}$

$= -\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

$= \sec\left(\pi - \frac{\pi}{4}\right)$

$= \sec\left(\frac{3\pi}{4}\right)$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$

and $\sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$

∴ The principal value of $\sec^{-1}(-\sqrt{2})$ is $\frac{3\pi}{4}$.

1 B. Question

Find the principal values of each of the following:

$\sec^{-1}(2)$

Answer

Let $\sec^{-1}(2) = y$

$\Rightarrow \sec y = 2$

$\Rightarrow \sec(\frac{\pi}{3})$

The range of principal value of $\sec^{-1}$ is $[0, \pi]-\{\frac{\pi}{2}\}$

And $\sec\left(\frac{\pi}{3}\right) = 2$

$\therefore$ The principal value of $\sec^{-1}(2)$ is $\frac{\pi}{3}$.

1 C. Question

Find the principal values of each of the following:

$$\sec^{-1}\left(2\sin\frac{3\pi}{4}\right)$$

Answer

Let us assume $2\sin\frac{3\pi}{4} = \theta$

We know $\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$

$\therefore 2\sin\frac{3\pi}{4} = 2\frac{1}{\sqrt{2}}$

$\Rightarrow 2\sin\frac{3\pi}{4} = \sqrt{2}$

$\therefore$ The question becomes $\sec^{-1}(\sqrt{2})$

Now,

Let $\sec^{-1}(\sqrt{2}) = y$

$\Rightarrow \sec y = \sqrt{2}$

$\Rightarrow \sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

The range of principal value of $\sec^{-1}$ is $[0, \pi]-\{\frac{\pi}{2}\}$

And $\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

$\therefore$ The principal value of $\sec^{-1}(2\sin\frac{3\pi}{4})$ is $\frac{\pi}{4}$.

1 D. Question

Find the principal values of each of the following:

$$\sec^{-1}\left(2\tan\frac{3\pi}{4}\right)$$

Answer

Let us assume $2\tan\frac{3\pi}{4} = \theta$

We know $\tan\frac{3\pi}{4} = -1$

$\therefore 2\tan\frac{3\pi}{4} = 2(-1)$

$\Rightarrow 2\tan\frac{3\pi}{4} = -2$

$\therefore$ The question converts to $\sec^{-1}(-2)$

Now,

Let $\sec^{-1}(-2) = y$

$\Rightarrow \sec y = -2$

$= -\sec\left(\frac{\pi}{3}\right) = 2$

$= \sec\left(\pi - \frac{\pi}{3}\right)$

$= \sec\left(\frac{2\pi}{3}\right)$

The range of principal value of $\sec^{-1}$is $[0, \pi]-\{\frac{\pi}{2}\}$

and $\sec\left(\frac{2\pi}{3}\right) = -2$

$\therefore$ The principal value of $\sec^{-1}(2\tan\frac{3\pi}{4})$ is $\frac{2\pi}{3}$

2 A. Question

For the principal values, evaluate the following:

$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

Answer

The Principal value for $\tan^{-1}\sqrt{3}$

Let $\tan^{-1}(\sqrt{3}) = y$

$\Rightarrow \tan y = \sqrt{3}$

The range of principal value of $\tan^{-1}$is $\{-\frac{\pi}{2}, \frac{\pi}{2}\}$

And $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

$\therefore$ The principal value of $\tan^{-1}(\sqrt{3})$ is $\frac{\pi}{3}$.

Now,

Principal value for $\sec^{-1}(-2)$

Let $\sec^{-1}(-2) = z$

$\Rightarrow \sec z = -2$

$= -\sec\left(\frac{\pi}{3}\right) = 2$

$= \sec\left(\pi - \frac{\pi}{3}\right)$

$= \sec\left(\frac{2\pi}{3}\right)$

The range of principal value of $\sec^{-1}$is $[0, \pi]-\{\frac{\pi}{2}\}$

and $\sec\left(\frac{2\pi}{3}\right) = -2$

Therefore, the principal value of $\sec^{-1}(-2)$ is $\frac{2\pi}{3}$.

$\therefore \tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \frac{\pi}{3} - \frac{2\pi}{3}$

$= \frac{-\pi}{3}$

$\therefore \tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{-\pi}{3}$.

2 B. Question

For the principal values, evaluate the following:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - 2\sec^{-1}\left(2\tan\frac{\pi}{6}\right)$$

Answer

Let,

$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = y$

$\Rightarrow \sin y = \frac{-\sqrt{3}}{2}$

$\Rightarrow -\sin y = \frac{\sqrt{3}}{2}$

$\Rightarrow -\sin\frac{\pi}{3}$

As we know $\sin(-\theta) = -\sin\theta$

$\therefore -\sin\frac{\pi}{3} = \sin\left(\frac{-\pi}{3}\right)$

The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\sin\left(\frac{-\pi}{3}\right) = \frac{-\sqrt{3}}{2}$

Therefore, the principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is $\frac{-\pi}{3}$ ….(1)

Let us assume $2\tan\frac{\pi}{6} = \theta$

We know $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\therefore 2\tan\frac{\pi}{6} = 2\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow 2\tan\frac{\pi}{6} = \frac{2}{\sqrt{3}}$

$\therefore$ The question converts to $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Now,

Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = z$

$\Rightarrow \sec z = \left(\frac{2}{\sqrt{3}}\right)$

$= \sec\left(\frac{\pi}{6}\right) = \left(\frac{2}{\sqrt{3}}\right)$

The range of principal value of $\sec^{-1}$is $[0, \pi]-\{\frac{\pi}{2}\}$

and $\sec\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$

Therefore, the principal value of $\sec^{-1}(2\tan\frac{\pi}{6})$ is $\frac{\pi}{3}$ .....(2)

$\therefore \text{Sin}^{-1}\left(\frac{-\sqrt{3}}{2}\right) - 2\sec^{-1}(2\tan\frac{\pi}{6})$

$= \frac{-\pi}{3} - \frac{2\pi}{3}$ (from (1) and (2))

$= \frac{-3\pi}{3}$

$= -\pi$

Therefore, the value of $\text{Sin}^{-1}\left(\frac{-\sqrt{3}}{2}\right) - 2\sec^{-1}(2\tan\frac{\pi}{6})$ is $-\pi$.

3 A. Question

Find the domain of

$\sec^{-1}(3x-1)$

Answer

The range of sec x is the domain of $\sec^{-1}x$

Now,

The range of sec x is $(-\infty, -1] \cup [1, \infty)$

∴ The domain of a given function would be

$3x-1 \leq -1$ and $3x-1 \geq 1$

$3x \leq 0$ and $3x \geq 2$

$x \leq 0$ and $x \geq \frac{2}{3}$

∴ The domain of the given function is $(-\infty,0]\cup[\frac{2}{3},\infty)$

3 B. Question

Find the domain of

$\sec^{-1}x-\tan^{-1}x$

Answer

Domain of $\sec^{-1}x$ is $(-\infty,-1]\cup[1,\infty)$

Domain of $\tan^{-1}x$ is $\mathbb{R}$

Union of (1) and (2) will be domain of given function

$(-\infty,-1]\cup[1,\infty)\cup\mathbb{R}$

⇒ $(-\infty,-1]\cup[1,\infty)$

∴ The domain of given function is $(-\infty,-1]\cup[1,\infty)$.

1 A. Question

Find the principal values of each of the following:

$\sec^{-1}(-\sqrt{2})$

Answer

Let $\sec^{-1}(-\sqrt{2}) = y$

⇒ $\sec y = -\sqrt{2}$

$= -\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

$= \sec\left(\pi - \frac{\pi}{4}\right)$

$= \sec\left(\frac{3\pi}{4}\right)$

The range of principal value of $\sec^{-1}$ is $[0, \pi]-\{\frac{\pi}{2}\}$

and $\sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$

∴ The principal value of $\sec^{-1}(-\sqrt{2})$ is $\frac{3\pi}{4}$.

Exercise 4.5

1 A. Question

Find the principal values of each of the following:

$\operatorname{cosec}^{-1}(-\sqrt{2})$

Answer

$\operatorname{cosec}^{-1}(-\sqrt{2}) = y$

⇒ $\operatorname{cosec} y = -\sqrt{2}$

⇒ $-\operatorname{cosec} y = \sqrt{2}$

⇒ $-\operatorname{cosec} \frac{\pi}{4} = \sqrt{2}$

As we know $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

∴ $-\operatorname{cosec} \frac{\pi}{4} = \operatorname{cosec}\left(\frac{-\pi}{4}\right)$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]-\{0\}$ and

$\operatorname{cosec}\left(\frac{-\pi}{4}\right) = -\sqrt{2}$

Therefore, the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$ is $\frac{-\pi}{4}$.

1 B. Question

Find the principal values of each of the following:

$\operatorname{cosec}^{-1}(-2)$

Answer

$\text{cosec}^{-1}{-2} = y$

⇒ cosec y = –2

⇒ –cosec y = 2

⇒ $-\text{cosec}\,\frac{\pi}{6} = 2$

As we know cosec(–θ) = –cosecθ

∴ $-\text{cosec}\,\frac{\pi}{6} = \text{cosec}\left(\frac{-\pi}{6}\right)$

The range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and

$\text{cosec}\left(\frac{-\pi}{6}\right) = -2$

Therefore, the principal value of $\text{cosec}^{-1}(-2)$ is $\frac{-\pi}{6}$.

1 C. Question

Find the principal values of each of the following:

$$\text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

Answer

Let $\text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$

⇒ $\text{cosec}\,y = \left(\frac{2}{\sqrt{3}}\right)$

$= \text{cosec}\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$

The range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

and $\text{cosec}\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$

Therefore, the principal value of $\text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{3}$.

1 D. Question

Find the principal values of each of the following:

$$\text{cosec}^{-1}\left(2\cos\frac{2\pi}{3}\right)$$

Answer

$\text{cosec}^{-1}(2\cos\frac{2\pi}{3})$

Let us assume $2\cos\frac{2\pi}{3} = \theta$

We know $\cos\frac{2\pi}{3} = \frac{-1}{2}$

∴ $2\cos\frac{2\pi}{3} = 2\left(\frac{-1}{2}\right)$

⇒ $2\cos\frac{2\pi}{3} = -1$

∴ The question converts to $\text{cosec}^{-1}(-1)$

Now,

$\text{cosec}^{-1}{-1} = y$

⇒ cosec y = –1

⇒ –cosec y = 1

⇒ $-\text{cosec}\,\frac{\pi}{4} = 1$

As we know cosec(–θ) = –cosecθ

∴ $-\text{cosec}\,\frac{\pi}{2} = \text{cosec}\left(\frac{-\pi}{2}\right)$

The range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$–{0} and

$\text{cosec}\left(\frac{-\pi}{2}\right) = -1$

Therefore, the principal value of $\text{cosec}^{-1}(2\cos\frac{2\pi}{3})$ is $\frac{-\pi}{2}$.

2. Question

Find the set of values of $\text{cosec}^{-1}(\sqrt{3}/2)$.

Answer

Let y = $\text{cosec}^{-1}(\sqrt{3}/2)$

We know that,

Domain of y = cosec^{-1} x is (– ∞, 1] U [1, ∞]

But √3/2 < 1

Therefore, it can not be a value of y.

Hence, Set of values of $\text{cosec}^{-1}(\sqrt{3}/2)$ is a null set.

3 A. Question

For the principal values, evaluate the following:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \text{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$$

Answer

Let,

$\text{Sin}^{-1}\left(\frac{-\sqrt{3}}{2}\right) = y$

⇒ $\sin y = \frac{-\sqrt{3}}{2}$

⇒ $-\sin y = \frac{\sqrt{3}}{2}$

⇒ $-\sin\frac{\pi}{3}$

As we know sin(–θ) = –sinθ

∴ $-\sin\frac{\pi}{3} = \sin\left(\frac{-\pi}{3}\right)$

The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\sin\left(\frac{-\pi}{3}\right) = \frac{-\sqrt{3}}{2}$

Therefore, the principal value of $\text{Sin}^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is $\frac{-\pi}{3}$ ....(1)

Let,

$\text{cosec}^{-1}\left(\frac{-\sqrt{3}}{2}\right) = z$

$\Rightarrow$ cosec z = $\frac{-\sqrt{3}}{2}$

$\Rightarrow$ –cosec z = $\frac{\sqrt{3}}{2}$

$\Rightarrow$ –cosec $\frac{\pi}{3}$

As we know cosec(–θ) = –cosecθ

∴ –cosec $\frac{\pi}{3}$ = cosec $\left(\frac{-\pi}{3}\right)$

The range of principal value of cosec^{-1} is is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$–{0} and

cosec $\left(\frac{-\pi}{3}\right) = \frac{-\sqrt{3}}{2}$

Therefore, the principal value of $\text{cosec}^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is $\frac{-\pi}{3}$ ....(2)

From (1) and (2) we get

$\Rightarrow \frac{-\pi}{3} + \frac{-\pi}{3}$

$= \frac{-2\pi}{3}$

3 B. Question

For the principal values, evaluate the following:

$$\sec^{-1}\left(\sqrt{2}\right) + 2\,\text{cosec}^{-1}\left(-\sqrt{2}\right)$$

Answer

Let $\sec^{-1}(-\sqrt{2}) = y$

$\Rightarrow$ sec y = –√2

= – $\sec\left(\frac{\pi}{4}\right)$ = √2

= $\sec\left(\pi - \frac{\pi}{4}\right)$

= $\sec\left(\frac{3\pi}{4}\right)$

The range of principal value of $\sec^{-1}$is [0, π]–$\{\frac{\pi}{2}\}$

and $\sec\left(\frac{3\pi}{4}\right)$ = –√2.

Let,

cosec^{-1}–√2 = z

⇒ cosec z = –√2

⇒ –cosec z = √2

⇒ –cosec $\frac{\pi}{4}$ = √2

As we know cosec(–θ) = –cosecθ

∴ –cosec $\frac{\pi}{4}$ = cosec $\left(\frac{-\pi}{4}\right)$

The range of principal value of $cosec^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$–{0} and

cosec $\left(\frac{-\pi}{4}\right)$ = –√2

Therefore, the principal value of $cosec^{-1}$(–√2) is $\frac{-\pi}{4}$.

$cosec^{-1}$–√2 = y

⇒ cosec y = –√2

⇒ –cosec y = √2

⇒ –cosec $\frac{\pi}{4}$ = √2

As we know cosec(–θ) = –cosecθ

∴ –cosec $\frac{\pi}{4}$ = cosec $\left(\frac{-\pi}{4}\right)$

The range of principal value of $cosec^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$–{0} and

cosec $\left(\frac{-\pi}{4}\right)$ = –√2

Therefore, the principal value of $cosec^{-1}$(–√2) is $\frac{-\pi}{4}$.

From (1) and (2) we get

⇒ $\frac{3\pi}{4} + 2 \times \frac{-\pi}{4}$

= $\frac{3\pi}{4} + \frac{-2\pi}{4}$

= $\frac{\pi}{4}$

3 C. Question

For the principal values, evaluate the following:

$$\sin^{-1}\left[\cos\left\{\text{cosec}^{-1}(-2)\right\}\right]$$

Answer

First of all we need to find the principal value for $cosec^{-1}$(–2)

Let,

$cosec^{-1}$–2 = y

⇒ cosec y = –2

⇒ –cosec y = 2

⇒ –cosec $\frac{\pi}{6}$ = 2

As we know cosec(–θ) = –cosecθ

∴ $-\text{cosec}\,\frac{\pi}{6} = \text{cosec}\left(\frac{-\pi}{6}\right)$

The range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$–{0} and

$\text{cosec}\left(\frac{-\pi}{6}\right) = -2$

Therefore, the principal value of $\text{cosec}^{-1}(-2)$ is $\frac{-\pi}{6}$.

∴ Now, the question changes to

$\text{Sin}^{-1}[\cos\frac{-\pi}{6}]$

Cos(–θ) = cos(θ)

∴ we can write the above expression as

$\text{Sin}^{-1}[\cos\frac{\pi}{6}]$

Let,

$\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$

$\Rightarrow \sin y = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin\frac{\pi}{3}$

The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Therefore, the principal value of $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{3}$.

Hence, the principal value of the given equation is $\frac{\pi}{3}$.

3 D. Question

For the principal values, evaluate the following:

$$\text{cosec}^{-1}\left(2\tan\frac{11\pi}{6}\right)$$

Answer

We can write,

$\tan\frac{11\pi}{6} = \tan\left(2\pi - \frac{\pi}{6}\right)$

tan(2π – θ)

= tan(–θ)

= –tanθ

∴ $\tan\frac{11\pi}{6}$ becomes $-\tan\frac{\pi}{6}$

$-\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$

$\Rightarrow 2\tan\frac{11\pi}{6} = -\frac{2}{\sqrt{3}}$

∴ The question converts to $\text{cosec}^{-1}(-\frac{2}{\sqrt{3}})$

Let $\text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$

⇒ $\text{cosec } y = \left(\frac{2}{\sqrt{3}}\right)$

$= \text{cosec}\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$

The range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

and $\text{cosec}\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$

Therefore, the principal value of $\text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{3}$.

Exercise 4.6

1 A. Question

Find the principal values of each of the following:

$\cot^{-1}(-\sqrt{3})$

Answer

Let $\cot^{-1}(-\sqrt{3}) = y$

⇒ $\cot y = -\sqrt{3}$

$= -\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$

$= \cot\left(\pi - \frac{\pi}{6}\right)$

$= \cot\left(\frac{5\pi}{6}\right)$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

and $\cot\left(\frac{5\pi}{6}\right) = -\sqrt{3}$

∴ The principal value of $\cot^{-1}(-\sqrt{3})$ is $\frac{5\pi}{6}$

1 B. Question

Find the principal values of each of the following:

$\cot^{-1}(\sqrt{3})$

Answer

Let $\cot^{-1}(\sqrt{3}) = y$

⇒ $\cot y = \sqrt{3}$

$= \cot\left(\frac{\pi}{6}\right) = \sqrt{3}$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

and $\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$

∴ The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$

1 C. Question

Find the principal values of each of the following:

$$\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$$

Answer

Let $\cot^{-1}(\frac{-1}{\sqrt{3}})$ = y

⇒ cot y = $\frac{-1}{\sqrt{3}}$

= – $\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$

= $\cot\left(\pi - \frac{\pi}{3}\right)$

= $\cot\left(\frac{2\pi}{3}\right)$

The range of principal value of $\cot^{-1}$is (0, π)

and $\cot\left(\frac{2\pi}{3}\right) = \frac{-1}{\sqrt{3}}$

∴ The principal value of $\cot^{-1}(\frac{-1}{\sqrt{3}})$ is $\frac{2\pi}{3}$

1 D. Question

Find the principal values of each of the following:

$$\cot^{-1}\left(\tan\frac{3\pi}{4}\right)$$

Answer

The value of

$\tan\frac{3\pi}{4}$ = –1

∴ The question becomes $\cot^{-1}(-1)$

Let $\cot^{-1}(-1)$ = y

⇒ cot y = –1

= – $\cot\left(\frac{\pi}{4}\right)$ = 1

= $\cot\left(\pi - \frac{\pi}{4}\right)$

= $\cot\left(\frac{3\pi}{4}\right)$

The range of principal value of $\cot^{-1}$is (0, π)

and $\cot\left(\frac{3\pi}{4}\right)$ = –1

∴ The principal value of $\cot^{-1}(\tan\frac{3\pi}{4})$ is $\frac{3\pi}{4}$.

2. Question

Find the domain of f(x) = cotx + $\cot^{-1}$ x.

Answer

Now the domain of cot x is ℝ

While the domain of $\cot^{-1}x$ is [0,π]

∴ The union of these two will give the domain of f(x)

⇒ ℝ ⋃ [0,π]

= [0,π]

∴ The domain of f(x) is [0,π]

3 A. Question

Evaluate each of the following:

$$\cot^{-1}\frac{1}{\sqrt{3}}-\operatorname{cosec}^{-1}(-2)+\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

Answer

Let $\cot^{-1}(\frac{-1}{\sqrt{3}})$ = y

⇒ cot y = $\frac{-1}{\sqrt{3}}$

= - $\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$

= $\cot\left(\pi-\frac{\pi}{3}\right)$

= $\cot\left(\frac{2\pi}{3}\right)$

The range of principal value of $\cot^{-1}$is (0, π)

and $\cot\left(\frac{2\pi}{3}\right) = \frac{-1}{\sqrt{3}}$

∴ The principal value of $\cot^{-1}(\frac{-1}{\sqrt{3}})$ is $\frac{2\pi}{3}$ ...(1)

Let,

$\operatorname{cosec}^{-1}$–2 = z

⇒ cosec z = –2

⇒ –cosec z = 2

⇒ –cosec $\frac{\pi}{6}$ = 2

As we know cosec(–θ) = –cosecθ

∴ –cosec $\frac{\pi}{6}$ = cosec $\left(\frac{-\pi}{6}\right)$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$–{0} and

cosec $\left(\frac{-\pi}{6}\right)$ = –2

Therefore, the principal value of $\operatorname{cosec}^{-1}$(–2) is $\frac{-\pi}{6}$...(2)

Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ = w

$\Rightarrow$ sec w = $\left(\frac{2}{\sqrt{3}}\right)$

= sec$\left(\frac{\pi}{6}\right)$ = $\left(\frac{2}{\sqrt{3}}\right)$

The range of principal value of $\sec^{-1}$is [0, π]–{$\frac{\pi}{2}$}

and sec$\left(\frac{\pi}{3}\right)$ = $\left(\frac{2}{\sqrt{3}}\right)$

Therefore, the principal value of $\sec^{-1}(\frac{2}{\sqrt{3}})$ is $\frac{\pi}{3}$...(3)

From (1), (2) and (3) we can write the above equation as

$= \frac{2\pi}{3} - \frac{-\pi}{6} + \frac{\pi}{3}$

$= \frac{4\pi+\pi+2\pi}{6}$

$= \frac{7\pi}{6}$

3 B. Question

Evaluate each of the following:

$$\cot^{-1}\left\{2\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right\}$$

Answer

For finding the solution we first of need to find the principal value of

$\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Let,

$\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ =y

$\Rightarrow$ sin y = $\frac{\sqrt{3}}{2}$

$\Rightarrow$ sin $\frac{\pi}{3}$

The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ and sin $\left(\frac{\pi}{3}\right)$ = $\frac{\sqrt{3}}{2}$

Therefore, the principal value of $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{3}$

∴ The above equation changes to $\cot^{-1}(2\cos\frac{\pi}{3})$

Now we need to find the value of $2\cos\frac{\pi}{3}$

∴ $\cos\frac{\pi}{3} = \frac{1}{2}$

$\Rightarrow 2\cos\frac{\pi}{3} = 1 \times \frac{1}{2}$

$\Rightarrow 2\cos\frac{\pi}{3} = 1$

Now the equation simplification to $\cot^{-1}(1)$

Let $\cot^{-1}(1) = y$

⇒ cot y = 1

$= \cot\left(\frac{\pi}{4}\right) = 1$

The range of principal value of $\cot^{-1}$is (0, π)

and $\cot\left(\frac{\pi}{4}\right) = 1$

∴ The principal value of $\cot^{-1}(2\cos(\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right))$ is $\frac{\pi}{4}$

3 C. Question

Evaluate each of the following:

$$\text{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right) + 2\cot^{-1}(-1)$$

Answer

Now first of the principal value of

$\text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Let $\text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$

⇒ $\text{cosec } y = \left(\frac{2}{\sqrt{3}}\right)$

$= \text{cosec}\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$

The range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$-{0}

and $\text{cosec}\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$

Therefore, the principal value of $\text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{3}$...(1)

Now, the value of $\cot^{-1}(-1)$

Let $\cot^{-1}(-1) = y$

⇒ cot y = –1

$= -\cot\left(\frac{\pi}{4}\right) = 1$

$= \cot\left(\pi - \frac{\pi}{4}\right)$

$= \cot\left(\frac{3\pi}{4}\right)$

The range of principal value of $\cot^{-1}$is (0, π)

and $\cot\left(\frac{3\pi}{4}\right) = -1$

Therefore, the principal value of $\cot^{-1}(-1)$ is $\frac{3\pi}{4}$ ...(2)

From (1) and (2) we can write the given equation as

$= \frac{\pi}{3} + 2 \times \frac{3\pi}{4}$

$= \frac{\pi}{3} + \frac{3\pi}{2}$

$= \frac{11\pi}{6}$

3 D. Question

Evaluate each of the following:

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$$

Answer

Let $\tan^{-1}(\frac{-1}{\sqrt{3}}) = y$

$\Rightarrow \tan y = \frac{-1}{\sqrt{3}}$

$= - \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

$= \tan\left(-\frac{\pi}{6}\right)$

∴ The principal value of $\tan^{-1}(\frac{-1}{\sqrt{3}})$ is $\frac{-\pi}{6}$ ...(1)

Let $\cot^{-1}(\frac{-1}{\sqrt{3}}) = z$

$\Rightarrow \cot z = \frac{-1}{\sqrt{3}}$

$= - \cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$

$= \cot\left(\pi - \frac{\pi}{3}\right)$

$= \cot\left(\frac{2\pi}{3}\right)$

The range of principal value of $\cot^{-1}$is $(0, \pi)$

and $\cot\left(\frac{2\pi}{3}\right) = \frac{-1}{\sqrt{3}}$

∴ The principal value of $\cot^{-1}(\frac{-1}{\sqrt{3}})$ is $\frac{2\pi}{3}$ ...(2)

$\sin\frac{-\pi}{2} = -1$

∴ $\tan^{-1}(-1)$

Let $\tan^{-1}(-1) = w$

$\Rightarrow \tan w = -1$

$= - \tan\left(\frac{\pi}{4}\right) = 1$

$= \tan\left(-\frac{\pi}{4}\right)$

∴ The principal value of $\tan^{-1}(-1)$ is $\frac{-\pi}{4}$...(3)

From(1),(2) and (3) we get

$= \frac{-\pi}{6} + \frac{2\pi}{3} + \frac{-\pi}{4}$

$= \frac{\pi}{4}$

Exercise 4.7

1 A. Question

Evaluate each of the following:

$$\sin^{-1}\left(\sin\frac{\pi}{6}\right)$$

Answer

The value of $\sin\frac{\pi}{6}$ is $\frac{1}{2}$

∴ The question becomes $\sin^{-1}\left(\frac{1}{2}\right)$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$

$\Rightarrow \sin y = \frac{1}{2}$

$= \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

The range of principal value of $\sin^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

Therefore, the value of $\sin^{-1}(\sin\frac{\pi}{6})$ is $\frac{\pi}{6}$.

Alternate Solution:

$\sin^{-1}(\sin x) = x$

Provided $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

∴ we can write $\sin^{-1}(\sin\frac{\pi}{6}) = \frac{\pi}{6}$

1 B. Question

Evaluate each of the following:

$$\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$$

Answer

The value of $\sin\frac{7\pi}{6}$ is $\frac{-1}{2}$

∴ The question becomes $\sin^{-1}\left(\frac{-1}{2}\right)$

Let $\sin^{-1}\left(\frac{-1}{2}\right) = y$

$\Rightarrow -\sin y = \frac{1}{2}$

$= -\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

As, $-\sin(\theta)$ is $\sin(-\theta)$.

$\Rightarrow -\sin\left(\frac{\pi}{6}\right) = \sin\left(\frac{-\pi}{6}\right)$

The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\sin\left(\frac{-\pi}{6}\right) = \frac{-1}{2}$

Therefore, the value of $\sin^{-1}(\sin\frac{7\pi}{6})$ is $\frac{-\pi}{6}$.

1 C. Question

Evaluate each of the following:

$$\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$$

Answer

The value of $\sin\frac{5\pi}{6}$ is $\frac{1}{2}$

∴ The question becomes $\sin^{-1}\left(\frac{1}{2}\right)$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$

$\Rightarrow \sin y = \frac{1}{2}$

$= \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

The range of principal value of $\sin^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

Therefore, the value of $\sin^{-1}(\sin\frac{5\pi}{6})$ is $\frac{\pi}{6}$.

1 D. Question

Evaluate each of the following:

$$\sin^{-1}\left(\sin\frac{13\pi}{7}\right)$$

Answer

We can write $(\sin\frac{13\pi}{7})$ as $\sin\left(2\pi - \frac{\pi}{7}\right)$

As we know $\sin(2\pi - \theta) = \sin(-\theta)$

So $\sin\left(2\pi - \frac{\pi}{7}\right)$ can be written as $\sin\left(\frac{\pi}{7}\right)$

∴ The equation becomes $\sin^{-1}(\sin\frac{\pi}{7})$

As $\sin^{-1}(\sin x) = x$

Provided $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

∴ we can write $\sin^{-1}(\sin\frac{\pi}{7}) = \frac{\pi}{7}$

1 E. Question

Evaluate each of the following:

$\sin^{-1}\left(\sin\frac{17\pi}{8}\right)$

Answer

We can write ($\sin\frac{17\pi}{8}$) as $\sin\left(2\pi+\frac{\pi}{8}\right)$

As we know sin(2π +θ) = sin(θ)

So $\sin\left(2\pi+\frac{\pi}{8}\right)$ can be written as $\sin\left(\frac{\pi}{8}\right)$

∴ The equation becomes $\sin^{-1}(\sin\frac{\pi}{8})$

As $\sin^{-1}(\sin x) = x$

Provided x ϵ $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

∴ we can write $\sin^{-1}(\sin\frac{\pi}{8}) = \frac{\pi}{8}$

1 F. Question

Evaluate each of the following:

$\sin^{-1}\left\{\left(\sin-\frac{17\pi}{8}\right)\right\}$

Answer

As we know sin(–θ) is –sin(θ)

∴ We can write ($\sin\frac{-17\pi}{8}$) as $-\sin\left(\frac{17\pi}{8}\right)$

Now $-\sin\left(\frac{-17\pi}{8}\right) = -\sin\left(2\pi+\frac{\pi}{8}\right)$

As we know sin(2π +θ) = sin(θ)

So $-\sin\left(2\pi+\frac{\pi}{8}\right)$ can be written as $-\sin\left(\frac{\pi}{8}\right)$

And $-\sin\left(\frac{\pi}{8}\right) = \sin\left(\frac{-\pi}{8}\right)$

The equation becomes $\sin^{-1}(\sin\frac{-\pi}{8})$

As $\sin^{-1}(\sin x) = x$

Provided x ϵ $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

∴ we can write $\sin^{-1}(\sin\frac{-\pi}{8}) = \frac{-\pi}{8}$

1 G. Question

Evaluate each of the following:

$\sin^{-1}(\sin 3)$

Answer

$\sin^{-1}(\sin x) = x$

Provided x ϵ $\left[\frac{-\pi}{2},\frac{\pi}{2}\right] \approx [-1.57, 1.57]$

And in our equation x is 3 which does not lie in the above range.

We know sin[π – x] = sin[x]

∴ sin(π – 3) = sin(3)

Also π–3 belongs in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

∴ $\sin^{-1}$(sin3) = π–3

1 H. Question

Evaluate each of the following:

$\sin^{-1}$(sin4)

Answer

$\sin^{-1}$(sin x) = x

Provided x ϵ $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ ≈ [–1.57,1.57]

And in our equation x is 4 which does not lie in the above range.

We know sin[π – x] = sin[–x]

∴ sin(π – 4) = sin(–4)

Also π–4 belongs in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

∴ $\sin^{-1}$(sin 4) = π – 4

1 I. Question

Evaluate each of the following:

$\sin^{-1}$ (sin12)

Answer

$\sin^{-1}$(sin x) = x

Provided x ϵ $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ ≈ [–1.57,1.57]

And in our equation x is 4 which does not lie in the above range.

We know sin[2nπ – x] = sin[–x]

∴ sin(2nπ – 12) = sin(–12)

Here n = 2

Also 2π–12 belongs in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

∴ $\sin^{-1}$(sin12) = 2π – 12

1 J. Question

Evaluate each of the following:

$\sin^{-1}$ (sin 2)

Answer

$\sin^{-1}$(sin x) = x

Provided x ϵ $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ ≈ [–1.57,1.57]

And in our equation x is 3 which does not lie in the above range.

We know sin[π – x] = sin[x]

∴ sin(π – 2) = sin(2)

Also π–2 belongs in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

∴ $\sin^{-1}(\sin 2) = \pi - 2$

2 A. Question

Evaluate each of the following:

$$\cos^{-1}\left\{\cos\left(-\frac{\pi}{4}\right)\right\}$$

Answer

As cos(–θ) is cos(θ)

∴ (cos $\left(\frac{-\pi}{4}\right)$) = (cos $\left(\frac{\pi}{4}\right)$)

Now,

cos $\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

∴ The question becomes $\cos^{-1}(\frac{1}{\sqrt{2}})$

Let $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = y$

⇒ cos y = $\frac{1}{\sqrt{2}}$

= $\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

The range of principal value of $\cos^{-1}$ is [0,π] and cos $\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

Therefore, the value of $\cos^{-1}$(cos $\left(\frac{-\pi}{4}\right)$) is $\frac{\pi}{4}$.

2 B. Question

Evaluate each of the following:

$$\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$$

Answer

The value of cos $\left(\frac{5\pi}{4}\right)$ is $\frac{-1}{\sqrt{2}}$

Now,

∴ The question becomes $\cos^{-1}(\frac{-1}{\sqrt{2}})$

Let $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = y$

⇒ cos y = $\frac{-1}{\sqrt{2}}$

= $-\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$= \cos\left(\pi - \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

$\Rightarrow \cos\left(\frac{3\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

The range of principal value of $\cos^{-1}$ is $[0,\pi]$ and $\cos\left(\frac{3\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

Therefore, the value of $\cos^{-1}(\cos\left(\frac{5\pi}{4}\right))$ is $\frac{3\pi}{4}$.

2 C. Question

Evaluate each of the following:

$$\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$$

Answer

The value of $\cos\left(\frac{4\pi}{3}\right)$ is $\frac{-1}{2}$

Now,

∴ The question becomes $\cos^{-1}(\frac{-1}{2})$

Let $\cos^{-1}\left(\frac{-1}{2}\right) = y$

$\Rightarrow \cos y = \frac{-1}{2}$

$= -\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$= \cos\left(\pi - \frac{\pi}{3}\right) = \frac{-1}{2}$

$\Rightarrow \cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2}$

The range of principal value of $\cos^{-1}$ is $[0,\pi]$ and $\cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2}$

Therefore, the value of $\cos^{-1}(\cos\left(\frac{4\pi}{3}\right))$ is $\frac{2\pi}{3}$.

2 D. Question

Evaluate each of the following:

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$$

Answer

The value of $\cos\left(\frac{13\pi}{6}\right)$ is $\frac{\sqrt{3}}{2}$

Now,

∴ The question becomes $\cos^{-1}(\frac{\sqrt{3}}{2})$

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$

$\Rightarrow \cos y = \frac{\sqrt{3}}{2}$

$= \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

The range of principal value of $\cos^{-1}$ is $[0,\pi]$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Therefore, the value of $\cos^{-1}(\cos\left(\frac{13\pi}{6}\right))$ is $\frac{\pi}{6}$.

2 E. Question

Evaluate each of the following:

$\cos^{-1}(\cos 3)$

Answer

As $\cos^{-1}(\cos x) = x$

Provided $x \in [0,\pi]$

$\therefore$ we can write $\cos^{-1}(\cos 3)$ as 3.

2 F. Question

Evaluate each of the following:

$\cos^{-1}(\cos 4)$

Answer

$\cos^{-1}(\cos x) = x$

Provided $x \in [0,\pi] \approx [0,3.14]$

And in our equation x is 4 which does not lie in the above range.

We know $\cos[2\pi - x] = \cos[x]$

$\therefore \cos(2\pi - 4) = \cos(4)$

Also $2\pi-4$ belongs in $[0,\pi]$

$\therefore \cos^{-1}(\cos 4) = 2\pi-4$

2 G. Question

Evaluate each of the following:

$\cos^{-1}(\cos 5)$

Answer

$\cos^{-1}(\cos x) = x$

Provided $x \in [0,\pi] \approx [0,3.14]$

And in our equation x is 5 which does not lie in the above range.

