5. Algebra of Matrices

Exercise 5.1

1. Question

If a matrix has 8 elements, what are the possible orders it can have? What if it has 5 elements?

Answer

If a matrix is of order m×n elements, it has mn elements. So, if the matrix has 8 elements, we will find the ordered pairs m and n.

mn = 8

Then, ordered pairs m and n can be

m×n be (8×1),(1×8),(4×2),(2×4)

Now, if it has 5 elements

Possible orders are (5×1), (1×5).

2 A. Question

If $A = \left[a_{ij}\right] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and $B = \left[b_{ij}\right] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$ then find

$a_{22} + b_{21}$

Answer

$A = [a_{ij}] = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ ....(1)

$B = [b_{ij}] = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$ ....(2)

Given, $A = [a_{ij}] = \begin{pmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{pmatrix}$ $B = [b_{ij}] = \begin{pmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{pmatrix}$

Now, Comparing with equation (1) and (2)

$a_{22} = 4$ and $b_{21} = -3$

$a_{22} + b_{21} = 4 + (-3) = 1$

2 B. Question

If $A = \left[a_{ij}\right] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and $B = \left[b_{ij}\right] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$ then find

$a_{11}\, b_{11} + a_{22}\, b_{22}$

Answer

$A = [a_{ij}] = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ ....(1)

$B = [b_{ij}] = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$ ....(2)

Given, $A = [a_{ij}] = \begin{pmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{pmatrix}$ $B = [b_{ij}] = \begin{pmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{pmatrix}$

Now, Comparing with equation (1) and (2)

$a_{11} = 2$, $a_{22} = 4$, $b_{11} = 2$, $b_{22} = 4$

$a_{11}\, b_{11} + a_{22}\, b_{22} = 2 \times 2 + 4 \times 4 = 4 + 16 = 20$

3. Question

Let A be a matrix of order 3 × 4. If R_1 denotes the first row of A and C_2 denotes its second column, then determine the orders of matrices R_1 and C_2.

Answer

Let A be a matrix of order 3×4.

$A = [a_{ij}]_{3\times 4}$

R_1 = first row of A = $[a_{11}, a_{12}, a_{13}, a_{14}]$

So, order of matrix $R_1 = 1\times 4$

C_2 = second column of A = $\begin{matrix} a_{12} \\ a_{22} \\ a_{32} \end{matrix}$

Order of $C_2 = 3\times 1$

4 A. Question

Construct a 2 ×3 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by :

$a_{ij} = i \times j$

Answer

Let $A = [a_{ij}]_{2\times 3}$

So, the elements in a 2×3 matrix are

a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}

$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$.................. (1)

$a_{11} = 1\times 1 = 1$ $a_{12} = 1\times 2 = 2$ $a_{13} = 1\times 3 = 3$

$a_{21} = 2\times 1 = 2$ $a_{22} = 2\times 2 = 4$ $a_{23} = 2\times 3 = 6$

So, from (1)

$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}$

4 B. Question

Construct a 2 ×3 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by :

$a_{ij} = 2i - j$

Answer

Let A = $[a_{ij}]_{2\times3}$

So, the elements in a 2×3 matrix are

a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}

A = $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$.................. (1)

a_{11} = 2×1 – 1 = 2 – 1 = 1 a_{12} = 2×1 – 2 = 2 – 2 = 0 a_{13} = 2×1 – 3 = 2 – 3 = – 1

a_{21} = 2×2 – 1 = 4 – 1 = 3 a_{22} = 2×2 – 2 = 4 – 2 = 2 a_{23} = 2×2 – 3 = 4 – 3 = 1

So, from (1)

A = $\begin{pmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{pmatrix}$

4 C. Question

Construct a 2 ×3 matrix A = $[a_{ij}]$ whose elements a_{ij} are given by :

a_{ij} = i + j

Answer

Let A = $[a_{ij}]_{2\times3}$

So, the elements in a 2×3 matrix are

a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}

A = $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ ...... (1)

a_{11} = 1 + 1 = 2 a_{12} = 1 + 2 = 3 a_{13} = 1 + 3 = 4

a_{21} = 2 + 1 = 3 a_{22} = 2 + 2 = 4 a_{23} = 2 + 3 = 5

So, from (1)

A = $\begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$

4 D. Question

Construct a 2 ×3 matrix A = $[a_{ij}]$ whose elements a_{ij} are given by :

$a_{ij} = \frac{(i+j)^2}{2}$

Answer

Let A = $[a_{ij}]_{2\times3}$

So, the elements in a 2×3 matrix are

a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}

A = $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ ...... (1)

$a_{11} = \frac{(1+1)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$

$a_{12} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{13} = \frac{(1+3)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$

$a_{21} = \frac{(2+1)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{22} = \frac{(2+2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$

$a_{23} = \frac{(2+3)^2}{2} = \frac{5^2}{2} = \frac{25}{2} = 12.5$

So, from (1)

$A = \begin{pmatrix} 2 & 4.5 & 8 \\ 4.5 & 8 & 12.5 \end{pmatrix}$

5 A. Question

Construct a 2 × 2 matrix A = [a_{jj}] whose elements a_{jj} are given by :

$$\frac{(i+j)^2}{2}$$

Answer

Let A = $[a_{ij}]_{2\times 2}$

So, the elements in a 2×2 matrix are

a_{11}, a_{12}, a_{21}, a_{22},

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ ...... (1)

$a_{11} = \frac{(1+1)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$

$a_{12} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{21} = \frac{(2+1)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{22} = \frac{(2+2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$

So, from (1)

$A = \begin{pmatrix} 2 & 4.5 \\ 4.5 & 8 \end{pmatrix}$

5 B. Question

Construct a 2 × 2 matrix A = [a_{jj}] whose elements a_{jj} are given by :

$$a_{jj} = \frac{(i-j)^2}{2}$$

Answer

Let A = $[a_{ij}]_{2\times 2}$

So, the elements in a 2×2 matrix are

a_{11}, a_{12}, a_{21}, a_{22},

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ ...... (1)

$a_{11} = \frac{(1-1)^2}{2} = \frac{0^2}{2} = 0$

$a_{12} = \frac{(1-2)^2}{2} = \frac{1^2}{2} = \frac{1}{2} = 0.5$

$a_{21} = \frac{(2-1)^2}{2} = \frac{1^2}{2} = \frac{1}{2} = 0.5$

$a_{22} = \frac{(2-2)^2}{2} = \frac{0^2}{2} = 0$

So, from (1)

$A = \begin{pmatrix} 0 & 0.5 \\ 0.5 & 0 \end{pmatrix}$

5 C. Question

Construct a 2 × 2 matrix A = $[a_{jj}]$ whose elements a_{jj} are given by :

$$a_{jj} = \frac{(i-2j)^2}{2}$$

Answer

Let A = $[a_{ij}]_{2\times2}$

So, the elements in a 2×2 matrix are

a_{11}, a_{12}, a_{21}, a_{22},

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ ...... (1)

$a_{11} = \frac{(1-2\times1)^2}{2} = \frac{1^2}{2} = 0.5$

$a_{12} = \frac{(1-2\times2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{21} = \frac{(2-2\times1)^2}{2} = \frac{0^2}{2} = 0$

$a_{22} = \frac{(2-2\times2)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$

So, from (1)

$A = \begin{pmatrix} 0.5 & 4.5 \\ 0 & 2 \end{pmatrix}$

5 D. Question

Construct a 2 × 2 matrix A = $[a_{jj}]$ whose elements a_{jj} are given by :

$$a_{jj} = \frac{(2i+j)^2}{2}$$

Answer

Let A = $[a_{ij}]_{2\times2}$

So, the elements in a 2×2 matrix are

a_{11}, a_{12}, a_{21}, a_{22},

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ ...... (1)

$a_{11} = \frac{(2\times1+1)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{12} = \frac{(2\times1+2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$

$a_{21} = \frac{(2\times2+1)^2}{2} = \frac{5^2}{2} = \frac{25}{2} = 12.5$

$a_{22} = \frac{(2\times2+2)^2}{2} = \frac{6^2}{2} = \frac{36}{2} = 18$

So, from (1)

$A = \begin{pmatrix} 4.5 & 8 \\ 12.5 & 18 \end{pmatrix}$

5 E. Question

Construct a 2 × 2 matrix A = [a_{jj}] whose elements a_{jj} are given by :

$$a_{jj} = \frac{\left|2i - 3\hat{j}\right|}{2}$$

Answer

Let A = $[a_{ij}]_{2\times2}$

So, the elements in a 2×2 matrix are

a_{11}, a_{12}, a_{21}, a_{22},

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ ......(1)

$a_{11} = \frac{|2\times1-3\times1|}{2} = \frac{1}{2} = 0.5$

$a_{12} = \frac{|2\times1-3\times2|}{2} = \frac{4}{2} = 2$

$a_{21} = \frac{|2\times2-3\times1|}{2} = \frac{4-3}{2} = \frac{1}{2} = 0.5$

$a_{22} = \frac{|2\times2-3\times2|}{2} = \frac{2}{2} = 1$

So, from (1)

$A = \begin{pmatrix} 0.5 & 2 \\ 0.5 & 1 \end{pmatrix}$

5 F. Question

Construct a 2 × 2 matrix A = [a_{jj}] whose elements a_{jj} are given by :

$$a_{jj} = \frac{\left|-3i + \hat{j}\right|}{2}$$

Answer

Let A = $[a_{ij}]_{2\times2}$

So, the elements in a 2×2 matrix are

a_{11}, a_{12}, a_{21}, a_{22},

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ ...... (1)

$a_{11} = \frac{|-3\times1+1|}{2} = \frac{2}{2} = 1$

$a_{12} = \frac{|-3\times1+2|}{2} = \frac{1}{2} = 0.5$

$a_{21} = \frac{|-3\times2+1|}{2} = \frac{5}{2} = 2.5$

$a_{22} = \frac{|-3\times2+2|}{2} = \frac{4}{2} = 2$

So, from (1)

$A = \begin{pmatrix} 1 & 0.5 \\ 2.5 & 2 \end{pmatrix}$

5 G. Question

Construct a 2 × 2 matrix A = [a_{jj}] whose elements a_{jj} are given by :

$a_{ij} = e^{2ix} \sin xj$

Answer

Let $A = [a_{ij}]_{2\times2}$

So, the elements in a 2×2 matrix are

a_{11}, a_{12}, a_{21}, a_{22},

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ ...... (1)

$a_{11} = e^{2\times1x} \sin x \times 1 = e^{2x} \sin x$

$a_{12} = e^{2\times1x} \sin x \times 2 = e^{2x} \sin 2x$

$a_{21} = e^{2\times2x} \sin x \times 1 = e^{4x} \sin x$

$a_{22} = e^{2\times2x} \sin x \times 2 = e^{4x} \sin 2x$

So, from (1)

$A = \begin{pmatrix} e^{2x} \sin x & e^{2x} \sin 2x \\ e^{4x} \sin x & e^{4x} \sin 2x \end{pmatrix}$

6 A. Question

Construct a 3×4 matrix A = [a_{jj}] whose elements a_{jj} are given by :

$a_{jj} = i + j$

Answer

Let $A = [a_{ij}]_{2\times3}$

So, the elements in a 3×4 matrix are

a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}

$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$ ......(1)

$a_{11} = 1 + 1 = 2$ $a_{12} = 1 + 2 = 3$ $a_{13} = 1 + 3 = 4$ $a_{14} = 1 + 4 = 5$

$a_{21} = 2 + 1 = 3$ $a_{22} = 2 + 2 = 4$ $a_{23} = 2 + 3 = 5$ $a_{24} = 2 + 4 = 6$

$a_{31} = 3 + 1 = 4$ $a_{32} = 3 + 2 = 5$ $a_{33} = 3 + 3 = 6$ $a_{34} = 3 + 4 = 7$

So, from (1)

$A = \begin{bmatrix} 2 & \cdots & 5 \\ \vdots & \ddots & \vdots \\ 4 & \cdots & 7 \end{bmatrix}$

6 B. Question

Construct a 3×4 matrix A = [a_{jj}] whose elements a_{jj} are given by :

a_{jj} = i – j

Answer

Let A = $[a_{ij}]_{2\times3}$

So, the elements in a 3×4 matrix are

a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23},a_{24},a_{31},a_{32},a_{33},a_{34}

$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$...... (1)

a_{11} = 1 – 1 = 0 a_{12} = 1 – 2 = – 1 a_{13} = 1 – 3 = – 2 a_{14} = 1 – 4 = – 3

a_{21} = 2 – 1 = 1 a_{22} = 2 – 2 = 0 a_{23} = 2 – 3 = – 1 $a_{24 =}$ 2 – 4 = – 2

a_{31} = 3 – 1 = 2 a_{32} = 3 – 2 = 1 a_{33} = 3 – 3 = 0 a_{34} = 3 – 4 = – 1

So, from (1)

$A = \begin{bmatrix} 0 & \cdots & -3 \\ \vdots & \ddots & \vdots \\ 2 & \cdots & -1 \end{bmatrix}$

6 C. Question

Construct a 3×4 matrix A = [a_{jj}] whose elements a_{jj} are given by :

a_{jj} = 2i

Answer

Let A = $[a_{ij}]_{2\times3}$

So, the elements in a 3×4 matrix are

a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23},a_{24},a_{31},a_{32},a_{33},a_{34}

$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$......(1)

a_{11} = 2×1 = 2 a_{12} = 2×1 = 2 a_{13} = 2×1 = 2 a_{14} = 2×1 = 2

a_{21} = 2×2 = 4 a_{22} = 2×2 = 4 a_{23} = 2×2 = 4 $a_{24 =}$ 2×2 = 4

a_{31} = 2×3 = 6 a_{32} = 2×3 = 6 a_{33} = 2×3 = 6 a_{34} = 2×3 = 6

So, from (1)

$A = \begin{bmatrix} 2 & \cdots & 2 \\ \vdots & \ddots & \vdots \\ 6 & \cdots & 6 \end{bmatrix}$

6 D. Question

Construct a 3×4 matrix A = [a_{jj}] whose elements a_{jj} are given by :

a_{jj} = j

Answer

Let A = $[a_{ij}]_{2\times3}$

So, the elements in a 3×4 matrix are

a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}

A = $\begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$ ......(1)

$a_{11} = 1$ $a_{12} = 2$ $a_{13} = 3$ $a_{14} = 4$

$a_{21} = 1$ $a_{22} = 2$ $a_{23} = 3$ $a_{24} = 4$

$a_{31} = 1$ $a_{32} = 2$ $a_{33} = 3$ $a_{34} = 4$

So, from (1)

A = $\begin{bmatrix} 1 & \cdots & 4 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 4 \end{bmatrix}$

6 E. Question

Construct a 3×4 matrix A = $[a_{jj}]$ whose elements a_{jj} are given by :

$$a_{jj} = \frac{1}{2}|-3i + j|$$

Answer

Let A = $[a_{ij}]_{2\times3}$

So, the elements in a 3×4 matrix are

a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}

A = $\begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$ ...... (1)

$a_{11} = \frac{1}{2}(-3 \times 1 + 1) = \frac{1}{2}(-3 + 1) = \frac{1}{2}(-2) = -1$

$a_{12} = \frac{1}{2}(-3 \times 1 + 2) = \frac{1}{2}(-3 + 2) = \frac{1}{2}(-1) = -\frac{1}{2}$

$a_{13} = \frac{1}{2}(-3 \times 1 + 3) = \frac{1}{2}(-3 + 3) = \frac{1}{2}(0) = 0$

$a_{14} = \frac{1}{2}(-3 \times 1 + 4) = \frac{1}{2}(-3 + 4) = \frac{1}{2}(1) = \frac{1}{2}$

$a_{21} = \frac{1}{2}(-3 \times 2 + 1) = \frac{1}{2}(-6 + 1) = \frac{1}{2}(-5) = -\frac{5}{2}$

$a_{22} = \frac{1}{2}(-3 \times 2 + 2) = \frac{1}{2}(-6 + 2) = \frac{1}{2}(-4) = -2$

$a_{23} = \frac{1}{2}(-3 \times 2 + 3) = \frac{1}{2}(-6 + 3) = \frac{1}{2}(-3) = -\frac{3}{2}$

$a_{24} = \frac{1}{2}(-3 \times 2 + 4) = \frac{1}{2}(-6 + 4) = \frac{1}{2}(-2) = -1$

$a_{31} = \frac{1}{2}(-3 \times 3 + 1) = \frac{1}{2}(-9 + 1) = \frac{1}{2}(-8) = -4$

$a_{32} = \frac{1}{2}(-3 \times 3 + 2) = \frac{1}{2}(-9 + 2) = \frac{1}{2}(-7) = -\frac{7}{2}$

$a_{33} = \frac{1}{2}(-3 \times 3 + 3) = \frac{1}{2}(-9 + 3) = \frac{1}{2}(-6) = -3$

$a_{34} = \frac{1}{2}(-3 \times 3 + 4) = \frac{1}{2}(-9 + 4) = \frac{1}{2}(-5) = -\frac{5}{2}$

So, from (1)

$A = \begin{bmatrix} -1 & \cdots & \frac{1}{2} \\ \vdots & \ddots & \vdots \\ -4 & \cdots & -\frac{5}{2} \end{bmatrix}$

7 A. Question

Construct a 4 × 3 matrix A = $[a_{jj}]$ whose elements a_{jj} are given by :

$a_{jj} = 2i + \frac{i}{j}$

Answer

Let A = $[a_{ij}]_{4\times3}$

So, the elements in a 4×3 matrix are

a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23},a_{31},a_{32},a_{33},a_{41}, a_{42}, a_{43}

$A = \begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{43} \end{bmatrix}$......(1)

$a_{11} = 2 \times 1 + \frac{1}{1} = 2 + 1 = 3$

$a_{12} = 2 \times 1 + \frac{1}{2} = 2 + \frac{1}{2} = \frac{5}{2}$

$a_{13} = 2 \times 1 + \frac{1}{3} = 2 + \frac{1}{3} = \frac{7}{3}$

$a_{21} = 2 \times 2 + \frac{2}{1} = 4 + 2 = 6$

$a_{22} = 2 \times 2 + \frac{2}{2} = 4 + 1 = 5$

$a_{23} = 2 \times 2 + \frac{2}{3} = 4 + \frac{2}{3} = \frac{14}{3}$

$a_{31} = 2 \times 3 + \frac{3}{1} = 6 + 3 = 9$

$a_{32} = 2 \times 3 + \frac{3}{2} = 6 + \frac{3}{2} = \frac{15}{2}$

$a_{33} = 2 \times 3 + \frac{3}{3} = 6 + 1 = 7$

$a_{41} = 2 \times 4 + \frac{4}{1} = 8 + 4 = 12$

$a_{42} = 2 \times 4 + \frac{4}{2} = 8 + 2 = 10$

$a_{43} = 2 \times 4 + \frac{4}{3} = 8 + \frac{4}{3} = \frac{28}{3}$

So, from (1)

$$A = \begin{bmatrix} 3 & \cdots & \frac{7}{3} \\ \vdots & \ddots & \vdots \\ 12 & \cdots & \frac{28}{3} \end{bmatrix}$$

7 B. Question

Construct a 4 × 3 matrix A = $[a_{jj}]$ whose elements a_{jj} are given by :

$$a_{jj} = \frac{i-j}{i+j}$$

Answer

Let A = $[a_{ij}]_{4\times 3}$

So, the elements in a 4×3 matrix are

a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, a_{41}, a_{42}, a_{43}

$$A = \begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{43} \end{bmatrix} \text{......(1)}$$

$a_{11} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

$a_{12} = \frac{1-2}{1+2} = \frac{-1}{3}$

$a_{13} = \frac{1-3}{1+3} = \frac{-2}{4} = -\frac{1}{2}$

$a_{21} = \frac{2-1}{2+1} = \frac{1}{3}$

$a_{22} = \frac{2-2}{2+2} = \frac{0}{4} = 0$

$a_{23} = \frac{2-3}{2+3} = \frac{-1}{5}$

$a_{31} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$

$a_{32} = \frac{3-2}{3+2} = \frac{1}{5}$

$a_{33} = \frac{3-3}{3+3} = \frac{0}{6} = 0$

$a_{41} = \frac{4-1}{4+1} = \frac{3}{5}$

$a_{42} = \frac{4-2}{4+2} = \frac{2}{6} = \frac{1}{3}$

$a_{43} = \frac{4-3}{4+3} = \frac{1}{7}$

So, from (1)

$$A = \begin{bmatrix} 0 & \cdots & -\frac{1}{2} \\ \vdots & \ddots & \vdots \\ \frac{3}{5} & \cdots & \frac{1}{7} \end{bmatrix}$$

7 C. Question

Construct a 4 × 3 matrix A = $[a_{jj}]$ whose elements a_{jj} are given by :

a_{jj} = i

Answer

Let A = $[a_{ij}]_{4\times3}$

So, the elements in a 4×3 matrix are

a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23},a_{31},a_{32},a_{33},a_{41}, a_{42}, a_{43}

A = $\begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{43} \end{bmatrix}$......(1)

$a_{11} = 1$

$a_{12} = 1$

$a_{13} = 1$

$a_{21} = 2$

$a_{22} = 2$

$a_{23} = 2$

$a_{31} = 3$

$a_{32} = 3$

$a_{33} = 3$

$a_{41} = 4$

$a_{42} = 4$

$a_{43} = 4$

So, from (1)

A = $\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 4 & \cdots & 4 \end{bmatrix}$

8. Question

Find x, y, a and b if

$$\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}.$$

Answer

Given two matrices are equal.

$$\begin{pmatrix} 3x + 4y & 2 & x-2y \\ a + b & 2a-b & -1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{pmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

∴$3x + 4y = 2$ ...... (1)

and $x – 2y = 4$ ...... (2)

and $a + b = 5$ ......(3)

$2a – b = – 5$ ......(4)

Multiplying equation (2) by 2 and adding to equation (1)

$3x + 4y + 2x – 4y = 2 + 8$

$\Rightarrow 5x = 10$

$\Rightarrow x = 2$

Now, Putting the value of x in equation (1)

$3\times2 + 4y = 2$

$\Rightarrow 6 + 4y = 2$

$\Rightarrow 4y = 2 - 6$

$\Rightarrow 4y = -4$

$\Rightarrow y = -1$

Adding equation (3) and (4)

$a + b + 2a - b = 5 + (-5)$

$\Rightarrow 3a = 5 - 5 = 0$

$\Rightarrow a = 0$

Now, Putting the value of a in equation (3)

$0 + b = 5$

$\Rightarrow b = 5$

$\therefore a = 0, b = 5, x = 2$ and $y = -1$

9. Question

Find x, y, a and b if

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

Answer

Given two matrices are equal.

$$\begin{pmatrix} 2a + b & a-2b \\ 5c-d & 4c + 3d \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

$\therefore 2a + b = 4$ ……(1)

And $a - 2b = -3$ …… (2)

And $5c - d = 11$ …… (3)

$4c + 3d = 24$ …… (4)

Multiplying equation (1) by 2 and adding to equation (2)

$4a + 2b + a - 2b = 8 - 3$

$\Rightarrow 5a = 5$

$\Rightarrow a = 1$

Now, Putting the value of a in equation (1)

$2\times1 + b = 4$

$\Rightarrow 2 + b = 4$

$\Rightarrow b = 4 - 2$

$\Rightarrow$ b = 2

Multiplying equation (3) by 3 and adding to equation (4)

15c – 3d + 4c + 3d = 33 + 24

$\Rightarrow$ 19c = 57

$\Rightarrow$ c = 3

Now, Putting the value of c in equation (4)

4×3 + 3d = 24

$\Rightarrow$ 12 + 3d = 24

$\Rightarrow$ 3d = 24 – 12

$\Rightarrow$ 3d = 12

$\Rightarrow$ d = 4

∴ a = 1, b = 2, c = 3 and d = 4

10. Question

Find the values of a, b, c and d from the following equations:

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

Answer

Given two matrices are equal.

$$\begin{pmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

∴2a + b = 4 …… (1)

And a – 2b = – 3 ……(2)

And 5c – d = 11 …… (3)

4c + 3d = 24 …… (4)

Multiplying equation (1) by 2 and adding to equation (2)

4a + 2b + a – 2b = 8 – 3

$\Rightarrow$ 5a = 5

$\Rightarrow$ a = 1

Now, Putting the value of a in equation (1)

2×1 + b = 4

$\Rightarrow$ 2 + b = 4

$\Rightarrow$ b = 4 – 2

$\Rightarrow$ b = 2

Multiplying equation (3) by 3 and adding to equation (4)

15c – 3d + 4c + 3d = 33 + 24

$\Rightarrow$ 19c = 57

⇒ c = 3

Now, Putting the value of c in equation (4)

4×3 + 3d = 24

⇒ 12 + 3d = 24

⇒ 3d = 24 – 12

⇒ 3d = 12

⇒ d = 4

∴ a = 1, b = 2, c = 3 and d = 4

11. Question

Find x, y and z so that A = B, where

$$A = \begin{bmatrix} x-2 & 3 & 2z \\ 18z & y+2 & 6z \end{bmatrix}, B = \begin{bmatrix} y & z & 6 \\ 6y & z & 2y \end{bmatrix}$$

Answer

Given two matrices are equal as A = B.

$$\begin{pmatrix} x-2 & 3 & 2z \\ 18z & y+2 & 6z \end{pmatrix} = \begin{pmatrix} y & z & 6 \\ 6y & x & 2y \end{pmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

∴x – 2 = y ...... (1)

z = 3

And y + 2 = z ...... (2)

2y = 6z

⇒ y = 3z ......(3)

Putting the value of z in equation (3)

∴ y = 3z = 3×3 = 9

Putting the value of y in equation (1)

x – 2 = 9

⇒ x – 2 = 9

⇒ x = 9 + 2

⇒ x = 11

∴ x = 11, y = 9, z = 3

12. Question

If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-\omega \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, find x, y, z, ω.

Answer

Given two matrices are equal.

$$\begin{pmatrix} x & 3x-y \\ 2x+z & 3y-\omega \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 4 & 7 \end{pmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

∴$x = 3$ …… (1)

And $3x - y = 2$ …… (2)

And $2x + z = 4$ …… (3)

$3y - \omega = 7$ …… (4)

Putting the value of x in equation (2)

$3 \times 3 - y = 2$

⇒ $9 - y = 2$

⇒ $y = 9 - 2$

⇒ $y = 7$

Now, putting the value of y in equation (4)

$3 \times 7 - \omega = 7$

⇒ $21 - \omega = 7$

⇒ $\omega = 21 - 7$

⇒ $\omega = 14$

Again, Putting the value of x in equation (3)

$2 \times 3 + z = 4$

⇒ $6 + z = 4$

⇒ $z = 4 - 6$

⇒ $z = -2$

∴ $x = 3$, $y = 7$, $z = -2$ and $\omega = 14$

13. Question

If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-\omega \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, find x, y, z,ω.

Answer

Given two matrices are equal.

$$\begin{pmatrix} x & 3x-y \\ 2x+z & 3y-\omega \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 4 & 7 \end{pmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

∴$x = 3$ ……(1)

And $3x - y = 2$……(2)

And $2x + z = 4$ …… (3)

$3y - \omega = 7$ ……(4)

Putting the value of x in equation (2)

$3 \times 3 - y = 2$

⇒ $9 - y = 2$

⇒ $y = 9 - 2$

$\Rightarrow y = 7$

Now, putting the value of y in equation (4)

$3\times7 - \omega = 7$

$\Rightarrow 21 - \omega = 7$

$\Rightarrow \omega = 21 - 7$

$\Rightarrow \omega = 14$

Again, Putting the value of x in equation (3)

$2\times3 + z = 4$

$\Rightarrow 6 + z = 4$

$\Rightarrow z = 4 - 6$

$\Rightarrow z = -2$

$\therefore x = 3, y = 7, z = -2$ and $\omega = 14$

14. Question

If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$

Obtain the values of a, b, c, x, y and z.

Answer

Given two matrices are equal.

$$\begin{pmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{pmatrix} = \begin{pmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{pmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

$\therefore x + 3 = 0$

$\Rightarrow x = 0 - 3 = -3$ …… (1)

And $z + 4 = 6$

$\Rightarrow z = 6 - 4 = 2$ …… (2)

And $2y - 7 = 3y - 2$

$\Rightarrow 2y - 3y = -2 + 7$

$\Rightarrow -y = 5$

$\Rightarrow y = -5$ …… (3)

$4x + 6 = 2x$ …… (4)

$a - 1 = -3$

$\Rightarrow a = -3 + 1 = -2$ …… (5)

$2c + 2 = 0$

$\Rightarrow 2c = -2$

$\Rightarrow c = -1$ …… (6)

$b - 3 = 2b + 4$

$\Rightarrow$ b – 2b = 4 + 3

$\Rightarrow$ – b = 7

$\Rightarrow$ b = – 7 ...... (7)

$\therefore$ x = – 3, y = – 5, z = 2 and a = – 2, b = – 7, c = – 1

15. Question

If $\begin{bmatrix} 2x+1 & 5x \\ 0 & y^2+1 \end{bmatrix} = \begin{bmatrix} x+3 & 10 \\ 0 & 26 \end{bmatrix}$, find the value of (x + y).

Answer

Given two matrices are equal.

$$\begin{pmatrix} 2x + 1 & 5x \\ 0 & y^2 + 1 \end{pmatrix} = \begin{pmatrix} x + 3 & 10 \\ 0 & 26 \end{pmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

$\therefore$2x + 1 = x + 3 ...... (1)

$\Rightarrow$ 2x – x = 3 – 1

$\Rightarrow$ x = 2

And 5x = 10 ...... (2)

y^2 + 1 = 26...... (3)

$\Rightarrow$ y^2 = 26 – 1

$\Rightarrow$ y^2 = 25

$\Rightarrow$ y = 5 or – 5

$\therefore$ x = 2, y = 5 or – 5

$\therefore$ x + y = 2 + 5 = 7

Or x + y = 2 – 5 = – 3

16. Question

If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & \omega \\ 0 & 6 \end{bmatrix}$, then find the values of x, y, z and ω.

Answer

Given two matrices are equal.

$$\begin{pmatrix} xy & 4 \\ z + 6 & x + y \end{pmatrix} = \begin{pmatrix} 8 & \omega \\ 0 & 6 \end{pmatrix}$$

We know that if two matrices are equal, then the elements of each matrix are also equal.

$\therefore$xy = 8 ...... (1)

And ω = 4 ......(2)

And z + 6 = 0

$\Rightarrow$ z = – 6 ......(3)

x + y = 6 ...... (4)

$\Rightarrow x + \frac{8}{x} = 6$ ...... (from (1))

$\Rightarrow \frac{(x^2 + 8)}{x} = 6$

$\Rightarrow x^2 + 8 = 6x$

$\Rightarrow x^2 - 6x + 8 = 0$

$\Rightarrow x^2 - 4x - 2x + 8$

$\Rightarrow x(x-4) - 2(x-4) = 0$

$\Rightarrow (x-2)(x-4) = 0$

x = 2 or x = 4

∴ x = 2 or 4, z = – 6 and ω = 4

17 A. Question

Give an example of

a row matrix which is also a column matrix

Answer

As we know that order of a row matrix = 1× n

and order of a column matrix = m×1

So, order of a row as well as column matrix = 1×1

Therefore, required matrix A = $[a_{ij}]_{1\times1}$

17 B. Question

Give an example of

a diagonal matrix which is not scalar

Answer

We know that a diagonal matrix has only a_{11}, a_{22} and a_{33} for a 3×3 matrix such that these elements are equal or different and all other entries 0 while scalar matrix has $a_{11} = a_{22} = a_{33}$ = k(say). So, a diagonal matrix which is not scalar must have $a_{11} \neq a_{22} \neq a_{33}$ for i≠j

Required matrix = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

17 C. Question

Give an example of

a triangular matrix.