We know $\cos[2\pi - x] = \cos[x]$

$\therefore \cos(2\pi - 5) = \cos(5)$

Also $2\pi-5$ belongs in $[0,\pi]$

$\therefore \cos^{-1}(\cos 5) = 2\pi-5$

2 H. Question

Evaluate each of the following:

$\cos^{-1}(\cos 12)$

Answer

$\cos^{-1}(\cos x) = x$

Provided x ϵ [0,π] ≈ [0,3.14]

And in our equation x is 4 which does not lie in the above range.

We know cos[2nπ – x] = cos[x]

∴ cos(2nπ – 12) = cos(12)

Here n = 2.

Also 4π–12 belongs in [0,π]

∴ $\cos^{-1}(\cos 12) = 4\pi - 12$

3 A. Question

Evaluate each of the following:

$$\tan^{-1}\left(\tan\frac{\pi}{3}\right)$$

Answer

As, $\tan^{-1}(\tan x) = x$

Provided x ϵ $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow \tan^{-1}(\tan\frac{\pi}{3})$

$= \frac{\pi}{3}$

3 B. Question

Evaluate each of the following:

$$\tan^{-1}\left(\tan\frac{6\pi}{7}\right)$$

Answer

Tan$\frac{6\pi}{7}$ can be written as $\tan\left(\pi - \frac{\pi}{7}\right)$

$\tan\left(\pi - \frac{\pi}{7}\right) = -\tan\frac{\pi}{7}$

∴ As, $\tan^{-1}(\tan x) = x$

Provided x ϵ $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\tan^{-1}(\tan\frac{6\pi}{7}) = -\frac{\pi}{7}$

3 C. Question

Evaluate each of the following:

$$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$$

Answer

The value of $\tan\frac{7\pi}{6} = \frac{1}{\sqrt{3}}$

∴ The question becomes $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Let,

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = y$

$\Rightarrow \tan y = \left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \tan\left(\frac{\pi}{6}\right) = \left(\frac{1}{\sqrt{3}}\right)$

The range of the principal value of $\tan^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(\frac{\pi}{6}\right) = \left(\frac{1}{\sqrt{3}}\right)$.

∴ The value of $\tan^{-1}(\tan\frac{7\pi}{6})$ is $\frac{\pi}{6}$.

3 D. Question

Evaluate each of the following:

$$\tan^{-1}\left(\tan\frac{9\pi}{4}\right)$$

Answer

The value of $\tan\frac{9\pi}{4} = 1$

∴ The question becomes $\tan^{-1}1$

Let,

$\tan^{-1}1 = y$

$\Rightarrow \tan y = 1$

$\Rightarrow \tan\left(\frac{\pi}{4}\right) = 1$

The range of the principal value of $\tan^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(\frac{\pi}{4}\right) = 1$.

∴ The value of $\tan^{-1}(\tan\frac{9\pi}{4})$ is $\frac{\pi}{4}$.

3 E. Question

Evaluate each of the following:

$\tan^{-1}(\tan 1)$

Answer

As, $\tan^{-1}(\tan x) = x$

Provided $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow \tan^{-1}(\tan 1)$

$= 1$

3 F. Question

Evaluate each of the following:

$\tan^{-1}$ (tan 2)

Answer

As, $\tan^{-1}(\tan x) = x$

Provided x ϵ $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Here our x is 2 which does not belong to our range

We know tan(π –θ) = –tan(θ)

∴ tan(θ –π) = tan(θ)

∴ tan(2–π) = tan(2)

Now 2–π is in the given range

∴ $\tan^{-1}$ (tan 2) = 2–π

3 G. Question

Evaluate each of the following:

$\tan^{-1}$ (tan 4)

Answer

As, $\tan^{-1}(\tan x) = x$

Provided x ϵ $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Here our x is 4 which does not belong to our range

We know tan(π –θ) = –tan(θ)

∴ tan(θ –π) = tan(θ)

∴ tan(4–π) = tan(4)

Now 4–π is in the given range

∴ $\tan^{-1}$ (tan 4) = 4–π

3 H. Question

Evaluate each of the following:

$\tan^{-1}$ (tan 12)

Answer

As, $\tan^{-1}(\tan x) = x$

Provided x ϵ $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Here our x is 12 which does not belong to our range

We know tan(nπ –θ) = –tan(θ)

∴ tan(θ –2nπ) = tan(θ)

Here n = 4

∴ tan(12–4π) = tan(12)

Now 12–4π is in the given range

∴ $\tan^{-1}$ (tan 12) = 12–4π.

4 A. Question

Evaluate each of the following:

$$\sec^{-1}\left(\sec\frac{\pi}{3}\right)$$

Answer

As $\sec^{-1}(\sec x) = x$

Provided $x \in [0,\pi]-\left\{\frac{\pi}{2}\right\}$

∴ we can write $\sec^{-1}\sec\left(\frac{\pi}{3}\right)$ as $\frac{\pi}{3}$.

4 B. Question

Evaluate each of the following:

$$\sec^{-1}\left(\sec\frac{2\pi}{3}\right)$$

Answer

As $\sec^{-1}(\sec x) = x$

Provided $x \in [0,\pi]-\left\{\frac{\pi}{2}\right\}$

∴ we can write $\sec^{-1}\sec\left(\frac{2\pi}{3}\right)$ as $\frac{2\pi}{3}$.

4 C. Question

Evaluate each of the following:

$$\sec^{-1}\left(\sec\frac{5\pi}{4}\right)$$

Answer

The value of $\sec\left(\frac{5\pi}{4}\right)$ is $-\sqrt{2}$.

∴ The question becomes $\sec^{-1}(-\sqrt{2})$.

Let $\sec^{-1}(-\sqrt{2}) = y$

$\Rightarrow \sec y = -\sqrt{2}$

$= -\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

$= \sec\left(\pi - \frac{\pi}{4}\right)$

$= \sec\left(\frac{3\pi}{4}\right)$

The range of principal value of $\sec^{-1}$is $[0, \pi]-\{\frac{\pi}{2}\}$

and $\sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$

∴ The principal value of $\sec^{-1}(-\sqrt{2})$ is $\frac{3\pi}{4}$.

4 D. Question

Evaluate each of the following:

$$\sec^{-1}\left(\sec\frac{7\pi}{3}\right)$$

Answer

The value of $\sec\left(\frac{7\pi}{3}\right)$ is 2

Let $\sec^{-1}(2) = y$

$\Rightarrow \sec y = 2$

$\Rightarrow \sec(\frac{\pi}{3})$

The range of principal value of $\sec^{-1}$is $[0, \pi]-\{\frac{\pi}{2}\}$

And $\sec\left(\frac{\pi}{3}\right) = 2$

$\therefore$ The principal value of $\sec^{-1}(\sec\left(\frac{7\pi}{3}\right))$ is $\frac{\pi}{3}$.

4 E. Question

Evaluate each of the following:

$$\sec^{-1}\left(\sec\frac{9\pi}{5}\right)$$

Answer

$\sec\left(\frac{9\pi}{5}\right)$ can be written as $\sec\left(2\pi - \frac{\pi}{5}\right)$

Also, we know $\sec(2\pi - \theta) = \sec(\theta)$

$\therefore \sec\left(2\pi - \frac{\pi}{5}\right) = \sec\left(\frac{\pi}{5}\right)$

$\therefore$ Now the given equation can be written as $\sec^{-1}\sec\left(\frac{\pi}{5}\right)$

As $\sec^{-1}(\sec x) = x$

Provided $x \in [0,\pi]-\left\{\frac{\pi}{2}\right\}$

$\therefore$ we can write $\sec^{-1}\sec\left(\frac{\pi}{5}\right)$ as $\frac{\pi}{5}$.

4 F. Question

Evaluate each of the following:

$$\sec^{-1}\left\{\sec\left(-\frac{7\pi}{3}\right)\right\}$$

Answer

As $\sec(-\theta)$ is $\sec(\theta)$

$\therefore \sec\left(\frac{-7\pi}{3}\right) = \sec\left(\frac{7\pi}{3}\right)$

The value of $\sec\left(\frac{7\pi}{3}\right)$ is 2.

Let $\sec^{-1}(2) = y$

⇒ sec y = 2

⇒ $\sec(\frac{\pi}{3})$

The range of principal value of $\sec^{-1}$is $[0, \pi]-\{\frac{\pi}{2}\}$

And $\sec\left(\frac{\pi}{3}\right) = 2$

∴ The value of $\sec^{-1}(\sec\left(\frac{-7\pi}{3}\right))$ is $\frac{\pi}{3}$.

4 G. Question

Evaluate each of the following:

$$\sec^{-1}\left\{\sec\left(-\frac{13\pi}{4}\right)\right\}$$

Answer

As sec(–θ) is sec(θ)

∴ $\sec\left(\frac{-7\pi}{3}\right) = \sec\left(\frac{7\pi}{3}\right)$

The value of $\sec\left(\frac{-13\pi}{4}\right)$ is $-\sqrt{2}$.

Let $\sec^{-1}(-\sqrt{2}) = y$

⇒ sec y = $-\sqrt{2}$

$= - \sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

$= \sec\left(\pi - \frac{\pi}{4}\right)$

$= \sec\left(\frac{3\pi}{4}\right)$

The range of principal value of $\sec^{-1}$is $[0, \pi]-\{\frac{\pi}{2}\}$

and $\sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$.

Therefore, the value of $\sec^{-1}\sec\left(\frac{-13\pi}{4}\right)$ is $\frac{3\pi}{4}$.

4 H. Question

Evaluate each of the following:

$$\sec^{-1}\left(\sec\frac{25\pi}{6}\right)$$

Answer

$\sec\left(\frac{25\pi}{6}\right) = \left(\frac{2}{\sqrt{3}}\right)$

∴ The question converts to $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Now,

Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = z$

$\Rightarrow \sec z = \left(\frac{2}{\sqrt{3}}\right)$

$= \sec\left(\frac{\pi}{6}\right) = \left(\frac{2}{\sqrt{3}}\right)$

The range of principal value of $\sec^{-1}$is $[0, \pi]-\{\frac{\pi}{2}\}$

and $\sec\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$

Therefore, the value of $\sec^{-1}\sec\left(\frac{25\pi}{6}\right)$ is $\frac{\pi}{3}$.

5 A. Question

Evaluate each of the following:

$$\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{\pi}{4}\right)$$

Answer

$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x$

Provided $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]-\{0\}$

$\therefore$ we can write $\operatorname{cosec}^{-1}(\operatorname{cosec}\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$.

5 B. Question

Evaluate each of the following:

$$\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{3\pi}{4}\right)$$

Answer

$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x$

Provided $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]-\{0\}$

$\therefore$ we can write $\operatorname{cosec}^{-1}(\operatorname{cosec}\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4}$.

5 C. Question

Evaluate each of the following:

$$\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{6\pi}{5}\right)$$

Answer

$\operatorname{cosec}\left(\frac{6\pi}{5}\right)$ can be written as $\operatorname{cosec}\left(\pi + \frac{\pi}{5}\right)$

$\operatorname{cosec}\left(\pi + \frac{\pi}{5}\right) = -\operatorname{cosec}\left(\frac{\pi}{5}\right)$

Also,

$-\operatorname{cosec}(\theta) = \operatorname{cosec}(-\theta)$

$\Rightarrow -\text{cosec}\left(\frac{\pi}{5}\right) = \text{cosec}\left(\frac{-\pi}{5}\right)$

Now the question becomes $\text{cosec}^{-1}(\text{cosec}\left(\frac{-\pi}{5}\right))$

$\text{cosec}^{-1}(\text{cosec } x) = x$

Provided $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

$\therefore$ we can write $\text{cosec}^{-1}(\text{cosec}\left(\frac{-\pi}{5}\right) = \frac{-\pi}{5}$.

5 D. Question

Evaluate each of the following:

$$\text{cosec}^{-1}\left(\text{cosec}\frac{11\pi}{6}\right)$$

Answer

The value of $\text{cosec}\left(\frac{11\pi}{6}\right) = -2$.

Let,

$\text{cosec}^{-1}{-2} = y$

$\Rightarrow \text{cosec } y = -2$

$\Rightarrow -\text{cosec } y = 2$

$\Rightarrow -\text{cosec } \frac{\pi}{6} = 2$

As we know $\text{cosec}(-\theta) = -\text{cosec}\theta$

$\therefore -\text{cosec } \frac{\pi}{6} = \text{cosec } \left(\frac{-\pi}{6}\right)$

The range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and

$\text{cosec } \left(\frac{-\pi}{6}\right) = -2$

Therefore, the value of $\text{cosec}^{-1}(\text{cosec}\left(\frac{11\pi}{6}\right))$ is $\frac{-\pi}{6}$.

5 E. Question

Evaluate each of the following:

$$\text{cosec}^{-1}\left(\text{cosec}\frac{13\pi}{6}\right)$$

Answer

The value of $\text{cosec}\left(\frac{13\pi}{6}\right)$ is 2.

$\therefore$ The question becomes $\text{cosec}^{-1}(2)$

Let,

$\text{cosec}^{-1}(2) = y$

$\therefore \text{cosec } y = 2$

$\Rightarrow \text{cosec}\left(\frac{\pi}{6}\right) = 2$

The range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$-{0} and

$\text{cosec}\left(\frac{\pi}{6}\right) = 2$

Therefore, the value of $\text{cosec}^{-1}(\text{cosec}\left(\frac{13\pi}{6}\right))$ is $\frac{\pi}{6}$.

5 F. Question

Evaluate each of the following:

$$\text{cosec}^{-1}\left\{\text{cosec}\left(-\frac{9\pi}{4}\right)\right\}$$

Answer

As we know cosec(–θ) = –cosecθ

$\therefore \text{cosec}\left(\frac{-9\pi}{4}\right) = -\text{cosec}\left(\frac{9\pi}{4}\right)$

$-\text{cosec}\left(\frac{9\pi}{4}\right)$ can be written as $-\text{cosec}\left(2\pi + \frac{\pi}{4}\right)$

Also,

cosec(2π+θ) = cosecθ

$\therefore -\text{cosec}\left(2\pi + \frac{\pi}{4}\right) = -\text{cosec}\left(\frac{\pi}{4}\right)$

As we know –cosec(θ) = cosec(–θ)

$\therefore -\text{cosec}\left(\frac{\pi}{4}\right) = \text{cosec}\left(\frac{-\pi}{4}\right)$

Now the question becomes $\text{cosec}^{-1}(\text{cosec}\left(\frac{-\pi}{4}\right))$

$\text{cosec}^{-1}(\text{cosec } x) = x$

Provided $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$-{0}

$\therefore$ we can write $\text{cosec}^{-1}(\text{cosec}\left(\frac{-\pi}{4}\right) = \frac{-\pi}{4}$.

6 A. Question

Evaluate each of the following:

$$\cot^{-1}\left(\cot\frac{\pi}{3}\right)$$

Answer

$\cot^{-1}(\cot x) = x$

Provided $x \in (0,\pi)$

$\therefore \cot^{-1}(\cot\frac{\pi}{3}) = \frac{\pi}{3}$.

6 B. Question

Evaluate each of the following:

$$\cot^{-1}\left(\cot\frac{4\pi}{3}\right)$$

Answer

$\cot\frac{4\pi}{3}$ can be written as $\cot\left(\pi+\frac{\pi}{3}\right)$

we know cot(π +θ) = cot(θ)

$\therefore \cot\left(\pi+\frac{\pi}{3}\right) = \cot\left(\frac{\pi}{3}\right)$

Now the question becomes $\cot^{-1}(\cot\frac{\pi}{3})$

$\cot^{-1}(\cot x) = x$

Provided x ϵ (0,π)

$\therefore \cot^{-1}(\cot\frac{4\pi}{3}) = \frac{\pi}{3}$.

6 C. Question

Evaluate each of the following:

$$\cot^{-1}\left(\cot\frac{9\pi}{4}\right)$$

Answer

The value of $\cot\frac{9\pi}{4}$ is 1.

∴ The question becomes $\cot^{-1}(1)$.

Let $\cot^{-1}(1) = y$

⇒ cot y = 1

$= \cot\left(\frac{\pi}{4}\right) = 1$

The range of principal value of $\cot^{-1}$is (0, π)

and $\cot\left(\frac{\pi}{4}\right) = 1$

∴ The value of $\cot^{-1}(\cot\frac{9\pi}{4})$ is $\frac{\pi}{4}$.

6 D. Question

Evaluate each of the following:

$$\cot^{-1}\left(\cot\frac{19\pi}{6}\right)$$

Answer

The value of $\cot\frac{19\pi}{6}$ is √3.

∴ The question becomes $\cot^{-1}(\sqrt{3})$.

Let $\cot^{-1}(\sqrt{3}) = y$

⇒ cot y = √3

$= \cot\left(\frac{\pi}{6}\right) = \sqrt{3}$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

and $\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$

∴ The principal value of $\cot^{-1}(\cot\frac{19\pi}{6})$ is $\frac{\pi}{6}$.

6 E. Question

Evaluate each of the following:

$$\cot^{-1}\left\{\cot\left(-\frac{8\pi}{3}\right)\right\}$$

Answer

$\cot(-\theta)$ is $-\cot(\theta)$

∴ The equation given above becomes $\cot^{-1}(-\cot\frac{8\pi}{3})$

$\cot\frac{8\pi}{3} = \frac{-1}{\sqrt{3}}$.

Therefore

Let $\cot^{-1}(\frac{1}{\sqrt{3}}) = y$

$\Rightarrow \cot y = \frac{1}{\sqrt{3}}$

$= \cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

and $\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$

∴ The value of $\cot^{-1}(\cot\frac{-8\pi}{3})$ is $\frac{\pi}{3}$.

6 F. Question

Evaluate each of the following:

$$\cot^{-1}\left\{\cot\left(-\frac{21\pi}{4}\right)\right\}$$

Answer

$\cot(-\theta)$ is $-\cot(\theta)$

∴ The equation given above becomes $\cot^{-1}(-\cot\frac{21\pi}{4})$

$\cot\frac{21\pi}{4} = 1$.

$\Rightarrow -\cot\frac{21\pi}{4} = -1$.

∴ we get $\cot^{-1}(-1)$

Let $\cot^{-1}(-1) = y$

$\Rightarrow \cot y = -1$

$= - \cot\left(\frac{\pi}{4}\right) = 1$

$= \cot\left(\pi - \frac{\pi}{4}\right)$

$= \cot\left(\frac{3\pi}{4}\right)$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

and $\cot\left(\frac{3\pi}{4}\right) = -1$

∴ The value of $\cot^{-1}(\cot\frac{-21\pi}{4})$ is $\frac{3\pi}{4}$.

7 A. Question

Write each of the following in the simplest form:

$$\cot^{-1}\left\{\frac{a}{\sqrt{x^2 - a^2}}\right\}, |x| > a$$

Answer

Let us assume $x = a \sec\theta$

$\theta = \sec^{-1}\frac{x}{a}$ ...(1)

∴ we can write

$\cot^{-1}\left\{\frac{a}{\sqrt{a^2 \sec^2\theta - a^2}}\right\}$

$= \cot^{-1}\left\{\frac{a}{\sqrt{a^2(\sec^2\theta - 1)}}\right\}$

$= \cot^{-1}\left\{\frac{a}{\sqrt{a^2 \tan^2\theta}}\right\}$

$= \cot^{-1}\left\{\frac{a}{a\tan\theta}\right\}$

$= \cot^{-1}\left\{\frac{1}{\tan\theta}\right\}$

$= \cot^{-1}(\cot\theta)$

$= \theta$.

From 1 we get the given equation simplification to $\sec^{-1}\frac{x}{a}$.

7 B. Question

Write each of the following in the simplest form:

$$\tan^{-1}\left\{x + \sqrt{1 + x^2}\right\}, x \in R$$

Answer

Put $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1}(x)$

$\tan^{-1}\{\tan\theta + \sqrt{1 + \tan^2\theta}\}$

$= \tan^{-1}\{\tan\theta + \sqrt{\sec^2\theta}\}$

$= \tan^{-1}\{\tan\theta + \sec\theta\}$

$= \tan^{-1}\left\{\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right\}$

$= \tan^{-1}\left\{\frac{1+\sin\theta}{\cos\theta}\right\}$

$\text{Sin}\theta = 2 \times \sin\frac{\theta}{2} \times \cos\frac{\theta}{2}$,$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

$= \tan^{-1}\left\{\frac{2\times\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}+\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}\right\}$

$= \tan^{-1}\left\{\frac{\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)^2}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\times\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}\right\}$

$= \tan^{-1}\left\{\frac{\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right\}$

Dividing by $\cos\frac{\theta}{2}$ we get,

$= \tan^{-1}\left\{\frac{\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}+\frac{\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)}{\left(-\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}+\frac{\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)}\right\}$

$= \tan^{-1}\left(\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}\right)$

$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\frac{\theta}{2}}{1-\tan\frac{\pi}{4}\tan\frac{\theta}{2}}\right)$

$\tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y}$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right)$

$= \frac{\pi}{4}+\frac{\theta}{2}$

From 1 we get

$= \frac{\pi}{4}+\frac{\tan^{-1}x}{2}$.

Therefore, the simplification of given equation is $\frac{\pi}{4}+\frac{\tan^{-1}x}{2}$.

7 C. Question

Write each of the following in the simplest form:

$\tan^{-1}\left\{\sqrt{1+x^2}-x\right\}, x \in R$

Answer

Put x = tanθ

$\Rightarrow \theta = \tan^{-1}(x)$

$\tan^{-1}\{\sqrt{1+\tan^2\theta} - \tan\theta\}$

$= \tan^{-1}\{\sqrt{\sec^2\theta}-\tan\theta\}$

$= \tan^{-1}\{\sec\theta - \tan\theta\}$

$= \tan^{-1}\left\{\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right\}$

$= \tan^{-1}\left\{\frac{1-\sin\theta}{\cos\theta}\right\}$

$\sin\theta = 2\times\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}$, $\cos\theta = \cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}$

$= \tan^{-1}\left\{\frac{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}-2\times\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}\right\}$

$= \tan^{-1}\left\{\frac{\left(\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\right)^2}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\times\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}\right\}$

$= \tan^{-1}\left\{\frac{\left(\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\right)}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right\}$

Dividing by $\cos\frac{\theta}{2}$ we get

$= \tan^{-1}\left\{\frac{\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}-\frac{\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)}{\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}+\frac{\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)}\right\}$

$= \tan^{-1}\left(\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}\right)$

$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan\frac{\theta}{2}}{1+\tan\frac{\pi}{4}\tan\frac{\theta}{2}}\right)$

$\tan(x-y) = \frac{\tan x-\tan y}{1+\tan x\tan y}$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right)$

$= \frac{\pi}{4}-\frac{\theta}{2}$

From 1 we get

$= \frac{\pi}{4}-\frac{\tan^{-1}x}{2}$.

Therefore, the simplification of given equation is $\frac{\pi}{4}-\frac{\tan^{-1}x}{2}$.

7 D. Question

Write each of the following in the simplest form:

$$\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}, x \neq 0$$

Answer

Assume $x = \tan\theta$

$= \tan^{-1}\left\{\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right\}$

$= \tan^{-1}\left\{\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right\}$

$= \tan^{-1}\left\{\frac{\sec\theta-1}{\tan\theta}\right\}$

$= \tan^{-1}\left\{\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right\}$

$= \tan^{-1}\left\{\frac{1-\cos\theta}{\sin\theta}\right\}$

Cos θ = 1 – 2 $\sin^2\frac{\theta}{2}$ and sinθ = $2 \times \sin\frac{\theta}{2} \times \cos\frac{\theta}{2}$

⇒ 1 – cosθ = 2 $\sin^2\frac{\theta}{2}$

$= \tan^{-1}\left\{\frac{2\sin^2\frac{\theta}{2}}{2\times\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}}\right\}$

$= \tan^{-1}\left\{\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right\}$

$= \tan^{-1}(\tan\frac{\theta}{2})$

$= \frac{\theta}{2}$

But θ = $\tan^{-1}$x.

$\therefore \frac{\theta}{2} = \frac{\tan^{-1}x}{2}$

Therefore, the simplification of given equation is $\frac{\tan^{-1}x}{2}$

7 E. Question

Write each of the following in the simplest form:

$$\tan^{-1}\left\{\frac{\sqrt{1+x^2}+1}{x}\right\}, x \neq 0$$

Answer

Assume x = tanθ

$= \tan^{-1}\left\{\frac{\sqrt{1+\tan^2\theta}+1}{\tan\theta}\right\}$

$= \tan^{-1}\left\{\frac{\sqrt{\sec^2\theta}+1}{\tan\theta}\right\}$

$= \tan^{-1}\left\{\frac{\sec\theta+1}{\tan\theta}\right\}$

$= \tan^{-1}\left\{\frac{\frac{1}{\cos\theta}+1}{\frac{\sin\theta}{\cos\theta}}\right\}$

$= \tan^{-1}\left\{\frac{1+\cos\theta}{\sin\theta}\right\}$

$\cos\theta = 2\cos^2\frac{\theta}{2} - 1$ and $\sin\theta = 2 \times \sin\frac{\theta}{2} \times \cos\frac{\theta}{2}$

$\Rightarrow 1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

$= \tan^{-1}\left\{\frac{2\cos^2\frac{\theta}{2}}{2\times\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}}\right\}$

$= \tan^{-1}\left\{\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}\right\}$

$= \tan^{-1}(\cot\frac{\theta}{2})$

$\cot\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \tan\left(-\frac{\theta}{2}\right)$

$= \tan^{-1}(\tan\frac{-\theta}{2})$

But $\theta = \tan^{-1}x$.

$\therefore \frac{-\theta}{2} = \frac{-\tan^{-1}x}{2}$

Therefore, the simplification of given equation is $\frac{-\tan^{-1}x}{2}$

7 F. Question

Write each of the following in the simplest form:

$\tan^{-1}\sqrt{\frac{a-x}{a+x}}, -a < x < a$

Answer

Put $x = a\cos\theta$

$\Rightarrow \tan^{-1}\sqrt{\frac{a-a\cos\theta}{a+a\cos\theta}}$

$\Rightarrow \tan^{-1}\sqrt{\frac{a(1-\cos\theta)}{a(1+\cos\theta)}}$

$\Rightarrow \tan^{-1}\sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)}}$

Rationalising it

$\tan^{-1}\sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)}} \times \sqrt{\frac{(1-\cos\theta)}{(1-\cos\theta)}}$

$\Rightarrow \tan^{-1}\sqrt{\frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}}$

$\Rightarrow \tan^{-1}\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

$\Rightarrow \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$ and $\sin\theta = 2 \times \sin\frac{\theta}{2} \times \cos\frac{\theta}{2}$

$\Rightarrow 1 - \cos\theta = 2\sin^2\frac{\theta}{2}$

$= \tan^{-1}\left\{\frac{2\sin^2\frac{\theta}{2}}{2\times\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}}\right\}$

$= \tan^{-1}\left\{\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right\}$

$= \tan^{-1}(\tan\frac{\theta}{2})$

$= \frac{\theta}{2}$

But $\theta = \cos^{-1}\frac{x}{a}$

∴ The given equation simplification to $\cos^{-1}\frac{x}{a}$.

7 G. Question

Write each of the following in the simplest form:

$$\tan^{-1}\left\{\frac{x}{a+\sqrt{a^2-x^2}}\right\}, -a < x < a$$

Answer

Assume x = a sinθ

$= \tan^{-1}\left\{\frac{a\sin\theta}{a+\sqrt{a^2-a^2\sin^2\theta}}\right\}$

$= \tan^{-1}\left\{\frac{a\sin\theta}{a+\sqrt{a^2(1-\sin^2\theta)}}\right\}$

$= \tan^{-1}\left\{\frac{a\sin\theta}{a+\sqrt{a^2(\cos^2\theta)}}\right\}$

$= \tan^{-1}\left\{\frac{a\sin\theta}{a+a\cos\theta}\right\}$

$= \tan^{-1}\left\{\frac{\sin\theta}{1+\cos\theta}\right\}$

$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$ and $\sin\theta = 2\times\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}$,$\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} = 1$

$= \tan^{-1}\left\{\frac{2\times\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}\right\}$

$= \tan^{-1}\left\{\frac{2\times\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right\}$

$= \tan^{-1}\left\{\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right\}$

$= \tan^{-1}(\tan\frac{\theta}{2})$

$= \frac{\theta}{2}$

But $\theta = \sin^{-1}\left(\frac{x}{a}\right)$

∴ The given equation simplification to $\sin^{-1}\left(\frac{x}{a}\right)$.

7 H. Question

Write each of the following in the simplest form:

$$\sin^{-1}\left\{\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right\}, \frac{1}{2}<x<\frac{1}{\sqrt{2}}$$

Answer

Assume x = sinθ

$= \sin^{-1}\left\{\frac{\sin\theta+\sqrt{1-\sin^2\theta}}{\sqrt{2}}\right\}$

$= \sin^{-1}\left\{\frac{\sin\theta+\cos\theta}{\sqrt{2}}\right\}$

$= \sin^{-1}\left\{\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta\right\}$

$= \sin^{-1}\left\{\cos\frac{\pi}{4}\sin\theta+\sin\frac{\pi}{4}\cos\theta\right\}$

sin(A+B) = sinAcosB+cosAsinB

∴ The above expression can be written as

$= \sin^{-1}\left\{\sin\frac{\pi}{4}+\theta\right\}$

$= \frac{\pi}{4}+\theta$

But $\theta = \sin^{-1}x$

∴ the above expression becomes $\frac{\pi}{4}+\sin^{-1}x$.

The given equation simplification to $\frac{\pi}{4}+\sin^{-1}x$.

7 I. Question

Write each of the following in the simplest form:

$$\sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}, 0<x<1$$

Answer

Put x = sin2θ

And we know $\sin^2\theta+\cos^2\theta = 1$

By putting these in above equation, we get

$= \sin^{-1}\left\{\frac{\sqrt{\sin^2\theta+\cos^2\theta+\sin 2\theta}+\sqrt{\sqrt{\sin^2\theta+\cos^2\theta-\sin 2\theta}}}{2}\right\}$

$= \sin^{-1}\left\{\frac{\sqrt{(\sin\theta+\cos\theta)^2}+\sqrt{(\sin\theta-\cos\theta)^2}}{2}\right\}$

$= \sin^{-1}\left\{\frac{\sin\theta+\cos\theta+\sin\theta-\cos\theta}{2}\right\}$

$= \sin^{-1}\left\{\frac{2\sin\theta}{2}\right\}$

$= \sin^{-1}(\sin\theta)$

$= \theta$

But $\theta = \frac{1}{2}\sin^{-1}x$

∴ The given equation simplification to $\frac{1}{2}\sin^{-1}x$.

7 J. Question

Write each of the following in the simplest form:

$$\sin^{-1}\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$$

Answer

Put $x = \cos\theta$

$= \sin^{-1}\left(2\tan^{-1}\left(\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)\right)$

$1 - \cos\theta = 2\sin^2\frac{\theta}{2}$ and $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

$= \sin^{-1}\left(2\tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\right)\right)$

$= \sin^{-1}\left(2\tan^{-1}\left(\sqrt{\tan^2\frac{\theta}{2}}\right)\right)$

$= \sin^{-1}(2\tan^{-1}(\tan\frac{\theta}{2}))$

$= \sin^{-1}(2 \times \frac{\theta}{2})$

$= \sin^{-1}(\theta)$

But $\theta = \cos^{-1}x$

∴ The above expression becomes $\sin^{-1}(\cos^{-1}x)$

Exercise 4.8

1 A. Question

Evaluate each of the following

$$\sin\left(\sin^{-1}\frac{7}{25}\right)$$

Answer

Let $\sin^{-1}\frac{7}{25} = y$

$\Rightarrow \sin y = \frac{7}{25}$

Where $y \in \left[0, \frac{\pi}{2}\right]$

⇒ $\sin\left(\sin^{-1}\frac{7}{25}\right)=\frac{7}{25}$ substituting $y=\sin^{-1}\frac{7}{25}$

1 B. Question

Evaluate each of the following

$\sin\left(\cos^{-1}\frac{5}{13}\right)$

Answer

Let $\cos^{-1}\frac{5}{13}=y$

⇒ $\cos y=\frac{5}{13}$ where $y\in\left[0,\frac{\pi}{2}\right]$

To find : $\sin\left(\cos^{-1}\frac{5}{13}\right)=\sin y$

As $\sin^2\theta + \cos^2\theta = 1$

⇒ $\sin y=\pm\sqrt{1-\cos^2 y}$

As $y\in\left[0,\frac{\pi}{2}\right]$

⇒ $\sin y=\sqrt{1-\cos^2 y}$

⇒ $\sin y=\sqrt{1-\left(\frac{5}{13}\right)^2}$

⇒ $\sin y=\sqrt{1-\frac{25}{169}}$

⇒ $\sin y=\sqrt{\frac{144}{169}}$

⇒ $\sin y=\frac{12}{13}$

⇒ $\sin\left[\cos^{-1}\left(\frac{5}{13}\right)\right]=\frac{12}{13}$

1 C. Question

Evaluate each of the following

$\sin\left(\tan^{-1}\frac{24}{7}\right)$

Answer

Let $\tan^{-1}\frac{24}{7}=y$

⇒ $\tan y=\frac{24}{7}$ where $y\in\left[0,\frac{\pi}{2}\right]$

To find : $\sin\left(\tan^{-1}\frac{24}{7}\right)=\sin y$

As $1+\cot^2\theta=\text{cosec}^2\theta$

⇒ $1+\cot^2 y=\text{cosec}^2 y$

Putting values

⇒ $1+\left(\frac{7}{24}\right)^2=\text{cosec}^2 y$

⇒ $1+\frac{49}{576}=\frac{1}{\sin^2 y}$

⇒ $\sin^2 y=\frac{576}{625}$

⇒ $\sin y=\frac{24}{25}\ \because\ y\in\left[0,\frac{\pi}{2}\right]$

⇒ $\sin\left(\tan^{-1}\frac{24}{7}\right)=\frac{24}{25}$

1 D. Question

Evaluate each of the following

$\sin\left(\sec^{-1}\frac{17}{8}\right)$

Answer

Let $\sec^{-1}\frac{17}{8}=y$

⇒ $\sec y=\frac{17}{8}$ where $y\in\left[0,\frac{\pi}{2}\right]$

To find : $\sin\left(\sec^{-1}\frac{17}{8}\right)=\sin y$

Now, $\cos y=\frac{1}{\sec y}$

⇒ $\cos y=\frac{8}{17}$

Now, $\sin y=\sqrt{1-\cos^2 y}$ where $y\in\left[0,\frac{\pi}{2}\right]$

$\Rightarrow \sin y = \sqrt{1-\left(\frac{8}{17}\right)^2}$

$\Rightarrow \sin y = \sqrt{\frac{225}{289}}$

$\Rightarrow \sin y = \frac{15}{17}$

$\Rightarrow \sin\left(\sec^{-1}\frac{17}{8}\right) = \frac{15}{17}$

1 E. Question

Evaluate each of the following

$\operatorname{cosec}\left(\cos^{-1}\frac{3}{5}\right)$

Answer

Let $\cos^{-1}\frac{3}{5} = y$

$\Rightarrow \cos y = \frac{3}{5}$ where $y \in \left[0, \frac{\pi}{2}\right]$

To find: $\operatorname{cosec}\left(\cos^{-1}\frac{3}{5}\right) = \operatorname{cosec} y$

As $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin y = \sqrt{1-\cos^2 y}$ where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \sin y = \sqrt{1-\left(\frac{3}{5}\right)^2}$

$\Rightarrow \sin y = \sqrt{\frac{16}{25}}$

$\Rightarrow \sin y = \frac{4}{5}$

$\Rightarrow \operatorname{cosec} y = \frac{5}{4}$

$\Rightarrow \operatorname{cosec}\left(\cos^{-1}\frac{3}{5}\right) = \frac{5}{4}$

1 F. Question

Evaluate each of the following

$\sec\left(\sin^{-1}\frac{12}{13}\right)$

Answer

Let $\sin^{-1}\frac{12}{13} = y$ where $y \in \left[0, \frac{\pi}{2}\right]$

⇒ $\sin y = \frac{12}{13}$

⇒ To find : $\sec\left(\sin^{-1}\frac{12}{13}\right) = \sec y$

As $\sin^2\theta + \cos^2\theta = 1$

⇒ $\cos y = \sqrt{1-\sin^2 y}$ where $y \in \left[0, \frac{\pi}{2}\right]$

⇒ $\cos y = \sqrt{1-\left(\frac{12}{13}\right)^2}$

⇒ $\cos y = \sqrt{1-\frac{144}{169}}$

⇒ $\cos y = \sqrt{\frac{25}{169}}$

⇒ $\cos y = \frac{5}{13}$

⇒ $\sec y = \frac{1}{\cos y}$

⇒ $\sec y = \frac{13}{5}$

⇒ $\sec\left(\sin^{-1}\frac{12}{13}\right) = \frac{13}{5}$

1 G. Question

Evaluate each of the following

$\tan\left(\cos^{-1}\frac{8}{17}\right)$

Answer

Let $\cos^{-1}\frac{8}{17} = y$ where $y \in \left[0, \frac{\pi}{2}\right]$

⇒ $\cos y = \frac{8}{17}$

To find: $\tan\left(\cos^{-1}\frac{8}{17}\right) = \tan y$

⇒ As $1+\tan^2\theta = \sec^2\theta$

⇒ $\tan y = \sqrt{\sec^2 y - 1}$ where $y \in \left[0, \frac{\pi}{2}\right]$

⇒ $\tan y = \sqrt{\left(\frac{1}{\cos^2 y}\right) - 1}$

⇒ $\tan y = \sqrt{\left(\frac{17}{8}\right)^2 - 1}$

⇒ $\tan y = \sqrt{\frac{289}{64} - 1}$

⇒ $\tan y = \sqrt{\frac{225}{64}}$

⇒ $\tan y = \frac{15}{8}$

⇒ $\tan\left(\cos^{-1}\frac{8}{17}\right) = \frac{15}{8}$

1 H. Question

Evaluate each of the following

$\cot\left(\cos^{-1}\frac{3}{5}\right)$

Answer

Let $\cos^{-1}\frac{3}{5} = y$ where $y \in \left[0, \frac{\pi}{2}\right]$

⇒ $\cos y = \frac{3}{5}$

To find: $\cot\left(\cos^{-1}\frac{3}{5}\right) = \cot y$

⇒ As $1+\tan^2\theta = \sec^2\theta$

⇒ $\tan y = \sqrt{\sec^2 y - 1}$ where $y \in \left[0, \frac{\pi}{2}\right]$

⇒ $\frac{1}{\cot y} = \sqrt{\left(\frac{1}{\cos^2 y}\right) - 1}$

$\Rightarrow \frac{1}{\cot y} = \sqrt{\left(\frac{5}{3}\right)^2 - 1}$

$\Rightarrow \frac{1}{\cot y} = \sqrt{\frac{16}{9}}$

$\Rightarrow \cot y = \frac{3}{4}$

$\Rightarrow \cot\left(\cos^{-1}\frac{3}{5}\right) = \frac{3}{4}$

1 I. Question

Evaluate each of the following

$\cos\left(\tan^{-1}\frac{24}{7}\right)$

Answer

Let $\tan^{-1}\frac{24}{7} = y$

$\Rightarrow \tan y = \frac{24}{7}$ where $y \in \left[0, \frac{\pi}{2}\right]$

To find: $\cos\left(\tan^{-1}\frac{24}{7}\right) = \cos y$

As $1+\tan^2\theta = \sec^2\theta$

$\Rightarrow 1 + \tan^2 y = \sec^2 y$

$\Rightarrow \sec y = \sqrt{1 + \tan^2 y}$ where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \sec y = \sqrt{1 + \left(\frac{24}{7}\right)^2}$

$\Rightarrow \sec y = \sqrt{\frac{625}{49}}$

$\Rightarrow \sec y = \frac{25}{7}$

$\Rightarrow \cos y = \frac{1}{\sec y}$

$\Rightarrow \cos y = \frac{7}{25}$

$\Rightarrow \cos\left(\tan^{-1}\frac{24}{7}\right) = \frac{7}{25}$

2 A. Question

Prove the following results:

$$\tan\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)=\frac{17}{6}$$

Answer

Let $\cos^{-1}\frac{4}{5}=x$ and $\tan^{-1}\frac{2}{3}=y$

⇒ $\cos x=\frac{4}{5}$ and $\tan y=\frac{2}{3}$

where $x, y \in \left[0,\frac{\pi}{2}\right]$

Now, LHS is reduced to : tan(x+y)

⇒ $\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x.\tan y}$ ..eq (i)..

As $\tan x=\sqrt{\sec^2 x-1}$ where $x\in\left[0,\frac{\pi}{2}\right]$

⇒ $\tan x=\sqrt{\frac{1}{\cos^2 x}-1}$

⇒ $\tan x=\sqrt{\left(\frac{5}{4}\right)^2-1}$

⇒ $\tan x=\sqrt{\frac{9}{16}}$

⇒ $\tan x=\frac{3}{4}$

Now putting the values of tan x and tan y in eq(i)

⇒ $\tan(x+y)=\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)}\right)$

⇒ $\tan(x+y)=\frac{17}{6}$

= RHS

2 B. Question

Prove the following results:

$$\cos\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)=\frac{6}{5\sqrt{13}}$$

Answer

Let $\sin^{-1}\frac{3}{5} = x$ and $\cot^{-1}\frac{3}{2} = y$

⇒ $\sin x = \frac{3}{5}$ and $\cot y = \frac{3}{2}$

where $x, y \in \left[0, \frac{\pi}{2}\right]$

Now, LHS is reduced to : cos(x+y)

⇒ $\cos(x+y) = \cos x.\cos y - \sin x.\sin y$ ..eq(i)

As $\cos x = \sqrt{1-\sin^2 x}$ where $x \in \left[0, \frac{\pi}{2}\right]$

⇒ $\cos x = \sqrt{1-\left(\frac{3}{5}\right)^2}$

⇒ $\cos x = \sqrt{1-\frac{9}{25}}$

⇒ $\cos x = \frac{4}{5}$

Also, $\operatorname{cosec} y = \sqrt{1+\cot^2 y}$ where $y \in \left[0, \frac{\pi}{2}\right]$

⇒ $\operatorname{cosec} y = \sqrt{1+\left(\frac{3}{2}\right)^2}$

⇒ $\operatorname{cosec} y = \frac{\sqrt{13}}{2}$

⇒ $\sin y = \frac{1}{\operatorname{cosec} y}$

⇒ $\sin y = \frac{2}{\sqrt{13}}$

⇒ $\cos y = \sqrt{1-\sin^2 y}$ where $y \in \left[0, \frac{\pi}{2}\right]$

⇒ $\cos y = \sqrt{1-\left(\frac{2}{\sqrt{13}}\right)^2}$

⇒ $\cos y = \frac{3}{\sqrt{13}}$

Putting the values in eq(i),

$\Rightarrow \cos(x+y) = \left(\frac{4}{5}\right)\left(\frac{3}{\sqrt{13}}\right) - \left(\frac{3}{5}\right)\left(\frac{2}{\sqrt{13}}\right)$

$\Rightarrow \cos(x+y) = \frac{6}{5\sqrt{13}}$

= RHS

2 C. Question

Prove the following results:

$\tan\left(\sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}\right) = \frac{63}{16}$

Answer

Let $\sin^{-1}\frac{5}{13} = x$ and $\cos^{-1}\frac{3}{5} = y$

$\Rightarrow \sin x = \frac{5}{13}$ and $\cos y = \frac{3}{5}$

where $x, y \in \left[0, \frac{\pi}{2}\right]$

Now, LHS is reduced to : tan(x+y)

$\Rightarrow \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x.\tan y}$ ..eq(i)

As $\cos x = \sqrt{1-\sin^2 x}$ where $x \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \cos x = \sqrt{1-\left(\frac{5}{13}\right)^2}$

$\Rightarrow \cos x = \frac{12}{13}$

Similarly,

$\sin y = \sqrt{1-\cos^2 y}$ where $x \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \sin y = \frac{4}{5}$

$\tan x = \frac{\sin x}{\cos x}$ and $\tan y = \frac{\sin y}{\cos y}$

$\Rightarrow \tan x = \frac{5}{12}$ and $\tan y = \frac{4}{3}$

Putting these values in eq(i)

$\Rightarrow \tan\left(x+y\right)=\dfrac{\dfrac{5}{12}+\dfrac{4}{3}}{1-\dfrac{5}{12}.\dfrac{4}{3}}$

$\Rightarrow \tan\left(x+y\right)=\dfrac{63}{16}$

= RHS

2 D. Question

Prove the following results:

$\sin\left(\cos^{-1}\dfrac{3}{5}+\sin^{-1}\dfrac{5}{13}\right)=\dfrac{63}{65}$

Answer

Let $\cos^{-1}\dfrac{3}{5}=x$ and $\sin^{-1}\dfrac{5}{13}=y$

$\Rightarrow \cos x=\dfrac{3}{5}$ and $\sin y=\dfrac{5}{13}$

where $x,y\in\left[0,\dfrac{\pi}{2}\right]$

Now, LHS is reduced to : sin(x+y)

$\Rightarrow \sin\left(x+y\right)=\sin x.\cos y+\cos x.\sin y$ ..eq(i)..

As $\sin^2 x+\cos^2 x=1$

$\Rightarrow \sin x=\sqrt{1-\cos^2 x}$ where $x\in\left[0,\dfrac{\pi}{2}\right]$

$\Rightarrow \sin x=\sqrt{1-\left(\dfrac{3}{5}\right)^2}$

$\Rightarrow \sin x=\dfrac{4}{5}$

Similarly,

$\cos y=\sqrt{1-\sin^2 y}$ where $x\in\left[0,\dfrac{\pi}{2}\right]$

$\Rightarrow \cos y=\dfrac{12}{13}$

Putting these values in eq(i)

$\Rightarrow \sin\left(x+y\right)=\dfrac{4}{5}.\dfrac{12}{13}+\dfrac{3}{5}.\dfrac{5}{13}$

$\Rightarrow \sin(x+y)=\frac{63}{65}$

= RHS

3. Question

Solve: $\cos(\sin^{-1}x)=\frac{1}{6}$

Answer

A. Let $\sin^{-1}x = y$

Where $y\in\left[0,\frac{\pi}{2}\right]$ because "cos y" is +ve

$\Rightarrow \sin y = x$

where "x" is +ve as $y\in\left[0,\frac{\pi}{2}\right]$

As $\sin^2 y+\cos^2 y = 1$

$\Rightarrow \cos y=\sqrt{1-\sin^2 y}$ where $y\in\left[0,\frac{\pi}{2}\right]$

$\Rightarrow \cos y=\sqrt{1-x^2}$

According to the question, $\cos(\sin^{-1}x)=\frac{1}{6}$

$\Rightarrow \cos y=\frac{1}{6}$

$\Rightarrow \sqrt{1-x^2}=\frac{1}{6}$

Squaring both sides,

$\Rightarrow 1-x^2=\frac{1}{36}$

$\Rightarrow x^2=\frac{35}{36}$

As $x > 0$

$\Rightarrow x=\frac{\sqrt{35}}{6}$

4. Question

Solve: $\cos\left[2\sin^{-1}(-x)\right]=0$

Answer

A. Let $\sin^{-1}(-x)=y$ where $y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\Rightarrow \sin y = -x$

According to question

$\Rightarrow \cos 2y = 0$

$\Rightarrow 1 - 2\sin^2 y = 0$

$\Rightarrow 1 - 2x^2 = 0$

$\Rightarrow x^2 = \frac{1}{2}$

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$

Exercise 4.9

1 A. Question

Evaluate:

$$\cos\left[\sin^{-1}\left(-\frac{7}{25}\right)\right]$$

Answer

Let $\sin^{-1}\left(-\frac{7}{25}\right) = x$

where $x \in \left[-\frac{\pi}{2}, 0\right]$

$\Rightarrow \sin x = -\frac{7}{25}$

To find: $\cos\left[\sin^{-1}\left(-\frac{7}{25}\right)\right] = \cos x$

As $\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos x = \sqrt{1 - \sin^2 x} \because x \in \left[-\frac{\pi}{2}, 0\right]$

$\Rightarrow \cos x = \sqrt{1 - \frac{49}{625}}$

$\Rightarrow \cos x = \sqrt{\frac{576}{625}}$

$\Rightarrow \cos x = \frac{24}{25}$

$\Rightarrow \cos\left[\sin^{-1}\left(-\frac{7}{25}\right)\right] = \frac{24}{25}$

1 B. Question

Evaluate:

$$\sec\left[\cot^{-1}\left(-\frac{5}{12}\right)\right]$$

Answer

Let $\cot^{-1}\left(-\frac{5}{12}\right)=x$

where $x\in\left(\frac{\pi}{2},\pi\right)$

$\Rightarrow \cot x=-\frac{5}{12}$

To find: $\sec\left[\cot^{-1}\left(-\frac{5}{12}\right)\right]=\sec x$

As $1+\tan^2 x=\sec^2 x$

$\Rightarrow 1+\frac{1}{\cot^2 x}=\sec^2 x$

$\Rightarrow \sec x=-\sqrt{1+\frac{1}{\cot^2 x}}$

As $x\in\left(\frac{\pi}{2},\pi\right)$

$\Rightarrow \sec x=-\sqrt{1+\left(\frac{12}{5}\right)^2}$

$\Rightarrow \sec x=-\frac{13}{5}$

$\Rightarrow \sec\left[\cot^{-1}\left(-\frac{5}{12}\right)\right]=-\frac{13}{5}$

1 C. Question

Evaluate:

$$\cot\left[\sec^{-1}\left(-\frac{13}{5}\right)\right]$$

Answer

Let $\sec^{-1}\left(-\frac{13}{5}\right)=x$ where $x\in\left(\frac{\pi}{2},\pi\right)$

$\Rightarrow \sec x=-\frac{13}{5}$

To find: $\cot\left[\sec^{-1}\left(-\frac{13}{5}\right)\right] = \cot x$

As $1 + \tan^2 x = \sec^2 x$

⇒ $\tan x = -\sqrt{\sec^2 x - 1}$

⇒ $\tan x = -\sqrt{\left(-\frac{13}{5}\right)^2 - 1}$

⇒ $\tan x = -\frac{12}{5}$

⇒ $\cot x = -\frac{5}{12}$

⇒ $\cot\left[\sec^{-1}\left(-\frac{13}{5}\right)\right] = -\frac{5}{12}$

2 A. Question

Evaluate:

$\tan\left[\cos^{-1}\left(-\frac{7}{25}\right)\right]$

Answer

Let $\cos^{-1}\left(-\frac{7}{25}\right) = x$ where $x \in \left(\frac{\pi}{2}, \pi\right)$

⇒ $\cos x = -\frac{7}{25}$

To find: $\tan\left[\cos^{-1}\left(-\frac{7}{25}\right)\right] = \tan x$

As $1 + \tan^2 x = \sec^2 x$

⇒ $\tan x = -\sqrt{\sec^2 x - 1}$ as $x \in \left(\frac{\pi}{2}, \pi\right)$

⇒ $\tan x = -\sqrt{\frac{1}{\cos^2 x} - 1}$

⇒ $\tan x = -\sqrt{\left(-\frac{25}{7}\right)^2 - 1}$

⇒ $\tan x = -\frac{24}{7}$

⇒ $\tan\left[\cos^{-1}\left(-\frac{7}{25}\right)\right] = -\frac{24}{7}$

2 B. Question

Evaluate:

$$\operatorname{cosec}\left[\cot^{-1}\left(-\frac{12}{5}\right)\right]$$

Answer

Let $\cot^{-1}\left(-\frac{12}{5}\right)=x$ where $x\in\left(\frac{\pi}{2},\pi\right)$

⇒ $\cot x=-\frac{12}{5}$

To find: $\operatorname{cosec}\left[\cot^{-1}\left(-\frac{12}{5}\right)\right]=\operatorname{cosec} x$

As $1+\cot^2 x=\operatorname{cosec}^2 x$

⇒ $\operatorname{cosec} x=\sqrt{1+\cot^2 x}$ as $x\in\left(\frac{\pi}{2},\pi\right)$

⇒ $\operatorname{cosec} x=\sqrt{1+\left(-\frac{12}{5}\right)^2}$

⇒ $\operatorname{cosec} x=\frac{13}{5}$

⇒ $\operatorname{cosec}\left[\cot^{-1}\left(-\frac{12}{5}\right)\right]=\frac{13}{5}$

2 C. Question

Evaluate:

$$\cos\left[\tan^{-1}\left(-\frac{3}{4}\right)\right]$$

Answer

Let $\tan^{-1}\left(-\frac{3}{4}\right)=x$ where $x\in\left[-\frac{\pi}{2},0\right]$

⇒ $\tan x=-\frac{3}{4}$

To find : $\cos\left[\tan^{-1}\left(-\frac{3}{4}\right)\right]=\cos x$

As $1+\tan^2 x=\sec^2 x$

⇒ $\sec x=\sqrt{1+\tan^2 x}$ as $x\in\left[-\frac{\pi}{2},0\right]$

$\Rightarrow \sec x = \sqrt{1+\left(-\frac{3}{4}\right)^2}$

$\Rightarrow \sec x = \frac{5}{4}$

$\Rightarrow \cos x = \frac{1}{\sec x}$

$\Rightarrow \cos x = \frac{4}{5}$

$\Rightarrow \cos\left[\tan^{-1}\left(-\frac{3}{4}\right)\right] = \frac{4}{5}$

3. Question

Evaluate: $\sin\left[\cos^{-1}\left(-\frac{3}{5}\right)+\cot^{-1}\left(-\frac{5}{12}\right)\right]$.

Answer

A. Let $\cos^{-1}\left(-\frac{3}{5}\right) = x$ and $\cot^{-1}\left(-\frac{5}{12}\right) = y$

$\Rightarrow \cos x = -\frac{3}{5}$ and $\cot y = -\frac{5}{12}$

where $x, y \in \left[\frac{\pi}{2}, \pi\right]$

To find: $\sin\left[\cos^{-1}\left(-\frac{3}{5}\right)+\cot^{-1}\left(-\frac{5}{12}\right)\right] = \sin(x+y)$

$\Rightarrow \sin(x+y) = \sin x.\cos y + \cos x.\sin y$ ..eq(i)

As $\sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin x = \sqrt{1-\cos^2 x}$ as $x \in \left[\frac{\pi}{2}, \pi\right]$

$\Rightarrow \sin x = \sqrt{1-\left(-\frac{3}{5}\right)^2}$

$\Rightarrow \sin x = \frac{4}{5}$

Also, $1+\cot^2 y = \operatorname{cosec}^2 y$

$\Rightarrow \operatorname{cosec} y = \sqrt{1+\cot^2 y}$

$\Rightarrow \operatorname{cosec} y = \sqrt{1+\left(-\frac{5}{12}\right)^2}$

$\Rightarrow \operatorname{cosec} y = \frac{13}{12}$

$\Rightarrow \sin y = \frac{1}{\operatorname{cosec} y}$

$\Rightarrow \sin y = \frac{12}{13}$

$\Rightarrow \cos y = \cot y . \sin y$

$\Rightarrow \cos y = -\frac{5}{12} \times \frac{12}{13} = -\frac{5}{13}$

Putting these values in eq(i)

$\Rightarrow \sin(x + y) = \frac{4}{5} . \left(-\frac{5}{13}\right) + \left(-\frac{3}{5}\right) . \frac{12}{13}$

$\Rightarrow \sin(x + y) = -\frac{56}{65}$

$\Rightarrow \sin\left[\cos^{-1}\left(-\frac{3}{5}\right) + \cot^{-1}\left(-\frac{5}{12}\right)\right] = -\frac{56}{65}$

Exercise 4.10

1 A. Question

Evaluate:

$\cot\left(\sin^{-1}\frac{3}{4} + \sec^{-1}\frac{4}{3}\right)$

Answer

$= \cot\left(\sin^{-1}\frac{3}{4} + \cos^{-1}\frac{3}{4}\right)$

$\left(\because \sec^{-1} x = \cos^{-1}\frac{1}{x}\right)$

We know, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$= \cot\frac{\pi}{2}$

= 0

1 B. Question

Evaluate:

$\sin\left(\tan^{-1} x + \tan^{-1}\frac{1}{x}\right)$ x<0

Answer

$= \sin\left(\tan^{-1} x + (\cot^{-1} x - \pi)\right)$

$\left(\because \tan^{-1}\theta = \cot^{-1}\frac{1}{\theta} - \pi \qquad \text{for } x < 0\right)$

$= \sin\left(\frac{\pi}{2} - \pi\right)$

$\left(\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}\right)$

$= \sin\left(-\frac{\pi}{2}\right)$

$= -\sin\frac{\pi}{2} = -1$

1 C. Question

Evaluate:

$\sin\left(\tan^{-1} x + \tan^{-1}\frac{1}{x}\right)$ x>0

Answer

$= \sin\left(\tan^{-1} x + \cot^{-1} x\right)$

$\left(\because \tan^{-1}\theta = \cot^{-1}\frac{1}{\theta} \qquad \text{for } x > 0\right)$

$= \sin\frac{\pi}{2}$

$\left(\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}\right)$

= 1

1 D. Question

Evaluate:

$\cot\left(\tan^{-1}\alpha + \cot^{-1}\alpha\right)$

Answer

$= \cot\left(\frac{\pi}{2}\right)$

$\left(\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}\right)$

= 0

1 E. Question

Evaluate:

$\cos(\sec^{-1} x + \operatorname{cosec}^{-1} x)$ |x|≥1

Answer

$= \cos\left(\cos^{-1}\frac{1}{x} + \sin^{-1}\frac{1}{x}\right)$

$\left(\because \sec^{-1}\theta = \cos^{-1}\frac{1}{\theta} \quad \text{and} \quad \operatorname{cosec}^{-1}\theta = \sin^{-1}\frac{1}{\theta}\right)$

$= \cos\frac{\pi}{2}$

$\left(\because \sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}\right)$

= 0

2. Question

2 If $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{4}$, then find the value of $\sin^{-1} x + \sin^{-1} y$.

Answer

A. $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{4}$

$\Rightarrow \left(\frac{\pi}{2} - \sin^{-1} x\right) + \left(\frac{\pi}{2} - \sin^{-1} y\right) = \frac{\pi}{4}$

$\left(\because \sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}\right)$

$\Rightarrow \pi - \left(\sin^{-1} x + \sin^{-1} y\right) = \frac{\pi}{4}$

$\Rightarrow \sin^{-1} x + \sin^{-1} y = \pi - \frac{\pi}{4}$

$\Rightarrow \sin^{-1} x + \sin^{-1} y = \frac{3\pi}{4}$

3. Question

If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{3}$ and $\cos^{-1} x - \cos^{-1} y = \frac{\pi}{6}$, then find x and y.

Answer

A. $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{3}$ ...eq(i)

$\cos^{-1} x - \cos^{-1} y = \frac{\pi}{6}$ ...eq(ii)

Subtracting (ii) from (i)

$\Rightarrow (\sin^{-1}x - \cos^{-1}x) + (\sin^{-1}y + \cos^{-1}y) = \frac{\pi}{3} - \frac{\pi}{6}$

$\Rightarrow (\sin^{-1}x - \cos^{-1}x) + \left(\frac{\pi}{2}\right) = \frac{\pi}{6}$

$\left(\because \sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}\right)$

$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1}x\right) - \cos^{-1}x = -\frac{\pi}{3}$

$\Rightarrow 2\cos^{-1}x = \frac{5\pi}{6}$

$\Rightarrow \cos^{-1}x = \frac{5\pi}{12}$

$\Rightarrow x = \cos\left(\frac{5\pi}{12}\right)$

$\Rightarrow x = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$\Rightarrow x = \cos\frac{\pi}{4}.\cos\frac{\pi}{6} - \sin\frac{\pi}{4}.\sin\frac{\pi}{6}$

$\Rightarrow x = \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2}$

$\Rightarrow x = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Now, putting the value of $\cos^{-1}x$ in eq(ii)

$\Rightarrow \frac{5\pi}{12} - \cos^{-1}y = \frac{\pi}{6}$

$\Rightarrow \cos^{-1}y = \frac{\pi}{4}$

$\Rightarrow y = \frac{1}{\sqrt{2}}$

$\Rightarrow x = \frac{\sqrt{3}-1}{2\sqrt{2}}$ and $y = \frac{1}{\sqrt{2}}$

4. Question

If $\cot\left(\cos^{-1}\frac{3}{5} + \sin^{-1}x\right) = 0$, then find the values of x.

Answer

A. $\cot\left(\cos^{-1}\frac{3}{5}+\sin^{-1}x\right)=0$

⇒ $\cos^{-1}\frac{3}{5}+\sin^{-1}x=n\pi+\frac{\pi}{2}$

⇒ $\sin^{-1}x=\left(n\pi+\frac{\pi}{2}\right)-\cos^{-1}\frac{3}{5}$

⇒ $x=\sin\left[\left(n\pi+\frac{\pi}{2}\right)-\cos^{-1}\frac{3}{5}\right]$

⇒ $x=\sin\left(n\pi+\frac{\pi}{2}\right)\cos\left(\cos^{-1}\frac{3}{5}\right)-\cos\left(n\pi+\frac{\pi}{2}\right)\sin\left(\cos^{-1}\frac{3}{5}\right)$

(using sin(A-B) = sinAcosB – cosAsinB)

⇒ $x=\pm\frac{3}{5}$

$\left(\text{The value of }\sin\left(n\pi+\frac{\pi}{2}\right)\text{switches between 1 and }-1\right)$

5. Question

$\left(\sin^{-1}x\right)^2+\left(\cos^{-1}x\right)^2=\frac{17\pi^2}{36}$. Find x

Answer

A. Using $a^2+b^2 = (a+b)^2 - 2ab$

⇒ $\left(\sin^{-1}x+\cos^{-1}x\right)^2-2\sin^{-1}x\cos^{-1}x=\frac{17\pi^2}{36}$

⇒ $\frac{\pi^2}{4}-2\sin^{-1}x\left(\frac{\pi}{2}-\sin^{-1}x\right)=\frac{17\pi^2}{36}$

Substituting $\sin^{-1}x$ with ‘a’

⇒ $2a^2-\pi a+\frac{\pi^2}{4}=\frac{17\pi^2}{36}$

⇒ $2a^2-\pi a-\frac{2\pi^2}{9}=0$

⇒ $18a^2-9\pi a-2\pi^2=0$

Using quadratic formulae

$$x=\frac{\left(-b\pm\sqrt{b^2-4ac}\right)}{2a}$$

⇒ $x=\frac{\pi(9\pm15)}{36}$

$\Rightarrow x = \frac{2\pi}{3}, -\frac{\pi}{6}$

6. Question

$\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$. Find x

Answer

A. $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$

$\Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1}x = n\pi + \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x = \left(n\pi + \frac{\pi}{2}\right) - \sin^{-1}\frac{1}{5}$

$\Rightarrow x = \cos\left[\left(n\pi + \frac{\pi}{2}\right) - \sin^{-1}\frac{1}{5}\right]$

$\Rightarrow x = \cos\left(n\pi + \frac{\pi}{2}\right)\cos\left(\sin^{-1}\frac{1}{5}\right) + \sin\left(n\pi + \frac{\pi}{2}\right)\sin\left(\sin^{-1}\frac{1}{5}\right)$

(using cos(A-B) = cosAcosB + sinAsinB)

$\Rightarrow x = \pm\frac{1}{5}$

(The value of $\sin\left(n\pi + \frac{\pi}{2}\right)$ switches between 1 and -1)

7. Question

Solve: $\sin^{-1}x = \frac{\pi}{6} + \cos^{-1}x$

Answer

A. $\sin^{-1}x = \frac{\pi}{6} + \cos^{-1}x$

$\Rightarrow \sin^{-1}x = \frac{\pi}{6} + \left(\frac{\pi}{2} - \sin^{-1}x\right)$

$\left(\because \sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}\right)$

$\Rightarrow 2\sin^{-1}x = \frac{2\pi}{3}$

$\Rightarrow \sin^{-1}x = \frac{\pi}{3}$

$\Rightarrow x = \frac{\sqrt{3}}{2}$

8. Question

Solve: $4\sin^{-1}x = \pi - \cos^{-1}x$

Answer

A. $4\sin^{-1}x = \pi - \cos^{-1}x$

$\Rightarrow 4\sin^{-1}x = \pi - \left(\frac{\pi}{2} - \sin^{-1}x\right)$

$\Rightarrow 3\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x = \frac{\pi}{6}$

$\Rightarrow x = \frac{1}{2}$

9. Question

Solve: $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$.

Answer

A. $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$

$\Rightarrow \tan^{-1}x + 2\left(\frac{\pi}{2} - \tan^{-1}x\right) = \frac{2\pi}{3}$

$\left(\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}\right)$

$\Rightarrow \tan^{-1}x = \frac{\pi}{3}$

$\Rightarrow x = \sqrt{3}$

10. Question

Solve: $5\tan^{-1}x + 3\cot^{-1}x = 2\pi$.

Answer

A. $5\tan^{-1}x + 3\cot^{-1}x = 2\pi$

$\Rightarrow 5\tan^{-1}x + 3\left(\frac{\pi}{2} - \tan^{-1}x\right) = 2\pi$

$\left(\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}\right)$

$\Rightarrow 2\tan^{-1}x = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}x = \frac{\pi}{4}$

$\Rightarrow x = 1$

Exercise 4.11

1 A. Question

Prove the following results:

$$\tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} = \tan^{-1}\frac{2}{9}$$

Answer

Given:- $\tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{1}{13}) = \tan^{-1}(\frac{2}{9})$

Take

LHS

$=\tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{1}{13})$

We know that, Formula

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

Thus,

$=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\times\frac{1}{13}}\right)$

$= \tan^{-1}\left(\frac{\frac{13+7}{91}}{\frac{91-1}{91}}\right)$

$=\tan^{-1}\left(\frac{20}{90}\right)$

$= \tan^{-1}\left(\frac{2}{9}\right)$

= RHS

Hence, Proved.

1 B. Question

Prove the following results:

$$\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$$

Answer

Given:- $\sin^{-1}(\frac{12}{13}) + \cos^{-1}\frac{4}{5} + \tan^{-1}(\frac{63}{16}) = \pi$

Take

LHS

$\sin^{-1}(\frac{12}{13}) + \cos^{-1}\frac{4}{5} + \tan^{-1}(\frac{63}{16})$

We know that, Formula

$\sin^{-1} x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$\cos^{-1} x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

Thus,

$=\tan^{-1}\left(\frac{\frac{12}{13}}{\sqrt{1-\left(\frac{12}{13}\right)^2}}\right) + \tan^{-1}\left(\frac{\sqrt{1-\left(\frac{4}{5}\right)^2}}{\frac{4}{5}}\right) + \tan^{-1}(\frac{63}{16})$

$=\tan^{-1}(\frac{12}{5}) + \tan^{-1}(\frac{3}{4}) + \tan^{-1}(\frac{63}{16})$

We know that, Formula

$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\frac{x+y}{1-xy}$

Thus,

$=\pi + \tan^{-1}(\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\times\frac{3}{4}}) + \tan^{-1}(\frac{63}{16})$

$=\pi + \tan^{-1}(-\frac{63}{16}) + \tan^{-1}(\frac{63}{16})$

We know that, Formula

$\tan^{-1}(-x) = -\tan^{-1} x$

$= \pi - \tan^{-1}(-\frac{63}{16}) + \tan^{-1}(\frac{63}{16})$

$= \pi$

So,

$\sin^{-1}(\frac{12}{13}) + \cos^{-1}\frac{4}{5} + \tan^{-1}(\frac{63}{16}) = \pi$

Hence, Proved.

1 C. Question

Prove the following results:

$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \sin^{-1}\frac{1}{\sqrt{5}}$

Answer

Given:- $\tan^{-1}(\frac{1}{4}) + \tan^{-1}(\frac{2}{9}) = \sin^{-1}(\frac{1}{\sqrt{5}})$

Take

LHS

$\tan^{-1}(\frac{1}{4}) + \tan^{-1}(\frac{2}{9})$

We know that, Formula

$$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\frac{x+y}{1-xy}$$

Thus,

$$= \tan^{-1}\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}$$

$$= \tan^{-1}\frac{\frac{17}{36}}{\frac{34}{36}}$$

$$=\tan^{-1}\frac{\frac{17}{36}}{\frac{34}{36}}$$

$$= \tan^{-1}\frac{1}{2}$$

Let,

$$\tan\theta = \frac{1}{2}$$

Therefore,

$$\sin\theta = \frac{1}{\sqrt{5}}$$

So,

$$\theta = \sin^{-1}\frac{1}{\sqrt{5}}$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right) = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \text{RHS}$$

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

Hence, Proved.