Answer

A triangular matrix is a square matrix,

A = $[a_{ij}]$ such that a_{ij} = 0 for all i>j

Required matrix = $\begin{pmatrix} 1 & 4 & 6 \\ 0 & -2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$

18. Question

The sales figure of two car dealers during January 2013 showed that dealer A sold 5 deluxe, 3 premium and 4 standard cars, while dealer B sold 7 deluxe, 2 premium and 3 premium and 4 standard cars, while dealer B sold 7 deluxe, 2 premium and 3 standard cars. Total sales over the 2 month period of January – February revealed that dealer A sold 8 deluxe 7 premium and 6 standard cars. In the same 2 month period, dealer B sold 10 deluxe, 5 premium and 7 standard cars. Write 2 x 3 matrices summarizing sales data for January and 2 – month period for each dealer.

Answer

By creating tables, we have

For January 2013

	Deluxe	Premium	Standard
Dealer A	5	3	4
Dealer B	7	2	3

For January to February

	Deluxe	Premium	Standard
Dealer A	8	7	6
Dealer B	10	5	7

Hence, we can form 2×3 matrices as

A = $\begin{pmatrix} 5 & 3 & 4 \\ 7 & 2 & 3 \end{pmatrix}$ and B = $\begin{pmatrix} 8 & 7 & 6 \\ 10 & 5 & 7 \end{pmatrix}$

19. Question

For what value of x and y are the following matrices equal?

$$A = \begin{bmatrix} 2x+1 & 2y \\ 0 & y^2-5y \end{bmatrix}, B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

Answer

Given two matrices are equal .i.e, A = B.

$$\begin{pmatrix} 2x+1 & 2y \\ 0 & y^2-5y \end{pmatrix} = \begin{pmatrix} x+3 & y^2+2 \\ 0 & -6 \end{pmatrix}$$

We know that if two matrices are equal, then the elements of each matrices are also equal.

∴2x + 1 = x + 3

⇒ 2x – x = 3 – 1

⇒ x = 2 ......(1)

And $2y = y^2 + 2$

⇒ $y^2 - 2y + 2 = 0$

⇒ $y = \frac{-2 \pm \sqrt{(4-8)}}{2}$

⇒ $y = \frac{-2 \pm 2i}{2}$

⇒ $y = \frac{2(-1 \pm i)}{2}$

⇒ $y = -1 \pm i$ (No real solutions) ...... (2)

And $y^2 - 5y = -6$

⇒ $y^2 - 5y + 6 = 0$

⇒ $y^2 - 3y - 2y + 6 = 0$

⇒ y(y – 3) – 2(y – 3) = 0

⇒ (y – 3)(y – 2) = 0

⇒ y = 3 or 2 ...... (3)

∴ From the above equations we can say that A and B can’t be equal for any value of y.

20. Question

Find the values of x and y if

$$\begin{bmatrix} x+10 & y^2+2y \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 3x+4 & 3 \\ 0 & y^2-5y \end{bmatrix}$$

Answer

Given two matrices are equal .i.e, A = B.

$$\begin{pmatrix} x + 10 & y^2 + 2y \\ 0 & -4 \end{pmatrix} = \begin{pmatrix} 3x + 4 & 3 \\ 0 & y^2 - 5y \end{pmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

∴x + 10 = 3x + 4

⇒ x – 3x = 4 – 10

⇒ – 2x = – 6

⇒ x = 3 ...... (1)

And y^2 + 2y = 3

⇒ y^2 + 2y – 3 = 0

⇒ y^2 + 3y – y – 3 = 0

⇒ y(y + 3) – 1(y + 3) = 0

⇒ (y + 3)(y – 1) = 0

⇒ y = – 3 or 1 ...... (2)

And y^2 – 5y = – 4

⇒ y^2 – 5y + 4 = 0

⇒ y^2 – 4y – y + 4 = 0

⇒ y(y – 4) – 1(y – 4) = 0

⇒ (y – 4)(y – 1) = 0

⇒ y = 4 or 1 ......(3)

∴ The common value is x = 3 and y = 1

21. Question

Find the values of a and b if A = B, where

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}, B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-10 \end{bmatrix}$$

Answer

Given two matrices are equal .i.e, A = B.

$\begin{pmatrix} a+4 & 3b \\ 8 & -6 \end{pmatrix} = \begin{pmatrix} 2a+2 & b^2+2 \\ 8 & b^2-10 \end{pmatrix}$

We know that if two matrices are equal then the elements of each matrices are also equal.

∴a + 4 = 2a + 2

⇒ a – 2a = 2 – 4

⇒ – a = – 2

⇒ a = 2 …… (1)

And 3b = b^2 + 2

⇒ b^2 – 3b + 2 = 0

⇒ b^2 – 2b – b + 2 = 0

⇒ b(b – 2) – 1(b – 2) = 0

⇒ (b – 2)(b – 1) = 0

⇒ b = 2 or 1 …… (2)

And – 6 = b^2 – 10

⇒ b^2 = – 10 + 6

⇒ b^2 = – 4

⇒ b = ±2i(No real solution) …… (3)

∴ a = 2, b = 2 or 1

Exercise 5.2

1 A. Question

Compute the following sums:

$$\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$$

Answer

$=\begin{bmatrix} 3-2 & -2+4 \\ 1+1 & 4+3 \end{bmatrix}$

$=\begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$

Hence, $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$

1 B. Question

Compute the following sums:

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix}$$

Answer

$=\begin{bmatrix} 2+1 & 1-2 & 3+3 \\ 0+2 & 3+6 & 5+1 \\ -1+0 & 2-3 & 5+1 \end{bmatrix}$

$=\begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$

Hence, $\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix}+\begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix}=\begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$

2 A. Question

Let $A=\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B=\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $C=\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$. Find each of the following :

i. 2A – 3B

ii. B – 4C

iii. 3A – C

iv. 3A – 2B + 3C

Answer

(i) $2A=2\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}=\begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix}$

$= 3B=3\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}=\begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix}$

$= 2A\text{-}3B=\begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix}-\begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix}=\begin{bmatrix} 4-3 & 8-9 \\ 6+6 & 4-15 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$

Hence, $2A\text{-}3B=\begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$

(ii) $4C=4\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}=\begin{bmatrix} -8 & 20 \\ 12 & 16 \end{bmatrix}$

$B\text{-}4C=\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}-\begin{bmatrix} -8 & 20 \\ 12 & 16 \end{bmatrix}$

$=\begin{bmatrix} 1+8 & 3-20 \\ -2-12 & 5-16 \end{bmatrix}=\begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$

Hence, $B\text{-}4C=\begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$

(iii) $3A= 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix}$

$= 3A\text{-}C=\begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix}-\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix}=\begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$

Hence, $3A\text{-}C=\begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$

(iv) $3A=3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}=\begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix}$

$= 2A= 2\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}=\begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix}$

$= 3C=3\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}=\begin{bmatrix}-6 & 15\\ 9 & 12\end{bmatrix}$

$= 3A-2B+3C=\begin{bmatrix}6 & 12\\ 9 & 6\end{bmatrix}-\begin{bmatrix}2 & 6\\ -4 & 10\end{bmatrix}+\begin{bmatrix}-6 & 15\\ 9 & 12\end{bmatrix}$

$= \begin{bmatrix}6-2-6 & 12-6+15\\ 9+4+9 & 6-10+12\end{bmatrix}=\begin{bmatrix}-2 & 21\\ 22 & 8\end{bmatrix}$

Hence, 3A-2B+3C= $\begin{bmatrix}-2 & 21\\ 22 & 8\end{bmatrix}$

3. Question

If $A = \begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}, B = \begin{bmatrix}-1 & 0 & 2\\ 3 & 4 & 1\end{bmatrix}, C = \begin{bmatrix}-1 & 2 & 3\\ 2 & 1 & 0\end{bmatrix}$, find

i. A + B and B + C

ii. 2B + 3A and 3C – 4B.

Answer

i. A+B is not possible because matrix A is an order of 2x2 and Matrix B is an order of 2x3, So the Sum of the matrix is only possible when their order is same.

ii. B+C=$\begin{bmatrix}-1 & 0\ 2\\ 3 & 4\ 1\end{bmatrix}+\begin{bmatrix}-1 & 2\ 3\\ 2 & 1\ 0\end{bmatrix}$

$= \begin{bmatrix}-1-1 & 0+2\ 2+3\\ 3+2 & 4+1\ 1+0\end{bmatrix}=\begin{bmatrix}-2 & 2\ 5\\ 5 & 5\ 1\end{bmatrix}$

Hence, B+C = $\begin{bmatrix}-2 & 2\ 5\\ 5 & 5\ 1\end{bmatrix}$

iii. 2B+3A also does not exist because the order of matrix B and matrix A is different , So we can not find the sum of these matrix.

iv. 3C-4B = $3\begin{bmatrix}-1 & 2\ 3\\ 2 & 1\ 0\end{bmatrix}-4\begin{bmatrix}-1 & 0 & 2\\ 3 & 4 & 1\end{bmatrix}$

$=\begin{bmatrix}-3 & 6\ 9\\ 6 & 3\ 0\end{bmatrix}-\begin{bmatrix}-4 & 0\ 8\\ 12 & 16\ 4\end{bmatrix}=\begin{bmatrix}-3+4 & 6-0 & 9-8\\ 6-12 & 3-16 & 10-4\end{bmatrix}$

$=\begin{bmatrix}1 & 6\ 1\\ -6 & -13\ 6\end{bmatrix}$

Hence, 3C-4B =$\begin{bmatrix}1 & 6\ 1\\ -6 & -13\ 6\end{bmatrix}$

4. Question

Let $A = \begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & 4\end{bmatrix}, B = \begin{bmatrix}0 & -2 & 5\\ 1 & -3 & 1\end{bmatrix}$ and $C = \begin{bmatrix}1 & -5 & 2\\ 6 & 0 & -4\end{bmatrix}$. Compute 2A – 3B + 4C.

Answer

2A=$2\begin{bmatrix}-1 & 0\ 2\\ 3 & 1\ 4\end{bmatrix}=\begin{bmatrix}-2 & 0\ 4\\ 6 & 2\ 8\end{bmatrix}$

$= 3B=3\begin{bmatrix}0 & -2\ 5\\ 1 & -3\ 1\end{bmatrix}=\begin{bmatrix}0 & -6\ 15\\ 13 & -9\ \ 3\end{bmatrix}$

$= 4C=4\begin{bmatrix}1 & -5\ -2\\ 6 & 0\ \ -4\end{bmatrix}=\begin{bmatrix}4 & -20 & -8\\ 24 & 0 & -16\end{bmatrix}$

$= 2A-3B+4C= \begin{bmatrix}-2 & 0\ 4\\ 6 & 2\ 8\end{bmatrix}-\begin{bmatrix}0 & -6\ 15\\ 13 & -9\ \ 3\end{bmatrix}+\begin{bmatrix}4 & -20 & -8\\ 24 & 0 & -16\end{bmatrix}$

$= \begin{bmatrix} -2-0+4 & 0+6-20 & 4-15-8 \\ 6-13+24 & 2+9+0 & 8-3-16 \end{bmatrix}$

$= \begin{bmatrix} 2 & -14 & -19 \\ 17 & 11 & -11 \end{bmatrix}$

Hence, 2A-3B+4C=$\begin{bmatrix} 2 & -14 & -19 \\ 17 & 11 & -11 \end{bmatrix}$

5. Question

If A = diag (2, -5, 9), B = diag (1, 1, -4) and C = diag (-6, 3, 4), find

i. A – 2B

ii. B + C – 2A

iii. 2A + 3B – 5C

Answer

i. A-2B = diag$(2\ \ -5\ 9)$ -2diag$(1\ 1\ \ -4)$

= diag$(2\ \ -5\ 9)$-diag$(2\ 2\ \ -8)$

= diag$(2-2\ \ -5-2\ 9+8)$

= diag$(0\ \ -7\ 17)$

Hence, A-2B=diag$(0,-7,17)$

ii. B+C-2A

=diag $(1\ 1\ \ -4)$+diag$(-6\ 3\ 4)$-2diag$(2\ \ -5\ 9)$

=diag$(1\ 1\ \ -4)$+diag$(-6\ 3\ 4)$-diag$(4\ \ -10\ 18)$

=diag$(1-6-4\ 1+3+10\ \ -4+4-18)$

=diag$(-9\ 14\ \ -18)$

Hence, B+C-2A=diag$(-9\ 14\ \ -18)$

iii. 2A+3B-5C

=2diag$(2\ \ -5\ 9)$+3diag$(1\ 1\ \ -4)$-5diag$(-6\ 3\ 4)$

=diag$(4\ \ -10\ 18)$+diag$(3\ 3\ \ -12)$-diag$(-30\ \ -15\ \ -20)$

=diag$(4+3+30\ \ -10+3+15\ 18-12+20)$

=diag$(37\ 8\ 26)$

Hence, 2A+3B-5C=diag$(37\ 8\ 26)$

6. Question

Given the matrices

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

Verify that (A + B) + C = A + (B + C).

Answer

L.H.S (A+B)+C

$$= (A+B)=\begin{bmatrix}2 & 1 & 1\\3 & -1 & 0\\0 & 2 & 4\end{bmatrix}+\begin{bmatrix}9 & 7 & -1\\3 & 5 & 4\\2 & 1 & 6\end{bmatrix}$$

$$= \begin{bmatrix}2+9 & 1+7 & 1-1\\3+3 & -1+5 & 0+4\\0+2 & 2+1 & 4+6\end{bmatrix}$$

$$= \begin{bmatrix}11 & 8 & 0\\6 & 4 & 4\\2 & 3 & 10\end{bmatrix}$$

$$= (A+B)+C = \begin{bmatrix}11 & 8 & 0\\6 & 4 & 4\\2 & 3 & 10\end{bmatrix} + \begin{bmatrix}2 & -4 & 3\\1 & -1 & 0\\9 & 4 & 5\end{bmatrix}$$

$$= \begin{bmatrix}11+2 & 8-4 & 0+3\\6+1 & 4-1 & 4+0\\2+9 & 3+4 & 10+5\end{bmatrix}=\begin{bmatrix}13 & 4 & 3\\7 & 3 & 4\\11 & 7 & 15\end{bmatrix}$$

$$= (A+B)+C= \begin{bmatrix}13 & 4 & 3\\7 & 3 & 4\\11 & 7 & 15\end{bmatrix}$$

R.H.S A+(B+C)

$$= (B+C)= \begin{bmatrix}9 & 7 & -1\\3 & 5 & 4\\2 & 1 & 6\end{bmatrix}+\begin{bmatrix}2 & -4 & 3\\1 & -1 & 0\\9 & 4 & 5\end{bmatrix}$$

$$= \begin{bmatrix}9+2 & 7-4 & -1+3\\3+1 & 5-1 & 4+0\\2+9 & 1+4 & 6+5\end{bmatrix}$$

$$= \begin{bmatrix}11 & 3 & 2\\4 & 4 & 4\\11 & 5 & 11\end{bmatrix}$$

$$= A+(B+C)= \begin{bmatrix}2 & 1 & 1\\3 & -1 & 0\\0 & 2 & 4\end{bmatrix}+\begin{bmatrix}11 & 3 & 2\\4 & 4 & 4\\11 & 5 & 11\end{bmatrix}$$

$$= \begin{bmatrix}2+11 & 1+3 & 1+2\\3+4 & -1+4 & 0+4\\0+11 & 2+5 & 4+11\end{bmatrix}$$

$$= \begin{bmatrix}13 & 4 & 3\\7 & 3 & 4\\11 & 7 & 15\end{bmatrix}$$

Hence, L.H.S=R.H.S

7. Question

Find matrices X and Y, if X + Y = $\begin{bmatrix}5 & 2\\0 & 9\end{bmatrix}$ and $X - Y = \begin{bmatrix}3 & 6\\0 & -1\end{bmatrix}$.

Answer

$$(X+Y)+(X-Y)= \begin{bmatrix}5 & 2\\0 & 9\end{bmatrix}+\begin{bmatrix}3 & 6\\0 & -1\end{bmatrix}$$

$$= 2X= \begin{bmatrix}5+3 & 2+6\\0+0 & 9-1\end{bmatrix}$$

= 2X= $\begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$

= X= $\frac{1}{2}\begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$

= X=$\begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$

= (X+Y)-(X-Y)= $\begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$-$\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

= 2Y= $\begin{bmatrix} 5-3 & 2-6 \\ -+0 & 9+1 \end{bmatrix}$

= 2Y= $\begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$

= Y= $\frac{1}{2}\begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$

= Y= $\begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$

Hence, The value of X=$\begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$ and Y=$\begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$

8. Question

Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X - Y\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

Answer

2X+Y=$\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

= Put the Value of Y

= 2X+$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$=$\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

= 2X=$\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$-$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

= 2X=$\begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$

= 2X=$\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$

= X=$\frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$

= X=$\begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$

Hence, The value of X=$\begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$

9. Question

Find matrices X and Y, if $2X - Y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and $X + 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$.

Answer

2(2X-Y)+(X+2Y)= $2\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}+\begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$

= 4X-2Y+X+2Y=$\begin{bmatrix} 12+3 & -12+2 & 0+5 \\ -8-2 & 4+1 & 2-7 \end{bmatrix}$

= 5X=$\begin{bmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{bmatrix}$

= X=$\frac{1}{5}\begin{bmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{bmatrix}$

= X=$\begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$

Now , We have to find Y , we will multiply the second equation by 2 and then Subtract from equation 1.

= (2X-Y)-2(X+2Y)=$\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}-2\begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$

= 2X-Y-2X-4Y=$\begin{bmatrix} 6-6 & -6-4 & 0-10 \\ -4+4 & 2-2 & 1+14 \end{bmatrix}$

= -5Y=$\begin{bmatrix} 0 & -10 & -10 \\ 0 & 0 & 15 \end{bmatrix}$

= Y=$-\frac{1}{5}\begin{bmatrix} 0 & -10 & -10 \\ 0 & 0 & 15 \end{bmatrix}$

= Y=$\begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$

Hence, The Value of X= $\begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$ and Y=$\begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$

10. Question

If $X - Y = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $X + Y = \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$, find X and Y.

Answer

(X-Y)+(X+Y)=$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}+\begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$

= X-Y+X+Y=$\begin{bmatrix} 1+3 & 1+5 & 1+1 \\ 1-1 & 1+1 & 0+4 \\ 1+11 & 0+8 & 0+0 \end{bmatrix}$

= 2X=$\begin{bmatrix} 4 & 6 & 2 \\ 0 & 2 & 4 \\ 12 & 8 & 0 \end{bmatrix}$

= X=$\frac{1}{2}\begin{bmatrix} 4 & 6 & 2 \\ 0 & 2 & 4 \\ 12 & 8 & 0 \end{bmatrix}$

= X=$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$

= (X-Y)-(X+Y)=$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}-\begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$

$= X-Y-X-Y=\begin{bmatrix} 1-3 & 1-5 & 1-1 \\ 1+1 & 1-1 & 0-4 \\ 1-11 & 0-8 & 0-0 \end{bmatrix}$

$= -2Y=\begin{bmatrix} -2 & -4 & 0 \\ 2 & 0 & -4 \\ -10 & -8 & 0 \end{bmatrix}$

$= Y=-\frac{1}{2}\begin{bmatrix} -2 & -4 & 0 \\ 2 & 0 & -4 \\ -10 & -8 & 0 \end{bmatrix}$

$= Y=\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$

Hence, The Value of $X=\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$, and $Y=\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$.

11. Question

Find matrix A, if $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix} + A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix}$.

Answer

$\begin{bmatrix} 1 & 2\,4 \\ -1 & 0\,9 \end{bmatrix}+A=\begin{bmatrix} 9 & -1\,1 \\ 4 & -2\,3 \end{bmatrix}$

$= A=\begin{bmatrix} 9 & -1\,1 \\ 4 & -2\,3 \end{bmatrix}-\begin{bmatrix} 1 & 2\,4 \\ -1 & 0\,9 \end{bmatrix}$

$= A=\begin{bmatrix} 9-1 & -1-2\,1-4 \\ 4+1 & -2-0\,3-9 \end{bmatrix}$

$= A=\begin{bmatrix} 8 & -3\,-3 \\ 5 & -2\,-6 \end{bmatrix}$

Hence, $A=\begin{bmatrix} 8 & -3\,-3 \\ 5 & -2\,-6 \end{bmatrix}$

12. Question

If $A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$, find matrix C such that 5A + 3B + 2C is a null matrix.

Answer

5A + 3B + 2C = 0

$= 5\begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}+3\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}+2\begin{bmatrix} x & y \\ z & w \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix}+\begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix}+\begin{bmatrix} 2x & 2y \\ 2z & 2w \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 45+3+2x & 5+15+2y \\ 35+21+2z & 40+36+2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= 2x + 3 + 45 = 0$ ...... (i)

$= 2y + 5 + 15 = 0$ ...... (ii)

$= 2z + 35 + 21 = 0$ ...... (iii)

$= 2w + 40 + 36 = 0$ ...... (iv)

= from equations (i),(ii),(iii),(iv), we get

= $2x = -48\ 2y = -20$

= $x = -\frac{48}{2}\ y = -\frac{20}{2}$

= $x = -24\ y = -10$

= $2z = -56\ 2w = -76$

= $z = -\frac{56}{2}\ w = -\frac{76}{2}$

= $z = -28\ w = -38$

Hence, C = $\begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$

13. Question

If $A = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}, B = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$, find matrix X such that 2A + 3X = 5B.

Answer

we have, 2A+3X=5B

= 3X= 5B-2A

= 3X= 5 $\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$-2 $\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$

= 3X=$\begin{bmatrix} 40 & 0 \\ 20 & -10 \\ 15 & 30 \end{bmatrix}$-$\begin{bmatrix} 4 & -4 \\ 8 & 4 \\ -10 & 2 \end{bmatrix}$

= 3X=$\begin{bmatrix} 40-4 & 0+4 \\ 20-8 & -10-4 \\ 15+10 & 30-2 \end{bmatrix}$

= 3X=$\begin{bmatrix} 36 & 4 \\ 12 & -14 \\ 25 & 28 \end{bmatrix}$

= X=$\begin{bmatrix} \frac{36}{3} & \frac{4}{3} \\ \frac{12}{3} & -\frac{14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$

Hence, X= $\begin{bmatrix} 12 & \frac{4}{3} \\ 4 & -\frac{14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$

14. Question

If $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$, find the matrix C such that A + B + C is zero matrix.

Answer

A+B+C=0.

= C=-A-B

= C=$-\begin{bmatrix}1 & -3 & 2\\2 & 0 & 2\end{bmatrix}$-$\begin{bmatrix}2 & -1 & -1\\1 & 0 & -1\end{bmatrix}$

= C=$\begin{bmatrix}-1-2 & 3+1 & -2+1\\-2-1 & 0-0 & -2+1\end{bmatrix}$

= C=$\begin{bmatrix}-3 & 4 & -1\\-3 & 0 & -1\end{bmatrix}$

Hence, C= $\begin{bmatrix}-3 & 4 & -1\\-3 & 0 & -1\end{bmatrix}$

15 A. Question

Find x, y satisfying the matrix equations

$$\begin{bmatrix}x-y & 2 & -2\\4 & x & 6\end{bmatrix}+\begin{bmatrix}3 & -2 & 2\\1 & 0 & -1\end{bmatrix}=\begin{bmatrix}6 & 0 & 0\\5 & 2x+y & 5\end{bmatrix}$$

Answer

$\begin{bmatrix}X-Y & 2 & -2\\4 & X & 6\end{bmatrix}+\begin{bmatrix}3 & -2 & 2\\1 & 0 & -1\end{bmatrix}=\begin{bmatrix}6 & 0 & 0\\5 & 2X+Y & 5\end{bmatrix}$

=$\begin{bmatrix}X-Y+3 & 2-2 & -2+2\\4+1 & X+0 & 6-1\end{bmatrix}=\begin{bmatrix}6 & 0 & 0\\5 & 2X+Y & 5\end{bmatrix}$

We know that, corresponding entries of equal matrices are equal.

= $\begin{bmatrix}X-Y+3 & 0 & 0\\5 & X & 5\end{bmatrix}=\begin{bmatrix}6 & 0 & 0\\5 & 2X+Y & 5\end{bmatrix}$

= $X-Y+3=6$-----(i) $X=2X+Y$ -----(ii)

= $X-Y=6-3$ $X-2X=Y$

= $X-Y=3$-----(iii) $X+Y=0$-----(iv)

Now, Add the eq(iii) and eq(iv) and we get,

= X-Y+X+Y=3

= 2X=3

= X=$\frac{3}{2}$

Now, Put the Value of X in eq (iv) and we get,

= $\frac{3}{2}$+Y=0

= Y=$-\frac{3}{2}$

Hence, X= $\frac{3}{2}$ and Y= $-\frac{3}{2}$

15 B. Question

Find x, y satisfying the matrix equations.

[x y + 2 z - 3] + [y 4 5] = [4 9, 12]

Answer

$\begin{bmatrix}X+Y & Y+2+4 & Z-3+5\end{bmatrix}$=$\begin{bmatrix}4 & 9 & 12\end{bmatrix}$

We know that , corresponding entries of equal matrices are equal.

= X+Y=4 ......(i)

= Y+6=9 ......(ii)

= Z+2=12 ......(iii)

On solving equation(i),(ii) and equation(iii) we get,

= Y=9-6

= Y=3

= Z=12-2

= Z=10

Put the value of Y in equation(i)...we get,

= X+3=4

= X=4-3

= X-1

Hence, X=1,Y=3 and Z=10

15 C. Question

Find x, y satisfying the matrix equations

$$x\begin{bmatrix}2\\1\end{bmatrix}+y\begin{bmatrix}3\\5\end{bmatrix}+\begin{bmatrix}-8\\-11\end{bmatrix}=O$$

Answer

To Find: Values of x and y

$$x\begin{bmatrix}2\\1\end{bmatrix}+y\begin{bmatrix}3\\5\end{bmatrix}+\begin{bmatrix}-8\\-11\end{bmatrix}=0$$

$$\begin{bmatrix}2x\\x\end{bmatrix}+\begin{bmatrix}3y\\5y\end{bmatrix}+\begin{bmatrix}-8\\-11\end{bmatrix}=0$$

So,

2x + 3y -8 = 0

2x + 3y = 8.......(1)

x + 5y – 11 = 0

x + 5y = 11......(2)

Multiplying equation 2 by 2, we get

2x + 10 y = 22........(3)

Subtracting equation 2 from 1, we get,

3y – 10 y = 11 – 22

-7y = -11

$$y=\frac{11}{7}$$

Putting this value in equation 1 we get,

$$x+5\times\frac{11}{7}=11$$

$x = 11 - \frac{55}{7}$

$x = \frac{22}{7}$

Therefore, the values are, $x = \frac{22}{7}, y = \frac{11}{7}$.

16. Question

If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find x and y.

Answer

$2\begin{bmatrix} 3 & 4 \\ 5 & X \end{bmatrix} + \begin{bmatrix} 1 & Y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$= \begin{bmatrix} 6 & 8 \\ 10 & 2X \end{bmatrix} + \begin{bmatrix} 1 & Y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

We know that, corresponding entries of equal matrices are equal.

= Y+8=0 2X+1=5

= Y=-8 2X=5-1 = Y=-8 X=$\frac{4}{2}$

= Y=-8 X=2

Hence, X=2 Y=-8

17. Question

Find the value of λ, a non-zero scalar, if $\lambda\begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix} + 2\begin{bmatrix} 1 & 2 & 3 \\ -1 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 10 \\ 4 & 2 & 14 \end{bmatrix}$.

Answer

$= \begin{bmatrix} \lambda & 0 & 2\lambda \\ 3\lambda & 4\lambda & 5\lambda \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ -2 & -6 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 10 \\ 4 & 2 & 14 \end{bmatrix}$

We know that, corresponding entries of equal matrices are equal.

$= \begin{bmatrix} \lambda + 2 & 0 + 4 & 2\lambda + 6 \\ 3\lambda - 2 & 4\lambda - 6 & 5\lambda + 4 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 10 \\ 4 & 2 & 14 \end{bmatrix}$

= λ+2=4

= λ=4-2

= λ=2

Since, 3λ-2=4

= 3λ=4+2

= 3λ+6

= $\lambda = \frac{6}{3}$

= λ=2

Hence, λ=2

18 A. Question

Find a matrix X such that 2A + B + X = O, where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$

Answer

we have 2A+B+X=0.

= X=-2A-B

= X=$-2\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$-$\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$

= X=$\begin{bmatrix} 2 & -4 \\ -6 & -8 \end{bmatrix}$-$\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$

= X=$\begin{bmatrix} 2-3 & -4+2 \\ -6-1 & -8-5 \end{bmatrix}$

= X=$\begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$

Hence, X=$\begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$

18 B. Question

If $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, then find the matrix X of order 3 x 2 such that 2A + 3X = 5B.

Answer

2A+3X=5B.

= So, we can write as 3X=5B-2A

= 3X=5$\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}-2\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$

= 3X=$\begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix}$-$\begin{bmatrix} 16 & 0 \\ 8 & -4 \\ 6 & 12 \end{bmatrix}$

= 3X=$\begin{bmatrix} 10-16 & -10-0 \\ 20-8 & 10+4 \\ -25-6 & 5-12 \end{bmatrix}$

= 3X=$\begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$

= X=$\frac{1}{3}\begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$

= X=$\begin{bmatrix} -2 & -\frac{10}{3} \\ 4 & \frac{14}{3} \\ -\frac{31}{3} & -\frac{7}{3} \end{bmatrix}$

Hence, X=$\begin{bmatrix} -2 & -\frac{10}{3} \\ 4 & \frac{14}{3} \\ -\frac{31}{3} & -\frac{7}{3} \end{bmatrix}$

19 A. Question

Find x, y, z and t, if

$$3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$$

Answer

$3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$

= $\begin{bmatrix} 3x & 3y \\ 3z & 3t \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+t & 2t+3 \end{bmatrix}$

We know, Corresponding entries in equal matrices are equal.

= $3x=x+4$ -----(i)

= $3x-x=4$

= $2x=4$

= $x=2$

Since, $3y=6+x+y$------(ii)

= put the value of x in equation(ii)

= $3y-y=6+2$

= $2y = 8$

Therefore, $y = 4$

Since, $3t = 2t+3$-----(iii)

= $3t - 2t=3$

Therefore $t = 3$

Since, $3z = t + z-1$-----(iv)

= put the value of t in equation(iv)

= $3z - z=3-1$

= $2z=2$

Therefore, $z = 1$

Hence, $x = 2, y = 4, z = 1, t = 3,$

19 B. Question

Find x, y, z and t, if

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

Answer

$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$

$= \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$

$= \begin{bmatrix} 2x+3 & 10+4 \\ 14+1 & 2y-6+2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$

$= 2x+3=7$ $2y-6+2=14$

$= 2x=7-3$ $2y=14+4$

$= 2x=4$ $2y=18$

$= x=2$ $y=9$

Hence, $x=2$, $y=9$

20. Question

If X and Y are 2 × 2 matrices, then solve the following matrix equations for X and Y.