2. Question

Find the value of $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$

Answer

Given:- $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$

Take

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$$

We know that, Formula

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

Thus,

$$= \tan^{-1}\frac{\frac{x}{y}-\left(\frac{x-y}{x+y}\right)}{1+\frac{x}{y}\times\left(\frac{x-y}{x+y}\right)}$$

$= \tan^{-1}\frac{x(x+y)-y(x-y)}{y(x+y)+x(x-y)}$

$=\tan^{-1}\frac{x^2+y^2}{x^2+y^2}$

$= \tan^{-1} 1$

$= \frac{\pi}{4}$

So,

$\tan^{-1}(\frac{x}{y}) - \tan^{-1}(\frac{x-y}{x+y}) = \frac{\pi}{4}$

3 A. Question

Solve the following equations for x:

$$\tan^{-1} 2x + \tan^{-1} 3x = n\pi + \frac{3\pi}{4}$$

Answer

Given:- $\tan^{-1}(2x) + \tan^{-1}(3x) = n\pi + \frac{3\pi}{4}$

Take

LHS

$\tan^{-1}(2x) + \tan^{-1}(3x) = n\pi + \frac{3\pi}{4}$

We know that, Formula

$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy}$

Thus,

$\Rightarrow \tan^{-1}\frac{2x+3x}{1-2x\times 3x} = n\pi + \frac{3\pi}{4}$

$\Rightarrow \tan^{-1}\frac{5x}{1-6x^2} = n\pi + \frac{3\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2} = \tan(n\pi + \frac{3\pi}{4})$

$\Rightarrow \frac{5x}{1-6x^2} = -1$

⇒ $5x = -1 + 6x^2$

⇒ $6x^2 - 5x - 1 = 0$

⇒ $6x^2 - 6x + x - 1 = 0$

⇒ $6x(x - 1) + 1(x - 1) = 0$

⇒ $(6x + 1)(x - 1) = 0$

⇒ $6x + 1 = 0$ or $x - 1 = 0$

$\Rightarrow x = -\frac{1}{6}$ or $x = 1$

Since,

$x = -\frac{1}{6} \in \left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

So,

$x = -\frac{1}{6}$ is the root of the given equation

Therefore,

$x = -\frac{1}{6}$

3 B. Question

Solve the following equations for x:

$$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$$

Answer

Given:- $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}(\frac{8}{31})$

Take

LHS

$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$

We know that, Formula

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

Thus,

$\Rightarrow \tan^{-1}\frac{(x+1)+(x-1)}{1-(x+1)\times(x-1)} = \tan^{-1}\frac{8}{31}$

$\Rightarrow \tan^{-1}\frac{2x}{1-(x^2-1)} = \tan^{-1}\frac{8}{31}$

$\Rightarrow \tan^{-1}\frac{2x}{1-x^2+1)} = \tan^{-1}\frac{8}{31}$

$\Rightarrow \frac{2x}{1-x^2+1)} = \frac{8}{31}$

⇒ $62x = 8 – 8x^2 + 8$

⇒ $4x^2 + 62x – 16 = 0$

⇒ $6x^2 + 31x – 8 = 0$

⇒ $4x(x + 8) – 1(x + 8) = 0$

⇒ $(4x – 1)(x + 8) = 0$

⇒ $6x + 1 = 0$ or $x – 1 = 0$

$\Rightarrow x = \frac{1}{4}$ or $x = -8$

Since,

$x = \frac{1}{4} \in (-\sqrt{2}, \sqrt{2})$

So,

$x = \frac{1}{4}$ is the root of the given equation

Therefore,

$$x = \frac{1}{4}$$

3 C. Question

Solve the following equations for x:

$$\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1} 3x$$

Answer

Given:- $\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1} 3x)$

Take

$$\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1} 3x$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$\Rightarrow \tan^{-1} \frac{(x+1)+(x-1)}{1-(x+1)\times(x-1)} + \tan^{-1}(x) = \tan^{-1} 3x$$

$$\Rightarrow \tan^{-1} \frac{2x}{1-(x^2-1)} + \tan^{-1}(x) = \tan^{-1} 3x$$

$$\Rightarrow \tan^{-1} \frac{2x}{1-x^2+1} + \tan^{-1}(x) = \tan^{-1} 3x$$

$$\Rightarrow \tan^{-1} \frac{2x}{2-x^2} + \tan^{-1}(x) = \tan^{-1} 3x$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$\Rightarrow \tan^{-1} \frac{\frac{2x}{2-x^2}+x}{1-x\left(\frac{2x}{2-x^2}\right)} = \tan^{-1} 3x$$

$$\Rightarrow \tan^{-1} \frac{\frac{2x+2x-x^3}{2-x^2}}{\frac{2-x^2-2x^2}{2-x^2}} = \tan^{-1} 3x$$

$$\Rightarrow \tan^{-1} \frac{4x-x^3}{2-3x^2} = \tan^{-1} 3x$$

$$\Rightarrow \frac{4x-x^3}{2-3x^2} = 3x$$

$\Rightarrow 4x - x^3 = 6x - 9x^3$

$\Rightarrow 9x^3 - x^3 + 4x - 6x = 0$

$\Rightarrow 8x^3 - 2x = 0$

$\Rightarrow 2x(4x^2 - 1) = 0$

$\Rightarrow x = 0 \text{ or } x = \frac{1}{2} \text{ or } x = -\frac{1}{2}$

All satisfies x value

So,

$x = 0 \text{ or } x = \frac{1}{2} \text{ or } x = -\frac{1}{2}$ is the root of the given equation

Therefore,

$x = 0, \pm\frac{1}{2}$

3 D. Question

Solve the following equations for x:

$\tan^{-1}\left(\frac{1-x}{1+x}\right) - \frac{1}{2}\tan^{-1} x = 0$, where x > 0

Answer

Given:- $\tan^{-1}(\frac{1-x}{1+x}) - \frac{1}{2}\tan^{-1}(x) = 0$

Take

$\tan^{-1}(\frac{1-x}{1+x}) - \frac{1}{2}\tan^{-1}(x) = 0$

We know that, Formula

$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy}$

Thus,

$\Rightarrow \tan^{-1} 1 - \tan^{-1} x = \frac{1}{2}\tan^{-1}(x)$

$\Rightarrow \frac{\pi}{4} = \frac{3}{2}\tan^{-1}(x)$

$\Rightarrow \tan^{-1}(x) = \frac{\pi}{6}$

$\Rightarrow x = \tan\frac{\pi}{6}$

$\Rightarrow x = \frac{1}{\sqrt{3}}$

3 E. Question

Solve the following equations for x:

$\cot^{-1} x - \cot^{-1}(x+2) = \frac{\pi}{12}$, where x > 0

Answer

Given:- $\cot^{-1}(x) - \cot^{-1}(x+2) = \frac{\pi}{12}$

Take

$\cot^{-1}(x) - \cot^{-1}(x+2) = \frac{\pi}{12}$

We know that, Formula

$\cot^{-1} x = \tan^{-1} \frac{1}{x}$

Thus,

$\Rightarrow \tan^{-1} \frac{1}{x} - \tan^{-1} \frac{1}{x+2} = \frac{\pi}{12}$

We know that, Formula

$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$

Thus,

$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{x}-\frac{1}{x+2}}{1+\frac{1}{x}\times\frac{1}{x+2}}\right) = \frac{\pi}{12}$

$\Rightarrow \frac{\frac{1}{x}-\frac{1}{x+2}}{1+\frac{1}{x}\times\frac{1}{x+2}} = \tan\frac{\pi}{12}$

$\Rightarrow \frac{2}{x^2+2x+1} = \tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

We know that, Formula

$\tan(x-y) = \frac{\tan x - \tan y}{1+(\tan x)(\tan y)}$

Thus,

$\Rightarrow \frac{2}{(x+1)^2} = \frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+(\tan\frac{\pi}{3})(\tan\frac{\pi}{4})}$

$\Rightarrow \frac{2}{(x+1)^2} = \frac{\sqrt{3}-1}{1+\sqrt{3}}$

By rationalisation

$\Rightarrow \frac{2}{(x+1)^2} = \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$

$\Rightarrow \frac{2}{(x+1)^2} = \frac{3-1}{(1+\sqrt{3})^2}$

$\Rightarrow (x+1)^2 = (1+\sqrt{3})^2$

$\Rightarrow x+1 = \pm(1+\sqrt{3})$

$\Rightarrow x+1 = 1+\sqrt{3}$ or $x+1 = -1-\sqrt{3}$

$\Rightarrow x = \sqrt{3}$ or $x = -2-\sqrt{3}$

as given, $x > 0$

Therefore

$x = \sqrt{3}$

3 F. Question

Solve the following equations for x:

$\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1}\left(\frac{8}{79}\right), x > 0$

Answer

Given:- $\tan^{-1}(x+2)+\tan^{-1}(x-2)=\tan^{-1}(\frac{8}{79})$

Take

$$\tan^{-1}(x+2)+\tan^{-1}(x-2)=\tan^{-1}\frac{8}{79}$$

We know that, Formula

$$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$$

Thus,

$\Rightarrow \tan^{-1}\frac{(x+2)+(x-2)}{1-(x+2)\times(x-2)}=\tan^{-1}\frac{8}{79}$

$\Rightarrow \tan^{-1}\frac{2x}{1-(x^2-4)}=\tan^{-1}\frac{8}{79}$

$\Rightarrow \tan^{-1}\frac{2x}{1-x^2+4}=\tan^{-1}\frac{8}{79}$

$\Rightarrow \frac{2x}{5-x^2}=\frac{8}{79}$

$\Rightarrow 40 - 8x^2 = 158x$

$\Rightarrow 8x^2 + 158x - 40 = 0$

$\Rightarrow 4x^2 + 79x - 20 = 0$

$\Rightarrow 4x^2 + 80x - x - 20 = 0$

$\Rightarrow 4x(x + 20) - 1(x + 20) = 0$

$\Rightarrow (4x - 1)(x + 20) = 0$

$\Rightarrow 4x - 1 = 0$ or $x + 20 = 0$

$\Rightarrow x=\frac{1}{4}$ or $x=-20$

Since,

$x > 0$

So,

$x=\frac{1}{4}$ is the root of the given equation

Therefore,

$$x=\frac{1}{4}$$

3 G. Question

Solve the following equations for x:

$$\tan^{-1}\frac{x}{2}+\tan^{-1}\frac{x}{3}=\frac{\pi}{4}, 0<x<\sqrt{6}$$

Answer

Given:- $\tan^{-1}(\frac{x}{2})+\tan^{-1}(\frac{x}{3})=\frac{\pi}{4}$

Take

$\tan^{-1}(\frac{x}{2}) + \tan^{-1}(\frac{x}{3}) = \frac{\pi}{4}$

We know that, Formula

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{x}{2}+\frac{x}{3}}{1-\frac{x}{2}\times\frac{x}{3}}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{3x+2x}{6-x^2}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{5x}{6-x^2}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{5x}{6-x^2} = \tan\frac{\pi}{4}$

$\Rightarrow \frac{5x}{6-x^2} = 1$

$\Rightarrow 5x = 6 - x^2$

$\Rightarrow x^2 + 5x - 6 = 0$

$\Rightarrow x^2 + 6x - x - 6 = 0$

$\Rightarrow x(x + 6) - 1(x + 6) = 0$

$\Rightarrow x = -6, 1$

as given

$0<x<\sqrt{6}$

Therefore

$x = 1$

3 H. Question

Solve the following equations for x:

$$\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{X+4}\right) = \frac{\pi}{4}$$

Answer

Given:- $\tan^{-1}(\frac{x-2}{x-4}) + \tan^{-1}(\frac{x+2}{x+4}) = \frac{\pi}{4}$

Take

$\tan^{-1}(\frac{x-2}{x-4}) + \tan^{-1}(\frac{x+2}{x+4}) = \frac{\pi}{4}$

We know that, Formula

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{x-2}{x-4}+\frac{x+2}{x+4}}{1-\frac{x-2}{x-4}\times\frac{x+2}{x+4}}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{(x-2)(x+4)+(x+2)(x-4)}{(x-4)(x+4)}}{\frac{(x-4)(x+4)-(x-2)(x+2)}{(x-4)(x+4)}}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{(x-2)(x+4)+(x+2)(x-4)}{(x-4)(x+4)-(x-2)(x+2)}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{x^2+2x-8+x^2-2x-8}{(x^2-16)-(x^2-4)}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{2x^2-16}{-12}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{x^2-8}{-6} = \tan\frac{\pi}{4}$

$\Rightarrow \frac{x^2-8}{-6} = 1$

$\Rightarrow x^2 - 8 = - 6$

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}$

3 I. Question

Solve the following equations for x:

$$\tan^{-1}(2+x)+\tan^{-1}(2-x)=\tan^{-1}\frac{2}{3},$$

$$\text{where } x<-\sqrt{3} \text{ or}, x>\sqrt{3}$$

Answer

Given:- $\tan^{-1}(2+x)+\tan^{-1}(2-x)=\tan^{-1}\left(\frac{2}{3}\right)$

Take

$\tan^{-1}(2+x)+\tan^{-1}(2-x)=\tan^{-1}\left(\frac{2}{3}\right)$

We know that, Formula

$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$

Thus,

$\Rightarrow \tan^{-1}\frac{(2+x)+(2-x)}{1-(2+x)\times(2-x)} = \tan^{-1}\left(\frac{2}{3}\right)$

$\Rightarrow \tan^{-1}\frac{4}{1-(4-x^2)} = \tan^{-1}\frac{2}{3}$

$\Rightarrow \tan^{-1}\frac{4}{1-4+x^2} = \tan^{-1}\frac{2}{3}$

$\Rightarrow \frac{4}{1-4+x^2} = \frac{2}{3}$

$\Rightarrow 2x^2 - 8 + 2 = 12$

$\Rightarrow 2x^2 = 18$

$\Rightarrow x = \pm 3$

Since,

$x < -\sqrt{3}$ or $x > \sqrt{3}$

So,

x= +3, –3 is the root of the given equation

Therefore,

x= +3, –3

3 J. Question

Solve the following equations for x:

$$\tan^{-1}\frac{x-2}{x-1}+\tan^{-1}\frac{x+2}{x+1}=\frac{\pi}{4}$$

Answer

Given:- $\tan^{-1}(\frac{x-2}{x-1})+\tan^{-1}(\frac{x+2}{x+1})=\frac{\pi}{4}$

Take

$$\tan^{-1}(\frac{x-2}{x-1})+\tan^{-1}(\frac{x+2}{x+1})=\frac{\pi}{4}$$

We know that, Formula

$$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-2}{x-1}+\frac{x+2}{x+1}}{1-\frac{x-2}{x-1}\times\frac{x+2}{x+1}}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{(x-2)(x+1)+(x+2)(x-1)}{(x-1)(x+1)}}{\frac{(x-1)(x+1)-(x-2)(x+2)}{(x-1)(x+1)}}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{(x-2)(x+1)+(x+2)(x-1)}{(x-1)(x+1)-(x-2)(x+2)}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{x^2+2x-2+x^2-2x-2}{(x^2-1)-(x^2-4)}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{2x^2-4}{-5}\right)=\frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2-4}{-5}=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2-4}{-5}=1$$

$\Rightarrow 2x^2 - 4 = - 5$

$\Rightarrow 2x^2 = 1$

$$\Rightarrow x=\pm\frac{1}{\sqrt{2}}$$

4. Question

Sum the following series:

$$\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{2}{9}+\tan^{-1}\frac{4}{33}+\ldots+\tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}}$$

Answer

Given:- $\tan^{-1}(\frac{1}{3})+\tan^{-1}\left(\frac{2}{9}\right)+\tan^{-1}\left(\frac{4}{33}\right)+\cdots+\tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right)$

Take

$$\tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right) = T_n \text{ (Let)}$$

$$\Rightarrow T_n = \tan^{-1}\left(\frac{2^n - 2^{n-1}}{1+2^n 2^{n-1}}\right)$$

We know that, Formula

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

$$\Rightarrow T_n = \tan^{-1}2^n - \tan^{-1}2^{n-1}$$

So,

$$T_1 = \tan^{-1}2^1 - \tan^{-1}2^0$$

$$T_2 = \tan^{-1}4 - \tan^{-1}2$$

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$$T_n = \tan^{-1}2^n - \tan^{-1}2^{n-1}$$

Adding all the terms, we get

$$= \tan^{-1}2^n - \tan^{-1}1$$

$$= \tan^{-1}2^n - \frac{\pi}{4}$$

Exercise 4.12

1. Question

Evaluate : $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right)$

Answer

Given:- $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right)$

Take

$$\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right)$$

We know that, Formula

$$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$$

$$= \cos\left(\sin^{-1}\left[\frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{3}{5}\right)^2}\right]\right)$$

$$= \cos\left(\sin^{-1}\left[\frac{3}{5}\times\frac{12}{13} + \frac{5}{13}\times\frac{4}{5}\right]\right)$$

$$= \cos\left(\sin^{-1}\left[\frac{56}{65}\right]\right)$$

We know that, Formula

$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$

Therefore,

$$=\cos\left(\cos^{-1}\sqrt{1-\left(\frac{56}{65}\right)^2}\right)$$

$$=\cos\left(\cos^{-1}\sqrt{\frac{33}{65}}\right)$$

$$=\frac{33}{65}$$

So,

$$\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right)=\frac{33}{65}$$

2 A. Question

Prove the following results:

$$\sin^{-1}\left(\frac{63}{65}\right)=\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)$$

Answer

Given:- $\sin^{-1}\frac{63}{65}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

Take

RHS

$$\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$$

We know that, Formula

$$\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$$

Thus,

$$=\sin^{-1}\frac{5}{13}+\sin^{-1}\sqrt{1-\frac{9}{25}}$$

$$=\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{4}{5}$$

By pathagorous theorem

$$=\tan^{-1}\frac{\frac{5}{13}}{\sqrt{1-\frac{25}{169}}}+\tan^{-1}\frac{\frac{4}{5}}{\sqrt{1-\frac{16}{25}}}$$

$$=\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{4}{3}$$

We know that, Formula

$$\tan^{-1}x+\tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

Thus,

$$=\tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}\right)$$

$= \tan^{-1}\left(\frac{63}{16}\right)$

Now,

LHS

$=\sin^{-1}\frac{63}{65}$

$$= \tan^{-1}\frac{\frac{63}{65}}{\sqrt{1-\left(\frac{25}{169}\right)^2}}$$

$=\tan^{-1}\left(\frac{63}{16}\right)$

So,

LHS = RHS

$$\sin^{-1}\frac{63}{65} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$$

2 B. Question

Prove the following results:

$$\sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{63}{16}$$

Answer

Given:- $\sin^{-1}\frac{63}{65} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

Take

LHS

$$\sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$$

We know that, Formula

$$\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$$

Thus,

$$=\sin^{-1}\frac{5}{13} + \sin^{-1}\sqrt{1-\frac{9}{25}}$$

$$=\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{4}{5}$$

By pathagorous theorem

$$=\tan^{-1}\frac{\frac{5}{13}}{\sqrt{1-\frac{25}{169}}} + \tan^{-1}\frac{\frac{4}{5}}{\sqrt{1-\frac{16}{25}}}$$

$$=\tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$$

We know that, Formula

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

Thus,

$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}\right)$

$= \tan^{-1}\left(\frac{63}{16}\right)$

Now,

RHS

$=\sin^{-1}\frac{63}{65}$

$$= \tan^{-1}\frac{\frac{63}{65}}{\sqrt{1-\left(\frac{25}{169}\right)^2}}$$

$=\tan^{-1}\left(\frac{63}{16}\right)$

So,

LHS = RHS

$$\sin^{-1}\frac{63}{65} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$$

2 C. Question

Prove the following results:

$$\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

Answer

Given:- $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

Take

LHS

$= \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$

$= \frac{9}{4}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right)$

We know that, Formula

$$\sin^{-1}x + \cos^{-1}y = \frac{\pi}{2}$$

Thus,

$= \frac{9}{4}\left(\cos^{-1}\frac{1}{3}\right)$

Now,

Assume that

$$\cos^{-1}\frac{1}{3} = x$$

Then,

$\Rightarrow \cos x = \frac{1}{3}$

And $\text{Sin}x = \sqrt{1 - \frac{1}{9}}$

$\Rightarrow \text{Sin}x = \frac{2\sqrt{2}}{3}$

Therefore,

$x = \sin^{-1}\frac{2\sqrt{2}}{3}$

⇒ LHS =RHS

$\Rightarrow \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

Hence Proved

3 A. Question

Prove the following results: Solve the following:

$\sin^{-1}x \sin^{-1}2x = \pi/3$

Answer

Given:- $\sin^{-1}x + \sin^{-1}2x = \frac{\pi}{3}$

Take

$\sin^{-1}x + \sin^{-1}2x = \frac{\pi}{3}$

$\Rightarrow \sin^{-1}x + \sin^{-1}2x = \sin^{-1}\frac{\sqrt{3}}{2}$

$\Rightarrow \sin^{-1}2x = \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}x$

We know that, Formula

$\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$

Thus,

$\Rightarrow \sin^{-1}2x = \sin^{-1}\left[\frac{\sqrt{3}}{2}\sqrt{1-x^2} - x\sqrt{1-\frac{\sqrt{3}}{2}^2}\right]$

$\Rightarrow \sin^{-1}2x = \sin^{-1}\left[\frac{\sqrt{3}}{2}\sqrt{1-x^2} - \frac{x}{2}\right]$

$\Rightarrow 2x = \left[\frac{\sqrt{3}}{2}\sqrt{1-x^2} - \frac{x}{2}\right]$

$\Rightarrow \frac{5x}{2} = \frac{\sqrt{3}}{2}\sqrt{1-x^2}$

$\Rightarrow 25x^2 = 3(1-x^2)$

$\Rightarrow x^2 = \frac{3}{28}$

$\Rightarrow x = \pm\frac{1}{2}\sqrt{\frac{3}{7}}$

3 B. Question

Prove the following results: Solve the following:

$\cos^{-1}x+\sin^{-1}x/1-\pi/6=0$

Answer

Given:- $\cos^{-1}x+\sin^{-1}\frac{x}{2}-\frac{\pi}{6} = 0$

Take

$$\cos^{-1}x+\sin^{-1}\frac{x}{2}-\frac{\pi}{6} = 0$$

$\Rightarrow \cos^{-1}x+\sin^{-1}\frac{x}{2}=\frac{\pi}{6}$

We know that, Formula

$$\cos^{-1}x=\sin^{-1}\sqrt{1-x^2}$$

Thus,

$\Rightarrow \cos^{-1}x+\sin^{-1}\frac{x}{2}=\sin^{-1}\frac{1}{2}-\sin^{-1}\sqrt{1-x^2}$

We know that, Formula

$$\sin^{-1}x+\sin^{-1}y= \sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]$$

Thus,

$\Rightarrow \sin^{-1}\frac{x}{2}=\sin^{-1}\left[\frac{1}{2}\sqrt{1-1+x^2}-\sqrt{1-x^2}\sqrt{1-\frac{1}{4}}\right]$

$\Rightarrow \sin^{-1}\frac{x}{2}=\sin^{-1}\left[\frac{x}{2}-\frac{\sqrt{3}\sqrt{1-x^2}}{2}\right]$

$\Rightarrow \frac{x}{2}= \frac{x}{2}-\frac{\sqrt{3}\sqrt{1-x^2}}{2}$

$\Rightarrow \frac{\sqrt{3}\sqrt{1-x^2}}{2}=0$

$\Rightarrow \sqrt{1-x^2}=0$

$\Rightarrow 1-x^2=0$

$\Rightarrow x^2= 1$

$\Rightarrow x = \pm1$

Exercise 4.13

1. Question

If $\cos^{-1} x/2 + \cos^{-1}y/3 = a$, then prove that $9x^2–12xy \cos a + 4y^2 = 36 \sin^2 a$.

Answer

Given:- $\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=a$

Take

$$\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=a$$

We know that, Formula

$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$

Thus,

$\Rightarrow \cos^{-1}\left[\frac{x}{2} \times \frac{y}{3} - \sqrt{1-\left(\frac{x}{2}\right)^2}\sqrt{1-\left(\frac{y}{3}\right)^2}\right] = a$

$\Rightarrow \left[\frac{xy}{6} - \frac{\sqrt{4-x^2}}{2} \times \frac{\sqrt{9-y^2}}{3}\right] = \cos a$

$\Rightarrow xy - \sqrt{4-x^2} \times \sqrt{9-y^2} = 6\cos a$

$\Rightarrow xy - 6\cos a = \sqrt{4-x^2}\sqrt{9-y^2}$

Now lets take square of both side, we get

$\Rightarrow (xy - 6\cos a)^2 = (4-x^2)(9-y^2)$

$\Rightarrow x^2y^2 + 36\cos^2 a - 12xy\cos a = 36 - 9x^2 - 4y^2 + x^2y^2$

$\Rightarrow 9x^2 + 4y^2 - 36 + 36\cos^2 a - 12xy\cos a = 0$

$\Rightarrow 9x^2 + 4y^2 - 12xy\cos a - 36(1 - \cos^2 a) = 0$

$\Rightarrow 9x^2 + 4y^2 - 12xy\cos a - 36\sin^2 a = 0$

$\Rightarrow 9x^2 + 4y^2 - 12xy\cos a = 36\sin^2 a$

Hence Proved

2. Question

Solve the equation: $\cos^{-1}$ a/x–$\cos^{-1}$ b/x =$\cos^{-1}$ 1/b - $\cos^{-1}$ 1/a

Answer

Given:- $\cos^{-1}\frac{a}{x} - \cos^{-1}\frac{b}{x} = \cos^{-1}\frac{1}{b} - \cos^{-1}\frac{1}{a}$

Take

$\cos^{-1}\frac{a}{x} - \cos^{-1}\frac{b}{x} = \cos^{-1}\frac{1}{b} - \cos^{-1}\frac{1}{a}$

$\Rightarrow \cos^{-1}\frac{a}{x} + \cos^{-1}\frac{1}{a} = \cos^{-1}\frac{1}{b} + \cos^{-1}\frac{b}{x}$

We know that, Formula

$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$

Thus,

$\Rightarrow \cos^{-1}\left[\frac{1}{x} - \sqrt{1-\left(\frac{a}{x}\right)^2}\sqrt{1-\left(\frac{1}{a}\right)^2}\right] = \cos^{-1}\left[\frac{1}{x} - \sqrt{1-\left(\frac{b}{x}\right)^2}\sqrt{1-\left(\frac{1}{b}\right)^2}\right]$

$\Rightarrow \frac{1}{x} - \sqrt{1-\left(\frac{a}{x}\right)^2}\sqrt{1-\left(\frac{1}{a}\right)^2} = \frac{1}{x} - \sqrt{1-\left(\frac{b}{x}\right)^2}\sqrt{1-\left(\frac{1}{b}\right)^2}$

$\Rightarrow \sqrt{1-\left(\frac{a}{x}\right)^2}\sqrt{1-\left(\frac{1}{a}\right)^2} = \sqrt{1-\left(\frac{b}{x}\right)^2}\sqrt{1-\left(\frac{1}{b}\right)^2}$

Squaring both side or removing square root, we get

$\Rightarrow \left(1-\left(\frac{a}{x}\right)^2\right)\left(1-\left(\frac{1}{a}\right)^2\right)=\left(1-\left(\frac{b}{x}\right)^2\right)\left(1-\left(\frac{1}{b}\right)^2\right)$

$\Rightarrow 1-\left(\frac{a}{x}\right)^2-\left(\frac{1}{a}\right)^2+\left(\frac{1}{x}\right)^2 = 1-\left(\frac{b}{x}\right)^2-\left(\frac{1}{b}\right)^2+\left(\frac{1}{x}\right)^2$

$\Rightarrow \left(\frac{b}{x}\right)^2-\left(\frac{a}{x}\right)^2=\left(\frac{1}{a}\right)^2-\left(\frac{1}{b}\right)^2$

$\Rightarrow (b^2 - a^2)a^2b^2 = x^2(b^2 - a^2)$

$\Rightarrow x^2 = a^2b^2$

$\Rightarrow x = ab$

3. Question

Solve: $\cos^{-1} \sqrt{3}x+\cos^{-1}x= \pi/2$

Answer

Given:- $\cos^{-1}\sqrt{3}x+\cos^{-1}x=\frac{\pi}{2}$

Take

$\cos^{-1}\sqrt{3}x+\cos^{-1}x=\frac{\pi}{2}$

We know that, Formula

$\cos^{-1}x+\cos^{-1}y= \cos^{-1}\left[xy-\sqrt{1-x^2}\sqrt{1-y^2}\right]$

Thus,

$\Rightarrow \cos^{-1}\left[\sqrt{3}x^2-\sqrt{1-(3x)^2}\sqrt{1-x^2}\right]= \frac{\pi}{2}$

$\Rightarrow \sqrt{3}x^2-\sqrt{1-(3x)^2}\sqrt{1-x^2}=\cos\frac{\pi}{2}$

$\Rightarrow \sqrt{3}x^2-\sqrt{1-(3x)^2}\sqrt{1-x^2}=0$

$\Rightarrow \sqrt{3}x^2=\sqrt{1-(3x)^2}\sqrt{1-x^2}$

Squaring both sides, we get

$\Rightarrow 3x^4 = 1 - x^2 - 3x^2 + 3x^4$

$\Rightarrow 1 - 4x^2 = 0$

$\Rightarrow x^2 = \frac{1}{4}$

$\Rightarrow x = \pm\frac{1}{2}$

4. Question

Prove that: $\cos^{-1} 4/5+\cos^{-1} 12/3=\cos^{-1} 33/65$

Answer

Given:- $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{3}=\cos^{-1}\frac{33}{65}$

Take

LHS

$\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{3}$

We know that, Formula

$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$

Thus,

$= \cos^{-1}\left[\frac{4}{5} \times \frac{12}{3} - \sqrt{1-\left(\frac{4}{5}\right)^2}\sqrt{1-\left(\frac{12}{3}\right)^2}\right]$

$= \cos^{-1}\left[\frac{48}{65} - \sqrt{1-\frac{16}{25}}\sqrt{1-\frac{144}{169}}\right]$

$= \cos^{-1}\left[\frac{48}{65} - \frac{3}{5} \times \frac{5}{13}\right]$

$= \cos^{-1}\left[\frac{48}{65} - \frac{15}{65}\right]$

$= \cos^{-1}\frac{33}{65}$

= RHS

So,

$\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{3} = \cos^{-1}\frac{33}{65}$

Hence Proved

Exercise 4.14

1 A. Question

Evaluate the following:

$$\tan\left\{2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right\}$$

Answer

Given:- $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right\}$

Now, as we know

$2\tan^{-1}(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right), \text{if } |x| < 1$

and $\frac{\pi}{4}$ can be written as $\tan^{-1}(1)$

$= \tan\left\{\tan^{-1}\left(\frac{2 \times \frac{1}{5}}{1-\frac{1}{25}}\right) - \tan^{-1}1\right\}$

$= \tan\left\{\tan^{-1}\left(\frac{5}{12}\right) - \tan^{-1}1\right\}$

We know that,

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$

$= \tan\left\{\tan^{-1}\left(\frac{\frac{5}{12}-1}{1+\frac{5}{12}}\right)\right\}$

$= \tan\left\{\tan^{-1}\left(\frac{-7}{17}\right)\right\}$

$= -\frac{7}{17}$

1 B. Question

Evaluate the following:

$\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)$

Answer

Given:- $\tan\left\{\frac{1}{2}\sin^{-1}\frac{3}{4}\right\}$

Let $\frac{1}{2}\sin^{-1}\frac{3}{4} = t\ (say)$

Therefore,

$\Rightarrow \sin^{-1}\frac{3}{4} = 2t$

$\Rightarrow \sin 2t = \frac{3}{4}$

Now, by Pythagoras theorem

$\Rightarrow \sin 2t = \frac{3}{4} = \frac{perpendicular}{hypotenuse}$

$\Rightarrow \cos 2t = \frac{\sqrt{4^2-3^2}}{4} = \frac{Base}{hypotenuse}$

$\Rightarrow \cos 2t = \frac{\sqrt{7}}{4}$

As given, and putting assume value, we get

$\tan\left\{\frac{1}{2}\sin^{-1}\frac{3}{4}\right\}$

$= \tan(t)$

We know that: Formula

$\tan(x) = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$

$= \sqrt{\frac{1-\cos 2t}{1+\cos 2t}}$

$= \sqrt{\frac{1-\frac{\sqrt{7}}{4}}{1+\frac{\sqrt{7}}{4}}}$

$= \sqrt{\frac{4-\sqrt{7}}{4+\sqrt{7}}}$

$= \sqrt{\frac{(4-\sqrt{7})(4-\sqrt{7})}{(4+\sqrt{7})(4-\sqrt{7})}}$; by rationalisation

$= \sqrt{\frac{(4-\sqrt{7})^2}{9}}$

$= \frac{4-\sqrt{7}}{3}$

Hence

$$\tan\left\{\frac{1}{2}\sin^{-1}\frac{3}{4}\right\} = \frac{4-\sqrt{7}}{3}$$

1 C. Question

Evaluate the following:

$$\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$$

Answer

Given:- $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$

We know that : Formula

$\cos^{-1}x = 2\sin^{-1}\left(\pm\sqrt{\frac{1-x}{2}}\right)$; choose that formula which actually simplifies function

Thus, given function changes to

$$\sin\left(\frac{1}{2}2\sin^{-1}\left(\pm\sqrt{\frac{1-\frac{4}{5}}{2}}\right)\right)$$

$= \sin\left(\sin^{-1}\left(\pm\sqrt{\frac{1}{2\times5}}\right)\right)$

$=\sin\left(\sin^{-1}\left(\pm\frac{1}{\sqrt{10}}\right)\right)$

As we know

$\sin(\sin^{-1}x) = x \text{ as } n \in [-1, 1]$

$= \pm\frac{1}{\sqrt{10}}$

Hence,

$$\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = \pm\frac{1}{\sqrt{10}}$$

1 D. Question

Evaluate the following:

$$\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$$

Answer

Given:- $\sin\left(2\tan^{-1}\left(\frac{2}{3}\right)\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$

We know that :- Formula (- obtain by Pythagoras theorem)

$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}(x)$; Formula of tan in terms of sine, so that it make simplification easier

And

$\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \tan^{-1}(x)$; Formula of tan in terms of cos, so that it make simplification easier

Now given function becomes,

$= \sin\left(\sin^{-1}\left(\frac{2\times\frac{2}{3}}{1+\frac{4}{9}}\right)\right) + \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+3}}\right)\right)$

$= \sin\left(\sin^{-1}\left(\frac{12}{13}\right)\right) + \cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$

$= \frac{12}{13} + \frac{1}{2}$

$= \frac{37}{26}$

Hence,

$\sin\left(2\tan^{-1}\left(\frac{2}{3}\right)\right) + \cos\left(\tan^{-1}\sqrt{3}\right) = \frac{37}{26}$

2 A. Question

Prove the following results:

$$2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$$

Answer

Given:- $2\sin^{-1}\frac{3}{5} = \tan^{-1}\left(\frac{24}{7}\right)$

Take

LHS

$=2\sin^{-1}\frac{3}{5}$

We know that, Formula

$\sin^{-1}(x) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

Thus,

$= 2 \times \tan^{-1}\left(\frac{\frac{3}{5}}{\sqrt{1-\frac{9}{25}}}\right)$

$=2 \times \tan^{-1}\left(\frac{\frac{3}{5}}{\frac{4}{5}}\right)$

$= 2 \times \tan^{-1}\left(\frac{3}{4}\right)$

Again we know that, Formula

$2\tan^{-1}(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, if $|x| < 1$

Thus,

$=\tan^{-1}(\frac{2\times\frac{3}{4}}{1-\frac{9}{16}})$

$= \tan^{-1}(\frac{\frac{3}{2}}{\frac{7}{16}})$

$=\tan^{-1}(\frac{24}{7})$

= RHS

So,

$2\sin^{-1}\frac{3}{5} = \tan^{-1}(\frac{24}{7})$

Hence Proved

2 B. Question

Prove the following results:

$$\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}\frac{3}{5}=\frac{1}{2}\sin^{-1}\frac{4}{5}$$

Answer

Given:- $\tan^{-1}(\frac{1}{4})+\tan^{-1}(\frac{2}{9}) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right) = \frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$

Take

LHS

$=\tan^{-1}(\frac{1}{4})+\tan^{-1}(\frac{2}{9})$

We know that, Formula

$\tan^{-1}x+\tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

Thus,

$=\tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}\right)$

$= \tan^{-1}\left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right)$

$=\tan^{-1}\left(\frac{17}{34}\right)$

$= \tan^{-1}\left(\frac{1}{2}\right)$

Multiplying and dividing by 2

$= \frac{1}{2}\left\{2\tan^{-1}\left(\frac{1}{2}\right)\right\}$

We know that, Formula

$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$= \frac{1}{2}\cos^{-1}\left(\frac{1-\frac{1}{4}}{1+\frac{1}{4}}\right)$

$= \frac{1}{2}\cos^{-1}\left(\frac{\frac{3}{4}}{\frac{5}{4}}\right)$

$= \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

= RHS

So,

$\tan^{-1}(\frac{1}{4}) + \tan^{-1}(\frac{2}{9}) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

Now,

$= \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

We know that, Formula

$=\cos^{-1} x = \sin^{-1}\sqrt{1 - x^2}$

Thus,

$= \frac{1}{2}\sin^{-1}\sqrt{1 - \frac{9}{25}}$

$= \frac{1}{2}\sin^{-1}\sqrt{\frac{16}{25}}$

$=\frac{1}{2}\sin^{-1}\frac{4}{5}$

= RHS

So,

$\tan^{-1}(\frac{1}{4}) + \tan^{-1}(\frac{2}{9}) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right) = \frac{1}{2}\sin^{-1}\frac{4}{5}$

Hence Proved

2 C. Question

Prove the following results:

$$\tan^{-1}\frac{2}{3} = \frac{1}{2}\tan^{-1}\frac{12}{5}$$

Answer

Given:- $\tan^{-1}(\frac{2}{3}) = \frac{1}{2}\tan^{-1}(\frac{12}{5})$

Take

LHS

$=\tan^{-1}(\frac{2}{3})$

Multiplying and dividing by 2

$= \frac{1}{2}\left\{2\tan^{-1}\left(\frac{2}{3}\right)\right\}$

We know that, Formula

$2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$

$= \frac{1}{2}\tan^{-1}\left(\frac{2\times\frac{2}{3}}{1-\frac{4}{9}}\right)$

$= \frac{1}{2}\tan^{-1}\left(\frac{\frac{4}{3}}{\frac{5}{9}}\right)$

$= \frac{1}{2}tan^{-1}\left(\frac{12}{5}\right)$

= RHS

So,

$tan^{-1}(\frac{2}{3}) = \frac{1}{2}tan^{-1}\left(\frac{12}{5}\right)$

Hence Proved

2 D. Question

Prove the following results:

$$\tan^{-1}\left(\frac{1}{7}\right)+2\tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$$

Answer

Given:- $\tan^{-1}(\frac{1}{7}) + 2\tan^{-1}(\frac{1}{3}) = \frac{\pi}{4}$

Take

LHS

$= \tan^{-1}(\frac{1}{7}) + 2\tan^{-1}(\frac{1}{3})$

We know that, Formula

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Thus,

$=\tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{2\times\frac{1}{3}}{1-\frac{1}{9}})$

$= \tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{\frac{2}{3}}{\frac{8}{9}})$

$= \tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{3}{4})$

We know that, Formula

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}}\right)$

$= \tan^{-1}\left(\frac{\frac{25}{28}}{\frac{25}{28}}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

= RHS

So,

$\tan^{-1}(\frac{1}{7}) + 2\tan^{-1}(\frac{1}{3}) = \frac{\pi}{4}$

Hence Proved

2 E. Question

Prove the following results:

$$\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} = \frac{\pi}{2}$$

Answer

Given:- $\sin^{-1}(\frac{4}{5}) + 2\tan^{-1}(\frac{1}{3}) = \frac{\pi}{2}$

Take

LHS

$= \sin^{-1}(\frac{4}{5}) + 2\tan^{-1}(\frac{1}{3})$

We know that, Formula

$$\sin^{-1}(x) = \tan^{-1}(\frac{x}{\sqrt{1-x^2}})$$

And,

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$$= \tan^{-1}(\frac{\frac{4}{5}}{\sqrt{1-\frac{16}{25}}}) + \tan^{-1}(\frac{2\times\frac{1}{3}}{1-\frac{1}{9}})$$

$$= \tan^{-1}(\frac{\frac{4}{5}}{\sqrt{\frac{9}{25}}}) + \tan^{-1}(\frac{\frac{2}{3}}{\frac{8}{9}})$$

$$= \tan^{-1}(\frac{4}{3}) + \tan^{-1}(\frac{3}{4})$$

We know that, Formula

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

$$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{3}{4}}{1-\frac{4}{3}\times\frac{3}{4}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{25}{12}}{0}\right)$$

$$= \tan^{-1}(\infty)$$

$$= \frac{\pi}{2}$$

= RHS

So,

$$\sin^{-1}(\frac{4}{5}) + 2\tan^{-1}(\frac{1}{3}) = \frac{\pi}{2}$$

Hence Proved

2 F. Question

Prove the following results:

$$2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$$

Answer

Given:- $2\sin^{-1}(\frac{3}{5}) - \tan^{-1}(\frac{17}{31}) = \frac{\pi}{4}$

Take

LHS

$$= 2\sin^{-1}(\frac{3}{5}) - \tan^{-1}(\frac{17}{31})$$

We know that, Formula

$$\sin^{-1}(x) = \tan^{-1}(\frac{x}{\sqrt{1-x^2}})$$

Thus,

$$=2\tan^{-1}(\frac{\frac{3}{5}}{\sqrt{1-\frac{16}{25}}}) - \tan^{-1}(\frac{17}{31})$$

$$=2\tan^{-1}(\frac{\frac{4}{5}}{\sqrt{\frac{9}{25}}}) - \tan^{-1}(\frac{17}{31})$$

$$= 2\tan^{-1}(\frac{3}{4}) - \tan^{-1}(\frac{17}{31})$$

We know that, Formula

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$$=\tan^{-1}(\frac{2\times\frac{3}{4}}{1-\frac{9}{16}}) - \tan^{-1}(\frac{17}{31})$$

$$= \tan^{-1}(\frac{\frac{3}{2}}{\frac{7}{16}}) - \tan^{-1}(\frac{17}{31})$$

$$= \tan^{-1}(\frac{24}{7}) - \tan^{-1}(\frac{17}{31})$$

We know that, Formula

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

$$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\times\frac{17}{31}}\right)$$

$= \tan^{-1}\left(\frac{\frac{744-119}{217}}{\frac{217+408}{217}}\right)$

$= \tan^{-1}\left(\frac{625}{625}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

= RHS

So,

$2\sin^{-1}(\frac{3}{5}) - \tan^{-1}(\frac{17}{31}) = \frac{\pi}{4}$

Hence Proved

2 G. Question

Prove the following results:

$$2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \tan^{-1}\left(\frac{4}{7}\right)$$

Answer

Given:- $2\tan^{-1}(\frac{1}{5}) + \tan^{-1}(\frac{1}{8}) = \tan^{-1}(\frac{4}{7})$

Take

LHS

$= 2\tan^{-1}(\frac{1}{5}) + \tan^{-1}(\frac{1}{8})$

We know that, Formula

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Thus,

$=\tan^{-1}(\frac{2\times\frac{1}{5}}{1-\frac{1}{25}}) + \tan^{-1}(\frac{1}{8})$

$= \tan^{-1}(\frac{\frac{2}{5}}{\frac{24}{25}}) + \tan^{-1}(\frac{1}{8})$

$= \tan^{-1}(\frac{5}{12}) + \tan^{-1}(\frac{1}{8})$

We know that, Formula

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{1}{8}}{1-\frac{5}{12}\times\frac{1}{8}}\right)$

$= \tan^{-1}\left(\frac{\frac{10+3}{24}}{\frac{96-5}{96}}\right)$

$= \tan^{-1}\left(\frac{13}{24}\times\frac{96}{91}\right)$

$= \tan^{-1}\left(\frac{4}{7}\right)$

= RHS

So,

$2\tan^{-1}(\frac{1}{5}) + \tan^{-1}(\frac{1}{8}) = \tan^{-1}(\frac{4}{7})$

Hence Proved

2 H. Question

Prove the following results:

$$2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$$

Answer

Given:- $2\tan^{-1}(\frac{3}{4}) - \tan^{-1}(\frac{17}{31}) = \frac{\pi}{4}$

Take

LHS

$= 2\tan^{-1}(\frac{3}{4}) - \tan^{-1}(\frac{17}{31})$

We know that, Formula

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$=\tan^{-1}(\frac{2\times\frac{3}{4}}{1-\frac{9}{16}}) - \tan^{-1}(\frac{17}{31})$

$= \tan^{-1}(\frac{3}{2}\times\frac{16}{7}) - \tan^{-1}(\frac{17}{31})$

$= \tan^{-1}(\frac{24}{7}) - \tan^{-1}(\frac{17}{31})$

We know that, Formula

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\times\frac{17}{31}}\right)$

$= \tan^{-1}\left(\frac{\frac{744-119}{217}}{\frac{217+408}{217}}\right)$

$= \tan^{-1}\left(\frac{625}{625}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

= RHS

So,

$2\tan^{-1}(\frac{3}{4}) - \tan^{-1}(\frac{17}{31}) = \frac{\pi}{4}$

Hence Proved

2 I. Question

Prove the following results:

$$2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$$

Answer

Given:- $2\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{7}) = \tan^{-1}(\frac{31}{17})$

Take

LHS

$= 2\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{7})$

We know that, Formula

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$= \tan^{-1}(\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}) + \tan^{-1}(\frac{1}{7})$

$= \tan^{-1}(\frac{\frac{2}{2}}{\frac{3}{4}}) + \tan^{-1}(\frac{1}{7})$

$= \tan^{-1}(\frac{4}{3}) + \tan^{-1}(\frac{1}{7})$

We know that, Formula

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{1}{7}\times\frac{4}{3}}\right)$

$= \tan^{-1}\left(\frac{\frac{31}{21}}{\frac{17}{21}}\right)$

$= \tan^{-1}\left(\frac{31}{17}\right)$

= RHS

So,

$2\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{7}) = \tan^{-1}(\frac{31}{17})$

Hence Proved

2 J. Question

Prove the following results:

$$4\tan^{-1}\left(\frac{1}{5}\right)-\tan^{-1}\left(\frac{1}{239}\right)=\frac{\pi}{4}$$

Answer

Given:- $4\tan^{-1}(\frac{1}{5}) - \tan^{-1}(\frac{1}{239}) = \frac{\pi}{4}$

Take

LHS

$= 4\tan^{-1}(\frac{1}{5}) - \tan^{-1}(\frac{1}{239})$

We know that, Formula

$$4\tan^{-1}x = \tan^{-1}\left(\frac{4x - 4x^3}{1 - 6x^2 + x^4}\right)$$

Thus,

$$=\tan^{-1}\left(\frac{4\times\frac{1}{5}-4\left(\frac{1}{5}\right)^3}{1-6\left(\frac{1}{5}\right)^2+\left(\frac{1}{5}\right)^4}\right) - \tan^{-1}(\frac{1}{239})$$

$= \tan^{-1}(\frac{120}{119}) - \tan^{-1}(\frac{1}{239})$

We know that, Formula

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

$$= \tan^{-1}\left(\frac{\frac{120}{119}-\frac{1}{239}}{1\frac{120}{119}\times\frac{1}{239}}\right)$$

$$= \tan^{-1}\left(\frac{120\times239-119}{119\times239+120}\right)$$

$$= \tan^{-1}\left(\frac{28561}{28561}\right)$$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

= RHS

So,

$$4\tan^{-1}(\frac{1}{5}) - \tan^{-1}(\frac{1}{239}) = \frac{\pi}{4}$$

Hence Proved

3. Question

If $\sin^{-1}\frac{2a}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\frac{2x}{1-x^2}$,

Then prove that $x = \frac{a-b}{1+ab}$.

Answer

Given:- $\sin^{-1}(\frac{2a}{1+a^2}) - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}(\frac{2x}{1-x^2})$

Take

$\Rightarrow \sin^{-1}(\frac{2a}{1+a^2}) - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}(\frac{2x}{1-x^2})$

We know that, Formula

$$2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

And

$$2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$\Rightarrow 2\tan^{-1}(a) - 2\tan^{-1}(b) = 2\tan^{-1}(x)$

$\Rightarrow 2(\tan^{-1}(a) - \tan^{-1}(b)) = 2\tan^{-1}(x)$

$\Rightarrow \tan^{-1}(a) - \tan^{-1}(b) = \tan^{-1}(x)$

We know that, Formula

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy}$$

Thus,

$\Rightarrow \tan^{-1}(\frac{a-b}{1+ab}) = \tan^{-1}(x)$

On comparing we get,

$\Rightarrow x = \frac{a-b}{1+ab}$

Hence Proved

4 A. Question

Prove that:

$$\tan^{-1}\left(\frac{1-x^2}{2x}\right)\cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{2}$$

Answer

Given:- $\tan^{-1}\frac{1-x^2}{2x} + \cot^{-1}\frac{1-x^2}{2x} = \frac{\pi}{2}$

Take

LHS

$= \tan^{-1}\frac{1-x^2}{2x} + \cot^{-1}\frac{1-x^2}{2x}$

We know that, Formula

$$\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$$

Thus,

$$=\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

We know that, Formula

$$\tan^{-1}x+\tan^{-1}y=\ \tan^{-1}\frac{x+y}{1-xy}$$

$$=\tan^{-1}\left(\frac{\left(\frac{1-x^2}{2x}\right)+\left(\frac{2x}{1-x^2}\right)}{1-\left(\frac{1-x^2}{2x}\right)\times\left(\frac{2x}{1-x^2}\right)}\right)$$

$$=\tan^{-1}\left(\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{\frac{2x(1-x^2)-2x(1-x^2)}{2x(1-x^2)}}\right)$$

$$=\tan^{-1}\left(\frac{1+x^4+2x^2}{0}\right)$$

$$=\tan^{-1}(\infty)$$

$$=\frac{\pi}{2}$$

= RHS

So,

$$\tan^{-1}\frac{1-x^2}{2x}+\cot^{-1}\frac{1-x^2}{2x}=\frac{\pi}{2}$$

Hence Proved

4 B. Question

Prove that:

$$\sin^{-1}\left\{\tan^{-1}\frac{1-x^2}{2x}+\cos^{-1}\frac{1-x^2}{1+x^2}\right\}=1$$

Answer

Given:- $\sin\left(\tan^{-1}\frac{1-x^2}{2x}+\cos^{-1}\frac{1-x^2}{1+x^2}\right)=1$

Take

LHS

$$=\sin\left(\tan^{-1}\frac{1-x^2}{2x}+\cos^{-1}\frac{1-x^2}{1+x^2}\right)$$

We know that, Formula

$$2\tan^{-1}x=\ \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

Thus,

$$=\sin\left(\tan^{-1}\frac{1-x^2}{2x}+2\tan^{-1}x\right)$$

Again,

$$2\tan^{-1}x=\ \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$= \sin\left(\tan^{-1}\frac{1-x^2}{2x} + \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right)$

We know that, Formula

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

$= \sin\left(\tan^{-1}\left(\frac{\frac{1-x^2}{2x}+\left(\frac{2x}{1-x^2}\right)}{1-\frac{1-x^2}{2x}\times\left(\frac{2x}{1-x^2}\right)}\right)\right)$

$= \sin\left(\tan^{-1}\left(\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{\frac{2x(1-x^2)-2x(1-x^2)}{2x(1-x^2)}}\right)\right)$

$= \sin\left(\tan^{-1}\left(\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{0}\right)\right)$

$= \sin(\tan^{-1}(\infty))$

$= \sin\left(\frac{\pi}{2}\right)$

= 1

= RHS

So,

$\sin^{-1}\left(\tan^{-1}\frac{1-x^2}{2x} + \cos^{-1}\frac{1-x^2}{1+x^2}\right) = 1$

Hence Proved

5. Question

If $\sin^{-1}\frac{2a}{1+a^2} - \sin^{-1}\frac{2b}{1+b^2} = \tan^{-1}x$, Prove that $x = \frac{ab}{1-ab}$

Answer

Given:- $\sin^{-1}(\frac{2a}{1+a^2}) + \sin^{-1}\frac{2b}{1+b^2} = 2\tan^{-1}(x)$

Take

$\sin^{-1}(\frac{2a}{1+a^2}) + \sin^{-1}\frac{2b}{1+b^2} = 2\tan^{-1}(x)$

We know that, Formula

$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Thus,

$\Rightarrow 2\tan^{-1}(a) + 2\tan^{-1}(b) = 2\tan^{-1}(x)$

$\Rightarrow 2(\tan^{-1}(a) + \tan^{-1}(b)) = 2\tan^{-1}(x)$

$\Rightarrow \tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}(x)$

We know that, Formula

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

Thus,

$\Rightarrow \tan^{-1}(\frac{a+b}{1-ab}) = \tan^{-1}(x)$

On comparing we get,

$\Rightarrow x = \frac{a+b}{1-ab}$

Hence Proved

6. Question

Show that $2\tan^{-1}x + \sin^{-1}\frac{2x}{1+x^2}$ is constant for x ≥1, find the constant.

Answer

Given:- $2\tan^{-1}(x) + \sin^{-1}(\frac{2x}{1+x^2})$

Take

$2\tan^{-1}(x) + \sin^{-1}(\frac{2x}{1+x^2})$

We know that, Formula

$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Thus,

$=2\tan^{-1}(x) + 2\tan^{-1}(x)$

$=4\tan^{-1}(x)$

Now as given,

For, x ≥1

$= 4\tan^{-1}(1)$

$= 4 \times \frac{\pi}{4}$

= π

= Constant

So,

$2\tan^{-1}(x) + \sin^{-1}(\frac{2x}{1+x^2}) = \pi$

7 A. Question

Find the values of each of the following:

$\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$

Answer

Given:- $\tan^{-1}\left(2\cos\left(2\sin^{-1}(\frac{1}{2})\right)\right)$

Take

$\tan^{-1}\left(2\cos\left(2\sin^{-1}(\frac{1}{2})\right)\right)$

We know that, Formula

$\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

Therefore,

$\cos(2 \times \frac{\pi}{6}) = \frac{1}{2}$

Thus,

$=\tan^{-1}\left(2\cos\left(2 \times \frac{\pi}{6}\right)\right)$

$=\tan^{-1}\left(2\cos\left(\frac{\pi}{3}\right)\right)$

$=\tan^{-1}\left(2 \times \frac{1}{2}\right)$

$=\tan^{-1}(1)$

$= \frac{\pi}{4}$

So,

$\tan^{-1}\left(2\cos\left(2\sin^{-1}(\frac{1}{2})\right)\right) = \frac{\pi}{4}$

7 B. Question

Find the values of each of the following:

$\cos\left(\sec^{-1}x - \text{cosec}^{-1}x\right), |x| \geq 1$

Answer

Given:- $\cos(\sec^{-1}x - \text{cosec}^{-1}x)$

Take

$\cos(\sec^{-1}x - \text{cosec}^{-1}x)$

We know that, Formula

$\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}$

Therefore,

$=\cos(\frac{\pi}{2})$

= 0

So,

$\Rightarrow \cos(\sec^{-1}x - \text{cosec}^{-1}x) = 0$

8 A. Question

Solve the following equations for x:

$$\tan^{-1}\frac{1}{4}+2\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$$

Answer

Given:- $\tan^{-1}\frac{1}{4}+2\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$

Take

$\Rightarrow \tan^{-1}\frac{1}{4}+2\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$

We know that, Formula

$2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Thus,

$\Rightarrow \tan^{-1}\frac{1}{4}+\tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\frac{1}{25}}\right)+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\frac{1}{4}+\tan^{-1}\left(\frac{\frac{2}{5}}{\frac{24}{25}}\right)+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\frac{1}{4}+\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$

We know that, Formula

$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{4}+\frac{5}{12}}{1-\frac{1}{4}\times\frac{5}{12}}\right)+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{8}{12}}{\frac{43}{48}}\right)+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{32}{43}\right)+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$

We know that, Formula

$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$

And $\tan^{-1}1=\frac{\pi}{4}$

Thus,

$\Rightarrow \tan^{-1}\left(\frac{\frac{32}{43}+\frac{1}{6}}{1-\frac{32}{43}\times\frac{1}{6}}\right)+\tan^{-1}\frac{1}{x}=\tan^{-1}1$

$\Rightarrow \tan^{-1}\left(\frac{\frac{235}{258}}{\frac{226}{258}}\right)+\tan^{-1}\frac{1}{x}=\tan^{-1}1$

$\Rightarrow \tan^{-1}\left(\frac{235}{226}\right)+\tan^{-1}\frac{1}{x}=\tan^{-1}1$

We know that, Formula

$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x + y}{1 - xy}$

Thus,

$\Rightarrow \tan^{-1}\left(\frac{\frac{235}{226}+\frac{1}{x}}{1-\frac{235}{226}\times\frac{1}{x}}\right) = \tan^{-1} 1$, here $\frac{235}{226x} < 1$

On comparing we get,

$\Rightarrow \frac{\frac{235}{226}+\frac{1}{x}}{1-\frac{235}{226}\times\frac{1}{x}} = 1$

$\Rightarrow \frac{235x+226}{226x-235} = 1$

⇒235x+226 = 226x-235

⇒235x – 226x = 226 + 235

$\Rightarrow x = -\frac{461}{9}$

8 B. Question

Solve the following equations for x:

$$3\sin^{-1}\frac{2s}{1+x^2} - 4\cos^{-1}\frac{1-x^2}{1+x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$$

Answer

Given:- $3\sin^{-1}\frac{2x}{1-x^2} - 4\cos^{-1}\frac{1+x^2}{1-x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$

Take

$\Rightarrow 3\sin^{-1}\frac{2x}{1-x^2} - 4\cos^{-1}\frac{1+x^2}{1-x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$

We know that, Formula

$2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$

$2\tan^{-1} x = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$

And

$2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$

Thus,

$\Rightarrow 3(2\tan^{-1}(x)) - 4(2\tan^{-1}(x)) + 2(2\tan^{-1}(x)) = \frac{\pi}{3}$

$\Rightarrow 6\tan^{-1}(x) - 8\tan^{-1}(x) + 4\tan^{-1}(x) = \frac{\pi}{3}$

$\Rightarrow 2\tan^{-1}(x) = \frac{\pi}{3}$

$\Rightarrow \tan^{-1}(x) = \frac{\pi}{6}$

$\Rightarrow x = \tan\frac{\pi}{6}$

$\Rightarrow x = \frac{1}{\sqrt{3}}$

8 C. Question

Solve the following equations for x:

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{2\pi}{3}, x>0$$

Answer

Given:- $\tan^{-1}\frac{2x}{1-x^2}+\cot^{-1}\frac{1-x^2}{2x}=\frac{2\pi}{3}, x>0$

Take

$\Rightarrow \tan^{-1}\frac{2x}{1-x^2}+\cot^{-1}\frac{1-x^2}{2x}=\frac{2\pi}{3}$

We know that, Formula

$\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$

Thus,

$\Rightarrow \tan^{-1}\frac{2x}{1-x^2}+\tan^{-1}\frac{2x}{1-x^2}=\frac{2\pi}{3}$

We know that, Formula

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Thus,

$\Rightarrow 2\tan^{-1}(x)+2\tan^{-1}(x)=\frac{2\pi}{3}$

$\Rightarrow 4\tan^{-1}(x)=\frac{2\pi}{3}$

$\Rightarrow \tan^{-1}(x)=\frac{\pi}{6}$

$\Rightarrow x = \tan\frac{\pi}{6}$

$\Rightarrow x = \frac{1}{\sqrt{3}}$

8 D. Question

Solve the following equations for x:

$$2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x), x\neq\frac{\pi}{2}$$

Answer

Given:- $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x), x \neq \frac{\pi}{2}$

Take

$2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x)$

We know that, Formula

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Thus, here x = sinx

$\Rightarrow \tan^{-1}\left(\frac{2\times \sin x}{1-\sin^2 x}\right) = \tan^{-1}(2\sec x)$

We know that, Formula

$\cos x = 1 - \sin^2 x$

$\Rightarrow \frac{2\sin x}{\cos^2 x} = 2\sec x$

$\Rightarrow \frac{\sin x}{\cos x} = 1$

$\Rightarrow \tan x = 1$

$\Rightarrow x = \frac{\pi}{4}$

Thus the solution is $x = n\pi + \frac{\pi}{4}$

8 E. Question

Solve the following equations for x:

$$\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \frac{1}{2}\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{2\pi}{3}$$

Answer

Given:- $\cos^{-1}\frac{x^2-1}{x^2+1} + \frac{1}{2}\tan^{-1}\frac{2x}{1-x^2} = \frac{2\pi}{3}$

Take

$\Rightarrow \cos^{-1}\frac{x^2-1}{x^2+1} + \frac{1}{2}\tan^{-1}\frac{2x}{1-x^2} = \frac{2\pi}{3}$

$\Rightarrow \cos^{-1}(-\frac{1-x^2}{x^2+1}) + \frac{1}{2}\tan^{-1}\frac{2x}{1-x^2} = \frac{2\pi}{3}$

We know that, Formula

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

And,

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Thus,

$\Rightarrow \pi - 2\tan^{-1}(x) + \frac{1}{2}\times 2\tan^{-1}(x) = \frac{2\pi}{3}$

$\Rightarrow \pi - \tan^{-1}(x) = \frac{2\pi}{3}$

$\Rightarrow \tan^{-1}(x) = \pi - \frac{2\pi}{3}$

$\Rightarrow x = \tan\frac{\pi}{3}$

$\Rightarrow x = \sqrt{3}$

8 F. Question

Solve the following equations for x:

$$\tan^{-1}\left(\frac{x-2}{x\ \ -1}\right)+\tan^{-1}\left(\frac{x+2}{x+1}\right)=\frac{\pi}{4}$$

Answer

Given:- $\tan^{-1}\frac{x-2}{x-1}+\tan^{-1}\frac{x+2}{x+1}=\frac{\pi}{4}$

Take

$\Rightarrow \tan^{-1}\frac{x-2}{x-1}+\tan^{-1}\frac{x+2}{x+1}=\frac{\pi}{4}$

We know that, Formula

$\tan^{-1}1=\frac{\pi}{4}$

Thus,

$\Rightarrow \tan^{-1}\frac{x-2}{x-1}+\tan^{-1}\frac{x+2}{x+1}=\tan^{-1}1$

$\Rightarrow \tan^{-1}\frac{x-2}{x-1}=\tan^{-1}1-\tan^{-1}\frac{x+2}{x+1}$

We know that, Formula

$\tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}$

$\Rightarrow \tan^{-1}\frac{x-2}{x-1}=\tan^{-1}\left(\frac{1-\frac{x+2}{x+1}}{1+1\times\frac{x+2}{x+1}}\right)$

$\Rightarrow \tan^{-1}\frac{x-2}{x-1}=\tan^{-1}\left(\frac{\frac{x+1-x-2}{x+1}}{\frac{x+1+x+2}{x+1}}\right)$

$\Rightarrow \tan^{-1}\frac{x-2}{x-1}=\tan^{-1}\left(\frac{-1}{2x+3}\right)$

On comparing we get,

$\Rightarrow \frac{x-2}{x-1}=\frac{-1}{2x+3}$

$\Rightarrow \frac{(2x+3)(x-2)}{(x-1)}=-1$

$\Rightarrow (2x+3)(x-2)= -(x-1)$

$\Rightarrow 2x^2 - 4x + 3x - 6 = - x + 1$

$\Rightarrow 2x^2 - x - 6 = - x + 1$

$\Rightarrow 2x^2= 7$

$\Rightarrow x = \pm\sqrt{\frac{7}{2}}$

9. Question

Prove that $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)=\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right)$

Answer

Given:- $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{\theta}{2}\right)\right)=\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right)$

Take

LHS

$= 2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{\theta}{2}\right)\right)$

We know that, Formula

$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Thus,

$= \cos^{-1}\left(\frac{1-\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{\theta}{2}\right)\right)^2}{1+\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{\theta}{2}\right)\right)^2}\right)$

$= \cos^{-1}\left(\frac{1-\frac{a-b}{a+b}\times\tan^2\left(\frac{\theta}{2}\right)}{1+\frac{a-b}{a+b}\times\tan^2\left(\frac{\theta}{2}\right)}\right)$

$= \cos^{-1}\left(\frac{\frac{a+b-(a-b)}{a+b}\times\tan^2\left(\frac{\theta}{2}\right)}{\frac{a+b+(a-b)}{a+b}\times\tan^2\left(\frac{\theta}{2}\right)}\right)$

$= \cos^{-1}\left(\frac{a+b-(a-b)\times\tan^2\left(\frac{\theta}{2}\right)}{a+b+(a-b)\times\tan^2\left(\frac{\theta}{2}\right)}\right)$

$=\cos^{-1}\left(\frac{a\left(1-\tan^2\left(\frac{\theta}{2}\right)\right)+b\left(1+\tan^2\left(\frac{\theta}{2}\right)\right)}{a\left(1+\tan^2\left(\frac{\theta}{2}\right)\right)+b\left(1-\tan^2\left(\frac{\theta}{2}\right)\right)}\right)$

Dividing numerator and denominator by $\left(1+\tan^2\left(\frac{\theta}{2}\right)\right)$, we get

$=\cos^{-1}\left(\frac{a\left(\frac{1-\tan^2\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}\right)+b}{a+b\left(\frac{1-\tan^2\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}\right)}\right)$

We know that, Formula

$\cos x = \frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$

Thus,

$= \cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right)$

= RHS

So,

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{\theta}{2}\right)\right) = \cos^{-1}\left(\frac{a\cos\theta + b}{a + b\cos\theta}\right)$$

Hence Proved

10. Question

prove that:

$\tan^{-1}\frac{2ab}{a^2-b^2} + \tan^{-1}\frac{2xy}{x^2-y^2} = \tan^{-1}\frac{2a\beta}{a^2-b^2}$, where a= ax – by and β=ay+bx.

Answer

Given:- $\tan^{-1}\frac{2ab}{a^2-b^2} + \tan^{-1}\frac{2xy}{x^2-y^2} = \tan^{-1}\frac{2\alpha\beta}{\alpha^2-\beta^2}$

Take

LHS

$= \tan^{-1}\frac{2ab}{a^2-b^2} + \tan^{-1}\frac{2xy}{x^2-y^2}$

We know that, Formula

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

Thus,

$$=\tan^{-1}\frac{\left(\frac{2ab}{a^2-b^2}\right)+\left(\frac{2xy}{x^2-y^2}\right)}{1-\left(\frac{2ab}{a^2-b^2}\right)\left(\frac{2xy}{x^2-y^2}\right)}$$

$$= \tan^{-1}\frac{\frac{2abx^2-2aby^2+2xya^2-2xyb^2}{(a^2-b^2)(x^2-y^2)}}{\frac{a^2x^2+b^2y^2-2abxy-a^2y^2-b^2x^2-2abxy}{(a^2-b^2)(x^2-y^2)}}$$

$$= \tan^{-1}\frac{2(abx^2-aby^2+xya^2-xyb^2)}{a^2x^2+b^2y^2-2abxy-a^2y^2-b^2x^2-2abxy}$$

Formula used:- $a^2 + b^2 + 2ab = (a+b)^2$

$$= \tan^{-1}\frac{2\{(bx+ay)(ax-by)\}}{(ax-by)^2-(a^2y^2+b^2x^2+2abxy)}$$

$$=\tan^{-1}\frac{2\{ax(bx+ay)+by(ay+bx)\}}{(ax-by)^2-(bx+ay)^2}$$

As given

a= ax–by and β= ay+bx

Thus,

$$\tan^{-1}\frac{2\alpha\beta}{\alpha^2-\beta^2}$$

= RHS

So,

$$\tan^{-1}\frac{2ab}{a^2-b^2} + \tan^{-1}\frac{2xy}{x^2-y^2} = \tan^{-1}\frac{2\alpha\beta}{\alpha^2-\beta^2}$$

Hence Proved

11. Question

For any a,b,x,y>0, prove that:

$$\frac{2}{3}\tan^{-1}\left(\frac{3ab^2-a^3}{b^3-3a^2b}\right)\frac{2}{3}\tan^{-1}\left(\frac{3xy^2-x^3}{y^3-3x^2y}\right)=\tan^{-1}\frac{2a\beta}{a^2-b^2}$$, where a=-ax by, β = bx + ay.

Answer

Given:- $\frac{2}{3}\tan^{-1}\frac{3ab^2-a^3}{b^3-3a^2b}+\frac{2}{3}\tan^{-1}\frac{3xy^2-x^3}{y^2-3x^2y}=\tan^{-1}\frac{2\alpha\beta}{\alpha^2-\beta^2}$

Take

LHS

$=\frac{2}{3}\tan^{-1}\frac{3ab^2-a^3}{b^3-3a^2b}+\frac{2}{3}\tan^{-1}\frac{3xy^2-x^3}{y^2-3x^2y}$

Dividing numerator and denominator of 1st term and 2nd term by b^3 and y^3 respectively.

$=\frac{2}{3}\tan^{-1}\frac{\frac{3ab^2-a^3}{b^3}}{\frac{b^3-3a^2b}{b^3}}+\frac{2}{3}\tan^{-1}\frac{\frac{3xy^2-x^3}{y^3}}{\frac{y^2-3x^2y}{y^3}}$

$=\frac{2}{3}\tan^{-1}\frac{3\left(\frac{a}{b}\right)-\left(\frac{a}{b}\right)^3}{1-3\left(\frac{a}{b}\right)^2}+\frac{2}{3}\tan^{-1}\frac{3\left(\frac{x}{y}\right)-\left(\frac{x}{y}\right)^3}{1-3\left(\frac{x}{y}\right)^2}$

We know that, Formula

$$3\tan^{-1}x=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

Thus,

$=\frac{2}{3}\left\{3\tan^{-1}\left(\frac{a}{b}\right)\right\}+\frac{2}{3}\left\{3\tan^{-1}\left(\frac{x}{y}\right)\right\}$

$=2\tan^{-1}\left(\frac{a}{b}\right)+2\tan^{-1}\left(\frac{x}{y}\right)$

$=2\left(\tan^{-1}\left(\frac{a}{b}\right)+\tan^{-1}\left(\frac{x}{y}\right)\right)$

We know that, Formula

$$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$$

Thus,

$=2\tan^{-1}\frac{\left(\frac{a}{b}\right)+\left(\frac{x}{y}\right)}{1-\left(\frac{a}{b}\right)\left(\frac{x}{y}\right)}$

$=2\tan^{-1}\frac{\left(\frac{ay+bx}{by}\right)}{\left(\frac{by-ax}{by}\right)}$

$=2\tan^{-1}\frac{ay+bx}{by-ax}$

As given,

ay + bx =β, -ax + by = α

$=2\tan^{-1}\frac{\beta}{\alpha}$

We know that, Formula

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$$= \tan^{-1}\left(\frac{2\times\frac{\beta}{\alpha}}{1-\left(\frac{\beta}{\alpha}\right)^2}\right)$$

$$= \tan^{-1}\left(\frac{2\beta}{\alpha}\times\frac{\alpha^2}{\alpha^2-\beta^2}\right)$$

$$=\tan^{-1}\frac{2\alpha\beta}{\alpha^2-\beta^2}$$

= RHS

So,

$$\frac{2}{3}\tan^{-1}\frac{3ab^2-a^3}{b^3-3a^2b}+\frac{2}{3}\tan^{-1}\frac{3xy^2-x^3}{y^2-3x^2y} = \tan^{-1}\frac{2\alpha\beta}{\alpha^2-\beta^2}$$

Hence Proved

MCQ

1. Question

Choose the correct answer

If $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\} = \alpha$, then x^2 =

A. sin 2α

B. sin α

C. cos 2α

D. cos α

Answer

We are given that,

$$\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\} = \alpha$$

We need to find the value of x^2.

Take,

$$\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\} = \alpha$$

Multiply on both sides by tangent.

$$\Rightarrow \tan\left[\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}\right] = \tan\alpha$$

Since, we know that $\tan(\tan^{-1} x) = x$.

So,

$$\Rightarrow \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}} = \tan\alpha$$

Or

$$\tan\alpha = \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$$

Now, we need to simplify it in order to find x^2. So, rationalize the denominator by multiplying and dividing by $\sqrt{1+x^2}-\sqrt{1-x^2}$.

$$\Rightarrow \tan\alpha = \left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right) \times \left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$$

$$= \frac{\left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)^2}{\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)\left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)}$$

Note the denominator is in the form: (x + y)(x – y), where

$(x + y)(x – y) = x^2 – y^2$

So,

$$\Rightarrow \tan\alpha = \frac{\left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)^2}{\left(\sqrt{1+x^2}\right)^2-\left(\sqrt{1-x^2}\right)^2} \ldots(i)$$

Numerator:

Applying the algebraic identity in the numerator, $(x – y)^2 = x^2 + y^2 – 2xy$.

We can write as,

$$\left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)^2 = \left(\sqrt{1+x^2}\right)^2 + \left(\sqrt{1-x^2}\right)^2 - 2\sqrt{1+x^2}\sqrt{1-x^2}$$

$$\Rightarrow \left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)^2 = (1+x^2) + (1-x^2) - 2\sqrt{(1+x^2)(1-x^2)}$$

Again using the identity, $(x + y)(x – y) = x^2 – y^2$.

$$\Rightarrow \left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)^2 = 1 + x^2 + 1 - x^2 - 2\sqrt{1-(x^2)^2}$$

$$\Rightarrow \left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)^2 = 2 - 2\sqrt{1-x^4} \ldots(ii)$$

Denominator:

Solving the denominator, we get

$$\left(\sqrt{1+x^2}\right)^2 - \left(\sqrt{1-x^2}\right)^2 = (1+x^2) - (1-x^2)$$

$$\Rightarrow \left(\sqrt{1+x^2}\right)^2 - \left(\sqrt{1-x^2}\right)^2 = 1 + x^2 - 1 + x^2$$

$$\Rightarrow \left(\sqrt{1+x^2}\right)^2 - \left(\sqrt{1-x^2}\right)^2 = 2x^2 \ldots(iii)$$

Substituting values of Numerator and Denominator from (ii) and (iii) in equation (i),

$$\Rightarrow \tan\alpha = \frac{2 - 2\sqrt{1-x^4}}{2x^2}$$

$\Rightarrow \tan\alpha = \frac{2\left(1-\sqrt{1-x^4}\right)}{2x^2}$

$\Rightarrow \tan\alpha = \frac{1-\sqrt{1-x^4}}{x^2}$

By cross-multiplication,

$\Rightarrow x^2 \tan \alpha = 1 - \sqrt{(1 - x^4)}$

$\Rightarrow \sqrt{(1 - x^4)} = 1 - x^2 \tan \alpha$

Squaring on both sides,

$\Rightarrow [\sqrt{(1 - x^4)}]^2 = [1 - x^2 \tan \alpha]^2$

$\Rightarrow 1 - x^4 = (1)^2 + (x^2 \tan \alpha)^2 - 2x^2 \tan \alpha$ [$\because$, $(x - y)^2 = x^2 + y^2 - 2xy$]

$\Rightarrow 1 - x^4 = 1 + x^4 \tan^2 \alpha - 2x^2 \tan \alpha$

$\Rightarrow x^4 \tan^2 \alpha - 2x^2 \tan \alpha + x^4 + 1 - 1 = 0$

$\Rightarrow x^4 \tan^2 \alpha - 2x^2 \tan \alpha + x^4 = 0$

Rearranging,

$\Rightarrow x^4 + x^4 \tan^2 \alpha - 2x^2 \tan \alpha = 0$

$\Rightarrow x^4 (1 + \tan^2 \alpha) - 2x^2 \tan \alpha = 0$

$\Rightarrow x^4 (\sec^2 \alpha) - 2x^2 \tan \alpha = 0$ [$\because$, $\sec^2 x - \tan^2 x = 1 \Rightarrow 1 + \tan^2 x = \sec^2 x$]

Taking x^2 common from both terms,

$\Rightarrow x^2 (x^2 \sec^2 \alpha - 2 \tan \alpha) = 0$

$\Rightarrow x^2 = 0$ or $(x^2 \sec^2 \alpha - 2 \tan \alpha) = 0$

But $x^2 \neq 0$ as according to the question, we need to find some value of x^2.