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}, 3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$$

Answer

$2X+3Y=\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$-----(i)

$= 3X+2Y= \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$-----(ii)

Multiply equation(i) by 3 and equation(ii) by 2, we get,

$= 6X+9Y=\begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix}$

$= 6X+4Y=\begin{bmatrix} -4 & 4 \\ 2 & -10 \end{bmatrix}$

Subtract these equation then we get,

$= 5Y=\begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 4 \\ 2 & -10 \end{bmatrix}$

$= 5y=\begin{bmatrix} 6+4 & 9-4 \\ 12-2 & 0+10 \end{bmatrix}$

$= 5Y=\begin{bmatrix} 10 & 5 \\ 10 & 10 \end{bmatrix}$

$= Y=\frac{1}{5}\begin{bmatrix} 10 & 5 \\ 10 & 10 \end{bmatrix}$

$= Y=\begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$

Now, put the value of Y in equation (i)

$= 2X+3\begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$

$= 2X=\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}-\begin{bmatrix} 6 & 3 \\ 6 & 6 \end{bmatrix}$

$= 2X=\begin{bmatrix} 2-6 & 3-3 \\ 4-6 & 0-6 \end{bmatrix}$

$= 2X=\begin{bmatrix} -4 & 0 \\ -2 & -6 \end{bmatrix}$

$= X=\frac{1}{2}\begin{bmatrix} -4 & 0 \\ -2 & -6 \end{bmatrix}$

$= X=\begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$

Hence, $Y=\begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$ and $X=\begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$

21. Question

In a certain city there are 30 colleges. Each college has 15 peons, 6 clerks, 1 typist and 1 section officer. Express the given information as a column matrix. Using scalar multiplication, find the total number of posts of each kind in all the colleges.

Answer

The Total number of post of each kind in 30 college. So,

$$= 30A=30\begin{bmatrix} 15 \\ 6 \\ 1 \\ 1 \end{bmatrix}$$

$$= 30A=\begin{bmatrix} 450 \\ 180 \\ 30 \\ 30 \end{bmatrix}$$

22. Question

The monthly incomes of Aryan and Babban are in the ration 3: 4 and their monthly expenditures are in the ratio 5: 7. If each saves 15000 per month, find their monthly incomes using the matrix method. This problem reflects which value?

Answer

Let us represent the situation through a matrix.

We will make two matrices: Income and Expenditure Matrices.

We know that Saving = Income – Expenditure.

Let the incomes of Aryan and Babban be 3x and 4x respectively and the expenditures be 5y and 7y respectively.

Income Matrix = $\begin{bmatrix} 3x \\ 4x \end{bmatrix}$

Expenditure Matrix = $\begin{bmatrix} 5y \\ 7y \end{bmatrix}$

Now, Saving = $\begin{bmatrix} 3x \\ 4x \end{bmatrix} - \begin{bmatrix} 5y \\ 7y \end{bmatrix}$

Given: Saving = 15000 each

Therefore, we have,

$$\begin{bmatrix} 15000 \\ 15000 \end{bmatrix} = \begin{bmatrix} 3x \\ 4x \end{bmatrix} - \begin{bmatrix} 5y \\ 7y \end{bmatrix}$$

So,

3 x – 5 y = 15000 ....(1)

4 x – 7 y = 15000 .....(2)

Solving equations 1 and 2, we get,

Multiplying eq(1) by 4 and eq(2) by 3 we get,

12 x – 20 y = 60000 ….(3)

12 x – 21 y = 45000 …..(4)

Eq(3) – Eq(4),

Y = 15000

Putting this value in eq(1) we get,

3 x – 4 × 15000 = 15000

3 x = 75000

X = 25000.

There monthly incomes are, 3 x = 3 × 15000 = 45000 and

4 x = 4 × 15000 = 60000.

Exercise 5.3

1 A. Question

Compute the indicated products:

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

Answer

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$= \begin{bmatrix} (a)(a) + (b)(b) & (a)(-b) + (b)(a) \\ (-b)(a) + (a)(b) & (-b)(-b) + (a)(a) \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 & -ab + ab \\ -ba + ab & a^2 + b^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}$$

1 B. Question

Compute the indicated products:

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix}$$

Answer

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1) + (-2)(-3) & (1)(2) + (-2)(2) & (-2)(2) + (-2)(-1) \\ (2)(1) + (3)(-3) & (2)(2) + (3)(2) & (2)(3) + (3)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+6 & 2-4 & 3+2 \\ 2-9 & 4+6 & 6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

1 C. Question

Compute the indicated products:

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

Answer

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(1)+(3)(0)+(4)(3) & (2)(-3)+(3)(2)+(4)(0) & (2)(5)+(3)(4)+(4)(5) \\ (3)(1)+(4)(0)+(5)(3) & (3)(-3)+(4)(2)+(5)(0) & (3)(5)+(4)(4)+(5)(5) \\ (4)(1)+(5)(0)+(6)(3) & (4)(-3)+(5)(2)+(6)(0) & (4)(5)+(5)(4)+(6)(5) \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

2 A. Question

Show that AB ≠ BA in each of the following cases:

$A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

Answer

given $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+20 \end{bmatrix}$$

$AB = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$ ......(1)

$$BA = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 10 + 6 & -2 + 7 \\ 15 + 24 & -3 + 28 \end{bmatrix}$$

$$BA = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \ldots\ldots(2)$$

From equation (1) and (2) we get

$AB \neq BA$

2 B. Question

Show that AB ≠ BA in each of the following cases:

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Answer

$$\text{given } A = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 + 0 + 6 & 1 - 2 + 9 & 0 + 2 + 12 \\ 0 + 0 + 0 & 0 - 1 + 0 & 0 + 1 + 0 \\ -1 + 0 + 0 & 1 - 1 + 0 & 0 + 1 + 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \ldots\ldots(1)$$

$$AB = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 + 0 + 0 & -2 + 1 + 0 & -3 + 0 + 0 \\ 0 + 0 + 1 & 0 - 1 + 1 & 0 + 0 + 0 \\ 2 + 0 + 4 & 4 + 3 + 4 & 6 + 0 + 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \ldots\ldots(2)$$

From (1) and (2) AB ≠ BA

2 C. Question

Show that AB ≠ BA in each of the following cases:

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

Answer

$$\text{Given } A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 3 + 0 & 1 + 0 + 0 & 0 + 0 + 0 \\ 0 + 1 + 0 & 1 + 0 + 0 & 0 + 0 + 0 \\ 0 + 1 + 0 & 4 + 0 + 0 & 0 + 0 + 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix} \text{......(1)}$$

$$BA = \begin{bmatrix} 0 & 10 & 1 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 1 + 0 & 0 + 1 + 0 & 0 + 0 + 0 \\ 1 + 0 + 0 & 3 + 0 + 0 & 0 + 0 + 0 \\ 0 + 5 + 4 & 0 + 5 + 1 & 0 + 0 + 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 0 \\ 9 & 6 & 0 \end{bmatrix} \text{......(2)}$$

From equation (1) and (2) we get AB $\neq$ BA

3 A. Question

Compute the products AB and BA whichever exists in each of the following cases:

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

Answer

$$\text{So} A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

since the order of A is 2$\times$2 and order of B is 2$\times$3,

so AB is possible but BA is not the possible order of AB is 2$\times$3.

$$AB = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1) + (-2)(2) & (1)(2) + (-2)(3) & (1)(3) + (-2)(1) \\ (2)(1) + (3)(2) & (2)(2) + (3)(3) & (2)(3) + (3)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 4 & 2 - 6 & 3 - 2 \\ 2 + 6 & 4 + 6 & 6 + 3 \end{bmatrix}$$

Hence

$$AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

And BA does not exits

3 B. Question

Compute the products AB and BA whichever exists in each of the following cases:

$$A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$$

Answer

here, $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

since the order of A is 3×2 and order of B is 2×3,

AB and BA both exit and order of AB = 3×3 and order of BA = 2×2

$$AB = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (3)(4) + (2)(0) & (3)(5) + (2)(1) & (3)(6) + (2)(2) \\ (-1)(4) + (0)(0) & (-1)(5) + (0)(0) & (-1)(6) + (0)(2) \\ (-1)(4) + (1)(0) & (-1)(5) + (1)(1) & (-1)(6) + (1)(2) \end{bmatrix}$$

$$= \begin{bmatrix} 12 + 0 & 15 + 2 & 18 + 4 \\ -4 + 0 & -5 + 0 & -6 + 0 \\ -4 + 0 & -5 + 0 & -6 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (4)(3) + (5)(-1) + (6)(-1) & (4)(2) + (5)(0) + (6)(1) \\ (0)(3) + (1)(-1) + (2)(-1) & (0)(2) + (1)(0) + (2)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 12 - 5 - 6 & 8 + 0 + 6 \\ 0 - 1 - 2 & 0 + 0 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

Hence

$$AB = \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}, BA = \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

3 C. Question

Compute the products AB and BA whichever exists in each of the following cases:

$A = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$

Answer

here $A = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$

since the order of A is 1×4 and order of B is 4× 1,

AB and BA both exit and order of AB = 1× 1 and order of BA = 4× 4

$$AB = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

$= [(1)(0) + (-1)(1) + (2)(3) + (3)(2)]$

$= [0 - 1 + 6 + 6]$

$AB = [11]$

$$BA = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & 3 \\ 3 & -3 & 6 & 9 \\ 2 & -2 & 4 & 6 \end{bmatrix}$$

Hence

$AB = [11]$

$$BA = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & 3 \\ 3 & -3 & 6 & 9 \\ 2 & -2 & 4 & 6 \end{bmatrix}$$

3 D. Question

Compute the products AB and BA whichever exists in each of the following cases:

$$[a, b]\begin{bmatrix} c \\ d \end{bmatrix} + [a\ b\ c\ d]\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Answer

$$[a \quad b]\begin{bmatrix} c \\ d \end{bmatrix} + [a \quad b\ c \quad d]]\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$= [ac + bd] + [a^2 + b^2 + c^2 + d^2]$

$= [ac + bd = a^2 + b^2 + c^2 + d^2]$

Hence,

$$[a \quad b]\begin{bmatrix} c \\ d \end{bmatrix} + [a \quad b\ c \quad d]]\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = [ac + bd = a^2 + b^2 + c^2 + d^2]$$

4 A. Question

Show that AB ≠ BA in each of the following cases:

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

Answer

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 2 & -4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4+1+6 & 6-2-9 & -2+1+4 \\ -6+0+6 & 9+0-9 & -3+0+4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -5 & 3 \\ 0 & 0 & 1 \end{bmatrix} \text{......(1)}$$

$$BA = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 2 & -4 \end{bmatrix}\begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+6-3 & -6-3+0 & 2-3+1 \\ -1+4-3 & -3-2+0 & 1-2+1 \\ -6+18-12 & -18-9+0 & 6-9+4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & -9 & 0 \\ 0 & -5 & 0 \\ 0 & -27 & 1 \end{bmatrix} \text{......(2)}$$

From equation (1) and (2) AB≠BA

4 B. Question

Show that AB ≠ BA in each of the following cases:

$$A = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

Answer

$$A = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10-12-1 & 20-16-3 & 10-8-2 \\ -11+15+0 & -22+20+0 & -11+10+0 \\ 9-15+1 & 18-20+3 & 9-10+2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \text{......(1)}$$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10-22+9 & -4+10-5 & -9+0+1 \\ 30-44+10 & -12+20-10 & -3+0+2 \\ 10-33+18 & -4+15-10 & -1+0+2 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \text{......(2)}$$

From equation (1) and (2) AB≠BA

5 A. Question

Evaluate the following:

$$\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

Answer

$$\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1+3 & 3-2 \\ -1-1 & -4+1 \end{bmatrix}\right)\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 4 & 1 \\ -2 & -3 \end{bmatrix}\right)\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2 & 12+4 & 20+6 \\ -2-6 & -6-12 & -10-18 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 16 & 26 \\ -8 & -18 & -28 \end{bmatrix}$$

Hence,

$$\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 6 & 16 & 26 \\ -8 & -18 & -28 \end{bmatrix}$$

5 B. Question

Evaluate the following:

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

Answer

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+0 & 0+0+3 & 2+0+6 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 & 10 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 10+12+60 \end{bmatrix}$$

$$= \begin{bmatrix} 82 \end{bmatrix}$$

Hence,

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\2 & 0 & 1\\0 & 1 & 2\end{bmatrix}\begin{bmatrix}2\\4\\6\end{bmatrix} = [82]$$

5 C. Question

Evaluate the following:

$$\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\left(\begin{bmatrix}1 & 0 & 2\\2 & 0 & 1\end{bmatrix} - \begin{bmatrix}0 & 1 & 2\\1 & 0 & 2\end{bmatrix}\right)$$

Answer

$$\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\left(\begin{bmatrix}1 & 0 & 2\\2 & 0 & 1\end{bmatrix} - \begin{bmatrix}0 & 1 & 2\\1 & 0 & 2\end{bmatrix}\right)$$

$$= \begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\left(\begin{bmatrix}1-0 & 0-1 & 2-2\\2-1 & 0-0 & 1-2\end{bmatrix}\right)$$

$$= \begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\1 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1-1 & -1+0 & 0+1\\0+2 & 0+0 & 0-2\\2+3 & -2+0 & 0-3\end{bmatrix}$$

$$= \begin{bmatrix}0 & -1 & 1\\2 & 0 & -2\\5 & -2 & -3\end{bmatrix}$$

Hence,

$$\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\left(\begin{bmatrix}1 & 0 & 2\\2 & 0 & 1\end{bmatrix} - \begin{bmatrix}0 & 1 & 2\\1 & 0 & 2\end{bmatrix}\right) = \begin{bmatrix}0 & -1 & 1\\2 & 0 & -2\\5 & -2 & -3\end{bmatrix}$$

6. Question

If $A = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}, B = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$ and $C = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$, then show that $A^2 = B^2 = C^2 = I_2$.

Answer

given $A = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, B = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}, c = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

$$A^2 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1+0 & 0+0\\0+0 & 0+1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$A^2 = I_2$ ......(1)

$$B^2 = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$$

$$= \begin{bmatrix}1+0 & 0+0\\0+0 & 0+1\end{bmatrix}$$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$B^2 = I_2$ ......(2)

$C^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 + 0 & 0 + 0 \\ 0 + 0 & 0 + 1 \end{bmatrix}$

$C^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$C^2 = I_2$ ......(3)

Hence,

From equation (1),(2) and (3),

$A^2 = B^2 = C^2 = I_2$

7. Question

If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $3A^2 - 2B + I$.

Answer

given $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$

$3A^2 - 2B + I$

$= 3\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} - 2\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= 3\begin{bmatrix} 4 - 3 & -2 - 2 \\ 6 + 6 & -3 + 4 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= 3\begin{bmatrix} 1 & -4 \\ 12 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 - 0 + 1 & -12 + 8 + 0 \\ 36 + 2 + 0 & 3 - 14 + 1 \end{bmatrix}$

$= \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$

Hence,

$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$

8. Question

If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, prove that (A – 2I) (A – 3I) = O.

Answer

given $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

(A-2I)(A-3I)

$= \left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$

$= \left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right)\left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right)$

$= \left(\begin{bmatrix} 4-2 & 2-0 \\ -1-0 & 1-2 \end{bmatrix}\right)\left(\begin{bmatrix} 4-3 & 2-0 \\ -1-0 & 1-3 \end{bmatrix}\right)$

$= \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 2-2 & 4-4 \\ -1+1 & -2+2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= 0$

Hence,

$(A-2I)(A-3I) = 0$

9. Question

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, Show that $A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$.

Answer

given $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = A^2.A$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 1+2 \\ 0+0 & 0+1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

Hence,

$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

10. Question

If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, show that $A^2 = O$

Answer

Given, A =

$A^2 = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$

$= \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= 0$

Hence,

$A^2 = 0$

11. Question

If $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$, find A^2

Answer

Given,

$A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$A^2 = A.A$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2 2\theta - \sin^2 2\theta & \cos 2\theta \sin 2\theta + \cos 2\theta \sin 2\theta \\ -\cos 2\theta \sin 2\theta - \cos 2\theta \sin 2\theta & -\sin^2 2\theta + \cos^2 2\theta \end{bmatrix}$

since $\cos^2\theta - \sin^2\theta = \cos 2\theta$

$= \begin{bmatrix} \cos 4\theta & 2\cos 2\theta \sin 2\theta \\ -2\cos 2\theta \sin 2\theta & \cos 4\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$ $|\sin 2\theta = 2\sin\theta \cos\theta$

$A^2 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$

12. Question

If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$, show that $AB = BA = O_{3 \times 3}$.

Answer

Given, $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

$$= \begin{bmatrix} -2-3+5 & 6+9-15 & 5+15-20 \\ 1+4-5 & -3-12+15 & -5-15+20 \\ -1-3+4 & 3+9-12 & 5+15-20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

AB = $0_{3\times3}$ ......(1)

$$BA = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+5 & 3+12-15 & 5+15-20 \\ 2+3-5 & -3-12+15 & -5-15+20 \\ -2-3+5 & 3+9-12 & 5+15-20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

BA = $0_{3\times3}$ ......(2)

From equation 1 and 2,

AB = BA = $0_{3\times3}$

13. Question

If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, show that AB = BA = $O_{3\times3}$.

Answer

Given, $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0+abc-abc & 0+b^2c-b^2c & 0+bc^2-bc^2 \\ -a^2c+0+a^2c & -abc+0+abc & -ac^2+0+ac^2 \\ a^2b-a^2b+0 & ab^2-ab^2+0 & abc-abc+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

AB = $0_{3\times3}$ ......(1)

$$BA = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+abc-abc & 0+b^2c-b^2c & 0+bc^2-bc^2 \\ -a^2c+0+a^2c & -abc+0+abc & -ac^2+0+ac^2 \\ a^2b-a^2b+0 & ab^2-ab^2+0 & abc-abc+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

BA = $0_{3\times3}$ ......(2)

From equation 1 and 2,

AB = BA = $0_{3\times3}$

14. Question

If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, show that AB = A and BA = B.

Answer

Given, $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} =$

$= \begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & 2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+18 & -4-12+12 \end{bmatrix}$

$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

AB = A

$BA = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

$= \begin{bmatrix} 4+2-4 & -6-8+12 & -10-10+16 \\ -2-3+4 & 3+12-12 & 5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

BA = B

15. Question

Let $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$, compute $A^2 - B^2$.

Answer

Given, $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}\begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$

$= \begin{bmatrix} 1+3+5 & -1-3-5 & 1+3-5 \\ -3-9+15 & 3+9+15 & -3-9+15 \\ -5+15+25 & 5-15+25 & -5+15+25 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -1 & 9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} \text{......(1)}$$

$$B^2 = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0+4+3 & 0-12+12 & 0-12+12 \\ 0-3+3 & 4+9-12 & 3+9-12 \\ 0+4-4 & -4-12+16 & -3-12+16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{......(2)}$$

Subtracting equation 2 from 1,

$$A^2 - B^2 = \begin{bmatrix} -1 & 9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

Hence,

$$A^2 - B^2 = \begin{bmatrix} -2 & 9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

16 A. Question

For the following matrices verify the associativity of matrix multiplication i.e. (AB) C = A(BC).

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Answer

Given $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}$ and $c = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$(AB)C = \left(\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \right) \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+0 & 0+4+0 \\ -1+0+0 & 0+0+3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4 \\ -1-3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \text{......(1)}$$

$$A(BC) = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 + 0 \\ -1 - 2 \\ 0 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 6 + 0 \\ -1 + 0 - 3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \text{......(2)}$$

From equation (1) and (2) we get,

$$(AB)C = A(BC)$$

16 B. Question

For the following matrices verify the associativity of matrix multiplication i.e. (AB) C = A(BC).

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Answer

given $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$(AB)C = \left(\begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 0 + 6 & -4 + 2 - 3 & 4 + 4 + 3 \\ 1 + 0 + 4 & -1 + 1 - 2 & 1 + 2 + 2 \\ 3 + 0 + 2 & -3 + 0 - 1 & 3 + 0 + 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -5 & 11 \\ 5 & -2 & 5 \\ 5 & -4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 - 15 + 0 & 20 + 0 + 0 & -10 + 5 + 11 \\ 5 - 6 + 0 & 10 + 0 + 0 & -5 - 2 + 5 \\ 5 - 12 + 0 & 10 + 0 + 0 & -5 - 4 + 4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \text{......(1)}$$

$$A(BC) = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 - 3 + 0 & 2 + 0 + 0 & -1 - 1 + 1 \\ 0 + 3 + 0 & 0 + 0 + 0 & 0 + 1 + 2 \\ 2 - 3 + 0 & 4 + 0 + 0 & -2 - 1 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 1 \\ 3 & 0 & 3 \\ -1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -8 + 6 - 3 & 8 + 0 + 12 & -4 + 6 - 6 \\ -2 + 3 - 2 & 2 + 0 + 8 & -1 + 3 - 4 \\ -6 + 0 - 1 & 6 + 0 + 4 & -3 + 0 - 2 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} ......(2)$$

From equation (1) and (2)

$$A(BC) = A(BC)$$

17 A. Question

For the following matrices verify the distributivity of matrix multiplication over matrix addition i.e. A(B + C) = AB + AC.

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

Answer

given $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$$A(B + C) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 + 0 & 0 + 1 \\ 2 + 1 & 1 - 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 - 3 & 1 + 0 \\ 0 + 6 & 0 + 0 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} ......(1)$$

$$AB = AC = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 - 2 & 0 - 1 \\ 0 + 4 & 0 + 2 \end{bmatrix} + \begin{bmatrix} 0 + -1 & 1 + 1 \\ 0 + 2 & 0 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 - 1 & -1 + 2 \\ 4 + 2 & 2 - 2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} ......(2)$$

Using equation (1) and (2),

$$A(B + C) = AB + AC$$

17 B. Question

For the following matrices verify the distributivity of matrix multiplication over matrix addition i.e. A(B + C) = AB + AC.

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}.$$

Answer

given $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, c = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$A(B + C) = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}(\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix})$

$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0 + 1 & 1 - 1 \\ 1 + 0 & 1 + 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 - 1 & 0 + 2 \\ 1 + 1 & 0 + 2 \\ -1 + 2 & 0 + 4 \end{bmatrix}$

$A(B + C) = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix}$ ......(1)

$AB + AC = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 + 1 & 2 - 1 \\ 0 + 1 & 1 + 1 \\ 0 + 2 & -1 + 2 \end{bmatrix} + \begin{bmatrix} 2 + 0 & -2 - 1 \\ 1 + 0 & -1 + 1 \\ -1 + 0 & 1 + 2 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1 + 2 & 1 - 3 \\ 1 + 1 & 2 + 0 \\ 2 - 1 & 1 + 3 \end{bmatrix}$

$AB + AC = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix}$ ......(2)

From equation (1) and (2),

$A(B + C) = AB + AC$

18. Question

If $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$, verify that A(B – C) = AB – AC

Answer

Given,

$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$$A(B - C) = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 6 \\ -1 & 0 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A(B - C) = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix}$$

$$AB - AC = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & -8 \\ 2 & 14 & -15 \\ -3 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 14 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix}$$

$$AB - AC = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix}$$

From equation (1) and (2),We get

$A(B - C) = AB - AC$

19. Question

Compute the elements a_{43} and a_{22} of the matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

Answer

Given,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 2 \\ 12 & 4 \\ -1 & 12 \\ 24 & 8 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & -9 & 11 & -14 & 6 \\ 12 & 0 & 4 & 8 & -24 \\ 36 & -37 & 49 & -50 & 2 \\ 24 & 0 & 8 & 16 & -48 \end{bmatrix}$$

Hence,

$a_{43} = 8 , a_{22} = 0$

20. Question

If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$ and I is the identity matrix of order 3, show that $A^3 = pI + qA + rA^2$.

Answer

Given,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$A^2 = A.A$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix}$$

$$A^3 = A^2.A$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+0+pq+pr^2 & pr+0+q^2+qr^2 & 0+p+qr+qr+r^2 \end{bmatrix}$$

$$= \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix}$$

$pI + qA + rA^2$

$$= p\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + q\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} + r\begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix}$$

$$= \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix}$$

Hence,

$A^3 = pI + qA + rA^2$

Hence proved.

21. Question

If ω is a complex cube root of unity, show that

$$\left(\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix} + \begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Answer

Given, ω is a complex cube root of unity

$$\left(\begin{bmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{bmatrix}\begin{bmatrix}\omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\\ \omega & \omega^2 & 1\end{bmatrix}\right)\begin{bmatrix}1\\ \omega\\ \omega^2\end{bmatrix}$$

$$=\left(\begin{bmatrix}1+\omega & \omega+\omega^2 & \omega^2+1\\ \omega+\omega^2 & \omega^2+1 & \omega+1\\ \omega^2+\omega & 1+\omega^2 & \omega+1\end{bmatrix}\right)\begin{bmatrix}1\\ \omega\\ \omega^2\end{bmatrix} \{\text{Since } 1+\omega+\omega^2=0 \text{ and } \omega^3=1\}$$

$$=\begin{bmatrix}-\omega^2 & -1 & -\omega\\ -1 & -\omega & -\omega^2\\ -1 & -\omega & -\omega^2\end{bmatrix}\begin{bmatrix}1\\ \omega\\ \omega^2\end{bmatrix}$$

$$=\begin{bmatrix}-\omega^2-\omega-\omega^3\\ -1-\omega^2-\omega^4\\ -1-\omega^2-\omega^4\end{bmatrix}$$

$$=\begin{bmatrix}-\omega(1+\omega+\omega^2)\\ -1-\omega^2-\omega.\omega^3\\ -1-\omega^2-\omega.\omega^3\end{bmatrix}$$

$$=\begin{bmatrix}-\omega(0)\\ -1-\omega^2-\omega\\ -1-\omega^2-\omega\end{bmatrix} \{\text{from reason (1)}\}$$

$$=\begin{bmatrix}0\\ -(1+\omega^2+\omega)\\ -(1+\omega^2+\omega)\end{bmatrix}$$

$$=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

22. Question

If $A=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$, show that $A^2 = A$.

Answer

Given:

$$A=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$$

$$A^2 = A.A$$

$$=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$$

$$=\begin{bmatrix}4+3-5 & -6-12+15 & -10-15+20\\ -2-4+5 & 3+16-15 & 5+20-20\\ 2+3-4 & -3-12+12 & -5-15+16\end{bmatrix}$$

$$=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}=A$$

Hence,

$$A^2 = A$$

23. Question

If $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$, show that $A^2 = I_3$

Answer

Given,

$A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$

$A^2 = A.A$

$= \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 16-3-12 & -4+0+4 & 16+4+12 \\ 12+0-12 & -3+0+4 & -12+0+12 \\ 12-3-9 & -3+0+3 & -12+4+9 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$

Hence,

$A^2 = I_3$

24 A. Question

If $\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$, find x.

Answer

Given,

$[1 \quad 1 \quad x] \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$

$\Rightarrow [1 + 2x + 0 \quad x + 0 + 2 \quad 2 + 1 + 0] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$

$\Rightarrow [2x + 4 \quad x + 2 \quad 2x + 4] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$

⇒ [2x + 1 + 2 + x + 3] = 0

⇒ 3x + 6 = 0

⇒ x = -2

24 B. Question

If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, find x.

Answer

Given,

$$\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}\begin{bmatrix}1 & -3\\-2 & 4\end{bmatrix}=\begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$$

By multiplication of matrices, we have

$$\begin{bmatrix}2*1\ +\ 3*(-2) & 2*(-3)\ +\ 3*4\\5\ +\ 7*(-2) & 5*(-3)\ +\ 7*4\end{bmatrix}=\begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$$

$$\begin{bmatrix}-4 & 6\\-9 & 13\end{bmatrix}=\begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$$

⇒ x = 13

⇒

25. Question

If $\begin{bmatrix}x & 4 & 1\end{bmatrix}\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix}=0,$ find x

Answer

Given,

$$\begin{bmatrix}x & 4 & 1\end{bmatrix}\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}2x\ +\ 4\ +\ 0 & x\ +\ 0\ +\ 2 & 2x\ +\ 8-4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}2x\ +\ 4 & x\ +\ 2 & 2x\ +\ 4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix}=0$$

⇒ [(2x + 4)x + 4(x + 2)-1(2x + 4)] = 0

⇒ $2x^2$ + 4x + 4x + 8-2x-4 = 0

⇒ $2x^2$ + 6x + 4 = 0

⇒ $2x^2$ + 2x + 4x + 4 = 0

⇒ 2x(x + 1) + 4(x + 1) = 0

⇒ (x + 1)(2x + 4) = 0

⇒ x = -1 or x = -2

Hence, x = -1 or x = -2

26. Question

If $\begin{bmatrix}1 & -1 & x\end{bmatrix}\begin{bmatrix}0 & 1 & -1\\2 & 1 & 3\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix}=0,$ find x.

Answer

Given: $\begin{bmatrix} 1 & -1 & x \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$

We will multiply the 1×3 matrix with a 3×3 matrix, we get

$[1 \times 0 + (-1 \times 2) + x \times 1 \quad 1 \times 1 + (-1 \times 1) + x \times 1 \quad 1 \times (-1) + (-1 \times$, [as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots +$

0
$a_{in}b_{nj}$]

$\Rightarrow \begin{bmatrix} 0 - 2 + x & x & (-1) - 3 + x \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x - 2 & x & x - 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$

Now we will multiply these two matrices, we get

$[(x - 2) \times 0 + x \times 1 + (x - 4) \times 1] = 0$

$\Rightarrow x + x - 4 = 0$

$\Rightarrow 2x = 4 \Rightarrow x = 2$

Therefore, the value of x satisfying the given matrix condition is 2

27. Question

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then prove that $A^2 - A + 2I = 0$.

Answer

Given: $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

To prove $A^2 - A + 2I = 0$

Now, we will find the matrix for A^2, we get

$A^2 = A \times A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (-2 \times 4) & 3 \times (-2) + (-2 \times -2) \\ 4 \times 3 + (-2 \times 4) & 4 \times (-2) + (-2 \times -2) \end{bmatrix}$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$\Rightarrow A^2 = \begin{bmatrix} 9 - 8 & -6 + 4 \\ 12 - 8 & -8 + 4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \ldots\ldots\ldots\ldots (i)$

Now, we will find the matrix for 2I, we get

$2I = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow 2I = \begin{bmatrix} 2 \times 1 & 2 \times 0 \\ 2 \times 0 & 2 \times 1 \end{bmatrix}$

$\Rightarrow 2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \ldots\ldots\ldots\ldots(ii)$

So,

$A^2 - A + 2I$

Substitute corresponding values from eqn(i) and eqn(ii), we get

$\Rightarrow = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 1 - 3 + 2 & -2 - (-2) + 0 \\ 4 - 4 + 0 & -4 - (-2) + 2 \end{bmatrix}$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$]

$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Therefore, $A^2 - A + 2I = 0$

Hence proved

28. Question

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find λ so that $A^2 = 5A + \lambda I$.