$\Rightarrow x^2 \sec^2 \alpha - 2 \tan \alpha = 0$

$\Rightarrow x^2 \sec^2 \alpha = 2 \tan \alpha$

In order to find the value of x^2, shift $\sec^2 \alpha$ to Right Hand Side (RHS).

$\Rightarrow x^2 = \frac{2\tan\alpha}{\sec^2\alpha}$

Putting $\sec^2\alpha = \frac{1}{\cos^2\alpha}$ and $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$,

$\Rightarrow x^2 = \frac{2\left(\frac{\sin\alpha}{\cos\alpha}\right)}{\frac{1}{\cos^2\alpha}}$

$\Rightarrow x^2 = 2 \times \frac{\sin\alpha}{\cos\alpha} \times \cos^2\alpha$

$\Rightarrow x^2 = 2 \sin \alpha \cos \alpha$

Using the trigonometric identity, $2 \sin x \cos x = \sin 2x$.

$\Rightarrow x^2 = \sin 2\alpha$

2. Question

Choose the correct answer

The value of $\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right\}$ is

A. $\frac{\sqrt{29}}{3}$

B. $\frac{29}{3}$

C. $\frac{\sqrt{3}}{29}$

D. $\frac{3}{29}$

Answer

We need to find the value of

$\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right\}$

Let,

$\cos^{-1}\frac{1}{5\sqrt{2}}=\mathrm{a}$ and $\sin^{-1}\frac{4}{\sqrt{17}}=\mathrm{b}$

$\Rightarrow \cos \mathrm{a}=\frac{1}{5\sqrt{2}}$ and $\sin \mathrm{b}=\frac{4}{\sqrt{17}}$

Let us find sin a and cos b.

For sin a,

We know the trigonometric identity, $\sin^2 a + \cos^2 a = 1$

$\Rightarrow \sin^2 a = 1 - \cos^2 a$

$\Rightarrow \sin a = \sqrt{(1 - \cos^2 a)}$

Substituting the value of cos a,

$\Rightarrow \sin \mathrm{a}=\sqrt{1-\left(\frac{1}{5\sqrt{2}}\right)^2}$

$=\sqrt{1-\frac{1}{50}}$

$=\sqrt{\frac{50-1}{50}}$

$=\sqrt{\frac{49}{50}}$

$=\frac{7}{5\sqrt{2}}$

We have $\sin \mathrm{a}=\frac{7}{5\sqrt{2}}$ and $\cos \mathrm{a}=\frac{1}{5\sqrt{2}}$.

So, we can find tan a.

$\because, \tan a = \frac{\sin a}{\cos a}$

$= \frac{\frac{7}{5\sqrt{2}}}{\frac{1}{5\sqrt{2}}}$

$= \frac{7}{5\sqrt{2}} \times 5\sqrt{2}$

⇒ tan a = 7 …(i)

For cos b,

We know the trigonometric identity,

$\sin^2 b + \cos^2 b = 1$

$\Rightarrow \cos^2 b = 1 - \sin^2 b$

$\Rightarrow \cos b = \sqrt{(1 - \sin^2 b)}$

Substituting the value of sin b,

$\Rightarrow \cos b = \sqrt{1 - \left(\frac{4}{\sqrt{17}}\right)^2}$

$= \sqrt{1 - \frac{16}{17}}$

$= \sqrt{\frac{17 - 16}{17}}$

$= \frac{1}{\sqrt{17}}$

We have $\sin b = \frac{4}{\sqrt{17}}$ and $\cos b = \frac{1}{\sqrt{17}}$.

So, we can find tan b.

$\because, \tan b = \frac{\sin b}{\cos b}$

$= \frac{\frac{4}{\sqrt{17}}}{\frac{1}{\sqrt{17}}}$

$= \frac{4}{\sqrt{17}} \times \sqrt{17}$

⇒ tan b = 4 …(ii)

We can write as,

$\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right\} = \tan\{a - b\}$

Now, we need to solve Right Hand Side (RHS).

We know the trigonometric identity,

$$\tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\tan b}$$

Substituting the values of tan a and tan b from (i) and (ii),

$$=\frac{7-4}{1+(7)(4)}$$

$$=\frac{3}{1+28}$$

$$=\frac{3}{29}$$

So,

$$\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right\}=\frac{3}{29}$$

3. Question

Choose the correct answer

$2\tan^{-1}|\operatorname{cosec}(\tan^{-1}x)-\tan(\cot^{-1}x)|$ is equal to

A. $\cot^{-1}x$

B. $\cot^{-1}\frac{1}{x}$

C. $\tan^{-1}x$

D. none of these

Answer

We need to find the value of 2 $\tan^{-1}|\operatorname{cosec}(\tan^{-1}x)-\tan(\cot^{-1}x)|$.

So, take

2 $\tan^{-1}|\operatorname{cosec}(\tan^{-1}x)-\tan(\cot^{-1}x)|$

Using property of inverse trigonometry,

$$\cot^{-1}x=\tan^{-1}\frac{1}{x}$$

$$\Rightarrow 2\tan^{-1}|\operatorname{cosec}(\tan^{-1}x)-\tan(\cot^{-1}x)|$$
$$=2\tan^{-1}\left|\operatorname{cosec}(\tan^{-1}x)-\tan\left(\tan^{-1}\frac{1}{x}\right)\right|$$

$$=2\tan^{-1}\left|\operatorname{cosec}(\tan^{-1}x)-\frac{1}{x}\right|$$

Now, let y = $\tan^{-1}x$

So, tan y = x

Substituting the value of $\tan^{-1}x$ and x in the equation,

$$\Rightarrow 2\tan^{-1}|\operatorname{cosec}(\tan^{-1}x)-\tan(\cot^{-1}x)|=2\tan^{-1}\left|\operatorname{cosec}y-\frac{1}{\tan y}\right|$$

Put,

$$\operatorname{cosec}y=\frac{1}{\sin y}\text{ and }\tan y=\frac{\sin y}{\cos y}$$

$\Rightarrow 2\tan^{-1}|\text{cosec}\,(\tan^{-1}x) - \tan(\cot^{-1}x)| = 2\tan^{-1}\left|\frac{1}{\sin y} - \frac{1}{\frac{\sin y}{\cos y}}\right|$

$= 2\tan^{-1}\left|\frac{1}{\sin y} - \frac{\cos y}{\sin y}\right|$

$= 2\tan^{-1}\left|\frac{1-\cos y}{\sin y}\right|$

Since, we know the trigonometric identity,

$1 - \cos 2y = 2\sin^2 y$

$\Rightarrow 1 - \cos y = 2\sin^2\frac{y}{2}$

Also, $\sin 2y = 2\sin y\cos y$

$\Rightarrow \sin y = 2\sin\frac{y}{2}\cos\frac{y}{2}$

We get,

$= 2\tan^{-1}\left|\frac{2\sin^2\frac{y}{2}}{2\sin\frac{y}{2}\cos\frac{y}{2}}\right|$

$= 2\tan^{-1}\left|\frac{\sin\frac{y}{2}}{\cos\frac{y}{2}}\right|$

Since,

$\tan y = \frac{\sin y}{\cos y}$

Then,

$= 2\tan^{-1}\left|\tan\frac{y}{2}\right|$

$= 2 \times \frac{y}{2}$

$\Rightarrow 2\tan^{-1}|\text{cosec}(\tan^{-1}x) - \tan(\cot^{-1}x)| = y$

Put $y = \tan^{-1}x$ as let above.

$\Rightarrow 2\tan^{-1}|\text{cosec}(\tan^{-1}x) - \tan(\cot^{-1}x)| = \tan^{-1}x$

4. Question

Choose the correct answer

If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$, then $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} =$

A. $\sin^2\alpha$

B. $\cos^2\alpha$

C. $\tan^2\alpha$

D. $\cot^2\alpha$

Answer

We are given that,

$$\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$$

We need to find the value of

$$\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}$$

By property of inverse trigonometry,

$\cos^{-1} a + \cos^{-1} b = \cos^{-1}(ab - \sqrt{(1-a^2)}\sqrt{(1-b^2)})$

So,

$$\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$$

$$\Rightarrow \cos^{-1}\left(\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)-\sqrt{1-\left(\frac{x}{a}\right)^2}\sqrt{1-\left(\frac{y}{b}\right)^2}\right)=\alpha$$

Simplifying further,

$$\Rightarrow \cos^{-1}\left(\frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}\right)=\alpha$$

Taking cosine on both sides,

$$\Rightarrow \cos\left[\cos^{-1}\left(\frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}\right)\right]=\cos\alpha$$

Using the property of inverse trigonometric function,

$\cos(\cos^{-1} x) = x$

$$\Rightarrow \frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}=\cos\alpha$$

$$\Rightarrow \frac{xy}{ab}-\cos\alpha=\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}$$

To simplify it further, take square on both sides.

$$\Rightarrow \left[\frac{xy}{ab}-\cos\alpha\right]^2=\left[\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}\right]^2$$

Using algebraic identity,

$(x - y)^2 = x^2 + y^2 - 2xy$

$$\Rightarrow \left(\frac{xy}{ab}\right)^2+\cos^2\alpha-\frac{2xy}{ab}\cos\alpha=\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)$$

Simplifying it further,

$$\Rightarrow \frac{x^2y^2}{a^2b^2} + \cos^2\alpha - \frac{2xy}{ab}\cos\alpha = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2y^2}{a^2b^2}$$

Shifting all terms at one side,

$$\Rightarrow \frac{x^2y^2}{a^2b^2} - \frac{x^2y^2}{a^2b^2} + \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\alpha = 1 - \cos^2\alpha$$

Using trigonometric identity,

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \cos^2 x$

We get,

$$\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$$

5. Question

Choose the correct answer

The positive integral solution of the equation $\tan^{-1} x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}$ is

A. x = 1, y = 2

B. x = 2, y = 1

C. x = 3, y = 2

D. x = –2, y = –1

Answer

We need to find the positive integral solution of the equation:

$$\tan^{-1} x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}$$

Using property of inverse trigonometry,

$$\cos^{-1} x = \tan^{-1} \frac{\sqrt{1-x^2}}{x}$$

Also,

$$\sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}}$$

Taking,

$$\tan^{-1} x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}$$

$$\Rightarrow \tan^{-1} x + \tan^{-1} \frac{\sqrt{1-\left(\frac{y}{\sqrt{1+y^2}}\right)^2}}{\frac{y}{\sqrt{1+y^2}}} = \tan^{-1} \frac{\frac{3}{\sqrt{10}}}{\sqrt{1-\left(\frac{3}{\sqrt{10}}\right)^2}}$$

$$\Rightarrow \tan^{-1}x + \tan^{-1}\frac{\sqrt{1-\frac{y^2}{1+y^2}}}{\frac{y}{\sqrt{1+y^2}}} = \tan^{-1}\frac{\frac{3}{\sqrt{10}}}{\sqrt{1-\frac{9}{10}}}$$

$$\Rightarrow \tan^{-1}x + \tan^{-1}\frac{\sqrt{\frac{1+y^2-y^2}{1+y^2}}}{\frac{y}{\sqrt{1+y^2}}} = \tan^{-1}\frac{\frac{3}{\sqrt{10}}}{\sqrt{\frac{10-9}{10}}}$$

$$\Rightarrow \tan^{-1}x + \tan^{-1}\frac{\sqrt{\frac{1}{1+y^2}}}{\frac{y}{\sqrt{1+y^2}}} = \tan^{-1}\frac{\frac{3}{\sqrt{10}}}{\sqrt{\frac{1}{10}}}$$

$$\Rightarrow \tan^{-1}x + \tan^{-1}\left(\frac{1}{\sqrt{1+y^2}} \times \frac{\sqrt{1+y^2}}{y}\right) = \tan^{-1}\left(\frac{3}{\sqrt{10}} \times \sqrt{10}\right)$$

$$\Rightarrow \tan^{-1}x + \tan^{-1}\frac{1}{y} = \tan^{-1}3$$

Using the property of inverse trigonometry,

$$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

Similarly,

$$\Rightarrow \tan^{-1}\left(\frac{x+\frac{1}{y}}{1-(x)\left(\frac{1}{y}\right)}\right) = \tan^{-1}3$$

Taking tangent on both sides of the equation,

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{x+\frac{1}{y}}{1-\frac{x}{y}}\right)\right] = \tan[\tan^{-1}3]$$

Using property of inverse trigonometry,

$\tan(\tan^{-1} A) = A$

Applying this property on both sides of the equation,

$$\Rightarrow \frac{x+\frac{1}{y}}{1-\frac{x}{y}} = 3$$

Simplifying the equation,

$$\Rightarrow \frac{\frac{xy+1}{y}}{\frac{y-x}{y}} = 3$$

$$\Rightarrow \frac{xy+1}{y} \times \frac{y}{y-x} = 3$$

$$\Rightarrow \frac{xy+1}{y-x} = 3$$

Cross-multiplying in the equation,

⇒ xy + 1 = 3(y – x)

⇒ xy + 1 = 3y – 3x

⇒ xy + 3x = 3y – 1

⇒ x(y + 3) = 3y – 1

$$\Rightarrow x = \frac{3y - 1}{y + 3}$$

We need to find positive integral solutions using the above result.

That is, we need to find solution which is positive as well as in integer form. A positive integer are all natural numbers.

That is, x, y > 0.

So, keep the values of y = 1, 2, 3, 4, … and find x.

x	$\frac{3(1)-1}{1+3} = \frac{1}{2}$	$\frac{3(2)-1}{2+3} = 1$	$\frac{3(3)-1}{3+3} = \frac{4}{3}$	$\frac{3(4)-1}{4+3} = \frac{11}{7}$	$\frac{3(5)-1}{5+3} = \frac{7}{4}$
y	1	2	3	4	5

Note that, only at y = 2, value is x is positive integer.

Thus, the positive integral solution of the given equation is x = 1, y = 2.

6. Question

Choose the correct answer

If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then x =

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $-\frac{1}{2}$

D. none of these

Answer

We are given that,

$\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$ …(i)

We need to find the value of x.

By using the property of inverse trigonometry,

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

We can find the value of $\sin^{-1}$ x in the terms of $\cos^{-1}$ x.

$\Rightarrow \sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$

Substituting the value of $\sin^{-1}$ x in equation (i),

$\left(\frac{\pi}{2} - \cos^{-1}x\right) - \cos^{-1}x = \frac{\pi}{6}$

Simplifying it further,

$\frac{\pi}{2} - \cos^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

$\Rightarrow \frac{\pi}{2} - 2\cos^{-1}x = \frac{\pi}{6}$

$\Rightarrow 2\cos^{-1}x = \frac{\pi}{2} - \frac{\pi}{6}$

$= \frac{3\pi - \pi}{6}$

$= \frac{2\pi}{6}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{6}$

Multiplying cosine on both sides of the equation,

$\Rightarrow \cos[\cos^{-1}x] = \cos\frac{\pi}{6}$

Using property of inverse trigonometry,

$\cos[\cos^{-1} x] = x$

$\Rightarrow x = \cos\frac{\pi}{6}$

And we know the value,

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Therefore,

$x = \frac{\sqrt{3}}{2}$

7. Question

Choose the correct answer

$\sin\left[\cot^{-1}\left\{\tan\left(\cos^{-1}x\right)\right\}\right]$ is equal to

A. x

B. $\sqrt{1-x^2}$

C. $\frac{1}{x}$

D. none of these

Answer

We need to find the value of

sin [$\cot^{-1}$ {tan ($\cos^{-1}$ x)}] ...(i)

We can solve such equation by letting the inner most trigonometric function (here, $\cos^{-1}$ x) as some variable, and solve systematically following BODMAS rule and other trigonometric identities.

Let $\cos^{-1}$ x = y

We can re-write the equation (i),

sin [$\cot^{-1}$ {tan ($\cos^{-1}$ x)}] = sin [$\cot^{-1}$ {tan y}]

Using trigonometric identity,

$\tan y = \cot\left(\frac{\pi}{2} - y\right)$

[∵, $\cot\left(\frac{\pi}{2} - y\right)$ lies in 1st Quadrant and sine, cosine, tangent and cot are positive in 1st Quadrant]

$\Rightarrow \sin[\cot^{-1}\{\tan(\cos^{-1} x)\}] = \sin\left[\cot^{-1}\left\{\cot\left(\frac{\pi}{2} - y\right)\right\}\right]$

Using property of inverse trigonometry,

$\cot^{-1}$(cot x) = x

$\Rightarrow \sin[\cot^{-1}\{\tan(\cos^{-1} x)\}] = \sin\left[\frac{\pi}{2} - y\right]$

Using trigonometric identity,

$\cos y = \sin\left(\frac{\pi}{2} - y\right)$

Substituting this value of $\sin\left(\frac{\pi}{2} - y\right)$,

$\Rightarrow \sin[\cot^{-1}\{\tan(\cos^{-1} x)\}] = \cos y$

We had let above that $\cos^{-1}$ x = y.

If,

$\cos^{-1}$ x = y

⇒ x = cos y

Therefore,

$\sin[\cot^{-1}\{\tan(\cos^{-1} x)\}] = x$

8. Question

Choose the correct answer

The number of solutions of the equation $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ is

A. 2

B. 3

C. 1

D. none of these

Answer

We need to find the number of solutions of the equation,

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

We shall apply the property of inverse trigonometry, that is,

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

So,

$$\tan^{-1}\left(\frac{2x+3x}{1-(2x)(3x)}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$$

Taking tangent on both sides of the equation,

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{5x}{1-6x^2}\right)\right] = \tan\frac{\pi}{4}$$

Using property of inverse trigonometry,

$\tan(\tan^{-1} A) = A$

Also,

$$\tan\frac{\pi}{4} = 1$$

We get,

$$\Rightarrow \frac{5x}{1-6x^2} = 1$$

Simplifying it,

$\Rightarrow 5x = 1 - 6x^2$

$\Rightarrow 6x^2 + 5x - 1 = 0$

Since, this is a quadratic equation, it is clear that it will have 2 solutions.

Let us check:

We have,

$6x^2 + 5x - 1 = 0$

$\Rightarrow 6x^2 + 6x - x - 1 = 0$

$\Rightarrow 6x(x + 1) - (x + 1) = 0$

$\Rightarrow (6x - 1)(x + 1) = 0$

$\Rightarrow (6x - 1) = 0$ or $(x + 1) = 0$

$\Rightarrow 6x = 1$ or $x = -1$

$$\Rightarrow x = \frac{1}{6} \text{ or } x = -1$$

Hence, there are 2 solutions of the given equation.

9. Question

Choose the correct answer

If $\alpha = \tan^{-1}\left(\tan\frac{5\pi}{4}\right)$ and $\beta = \tan^{-1}\left(-\tan\frac{2\pi}{3}\right)$, then

A. 4α = 3 β

B. 3α = 4β

C. $\alpha - \beta = \frac{7\pi}{12}$

D. none of these

Answer

We are given that,

$\alpha = \tan^{-1}\left(\tan\frac{5\pi}{4}\right)$ and $\beta = \tan^{-1}\left(-\tan\frac{2\pi}{3}\right)$

Take,

$\alpha = \tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

We can write $\frac{5\pi}{4}$ as,

$\frac{5\pi}{4} = \pi + \frac{\pi}{4}$

Then,

$\alpha = \tan^{-1}\left(\tan\left(\pi + \frac{\pi}{4}\right)\right)$

Also, by trigonometric identity

$\tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4}$

[∵, $\tan\left(\pi + \frac{\pi}{4}\right)$ lies in III Quadrant and tangent is positive in III Quadrant]

$\Rightarrow \alpha = \tan^{-1}\left(\tan\frac{\pi}{4}\right)$

Using the property of inverse trigonometry, that is, $\tan^{-1}(\tan A) = A$.

$\Rightarrow \alpha = \frac{\pi}{4}$

Now, take

$\beta = \tan^{-1}\left(-\tan\frac{2\pi}{3}\right)$

We can write $\frac{2\pi}{3}$ as,

$\frac{2\pi}{3} = \pi - \frac{\pi}{3}$

Then,

$\beta = \tan^{-1}\left(-\tan\left(\pi - \frac{\pi}{3}\right)\right)$

By trigonometric identity,

$\tan\left(\pi - \frac{\pi}{3}\right) = -\tan\frac{\pi}{3}$

[∵, $\tan\left(\pi - \frac{\pi}{3}\right)$ lies in II Quadrant and tangent is negative in II Quadrant]

$\Rightarrow \beta = \tan^{-1}\left(-\left(-\tan\frac{\pi}{3}\right)\right)$

$\Rightarrow \beta = \tan^{-1}\left(\tan\frac{\pi}{3}\right)$

Using the property of inverse trigonometry, that is, $\tan^{-1}(\tan A) = A$.

$\Rightarrow \beta = \frac{\pi}{3}$

We have,

$\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{3}$

⇒ 4α = π and 3β = π

Since, the values of 4α and 3β are same, that is,

4α = 3β = π

Therefore,

4α = 3β

10. Question

Choose the correct answer

The number of real solutions of the equation $\sqrt{1+\cos 2x} = \sqrt{2}\sin^{-1}(\sin x), -\pi \le x \le \pi$ is

A. 0

B. 1

C. 2

D. infinite

Answer

We are given with equation:

$\sqrt{(1 + \cos 2x)} = \sqrt{2}\sin^{-1}(\sin x)$ …(i)

Where $-\pi \le x \le \pi$

We need to find the number of real solutions of the given equation.

Using trigonometric identity,

$\cos 2x = \cos^2 x - \sin^2 x$

$\Rightarrow \cos 2x = \cos^2 x - (1 - \cos^2 x)$ [∵, $\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x$]

$\Rightarrow \cos 2x = \cos^2 x - 1 + \cos^2 x$

$\Rightarrow \cos 2x = 2\cos^2 x - 1$

$\Rightarrow 1 + \cos 2x = 2\cos^2 x$

Substituting the value of (1 + cos 2x) in equation (i),

$\sqrt{(2\cos^2 x)} = \sqrt{2}\sin^{-1}(\sin x)$

⇒ $\sqrt{2}$ |cos x| = $\sqrt{2}$ $\sin^{-1}$(sin x)

$\sqrt{2}$ will get cancelled from each sides,

⇒ |cos x| = $\sin^{-1}$(sin x)

Take interval $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$:

|cos x| is positive in interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, hence |cos x| = cos x.

And, sin x is also positive in interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, hence $\sin^{-1}$(sin x) = x.

So, |cos x| = $\sin^{-1}$(sin x)

⇒ cos x = x

If we draw y = cos x and y = x on the same graph, we will notice that they intersect at one point, thus giving us 1 solution.

∴, There is 1 solution of the given equation in interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Take interval $x \in \left[-\pi, -\frac{\pi}{2}\right)$:

|cos x| is negative in interval $\left[-\pi, -\frac{\pi}{2}\right)$, hence |cos x| = -cos x.

And, sin x is also negative in interval $\left[-\pi, -\frac{\pi}{2}\right)$, hence $\sin^{-1}$(sin (π + x)) = π + x.

So, |cos x| = $\sin^{-1}$(sin x)

⇒ -cos x = π + x

⇒ cos x = -π – x

If we draw y = cos x and y = -π – x on the same graph, we will notice that they intersect at one point, thus giving us 1 solution.

∴, There is 1 solution of the given equation in interval $\left[-\pi, -\frac{\pi}{2}\right)$.

Take interval $x \in \left(\frac{\pi}{2}, \pi\right]$:

|cos x| is negative in interval $\left(\frac{\pi}{2}, \pi\right]$, hence |cos x| = -cos x.

And, sin x is positive in interval $\left(\frac{\pi}{2}, \pi\right]$, hence $\sin^{-1}$(sin (-π – x)) = -π – x.

So, |cos x| = $\sin^{-1}$(sin x)

⇒ -cos x = -π – x

⇒ -cos x = -(π + x)

⇒ cos x = π + x

If we draw y = cos x and y = π + x on the same graph, we will notice that they doesn’t intersect at any point, thus giving us no solution.

∴, There is 0 solution of the given equation in interval $\left(\frac{\pi}{2}, \pi\right]$.

Hence, we get 2 solutions of the given equation in interval [-π, π].

11. Question

Choose the correct answer

If x < 0, y < 0 such that xy = 1, then $\tan^{-1}x + \tan^{-1}y$ equals

A. $\frac{\pi}{2}$

B. $-\frac{\pi}{2}$

C. –π

D. none of these

Answer

We are given that,

xy = 1, x < 0 and y < 0

We need to find the value of $\tan^{-1} x + \tan^{-1} y$.

Using the property of inverse trigonometry,

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

We already know the value of xy, that is, xy = 1.

Also, we know that x, y < 0.

Substituting xy = 1 in denominator,

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-1}\right)$$

$$= \tan^{-1}\left(\frac{x+y}{0}\right)$$

And since (x + y) = negative value = integer = -a (say).

$$= \tan^{-1}\left(-\frac{a}{0}\right)$$

$\Rightarrow \tan^{-1} x + \tan^{-1} y = \tan^{-1} -\infty$ ...(i)

Using value of inverse trigonometry,

$$\tan^{-1} -\infty = -\frac{\pi}{2}$$

Substituting the value of $\tan^{-1} -\infty$ in the equation (i), we get

$$\tan^{-1}x + \tan^{-1}y = -\frac{\pi}{2}$$

12. Question

Choose the correct answer

If $u = \cot^{-1}\left\{\sqrt{\tan\theta}\right\} - \tan^{-1}\left\{\sqrt{\tan\theta}\right\}$ then, $\tan\left(\frac{\pi}{4} - \frac{u}{2}\right)$ =

A. $\sqrt{\tan\theta}$

B. $\sqrt{\cot\theta}$

C. tan θ

D. cot θ

Answer

We are given with

$u = \cot^{-1}\{\sqrt{\tan\theta}\} - \tan^{-1}\{\sqrt{\tan\theta}\}$

We need to find the value of $\tan\left(\frac{\pi}{4} - \frac{u}{2}\right)$.

Let $\sqrt{\tan\theta} = x$

Then, $u = \cot^{-1}\{\sqrt{\tan\theta}\} - \tan^{-1}\{\sqrt{\tan\theta}\}$ can be written as

$u = \cot^{-1} x - \tan^{-1} x$ ...(i)

We know by the property of inverse trigonometry,

$$\cot^{-1} x + \tan^{-1} x = \frac{\pi}{2}$$

Or,

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

Substituting the value of $\cot^{-1} x$ in equation (i), we get

$u = (\cot^{-1} x) - \tan^{-1} x$

$$\Rightarrow u = \left(\frac{\pi}{2} - \tan^{-1} x\right) - \tan^{-1} x$$

$$= \frac{\pi}{2} - \tan^{-1} x - \tan^{-1} x$$

$$= \frac{\pi}{2} - 2\tan^{-1} x$$

Rearranging the equation,

$$\Rightarrow u + 2\tan^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow 2\tan^{-1} x = \frac{\pi}{2} - u$$

Now, divide by 2 on both sides of the equation.

$$\Rightarrow \frac{2\tan^{-1} x}{2} = \frac{\frac{\pi}{2} - u}{2}$$

$$\Rightarrow \tan^{-1} x = \frac{\frac{\pi}{2}}{2} - \frac{u}{2}$$

$$= \frac{\pi}{2} \times \frac{1}{2} - \frac{u}{2}$$

$$= \frac{\pi}{4} - \frac{u}{2}$$

Taking tangent on both sides, we get

$$\Rightarrow \tan(\tan^{-1} x) = \tan\left(\frac{\pi}{4} - \frac{u}{2}\right)$$

Using property of inverse trigonometry,

$\tan(\tan^{-1} x) = x$

$\Rightarrow x = \tan\left(\frac{\pi}{4} - \frac{u}{2}\right)$

Recall the value of x. That is, x = √tan θ

$\Rightarrow \tan\left(\frac{\pi}{4} - \frac{u}{2}\right) = \sqrt{\tan\theta}$

13. Question

Choose the correct answer

If $\cos^{-1}\frac{x}{3} + \cos^{-1}\frac{y}{2} = \frac{\theta}{2}$, then $4x^2 - 12xy\cos\frac{\theta}{2} + 9y^2 =$

A. 36

B. 36 – 36 cos θ

C. 18 – 18 cos θ

D. 18 + 18 cos θ

Answer

We are given with,

$\cos^{-1}\frac{x}{3} + \cos^{-1}\frac{y}{2} = \frac{\theta}{2}$ ...(i)

We need to find the value of

$4x^2 - 12\,xy\cos\frac{\theta}{2} + 9y^2$

Take Left Hand Side (LHS) of equation (i),

Using the property of inverse trigonometry,

$\cos^{-1}A + \cos^{-1}B = \cos^{-1}\left(AB - \sqrt{1 - A^2}\sqrt{1 - B^2}\right)$

Putting $A = \frac{x}{3}$ and $B = \frac{y}{2}$,

$LHS = \cos^{-1}\frac{x}{3} + \cos^{-1}\frac{y}{2}$

$\Rightarrow LHS = \cos^{-1}\left(\left(\frac{x}{3}\right)\left(\frac{y}{2}\right) - \sqrt{1 - \left(\frac{x}{3}\right)^2}\sqrt{1 - \left(\frac{y}{2}\right)^2}\right)$

$\Rightarrow LHS = \cos^{-1}\left(\frac{xy}{6} - \sqrt{1 - \frac{x^2}{9}}\sqrt{1 - \frac{y^2}{4}}\right)$

$\Rightarrow LHS = \cos^{-1}\left(\frac{xy}{6} - \sqrt{\frac{9 - x^2}{9}}\sqrt{\frac{4 - y^2}{4}}\right)$

Equate LHS to RHS.

$\cos^{-1}\left(\frac{xy}{6} - \sqrt{\frac{9 - x^2}{9}}\sqrt{\frac{4 - y^2}{4}}\right) = \frac{\theta}{2}$

Taking cosine on both sides,

$$\Rightarrow \cos\left[\cos^{-1}\left(\frac{xy}{6} - \sqrt{\frac{9-x^2}{9}}\sqrt{\frac{4-y^2}{4}}\right)\right] = \cos\frac{\theta}{2}$$

Using property of inverse trigonometry,

$\cos(\cos^{-1} A) = A$

$$\Rightarrow \frac{xy}{6} - \sqrt{\frac{9-x^2}{9}}\sqrt{\frac{4-y^2}{4}} = \cos\frac{\theta}{2}$$

Simplifying the equation,

$$\Rightarrow \frac{xy}{6} - \frac{\sqrt{9-x^2}}{3}\frac{\sqrt{4-y^2}}{2} = \cos\frac{\theta}{2}$$

$$\Rightarrow \frac{xy}{6} - \frac{\sqrt{9-x^2}\sqrt{4-y^2}}{6} = \cos\frac{\theta}{2}$$

$$\Rightarrow xy - \sqrt{9-x^2}\sqrt{4-y^2} = 6\cos\frac{\theta}{2}$$

$$\Rightarrow xy - 6\cos\frac{\theta}{2} = \sqrt{9-x^2}\sqrt{4-y^2}$$

Squaring on both sides,

$$\Rightarrow \left[xy - 6\cos\frac{\theta}{2}\right]^2 = \left[\sqrt{9-x^2}\ \sqrt{4-y^2}\right]^2$$

Using algebraic identity,

$(A - B)^2 = A^2 + B^2 - 2AB$

$$\Rightarrow (xy)^2 + \left(6\cos\frac{\theta}{2}\right)^2 - 2(xy)\left(6\cos\frac{\theta}{2}\right) = (9-x^2)(4-y^2)$$

$$\Rightarrow x^2y^2 + 36\cos^2\frac{\theta}{2} - 12xy\cos\frac{\theta}{2} = 36 - 9y^2 - 4x^2 + x^2y^2$$

$$\Rightarrow x^2y^2 - x^2y^2 + 4x^2 + 9y^2 - 12xy\cos\frac{\theta}{2} = 36 - 36\cos^2\frac{\theta}{2}$$

$$\Rightarrow 4x^2 - 12xy\cos\frac{\theta}{2} + 9y^2 = 36 - 36\cos^2\frac{\theta}{2}$$

Using trigonometric identity,

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ ...(ii)

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$ ...(iii)

Putting value of $\sin^2\theta$ from equation (iii) in equation (ii), we get

$\cos 2\theta = \cos^2\theta - (1 - \cos^2\theta)$

Or, $\cos 2\theta = \cos^2\theta - 1 + \cos^2\theta$

Or, $\cos 2\theta = 2\cos^2\theta - 1$

Or, $2\cos^2\theta = \cos 2\theta + 1$

Replace θ by θ/2.

$$2\cos^2\frac{\theta}{2} = \cos\frac{2\times\theta}{2} + 1$$

$\Rightarrow 2\cos^2\frac{\theta}{2} = \cos\theta + 1$

Substituting the value of $2\cos^2\frac{\theta}{2}$ in

$$4x^2 - 12xy\cos\frac{\theta}{2} + 9y^2 = 36 - 36\cos^2\frac{\theta}{2}$$

$$\Rightarrow 4x^2 - 12xy\cos\frac{\theta}{2} + 9y^2 = 36 - 18\left(2\cos^2\frac{\theta}{2}\right)$$

$$\Rightarrow 4x^2 - 12xy\cos\frac{\theta}{2} + 9y^2 = 36 - 18(\cos\theta + 1)$$

$$\Rightarrow 4x^2 - 12xy\cos\frac{\theta}{2} + 9y^2 = 36 - 18\cos\theta - 18$$

$$\Rightarrow 4x^2 - 12xy\cos\frac{\theta}{2} + 9y^2 = 18 - 18\cos\theta$$

14. Question

Choose the correct answer

If $\alpha = \tan^{-1}\left(\frac{\sqrt{3}\,x}{2y - x}\right)$, $\beta = \tan^{-1}\left(\frac{2x - y}{\sqrt{3}\,y}\right)$, then α - β =

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $-\frac{\pi}{3}$

Answer

We are given with,

$$\alpha = \tan^{-1}\left(\frac{\sqrt{3}x}{2y - x}\right)$$

$$\beta = \tan^{-1}\left(\frac{2x - y}{\sqrt{3}y}\right)$$

We need to find the value of α – β.

So,

$$\alpha - \beta = \tan^{-1}\left(\frac{\sqrt{3}x}{2y - x}\right) - \tan^{-1}\left(\frac{2x - y}{\sqrt{3}y}\right)$$

Using the property of inverse trigonometry,

$$\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A - B}{1 + AB}\right)$$

So,

$$\alpha - \beta = \tan^{-1}\left(\frac{\left(\frac{\sqrt{3}x}{2y-x}\right) - \left(\frac{2x-y}{\sqrt{3}y}\right)}{1 + \left(\frac{\sqrt{3}x}{2y-x}\right)\left(\frac{2x-y}{\sqrt{3}y}\right)}\right)$$

$$\Rightarrow \alpha - \beta = \tan^{-1}\left(\frac{\frac{\sqrt{3}x \times \sqrt{3}y - (2x-y)(2y-x)}{\sqrt{3}y(2y-x)}}{\frac{\sqrt{3}y(2y-x) + \sqrt{3}x(2x-y)}{\sqrt{3}y(2y-x)}}\right)$$

$$\Rightarrow \alpha - \beta = \tan^{-1}\left(\frac{\sqrt{3}x \times \sqrt{3}y - (2x-y)(2y-x)}{\sqrt{3}y(2y-x)} \times \frac{\sqrt{3}y(2y-x)}{\sqrt{3}y(2y-x) + \sqrt{3}x(2x-y)}\right)$$

$$\Rightarrow \alpha - \beta = \tan^{-1}\left(\frac{3xy - 4xy + 2x^2 + 2y^2 - xy}{2\sqrt{3}y^2 - \sqrt{3}xy + 2\sqrt{3}x^2 - \sqrt{3}xy}\right)$$

$$\Rightarrow \alpha - \beta = \tan^{-1}\left(\frac{2x^2 + 2y^2 - 2xy}{2\sqrt{3}x^2 + 2\sqrt{3}y^2 - 2\sqrt{3}xy}\right)$$

Simplifying it further,

$$\Rightarrow \alpha - \beta = \tan^{-1}\left(\frac{2x^2 + 2y^2 - 2xy}{\sqrt{3}(2x^2 + 2y^2 - 2xy)}\right)$$

The term $(2x^2 + 2y^2 - 2xy)$ gets cancelled from numerator and denominator.

$$\Rightarrow \alpha - \beta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Using the value of inverse trigonometry,

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$\therefore, \alpha - \beta = \frac{\pi}{6}$$

15. Question

Choose the correct answer

Let $f(x) = e^{\cos^{-1}\{\sin(x+\pi/3)\}}$. Then f(8 π/9) =

A. $e^{5\pi/18}$

B. $e^{13\pi/18}$

C. $e^{-2\pi/18}$

D. none of these

Answer

We are given with,

$$f(x) = e^{\cos^{-1}\left\{\sin\left(x+\frac{\pi}{3}\right)\right\}}$$

We need to find $f\left(\frac{8\pi}{9}\right)$.

We just need to find put $x = \frac{8\pi}{9}$ in f(x).

So,

$$f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\sin\left(\frac{8\pi}{9}+\frac{\pi}{3}\right)\right\}}$$

Simplify the equation,

$$f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\sin\left(\frac{8\pi+3\pi}{9}\right)\right\}}$$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\sin\left(\frac{11\pi}{9}\right)\right\}}$$

Using trigonometric identity,

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\cos\left(\frac{\pi}{2}-\frac{11\pi}{9}\right)\right\}}$$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\cos\left(\frac{9\pi-22\pi}{18}\right)\right\}}$$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\cos\left(-\frac{13\pi}{18}\right)\right\}}$$

Using trigonometric identity,

$\cos(-\theta) = \cos\theta$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\cos\left(\frac{13\pi}{18}\right)\right\}}$$

Using property of inverse trigonometry,

$\cos^{-1}(\cos\theta) = \theta$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\left(\frac{13\pi}{18}\right)}$$

16. Question

Choose the correct answer

$\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{2}{11}$ is equal to

A. 0

B. $\frac{1}{2}$

C. –1

D. none of these

Answer

We need to find the value of

$$\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{2}{11}$$

Using property of inverse trigonometry,

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A + B}{1 - AB}\right)$$

Replacing the values of A by $\frac{1}{11}$ and B by $\frac{2}{11}$,

$$\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{2}{11} = \tan^{-1}\left(\frac{\frac{1}{11} + \frac{2}{11}}{1 - \left(\frac{1}{11}\right)\left(\frac{2}{11}\right)}\right)$$

Solving it further,

$$= \tan^{-1}\left(\frac{\frac{1 + 2}{11}}{1 - \frac{2}{121}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{11}}{\frac{121 - 2}{121}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{11}}{\frac{119}{121}}\right)$$

$$= \tan^{-1}\left(\frac{3}{11} \times \frac{121}{119}\right)$$

$$= \tan^{-1}\left(\frac{3 \times 11}{119}\right)$$

$$= \tan^{-1}\left(\frac{33}{119}\right)$$

$$= 0.27$$

Thus, none of this match the result.

17. Question

Choose the correct answer

If $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$, then $9x^2$ – $12xy \cos \theta + 4y^2$ is equal to

A. 36

B. $-36 \sin^2 \theta$

C. $36 \sin^2 \theta$

D. $36 \cos^2 \theta$

Answer

We are given with,

$$\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$$

We need to find the value of $9x^2 - 12xy \cos \theta + 4y^2$.

Using property of inverse trigonometry,

$$\cos^{-1} A + \cos^{-1} B = \cos^{-1}\left(AB - \sqrt{1 - A^2}\sqrt{1 - B^2}\right)$$

Take Left Hand Side (LHS) of:

$$\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$$

Replace A by $\frac{x}{2}$ and B by $\frac{y}{3}$.

$$\text{LHS}=\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}$$

$$\Rightarrow \text{LHS}=\cos^{-1}\left(\left(\frac{x}{2}\right)\left(\frac{y}{3}\right)-\sqrt{1-\left(\frac{x}{2}\right)^2}\sqrt{1-\left(\frac{y}{3}\right)^2}\right)$$

$$=\cos^{-1}\left(\frac{xy}{6}-\sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}\right)$$

$$=\cos^{-1}\left(\frac{xy}{6}-\sqrt{\frac{4-x^2}{4}}\sqrt{\frac{9-y^2}{9}}\right)$$

Further solving,

$$=\cos^{-1}\left(\frac{xy}{6}-\frac{\sqrt{4-x^2}}{2}\frac{\sqrt{9-y^2}}{3}\right)$$

We shall equate LHS to RHS,

$$\cos^{-1}\left(\frac{xy}{6}-\frac{\sqrt{4-x^2}}{2}\frac{\sqrt{9-y^2}}{3}\right)=\theta$$

Taking cosine on both sides,

$$\cos\left[\cos^{-1}\left(\frac{xy}{6}-\frac{\sqrt{4-x^2}}{2}\frac{\sqrt{9-y^2}}{3}\right)\right]=\cos\theta$$

Using property of inverse trigonometry,

$\cos(\cos^{-1} A) = A$

So,

$$\Rightarrow \frac{xy}{6}-\frac{\sqrt{4-x^2}}{2}\frac{\sqrt{9-y^2}}{3}=\cos\theta$$

$$\Rightarrow \frac{xy}{6}-\frac{\sqrt{4-x^2}\sqrt{9-y^2}}{6}=\cos\theta$$

$$\Rightarrow \frac{xy-\sqrt{4-x^2}\sqrt{9-y^2}}{6}=\cos\theta$$

By cross-multiplying,

$\Rightarrow xy - \sqrt{(4 - x^2)}\,\sqrt{(9 - y^2)} = 6 \cos \theta$

Rearranging it,

$\Rightarrow xy - 6 \cos \theta = \sqrt{(4 - x^2)}\,\sqrt{(9 - y^2)}$

Squaring on both sides,

$\Rightarrow [xy - 6 \cos \theta]^2 = [\sqrt{(4 - x^2)}\,\sqrt{(9 - y^2)}]^2$

Using algebraic identity,

$(a - b)^2 = a^2 + b^2 - 2ab$

$\Rightarrow (xy)^2 + (6 \cos \theta)^2 - 2(xy)(6 \cos \theta) = (4 - x^2)(9 - y^2)$

$\Rightarrow x^2y^2 + 36 \cos^2 \theta - 12xy \cos \theta = 36 - 9x^2 - 4y^2 + x^2y^2$

$\Rightarrow x^2y^2 - x^2y^2 + 9x^2 - 12xy \cos \theta + 4y^2 = 36 - 36 \cos^2 \theta$

$\Rightarrow 9x^2 - 12xy \cos \theta + 4y^2 = 36 (1 - \cos^2 \theta)$

Using trigonometric identity,

$\sin^2 \theta + \cos^2 \theta = 1$

$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$

Substituting the value of $(1 - \cos^2 \theta)$, we get

$\Rightarrow 9x^2 - 12xy \cos \theta + 4y^2 = 36 \sin^2 \theta$

18. Question

Choose the correct answer

If $\tan^{-1} 3 + \tan^{-1} x = \tan^{-1} 8$, then x =

A. 5

B. $\frac{1}{5}$

C. .$\frac{5}{14}$.

D. $\frac{14}{5}$

Answer

We are given with,

$\tan^{-1} 3 + \tan^{-1} x = \tan^{-1} 8$

We need to find the value of x.

Using property of inverse trigonometry,

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A + B}{1 - AB}\right)$$

Let us replace A by 3 and B by x.

$$\tan^{-1} 3 + \tan^{-1} x = \tan^{-1}\left(\frac{3 + x}{1 - (3)(x)}\right)$$

$$= \tan^{-1}\left(\frac{3 + x}{1 - 3x}\right)$$

Since, according to the question

$\tan^{-1} 3 + \tan^{-1} x = \tan^{-1} 8$

So,

$$\Rightarrow \tan^{-1}\left(\frac{3 + x}{1 - 3x}\right) = \tan^{-1} 8$$

Taking tangent on both sides,

$\Rightarrow \tan\left[\tan^{-1}\left(\frac{3+x}{1-3x}\right)\right] = \tan[\tan^{-1} 8]$

Using property of inverse trigonometry,

$\tan(\tan^{-1} A) = A$

$\Rightarrow \frac{3+x}{1-3x} = 8$

Now, in order to find x, we need to solve the linear equation.

By cross-multiplying,

⇒ 3 + x = 8(1 – 3x)

⇒ 3 + x = 8 – 24x

⇒ 24x + x = 8 – 3

⇒ 25x = 5

$\Rightarrow x = \frac{5}{25}$

$\Rightarrow x = \frac{1}{5}$

19. Question

Choose the correct answer

The value of $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$ is

A. $\frac{3\pi}{5}$

B. $-\frac{\pi}{10}$

C. $\frac{\pi}{10}$

D. $\frac{7\pi}{5}$

Answer

We need to find the value of $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$.

$\sin^{-1}\left(\cos\frac{33\pi}{5}\right) = \sin^{-1}\left(\cos\left(6\pi + \frac{3\pi}{5}\right)\right)$

$\left[\because, \cos\frac{33\pi}{5} = \cos\left(6\pi + \frac{3\pi}{5}\right)\right]$

Using the trigonometric identity,

$\cos(6\pi + \theta) = \cos\theta$

As the function lies in I Quadrant and so it will be positive.

$\Rightarrow \sin^{-1}\left(\cos\frac{33\pi}{5}\right) = \sin^{-1}\left(\cos\frac{3\pi}{5}\right)$

Using the trigonometric identity,

$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

$\Rightarrow \sin^{-1}\left(\cos\frac{33\pi}{5}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right)$

Using property of inverse trigonometry,

$\sin^{-1}(\sin A) = A$

$= \frac{\pi}{2} - \frac{3\pi}{5}$

$= \frac{5\pi - 6\pi}{10}$

$= -\frac{\pi}{10}$

20. Question

Choose the correct answer

The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is

A. $\frac{\pi}{2}$

B. $\frac{5\pi}{3}$

C. $\frac{10\pi}{3}$

D. 0

Answer

We need to find the value of:

$\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

Let us simplify the trigonometric function.

We can write as:

$\cos\frac{5\pi}{3} = \cos\left(2\pi - \frac{\pi}{3}\right)$

Similarly,

$\sin\frac{5\pi}{3} = \sin\left(2\pi - \frac{\pi}{3}\right)$

$$\Rightarrow \cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right)$$
$$= \cos^{-1}\left(\cos\left(2\pi - \frac{\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(2\pi - \frac{\pi}{3}\right)\right)$$

Since, $\cos\left(2\pi - \frac{\pi}{3}\right)$ lies on IV Quadrant and cosine is positive in IV Quadrant.

$\therefore, \cos\left(2\pi - \frac{\pi}{3}\right) = \cos\frac{\pi}{3}$

And since, $\sin\left(2\pi - \frac{\pi}{3}\right)$ lies on IV Quadrant and sine is negative in IV Quadrant.

$\therefore, \sin\left(2\pi - \frac{\pi}{3}\right) = -\sin\frac{\pi}{3}$

$\Rightarrow \cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \cos^{-1}\left(\cos\frac{\pi}{3}\right) + \sin^{-1}\left(-\sin\frac{\pi}{3}\right)$

$= \cos^{-1}\left(\cos\frac{\pi}{3}\right) - \sin^{-1}\left(\sin\frac{\pi}{3}\right)$

Using property of inverse trigonometry,

$\sin^{-1}(\sin A) = A$ and $\cos^{-1}(\cos A) = A$

$\Rightarrow \cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \frac{\pi}{3} - \frac{\pi}{3}$

=0

21. Question

Choose the correct answer

$\sin\left\{2\cos^{-1}\left(\frac{-3}{5}\right)\right\}$ is equal to

A. $\frac{6}{25}$

B. $\frac{24}{25}$

C. $\frac{4}{5}$

D. $-\frac{24}{25}$

Answer

We need to find the value of:

$\sin\left\{2\cos^{-1}\left(-\frac{3}{5}\right)\right\}$

Let $\cos^{-1}\left(-\frac{3}{5}\right) = x$

Take cosine on both sides, we get

$\cos\left[\cos^{-1}\left(-\frac{3}{5}\right)\right] = \cos x$

Using property of inverse trigonometry,

$\cos(\cos^{-1} A) = A$

$\Rightarrow -\frac{3}{5} = \cos x$

We have the value of cos x, let us find the value of sin x.

By trigonometric identity,

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \cos^2 x$

$\Rightarrow \sin x = \sqrt{1-\cos^2 x}$

Putting $\cos x = -\frac{3}{5}$,

$= \sqrt{1-\left(-\frac{3}{5}\right)^2}$

$= \sqrt{1-\frac{9}{25}}$

$= \sqrt{\frac{25-9}{25}}$

$= \sqrt{\frac{16}{25}}$

$= \frac{4}{5}$

Now,

$\sin\left\{2\cos^{-1}\left(-\frac{3}{5}\right)\right\} = \sin 2x$

Using the trigonometric identity,

$\sin 2x = 2 \sin x \cos x$

$\Rightarrow \sin\left\{2\cos^{-1}\left(-\frac{3}{5}\right)\right\} = 2\sin x\cos x$

Putting the value of $\sin x = \frac{4}{5}$ and $\cos x = -\frac{3}{5}$,

$= 2 \times \frac{4}{5} \times -\frac{3}{5}$

$= -\frac{24}{25}$

22. Question

Choose the correct answer

If $\theta = \sin^{-1}\{\sin(-600°)\}$, then one of the possible values of θ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $-\frac{2\pi}{3}$

Answer

We are given that,

$\theta = \sin^{-1} \{\sin (-600°)\}$

We know that,

sin (2π – θ) = sin (4π – θ) = sin (6π – θ) = sin (8π – θ) = … = -sin θ

As, sin (2π – θ), sin (4π – θ), sin (6π – θ), … all lie in IV Quadrant where sine function is negative.

So,

If we replace θ by 600°, then we can write as

sin (4π – 600°) = -sin 600°

Or,

sin (4π – 600°) = sin (-600°)

Or,

sin (720° – 600°) = sin (-600°) …(i)

[∵, 4π = 4 × 180° = 720° < 600°]

Thus, we have

$\theta = \sin^{-1} \{\sin (-600°)\}$

$\Rightarrow \theta = \sin^{-1} \{\sin (720° – 600°)\}$ [from equation (i)]

$\Rightarrow \theta = \sin^{-1} \{\sin 120°\}$ …(ii)

We know that,

sin (π – θ) = sin (3π – θ) = sin (5π – θ) = … = sin θ

As, sin (π – θ), sin (3π – θ), sin (5π – θ), … all lie in II Quadrant where sine function is positive.

So,

If we replace θ by 120°, then we can write as

sin (π – 120°) = sin 120°

Or,

sin (180° - 120°) = sin 120° …(iii)

[∵, π = 180° < 120°]

Thus, from equation (ii),

$\theta = \sin^{-1} \{\sin 120°\}$

$\Rightarrow \theta = \sin^{-1} \{\sin (180° - 120°)\}$ [from equation (iii)]

$\Rightarrow \theta = \sin^{-1} \{\sin 60°\}$

Using property of inverse trigonometry,

$\sin^{-1} (\sin A) = A$

$\Rightarrow \theta = 60°$

$\Rightarrow \theta = \frac{\pi}{3}$

23. Question

Choose the correct answer

If $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$, then x is equal to

A. $\frac{1}{\sqrt{3}}$

B. $-\frac{1}{\sqrt{3}}$

C. $\sqrt{3}$

D. $-\frac{\sqrt{3}}{4}$

Answer

We are given that,

$$3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$$

We need to find the value of x.

We know that by trigonometric identity, we can represent sin θ, cos θ and tan θ in terms of tan θ.

Note,

$$\sin 2\theta = \left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$\cos 2\theta = \left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$\tan 2\theta = \left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

So, in the equation given in the question, let x = tan θ.

Re-writing the equation,

$$3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$$

$$\Rightarrow 3\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) - 4\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + 2\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \frac{\pi}{3}$$

Substituting the values of trigonometric identities,

$$\Rightarrow 3\sin^{-1}(\sin 2\theta) - 4\cos^{-1}(\cos 2\theta) + 2\tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$$

Using the property of inverse trigonometry, we have

$\sin^{-1}(\sin A) = A$, $\cos^{-1}(\cos A) = A$ and $\tan^{-1}(\tan A) = A$

$$\Rightarrow 3 \times 2\theta - 4 \times 2\theta + 2 \times 2\theta = \frac{\pi}{3}$$

$\Rightarrow 6\theta - 8\theta + 4\theta = \frac{\pi}{3}$

$\Rightarrow 2\theta = \frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{3} \times \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

Now, in order to find the value of x, recall

x = tan θ

Substitute the value of θ derived above,

$\Rightarrow x = \tan\frac{\pi}{6}$

$\Rightarrow x = \frac{1}{\sqrt{3}}$

24. Question

Choose the correct answer

If $4\cos^{-1}x + \sin^{-1}x = \pi$, then the value of x is

A. $\frac{3}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{2}{\sqrt{3}}$

Answer

We are given that,

$4\cos^{-1}x + \sin^{-1}x = \pi$ ...(i)

We need to find the value of x.

Using the property of inverse trigonometry,

$\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\theta = \frac{\pi}{2} - \cos^{-1}\theta$

Replacing θ by x, we get

$\Rightarrow \sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$

Substituting the value of $\sin^{-1}x$ in (i),

$4\cos^{-1}x + \sin^{-1}x = \pi$

$\Rightarrow 4\cos^{-1}x + \left(\frac{\pi}{2} - \cos^{-1}x\right) = \pi$

$\Rightarrow 4\cos^{-1}x + \frac{\pi}{2} - \cos^{-1}x = \pi$

$\Rightarrow 3\cos^{-1}x = \pi - \frac{\pi}{2}$

$\Rightarrow 3\cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{2} \times \frac{1}{3}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{6}$

Taking cosines on both sides,

$\Rightarrow \cos[\cos^{-1}x] = \cos\frac{\pi}{6}$

$\Rightarrow x = \cos\frac{\pi}{6}$

$\Rightarrow x = \frac{\sqrt{3}}{2}$

25. Question

Choose the correct answer

If $\tan^{-1}\frac{x+1}{x-1} + \tan^{-1}\frac{x-1}{x} = \tan^{-1}(-7)$, then the value of x is

A. 0

B. -2

C. 1

D. 2

Answer

We are given that,

$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$ ...(i)

We need to find the value of x.

Using the property of inverse trigonometry,

$$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

Replace A by $\frac{x+1}{x-1}$ and B by $\frac{x-1}{x}$.

$$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}\left(\frac{\left(\frac{x+1}{x-1}\right) + \left(\frac{x-1}{x}\right)}{1 - \left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)}\right)$$

Putting this value in equation (i),

$$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$$

$$\Rightarrow \tan^{-1}\left(\frac{\left(\frac{x+1}{x-1}\right)+\left(\frac{x-1}{x}\right)}{1-\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)}\right)=\tan^{-1}(-7)$$

Taking tangent on both sides,

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\left(\frac{x+1}{x-1}\right)+\left(\frac{x-1}{x}\right)}{1-\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)}\right)\right]=\tan[\tan^{-1}(-7)]$$

Using the property of inverse trigonometry,

$\tan(\tan^{-1} A) = A$

$$\Rightarrow \frac{\left(\frac{x+1}{x-1}\right)+\left(\frac{x-1}{x}\right)}{1-\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)}=-7$$

Cross-multiplying, we get

$$\Rightarrow \left(\frac{x+1}{x-1}\right)+\left(\frac{x-1}{x}\right)=-7\left[1-\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)\right]$$

Simplifying the equation in order to find the value of x,

$$\Rightarrow \frac{x(x+1)+(x-1)(x-1)}{x(x-1)}=-7\left[\frac{x(x-1)-(x+1)(x-1)}{x(x-1)}\right]$$

Let us cancel the denominator from both sides of the equation.

$\Rightarrow x(x + 1) + (x - 1)(x - 1) = -7[x(x - 1) - (x + 1)(x - 1)]$

$\Rightarrow x^2 + x + (x - 1)^2 = -7[x^2 - x - (x + 1)(x - 1)]$

Using the algebraic identity,

$(a - b) = a^2 + b^2 - 2ab$

And, $(a + b)(a - b) = a^2 - b^2$

$\Rightarrow x^2 + x + x^2 + 1 - 2x = -7[x^2 - x - (x^2 - 1)]$

$\Rightarrow 2x^2 - x + 1 = -7[x^2 - x - x^2 + 1]$

$\Rightarrow 2x^2 - x + 1 = -7[1 - x]$

$\Rightarrow 2x^2 - x + 1 = -7 + 7x$

$\Rightarrow 2x^2 - x - 7x + 1 + 7 = 0$

$\Rightarrow 2x^2 - 8x + 8 = 0$

$\Rightarrow 2(x^2 - 4x + 4) = 0$

$\Rightarrow x^2 - 4x + 4 = 0$

We need to solve the quadratic equation to find the value of x.

$\Rightarrow x^2 - 2x - 2x + 4 = 0$

$\Rightarrow x(x - 2) - 2(x - 2) = 0$

$\Rightarrow (x - 2)(x - 2) = 0$

$\Rightarrow x = 2$ or $x = 2$

Hence, $x = 2$.

26. Question

Choose the correct answer

If $\cos^{-1}x > \sin^{-1}x$, then

A. $\frac{1}{\sqrt{2}} < x \le 1$

B. $0 \le < \frac{1}{\sqrt{2}}$

C. $-1 \le x < \frac{1}{\sqrt{2}}$

D. x > 0

Answer

We are given that,

$\cos^{-1} x > \sin^{-1} x$

We need to find the range of x.

Using the property of inverse trigonometry,

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

Or,

$$\Rightarrow \sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$$

So, re-writing the inequality,

$\cos^{-1} x > \sin^{-1} x$

$$\Rightarrow \cos^{-1}x > \frac{\pi}{2} - \cos^{-1}x$$

Adding $\cos^{-1}$ x on both sides of the inequality,

$$\Rightarrow \cos^{-1}x + \cos^{-1}x > \frac{\pi}{2} - \cos^{-1}x + \cos^{-1}x$$

$$\Rightarrow 2\cos^{-1}x > \frac{\pi}{2}$$

Dividing both sides of the inequality by 2,

$$\Rightarrow \frac{2\cos^{-1}x}{2} > \frac{\pi}{2} \times \frac{1}{2}$$

$$\Rightarrow \cos^{-1}x > \frac{\pi}{4}$$

Taking cosine on both sides of the inequality,

$$\Rightarrow \cos[\cos^{-1}x] > \cos\frac{\pi}{4}$$

$$\Rightarrow x > \frac{1}{\sqrt{2}}$$

$\frac{1}{\sqrt{2}}$ is the minimum value of x, while the maximum value of cosine function is 1.

$\Rightarrow \frac{1}{\sqrt{2}} < x < 1$

27. Question

Choose the correct answer

In a ΔABC, If C is a right angle, then $\tan^{-1}\left(\frac{a}{b+c}\right) + \tan^{-1}\left(\frac{b}{c+a}\right) =$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{5\pi}{2}$

D. $\frac{\pi}{6}$

Answer

We are given that,

ΔABC is a right-angled triangle at C.

Let the sides of the ΔABC be

AC = b

BC = a

AB = c

By Pythagoras theorem, where C is the right angle,

$(AC)^2 + (BC)^2 = (AB)^2$

$\Rightarrow b^2 + a^2 = c^2$

Or,

$a^2 + b^2 = c^2$ ...(i)

Using the property of inverse trigonometry,

$\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

Replacing A by $\left(\frac{a}{b+c}\right)$ and B by $\left(\frac{b}{c+a}\right)$,

$$\Rightarrow \tan^{-1}\left(\frac{a}{b+c}\right) + \tan^{-1}\left(\frac{b}{c+a}\right) = \tan^{-1}\left(\frac{\left(\frac{a}{b+c}\right) + \left(\frac{b}{c+a}\right)}{1 - \left(\frac{a}{b+c}\right)\left(\frac{b}{c+a}\right)}\right)$$

$$= \tan^{-1}\left(\frac{\frac{a(c+a) + b(b+c)}{(b+c)(c+a)}}{\frac{(b+c)(c+a) - ab}{(b+c)(c+a)}}\right)$$

$$= \tan^{-1}\left(\left(\frac{ac + a^2 + b^2 + bc}{(b+c)(c+a)}\right) \times \left(\frac{(b+c)(c+a)}{bc + ab + c^2 + ac - ab}\right)\right)$$

$\Rightarrow \tan^{-1}\left(\frac{a}{b+c}\right) + \tan^{-1}\left(\frac{b}{c+a}\right) = \tan^{-1}\left(\frac{a^2+b^2+ac+bc}{c^2+ac+bc}\right)$

Substituting the value of $a^2 + b^2$ from equation (i),

$= \tan^{-1}\left(\frac{c^2+ac+bc}{c^2+ac+bc}\right)$

$=\tan^{-1}$

$\Rightarrow \tan^{-1}\left(\frac{a}{b+c}\right) + \tan^{-1}\left(\frac{b}{c+a}\right) = \frac{\pi}{4}$

28. Question

Choose the correct answer

The value of $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{2\sqrt{2}}$

D. $\frac{1}{3\sqrt{3}}$

Answer

We need to find the value of

$\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\frac{\sqrt{63}}{8} = x$

Now, take sine on both sides,

$\sin\left[\sin^{-1}\frac{\sqrt{63}}{8}\right] = \sin x$

Using the property of inverse trigonometry,

$\sin(\sin^{-1} A) = A$

$\Rightarrow \sin x = \frac{\sqrt{63}}{8}$

Let us find the value of cos x.

We know by trigonometric identity, that

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \cos x = \sqrt{1 - \sin^2 x}$

Put the value of sin x,

$= \sqrt{1 - \left(\frac{\sqrt{63}}{8}\right)^2}$

$= \sqrt{1 - \frac{63}{64}}$

$= \sqrt{\frac{64 - 63}{64}}$

$= \sqrt{\frac{1}{64}}$

$= \frac{1}{8}$

We have,

$\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \sin\left(\frac{1}{4}x\right)$

$\Rightarrow \sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \sin\frac{x}{4}$ ...(i)

Using the trigonometric identity,

$\cos 2x = \cos^2 x - \sin^2 x$

$\Rightarrow \cos 2x = (1 - \sin^2 x) - \sin^2 x$ [$\because$, $\sin^2 x + \cos^2 x = 1$]

$\Rightarrow \cos 2x = 1 - \sin^2 x - \sin^2 x$

$\Rightarrow \cos 2x = 1 - 2\sin^2 x$

Or,

$2\sin^2 x = 1 - \cos 2x$

$\Rightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$

$\Rightarrow \sin x = \sqrt{\frac{1 - \cos 2x}{2}}$

Replacing x by x/4,

$\Rightarrow \sin\frac{x}{4} = \sqrt{\frac{1 - \cos\left(2 \times \frac{x}{4}\right)}{2}}$

$= \sqrt{\frac{1 - \cos\frac{x}{2}}{2}}$

Substituting the value of $\sin\frac{x}{4}$ in equation (i),

$\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \sqrt{\frac{1 - \cos\frac{x}{2}}{2}}$ ...(ii)

Using the trigonometric identity,

$\cos 2x = \cos^2 x - \sin^2 x$

$\Rightarrow \cos 2x = \cos^2 x - (1 - \cos^2 x)$ [$\because$, $\sin^2 x + \cos^2 x = 1$]

$\Rightarrow \cos 2x = \cos^2 x - 1 + \cos^2 x$

$\Rightarrow \cos 2x = 2\cos^2 x - 1$

Or,

$2\cos^2 x = 1 + \cos 2x$

$$\Rightarrow \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\Rightarrow \cos x = \sqrt{\frac{1 + \cos 2x}{2}}$$

Replacing x by x/2,

$$\Rightarrow \cos\frac{x}{2} = \sqrt{\frac{1 + \cos\left(2 \times \frac{x}{2}\right)}{2}}$$

$$= \sqrt{\frac{1 + \cos x}{2}}$$

Substituting the value of $\cos\frac{x}{2}$ in equation (ii),

$$\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \sqrt{\frac{1 - \sqrt{\frac{1 + \cos x}{2}}}{2}}$$

Put the value of cos x as found above, cos x = 1/8.

$$= \sqrt{\frac{1 - \sqrt{\frac{1 + \frac{1}{8}}{2}}}{2}}$$

$$= \sqrt{\frac{\left(1 - \sqrt{\frac{\frac{8+1}{8}}{2}}\right)}{2}}$$

$$= \sqrt{\frac{1 - \sqrt{\frac{\frac{9}{8}}{2}}}{2}}$$

$$= \sqrt{\frac{1 - \sqrt{\frac{9}{16}}}{2}}$$

$$= \sqrt{\frac{\left(1 - \frac{3}{4}\right)}{2}}$$

$$= \sqrt{\frac{\frac{4-3}{4}}{2}}$$

$$= \sqrt{\frac{\frac{1}{4}}{2}}$$

$$= \sqrt{\frac{1}{8}}$$

$$= \frac{1}{2\sqrt{2}}$$

29. Question

Choose the correct answer

$$\cot\left(\frac{\pi}{4} - 2\cot^{-1} 3\right) =$$

A. 4

B. 6

C. 5

D. none of these

Answer

We need to find the value of

$$\cot\left(\frac{\pi}{4} - 2\cot^{-1} 3\right)$$

Let 2 $\cot^{-1}$ 3 = y

Then,

$$\cot^{-1} 3 = \frac{y}{2}$$

$$\Rightarrow \cot\frac{y}{2} = 3$$

Substituting 2 $\cot^{-1}$ 3 = y,

$$\cot\left(\frac{\pi}{4} - 2\cot^{-1} 3\right) = \cot\left(\frac{\pi}{4} - y\right)$$

Using the trigonometric identity,

$$\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

So,

$$\Rightarrow \cot\left(\frac{\pi}{4} - 2\cot^{-1} 3\right) = \frac{\cot\frac{\pi}{4}\cot y + 1}{\cot y - \cot\frac{\pi}{4}}$$

We know that,

$\cot\frac{\pi}{4} = 1$

$\Rightarrow \cot\left(\frac{\pi}{4} - 2\cot^{-1}3\right) = \frac{\cot y+1}{\cot y-1}$ ...(i)

We know that, by trigonometric identity,

$$\tan 2y = \frac{2\tan y}{1 - \tan^2 y}$$

Take reciprocal of both sides,

$$\frac{1}{\tan 2y} = \frac{1 - \tan^2 y}{2\tan y}$$

$$\Rightarrow \cot 2y = \frac{1 - \tan^2 y}{2\tan y}$$

$$\left[\because, \frac{1}{\tan 2y} = \cot 2y\right]$$

$$\Rightarrow \cot 2y = \frac{1 - \frac{1}{\cot^2 y}}{2 \times \frac{1}{\cot y}}$$

$$= \frac{\frac{\cot^2 y - 1}{\cot^2 y}}{2 \times \frac{1}{\cot y}}$$

$$= \frac{\cot^2 y - 1}{2\cot y}$$

Put y = y/2.

$$\Rightarrow \cot y = \frac{\cot^2\frac{y}{2} - 1}{2\cot\frac{y}{2}}$$

Putting the value of cot y in equation (i),

$$\Rightarrow \cot\left(\frac{\pi}{4} - 2\cot^{-1}3\right) = \frac{\frac{\cot^2\frac{y}{2} - 1}{2\cot\frac{y}{2}} + 1}{\frac{\cot^2\frac{y}{2} - 1}{2\cot\frac{y}{2}} - 1}$$

$$= \frac{\frac{\cot^2\frac{y}{2} - 1 + 2\cot\frac{y}{2}}{2\cot\frac{y}{2}}}{\frac{\cot^2\frac{y}{2} - 1 - 2\cot\frac{y}{2}}{2\cot\frac{y}{2}}}$$

$$= \frac{\cot^2\frac{y}{2} + 2\cot\frac{y}{2} - 1}{\cot^2\frac{y}{2} - 2\cot\frac{y}{2} - 1}$$

Put the value of $\cot\frac{y}{2} = 3$ derived above and also $\cot^2\frac{y}{3} = 3^2 = 9$.

$$= \frac{9 + 2 \times 3 - 1}{9 - 2 \times 3 - 1}$$

$= \frac{9+6-1}{9-6-1}$

$= \frac{14}{2}$

=7

30. Question

Choose the correct answer

If $\tan^{-1}(\cot\theta) = 2\theta$, then θ =

A. $\pm\frac{\pi}{3}$

B. $\pm\frac{\pi}{4}$

C. $\pm\frac{\pi}{6}$

D. none of these

Answer

We are given that,

$\tan^{-1}(\cot\theta) = 2\theta$

We need to find the value of θ.

We have,

$\tan^{-1}(\cot\theta) = 2\theta$

Taking tangent on both sides,

$\Rightarrow \tan[\tan^{-1}(\cot\theta)] = \tan 2\theta$

Using property of inverse trigonometry,

$\tan(\tan^{-1} A) = A$

$\Rightarrow \cot\theta = \tan 2\theta$

Or,

$\Rightarrow \tan 2\theta = \cot\theta$

Using the trigonometric identity,

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \cot\theta$

Using the trigonometric identity,

$\cot\theta = \frac{1}{\tan\theta}$

$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{1}{\tan\theta}$

By cross-multiplying,

$\Rightarrow \tan\theta \times 2\tan\theta = 1 - \tan^2\theta$

$\Rightarrow 2\tan^2\theta = 1 - \tan^2\theta$

$\Rightarrow 2\tan^2\theta + \tan^2\theta = 1$

$\Rightarrow 3\tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \frac{1}{3}$

$\Rightarrow \tan\theta = \pm\frac{1}{\sqrt{3}}$

And $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

$\Rightarrow \tan\theta = \pm\tan\frac{\pi}{6}$

Thus,

$\theta = \pm\frac{\pi}{6}$

31. Question

Choose the correct answer

If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where a, x ∈ (0, 1) then, the value of x is

A. 0

B. $\frac{a}{2}$

C. a

D. $\frac{2a}{1-a^2}$

Answer

We are given that,

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Where, a, x ∈ (0, 1).

We need to find the value of x.

Using property of inverse trigonometry,

$$2\tan^{-1}a = \sin^{-1}\left(\frac{2a}{1+a^2}\right)$$

$$= \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)$$

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Then, we can write as

$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

⇒ $2\tan^{-1} a + 2\tan^{-1} a = 2\tan^{-1} x$

⇒ $4\tan^{-1} a = 2\tan^{-1} x$

Dividing both sides by 2,

$$\Rightarrow \frac{4\tan^{-1} a}{2} = \frac{2\tan^{-1} x}{2}$$

⇒ $2\tan^{-1} a = \tan^{-1} x$

Using property of inverse trigonometry,

$$2\tan^{-1} a = \tan^{-1}\left(\frac{2a}{1-a^2}\right)$$

Then,

$$\Rightarrow \tan^{-1}\left(\frac{2a}{1-a^2}\right) = \tan^{-1} x$$

Taking tangent on both sides,

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{2a}{1-a^2}\right)\right] = \tan[\tan^{-1} x]$$

$$\Rightarrow \frac{2a}{1-a^2} = x$$

Or,

$$\Rightarrow x = \frac{2a}{1-a^2}$$

32. Question

Choose the correct answer

The value of $\sin\left(2\left(\tan^{-1} 0.75\right)\right)$ is equal to

A. 0.75

B. 1.5

C. 0.96

D. $\sin^{-1} 1.5$

Answer

We need to find the value of $\sin(2(\tan^{-1} 0.75))$.