Answer

Given: $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2 = 5A + \lambda I$

Now, we will find the matrix for A^2, we get

$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (1 \times -1) & 3 \times 1 + 1 \times 2 \\ (-1 \times 3) + 2 \times (-1) & (-1 \times 1) + 2 \times 2 \end{bmatrix}$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$\Rightarrow A^2 = \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \ldots\ldots\ldots(i)$

Now, we will find the matrix for 5A, we get

$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times 1 \\ 5 \times (-1) & 5 \times 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} \ldots\ldots\ldots\ldots(ii)$

So,

$A^2 = 5A + \lambda I$

Substitute corresponding values from eqn(i) and eqn(ii), we get

$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 + \lambda & 5 + 0 \\ -5 + 0 & 10 + \lambda \end{bmatrix}$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$]

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal,

Hence, $8 = 15 + \lambda \Rightarrow \lambda = -7$ and $3 = 10 + \lambda \Rightarrow \lambda = -7$

So the value of λ so that $A^2 = 5A + \lambda I$ is – 7

29. Question

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I_2 = 0$.

Answer

Given: $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

I_2 is an identity matrix of size 2, so $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

To show that $A^2 - 5A + 7I_2 = 0$

Now, we will find the matrix for A^2, we get

$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (1 \times -1) & 3 \times 1 + 1 \times 2 \\ (-1 \times 3) + 2 \times (-1) & (-1 \times 1) + 2 \times 2 \end{bmatrix}$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$\Rightarrow A^2 = \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \ldots\ldots\ldots.(i)$

Now, we will find the matrix for 5A, we get

$5A = 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times 1 \\ 5 \times (-1) & 5 \times 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} \ldots\ldots\ldots\ldots.(ii)$

Now,

$7I_2 = 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \ldots\ldots.(iii)$

So,

$A^2 - 5A + 7I_2$

Substitute corresponding values from eqn(i), (ii) and (iii), we get

$\Rightarrow = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5-(-5)+0 & 3-10+7 \end{bmatrix}$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$]

$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Therefore, $A^2 - 5A + 7I_2 = 0$

Hence proved

30. Question

If $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$, show that $A^2 - 2A + 3I_2 = 0$.

Answer

Given: $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

I_2 is an identity matrix of size 2, so $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

To show that $A^2 - 2A + 3I_2 = 0$

Now, we will find the matrix for A^2, we get

$A^2 = A \times A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 2\times 2 + (3\times -1) & 2\times 3 + 3\times 0 \\ (-1\times 2) + 0\times(-1) & (-1\times 3) + 0\times 0 \end{bmatrix}$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$\Rightarrow A^2 = \begin{bmatrix} 4-3 & 6+0 \\ -2+0 & -3+0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} \ldots\ldots\ldots.(\mathrm{i})$

Now, we will find the matrix for 2A, we get

$2A = 2\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$\Rightarrow 2A = \begin{bmatrix} 2\times 2 & 2\times 3 \\ 2\times(-1) & 2\times 0 \end{bmatrix}$

$\Rightarrow 2A = \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} \ldots\ldots\ldots\ldots.(\mathrm{ii})$

Now,

$3I_2 = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \ldots\ldots..(\mathrm{iii})$

So,

$A^2 - 2A + 3I_2$

Substitute corresponding values from eqn(i), (ii) and (iii), we get

$$\Rightarrow = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 1-4+3 & 6-6+0 \\ -2-(-2)+0 & -3-0+3 \end{bmatrix}$$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$]

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Therefore, $A^2 - 2A + 3I_2 = 0$

Hence proved

31. Question

Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, satisfies the equation $A^3 - 4A^2 + A = 0$.

Answer

Given: $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

To show that $A^3 - 4A^2 + A = 0$

Now, we will find the matrix for A^2, we get

$$A^2 = (A \times A) = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 2\times2 + (3\times1) & 2\times3 + 3\times2 \\ 1\times2 + 2\times1 & 1\times3 + 2\times2 \end{bmatrix}$$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$$\Rightarrow A^2 = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \ldots\ldots\ldots\ldots.(i)$$

Now, we will find the matrix for A^3, we get

$$A^3 = A^2 \times A = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 7\times2 + 12\times1 & 7\times3 + 12\times2 \\ 4\times2 + 7\times1 & 4\times3 + 7\times2 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} \ldots\ldots\ldots\ldots.(ii)$$

So,

$A^3 - 4A^2 + A$

Substitute corresponding values from eqn(i) and (ii), we get

$$\Rightarrow = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$\Rightarrow = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 4\times 7 & 4\times 12 \\ 4\times 4 & 4\times 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 26 - 28 + 2 & 45 - 48 + 3 \\ 15 - 16 + 1 & 26 - 28 + 2 \end{bmatrix}$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$]

$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Therefore,$A^3 - 4A^2 + A = 0$

Hence matrix A satisfies the given equation.

22. Question

Show that the matrix $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ is a root of the equation $A^2 - 12A - I = 0$.

Answer

Given: $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

I is an identity matrix so $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

To show that $A^2 - 12A - I = 0$

Now, we will find the matrix for A^2, we get

$A^2 = A \times A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 5\times 5 + 3\times 12 & 5\times 3 + 3\times 7 \\ 12\times 5 + 7\times 12 & 12\times 3 + 7\times 7 \end{bmatrix}$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$\Rightarrow A^2 = \begin{bmatrix} 25 + 36 & 15 + 21 \\ 60 + 84 & 36 + 49 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} \ldots\ldots\ldots.(i)$

Now, we will find the matrix for 12A, we get

$12A = 12\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

$\Rightarrow 12A = \begin{bmatrix} 12\times 5 & 12\times 3 \\ 12\times 12 & 12\times 7 \end{bmatrix}$

$\Rightarrow 12A = \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} \ldots\ldots\ldots\ldots.(ii)$

So,

$A^2 - 12A - I$

Substitute corresponding values from eqn(i) and (ii), we get

$\Rightarrow = \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 61-60-1 & 36-36-0 \\ 144-144-0 & 85-84-1 \end{bmatrix}$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$]

$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Therefore, $A^2 - 12A - I = 0$

Hence matrix A is the root of the given equation.

33. Question

If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, find $A^2 - 5A - 14I$.

Answer

Given: $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

I is identity matrix so $14I = 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$

To find $A^2 - 5A - 14I$

Now, we will find the matrix for A^2, we get

$A^2 = A \times A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (-5 \times -4) & 3 \times (-5) + (-5 \times 2) \\ (-4 \times 3) + (2 \times -4) & (-4 \times -5) + 2 \times 2 \end{bmatrix}$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$\Rightarrow A^2 = \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} \ldots\ldots\ldots (i)$

Now, we will find the matrix for 5A, we get

$5A = 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times (-5) \\ 5 \times (-4) & 5 \times 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} \ldots\ldots\ldots\ldots (ii)$

So,

$A^2 - 5A - 14I$

Substitute corresponding values from eqn(i) and (ii), we get

$\Rightarrow = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20-0 & 24-10-14 \end{bmatrix}$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$]

$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Therefore, $A^2 - 5A - 14I = 0$

34. Question

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Use this to find A^4.

Answer

Given: $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

I is identity matrix so $7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

To show that $A^2 - 5A + 7I = 0$

Now, we will find the matrix for A^2, we get

$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (1 \times -1) & 3 \times 1 + 1 \times 2 \\ (-1 \times 3) + (2 \times -1) & (-1 \times 1) + 2 \times 2 \end{bmatrix}$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$\Rightarrow A^2 = \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \ldots\ldots\ldots.(i)$

Now, we will find the matrix for 5A, we get

$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times 1 \\ 5 \times (-1) & 5 \times 2 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} \ldots\ldots\ldots\ldots.(ii)$

So,

$A^2 - 5A + 7I$

Substitute corresponding values from eqn(i) and (ii), we get

$\Rightarrow = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 8 - 15 - 7 & 5 - 5 - 0 \\ -5 + 5 - 0 & 3 - 10 - 7 \end{bmatrix}$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$]

$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Therefore, $A^2 - 5A + 7I = 0$

Hence proved

We will find A^4

$A^2 - 5A + 7I = 0$

Multiply both sides by A^2, we get

$A^2(A^2 - 5A + 7I) = A^2(0)$

$\Rightarrow A^4 - 5A^2.A + 7I.A^2$

$\Rightarrow A^4 = 5A^2.A - 7I.A^2$

$\Rightarrow A^4 = 5A^2A - 7A^2$

As multiplying by the identity matrix, I don't change anything. Now will substitute the corresponding values we get

$$\Rightarrow A^4 = 5\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 7\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\Rightarrow A^4 = 5\begin{bmatrix} 8\times 3 + (5\times -1) & 8\times 1 + 5\times 2 \\ (-5\times 3) + 3\times(-1) & (-5\times 1) + 3\times 2 \end{bmatrix} - 7\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\Rightarrow A^4 = 5\begin{bmatrix} 24-5 & 8+10 \\ -15-3 & -5+6 \end{bmatrix} - 7\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\Rightarrow A^4 = 5\begin{bmatrix} 19 & 18 \\ -18 & 1 \end{bmatrix} - 7\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 5\times 19 & 5\times 18 \\ 5\times(-18) & 5\times 1 \end{bmatrix} - \begin{bmatrix} 7\times 8 & 7\times 5 \\ 7\times(-5) & 7\times 3 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 95 & 90 \\ -90 & 5 \end{bmatrix} - \begin{bmatrix} 56 & 35 \\ -35 & 21 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 95-56 & 90-35 \\ -90+35 & 5-21 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

Hence this is the value of A^4

35. Question

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find k such that $A^2 = kA - 2I_2$.

Answer

Given: $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

I_2 is an identity matrix of size 2, so $2I_2 = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

Also given, $A^2 = kA - 2I_2$

Now, we will find the matrix for A^2, we get

$$A^2 = A\times A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3\times 3 + (-2\times 4) & 3\times(-2) + (-2\times -2) \\ (4\times 3) + (-2\times 4) & (4\times -2) + (-2\times -2) \end{bmatrix}$$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$$\Rightarrow A^2 = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \ldots\ldots\ldots.(i)$$

Now, we will find the matrix for kA, we get

$$kA = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow kA = \begin{bmatrix} k\times 3 & k\times(-2) \\ k\times 4 & k\times(-2) \end{bmatrix}$$

$$\Rightarrow kA = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} \ldots\ldots\ldots\ldots.(ii)$$

So,

$A^2 = kA - 2I_2$

Substitute corresponding values from eqn(i) and (ii), we get

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k-0 \\ 4k-0 & -2k-2 \end{bmatrix}$$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$],

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

Hence, $3k - 2 = 1 \Rightarrow k = 1$

Therefore, the value of k is 1

36. Question

If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k such that $A^2 - 8A + kI - 0$.

Answer

Given: $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

I is identity matrix, so $kI = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

Also given, $A^2 - 8A + kI = 0$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1\times 1 + 0 & 0 + 0 \\ (-1\times 1) + 7\times(-1) & 0 + 7\times 7 \end{bmatrix}$$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} \ldots\ldots\ldots.(i)$$

Now, we will find the matrix for 8A, we get

$$8A = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$\Rightarrow 8A = \begin{bmatrix} 8\times 1 & 8\times 0 \\ 8\times(-1) & 8\times 7 \end{bmatrix}$

$\Rightarrow 8A = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} \ldots\ldots\ldots\ldots(ii)$

So,

$A^2 - 8A + kI = 0$

Substitute corresponding values from eqn(i) and (ii), we get

$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 1-8+k & 0-0+0 \\ -8+8+0 & 49-56+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$],

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

Hence, $1-8+k = 0 \Rightarrow k = 7$

Therefore, the value of k is 7

37. Question

If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = x^2 - 2x - 3$, show that $f(A) = 0$.

Answer

Given: $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = x^2 - 2x - 3$

To show that $f(A) = 0$

Substitute $x = A$ in $f(x)$, we get

$f(A) = A^2 - 2A - 3I \ldots\ldots\ldots(i)$

I is identity matrix, so $3I = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Now, we will find the matrix for A^2, we get

$A^2 = A\times A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1\times 1 + 2\times 2 & 1\times 2 + 2\times 1 \\ 2\times 1 + 1\times 2 & 2\times 2 + 1\times 1 \end{bmatrix}$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$\Rightarrow A^2 = \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \ldots\ldots\ldots(ii)$

Now, we will find the matrix for 2A, we get

$2A = 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow 2A = \begin{bmatrix} 2\times 1 & 2\times 2 \\ 2\times 2 & 2\times 1 \end{bmatrix}$

$\Rightarrow 2A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} \ldots\ldots\ldots\ldots (iii)$

Substitute corresponding values from eqn(ii) and (iii) in eqn(i), we get

$f(A) = A^2 - 2A - 3I$

$\Rightarrow f(A) = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow f(A) = \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix}$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$],

$\Rightarrow f(A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

So, $\Rightarrow f(A) = 0$

Hence Proved

38. Question

If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find λ, μ so that $A^2 = \lambda A + \mu I$

Answer

Given: $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2 = \lambda A + \mu I$

So $\mu I = \mu \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$

Now, we will find the matrix for A^2, we get

$A^2 = A \times A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 2\times 2 + 3\times 1 & 2\times 3 + 3\times 2 \\ 1\times 2 + 2\times 1 & 1\times 3 + 2\times 2 \end{bmatrix}$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$\Rightarrow A^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \ldots\ldots\ldots (i)$

Now, we will find the matrix for λA, we get

$\lambda A = \lambda \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow \lambda A = \begin{bmatrix} \lambda\times 2 & \lambda\times 3 \\ \lambda\times 1 & \lambda\times 2 \end{bmatrix}$

$\Rightarrow \lambda A = \begin{bmatrix} 2\lambda & 3\lambda \\ \lambda & 2\lambda \end{bmatrix} \ldots\ldots\ldots\ldots (ii)$

But given, $A^2 = \lambda A + \mu I$

Substitute corresponding values from eqn(i) and (ii), we get

$\Rightarrow \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2\lambda & 3\lambda \\ \lambda & 2\lambda \end{bmatrix} + \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2\lambda + \mu & 3\lambda + 0 \\ \lambda + 0 & 2\lambda + \mu \end{bmatrix}$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$],

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

Hence, $\lambda + 0 = 4 \Rightarrow \lambda = 4$

And also, $2\lambda + \mu = 7$

Substituting the obtained value of λ in the above equation, we get

$2(4) + \mu = 7 \Rightarrow 8 + \mu = 7 \Rightarrow \mu = -1$

Therefore, the value of λ and μ are 4 and – 1 respectively

39. Question

Find the value of x for which the matrix product $\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix}$ equal to an identity matrix.

Answer

We know, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is identity matrix of size 3.

So according to the given criteria

$$\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now we will multiply the two matrices on LHS using the formula $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$, we get

$$\begin{bmatrix} 2\times(-x) + 0 + 7\times x & 2\times 14x + 0 + 7\times(-4x) & 2\times 7x + 0 + 7\times(-2x) \\ 0 + 0 + 0 & 0 + 1\times 1 + 0 & 0 + 0 + 0 \\ 1\times(-x) + 0 + 1\times x & 1\times 14x + (-2\times 1) + (1\times -4x) & 1\times 7x + 0 + 1\times(-2x) \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x - 2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

So we get $5x = 1 \Rightarrow x = \frac{1}{5}$

So the value of x is $\frac{1}{5}$

40 A. Question

Solve the matrix equations:

$$[x \;\; 1]\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x \\ 5 \end{bmatrix} = 0$$

Answer

$$[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x \\ 5 \end{bmatrix} = 0$$

Now we will multiply the two first matrices on LHS using the formula $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$, we get

$[x \times 1 + 1 \times (-2) \quad 0 + 1 \times (-3)] \begin{bmatrix} x \\ 5 \end{bmatrix} = 0$

$\Rightarrow [x-2 \quad -3] \begin{bmatrix} x \\ 5 \end{bmatrix} = 0$

Again multiply these two LHS matrices, we get

$\Rightarrow [(x-2) \times x + (-3) \times 5] = 0$

⇒ $x^2 - 2x - 15 = 0$. This is form of quadratic equation, we will solve this by splitting the middle term, we get

⇒ $x^2 - 5x + 3x - 15 = 0$

⇒ $x(x - 5) + 3(x - 5) = 0$

⇒ $(x - 5)(x + 3) = 0$

⇒ $x - 5 = 0$ or $x + 3 = 0$

This gives, $x = 5$ or $x = -3$ is the required solution of the matrices.

40 B. Question

Solve the matrix equations:

$$[1 \ 2 \ 1] \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

Answer

$$[1 \quad 2 \quad 1] \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

Now we will multiply the two first matrices on LHS using the formula $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$, we get

$$[1 \times 1 + 2 \times 2 + 1 \times 1 \quad 1 \times 2 + 0 + 0 \quad 0 + 2 \times 1 + 1 \times 2] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow [6 \quad 2 \quad 4] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

Again multiply these two LHS matrices, we get

$\Rightarrow [0 + 2 \times 2 + 4x] = 0$

⇒ $4 + 4x = 0$ we will solve this linear equation, we get

⇒ $4x = -4$

This gives, $x = -1$ is the required solution of the matrices.

40 C. Question

Solve the matrix equations:

$$[x - 5 - 1] \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

Answer

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

Now we will multiply the two first matrices on LHS using the formula $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$, we get

$$\begin{bmatrix} x \times 1 + 0 + (-1 \times 2) & 0 + (-5 \times 2) + 0 & 2x + (-5 \times 1) + (-1 \times 3) \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}$$

$= 0$

$$\Rightarrow \begin{bmatrix} x - 2 & -10 & 2x - 8 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

Again multiply these two LHS matrices, we get

$\Rightarrow [(x-2)x + (-10 \times 4) + (2x-8) \times 1] = 0$

$\Rightarrow x^2 - 2x - 40 + 2x - 8 = 0$

$\Rightarrow x^2 - 48 = 0$. This is form of quadratic equation, we will solve this, we get

$\Rightarrow x^2 = 48 \Rightarrow x^2 = 16 \times 3$

This gives, $x = \pm 4\sqrt{3}$ is the required solution of the matrices.

40 D. Question

Solve the matrix equations:

$$\begin{bmatrix} 2x & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix} = 0$$

Answer

$$\begin{bmatrix} 2x & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix} = 0$$

Now we will multiply the two first matrices on LHS using the formula $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$, we get

$$\begin{bmatrix} 2x \times 1 + 3 \times (-3) & 2x \times 2 + 0 \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x - 9 & 4x \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix} = 0$$

Again multiply these two LHS matrices, we get

$\Rightarrow [(2x-9) \times x + 4x \times 8] = 0$

$\Rightarrow x^2 - 9x + 32x = 0$

$\Rightarrow x^2 - 23x$. This is form of quadratic equation, we will solve this, we get

$\Rightarrow x(x - 23) = 0$

$\Rightarrow x = 0$ or $x - 23 = 0$

This gives, $x = 0$ or $x = 23$ is the required solution of the matrices.

41. Question

If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$, compute $A^2 - 4A + 3I_3$.

Answer

Given: $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$

To find the value of $A^2 - 4A + 3I_3$

I_3 is an identity matrix of size 3, so $3I_3 = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 \times 1 + 2 \times 3 + 0 & 1 \times 2 + 2 \times (-4) + 0 & 0 + 2 \times 5 + 0 \\ 3 \times 1 + 3 \times (-4) + 0 & 3 \times 2 + (-4 \times -4) + 5 \times (-1) & 0 + (-4 \times 5) + 5 \times 3 \\ 0 + (-1 \times 3) + 0 & 0 + (-1 \times -4) + 3 \times (-1) & 0 + (-1 \times 5) + 3 \times 3 \end{bmatrix}$$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$$\Rightarrow A^2 = \begin{bmatrix} 1 + 6 & 2 - 8 & 10 \\ 3 - 12 & 6 + 16 - 5 & -20 + 15 \\ -3 & 4 - 3 & -5 + 9 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 7 & -6 & 10 \\ -9 & 17 & -5 \\ -3 & 1 & 4 \end{bmatrix} \ldots\ldots\ldots (i)$$

Now, we will find the matrix for 4A, we get

$$4A = 4\begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow 4A = \begin{bmatrix} 4 \times 1 & 4 \times 2 & 0 \\ 4 \times 3 & 4 \times (-4) & 4 \times 5 \\ 0 & 4 \times (-1) & 4 \times 3 \end{bmatrix}$$

$$\Rightarrow 4A = \begin{bmatrix} 4 & 8 & 0 \\ 12 & -16 & 20 \\ 0 & -4 & 12 \end{bmatrix} \ldots\ldots\ldots\ldots (ii)$$

So, Substitute corresponding values from eqn(i) and (ii)in equation $A^2 - 4A + 3I_3$, we get

$$\Rightarrow = \begin{bmatrix} 7 & -6 & 10 \\ -9 & 17 & -5 \\ -3 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 0 \\ 12 & -16 & 20 \\ 0 & -4 & 12 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 7 - 4 + 3 & -6 - 8 + 0 & 10 - 0 + 0 \\ -9 - 12 + 0 & 17 + 16 + 3 & -5 - 20 + 0 \\ -3 + 0 + 0 & 1 + 4 + 0 & 4 - 12 + 3 \end{bmatrix}$$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$],

$$\Rightarrow = \begin{bmatrix} 6 & 14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}$$

Hence the value of $A^2 - 4A + 3I_3 = \begin{bmatrix} 6 & 14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}$

42. Question

If f(x) = x^2 – 2x, find f(A), where $A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$.

Answer

Given: $A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$ and f(x) = x^2 – 2x

To find the value of f(A)

We will substitute x = A in the given equation we get

f(A) = A^2 – 2A................(i)

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 + 1 \times 4 + 0 & 0 + 1 \times 5 + 2 \times 2 & 0 + 0 + 3 \times 2 \\ 0 + 5 \times 4 + 0 & 4 \times 1 + 5 \times 5 + 0 & 4 \times 2 + 0 + 0 \\ 0 + 2 \times 4 + 0 & 0 + 2 \times 5 + 3 \times 2 & 0 + 0 + 3 \times 3 \end{bmatrix}$$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$$\Rightarrow A^2 = \begin{bmatrix} 4 & 5 + 4 & 6 \\ 20 & 4 + 25 & 8 \\ 8 & 10 + 6 & 9 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4 & 9 & 6 \\ 20 & 29 & 8 \\ 8 & 16 & 9 \end{bmatrix} \ldots\ldots\ldots.(i)$$

Now, we will find the matrix for 2A, we get

$$2A = 2\begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 0 & 2 \times 1 & 2 \times 2 \\ 2 \times 4 & 2 \times 5 & 0 \\ 0 & 2 \times 2 & 2 \times 3 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 0 & 2 & 4 \\ 8 & 10 & 0 \\ 0 & 4 & 6 \end{bmatrix} \ldots\ldots\ldots\ldots.(ii)$$

So, Substitute corresponding values from eqn(i) and (ii) in equation f(A) = A^2 – 2A, we get

$$\Rightarrow f(A) = \begin{bmatrix} 4 & 9 & 6 \\ 20 & 29 & 8 \\ 8 & 16 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 8 & 10 & 0 \\ 0 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 4 - 0 & 9 - 2 & 6 - 4 \\ 20 - 8 & 29 - 10 & 8 - 0 \\ 8 - 0 & 16 - 4 & 9 - 6 \end{bmatrix}$$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$],

$$\Rightarrow f(A) = \begin{bmatrix} 4 & 7 & 2 \\ 12 & 19 & 8 \\ 8 & 12 & 3 \end{bmatrix}$$

Hence the value of $f(A) = \begin{bmatrix} 4 & 7 & 2 \\ 12 & 19 & 8 \\ 8 & 12 & 3 \end{bmatrix}$

43. Question

If f(x) = $x^3 + 4x^2 - x$, find f(A), where $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$.

Answer

Given: $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$ and f(x) = $x^3 + 4x^2 - x$

To find the value of f(A)

We will substitute x = A in the given equation we get

f(A) = $A^3 + 4A^2 - A$ ....(i)

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 + 1 \times 2 + 2 \times 1 & 0 + 1 \times (-3) + 2 \times (-1) & 0 + 0 + 0 \\ 0 + (-3 \times 2) + 0 & 2 \times 1 + (-3) \times (-3) + 0 & 2 \times 2 + 0 + 0 \\ 0 + (-1 \times 2) + 0 & 1 \times 1 + (-1) \times (-3) + 0 & 1 \times 2 + 0 + 0 \end{bmatrix}$$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$$\Rightarrow A^2 = \begin{bmatrix} 2 + 2 & -3 - 2 & 0 \\ -6 & 2 + 9 & 4 \\ -2 & 1 + 3 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix} \ldots\ldots\ldots.(i)$$

Now, we will find the matrix for A^3, we get

$$A^3 = A^2 \times A = \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 0 + (-5 \times 2) + 0 & 4 \times 1 + (-5) \times (-3) + 0 & 4 \times 2 + 0 + 0 \\ 0 + 11 \times 2 + 4 \times 1 & (-6 \times 1) + 11 \times (-3) + 4 \times (-1) & (-6 \times 2) + 0 + 0 \\ 0 + 4 \times 2 + 2 \times 1 & (-2 \times 1) + 4 \times (-3) + 2 \times (-1) & (-2 \times 2) + 0 + 0 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix} \ldots\ldots\ldots\ldots.(ii)$$

So, Substitute corresponding values from eqn(i) and (ii) in equation f(A) = $A^3 + 4A^2 - A$, we get

$$\Rightarrow f(A) = \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix} + 4\begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix} + \begin{bmatrix} 4\times 4 & 4\times(-5) & 0 \\ 4\times(-6) & 4\times 11 & 4\times 4 \\ 4\times(-2) & 4\times 4 & 4\times 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix} + \begin{bmatrix} 16 & -20 & 0 \\ -24 & 44 & 16 \\ -8 & 16 & 8 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} -10 + 16 - 0 & 19 - 20 - 1 & 8 + 0 - 2 \\ 26 - 24 - 2 & -43 + 44 + 3 & -12 + 16 - 0 \\ 10 - 8 - 1 & -16 + 16 + 1 & -4 + 8 - 0 \end{bmatrix}$$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$],

$$\Rightarrow f(A) = \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

Hence the value of $f(A) = \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix}$

44. Question

If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that A is a root of the polynomial $f(x) = x^3 - 6x^2 + 7x + 2$.

Answer

Given: $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $f(x) = x^3 - 6x^2 + 7x + 2$

To find the value of f(A)

We will substitute x = A in the given equation we get

$f(A) = A^3 - 6A^2 + 7A + 2I$................(i)

Here I is identity matrix

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1\times 1 + 0 + 2\times 2 & 0 + 0 + 0 & 1\times 2 + 0 + 2\times 3 \\ 0 + 0 + 2\times 1 & 0 + 2\times 2 + 0 & 0 + 2\times 1 + 1\times 3 \\ 2\times 1 + 0 + 3\times 2 & 0 + 0 + 0 & 2\times 2 + 0 + 3\times 3 \end{bmatrix}$$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$$\Rightarrow A^2 = \begin{bmatrix} 1 + 4 & 0 & 2 + 6 \\ 2 & 4 & 2 + 3 \\ 2 + 6 & 0 & 4 + 9 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \ldots\ldots\ldots.(i)$$

Now, we will find the matrix for A^3, we get

$$A^3 = A^2 \times A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 5\times1 + 0 + 8\times2 & 0 + 0 + 0 & 5\times2 + 0 + 8\times3 \\ 2\times1 + 0 + 5\times2 & 0 + 4\times2 + 0 & 2\times2 + 4\times1 + 5\times3 \\ 8\times1 + 0 + 13\times2 & 0 + 0 + 0 & 8\times2 + 0 + 13\times3 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 5 + 16 & 0 & 10 + 24 \\ 2 + 10 & 8 & 4 + 4 + 15 \\ 8 + 26 & 0 & 16 + 39 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} \ldots\ldots\ldots\ldots.(ii)$$

So, Substitute corresponding values from eqn(i) and (ii) in equation f(A) = A^3 – $6A^2$ + 7A + 2I, we get

$$\Rightarrow f(A) = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 6\times5 & 0 & 6\times8 \\ 6\times2 & 6\times4 & 6\times5 \\ 6\times8 & 0 & 6\times13 \end{bmatrix} + \begin{bmatrix} 7\times1 & 0 & 7\times2 \\ 0 & 7\times2 & 7\times1 \\ 7\times2 & 0 & 7\times3 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 21 - 30 + 7 + 2 & 0 - 0 + 0 + 0 & 34 - 48 + 14 + 0 \\ 12 - 12 + 0 + 0 & 8 - 24 + 14 + 2 & 23 - 30 + 7 + 0 \\ 34 - 48 + 14 + 0 & 0 - 0 + 0 + 0 & 55 - 78 + 21 + 2 \end{bmatrix}$$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$],

$$\Rightarrow f(A) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Hence the A is the root of the given polynomial.

45. Question

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then prove that A^2 – 4A – 5I = 0.

Answer

Given: $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

To prove A^2 – 4A – 5I = 0

Here I is the identity matrix

Now, we will find the matrix for A^2, we get

$A^2 = A \times A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$\Rightarrow A^2$

$= \begin{bmatrix} 1\times1 + 2\times2 + 2\times2 & 1\times2 + 2\times1 + 2\times2 & 1\times2 + 2\times2 + 2\times1 \\ 2\times1 + 1\times2 + 2\times2 & 2\times2 + 1\times1 + 2\times2 & 2\times2 + 1\times2 + 2\times1 \\ 2\times1 + 2\times2 + 1\times2 & 2\times2 + 2\times1 + 1\times2 & 2\times2 + 2\times2 + 1\times1 \end{bmatrix}$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$\Rightarrow A^2 = \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} \ldots\ldots\ldots(i)$

So, Substitute corresponding values from eqn(i) in equation

$A^2 - 4A - 5I$, we get

$\Rightarrow = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4\times1 & 4\times2 & 4\times2 \\ 4\times2 & 4\times1 & 4\times2 \\ 4\times2 & 4\times2 & 4\times1 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix}$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$],

$\Rightarrow = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$

Hence the $A^2 - 4A - 5I = 0$ (Proved)

46. Question

If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, show that $A^2 - 7A + 10I_3 = 0$.

Answer

Given: $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

To prove $A^2 - 7A + 10I_3 = 0$

Here I_3 is an identity matrix of size 3

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + 2 \times 1 + 0 & 3 \times 2 + 2 \times 4 + 0 & 0 + 0 + 0 \\ 1 \times 3 + 4 \times 1 + 0 & 1 \times 2 + 4 \times 4 + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 5 \times 5 \end{bmatrix}$$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$$\Rightarrow A^2 = \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} \ldots\ldots\ldots (i)$$

So, Substitute corresponding values in equation

$A^2 - 7A + 10I_3$, we get

$$\Rightarrow = \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - 7\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} + 10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 7 \times 3 & 7 \times 2 & 0 \\ 7 \times 1 & 7 \times 4 & 0 \\ 0 & 0 & 7 \times 5 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 11 - 21 + 10 & 14 - 14 + 0 & 0 \\ 7 - 7 - 0 & 18 - 28 + 10 & 0 \\ 0 & 0 & 25 - 35 + 10 \end{bmatrix}$$

[as $r_{ij} = a_{ij} + b_{ij} + c_{ij}$],

$$\Rightarrow = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Hence the $A^2 - 7A + 10I_3 = 0$ (Proved)

47. Question

Without using the concept of the inverse of a matrix, find the matrix $\begin{bmatrix} x & y \\ z & u \end{bmatrix}$ such that

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

Answer

Given: $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

Multiplying we get,

$$\begin{bmatrix} 5x - 7z & 5y - 7u \\ -2x + 3z & -2y + 3u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

From above we can see that,

$5x - 7z = -16$ ...(1)

$-2x + 3z = 7$ ....(2)

$5y - 7u = -6$ .....(3)

–2y + 3 u = 2 ……(4)

Now we have to solve these equations to find values of x, y, z and u

Multiplying eq (1) by 2 and eq (2) by 5 and adding the equations we get,

10 x – 14 z + 10 x + 15 z = –32 + 35

Z = 3

Putting this value in eq(1) we get,

5 x – 21 = – 16

5 x = 5

X = 1

Now, multiplying eq(3) by 2 and eq(4) by 5 and adding we get,

10 y – 14 u + 10 y + 15 u = –12 + 10

u = –2

Putting value of u in equation (3) we get,

5 y + 14 = = – 6

5 y = – 20

Y = –4

Therefore now we have,

$$\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$$

48 A. Question

Find the matrix A such that

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Answer

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

We know that the two matrices are eligible for their product only when the number of columns of first matrix is equal to the number of rows of the second matrix.

So, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is 2×2 matrix, and $\begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$ is 3×2 matrix

Now in order to get a 3×2 matrix as solution 2×2 matrix should be multiplied by 2×3 matrix. Hence matrix A is 2×3 matrix.

Let, $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

So the given question becomes,

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Now we will multiply the two matrices on LHS, we get

$$\Rightarrow \begin{bmatrix} 1 \times a + 1 \times d & 1 \times b + 1 \times e & 1 \times c + 1 \times f \\ 0 + 1 \times d & 0 + 1 \times e & 0 + 1 \times f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$\Rightarrow \begin{bmatrix} a + d & b + e & c + f \\ d & e & f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$

To satisfy the above equality condition, corresponding entries of the matrices should be equal, i.e.,

d = 1, e = 0, f = 1

a + d = 3 ⇒ a + 1 = 3 ⇒ a = 2

b + e = 3 ⇒ b + 0 = 3 ⇒ b = 3

c + f = 5 ⇒ c + 1 = 5 ⇒ c = 4

Now substituting these values in matrix A, we get

$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 1 \end{bmatrix}$is the matrix A.

48 B. Question

Find the matrix A such that

$$A\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Answer

$A\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

We know that the two matrices are eligible for their product only when the number of columns of first matrix is equal to the number of rows of the second matrix.

The matrix given on the RHS of the equation is a 2×3 matrix and the one given on the LHS of the equation is a 2×3 matrix.

Therefore, A has to be a 2×2 matrix.

Let, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

So the given question becomes,

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Now we will multiply the two matrices on LHS, we get

$\Rightarrow \begin{bmatrix} a \times 1 + b \times 4 & a \times 2 + b \times 5 & a \times 3 + b \times 6 \\ c \times 1 + d \times 4 & c \times 2 + d \times 5 & c \times 3 + d \times 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$\Rightarrow \begin{bmatrix} a + 4b & 2a + 5b & 3a + 6b \\ c + 4d & 2c + 5d & 3c + 6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

To satisfy the above equality condition, corresponding entries of the matrices should be equal, i.e.,

a + 4b = – 7, 2a + 5b = – 8, 3a + 6b = – 9

c + 4d = 2, 2c + 5d = 4, 3c + 6d = 6

Now, a + 4b = – 7 ⇒ a = – 7 – 4b............(i)

∴ 2a + 5b = – 8

⇒ 2(– 7 – 4b) + 5b = – 8 (by substituting the value of a from eqn(i))

⇒ – 14 – 8b + 5b = – 8

⇒ 3b = – 14 + 8

⇒ b = – 2

Hence substitute the value of b in eqn(i), we get

a = – 7 – 4b

⇒ a = – 7 – 4(– 2) = – 7 + 8 = 1

⇒ a = 1

Now, c + 4d = 2 ⇒ c = 2 – 4d…………(ii)

∴ 2c + 5d = 4

⇒ 2(2 – 4d) + 5d = 4 by substituting the value of a from eqn(ii))

⇒ 4 – 8d + 5d = 4

⇒ 3d = 0

⇒ d = 0

Hence substitute the value of d in eqn(ii), we get

c = 2 – 4d

⇒ c = 2 – 4(0)

⇒ c = 2

Now substituting these values in matrix A, we get

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$is the matrix A.

48 C. Question

Find the matrix A such that

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

Answer

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

We know that the two matrices are eligible for their product only when the number of columns of first matrix is equal to the number of rows of the second matrix.

The matrix given on the RHS of the equation is a 3×3 matrix and the one given on the LHS of the equation is a 1×3 matrix.

Therefore, A has to be a 1×3 matrix.

Let, $A = \begin{bmatrix} a & b & c \end{bmatrix}$

So the given question becomes,

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

Now we will multiply the two matrices on LHS, we get

$$\Rightarrow \begin{bmatrix} 4\times a & 4\times b & 4\times c \\ 1\times a & 1\times b & 1\times c \\ 3\times a & 3\times b & 3\times c \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$$\Rightarrow \begin{bmatrix} 4a & 4b & 4c \\ a & b & c \\ 3a & 3b & 3c \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

To satisfy the above equality condition, corresponding entries of the matrices should be equal, i.e.,

$4a = -4 \Rightarrow a = -1$

$4b = 8 \Rightarrow b = 2$

$4c = 4 \Rightarrow c = 1$

Now substituting these values in matrix A, we get

$A = [a \quad b \quad c] = [-1 \quad 2 \quad 1]$ is the matrix A.

48 D. Question

Find the matrix A such that

$$[2 \ \ 1 \ \ 3]\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$$

Answer

$$[2 \quad 1 \quad 3]\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$$

We will multiply the first two matrices, on the LHS, we get

$$[2\times(-1) + 1\times(-1) + 0 \quad 0 + 1\times 1 + 3\times 1 \quad 2\times(-1) + 3\times 1]\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$$\Rightarrow [-3 \quad 4 \quad 1]\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$$

Again multiply the two matrices on the LHS, we get

$[(-3)\times 1 + 0 + 1\times(-1)] = A$

$\Rightarrow A = [-4]$ is the matrix A.

48 E. Question

Find the matrix A such that

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

Answer

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

We know that the two matrices are eligible for their product only when the number of columns of first matrix is equal to the number of rows of the second matrix.

The matrix given on the RHS of the equation is a 3×3 matrix and the one given on the LHS of the equation is a 3×2 matrix.

Therefore, A has to be a 2×3 matrix.

Let, $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

So the given question becomes,

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

Now we will multiply the two matrices on LHS, we get

$$\Rightarrow \begin{bmatrix} 2 \times a + (-1) \times d & 2 \times b + (-1) \times e & 2 \times c + (-1) \times f \\ 1 \times a + 0 & 1 \times b + 0 & 1 \times c + 0 \\ (-3) \times a + 4 \times d & (-3) \times b + 4 \times e & (-3) \times c + 4 \times f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

[as $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$]

$$\Rightarrow \begin{bmatrix} 2a - d & 2b - e & 2c - f \\ a & b & c \\ -3a + 4d & -3b + 4e & -3c + 4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

To satisfy the above equality condition, corresponding entries of the matrices should be equal, i.e.,

2a – d = – 1…(i),

2b – e = – 8….(ii),

2c – f = – 10…..(iii)

a = 1, b = – 2,c = – 5

– 3a + 4d = 9……..(iv),

– 3b + 4e = 22………(v),

– 3c + 4f = 15……..(vi)

Substitute the value of a in eqn(i), we get

2a – d = – 1 ⇒ 2(1) – d = – 1 ⇒ d = 3

Substitute the value of b in eqn(ii), we get

2b – e = – 8 ⇒ 2(– 2) – e = – 8 ⇒ – 4 – e = – 8 ⇒ e = 4

Substitute the value of c in eqn(iii), we get

2c – f = – 10 ⇒ 2(– 5) – f = – 10 ⇒ – 10 – f = – 10 ⇒ f = 0

Now substituting these values in matrix A, we get

$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$ is the matrix A.

48 F. Question

Find the matrix A such that

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \\ 11 & 10 & 9 \end{bmatrix}$$

Answer

$$A \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \\ 11 & 10 & 9 \end{bmatrix}$$

On multiplying A with 2 × 3 matrix we get 3 × 3 matrix

Therefore, A must be a matrix of order 3 × 2

Let A = $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$

$$A \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$A \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \\ e+4f & 2e+5f & 3e+6f \end{bmatrix}$$

$$\begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \\ e+4f & 2e+5f & 3e+6f \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \\ 11 & 10 & 9 \end{bmatrix}$$

So we have,

a + 4 b = – 7 ....(1)

2 a + 5 b = – 8 ...(2)

c + 4 d = 2 ....(3)

2 c + 5 d = 4 ..(4)

e + 4 f = 11 ...(5)

2 e + 5 f = 10 ...(6)

a = 1, b = –2, c = 2, d = 0, e = –5 and f = 4 on solving the above equations.

Hence, A = $\begin{bmatrix} 1 & -2 \\ 2 & 0 \\ -5 & 4 \end{bmatrix}$

49. Question

Find a 2 × 2 matrix A such that $A \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6I_2$.

Answer

Given A is a 2×2 matrix,

So let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Here I_2 is an identity matrix of size 2, $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

So the given equation becomes,

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a \times 1 + b \times 1 & a \times (-2) + b \times 4 \\ c \times 1 + d \times 1 & c \times (-2) + d \times 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a + b & -2a + 4b \\ c + d & -2c + 4d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

To satisfy the above equality condition, corresponding entries of the matrices should be equal, i.e.,

a + b = 6…….(i)

– 2a + 4b = 0 ⇒ 2a = 4b ⇒ a = 2b……..(ii)

c + d = 0 ⇒ c = – d……(iii)

– 2c + 4d = 6 ……(iv)

Substitute the values of eqn(ii) in eqn (i), we get

a + b = 6 ⇒ 2b + b = 6 ⇒ b = 2

So eqn(ii) becomes, a = 2b = 2(2) = 4⇒ a = 4

Substitute the values of eqn(iii) in eqn (iv), we get

– 2c + 4d = 6 ⇒ – 2(– d) + 4d = 6 ⇒ 2d + 4d = 6 ⇒ 6d = 6 ⇒ d = 1

So eqn(iii) becomes, c = – d⇒ c = – 1

Now substituting these values in matrix A, we get

$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$is the matrix A.

50. Question

If $A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$, find A^{16}.

Answer

Given: $A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$

We will find A^2,

$A^2 = A \times A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 0 + 0 & 0 + 0 \\ 0 + 0 & 0 + 0 \end{bmatrix}$

$\Rightarrow A^2 = 0$

Hence, $A^{16} = (A^2)^8 = (0)^8 = 0$

Hence A^{16} is a nill matrix.

51. Question

If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$, then show that $(A + B)^2 = A^2 + B^2$.

Answer

Given $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$.

We need to prove $(A + B)^2 = A^2 + B^2$.

Let us evaluate the LHS and the RHS one at a time.

To find the LHS, we will first calculate A + B.

$$A + B = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow A + B = \begin{bmatrix} 0 + 0 & -x + 1 \\ x + 1 & 0 + 0 \end{bmatrix}$$

$$\therefore A + B = \begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}$$

We know $(A + B)^2 = (A + B)(A + B)$.

$$\Rightarrow (A + B)^2 = \begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}$$

$$\Rightarrow (A + B)^2 = \begin{bmatrix} (0)(0) + (-x + 1)(x + 1) & (0)(-x + 1) + (-x + 1)(0) \\ (x + 1)(0) + (0)(x + 1) & (x + 1)(-x + 1) + (0)(0) \end{bmatrix}$$

$$\Rightarrow (A + B)^2 = \begin{bmatrix} 0 + (1 - x^2) & 0 + 0 \\ 0 + 0 & (1 - x^2) + 0 \end{bmatrix}$$

$$\Rightarrow (A + B)^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix}$$

$$\Rightarrow (A + B)^2 = \begin{bmatrix} 1 - (-1) & 0 \\ 0 & 1 - (-1) \end{bmatrix} (\because x^2 = -1)$$

$$\therefore (A + B)^2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

To find the RHS, we will first calculate A^2 and B^2.

We know $A^2 = A \times A$.

$$\Rightarrow A^2 = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} (0)(0) + (-x)(x) & (0)(-x) + (-x)(0) \\ (x)(0) + (0)(x) & (x)(-x) + (0)(0) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 + (-x^2) & 0 + 0 \\ 0 + 0 & (-x^2) + 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (\because x^2 = -1)$$

Similarly, we also have $B^2 = B \times B$.

$$\Rightarrow B^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow B^2 = \begin{bmatrix} (0)(0) + (1)(1) & (0)(1) + (1)(0) \\ (1)(0) + (0)(1) & (1)(1) + (0)(0) \end{bmatrix}$$

$$\Rightarrow B^2 = \begin{bmatrix} 0 + 1 & 0 + 0 \\ 0 + 0 & 1 + 0 \end{bmatrix}$$

$$\therefore B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now, the RHS is $A^2 + B^2$.

$$\Rightarrow A^2 + B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 + B^2 = \begin{bmatrix} 1+1 & 0+0 \\ 0+0 & 1+1 \end{bmatrix}$$

$$\therefore A^2 + B^2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = (A+B)^2$$

Thus, $(A + B)^2 = A^2 + B^2$.

52. Question

If $A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, then verify that $A^2 + A = (A + I)$, where I is the identity matrix.

Answer

Given $A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

We need to prove $A^2 + A = A (A + I)$.

Let us evaluate the LHS and the RHS one at a time.

To find the LHS, we will first calculate A^2.

We know $A^2 = A \times A$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1+0+0 & 0+0+(-3) & -3+0+(-3) \\ 2+2+0 & 0+1+3 & -6+3+3 \\ 0+2+0 & 0+1+1 & 0+3+1 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 1 & -3 & -6 \\ 4 & 4 & 0 \\ 2 & 2 & 4 \end{bmatrix}$$

Now, the LHS is $A^2 + A$.

$$\Rightarrow A^2 + A = \begin{bmatrix} 1 & -3 & -6 \\ 4 & 4 & 0 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 + A = \begin{bmatrix} 1+1 & -3+0 & -6+(-3) \\ 4+2 & 4+1 & 0+3 \\ 2+0 & 2+1 & 4+1 \end{bmatrix}$$

$$\therefore A^2 + A = \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

To find the RHS, we will first calculate A + I.

$$A + I = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A + I = \begin{bmatrix} 1+1 & 0+0 & -3+0 \\ 2+0 & 1+1 & 3+0 \\ 0+0 & 1+0 & 1+1 \end{bmatrix}$$

$$\therefore A + I = \begin{bmatrix} 2 & 0 & -3 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

Now, the RHS is A(A + I).

$$\Rightarrow A(A+I) = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -3 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow A(A+I) = \begin{bmatrix} 2+0+0 & 0+0+(-3) & -3+0+(-6) \\ 4+2+0 & 0+2+3 & -6+3+6 \\ 0+2+0 & 0+2+1 & 0+3+2 \end{bmatrix}$$

$$\therefore A(A+I) = \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix} = A^2 + A$$

Thus, $A^2 + A = A (A + I)$.

53. Question

If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, obtain A^3.

Answer

Given $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$.

We need to find $A^2 - 5A - 14I$.

We know $A^2 = A \times A$.

$$\Rightarrow A^2 = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} (3)(3)+(-5)(-4) & (3)(-5)+(-5)(2) \\ (-4)(3)+(2)(-4) & (-4)(-5)+(2)(2) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

Now, we evaluate $-5A = -5 \times A$.

$$-5A = -5 \times \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow -5A = \begin{bmatrix} 3(-5) & -5(-5) \\ -4(-5) & 2(-5) \end{bmatrix}$$

$$\therefore -5A = \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix}$$

Finally, matrix $-14I = -14 \times I$.

$$-14I = -14 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow -14I = \begin{bmatrix} 1(-14) & 0(-14) \\ 0(-14) & 1(-14) \end{bmatrix}$$

$\therefore -14I = \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$

The given expression is $A^2 - 5A - 14I$.

$\Rightarrow A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} + \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$

$\Rightarrow A^2 - 5A - 14I = \begin{bmatrix} 29 + (-15) + (-14) & -25 + 25 + 0 \\ -20 + 20 + 0 & 24 + (-10) + (-14) \end{bmatrix}$

$\therefore A^2 - 5A - 14I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

On multiplying both sides with matrix A, we get

$A(A^2 - 5A - 14I) = O$

$\Rightarrow A^3 - 5A^2 - 14(A \times I) = O$

$\Rightarrow A^3 - 5A^2 - 14A = O$

$\Rightarrow A^3 = 5A^2 + 14A$

$\Rightarrow A^3 = 5 \times \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14 \times \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} 29(5) & -25(5) \\ -20(5) & 24(5) \end{bmatrix} + \begin{bmatrix} 3(14) & -5(14) \\ -4(14) & 2(14) \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} 145 + 42 & -125 + (-70) \\ -100 + (-56) & 120 + 28 \end{bmatrix}$

$\therefore A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$

Thus, $A^2 - 5A - 14I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$.

54 A. Question

If $P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then show that P(x) P(y) = P(x + y) = P(y) P(x).

Answer

Given $P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$.

We need to prove that P(x)P(y) = P(x + y) = P(y)P(x).

First, we will evaluate P(x)P(y).

$P(x)P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$

$\Rightarrow P(x)P(y) = \begin{bmatrix} \cos x \cos y - \sin x \sin y & \cos x \sin y + \sin x \cos y \\ -\sin x \cos y - \cos x \sin y & -\sin x \sin y + \cos x \cos y \end{bmatrix}$

$\Rightarrow P(x)P(y) = \begin{bmatrix} \cos x \cos y - \sin x \sin y & \sin x \cos y + \cos x \sin y \\ -(\sin x \cos y + \cos x \sin y) & \cos x \cos y - \sin x \sin y \end{bmatrix}$

$\therefore P(x)P(y) = \begin{bmatrix} \cos(x + y) & \sin(x + y) \\ -\sin(x + y) & \cos(x + y) \end{bmatrix} = P(x + y)$

Now, we will evaluate P(y)P(x).

$$P(y)P(x)=\begin{bmatrix}\cos y & \sin y\\ -\sin y & \cos y\end{bmatrix}\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$$

$$\Rightarrow P(y)P(x)=\begin{bmatrix}\cos y\cos x-\sin y\sin x & \cos y\sin x+\sin y\cos x\\ -\sin y\cos x-\cos y\sin x & -\sin y\sin x+\cos y\cos x\end{bmatrix}$$

$$\Rightarrow P(y)P(x)=\begin{bmatrix}\cos x\cos y-\sin x\sin y & \sin x\cos y+\cos x\sin y\\ -(\sin x\cos y+\cos x\sin y) & \cos x\cos y-\sin x\sin y\end{bmatrix}$$

$$\therefore P(y)P(x)=\begin{bmatrix}\cos(x+y) & \sin(x+y)\\ -\sin(x+y) & \cos(x+y)\end{bmatrix}=P(x+y)$$

Thus, P(x)P(y) = P(x + y) = P(y)P(x).

54 B. Question

If $P=\begin{bmatrix}x & 0 & 0\\ 0 & y & 0\\ 0 & 0 & z\end{bmatrix}$ and $Q=\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$, prove that $PQ=\begin{bmatrix}xa & 0 & 0\\ 0 & yb & 0\\ 0 & 0 & zc\end{bmatrix}=QP$

Answer

Given $P=\begin{bmatrix}x & 0 & 0\\ 0 & y & 0\\ 0 & 0 & z\end{bmatrix}$ and $Q=\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$

We need to prove that $PQ=\begin{bmatrix}xa & 0 & 0\\ 0 & yb & 0\\ 0 & 0 & zc\end{bmatrix}=QP$.

First, we will evaluate PQ.

$$PQ=\begin{bmatrix}x & 0 & 0\\ 0 & y & 0\\ 0 & 0 & z\end{bmatrix}\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$$

$$\Rightarrow PQ=\begin{bmatrix}x\times a+0+0 & 0+0+0 & 0+0+0\\ 0+0+0 & 0+y\times b+0 & 0+0+0\\ 0+0+0 & 0+0+0 & 0+0+z\times c\end{bmatrix}$$

$$\therefore PQ=\begin{bmatrix}xa & 0 & 0\\ 0 & yb & 0\\ 0 & 0 & zc\end{bmatrix}$$

Now, we will evaluate QP.

$$QP=\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}\begin{bmatrix}x & 0 & 0\\ 0 & y & 0\\ 0 & 0 & z\end{bmatrix}$$

$$\Rightarrow QP=\begin{bmatrix}a\times x+0+0 & 0+0+0 & 0+0+0\\ 0+0+0 & 0+b\times y+0 & 0+0+0\\ 0+0+0 & 0+0+0 & 0+0+c\times z\end{bmatrix}$$

$$\therefore QP=\begin{bmatrix}xa & 0 & 0\\ 0 & yb & 0\\ 0 & 0 & zc\end{bmatrix}=PQ$$

Thus, $PQ=\begin{bmatrix}xa & 0 & 0\\ 0 & yb & 0\\ 0 & 0 & zc\end{bmatrix}=QP$

55. Question

If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = O$.

Answer

Given $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$.

We need to find $A^2 - 5A + 4I$.

We know $A^2 = A \times A$.

$$\Rightarrow A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4+0+1 & 0+0+(-1) & 2+0+0 \\ 4+2+3 & 0+1+(-3) & 2+3+0 \\ 2+(-2)+0 & 0+(-1)+0 & 1+(-3)+0 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

Now, we evaluate $-5A = -5 \times A$.

$$-5A = -5 \times \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow -5A = \begin{bmatrix} 2(-5) & 0(-5) & 1(-5) \\ 2(-5) & 1(-5) & 3(-5) \\ 1(-5) & -1(-5) & 0(-5) \end{bmatrix}$$

$$\therefore -5A = \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}$$

Finally, matrix $4I = 4 \times I$.

$$4I = 4 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 4I = \begin{bmatrix} 1(4) & 0(4) & 0(4) \\ 0(4) & 1(4) & 0(4) \\ 0(4) & 0(4) & 1(4) \end{bmatrix}$$

$$\therefore 4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

The given expression is $A^2 - 5A + 4I$.

$$\Rightarrow A^2 - 5A + 4I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\Rightarrow A^2 - 5A + 4I = \begin{bmatrix} 5+(-10)+4 & -1+0+0 & 2+(-5)+0 \\ 9+(-10)+0 & -2+(-5)+4 & 5+(-15)+0 \\ 0+(-5)+0 & -1+5+0 & -2+0+4 \end{bmatrix}$$

$\therefore A^2 - 5A + 4I = \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$

Given that $A^2 - 5A + 4I + X = O$

$\Rightarrow X = -(A^2 - 5A + 4I)$

$\Rightarrow X = -\begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$

$\therefore X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$

Thus, $A^2 - 5A + 4I = \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$ and $X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$

56. Question

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for all positive integers n.

Answer

Given $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

We need to prove that $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$.

We will prove this result using the principle of mathematical induction.

<u>Step 1</u>: When n = 1, we have $A^n = A^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = A$

Hence, the equation is true for n = 1.

<u>Step 2</u>: Let us assume the equation true for some n = k, where k is a positive integer.

$\Rightarrow A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

To prove the given equation using mathematical induction, we have to show that $A^{k+1} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$.

We know $A^{k+1} = A^k \times A$.

$\Rightarrow A^{k+1} = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^{k+1} = \begin{bmatrix} (1)(1) + (k)(0) & (1)(1) + (k)(1) \\ (0)(1) + (1)(0) & (0)(1) + (1)(1) \end{bmatrix}$

$\Rightarrow A^{k+1} = \begin{bmatrix} 1 & 1+k \\ 0 & 1 \end{bmatrix}$

$\therefore A^{k+1} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$

Hence, the equation is true for n = k + 1 under the assumption that it is true for n = k.

Therefore, by the principle of mathematical induction, the equation is true for all positive integer values of n.

Thus, $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for all positive integers n.

57. Question

If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} a^n & b\left(\dfrac{a^n - 1}{a - 1}\right) \\ 0 & 1 \end{bmatrix}$ for every positive integer n.

Answer

Given $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$.

We need to prove that $A^n = \begin{bmatrix} a^n & b\left(\frac{a^n - 1}{a - 1}\right) \\ 0 & 1 \end{bmatrix}$.

We will prove this result using the principle of mathematical induction.

<u>Step 1</u>: When n = 1, we have $A^n = A^1$

$$\Rightarrow A^N = \begin{bmatrix} a^1 & b\left(\dfrac{a^1 - 1}{a - 1}\right) \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^N = \begin{bmatrix} a & b\left(\dfrac{a - 1}{a - 1}\right) \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^N = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} = A$$

Hence, the equation is true for n = 1.

<u>Step 2</u>: Let us assume the equation true for some n = k, where k is a positive integer.

$$\Rightarrow A^k = \begin{bmatrix} a^k & b\left(\dfrac{a^k - 1}{a - 1}\right) \\ 0 & 1 \end{bmatrix}$$

To prove the given equation using mathematical induction, we have to show that $A^{k+1} = \begin{bmatrix} a^{k+1} & b\left(\frac{a^{k+1} - 1}{a - 1}\right) \\ 0 & 1 \end{bmatrix}$.

We know $A^{k+1} = A^k \times A$.

$$\Rightarrow A^{k+1} = \begin{bmatrix} a^k & b\left(\dfrac{a^k - 1}{a - 1}\right) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} (a^k)(a) + b\left(\dfrac{a^k - 1}{a - 1}\right)(0) & (a^k)(b) + b\left(\dfrac{a^k - 1}{a - 1}\right)(1) \\ (0)(a) + (1)(0) & (0)(b) + (1)(1) \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} a^{k+1} & a^k b + b\left(\dfrac{a^k - 1}{a - 1}\right) \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} a^{k+1} & b\left(a^k + \dfrac{a^k - 1}{a - 1}\right) \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} a^{k+1} & b\left(\dfrac{(a - 1)a^k + a^k - 1}{a - 1}\right) \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} a^{k+1} & b\left(\frac{a^{k+1} - a^k + a^k - 1}{a-1}\right) \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{k+1} = \begin{bmatrix} a^{k+1} & b\left(\frac{a^{k+1} - 1}{a-1}\right) \\ 0 & 1 \end{bmatrix}$$

Hence, the equation is true for n = k + 1 under the assumption that it is true for n = k.

Therefore, by the principle of mathematical induction, the equation is true for all positive integer values of n.

Thus, $A^n = \begin{bmatrix} a^n & b\left(\frac{a^n-1}{a-1}\right) \\ 0 & 1 \end{bmatrix}$ for every positive integer n.

58. Question

If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, then prove by principle of mathematical induction that $A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}$ for all n ∈ N.

Answer

Given $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$.

We need to prove that $A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}$ using the principle of mathematical induction.

Step 1: When n = 1, we have $A^n = A^1$

$$\Rightarrow A^N = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore A^N = A$$

Hence, the equation is true for n = 1.

Step 2: Let us assume the equation true for some n = k, where k is a positive integer.

$$\Rightarrow A^k = \begin{bmatrix} \cos k\theta & i\sin k\theta \\ i\sin k\theta & \cos k\theta \end{bmatrix}$$

To prove the given equation using mathematical induction, we have to show that $A^{k+1} = \begin{bmatrix} \cos(k+1)\theta & i\sin(k+1)\theta \\ i\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$.

We know $A^{k+1} = A^k \times A$.

$$\Rightarrow A^{k+1} = \begin{bmatrix} \cos k\theta & i\sin k\theta \\ i\sin k\theta & \cos k\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} (\cos k\theta)(\cos\theta) + (i\sin k\theta)(i\sin\theta) & (\cos k\theta)(i\sin\theta) + (i\sin k\theta)(\cos\theta) \\ (i\sin k\theta)(\cos\theta) + (\cos k\theta)(i\sin\theta) & (i\sin k\theta)(i\sin\theta) + (\cos k\theta)(\cos\theta) \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} \cos k\theta\cos\theta + i^2\sin k\theta\sin\theta & i\cos k\theta\sin\theta + i\sin k\theta\cos\theta \\ i\sin k\theta\cos\theta + i\cos k\theta\sin\theta & i^2\sin k\theta\sin\theta + \cos k\theta\cos\theta \end{bmatrix}$$

However, we have $i^2 = -1$

$$\Rightarrow A^{k+1} = \begin{bmatrix} \cos k\theta\cos\theta - \sin k\theta\sin\theta & i(\cos k\theta\sin\theta + \sin k\theta\cos\theta) \\ i(\sin k\theta\cos\theta + \cos k\theta\sin\theta) & -\sin k\theta\sin\theta + \cos k\theta\cos\theta \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} \cos k\theta \cos\theta - \sin k\theta \sin\theta & i(\sin k\theta \cos\theta + \cos k\theta \sin\theta) \\ i(\sin k\theta \cos\theta + \cos k\theta \sin\theta) & \cos k\theta \cos\theta - \sin k\theta \sin\theta \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} \cos(k\theta + \theta) & i\sin(k\theta + \theta) \\ i\sin(k\theta + \theta) & \cos(k\theta + \theta) \end{bmatrix}$$

$$\therefore A^{k+1} = \begin{bmatrix} \cos(k+1)\theta & i\sin(k+1)\theta \\ i\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Hence, the equation is true for n = k + 1 under the assumption that it is true for n = k.

Therefore, by the principle of mathematical induction, the equation is true for all positive integer values of n.

Thus, $A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}$ for all n ϵ N.

59. Question

If $A = \begin{bmatrix} \cos\alpha + \sin\alpha & \sqrt{2}\sin\alpha \\ -\sqrt{2}\sin\alpha & \cos\alpha - \sin\alpha \end{bmatrix}$, prove that $A^n = \begin{bmatrix} \cos n\alpha + \sin n\alpha & \sqrt{2}\sin n\alpha \\ -\sqrt{2}\sin n\alpha & \cos n\alpha - \sin n\alpha \end{bmatrix}$ for all $n \in N$.

Answer

Given $A = \begin{bmatrix} \cos\alpha + \sin\alpha & \sqrt{2}\sin\alpha \\ -\sqrt{2}\sin\alpha & \cos\alpha - \sin\alpha \end{bmatrix}$.

We need to prove that $A^n = \begin{bmatrix} \cos n\alpha + \sin n\alpha & \sqrt{2}\sin n\alpha \\ -\sqrt{2}\sin n\alpha & \cos n\alpha - \sin n\alpha \end{bmatrix}$.

We will prove this result using the principle of mathematical induction.