We can re-write the equation,

$\sin(2(\tan^{-1} 0.75)) = \sin(2\tan^{-1} 0.75)$

Using the property of inverse trigonometry,

$$2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Replace x by 0.75.

$$2\tan^{-1} 0.75 = \sin^{-1}\left(\frac{2\times 0.75}{1+0.75^2}\right)$$

So,

sin (2(tan^{-1} 0.75)) = sin (2 tan^{-1} 0.75)

$\Rightarrow \sin\left(2(\tan^{-1} 0.75)\right) = \sin\left(\sin^{-1}\left(\frac{2 \times 0.75}{1 + 0.75^2}\right)\right)$

$= \sin\left(\sin^{-1}\left(\frac{1.5}{1 + 0.5626}\right)\right)$

$= \sin\left(\sin^{-1}\left(\frac{1.5}{1.5626}\right)\right)$

⇒ sin (2(tan^{-1} 0.75)) = sin (sin^{-1} 0.96)

Using the property of inverse trigonometry,

sin(sin^{-1} A) = A

⇒ sin (2(tan^{-1} 0.75)) = 0.96

33. Question

Choose the correct answer

If x > 1, then $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

A. 4tan^{-1}x

B. 0

C. $\frac{\pi}{2}$

D. π

Answer

We are given that, x > 1.

We need to find the value of

$2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Using the property of inverse trigonometry,

$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

We can substitute $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ by 2 tan^{-1} x.

So,

$2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x + 2\tan^{-1}x$

=4 tan^{-1} x

34. Question

Choose the correct answer

The domain of $\cos^{-1}(x^2 - 4)$ is

A. [3, 5]

B. [–1, 1]

C. $\left[-\sqrt{5}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{5}\right]$

D. $\left[-\sqrt{5}, -\sqrt{3}\right] \cap \left[-\sqrt{5}, \sqrt{3}\right]$

Answer

We need to find the domain of $\cos^{-1}(x^2 - 4)$.

We must understand that, the domain of definition of a function is the set of "input" or argument values for which the function is defined.

We know that, domain of an inverse cosine function, $\cos^{-1} x$ is,

$x \in [-1, 1]$

Then,

$(x^2 - 4) \in [-1, 1]$

Or,

$-1 \leq x^2 - 4 \leq 1$

Adding 4 on all sides of the inequality,

$-1 + 4 \leq x^2 - 4 + 4 \leq 1 + 4$

$\Rightarrow 3 \leq x^2 \leq 5$

Now, since x has a power of 2, so if we take square roots on all sides of the inequality then the result would be

$\Rightarrow \pm\sqrt{3} \leq x \leq \pm\sqrt{5}$

But this obviously isn't continuous.

So, we can write as

$x \in \left[-\sqrt{5}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{5}\right]$

35. Question

Choose the correct answer

The value of $\tan\left(\cos^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4}\right)$ is

A. $\frac{19}{8}$

B. $\frac{8}{19}$

C. $\frac{19}{12}$

D. $\frac{3}{4}$

Answer

We need to find the value of

$$\tan\left(\cos^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{4}\right)$$

Using the property of inverse trigonometry,

$$\cos^{-1}x=\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

Just replace x by 3/5,

$$\cos^{-1}\frac{3}{5}=\tan^{-1}\left(\frac{\sqrt{1-\left(\frac{3}{5}\right)^2}}{\frac{3}{5}}\right)$$

So,

$$\tan\left(\cos^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{4}\right)=\tan\left(\tan^{-1}\left(\frac{\sqrt{1-\frac{9}{25}}}{\frac{3}{5}}\right)+\tan^{-1}\frac{1}{4}\right)$$

$$=\tan\left(\tan^{-1}\left(\frac{\sqrt{\frac{25-9}{25}}}{\frac{3}{5}}\right)+\tan^{-1}\frac{1}{4}\right)$$

$$=\tan\left(\tan^{-1}\left(\frac{\sqrt{\frac{16}{25}}}{\frac{3}{5}}\right)+\tan^{-1}\frac{1}{4}\right)$$

$$=\tan\left(\tan^{-1}\left(\frac{\frac{4}{5}}{\frac{3}{5}}\right)+\tan^{-1}\frac{1}{4}\right)$$

$$=\tan\left(\tan^{-1}\left(\frac{4}{5}\times\frac{5}{3}\right)+\tan^{-1}\frac{1}{4}\right)$$

$$=\tan\left(\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{4}\right)$$

Using property of inverse trigonometry,

$$\tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

$$=\tan\left(\tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{4}}{1-\left(\frac{4}{3}\right)\left(\frac{1}{4}\right)}\right)\right)$$

$$=\tan\left(\tan^{-1}\left(\frac{\frac{16+3}{12}}{\frac{12-4}{12}}\right)\right)$$

$$=\tan\left(\tan^{-1}\left(\frac{\frac{19}{12}}{\frac{8}{12}}\right)\right)$$

$$=\tan\left(\tan^{-1}\left(\frac{19}{12}\times\frac{12}{8}\right)\right)$$

$= \tan\left(\tan^{-1}\frac{19}{8}\right)$

Using the property of inverse trigonometry,

$\tan(\tan^{-1} A) = A$

$\Rightarrow \tan\left(\cos^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4}\right) = \frac{19}{8}$

Very short answer

1. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$.

Answer

Let $\sin^{-1}(-\sqrt{3}/2) = x$ and $\cos^{-1}(-1/2) = y$

$\Rightarrow \sin x = (-\sqrt{3}/2)$ and $\cos y = -1/2$

We know that the range of the principal value branch of $\sin^{-1}$ is $(-\pi/2, \pi/2)$ and $\cos^{-1}$ is $(0, \pi)$.

We also know that $\sin(-\pi/3) = (-\sqrt{3}/2)$ and $\cos(2\pi/3) = -1/2$

∴ Value of $\sin^{-1}(-\sqrt{3}/2) + \cos^{-1}(-1/2) = -\pi/3 + 2\pi/3$

$= \pi/3$

2. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the difference between maximum and minimum values of $\sin^{-1}x$ for $x \in [-1, 1]$.

Answer

Let $f(x) = \sin^{-1} x$

For x to be defined, $-1 \le x \le 1$

For $-1 \le x \le 1$, $\sin^{-1}(-1) \le \sin^{-1} x \le \sin^{-1}(1)$

$\Rightarrow -\pi/2 \le \sin^{-1} x \le \pi/2$

$\Rightarrow -\pi/2 \le f(x) \le \pi/2$

Maximum value $= \pi/2$ and minimum value $= -\pi/2$

∴ The difference between maximum and minimum values of $\sin^{-1} x = \pi/2 - (-\pi/2) = 2\pi/2$

$= \pi$

3. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then write the value f x + y + z.

Answer

Given $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = 3\pi/2$

We know that maximum and minimum values of $\sin^{-1} x$ are $\pi/2$ and $-\pi/2$ respectively.

$\Rightarrow \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi/2 + \pi/2 + \pi/2$

$\Rightarrow \sin^{-1} x = \pi/2, \sin^{-1} y = \pi/2, \sin^{-1} z = \pi/2$

$\Rightarrow x = 1, y = 1, z = 1$

$\Rightarrow x + y + z = 1 + 1 + 1 = 3$

$\therefore x + y + z = 3$

4. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If x > 1, then write the value of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ in terms of $\tan^{-1} x$.

Answer

Given x > 1

$\Rightarrow \tan\theta > 1$

$\Rightarrow \frac{\pi}{4} < \theta < \frac{\pi}{2}$

Multiplying by -2,

$\Rightarrow -\pi < -2\theta < -\frac{\pi}{2}$

Subtracting with π,

$\Rightarrow 0 < \pi - 2\pi < \frac{\pi}{2}$

We know that $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

Put $\tan\theta = x$

$\Rightarrow \sin 2\theta = \frac{2x}{1+x^2}$

For x > 1,

$\Rightarrow \sin(\pi - 2\theta) = \frac{2x}{1+x^2}$

$\Rightarrow \pi - 2\theta = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Since $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1} x$

$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi - 2\tan^{-1} x$

5. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If x < 0, then write the value of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ in terms of $\tan^{-1} x$.

Answer

Given x < 0

⇒ -∞ < x < 0

Let x = tan θ

⇒ -∞ < tan θ < 0

$\Rightarrow -\frac{\pi}{2} < \theta < 0$

Multiplying by -2,

⇒ -π < -2θ < 0

We know that $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

Put tan θ = x

$\Rightarrow \cos(-2\theta) = \frac{1-x^2}{1+x^2}$

$\Rightarrow -2\theta = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Since x = tan θ

⇒ θ = $\tan^{-1} x$

$\therefore \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = -2\tan^{-1} x$

6. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Writ the value of $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$ for x > 0.

Answer

Given $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$ for x > 0

We know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$,if xy>1

$\Rightarrow \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}\left[\frac{x+\frac{1}{x}}{1-x+\frac{1}{x}}\right]$

$= \tan^{-1}\left[\frac{\frac{x^2+1}{x}}{0}\right]$

= $\tan^{-1}$ (∞)

$= \frac{\pi}{2}$

$\therefore \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$

7. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$ for x < 0.

Answer

Given $\tan^{-1} x + \tan^{-1} (1/x)$ for x < 0

We know that $\tan^{-1} x + \tan^{-1} y = -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$,if x<0, y<0

$\Rightarrow \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = -\pi + \tan^{-1}\left[\frac{x+\frac{1}{x}}{1-x+\frac{1}{x}}\right]$

$= -\pi + \tan^{-1}\left[\frac{\frac{x^2+1}{x}}{0}\right]$

$= -\pi + \tan^{-1} (\infty)$

$= -\pi + \pi/2$

$= -\pi/2$

$\therefore \tan^{-1} x + \tan^{-1} (1/x) = -\pi/2$

8. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

What is the value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?

Answer

We know that $\sin^{-1} (\sin \theta) = \pi - \theta$, if $\theta \in [\pi/2, 3\pi/2]$ and $\cos^{-1} (\cos \theta) = \theta$, if $\theta \in [0, \pi]$

Given $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$= \frac{2\pi}{3} + \left(\pi - \frac{2\pi}{3}\right)$

$= \pi$

$\therefore \cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \pi$

9. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $-1 < x < 0$, then write the value of $\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

Answer

Given -1 < x < 0

We know that $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x, \text{if} -1 \le x \le 1$ and $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = -2\tan^{-1}x, \text{if} -\infty < x \le 0$

Given $\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

= 2 $\tan^{-1}$ x – 2 $\tan^{-1}$ x

= 0

10. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Writ the value of sin ($\cot^{-1}$ x).

Answer

Given sin ($\cot^{-1}$ x)

Let $\cot^{-1}$ x = θ

⇒ x = cot θ

We know that 1 + $\cot^2$ θ = cosec^2 θ

⇒ 1 + x^2 = cosec^2 θ

We know that cosec θ = 1/sin θ

$$\Rightarrow 1 + x^2 = \frac{1}{\sin^2\theta}$$

$$\Rightarrow \sin^2\theta = \frac{1}{1 + x^2}$$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{1 + x^2}}$$

$$\therefore \sin(\cot^{-1}x) = \frac{1}{\sqrt{1 + x^2}}$$

11. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

Answer

Let $\cos^{-1}$ (1/2) = x and $\sin^{-1}$ (1/2) = y

⇒ cos x = 1/2 and sin y = 1/2

We know that the range of the principal value branch of $\sin^{-1}$ is (-π/2, π/2) and $\cos^{-1}$ is (0, π).

We also know that sin (π/ 6) = 1/2 and cos (π/3) = 1/2

⇒ Value of $\cos^{-1}$ (1/2) + 2$\sin^{-1}$ (1/2) = π/3 + 2(π/6)

= π/3 + π/3

$= 2\pi/3$

∴ Value of $\cos^{-1}(1/2) + 2\sin^{-1}(1/2) = 2\pi/3$

12. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the range of $\tan^{-1}x$.

Answer

We know that range of $\tan^{-1} x = (-\pi/2, \pi/2)$

13. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1}(\cos 1540°)$.

Answer

Given $\cos^{-1}(\cos 1540°)$

$= \cos^{-1}\{\cos (1440° + 100°)\}$

$= \cos^{-1}\{\cos (360° \times 4 + 100°)\}$

We know that $\cos (2\pi + \theta) = \cos \theta$

$= \cos^{-1}\{\cos 100°\}$

We know that $\cos^{-1}(\cos \theta) = \theta$ if $\theta \in [0, \pi]$

$= 100°$

∴ $\cos^{-1}(\cos 1540°) = 100°$

14. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin^{-1}(\sin(-600°))$.

Answer

Given $\sin^{-1}(\sin (-600°))$

$= \sin^{-1}(\sin (-600 + 360 \times 2))$

We know that $\sin (2n\pi + \theta) = \sin \theta$

$= \sin (\sin 120°)$

We know that $\sin^{-1}(\sin \theta) = \pi - \theta$, if $\theta \in [\pi/2, 3\pi/2]$

$= 180° - 120°$

$= 60°$

∴ $\sin^{-1}(\sin (-600°)) = 60°$

15. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos\left(2\sin^{-1}\frac{1}{3}\right)$.

Answer

Given cos (2sin^{-1} 1/3)

We know that $\sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$

$$= \cos\left(2\tan^{-1}\frac{\frac{1}{3}}{\sqrt{1-\left(\frac{1}{3}\right)^2}}\right)$$

$$= \cos\left(2\tan^{-1}\frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}}\right)$$

$$= \cos\left(2\tan^{-1}\frac{1}{2\sqrt{2}}\right)$$

We know that $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}$

$$= \cos\left(\cos^{-1}\frac{1-\left(\frac{1}{2\sqrt{2}}\right)^2}{1+\left(\frac{1}{2\sqrt{2}}\right)^2}\right)$$

$$= \frac{1-\frac{1}{8}}{1+\frac{1}{8}}$$

$$= \frac{7}{9}$$

$$\therefore \cos\left(2\sin^{-1}\frac{1}{3}\right) = \frac{7}{9}$$

16. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of sin^{-1}(1550°).

Answer

Given sin^{-1} (sin 1550°)

= sin^{-1} (sin (1440° + 110°))

= sin^{-1} (sin (360° × 4 + 110°))

We know that sin (2nπ + θ) = sin θ

= sin^{-1} (sin 110°)

We know that sin^{-1} (sin θ) = π – θ, if θ ∈ [π/2, 3π/2]

= 180° - 110°

= 70°

∴ $\sin^{-1}(\sin 1550°) = 70°$

17. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Evaluate: $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$.

Answer

Given $\sin(1/2 \cos^{-1} 4/5)$

We know that $\cos^{-1}x = 2\tan^{-1}\sqrt{\frac{1-x}{1+x}}$

$$= \sin\left(\frac{1}{2} \times 2\tan^{-1}\sqrt{\frac{1-\frac{4}{5}}{1+\frac{4}{5}}}\right)$$

$$= \sin\left(\tan^{-1}\frac{1}{3}\right)$$

We know that $\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$

$$= \sin\left(\sin^{-1}\frac{\frac{1}{3}}{\sqrt{1+\left(\frac{1}{3}\right)^2}}\right)$$

$$= \frac{\frac{1}{3}}{\frac{\sqrt{10}}{3}}$$

$$= \frac{1}{\sqrt{10}}$$

$$\therefore \sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = \frac{1}{\sqrt{10}}$$

18. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Evaluate: $\sin\left(\tan^{-1}\frac{3}{4}\right)$.

Answer

Given $\sin(\tan^{-1} 3/4)$

We know that $\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$

$$= \sin\left(\sin^{-1}\frac{\frac{3}{4}}{\sqrt{1+\left(\frac{3}{4}\right)^2}}\right)$$

We know that $\sin(\sin^{-1}\theta) = \theta$

$= \frac{\frac{3}{4}}{\frac{5}{4}}$

$= \frac{3}{5}$

$\therefore \sin\left(\tan^{-1}\frac{3}{4}\right) = \frac{3}{5}$

19. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1}\left(\tan\frac{3\pi}{4}\right)$.

Answer

Given $\cos^{-1}$ (tan 3π/4)

= $\cos^{-1}$ (tan (π - π/4))

We know that tan (π – θ) = - tan θ

= $\cos^{-1}$ (-tan π/4)

= $\cos^{-1}$ (-1)

We know that $\cos^{-1}$ x = π

∴ $\cos^{-1}$ (tan 3π/4) = π

20. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos\left(2\sin^{-1}\frac{1}{2}\right)$.

Answer

Given cos (2$\sin^{-1}$ 1/2)

= cos (2× π/6)

= cos (π/3)

= 1/2

∴ cos (2 $\sin^{-1}$ 1/2) = 1/2

21. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1}$(cos 350°) – $\sin^{-1}$(sin 350°).

Answer

Given $\cos^{-1}$ (cos 350°) – $\sin^{-1}$ (sin 350°)

= $\cos^{-1}$ [cos (360° - 10°)] – $\sin^{-1}$ (sin (360° - 10°)]

We know that cos (2π – θ) = cos θ and sin (2π – θ) = -sin θ

$= \cos^{-1}(\cos 10°) - \sin^{-1}(-\sin 10°)$

We know that $\cos^{-1}(\cos \theta)$, if $\theta \in [0, \pi]$ and $\sin(-\theta) = -\sin \theta$

$= 10° - \sin^{-1}(\sin(-10°))$

We know that $\sin^{-1}(\sin \theta) = \theta$, if $\theta \in [-\pi/2, \pi/2]$

$= 10° - (-10°)$

$= 10° + 10°$

$= 20°$

$\therefore \cos^{-1}(\cos 350°) - \sin^{-1}(\sin 350°) = 20°$

22. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^2\left(\frac{1}{2}\cos^{-1}\frac{3}{5}\right)$.

Answer

Given $\cos^2(1/2 \cos^{-1} 3/5)$

We know that $cos^{-1} x = 2\, cos^{-1} \sqrt{\frac{1+x}{2}}$

$$= cos^2\left(\frac{1}{2} \times 2\, cos^{-1} \sqrt{\frac{1+\frac{3}{5}}{2}}\right)$$

$$= (cos\left(cos^{-1}\sqrt{\frac{8}{10}}\right)$$

$$= \left(\sqrt{\frac{8}{10}}\right)^2$$

$$= \frac{4}{5}$$

$$\therefore cos^2\left(\frac{1}{2} cos^{-1}\frac{3}{5}\right) = \frac{4}{5}$$

23. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, then write the value of x + y + xy.

Answer

Given $\tan^{-1} x + \tan^{-1} y = \pi/4$

We know that $tan^{-1} x + tan^{-1} y = tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$\Rightarrow tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4}$

$\Rightarrow tan^{-1}\left(\frac{x+y}{1-xy}\right) = tan^{-1}(1)$

$\Rightarrow \frac{x+y}{1-xy} = 1$

⇒ x + y = 1 – xy

⇒ x + y + xy = 1

∴ x + y + xy = 1

24. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1}$ (cos 6).

Answer

Given $\cos^{-1}$ (cos 6)

We know that $\cos^{-1}$ (cos θ) = 2π – θ, if θ ∈ [π, 2π]

= 2π – 6

∴ $\cos^{-1}$ (cos 6) = 2π - 6

25. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin^{-1}\left(\cos\frac{\pi}{9}\right)$.

Answer

Given $\sin^{-1}$ (cos π/9)

We know that cos θ = sin (π/2 – θ)

= $\sin^{-1}$ (sin (π/2 - π/9))

= $\sin^{-1}$ (sin 7π/18)

We know that $\sin^{-1}$ (sin θ) = θ

= 7π/18

∴ $\sin^{-1}$ (cos π/9) = 7π/18

26. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right\}$.

Answer

Given sin (π/3 – $\sin^{-1}$ (-1/2))

We know that $\sin^{-1}(-\theta) = -\sin^{-1}\theta$

$= \sin(\pi/3 + \sin^{-1}(1/2)\ 0)$

$= \sin(\pi/3 + \pi/6)$

$= \sin(\pi/2)$

$= 1$

$\therefore \sin(\pi/3 - \sin^{-1}(-1/2)) = 1$

27. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan^{-1}\left\{\tan\left(\frac{15\pi}{4}\right)\right\}$.

Answer

Given $\tan^{-1}\{\tan(15\pi/4)\}$

$= \tan^{-1}\{\tan(4\pi - \pi/4)\}$

We know that $\tan(2\pi - \theta) = -\tan\theta$

$= \tan^{-1}(-\tan\pi/4)$

$= \tan^{-1}(-1)$

$= -\pi/4$

$\therefore \tan^{-1}\{\tan(15\pi/4)\} = -\pi/4$

28. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $2\sin^{-1}\frac{1}{2} + \cos^{-1}\left(-\frac{1}{2}\right)$.

Answer

Given $2\sin^{-1} 1/2 + \cos^{-1}(-1/2)$

$= \pi/6 + (\pi - \pi/3)$

$= \frac{\pi - 6\pi - 2\pi}{6}$

$= \frac{5\pi}{6}$

$\therefore 2\sin^{-1} 1/2 + \cos^{-1}(-1/2) = 5\pi/6$

29. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan^{-1}\frac{a}{b} - \tan^{-1}\left(\frac{a-b}{a+b}\right)$.

Answer

Given $tan^{-1}\frac{a}{b} - tan^{-1}\left(\frac{a-b}{a+b}\right)$

$$= tan^{-1}\left[\frac{\frac{a}{b} - \frac{a-b}{a+b}}{1 + \left(\frac{a}{b}\right)\left(\frac{a-b}{a+b}\right)}\right]$$

$$= tan^{-1}\left[\frac{\frac{a^2 + ab - ab + b^2}{b(a+b)}}{\frac{ba + b^2 + a^2 - ab}{b(a+b)}}\right]$$

$$= tan^{-1}\left[\frac{a^2 + b^2}{a^2 + b^2}\right]$$

= $\tan^{-1}$ (1)

= π/4

30. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1}\left(\cos\frac{2\pi}{4}\right)$.

Answer

Given $\cos^{-1}$ (cos 2π/4)

We know that $\cos^{-1}$ (cos θ) = θ

= 2π/4

= π/2

∴ $\cos^{-1}$ (cos 2π/4) = π/2

31. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1}x$.

Answer

Given LHS = $\sin^{-1}$ (2x - √ (1 – x^2))

Let x = sin θ

= $\sin^{-1}$ (2sin θ √ (1 – $\sin^2$ θ))

We know that 1 – $\sin^2$ θ = $\cos^2$ θ

= $\sin^{-1}$ (2 sin θ cos θ)

= $\sin^{-1}$ ($\sin^2$ θ)

= 2θ

= 2 $\sin^{-1}$ x

= RHS

∴ $\sin^{-1}(2x - \sqrt{(1 - x^2)}) = 2\sin^{-1} x$

32. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Evaluate: $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.

Answer

Given $\sin^{-1}(\sin 3\pi/5)$

We know that $\sin^{-1}(\sin\theta) = \pi - \theta$, if $\theta \in [\pi/2, 3\pi/2]$

$= \pi - 3\pi/5$

$= 2\pi/5$

∴ $\sin^{-1}(\sin 3\pi/5) = 2\pi/5$

33. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $\tan^{-1}\left(\sqrt{3}\right) + \cot^{-1} x = \frac{\pi}{2}$, find x.

Answer

Given $\tan^{-1}(\sqrt{3}) + \cot^{-1} x = \pi/2$

⇒ $\tan^{-1}(\sqrt{3}) = \pi/2 - \cot^{-1} x$

We know that $\tan^{-1} x + \cot^{-1} x = \pi/2$

⇒ $\tan^{-1}\sqrt{3} = \tan^{-1} x$

∴ $x = \sqrt{3}$

34. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $\sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1} x = \frac{\pi}{2}$, then find x.

Answer

Given $\sin^{-1}(1/3) + \cos^{-1} x = \pi/2$

⇒ $\sin^{-1}(1/3) = \pi/2 - \cos^{-1} x$

We know that $\sin^{-1} x + \cos^{-1} x = \pi/2$

⇒ $\sin^{-1}(1/3) = \sin^{-1} x$

∴ $x = 1/3$

35. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin^{-1}\left(\frac{1}{3}\right)-\cos^{-1}\left(-\frac{1}{3}\right)$.

Answer

Given $\sin^{-1}$ (1/3) – $\cos^{-1}$ (-1/3)

We know that $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

$$= sin^{-1}\left(\frac{1}{3}\right) - \left[\pi - cos^{-1}\left(\frac{1}{3}\right)\right]$$

$$= sin^{-1}\left(\frac{1}{3}\right) - \pi + cos^{-1}\left(\frac{1}{3}\right)$$

$$= sin^{-1}\left(\frac{1}{3}\right) + cos^{-1}\left(\frac{1}{3}\right) - \pi$$

$$= \frac{\pi}{2} - \pi$$

$$= -\frac{\pi}{2}$$

$$\therefore sin^{-1}\left(\frac{1}{3}\right) - cos^{-1}\left(-\frac{1}{3}\right) = -\frac{\pi}{2}$$

36. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $4\sin^{-1}x + \cos^{-1}x = \pi$, then what is the value of x?

Answer

Given $4\sin^{-1}x + \cos^{-1}x = \pi$

We know that $\sin^{-1}x + \cos^{-1}x = \pi/2$

$$\Rightarrow 4\,sin^{-1}x + \frac{\pi}{2} - sin^{-1}x = \pi$$

$$\Rightarrow 3\,sin^{-1}x = \pi - \frac{\pi}{2}$$

$$\Rightarrow 3\,sin^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow sin^{-1}x = \frac{\pi}{6}$$

$$\Rightarrow sin^{-1}x = sin^{-1}\frac{1}{2}$$

$\therefore$ x = 1/2

37. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If x < 0, y < 0 such that xy = 1, then write the value of $\tan^{-1}x + \tan^{-1}y$.

Answer

Given if x < 0, y < 0 such that xy = 1

Also given $\tan^{-1}x + \tan^{-1}y$

We know that $tan^{-1} x + tan^{-1} y = tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$= -\pi + tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$= -\pi + tan^{-1}\left(\frac{x+y}{1-1}\right)$

= -π + tan^{-1} (∞)

$= -\pi + \frac{\pi}{2}$

$= -\frac{\pi}{2}$

$\therefore tan^{-1} x + tan^{-1} y = -\frac{\pi}{2}$

38. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

What is the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?

Answer

Given sin^{-1} (-√3/2)

We know that sin^{-1} (-θ) = -sin^{-1} (θ)

= - sin^{-1} (√3/2)

= -π/3

∴ sin^{-1} (-√3/2) = -π/3

39. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.

Answer

Given sin^{-1} (-1/2)

We know that sin^{-1} (-θ) = -sin^{-1} (θ)

= - sin^{-1} (1/2)

= π/6

∴ sin^{-1} (-1/2) = π/6

40. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

Answer

We know that $\sin^{-1}(\sin\theta) = \pi - \theta$, if $\theta \in [\pi/2, 3\pi/2]$ and $\cos^{-1}(\cos\theta) = \theta$, if $\theta \in [0, \pi]$

Given $cos^{-1}\left(cos\frac{2\pi}{3}\right) + sin^{-1}\left(sin\frac{2\pi}{3}\right)$

$$= \frac{2\pi}{3} + \left(\pi - \frac{2\pi}{3}\right)$$

$= \pi$

$$\therefore cos^{-1}\left(cos\frac{2\pi}{3}\right) + sin^{-1}\left(sin\frac{2\pi}{3}\right) = \pi$$

41. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan\left(2\tan^{-1}\frac{1}{5}\right)$.

Answer

Let $\tan\theta = 1/5$

Given $\tan(2\tan^{-1} 1/5) = \tan 2\theta$

We know that $tan\,2\theta = \frac{2\,tan\,\theta}{1-tan^2\theta}$

$$= \frac{2 \times \frac{1}{5}}{1 - \frac{1}{25}}$$

$$= \frac{\frac{2}{5}}{\frac{24}{25}}$$

$$= \frac{5}{12}$$

$$\therefore tan(2\,tan^{-1}\frac{1}{5}) = \frac{5}{12}$$

42. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$.

Answer

Given $\tan^{-1}(1) + \cos^{-1}(-1/2)$

We know that $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

$$= \frac{\pi}{4} + \left[\pi - \frac{\pi}{3}\right]$$

$$= \frac{\pi}{4} + \frac{2\pi}{3}$$

$= \frac{3\pi + 8\pi}{12}$

$= \frac{11\pi}{12}$

$\therefore tan^{-1}(1) + cos^{-1}\left(-\frac{1}{2}\right) = \frac{11\pi}{12}$

43. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan^{-1}\left\{2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\}$.

Answer

Given $\tan^{-1}$ {2 sin (2 $\cos^{-1}$ √3/2)}

= $\tan^{-1}$ {2 sin (2 $\cos^{-1}$ cos π/6)}

= $\tan^{-1}$ {2 sin (2 × π/6)}

= $\tan^{-1}$ {2 sin (π/3)}

= $\tan^{-1}$ {2 × √3/2}

= $\tan^{-1}$ {√3}

= π/3

∴ $\tan^{-1}$ {2 sin (2 $\cos^{-1}$ √3/2)} = π/3

44. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the principal value of $\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3}$.

Answer

Given $\tan^{-1}$ √3 + $\cot^{-1}$ √3

We know that $\tan^{-1}$ √3 = π/3 and $\cot^{-1}$ √3 = π/6

$= \frac{\pi}{3} + \frac{\pi}{6}$

$= \frac{2\pi + \pi}{6}$

$= \frac{3\pi}{6}$

= π/2

45. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the principal value of $\cos^{-1}$(cos 680°).

Answer

Given $\cos^{-1}(\cos 680°)$

$= \cos^{-1}(\cos (720° - 40°))$

$= \cos^{-1}(\cos (2 \times 360° - 40°))$

$= \cos^{-1}(\cos 40°)$

$= 40°$

$\therefore \cos^{-1}(\cos 680°) = 40°$

46. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.

Answer

Given $\sin^{-1}(\sin 3\pi/5)$

$= \sin^{-1}[\sin (\pi - 2\pi/5)]$

$= \sin^{-1}(\sin 2\pi/5)$

$= 2\pi/5$

$\therefore \sin^{-1}(\sin 3\pi/5) = 2\pi/5$

47. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sec^{-1}\left(\frac{1}{2}\right)$.

Answer

We know that the value of $\sec^{-1}(1/2)$ is undefined as it is outside the range i.e. R – (-1, 1).

48. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1}\left(\cos\frac{14\pi}{3}\right)$.

Answer

Given $\cos^{-1}(\cos 14\pi/3)$

$= \cos^{-1}[\cos (4\pi + 2\pi/3)]$

$= \cos^{-1}(\cos 2\pi/3)$

$= 2\pi/3$

$\therefore \cos^{-1}(\cos 14\pi/3) = 2\pi/3$

49. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos\left(\sin^{-1}x+\cos^{-1}x\right), |x| \le 1$.

Answer

Given $|x| \le 1$

$\Rightarrow \pm x \le 1$

$\Rightarrow x \le 1$ or $-x \le 1$

$\Rightarrow x \le 1$ or $x \ge -1$

$\Rightarrow x \in [-1, 1]$

Now also given $\cos(\sin^{-1} x + \cos^{-1} x)$

We know that $\sin^{-1} x + \cos^{-1} x = \pi/2$

$\therefore \cos(\sin^{-1} x + \cos^{-1} x) = \cos(\pi/2) = 0$

50. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of the expression $\tan\left(\frac{\sin^{-1}x+\cos^{-1}x}{2}\right)$, when $x=\frac{\sqrt{3}}{2}$.

Answer

Given $tan\left(\frac{sin^{-1}x+cos^{-1}x}{2}\right) when\ x=\frac{\sqrt{3}}{2}$

$\Rightarrow tan\left(\frac{sin^{-1}x+cos^{-1}x}{2}\right)=tan\left(\frac{sin^{-1}\frac{\sqrt{3}}{2}+cos^{-1}\frac{\sqrt{3}}{2}}{2}\right)$

We know that $\sin^{-1} x + \cos^{-1} x = \pi/2$

$= \tan(\pi/4)$

$\therefore tan\left(\frac{sin^{-1}x+cos^{-1}x}{2}\right)=1$

51. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the principal value of $\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{1}{2}\right)\right\}$.

Answer

Given $sin^{-1}\left\{cos\left(sin^{-1}\frac{1}{2}\right)\right\}$

$=sin^{-1}\left\{cos\left(sin^{-1}\left(sin\frac{\pi}{3}\right)\right)\right\}$

$=sin^{-1}\left\{cos\left(\frac{\pi}{3}\right)\right\}$

$= \sin^{-1}(1/2)$

$= sin^{-1}\left(sin\frac{\pi}{3}\right)$

$= \frac{\pi}{3}$

52. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

The set of values of $\operatorname{cosec}^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

Answer

We know that the value of $\operatorname{cosec}^{-1}$ (√3/2) is undefined as it is outside the range i.e. R-(-1,1).

53. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan^{-1}\left(\frac{1}{x}\right)$ for x < 0 in terms of $\cot^{-1}(x)$.

Answer

Given $\tan^{-1}$ (1/x)

$= tan^{-1}\left(-\frac{1}{x}\right) for\ x < 0$

$= -\, tan^{-1}\left(\frac{1}{x}\right)$

= $\cot^{-1}$ x

= - (π – $\cot^{-1}$ x)

= - π + $\cot^{-1}$ x

54. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cot^{-1}(-x)$ for all x ∈ R in terms of $\cot^{-1}x$.

Answer

We know that $\cot^{-1}$ (-x) = π – $\cot^{-1}$ (x)

∴ The value of $\cot^{-1}$ (-x) for all x ∈ R in term of $\cot^{-1}$ x is π – $\cot^{-1}$ (x).

55. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos\left(\frac{\tan^{-1}x+\cot^{-1}x}{3}\right)$, when $x = -\frac{1}{\sqrt{3}}$.

Answer

Given $cos\left(\frac{tan^{-1}x+cot^{-1}x}{3}\right)\ when\ x = -\frac{1}{\sqrt{3}}$

We know that $\tan^{-1} x + \cot^{-1} x = \pi/2$

$= \cos (\pi/6)$

$= \sqrt{3}/2$

56. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $\cos\left(\tan^{-1} x + \cot^{-1} \sqrt{3}\right) = 0$, find the value of x.

Answer

Given $\cos (\tan^{-1} x + \cot^{-1} \sqrt{3}) = 0$

$\Rightarrow \cos (\tan^{-1} x + \cot^{-1} \sqrt{3}) = \cos (\pi/2)$

$\Rightarrow \tan^{-1} x + \cot^{-1} \sqrt{3} = \pi/2$

We know that $\tan^{-1} x + \cot^{-1} x = \pi/2$

$\therefore x = \sqrt{3}$

57. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Find the value of $2 \sec^{-1} 2 + \sin^{-1}\left(\frac{1}{2}\right)$.

Answer

Given $2 \sec^{-1} 2 + \sin^{-1} (1/2)$

$= 2 \sec^{-1} (\sec \pi/3) + \sin^{-1} (\sin \pi/6)$

$= 2 (\pi/3) + \pi/6$

$= 5\pi/6$

58. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $\cos\left(\sin^{-1} \frac{2}{5} + \cos^{-1} x\right) = 0$, find the value of x.

Answer

Given $\cos (\sin^{-1} 2/5 + \cos^{-1} x) = 0$

$\Rightarrow \cos (\sin^{-1} 2/5 + \cos^{-1} x) = \cos (\pi/2)$

$\Rightarrow \sin^{-1} 2/5 + \cos^{-1} x = \pi/2$

We know that $\sin^{-1} x + \cos^{-1} x = \pi/2$

$\therefore x = 2/5$

59. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

Answer

Given $\cos^{-1}$ (cos 13π/6)

= $\cos^{-1}$ [cos (2π + π/6)]

= $\cos^{-1}$ (cos π/6)

= π/6

∴ $\cos^{-1}$ (cos 13π/6) = π/6

60. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Find the value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$.

Answer

Given $\tan^{-1}$ (tan 9π/8)

= $\tan^{-1}$ [tan (π + π/8)]

= $\tan^{-1}$ (tan π/8)

= π/8

∴ $\tan^{-1}$ (tan 9π/8) = π/8