<u>Step 1</u>: When n = 1, we have $A^n = A^1$

$$\Rightarrow A^N = \begin{bmatrix} \cos\alpha + \sin\alpha & \sqrt{2}\sin\alpha \\ -\sqrt{2}\sin\alpha & \cos\alpha - \sin\alpha \end{bmatrix}$$

$$\therefore A^N = A$$

Hence, the equation is true for n = 1.

<u>Step 2</u>: Let us assume the equation true for some n = k, where k is a positive integer.

$$\Rightarrow A^k = \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2}\sin k\alpha \\ -\sqrt{2}\sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix}$$

To prove the given equation using mathematical induction, we have to show that
$A^{k+1} = \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2}\sin(k+1)\alpha \\ -\sqrt{2}\sin(k+1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix}$.

We know $A^{k+1} = A^k \times A$.

$$\Rightarrow A^{k+1} = \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2}\sin k\alpha \\ -\sqrt{2}\sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha + \sin\alpha & \sqrt{2}\sin\alpha \\ -\sqrt{2}\sin\alpha & \cos\alpha - \sin\alpha \end{bmatrix}$$

We evaluate each value of this matrix independently.

(a) The value at index (1, 1)

$$A_{11}^{k+1} = (\cos k\alpha + \sin k\alpha)(\cos\alpha + \sin\alpha) + \left(\sqrt{2}\sin k\alpha\right)\left(-\sqrt{2}\sin\alpha\right)$$

$$\Rightarrow A_{11}^{k+1} = \cos k\alpha \cos\alpha + \cos k\alpha \sin\alpha + \sin k\alpha \cos\alpha + \sin k\alpha \sin\alpha - 2\sin k\alpha \sin\alpha$$

$\Rightarrow A_{11}^{k+1} = \cos k\alpha \cos \alpha + \cos k\alpha \sin \alpha + \sin k\alpha \cos \alpha - \sin k\alpha \sin \alpha$

$\Rightarrow A_{11}^{k+1} = \cos k\alpha \cos \alpha - \sin k\alpha \sin \alpha + \sin k\alpha \cos \alpha + \cos k\alpha \sin \alpha$

$\Rightarrow A_{11}^{k+1} = \cos(k\alpha + \alpha) + \sin(k\alpha + \alpha)$

$\therefore A_{11}^{k+1} = \cos(k+1)\alpha + \sin(k+1)\alpha$

(b) The value at index (1, 2)

$A_{12}^{k+1} = (\cos k\alpha + \sin k\alpha)(\sqrt{2}\sin \alpha) + (\sqrt{2}\sin k\alpha)(\cos \alpha - \sin \alpha)$

$\Rightarrow A_{12}^{k+1} = \sqrt{2}\cos k\alpha \sin \alpha + \sqrt{2}\sin k\alpha \sin \alpha + \sqrt{2}\sin k\alpha \cos \alpha - \sqrt{2}\sin k\alpha \sin \alpha$

$\Rightarrow A_{12}^{k+1} = \sqrt{2}\cos k\alpha \sin \alpha + \sqrt{2}\sin k\alpha \cos \alpha$

$\Rightarrow A_{12}^{k+1} = \sqrt{2}(\cos k\alpha \sin \alpha + \sin k\alpha \cos \alpha)$

$\Rightarrow A_{12}^{k+1} = \sqrt{2}(\sin k\alpha \cos \alpha + \cos k\alpha \sin \alpha)$

$\Rightarrow A_{12}^{k+1} = \sqrt{2}\sin(k\alpha + \alpha)$

$\therefore A_{12}^{k+1} = \sqrt{2}\sin(k+1)\alpha$

(c) The value at index (2, 1)

$A_{21}^{k+1} = (-\sqrt{2}\sin k\alpha)(\cos \alpha + \sin \alpha) + (\cos k\alpha - \sin k\alpha)(-\sqrt{2}\sin \alpha)$

$\Rightarrow A_{21}^{k+1} = -\sqrt{2}\sin k\alpha \cos \alpha - \sqrt{2}\sin k\alpha \sin \alpha - \sqrt{2}\cos k\alpha \sin \alpha + \sqrt{2}\sin k\alpha \sin \alpha$

$\Rightarrow A_{21}^{k+1} = -\sqrt{2}\sin k\alpha \cos \alpha - \sqrt{2}\cos k\alpha \sin \alpha$

$\Rightarrow A_{21}^{k+1} = -\sqrt{2}(\sin k\alpha \cos \alpha + \cos k\alpha \sin \alpha)$

$\Rightarrow A_{21}^{k+1} = -\sqrt{2}\sin(k\alpha + \alpha)$

$\therefore A_{21}^{k+1} = -\sqrt{2}\sin(k+1)\alpha$

(d) The value at index (2, 2)

$A_{22}^{k+1} = (-\sqrt{2}\sin k\alpha)(\sqrt{2}\sin \alpha) + (\cos k\alpha - \sin k\alpha)(\cos \alpha - \sin \alpha)$

$$\Rightarrow A_{22}^{k+1} = -2\sin k\alpha \sin \alpha + \cos k\alpha \cos \alpha - \cos k\alpha \sin \alpha - \sin k\alpha \cos \alpha \\ + \sin k\alpha \sin \alpha$$

$\Rightarrow A_{22}^{k+1} = -\sin k\alpha \sin \alpha + \cos k\alpha \cos \alpha - \cos k\alpha \sin \alpha - \sin k\alpha \cos \alpha$

$\Rightarrow A_{22}^{k+1} = \cos k\alpha \cos \alpha - \sin k\alpha \sin \alpha - \sin k\alpha \cos \alpha - \cos k\alpha \sin \alpha$

$\Rightarrow A_{22}^{k+1} = \cos k\alpha \cos \alpha - \sin k\alpha \sin \alpha - (\sin k\alpha \cos \alpha + \cos k\alpha \sin \alpha)$

$\Rightarrow A_{22}^{k+1} = \cos(k\alpha + \alpha) - \sin(k\alpha + \alpha)$

$\therefore A_{22}^{k+1} = \cos(k+1)\alpha - \sin(k+1)\alpha$

So, the matrix A^{k+1} is

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2}\sin(k+1)\alpha \\ -\sqrt{2}\sin(k+1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix}$$

Hence, the equation is true for n = k + 1 under the assumption that it is true for n = k.

Therefore, by the principle of mathematical induction, the equation is true for all positive integer values of n.

Thus, $A^n = \begin{bmatrix} \cos n\alpha + \sin n\alpha & \sqrt{2}\sin n\alpha \\ -\sqrt{2}\sin n\alpha & \cos n\alpha - \sin n\alpha \end{bmatrix}$ for all n ϵ N.

60. Question

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, then use the principle of mathematical induction to show that $A^n = \begin{bmatrix} 1 & n & n(n+1)/2 \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$ for every positive integer n.

Answer

Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

We need to prove that $A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$ using the principle of mathematical induction.

Step 1: When n = 1, we have $A^n = A^1$

$$\Rightarrow A^N = \begin{bmatrix} 1 & 1 & \frac{1(1+1)}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^N = \begin{bmatrix} 1 & 1 & \frac{2}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A^N = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = A$$

Hence, the equation is true for n = 1.

Step 2: Let us assume the equation true for some n = k, where k is a positive integer.

$$\Rightarrow A^k = \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix}$$

To prove the given equation using mathematical induction, we have to show that

$$A^{k+1} = \begin{bmatrix} 1 & k+1 & \frac{(k+1)(k+1+1)}{2} \\ 0 & 1 & k+1 \\ 0 & 0 & 1 \end{bmatrix}.$$

We know $A^{k+1} = A^k \times A$.

$$\Rightarrow A^{k+1} = \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} 1+0+0 & 1+k+0 & 1+k+\frac{k(k+1)}{2} \\ 0+0+0 & 0+1+0 & 0+1+k \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} 1 & 1 & (k+1) + \frac{k(k+1)}{2} \\ 0 & 1 & 1+k \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} 1 & 1 & (k+1)\left(1+\frac{k}{2}\right) \\ 0 & 1 & k+1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} 1 & 1 & \frac{(k+1)(k+2)}{2} \\ 0 & 1 & k+1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{k+1} = \begin{bmatrix} 1 & k+1 & \frac{(k+1)(k+1+1)}{2} \\ 0 & 1 & k+1 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, the equation is true for n = k + 1 under the assumption that it is true for n = k.

Therefore, by the principle of mathematical induction, the equation is true for all positive integer values of n.

Thus, $A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$ for every positive integer n.

61. Question

If B, C are n rowed matrices and if A = B + C, BC = CB, C^2 = O, then show that for every n ∈ N, $A^{n+1} = B^n(B + (n + 1)C)$.

Answer

Given A = B + C, BC = CB and C^2 = O.

We need to prove that $A^{n+1} = B^n(B + (n + 1)C)$.

We will prove this result using the principle of mathematical induction.

<u>Step 1</u>: When n = 1, we have $A^{n+1} = A^{1+1}$

$\Rightarrow A^{n+1} = B^1(B + (1 + 1)C)$

$\therefore A^{n+1} = B(B + 2C)$

For the given equation to be true for n = 1, A^{n+1} must be equal to A^2.

It is given that A = B + C and we know $A^2 = A \times A$.

$\Rightarrow A^2 = (B + C)(B + C)$

$\Rightarrow A^2 = B(B + C) + C(B + C)$

$\Rightarrow A^2 = B^2 + BC + CB + C^2$

However, BC = CB and C^2 = O.

$\Rightarrow A^2 = B^2 + CB + CB + O$

$\Rightarrow A^2 = B^2 + 2CB$

$\therefore A^2 = B(B + 2C)$

Hence, $A^{n+1} = A^2$ and the equation is true for n = 1.

Step 2: Let us assume the equation true for some n = k, where k is a positive integer.

$\Rightarrow A^{k+1} = B^k(B + (k + 1)C)$

To prove the given equation using mathematical induction, we have to show that $A^{k+2} = B^{k+1}(B + (k + 2)C)$.

We know $A^{k+2} = A^{k+1} \times A$.

$\Rightarrow A^{k+2} = [B^k(B + (k + 1)C)](B + C)$

$\Rightarrow A^{k+2} = [B^{k+1} + (k + 1)B^kC)](B + C)$

$\Rightarrow A^{k+2} = B^{k+1}(B + C) + (k + 1)B^kC(B + C)$

$\Rightarrow A^{k+2} = B^{k+1}(B + C) + (k + 1)B^kCB + (k + 1)B^kC^2$

However, BC = CB and $C^2 = O$.

$\Rightarrow A^{k+2} = B^{k+1}(B + C) + (k + 1)B^kBC + (k + 1)B^kO$

$\Rightarrow A^{k+2} = B^{k+1}(B + C) + (k + 1)B^{k+1}C + O$

$\Rightarrow A^{k+2} = B^{k+1}(B + C) + B^{k+1}[(k + 1)C]$

$\Rightarrow A^{k+2} = B^{k+1}[(B + C) + (k + 1)C]$

$\Rightarrow A^{k+2} = B^{k+1}[B + (1 + k + 1)C]$

$\therefore A^{k+2} = B^{k+1}[B + (k + 2)C]$

Hence, the equation is true for n = k + 1 under the assumption that it is true for n = k.

Therefore, by the principle of mathematical induction, the equation is true for all positive integer values of n.

Thus, $A^{n+1} = B^n(B + (n + 1)C)$ for every $n \in N$.

62. Question

If A = diag(a b c), show that $A^n = \text{diag}(a^n\ b^n\ c^n)$ for all positive integers n.

Answer

Given $A = \text{diag}(a\,b\,c) = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.

We need to prove that $A^n = \text{diag}(a^n\ b^n\ c^n) = \begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}$.

We will prove this result using the principle of mathematical induction.

Step 1: When n = 1, we have $A^n = A^1$

$$\Rightarrow A^N = \begin{bmatrix} a^1 & 0 & 0 \\ 0 & b^1 & 0 \\ 0 & 0 & c^1 \end{bmatrix}$$

$$\therefore A^N = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = A$$

Hence, the equation is true for n = 1.

Step 2: Let us assume the equation true for some n = k, where k is a positive integer.

$$\Rightarrow A^k = \begin{bmatrix} a^k & 0 & 0 \\ 0 & b^k & 0 \\ 0 & 0 & c^k \end{bmatrix}$$

To prove the given equation using mathematical induction, we have to show that $A^{k+1} = \begin{bmatrix} a^{k+1} & 0 & 0 \\ 0 & b^{k+1} & 0 \\ 0 & 0 & c^{k+1} \end{bmatrix}$.

We know $A^{k+1} = A^k \times A$.

$$\Rightarrow A^{k+1} = \begin{bmatrix} a^k & 0 & 0 \\ 0 & b^k & 0 \\ 0 & 0 & c^k \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} a^k \times a + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + b^k \times b + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + c^k \times c \end{bmatrix}$$

$$\Rightarrow A^{k+1} = \begin{bmatrix} a^{k+1} & 0 & 0 \\ 0 & b^{k+1} & 0 \\ 0 & 0 & c^{k+1} \end{bmatrix}$$

$\therefore A^{k+1} = \text{diag}(a^{k+1}\ b^{k+1}\ c^{k+1})$

Hence, the equation is true for $n = k + 1$ under the assumption that it is true for $n = k$.

Therefore, by the principle of mathematical induction, the equation is true for all positive integer values of n.

Thus, $A^n = \text{diag}(a^n\ b^n\ c^n)$ for all positive integers n.

63. Question

If A is a square matrix, using mathematical induction prove that $(A^T)^n = (A^n)^T$ for all $n \in N$.

Answer

Given A is a square matrix.

We need to prove that $(A^T)^n = (A^n)^T$.

We will prove this result using the principle of mathematical induction.

<u>Step 1</u>: When n = 1, we have $(A^T)^1 = A^T$

$\therefore (A^T)^1 = (A^1)^T$

Hence, the equation is true for n = 1.

<u>Step 2</u>: Let us assume the equation true for some n = k, where k is a positive integer.

$\Rightarrow (A^T)^k = (A^k)^T$

To prove the given equation using mathematical induction, we have to show that $(A^T)^{k+1} = (A^{k+1})^T$.

We know $(A^T)^{k+1} = (A^T)^k \times A^T$.

$\Rightarrow (A^T)^{k+1} = (A^k)^T \times A^T$

We have $(AB)^T = B^T A^T$.

$\Rightarrow (A^T)^{k+1} = (AA^k)^T$

$\Rightarrow (A^T)^{k+1} = (A^{1+k})^T$

$\therefore (A^T)^{k+1} = (A^{k+1})^T$

Hence, the equation is true for $n = k + 1$ under the assumption that it is true for $n = k$.

Therefore, by the principle of mathematical induction, the equation is true for all positive integer values of n.

Thus, $(A^T)^n = (A^n)^T$ for all $n \in N$.

64. Question

A matrix X has a + b rows and a + 2 columns while the matrix Y has b + 1 rows and a + 3 columns. Both matrices XY and YX exist. Find a and b. Can you say XY and YX are of the same type? Are they equal?

Answer

X has a + b rows and a + 2 columns.

⇒ Order of X = (a + b) × (a + 2)

Y has b + 1 rows and a + 3 columns.

⇒ Order of Y = (b + 1) × (a + 3)

Recall that the product of two matrices A and B is defined only when the number of columns of A is equal to the number of rows of B.

It is given that the matrix XY exists.

⇒ Number of columns of X = Number of rows of Y

⇒ a + 2 = b + 1

∴ a = b – 1

The matrix YX also exists.

⇒ Number of columns of Y = Number of rows of X

⇒ a + 3 = a + b

∴ b = 3

We have a = b – 1

⇒ a = 3 – 1

∴ a = 2

Thus, a = 2 and b = 3.

Hence, order of X = 5 × 4 and order of Y = 4 × 5.

Order of XY = Number of rows of X × Number of columns of Y

⇒ Order of XY = 5 × 5

Order of YX = Number of rows of Y × Number of columns of X

⇒ Order of XY = 4 × 4

As the orders of the two matrices XY and YX are different, they are not of the same type and thus unequal.

65 A. Question

Give examples of matrices

A and B such that AB ≠ BA.

Answer

We need to find matrices A and B such that AB ≠ BA.

Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$.

First, we will find AB.

$$AB = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} (1)(1) + (2)(2) & (1)(0) + (2)(1) \\ (0)(1) + (3)(2) & (0)(0) + (3)(1) \end{bmatrix}$$

$\Rightarrow AB = \begin{bmatrix} 1+4 & 0+2 \\ 0+6 & 0+3 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 5 & 2 \\ 6 & 3 \end{bmatrix}$

Now, we will find BA.

$BA = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow BA = \begin{bmatrix} (1)(1)+(0)(0) & (1)(2)+(0)(3) \\ (2)(1)+(1)(0) & (2)(2)+(1)(3) \end{bmatrix}$

$\Rightarrow BA = \begin{bmatrix} 1+0 & 2+0 \\ 2+0 & 4+3 \end{bmatrix}$

$\therefore BA = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$

Thus, AB ≠ BA when $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$.

65 B. Question

Give examples of matrices

A and B such that AB = O but A ≠ O, B ≠ O.

Answer

We need to find matrices A and B such that AB = O but A ≠ O, B ≠ O.

Let $A = \begin{bmatrix} 1 & 0 \\ 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$.

Now, we will find AB.

$AB = \begin{bmatrix} 1 & 0 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow AB = \begin{bmatrix} (1)(0)+(0)(2) & (1)(0)+(0)(1) \\ (4)(0)+(0)(2) & (4)(0)+(0)(1) \end{bmatrix}$

$\Rightarrow AB = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Thus, AB ≠ O when $A = \begin{bmatrix} 1 & 0 \\ 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$.

65 C. Question

Give examples of matrices

A and B such that AB = O but BA ≠ O.

Answer

We need to find matrices A and B such that AB = O but BA ≠ O.

Let $A = \begin{bmatrix} -1 & 0 \\ 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix}$.

First, we will find AB.

$AB = \begin{bmatrix} -1 & 0 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix}$

$$\Rightarrow AB = \begin{bmatrix} (-1)(0)+(0)(3) & (-1)(0)+(0)(1) \\ (4)(0)+(0)(3) & (4)(0)+(0)(1) \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Now, we will find BA.

$$BA = \begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} (0)(-1)+(0)(3) & (0)(0)+(0)(0) \\ (3)(-1)+(1)(4) & (3)(0)+(1)(0) \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 0+0 & 0+0 \\ -3+4 & 0+0 \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \neq O$$

Thus, AB = O and BA ≠ O when $A = \begin{bmatrix} -1 & 0 \\ 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix}$.

65 D. Question

Give examples of matrices

A, B and C such that AB = AC but B ≠ C, A ≠ O.

Answer

We need to find matrices A, B and C such that AB = AC but B ≠ C, A ≠ O.

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

First, we will find AB.

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} (1)(1)+(0)(1) & (1)(1)+(0)(1) \\ (0)(1)+(0)(1) & (0)(1)+(0)(1) \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 1+0 & 1+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Now, we will find AC.

$$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow AC = \begin{bmatrix} (1)(1)+(0)(1) & (1)(1)+(0)(0) \\ (0)(1)+(0)(1) & (0)(1)+(0)(0) \end{bmatrix}$$

$$\Rightarrow AC = \begin{bmatrix} 1+0 & 1+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$\therefore AC = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Thus, AB = AC but B ≠ C, A ≠ O when $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

66. Question

Let A and B be square matrices of the same order. Does $(A + B)^2 = A^2 + 2AB + B^2$ hold? If not, why?

Answer

Given that A and B are square matrices of the same order.

We need to check if $(A + B)^2 = A^2 + 2AB + B^2$.

We know $(A + B)^2 = (A + B)(A + B)$

$\Rightarrow (A + B)^2 = A(A + B) + B(A + B)$

$\therefore (A + B)^2 = A^2 + AB + BA + B^2$

For the equation $(A + B)^2 = A^2 + 2AB + B^2$ to hold, we need AB = BA that is the matrices A and B must satisfy the commutative property for multiplication.

However, here it is not mentioned that AB = BA.

Therefore, $AB \neq BA$.

Thus, $(A + B)^2 \neq A^2 + 2AB + B^2$.

67. Question

If A and B are square matrices of the same order, explain, why in general

(i) $(A + B)^2 \neq A^2 + 2AB + B^2$

(ii) $(A - B)^2 \neq A^2 - 2AB + B^2$

(iii) $(A + B)(A - B) = A^2 - B^2$

Answer

(i) Given that A and B are square matrices of the same order.

We know $(A + B)^2 = (A + B)(A + B)$

$\Rightarrow (A + B)^2 = A(A + B) + B(A + B)$

$\therefore (A + B)^2 = A^2 + AB + BA + B^2$

For the equation $(A + B)^2 = A^2 + 2AB + B^2$ to be valid, we need AB = BA.

As the multiplication of two matrices does not satisfy the commutative property in general, $AB \neq BA$.

Thus, $(A + B)^2 \neq A^2 + 2AB + B^2$.

(ii) Given that A and B are square matrices of the same order.

We know $(A - B)^2 = (A - B)(A - B)$

$\Rightarrow (A - B)^2 = A(A - B) - B(A - B)$

$\therefore (A - B)^2 = A^2 - AB - BA + B^2$

For the equation $(A - B)^2 = A^2 - 2AB + B^2$ to be valid, we need AB = BA.

As the multiplication of two matrices does not satisfy the commutative property in general, $AB \neq BA$.

Thus, $(A - B)^2 \neq A^2 - 2AB + B^2$.

(iii) Given that A and B are square matrices of the same order.

We have $(A + B)(A - B) = A(A - B) + B(A - B)$

$\therefore (A + B)(A - B) = A^2 - AB + BA - B^2$

For the equation $(A + B)(A - B) = A^2 - B^2$ to be valid, we need $AB = BA$.

As the multiplication of two matrices does not satisfy the commutative property in general, $AB \neq BA$.

Thus, $(A + B)(A - B) \neq A^2 - B^2$.

68. Question

Let A and B be square matrices of the order 3 × 3. Is $(AB)^2 = A^2B^2$? Give reasons.

Answer

Given that A and B are square matrices of the order 3 × 3.

We know $(AB)^2 = (AB)(AB)$

$\Rightarrow (AB)^2 = A \times B \times A \times B$

$\Rightarrow (AB)^2 = A(BA)B$

If the matrices A and B satisfy the commutative property for multiplication, then $AB = BA$.

We found $(AB)^2 = A(BA)B$.

Hence, when $AB = BA$, we have $(AB)^2 = A(AB)B$.

$\Rightarrow (AB)^2 = A \times A \times B \times B$

$\Rightarrow (AB)^2 = A^2B^2$

Therefore, $(AB)^2 = A^2B^2$ holds only when $AB = BA$.

Thus, $(AB)^2 \neq A^2B^2$ unless the matrices A and B satisfy the commutative property for multiplication.

69. Question

If A and B are square matrices of the same order such that $AB = BA$, then show that $(A + B)^2 = A^2 + 2AB + B^2$.

Answer

Given that A and B are square matrices of the same order such that $AB = BA$.

We need to prove that $(A + B)^2 = A^2 + 2AB + B^2$.

We know $(A + B)^2 = (A + B)(A + B)$

$\Rightarrow (A + B)^2 = A(A + B) + B(A + B)$

$\Rightarrow (A + B)^2 = A^2 + AB + BA + B^2$

However, here it is mentioned that $AB = BA$.

$\Rightarrow (A + B)^2 = A^2 + AB + AB + B^2$

$\therefore (A + B)^2 = A^2 + 2AB + B^2$

Thus, $(A + B)^2 = A^2 + 2AB + B^2$ when $AB = BA$.

70. Question

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix}$.

Verify that $AB = AC$ though $B \neq C$, $A \neq 0$.

Answer

Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix}$.

We need to verify that AB = AC.

Let us evaluate the LHS and the RHS one at a time.

We will first calculate AB.

$$AB = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} (1)(3) + (1)(5) + (1)(-2) & (1)(1) + (1)(2) + (1)(4) \\ (3)(3) + (3)(5) + (3)(-2) & (3)(1) + (3)(2) + (3)(4) \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 3 + 5 + (-2) & 1 + 2 + 4 \\ 9 + 15 + (-6) & 3 + 6 + 12 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 6 & 7 \\ 18 & 21 \end{bmatrix}$$

Now, the RHS is AC.

$$AC = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix}$$

$$\Rightarrow AC = \begin{bmatrix} (1)(4) + (1)(-3) + (1)(5) & (1)(2) + (1)(5) + (1)(0) \\ (3)(4) + (3)(-3) + (3)(5) & (3)(2) + (3)(5) + (3)(0) \end{bmatrix}$$

$$\Rightarrow AC = \begin{bmatrix} 4 + (-3) + 5 & 2 + 5 + 0 \\ 12 + (-9) + 15 & 6 + 15 + 0 \end{bmatrix}$$

$$\therefore AC = \begin{bmatrix} 6 & 7 \\ 18 & 21 \end{bmatrix}$$

Thus, AB = AC even though B ≠ C and A ≠ O.

71. Question

Three shopkeepers, A, B and C go to a store to buy stationary. A purchases 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs 40 paise, a pen costs Rs 1.25 and a pencil costs 35 paise. Use matrix multiplication to calculate each individual's bill.

Answer

Given the purchase details of three shopkeepers A, B and C.

A: 12 dozen notebooks, 5 dozen pens and 6 dozen pencils

B: 10 dozen notebooks, 6 dozen pens and 7 dozen pencils

C: 11 dozen notebooks, 13 dozen pens and 8 dozen pencils

Hence, the items purchased by A, B and C can be represented in matrix form with rows denoting the shopkeepers and columns denoting the number of dozens of items as –

$$X = \begin{bmatrix} 12 & 5 & 6 \\ 10 & 6 & 7 \\ 11 & 13 & 8 \end{bmatrix}$$

The price of each of the items is also given.

Cost of one notebook = 40 paise

⇒ Cost of one dozen notebooks = 12 × 40 paise

⇒ Cost of one dozen notebooks = 480 paise

∴ Cost of one dozen notebooks = Rs 4.80

Cost of one pen = Rs 1.25

⇒ Cost of one dozen pens = 12 × Rs 1.25

∴ Cost of one dozen pens = Rs 15

Cost of one pencil = 35 paise

⇒ Cost of one dozen notebooks = 12 × 35 paise

⇒ Cost of one dozen notebooks = 420 paise

∴ Cost of one dozen notebooks = Rs 4.20

Hence, the cost of purchasing one dozen of the items can be represented in matrix form with each row corresponding to an item as –

$$Y = \begin{bmatrix} 4.80 \\ 15 \\ 4.20 \end{bmatrix}$$

Now, the individual bill for each shopkeeper can be found by taking the product of the matrices X and Y.

$$XY = \begin{bmatrix} 12 & 5 & 6 \\ 10 & 6 & 7 \\ 11 & 13 & 8 \end{bmatrix} \begin{bmatrix} 4.80 \\ 15 \\ 4.20 \end{bmatrix}$$

$$\Rightarrow XY = \begin{bmatrix} 12 \times 4.80 + 5 \times 15 + 6 \times 4.20 \\ 10 \times 4.80 + 6 \times 15 + 7 \times 4.20 \\ 11 \times 4.80 + 13 \times 15 + 8 \times 4.20 \end{bmatrix}$$

$$\Rightarrow XY = \begin{bmatrix} 57.60 + 75 + 25.20 \\ 48 + 90 + 29.40 \\ 52.80 + 195 + 33.60 \end{bmatrix}$$

$$\therefore XY = \begin{bmatrix} 157.80 \\ 167.40 \\ 281.40 \end{bmatrix}$$

Thus, the bills of shopkeepers A, B and C are Rs 157.80, Rs 167.40 and Rs 281.40 respectively.

72. Question

The cooperative stores of a particular school has 10 dozen physics books, 8 dozen chemistry books and 5 dozen mathematics books. Their selling prices are Rs 8.30, Rs 3.45 and Rs 4.50 each respectively. Find the total amount the store will receive from selling all the items.

Answer

Given the details of stock of various types of books.

Physics: 10 dozen books

Chemistry: 8 dozen books

Mathematics: 5 dozen books

Hence, the number of dozens of books available in the store can be represented in matrix form with each column corresponding to a different subject as -

$$X = \begin{bmatrix} 10 & 8 & 5 \end{bmatrix}$$

The price of each of the items is also given.

Cost of one physics book = Rs 8.30

⇒ Cost of one dozen physics books = 12 × Rs 8.30

∴ Cost of one dozen physics books = Rs 99.60

Cost of one chemistry book = Rs 3.45

⇒ Cost of one dozen chemistry books = 12 × Rs 3.45

∴ Cost of one dozen chemistry books = Rs 41.40

Cost of one mathematics book = Rs 4.50

⇒ Cost of one dozen mathematics books = 12 × Rs 4.50

∴ Cost of one dozen mathematics books = Rs 54

Hence, the cost of purchasing a dozen books of each subject can be represented in matrix form with each row corresponding to a different subject as –

$$Y = \begin{bmatrix} 99.60 \\ 41.40 \\ 54 \end{bmatrix}$$

Now, the amount received by the store upon selling all the available books can be found by taking the product of the matrices X and Y.

$$XY = \begin{bmatrix} 10 & 8 & 5 \end{bmatrix} \begin{bmatrix} 99.60 \\ 41.40 \\ 54 \end{bmatrix}$$

⇒ XY = [10 × 99.60 + 8 × 41.40 + 5 × 54]

⇒ XY = [996 + 331.20 + 270]

∴ XY = [1597.20]

Thus, the total amount the store will receive from selling all the items is Rs 1597.20.

73. Question

In a legislative assembly election, a political group hired a public relations firm to promote its candidates in three ways; telephone, house calls and letters. The cost per contact (in paise) is given matrix A as

$$\begin{array}{cc} & \text{Cost per contact} \\ A = & \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Telephone} \\ \text{House call} \\ \text{Letter} \end{array}$$

The number of contacts of each type made in two cities X and Y is given in matrix B as

$$\begin{array}{cc} & \begin{array}{ccc} \text{Telephone} & \text{Housecall} & \text{Letter} \end{array} \\ B = & \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 1000 \end{bmatrix} \end{array} \begin{array}{l} \\ \to X \\ \to Y \end{array}$$

Find the total amount spent by the group in the two cities X and Y.

Answer

Given matrix A containing the costs per contact in paisa for different types of contacting the public.

$$A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix}$$

Matrix B contains the number of contacts of each type made in two cities X and Y.

$$B = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}$$

Now, the total amount spent by the political group in the two cities for contacting the public can be obtained by taking the product of the matrices B and A.

$$BA = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}\begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1000 \times 40 + 500 \times 100 + 5000 \times 50 \\ 3000 \times 40 + 1000 \times 100 + 10000 \times 50 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 40000 + 50000 + 250000 \\ 120000 + 100000 + 500000 \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} 340000 \\ 720000 \end{bmatrix}$$

Amount spent in City X = 340000 paisa = Rs 3400

Amount spent in City Y = 720000 paisa = Rs 7200

Total amount spent = Rs (3400 + 7200) = Rs 10600

Thus, the total amount spent by the party in both the cities is Rs 10600 with Rs 3400 spent in city X and Rs 7200 spent in city Y.

74. Question

A trust fund has Rs 30000 that must be invested in two different types of bonds. The first bond pays 5% interest per year and the second bond pays 7% per year. Using matrix multiplication, determine how to divide Rs 30000 among the two types of bonds if the trust fund must obtain an annual total interest of (i) Rs 1800 and (ii) Rs 2000.

Answer

Given that Rs 30000 must be invested into two types of bonds with 5% and 7% interest rates.

Let Rs x be invested in bonds of the first type.

Thus, Rs (30000 – x) will be invested in the other type.

Hence, the amount invested in each type of the bonds can be represented in matrix form with each column corresponding to a different type of bond as -

$$X = \begin{bmatrix} x & 30000 - x \end{bmatrix}$$

(i) Annual interest obtained is Rs 1800.

We know the formula to calculate the interest on a principal of Rs P at a rate R% per annum for t years is given by,

$$\text{Interest} = \frac{PTR}{100}$$

Here, the time is one year and thus T = 1.

Hence, the interest obtained after one year can be expressed in matrix representation as -

$$\begin{bmatrix} x & 30000 - x \end{bmatrix}\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = [1800]$$

$$\Rightarrow \left[x \times \frac{5}{100} + (30000 - x) \times \frac{7}{100}\right] = [1800]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 1800$$

⇒ 5x + 7(30000 – x) = 1800 × 100

⇒ 5x + 210000 – 7x = 180000

⇒ –2x = 180000 – 210000

⇒ –2x = –30000

∴ x = 15000

Amount invested in the first bond = x = Rs 15000

Amount invested in the second bond = 30000 – x

⇒ Amount invested in the second bond = 30000 – 15000

∴ Amount invested in the second bond = Rs 15000

Thus, the trust has to invest Rs 15000 each in both the bonds in order to obtain an annual interest of Rs 1800.

(ii) Annual interest obtained is Rs 2000.

As in the previous case, the interest obtained after one year can be expressed in matrix representation as -

$$[\text{x} \quad 30000-\text{x}]\begin{bmatrix}\frac{5}{100}\\ \frac{7}{100}\end{bmatrix}=[2000]$$

$$\Rightarrow \left[\text{x}\times\frac{5}{100}+(30000-\text{x})\times\frac{7}{100}\right]=[2000]$$

$$\Rightarrow \frac{5\text{x}}{100}+\frac{7(30000-\text{x})}{100}=2000$$

⇒ 5x + 7(30000 – x) = 2000 × 100

⇒ 5x + 210000 – 7x = 200000

⇒ –2x = 200000 – 210000

⇒ –2x = –10000

∴ x = 5000

Amount invested in the first bond = x = Rs 5000

Amount invested in the second bond = 30000 – x

⇒ Amount invested in the second bond = 30000 – 5000

∴ Amount invested in the second bond = Rs 25000

Thus, the trust has to invest Rs 5000 in the first bond and Rs 25000 in the second bond in order to obtain an annual interest of Rs 2000.

75. Question

75. To promote making of toilets for women, an organization tried to generate awareness through (i) house calls (ii) letters, and (iii) announcements. The cost for each mode per attempt is given below :

(i) ₹ 50

(ii) ₹ 20

(iii) ₹ 40

The number of attempts made in three villages X, Y and Z are given below :

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organization for three villages separately, using matrices.

Answer

Given costs in Rs for making different types of attempts.

Cost for one house call = Rs 50

Cost for one letter = Rs 20

Cost for one announcement = Rs 40

Hence, the costs per contact in Rs for different types of contacting the people can be expressed in matrix form as -

$$A = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

Let matrix B contain the number of attempts of each type made in the three villages X, Y and Z.

From the given information, the number of attempts made can be expressed in the matrix form as -

$$B = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}$$

Now, the total cost incurred by the organization in the three villages for creating awareness can be obtained by taking the product of the matrices B and A.

$$BA = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 400 \times 50 + 300 \times 20 + 100 \times 40 \\ 300 \times 50 + 250 \times 20 + 75 \times 40 \\ 500 \times 50 + 400 \times 20 + 150 \times 40 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 20000 + 6000 + 4000 \\ 15000 + 5000 + 3000 \\ 25000 + 8000 + 6000 \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$$

Cost incurred in village X = Rs 30000

Cost incurred in village Y = Rs 23000

Cost incurred in village Z = Rs 39000

Thus, the cost incurred by the organization in villages X, Y and Z is Rs 30000, Rs 23000 and Rs 39000 respectively.

76. Question

There are 2 families A and B. There are 4 men, 6 women and 2 children in family A, and 2 men, 2 women and 4 children in family B. The recommended daily amount of calories is 2400 for men, 1990 for women, 1800 for children and 45 grams of proteins for men, 55 grams for women and 33 grams for children. Represent the above information using a matrix. Using matrix multiplication, calculate the total requirement of calories and proteins for each of the two families. What awareness can you create among people about the planned diet from this question?

Answer

Given the details of two families A and B.

A: 4 men, 6 women and 2 children

B: 2 men, 2 women and 4 children

Hence, the number of people in both the families A and B can be represented in matrix form with rows denoting the family and columns denoting the number of people of each type as -

$$X = \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

The calorie and protein requirements for different types of people are also given.

Men: 2400 calories, 45gm proteins

Women: 1900 calories, 55gm proteins

Children: 1800 calories, 33gm proteins

Hence, the required calories and proteins can be represented in matrix form with each row corresponding to different type of people as –

$$Y = \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

Now, the required number of calories and proteins for each of the two families can be obtained by taking the product of the matrices X and Y.

$$XY = \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$\Rightarrow XY = \begin{bmatrix} 4 \times 2400 + 6 \times 1900 + 2 \times 1800 & 4 \times 45 + 6 \times 55 + 2 \times 33 \\ 2 \times 2400 + 2 \times 1900 + 4 \times 1800 & 2 \times 45 + 2 \times 55 + 4 \times 33 \end{bmatrix}$$

$$\Rightarrow XY = \begin{bmatrix} 9600 + 11400 + 3600 & 180 + 330 + 66 \\ 4800 + 3800 + 7200 & 90 + 110 + 132 \end{bmatrix}$$

$$\therefore XY = \begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix}$$

Thus, the requirement of calories and protein is as follows –

Family A: 24600 calories and 576 grams protein

Family B: 15800 calories and 332 grams protein

It can be said that a balanced diet with proper amounts of calories and protein must be consumed by the people of all ages in order to lead a healthy life.

77. Question

In a parliament election, a political party hired a public relations firm to promote its candidates in three ways – telephone, house calls and letters. The cost per contact (in paisa) is given in matrix A as

$$A = \begin{array}{c} \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix} \end{array} \begin{array}{l} \text{Telephone} \\ \text{House call} \\ \text{Letter} \end{array}$$

The number of contacts of each type made in two cities X and Y is given in matrix B as

$$\begin{array}{cc} & \begin{array}{ccc} \text{Telephone} & \text{Housecall} & \text{Letter} \end{array} \\ B = & \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 1000 \end{bmatrix} \begin{array}{l} \text{City X} \\ \text{City Y} \end{array} \end{array}$$

Find the total amount spent by the party in the two cities.

What should one consider before casting his/her vote – party's promotional activity or their social activities?

Answer

Given matrix A contains the costs per contact in paisa for different types of contacting the public.

$$A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix}$$

Matrix B contains the number of contacts of each type made in two cities X and Y.

$$B = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}$$

Now, the total amount spent by the political party in the two cities for contacting the public can be obtained by taking the product of the matrices B and A.

$$BA = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}\begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1000 \times 140 + 500 \times 200 + 5000 \times 150 \\ 3000 \times 140 + 1000 \times 200 + 10000 \times 150 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 140000 + 100000 + 750000 \\ 420000 + 200000 + 1500000 \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} 990000 \\ 2120000 \end{bmatrix}$$

Amount spent in City X = 990000 paisa = Rs 9900

Amount spent in City Y = 2120000 paisa = Rs 21200

Total amount spent = Rs (9900 + 21200) = Rs 31110

Thus, the total amount spent by the party in both the cities is Rs 31110 with Rs 9990 spent in city X and Rs 21200 spent in city Y.

One must surely consider the party's social activities instead of their promotional activities before casting his/her vote.

78. Question

The monthly incomes of Aryan and Babbar are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. If each saves Rs 15000 per month, find their monthly incomes using matrix method. This problem reflects which value?

Answer

Let the monthly incomes of Aryan and Babbar be 3x and 4x respectively.

Let their monthly expenditures be 5y and 7y respectively.

Given that both of them save Rs 15000 per month.

We know that the savings is the difference between the income and the expenditure.

Thus, we have two equations –

3x – 5y = 15000

4x – 7y = 15000

Recall that the solution to the system of equations that can be written in the form AX = B is given by $X = A^{-1}B$.

Here, $A = \begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$

We know the inverse of a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by

$$A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

|A| = (3)(–7) – (4)(–5) = –21 + 20 = –1

$$\Rightarrow A^{-1} = \frac{1}{-1}\begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}$$

We have X = A^{-1}B.

$$\Rightarrow X = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} (7)(15000) + (-5)(15000) \\ (4)(15000) + (-3)(15000) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} (7-5)15000 \\ (4-3)15000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \times 15000 \\ 1 \times 15000 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30000 \\ 15000 \end{bmatrix}$$

Monthly income of Aryan = 3x = 3 × Rs 30000 = Rs 90000

Monthly income of Babbar = 4x = 4 × Rs 30000 = Rs 120000

Thus, the monthly incomes of Aryan and Babbar are Rs 90000 and Rs 120000 respectively.

This problem tells us that savings are important and our income must not be spent wastefully.

79. Question

A trust invested some money in two types of bonds. The first bond pays 10% interest and the second bond pays 12%. The trust received Rs 2800 as interest. However, if trust had interchanged money in bonds, they would have got Rs 100 less as interest. Using matrix method, find the amount invested by the trust.

Answer

Given that some amount is invested into two types of bonds with 10% and 12% interest rates.

Let the amount invested in bonds of the first type and the second type be Rs x and Rs y respectively.

Hence, the amount invested in each type of the bonds can be represented in matrix form with each column corresponding to a different type of bond as -

$$X = \begin{bmatrix} x & y \end{bmatrix}$$

The annual interest obtained is Rs 2800.

We know the formula to calculate the interest on a principal of Rs P at a rate R% per annum for t years is given by,

$$\text{Interest} = \frac{PTR}{100}$$

Here, the time is one year and thus T = 1.

Hence, the interest obtained after one year can be expressed in matrix representation as -

$$[x \quad y]\begin{bmatrix}\frac{10}{100}\\ \frac{12}{100}\end{bmatrix}=[2800]$$

$$\Rightarrow \left[x\times\frac{10}{100}+y\times\frac{12}{100}\right]=[2800]$$

$$\Rightarrow \frac{10x}{100}+\frac{12y}{100}=2800$$

⇒ 10x + 12y = 2800 × 100

⇒ 10x + 12y = 280000

∴ 5x + 6y = 140000 …… (1)

However, on reversing the invested amounts, the interest received is Rs 100 less than the earlier value (Rs 2800).

Now, the amount invested in the second bond is Rs x and that in the first bond is Rs y with the annual interest obtained being Rs 2700.

Hence, the interest obtained by exchanging the invested amount of the two bonds after one year can be expressed in matrix representation as –

$$[y \quad x]\begin{bmatrix}\frac{10}{100}\\ \frac{12}{100}\end{bmatrix}=[2700]$$

$$\Rightarrow \left[y\times\frac{10}{100}+x\times\frac{12}{100}\right]=[2700]$$

$$\Rightarrow \frac{10y}{100}+\frac{12x}{100}=2700$$

⇒ 10y + 12x = 2700 × 100

⇒ 12x + 10y = 270000

∴ 6x + 5y = 135000 …… (2)

Recall that the solution to the system of equations that can be written in the form AX = B is given by X = $A^{-1}B$.

Here, $A=\begin{bmatrix}5 & 6\\ 6 & 5\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $B=\begin{bmatrix}140000\\ 135000\end{bmatrix}$

We know the inverse of a matrix $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is given by

$$A^{-1}=\frac{1}{|A|}\text{adj}(A)=\frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

|A| = (5)(5) – (6)(6) = 25 – 36 = –11

$$\Rightarrow A^{-1}=\frac{1}{-11}\begin{bmatrix}5 & -6\\ -6 & 5\end{bmatrix}$$

We have X = $A^{-1}B$.

$$\Rightarrow X=\frac{1}{-11}\begin{bmatrix}5 & -6\\ -6 & 5\end{bmatrix}\begin{bmatrix}140000\\ 135000\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-11}\begin{bmatrix}(5)(140000)+(-6)(135000)\\ (-6)(140000)+(5)(135000)\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} 700000 - 810000 \\ -840000 + 675000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} -110000 \\ -165000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-110000}{-11} \\ \frac{-165000}{-11} \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10000 \\ 15000 \end{bmatrix}$$

Amount invested in the first bond = x = Rs 10000

Amount invested in the second bond = y = Rs 15000

Thus, the trust invested Rs 10000 in the first bond and Rs 15000 in the second bond.

Exercise 5.4

1 A. Question

Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that

$(2A)^T = 2A^T$

Answer

Given,

$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$

$(2A)^T = 2A^T$

Put the value of A

$$\Rightarrow \left(2\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}\right)^T = 2\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 4 & -6 \\ -14 & 10 \end{bmatrix}^T = 2\begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix} = \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix}$$

L.H.S = R.H.S

Hence verified.

1 B. Question

Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that

$(A + B)^T = A^T + B^T$

Answer

$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$

$(A + B)^T = A^T + B^T$

$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$

$\Rightarrow \begin{bmatrix} 2+1 & -3+0 \\ -7+2 & 5-4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3 & -3 \\ -5 & 1 \end{bmatrix}^T = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$

L.H.S = R.H.S

Hence proved.

1 C. Question

Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that

$(A - B)^T = A^T - B^T$

Answer

$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$

$(A - B)^T = A^T - B^T$

$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$

$\Rightarrow \begin{bmatrix} 2-1 & -3-0 \\ -7-2 & 5+4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -3 \\ -9 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$

L.H.S = R.H.S

1 D. Question

Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that

$(AB)^T = B^T A^T$

Answer

$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$

$(AB)^T = B^T A^T$

$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$

$\begin{bmatrix} 2-6 & 0+12 \\ -7+10 & 0-20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$

$\begin{bmatrix} -4 & 12 \\ 3 & -20 \end{bmatrix}^T = \begin{bmatrix} 2-6 & -7+10 \\ 0+12 & 0-20 \end{bmatrix}$

$$\begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix}$$

So, $(AB)^T = B^T A^T$

2. Question

If $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and B = [1 0 4], verify that $(AB)^T = B^T A^T$.

Answer

Given,

$$A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}^T \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & 12 \\ 5 & 0 & 20 \\ 2 & 0 & 8 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} \begin{bmatrix} 3 & 5 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$$

L.H.S = R.H.S

So, $(AB)^T = B^T A^T$

3 A. Question

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$. Find A^T, B^T and verify that

$(A + B)^T = A^T + B^T$

Answer

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$(A + B)^T = A^T + B^T$$

$$\left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T$$

$$\left(\begin{bmatrix} 1+1 & -1+2 & 0+3 \\ 2+2 & 1+1 & 3+3 \\ 1+0 & 2+1 & 1+1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 1+1 & 2+2 & 1+0 \\ -1+2 & 1+1 & 2+1 \\ 0+3 & 3+3 & 1+1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix}$$

L.H.S = R.H.S

So, $(A+B)^T = A^T + B^T$

3 B. Question

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$. Find A^T, B^T and verify that

$(AB)^T = B^T A^T$

Answer

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$(AB)^T = B^T A^T$

$$\left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$\begin{bmatrix} 1-2+0 & 2-1+0 & 3-3+0 \\ 2+2+0 & 4+1+3 & 6+3+3 \\ 1+4+0 & 2+2+1 & 3+6+1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 4 & 8 & 12 \\ 5 & 5 & 10 \end{bmatrix}^T = \begin{bmatrix} 1-2+0 & 2+2+0 & 1+4+0 \\ 2-1+0 & 4+1+3 & 2+2+1 \\ 3-3+0 & 6+3+3 & 3+6+1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix}$$

L.H.S = R.H.S

So, $(AB)^T = B^T A^T$

3 C. Question

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$. Find A^T, B^T and verify that

$(2A)^T = 2A^T$

Answer

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$(2A)^T = 2A^T$

$$\Rightarrow \left(2\begin{bmatrix}1 & -1 & 0\\2 & 1 & 3\\1 & 2 & 1\end{bmatrix}\right)^T = 2\begin{bmatrix}1 & -1 & 0\\2 & 1 & 3\\1 & 2 & 1\end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix}2 & -2 & 0\\4 & 2 & 6\\2 & 4 & 2\end{bmatrix}^T = 2\begin{bmatrix}1 & 2 & 1\\-1 & 1 & 2\\0 & 3 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2 & 4 & 2\\-2 & 2 & 4\\0 & 6 & 2\end{bmatrix} = \begin{bmatrix}2 & 4 & 2\\-2 & 2 & 4\\0 & 6 & 2\end{bmatrix}$$

L.H.S = R.H.S

So,

$(2A)^T = 2A^T$

4. Question

If $A = \begin{bmatrix}-2\\4\\5\end{bmatrix}$, B= [1 3 -6], verify that $(AB)^T = B^T A^T$.

Answer

Given,

$A = \begin{bmatrix}-2\\4\\5\end{bmatrix}, B = \begin{bmatrix}1 & 3 & -6\end{bmatrix}$

$(AB)^T = B^T A^T$

$$\Rightarrow \left(\begin{bmatrix}-2\\4\\5\end{bmatrix}\begin{bmatrix}1 & 3 & -6\end{bmatrix}\right)^T = \begin{bmatrix}1 & 3 & -6\end{bmatrix}^T \begin{bmatrix}-2\\4\\5\end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix}-2 & -6 & -12\\4 & 12 & -24\\5 & 15 & -30\end{bmatrix}^T = \begin{bmatrix}1\\3\\-6\end{bmatrix}\begin{bmatrix}-2 & 4 & 5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}-2 & 4 & 5\\-6 & 12 & 15\\-12 & -24 & -30\end{bmatrix} = \begin{bmatrix}-2 & 4 & 5\\-6 & 12 & 15\\-12 & -24 & -30\end{bmatrix}$$

L.H.S = R.H.S

So,

$(AB)^T = B^T A^T$

5. Question

If $A = \begin{bmatrix}2 & 4 & -1\\-1 & 0 & 2\end{bmatrix}, B = \begin{bmatrix}3 & 4\\-1 & 2\\2 & 1\end{bmatrix}$, find $(AB)^T$.

Answer

Given,

$A=\begin{bmatrix}2&4&-1\\-1&0&2\end{bmatrix}, B=\begin{bmatrix}3&4\\-1&2\\2&1\end{bmatrix}$

$(AB)^T=?$

$\Rightarrow\left(\begin{bmatrix}2&4&-1\\-1&0&2\end{bmatrix}\begin{bmatrix}3&4\\-1&2\\2&1\end{bmatrix}\right)^T$

$\Rightarrow\begin{bmatrix}6-4-2&8+8-1\\-3-0+4&-4+0+2\end{bmatrix}^T$

$\Rightarrow\begin{bmatrix}0&15\\1&-2\end{bmatrix}^T$

$\Rightarrow\begin{bmatrix}0&1\\15&-2\end{bmatrix}$

So,

$(AB)^T=\begin{bmatrix}0&1\\15&-2\end{bmatrix}$

6 A. Question

For two matrices A and B, $A=\begin{bmatrix}2&1&3\\4&1&0\end{bmatrix}, B=\begin{bmatrix}1&-1\\0&2\\5&0\end{bmatrix}$ verify that $(AB)^T = B^TA^T$.

Answer

Given,

$A=\begin{bmatrix}2&1&3\\4&1&0\end{bmatrix}, B=\begin{bmatrix}1&-1\\0&2\\5&0\end{bmatrix}$

$(AB)^T=B^TA^T$

$\Rightarrow\left(\begin{bmatrix}2&1&3\\4&1&0\end{bmatrix}\begin{bmatrix}1&-1\\0&2\\5&0\end{bmatrix}\right)^T=\begin{bmatrix}1&-1\\0&2\\5&0\end{bmatrix}^T\begin{bmatrix}2&1&3\\4&1&0\end{bmatrix}^T$

$\Rightarrow\begin{bmatrix}2+0+15&-2+2+0\\4+0+0&-4+2+0\end{bmatrix}^T=\begin{bmatrix}1&0&5\\-1&2&0\end{bmatrix}\begin{bmatrix}2&4\\1&1\\3&0\end{bmatrix}$

$\Rightarrow\begin{bmatrix}17&0\\4&-2\end{bmatrix}^T=\begin{bmatrix}2+0+15&4+0+0\\-2+2+0&-4+2+0\end{bmatrix}$

$\Rightarrow\begin{bmatrix}17&4\\0&-2\end{bmatrix}=\begin{bmatrix}17&4\\0&-2\end{bmatrix}$

L.H.S = R.H.S

So,

$(AB)^T=B^TA^T$

6 B. Question

For the matrices, A and B, verify that $(AB)^T = B^TA^T$, where $A=\begin{bmatrix}1&3\\2&4\end{bmatrix}, B=\begin{bmatrix}1&4\\2&5\end{bmatrix}$

Answer

Given,

$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$

$(AB)^T = B^T A^T$

$\Rightarrow \left(\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}^T \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^T$

$\Rightarrow \begin{bmatrix} 1+6 & 4+15 \\ 2+8 & 8+20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix}^T = \begin{bmatrix} 1+6 & 2+8 \\ 4+15 & 8+20 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$

L.H.S = R.H.S

So,

$(AB)^T = B^T A^T$

7. Question

If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, find $A^T - B^T$.

Answer

Given,

$A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix},$

$A^T - B^T = ?$

Transpose matrix of B,

$B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$A^T - B^T = \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix}$

$A^T - B^T = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

8. Question

If $A = \begin{bmatrix} \cos\alpha & \cos\alpha \\ -\sin\alpha & \sin\alpha \end{bmatrix}$, then verify that $A^TA = I_2$.

Answer

Given,

$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A^T = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A^T = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A^T A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A^T A = \begin{bmatrix} (\cos\alpha)(\cos\alpha) + (-\sin\alpha)(-\sin\alpha) & (\cos\alpha)(\sin\alpha) + (-\sin\alpha)(\cos\alpha) \\ (\sin\alpha)(\cos\alpha) + (\cos\alpha)(-\sin\alpha) & (\sin\alpha)(\sin\alpha) + (\cos\alpha)(\cos\alpha) \end{bmatrix}$

$A^T A = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$A^T A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $(\sin^2\alpha + \cos^2\alpha = 1)$

Hence verified $A^T A = I_2$

9. Question

If $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, verify that $A^TA = I_2$.

Answer

Given,

$A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$A^T = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$

$A^T A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$A^T A = \begin{bmatrix} (\sin\alpha)(\sin\alpha) + (-\cos\alpha)(-\cos\alpha) & (\sin\alpha)(\cos\alpha) + (-\cos\alpha)(\sin\alpha) \\ (\cos\alpha)(\sin\alpha) + (\sin\alpha)(-\cos\alpha) & (\cos\alpha)(\cos\alpha) + (\sin\alpha)(\sin\alpha) \end{bmatrix}$

$A^T A = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$A^T A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $(\sin^2\alpha + \cos^2\alpha = 1)$

Hence, $A^T A = I_2$

10. Question

If $l_i, m_i, n_i ; i = 1, 2, 3$ denote the direction cosines of three mutually perpendicular vectors in space, prove that $AA^T = I$, where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$.

Answer

Given,

l_i, m_i, n_i are direction cosines of three mutually perpendicular vectors

$$\Rightarrow \left.\begin{matrix} l_1l_2 + m_1m_2 + n_1n_2 = 0 \\ l_2l_3 + m_2m_3 + n_2n_3 = 0 \\ l_1l_3 + m_1m_3 + n_1n_3 = 0 \end{matrix}\right\} \ldots (A)$$

$$\Rightarrow \left.\begin{matrix} l_1^2 + m_1^2 + n_1^2 = 1 \\ l_2^2 + m_2^2 + n_2^2 = 1 \\ l_3^2 + m_3^2 + n_3^2 = 1 \end{matrix}\right\} \ldots (B)$$

And given,

$$A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}\begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}^T$$

$$AA^T = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}\begin{bmatrix} l_1 & l_2 & n_1 \\ m_1 & m_2 & n_2 \\ n_1 & n_2 & n_3 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} l_1^2 + m_1^2 + n_1^2 & l_1l_2 + m_1m_2 + n_1n_2 & l_1l_3 + m_1m_3 + n_1n_3 \\ l_1l_2 + m_1m_2 + n_1n_2 & l_2^2 + m_2^2 + n_2^2 & l_2l_3 + m_2m_3 + n_2n_3 \\ l_1l_3 + m_1m_3 + n_1n_3 & l_2l_3 + m_2m_3 + n_2n_3 & l_3^2 + m_3^2 + n_3^2 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, $AA^T = I$

Exercise 5.5

1. Question

If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, prove that A – A^T is a skew-symmetric matrix.

Answer

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$(A - A^T) = \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^T\right)$$

$$= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2-2 & 3-4 \\ 4-3 & 5-5 \end{bmatrix}$$

$$(A - A^T) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \ldots\text{(i)}$$

$$-(A - A^T)^T = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} T$$

$$= -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$-(A - A^T) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \ldots\text{(ii)}$$

From (i) and (ii) we can see that

a skew-symmetric matrix is a square matrix whose transpose equal to its negative, that is,

X = – X^T

So, A – A^T is a skew-symmetric.

2. Question

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, show that A – A^T is a skew-symmetric matrix.

Answer

Given,

$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$(A - A^T) = \left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^T\right)$

$= \left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right)$

$= \begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix}$

$(A - A^T) = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$ ...(i)

$-(A - A^T)^T = -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}^T$

$= -\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$

$-(A - A^T) = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$ ...(ii)

From (i) and (ii) we can see that

a skew-symmetric matrix is a square matrix whose transpose equals its negative, that is,

X = – X^T

So, A – A^T is a skew-symmetric matrix.

3. Question

If the matrix $A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$ is a symmetric matrix, find x, y, z and t.

Answer

Given,

$A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$ is a symmetric matrix.

We know that $A = [a_{ij}]_{m \times n}$ is a symmetric matrix if $a_{ij} = a_{ji}$

So,

$x = a_{13} = a_{31} = 4$

$y = a_{21} = a_{12} = 2$

$z = a_{22} = a_{22} = z$

$t = a_{32} = a_{23} = -3$

Hence,

X=4, y=2, t=-3 and z can have any value.

4. Question

Let $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$. Find matrices X and Y such that X + Y = A, where X is a symmetric and Y is a skew symmetric matrix.

Answer

Given, $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$ Then $A^T = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}$

$$X = \frac{1}{2}(A + A^T)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 3+3 & 2+1 & 7-2 \\ 1+2 & 4+4 & 3+5 \\ -2+7 & 5+3 & 8+8 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 3 & 5 \\ 3 & 8 & 8 \\ 5 & 8 & 16 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}$$

Now,

$$Y = \frac{1}{2}(A - A^T)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 3-3 & 2-1 & 7+2 \\ 1-2 & 4-4 & 3-5 \\ -2-7 & 5-3 & 8-8 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 1 & 9 \\ -1 & 0 & -2 \\ -9 & 2 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{9}{2} & 1 & 0 \end{bmatrix}$$

Now,

$$X^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} = X$$

$\Rightarrow$ X is a symmetric matrix.

Now,

$$-Y^T = -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{9}{2} & 1 & 0 \end{bmatrix}^T = -\begin{bmatrix} 0 & -\frac{1}{2} & -\frac{9}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{9}{2} & -1 & 0 \end{bmatrix}$$

$$-Y^T = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{9}{2} & 1 & 0 \end{bmatrix}$$

$$-Y^T = Y$$

$\therefore$ Y is a skew symmetric matrix.

And,

$$X + Y = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{9}{2} & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & \frac{3}{2}+\frac{1}{2} & \frac{5}{2}+\frac{9}{2} \\ \frac{3}{2}-\frac{1}{2} & 4+0 & 4-1 \\ \frac{5}{2}-\frac{9}{2} & 4+1 & 8+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} = A$$

Hence, X+Y=A

5. Question

Express the matrix $A=\begin{bmatrix}4 & 2 & -1\\ 3 & 5 & 7\\ 1 & -2 & 1\end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Answer

Given, $A=\begin{bmatrix}4 & 2 & -1\\ 3 & 5 & 7\\ 1 & -2 & 1\end{bmatrix}$ Then $A^T=\begin{bmatrix}4 & 3 & 1\\ 2 & 5 & -2\\ -1 & 7 & 1\end{bmatrix}$

$$X=\frac{1}{2}(A+A^T)$$

$$=\frac{1}{2}\left(\begin{bmatrix}4 & 2 & -1\\ 3 & 5 & 7\\ 1 & -2 & 1\end{bmatrix}+\begin{bmatrix}4 & 3 & 1\\ 2 & 5 & -2\\ -1 & 7 & 1\end{bmatrix}\right)$$

$$=\frac{1}{2}\begin{bmatrix}4+4 & 2+3 & -1+1\\ 3+2 & 5+5 & 7-2\\ 1-1 & -2+7 & 1+1\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}8 & 5 & 0\\ 5 & 10 & 5\\ 0 & 5 & 2\end{bmatrix}$$

$$X=\begin{bmatrix}4 & \frac{5}{2} & 0\\ \frac{5}{2} & 5 & \frac{5}{2}\\ 0 & \frac{5}{2} & 1\end{bmatrix}$$

$$X^T=\begin{bmatrix}4 & \frac{5}{2} & 0\\ \frac{5}{2} & 5 & \frac{5}{2}\\ 0 & \frac{5}{2} & 1\end{bmatrix}^T=\begin{bmatrix}4 & \frac{5}{2} & 0\\ \frac{5}{2} & 5 & \frac{5}{2}\\ 0 & \frac{5}{2} & 1\end{bmatrix}=X$$

$\therefore$ X is a symmetric matrix.

$$Y=\frac{1}{2}(A-A^T)$$

$$=\frac{1}{2}\left(\begin{bmatrix}4 & 2 & -1\\ 3 & 5 & 7\\ 1 & -2 & 1\end{bmatrix}-\begin{bmatrix}4 & 3 & 1\\ 2 & 5 & -2\\ -1 & 7 & 1\end{bmatrix}\right)$$

$$=\frac{1}{2}\begin{bmatrix}4-4 & 2-3 & -1-1\\ 3-2 & 5-5 & 7+2\\ 1+1 & -2-7 & 1-1\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}0 & -1 & -2\\ 1 & 0 & 9\\ 2 & -9 & 0\end{bmatrix}$$

$$Y=\begin{bmatrix}0 & -\frac{1}{2} & -1\\ \frac{1}{2} & 0 & \frac{9}{2}\\ 1 & -\frac{9}{2} & 0\end{bmatrix}$$

$$Y^T = \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix} = Y$$

$\Rightarrow$ Y is a skew symmetric matrix.

Now,

$$X + Y = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & \frac{5}{2}-\frac{1}{2} & 0-1 \\ \frac{5}{2}+\frac{1}{2} & 5+0 & \frac{5}{2}+\frac{9}{2} \\ 0+1 & \frac{5}{2}-\frac{9}{2} & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} = A$$

Hence, X + Y = A.

6. Question

Define a symmetric matrix. Prove that for $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$, $A + A^T$ is a symmetric matrix where A^T is the transpose of A.

Answer

A square matrix ‘A’ is called a symmetric matrix, if A = A^T.

Here,

$$A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 4+5 \\ 5+4 & 6+6 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix} \ldots \text{(i)}$$

$$(A + A^T)^T = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}^T$$

$$(A + A^T)^T = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix} \ldots \text{(ii)}$$

From equation (i) and (ii),

$(A + A^T)^T = (A + A^T)$

So, A + A^T is a symmetric matrix.

7. Question

Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Answer

Given, $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}, A^T = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

Let,

$X = \frac{1}{2}(A + A^T)$

$= \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 3+3 & -4+1 \\ 1-4 & -1-1 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 6 & -3 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix}$

Now,

$X^T = \begin{bmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix}^T = \begin{bmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix} = X$

Hence, X is a symmetric matrix.

Now let,

$Y = \frac{1}{2}(A - A^T)$

$= \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$

Now,

$-Y^T = -\begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = Y$

$\Rightarrow$ Y is a skew symmetric.

Now,

$$X + Y = \begin{bmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & -\frac{3}{2}-\frac{5}{2} \\ -\frac{3}{2}+\frac{5}{2} & -1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

X+Y = A

8. Question

Express the matrix $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix and verify your result.

Answer

Given, $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ Then, $A^T = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

Let,

$$X = \frac{1}{2}(A + A^T)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 3+3 & -2+3 & -4-1 \\ 3-2 & -2-2 & -5+1 \\ -1-4 & 1-5 & 2+2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} = X$$

$\Rightarrow$ X is a symmetric matrix.

And,

$$Y = \frac{1}{2}(A - A^T)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 3-3 & -2-3 & -4+1 \\ 3+2 & -2+2 & -5-1 \\ -1+4 & 1+5 & 2-2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

$$-Y^T = -\begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = Y$$

$\Longrightarrow$ Y is a skew symmetric matrix.

$$X + Y = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & \frac{1}{2}-\frac{3}{2} & -\frac{5}{2}-\frac{3}{2} \\ \frac{1}{2}+\frac{5}{2} & -2+0 & -2-3 \\ -\frac{5}{2}+\frac{3}{2} & -2+3 & 2+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = A$$

$$X + Y = A$$

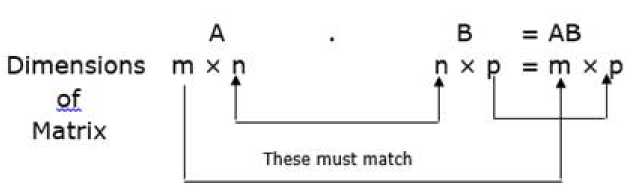
Very short answer

1. Question

If A is an m × n matrix and B is n × p matrix does AB exist? If yes, write its order.

Answer

Given: A = m × n matrix and B = n × p matrix



∴ the product AB is defined and the size of the product matrix AB is m × p.

2. Question

If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$. Write the order of AB and BA.

Answer

Given:

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \;\&\; B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

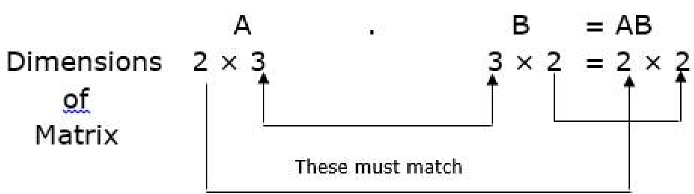
In matrix A, there are 2 rows and 3 columns.

∴ A is a 2 × 3 matrix

In matrix B, there are 3 rows and 2 columns.

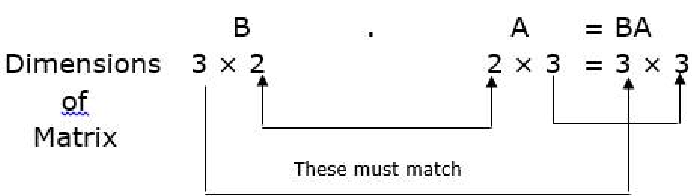
∴ B is a 3 × 2 matrix

So, the product matrix AB will be



∴ the order of AB is 2 × 2 matrix

and the order of product matrix BA will be



∴ the order of BA is 3 × 3 matrix.

3. Question

If $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$, write AB.

Answer

Given:

$$A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} \;\&\; B = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

So, AB will be

$$AB = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -16 + 9 \\ -4 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} -7 \\ 2 \end{bmatrix}$$

4. Question

If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, write AA^T.

Answer

Given:

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1}$$

Now, firstly we find the A^T

$$A^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3}$$

So, the product AA^T will be

$$AA^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}_{3\times 3}$$

5. Question

Give an example of two non-zero 2 × 2 matrices A and B such that AB = O.

Answer

Example 1:

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are the two non – zero matrices

Now, we will check that AB = 0 or not

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 1\times 0 + 0\times 0 & 1\times 0 + 0\times 1 \\ 0\times 0 + 0\times 0 & 0\times 0 + 0\times 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are the two non – zero matrices such that AB = 0

Example 2:

Let $A = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$ are the two non – zero matrices

Now, we will check that AB = 0 or not

$$AB = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 0\times 1 + 0\times (-1) & 0\times 1 + 0\times (-1) \\ 1\times 1 + 1\times (-1) & 1\times 1 + 1\times (-1) \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 0 & 0 \\ 1-1 & 1-1 \end{bmatrix}$$

$\Rightarrow AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence, $A = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$ are the two non – zero matrices such that AB = 0

6. Question

If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, find A + A^T.

Answer

Given:

$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

To find: A + A^T

Firstly, we find the A^T

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2 $\times$ 2 matrix, then the transpose of a matrix is $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

So,

$A^T = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$

$\therefore A + A^T = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$

$= \begin{bmatrix} 2+2 & 3+5 \\ 5+3 & 7+7 \end{bmatrix}$

$= \begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix}$

7. Question

If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, write A^2.

Answer

Given:

$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

To find: A^2

$\therefore A^2 = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$

$= \begin{bmatrix} 2\times2+3\times3 & 2\times5+3\times7 \\ 5\times2+7\times3 & 5\times5+7\times7 \end{bmatrix}$

$= \begin{bmatrix} 4+9 & 10+21 \\ 10+21 & 25+49 \end{bmatrix}$

$= \begin{bmatrix} 13 & 31 \\ 31 & 74 \end{bmatrix}$

8. Question

If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, find x satisfying $0 < x < \frac{\pi}{2}$ when A + A^T = I.

Answer

Given:

$A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

To find: x

Firstly, we find the A^T

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2 $\times$ 2 matrix, then the transpose of a matrix is $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

So,

$A^T = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$\therefore A + A^T = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} + \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$= \begin{bmatrix} \cos x + \cos x & -\sin x + \sin x \\ \sin x + (-\sin x) & \cos x + \cos x \end{bmatrix}$

$= \begin{bmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{bmatrix}$

It is given that A + A^T = I when $0 < x < \frac{\pi}{2}$

So,

$\begin{bmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Comparing both the matrices, we get

2cosx = 1

$\Rightarrow \cos x = \frac{1}{2}$

$\Rightarrow x = \cos^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow x = \cos^{-1}\left(\cos\frac{\pi}{3}\right) \left[\because 0 < x < \frac{\pi}{2}\right]$

$\Rightarrow x = \frac{\pi}{3}$

9. Question

If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, find AA^T.

Answer

Given:

$A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

To find: AA^T

Firstly, we find the A^T

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2×2 matrix, then the transpose of a matrix is $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

So,

$$A^T = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\therefore AA^T = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \times \cos x + (-\sin x) \times (-\sin x) & \sin x \times \cos x + (-\sin x) \times \cos x \\ \sin x \times \cos x + \cos x \times (-\sin x) & \sin x \times \sin x + \cos x \times \cos x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin x \cos x - \sin x \cos x \\ \sin x \cos x - \sin x \cos x & \sin^2 x + \cos^2 x \end{bmatrix}$$

$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ [$\because \cos^2 x + \sin^2 x = 1$]

$\Rightarrow AA^T = I$

10. Question

If $\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2\begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = I$, where I is 2×2 unit matrix. Find x and y.

Answer

$$\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2\begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = I$$

Here, it is given that I is a 2×2 unit matrix

So,

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, given equation becomes

$$\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2\begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + \begin{bmatrix} 2x & 0 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 + 2x & 0 \\ y + 2 & 5 - 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 + 2x & 0 \\ y + 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing the matrices, we get

$1 + 2x = 1$ …(i)

and $y + 2 = 0$ …(ii)

Solving eq. (i), we get

$1 + 2x = 1$

$\Rightarrow 2x = 0$

$\Rightarrow x = 0$

Solving eq. (ii), we get

y + 2 = 0

⇒ y = -2

Hence, the value of x = 0 and y = -2

11. Question

If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, satisfies the matrix equation A^2 = kA, write the value of k.

Answer

Given:

$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

and it satisfies the matrix equation A^2 = kA

Firstly, we find the A^2

$A^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \times 1 + (-1) \times (-1) & 1 \times (-1) + (-1) \times 1 \\ -1 \times 1 + 1 \times (-1) & -1 \times (-1) + 1 \times 1 \end{bmatrix}$

$= \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$= 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

= 2 A

∴ k = 2

Hence, the value of k = 2

12. Question

If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies A^4 = λA, then write the value of λ.

Answer

Given:

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

and it satisfies the matrix equation A^4 = λA

Firstly, we find the A^4

$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \times 1 + 1 \times 1 & 1 \times 1 + 1 \times 1 \\ 1 \times 1 + 1 \times 1 & 1 \times 1 + 1 \times 1 \end{bmatrix}$

$= \begin{bmatrix} 1+1 & 1+1 \\ 1+1 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$= 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

= 2 A

So, $A^4 = A^2 \times A^2$

$\Rightarrow A^4 = 2A \times 2A$

$\Rightarrow A^4 = 4A^2$

$\Rightarrow A^4 = 4 \times 2A$ [$\because A^2 = 2A$]

$\Rightarrow A^4 = 8A$

$\therefore \lambda = 8$

Hence, the value of $\lambda = 8$

13. Question

If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, find A^2.

Answer

$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Now, we have to find the A^2

$A \times A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} -1 \times (-1) & 0 & 0 \\ 0 & -1 \times (-1) & 0 \\ 0 & 0 & -1 \times (-1) \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow A^2 = I$

14. Question

If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, find A^3.

Answer

$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Now, we have to find the A^3

$$A \times A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} -1 \times (-1) & 0 & 0 \\ 0 & -1 \times (-1) & 0 \\ 0 & 0 & -1 \times (-1) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, we will find A^3

$$\Rightarrow A^2 \times A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 1 \times (-1) & 0 & 0 \\ 0 & 1 \times (-1) & 0 \\ 0 & 0 & 1 \times (-1) \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$\Rightarrow A^3 = A$

15. Question

If $A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$, find A^4.

Answer

Given:

$$A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

To find: A^4

$A^4 = A \times A \times A \times A$

$\Rightarrow A^4 = A^2 \times A^2$

So,

$$A^2 = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} -3 \times (-3) & 0 \\ 0 & -3 \times (-3) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$\Rightarrow A^2 = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow A^2 = 9I$

$\because A^4 = A^2 \times A^2$

$= 9I \times 9I$

$= 81\, I^2$

= 81 I [∵ I^2 = I]

16. Question

If [x 2]$\begin{bmatrix} 3 \\ 4 \end{bmatrix} = 2$, find x.

Answer

Given:

$[x \quad 2]\begin{bmatrix} 3 \\ 4 \end{bmatrix} = 2$

Here, we have to find the x

Solving the given matrix, we get

$[x \quad 2]\begin{bmatrix} 3 \\ 4 \end{bmatrix} = 2$

⇒ [3x + 8] = 2

⇒ 3x = 2 – 8

⇒ 3x = -6

⇒ x = -2

17. Question

If A = [a_{ij}] is a 2 × 2 matrix such that a_{ij} = i + 2j, write A.

Answer

Given: A = [a_{ij}] is a 2 × 2 matrix

$\therefore A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ ...(i)

Given that a_{ij} = i + 2j

So, a_{11} = 1 + 2 × 1 = 1 + 2 = 3

a_{12} = 1 + 2 × 2 = 1 + 4 = 5

a_{21} = 2 + 2 × 1 = 2 + 2 = 4

a_{22} = 2 + 2 × 2 = 2 + 4 = 6

Putting the values in eq. (i), we get

$\therefore A = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

18. Question

Write matrix A satisfying $A + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$.

Answer

Given:

$A + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$

To find: A

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Solving for matrix A, we get

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+2 & b+3 \\ c-1 & d+4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$

Comparing the values, we get

a + 2 = 3 ...(i)

b + 3 = -6 ...(ii)

c – 1 = -3 ...(iii)

and d + 4 = 8 ...(iv)

Solving eq. (i), we get

a + 2 = 3

⇒ a = 1

Solving eq. (ii), we get

b + 3 = -6

⇒ b = -6 – 3

⇒ b = -9

Solving eq. (iii), we get

c – 1 = -3

⇒ c = -3 + 1

⇒ c = -2

Solving eq. (iv), we get

d + 4 = 8

⇒ d = 8 – 4

⇒ d = 4

Putting the value of a, b , c and d to get the matrix A, we get

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & -9 \\ -2 & 4 \end{bmatrix}$

19. Question

If A = $[a_{ij}]$ is a square matrix such that $a_{ij} = i^2 - j^2$, then write whether A is symmetric or skew-symmetric.

Answer

Given: A = $[a_{ij}]$ is a square matrix such that $a_{ij} = i^2 - j^2$

Suppose A is a 2 × 2 square matrix i.e.

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Here,

$a_{ij} = i^2 - j^2$

So, $a_{12} = (1)^2 - (2)^2 = 1 - 4 = -3$

and $a_{21} = (2)^2 - (1)^2 = 4 - 1 = 3$

For diagonal elements, i = j, we have

$a_{11} = (1)^2 - (1)^2 = 0$

and $a_{22} = (2)^2 - (2)^2 = 0$

So, Matrix A becomes

$A = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

Now, we have to check A is symmetric or skew – symmetric.

We know that, if a matrix is symmetric then $A^T = A$

and if a matrix is skew – symmetric then $A^T = -A$

So, firstly we find the A^T

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2×2 matrix, then the transpose of a matrix is $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

So,

$A^T = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$\Rightarrow A^T = -\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

$\Rightarrow A^T = -A$

∴ A is a skew – symmetric matrix.

20. Question

For any square matrix write whether AA^T is symmetric or skew-symmetric.

Answer

Here, we have any square matrix

To Find: AA^T is symmetric or skew – symmetric

Proof: Firstly, we take the transpose of AA^T, so we get

$(AA^T)^T = (A^T)^T A^T$ [$\because (AB)^T = B^T A^T$]

$\Rightarrow (AA^T)^T = AA^T$ [$\because (A^T)^T = A$]

∴ AA^T is a symmetric matrix

21. Question

If $A = [a_{ij}]$ is a skew-symmetric matrix, then write the value of $\sum_i a_{ij}$.

Answer

Given: $A = [a_{ij}]$ is a skew – symmetric matrix

$\Rightarrow a_{ij} = -a_{ji}$ ...(i)

[for all values of i, j]

For diagonal elements,

⇒ $a_{ii} = - a_{ii}$ [for all values of i]

⇒ $a_{ii} + a_{ii} = 0$

⇒ $2a_{ii} = 0$

⇒ $a_{ii} = 0$ …(ii)

Now,

$$\sum_i \sum_j a_{ij} = a_{11} + a_{12} + a_{13} \ldots + a_{21} + a_{22} + a_{23} \ldots + a_{31} + a_{32} + a_{33} \ldots$$

$= 0 + a_{12} + a_{13} + \ldots + (-a_{12}) + 0 + a_{23} + \ldots + (-a_{13}) + (- a_{23}) + 0 + \ldots$

[from (i) and (ii)]

= 0

Thus,

$$\sum_i \sum_j a_{ij} = 0$$

Hence Proved.

22. Question

If $A = [a_{ij}]$ is a skew-symmetric matrix, then write the value of $\sum_i \sum_j a_{ij}$.

Answer

Given: $A = [a_{ij}]$ is a skew – symmetric matrix

⇒ $a_{ij} = - a_{ji}$ …(i)

[for all values of i, j]

For diagonal elements,

⇒ $a_{ii} = - a_{ii}$ [for all values of i]

⇒ $a_{ii} + a_{ii} = 0$

⇒ $2a_{ii} = 0$

⇒ $a_{ii} = 0$ …(ii)

Now,

$$\sum_i \sum_j a_{ij} = a_{11} + a_{12} + a_{13} \ldots + a_{21} + a_{22} + a_{23} \ldots + a_{31} + a_{32} + a_{33} \ldots$$

$= 0 + a_{12} + a_{13} + \ldots + (-a_{12}) + 0 + a_{23} + \ldots + (-a_{13}) + (- a_{23}) + 0 + \ldots$

[from (i) and (ii)]

= 0

Thus,

$$\sum_i \sum_j a_{ij} = 0$$

Hence Proved.

23. Question

If A and B are symmetric matrices, then write the condition for which AB is also symmetric.

Answer

If A and B are symmetric matrices, then AB is symmetric if and only if A and B commute .i.e.

$AB = (AB)^T = B^T A^T = BA$

$[\because B^T = B \text{ and } A^T = A]$

24. Question

If B is a skew-symmetric matrix, write whether the matrix AB A^T is symmetric or skew-symmetric.

Answer

B is a skew – symmetric matrix, then

$B^T = -B$ ...(i)

Consider

$(ABA^T)^T = (A^T)^T B^T A^T$

$[\because (AB)^T = B^T A^T]$

$\Rightarrow (ABA^T)^T = AB^T A^T$ $[\because (A^T)^T = A]$

$\Rightarrow (ABA^T)^T = A(-B)A^T$ [from (i)]

$\Rightarrow (ABA^T)^T = -ABA^T$

$\therefore ABA^T$ is a skew – symmetric matrix

25. Question

If B is a symmetric matrix, write whether the matrix AB A^T is symmetric or skew-symmetric.

Answer

B is a symmetric matrix, then

$B^T = B$ ...(i)

Consider

$(ABA^T)^T = (A^T)^T B^T A^T$

$[\because (AB)^T = B^T A^T]$

$\Rightarrow (ABA^T)^T = AB^T A^T$ $[\because (A^T)^T = A]$

$\Rightarrow (ABA^T)^T = A(B)A^T$ [from (i)]

$\Rightarrow (ABA^T)^T = ABA^T$

$\therefore ABA^T$ is a symmetric matrix

26. Question

If A is a skew-symmetric and $n \in N$ such that $(A^n)^T = \lambda A^n$, write the value of λ.

Answer

Let A is a skew – symmetric matrix, then

$A^T = -A$ ...(i)

Consider

$\Rightarrow (A^n)^T = \lambda A^n$ [given]

$\Rightarrow (A^T)^n = \lambda A^n$

$\Rightarrow (-A)^n = \lambda A^n$ [from (i)]

$\Rightarrow (-1)^n (A)^n = \lambda A^n$

Comparing both the sides, we get

$\lambda = (-1)^n$

27. Question

If A is a symmetric matrix and n ∈ N, write whether A^n is symmetric or skew-symmetric or neither of these two.

Answer

Given that A is a symmetric matrix

$\therefore A = A^T$ ...(i)

Now, we have to check A^n is symmetric or skew – symmetric

$(A^n)^T = (A\times A\times A\times A\ldots A)^T$ [for all $n \in N$]

$\Rightarrow (A^n)^T = (A^T \times A^T \ldots A^T)$

[$\because (AB)^T = B^T A^T$]

$= A \times A \ldots A$ [from (i)]

$= A^n$

$\Rightarrow (A^n)^T = A^n$

Case 1: If n is an even natural number, then

$(A^n)^T = A^n$

So, A^n is a symmetric matrix

Case 2: If n is odd natural number, then

$(A^n)^T = A^n$

So, A^n is a symmetric matrix

28. Question

If A is a skew-symmetric matrix and n is an even natural number, write whether A^n is symmetric or skew-symmetric or neither of these two.

Answer

Let A is a skew – symmetric matrix, then

$A^T = - A$ ...(i)

Now, we have to check A^n is symmetric or skew – symmetric

$(A^n)^T = (A^T)^n$ [for all $n \in N$]

$\Rightarrow (A^n)^T = (- A)^n$ [from (i)]

$\Rightarrow (A^n)^T = (-1)^n (A)^n$

Given that n is an even natural number, then

$(A^n)^T = A^n$

[$\because$ $(-1)^2 = 1$, $(-1)^4 = 1$,… $(-1)^n = 1$]

So, A^n is a symmetric matrix

29. Question

If A is a skew-symmetric matrix and n is an odd natural number, write whether A^n is symmetric or skew-symmetric or neither of the two.

Answer

Let A is a skew – symmetric matrix, then

$A^T = - A$ …(i)

Now, we have to check A^n is symmetric or skew – symmetric

$(A^n)^T = (A^T)^n$ [for all n ∈ N]

$\Rightarrow (A^n)^T = (- A)^n$ [from (i)]

$\Rightarrow (A^n)^T = (-1)^n (A)^n$

Given that n is odd natural number, then

$(A^n)^T = - A^n$

[$\because$ $(-1)^3 = - 1$, $(-1)^5 = - 1$,… $(-1)^n = - 1$]

So, A^n is a skew - symmetric matrix

30. Question

If A and B are symmetric matrices of the same order, write whether AB – BA is symmetric or skew-symmetric or neither of the two.

Answer

A and B are symmetric matrices,

$\therefore$ A' = A and B' = B …(i)

Consider (AB – BA)' = (AB)' – (BA)' [(a – b)' = a' – b']

= B'A' – A'B' [(AB)' = B'A']

= BA – AB [from (i)]

= - (AB – BA)

$\therefore$ (AB – BA)' = - (AB – BA)

Hence, (AB – BA) is a skew symmetric matrix.

31. Question

Write a square matrix which is both symmetric as well as skew-symmetric.

Answer

We must understand what symmetric matrix is.

A symmetric matrix is a square matrix that is equal to its transpose.

A symmetric matrix $\Leftrightarrow A = A^T$

Now, let us understand what skew-symmetric matrix is.

A skew-symmetric matrix is a square matrix whose transpose equals its negative, that, it satisfies the condition

A skew symmetric matrix ⇔ $A^T = -A$

And,

A square matrix is a matrix with the same number of rows and columns. An n-by-n matrix is known as a square matrix of order n.

We need to find a square matrix which is both symmetric as well as skew symmetric.

Take a 2 × 2 null matrix.

Say,

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Let us take transpose of the matrix A.

We know that, the transpose of a matrix is a new matrix whose rows are the columns of the original.

So,

$$A^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since, $A = A^T$.

∴, A is symmetric.

Take the same matrix and multiply it with -1.

$$-A = -1 \times \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow -A = -\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow -A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Let us take transpose of the matrix –A.

So,

$$-A^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since,

$A^T = -A$

∴, A is skew-symmetric.

Thus, A (a null matrix) is both symmetric as well as skew-symmetric.

32. Question

Find the value of x and y, if $2\begin{bmatrix} 1 & 3 \\ 0 & \text{x} \end{bmatrix} + \begin{bmatrix} \text{y} & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.

Answer

We are given that,

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

We need to find the value of x and y.

Taking Left Hand Side (LHS) matrix of the equation,

$$LHS = 2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix} + \begin{bmatrix}y & 0\\1 & 2\end{bmatrix}$$

Multiplying the scalar, 2 by each element of the matrix $\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}$,

$$\Rightarrow LHS = \begin{bmatrix}2\times 1 & 2\times 3\\2\times 0 & 2\times x\end{bmatrix} + \begin{bmatrix}y & 0\\1 & 2\end{bmatrix}$$

$$\Rightarrow LHS = \begin{bmatrix}2 & 6\\0 & 2x\end{bmatrix} + \begin{bmatrix}y & 0\\1 & 2\end{bmatrix}$$

Adding the corresponding elements,

$$LHS = \begin{bmatrix}2+y & 6+0\\0+1 & 2x+2\end{bmatrix}$$

$$\Rightarrow LHS = \begin{bmatrix}2+y & 6\\1 & 2x+2\end{bmatrix}$$

Equate LHS to Right Hand Side (RHS) equation,

$$\begin{bmatrix}2+y & 6\\1 & 2x+2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

We know that if we have,

$$\begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix} = \begin{bmatrix}b_{11} & b_{12}\\b_{21} & b_{22}\end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

Similarly, the corresponding elements of two matrices are equal,

2 + y = 5 …(i)

6 = 6

1 = 1

2x + 2 = 8 …(ii)

We have equations (i) and (ii) to solve for x and y.

From equation (i),

2 + y = 5

⇒ y = 5 – 2

⇒ y = 3

From equation (ii),

2x + 2 = 8

⇒ 2x = 8 – 2

⇒ 2x = 6

$$\Rightarrow x = \frac{6}{2}$$

⇒ x = 3

Thus, we have x = 3 and y = 3.

33. Question

If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$, find x and y.

Answer

We are given with,

$$\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$$

We need to find the values of x and y.

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

So, if we have

$$\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$$

Corresponding elements of two matrices are equal.

That is,

x + 3 = 5 …(i)

4 = 4

y – 4 = 3 …(ii)

x + y = 9 …(iii)

To solve for x and y, we have three equations (i), (ii) and (iii).

From equation (i),

x + 3 = 5

⇒ x = 5 – 3

⇒ x = 2

From equation (ii),

y – 4 = 3

⇒ y = 3 + 4

⇒ y = 7

We need not solve equation (iii) as we have got the values of x and y.

Thus, the values of x = 2 and y = 7.

34. Question

Find the value of x from the following: $\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$.

Answer

We are given with matrix equation,

$$\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$$

We need to find the values of x and y.

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

So, if we have

$$\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$$

Corresponding elements of two matrices are equal.

That is,

2x – y = 6 ...(i)

5 = 5

3 = 3

y = -2 ...(ii)

To solve for x and y, we have equations (i) and (ii).

From equation (ii),

y = -2

Substituting y = -2 in equation (i), we get

2x – y = 6

⇒ 2x – (-2) = 6

⇒ 2x + 2 = 6

⇒ 2x = 6 – 2

⇒ 2x = 4

$\Rightarrow x = \frac{4}{2}$

⇒ x = 2

Thus, we get x = 2 and y = -2.

35. Question

Find the value of y, if $\begin{bmatrix} x-y & 2 \\ x & 5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$.

Answer

We are given that,

$$\begin{bmatrix} x-y & 2 \\ x & 5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$$

We need to find the values of x and y.

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

So, if we have

$$\begin{bmatrix} x - y & 2 \\ x & 5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$$

Corresponding elements of two matrices are equal.

That is,

x – y = 2 …(i)

2 = 2

x = 3 …(ii)

5 = 5

To solve for x and y, we have equations (i) and (ii).

From equation (ii),

x = 3

Substituting x = 3 in equation (i), we get

3 – y = 2

⇒ y = 3 – 2

⇒ y = 1

Thus, we get x = 3 and y = 1.

36. Question

Find the value of x, if $\begin{bmatrix} 3x + y & -y \\ 2y - x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$.

Answer

We are given that,

$$\begin{bmatrix} 3x + y & -y \\ 2y - x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

We need to find the values of x and y.

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

So, if we have

$$\begin{bmatrix} 3x + y & -y \\ 2y - x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

Corresponding elements of two matrices are equal.

That is,

3x + y = 1 ...(i)

-y = 2 ...(ii)

2y – x = -5 ...(iii)

3 = 3

To solve for x and y, we have equations (i), (ii) and (iii).

From equation (ii),

-y = 2

Multiplying both sides by -1,

-1 × -y = -1 × 2

⇒ y = -2

Substituting y = -2 in either of the equations (i) or (iii), say (i)

3x + y = 1

⇒ 3x + (-2) = 1

⇒ 3x – 2 = 1

⇒ 3x = 1 + 2

⇒ 3x = 3

⇒ $x = \frac{3}{3}$

⇒ x = 1

Thus, we get x = 1 and y = -2.

37. Question

If matrix $\mathrm{A} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, write AA^T.

Answer

We are given that,

$A = [1 \quad 2 \quad 3]$

We need to compute AA^T.

We know that the transpose of a matrix is a new matrix whose rows are the columns of the original.

So, transpose of matrix A will be given as

$$A^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Multiplying A by A^T,

$$AA^T = [1 \quad 2 \quad 3] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

In multiplication of matrices,

$$[a_{11} \quad a_{12} \quad a_{13}] \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix}$$

Dot multiply the matching members of 1st row of first matrix and 1st column of second matrix and then sum up.

$(a_{11}\ a_{12}\ a_{13})(b_{11}\ b_{21}\ b_{31}) = a_{11} \times b_{11} + a_{12} \times b_{21} + a_{13} \times b_{31}$

So,

(1 2 3)(1 2 3) = 1 × 1 + 2 × 2 + 3 × 3

⇒ (1 2 3)(1 2 3) = 1 + 4 + 9

⇒ (1 2 3)(1 2 3) = 14

Thus,

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = [14]$$

38. Question

If $\begin{bmatrix} 2x+y & 3y \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \end{bmatrix}$, then find x.

Answer

We are given that,

$$\begin{bmatrix} 2x+y & 3y \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \end{bmatrix}$$

We need to find the value of x.

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

So, if we have

$$\begin{bmatrix} 2x+y & 3y \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \end{bmatrix}$$

Corresponding elements of two elements are equal.

That is,

2x + y = 6 …(i)

3y = 0 …(ii)

To solve for x, we have equations (i) and (ii).

We can’t solve for x using only equation (i) as equation (i) contains x as well as y. We need to find the value of y from equation (ii) first.

From equation (ii),

3y = 0

$\Rightarrow y = \frac{0}{3}$

⇒ y = 0

Substituting y = 0 in equation (i),

2x + y = 6

⇒ 2x + (0) = 6

⇒ 2x = 6 – 0

⇒ 2x = 6

⇒ $x = \frac{6}{2}$

⇒ x = 3

Thus, we get x = 3.

39. Question

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find A + A^T.

Answer

We are given that,

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

We need to find the value of A + A^T.

We know that the transpose of a matrix is a new matrix whose rows are the columns of the original.

We have,

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Here,

1st row of A = (1 2)

2nd row of A = (3 4)

Transpose of this matrix A, A^T will be given as

1st column of A^T = 1st row of A = (1 2)

2nd column of A^T = 2nd row of A = (3 4)

Then,

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

For addition of two matrices, say X and Y, where

$$X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \ and \ Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

Add the corresponding elements of matrices X and Y.

$$X + Y = \begin{bmatrix} x_{11} + y_{11} & x_{12} + y_{12} \\ x_{21} + y_{21} & x_{22} + y_{22} \end{bmatrix}$$

Similarly, we need to add these two matrices, A and A^T.

$$A + A^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Adding the corresponding elements of the matrices A and A^T,

$\Rightarrow A + A^T = \begin{bmatrix} 1+1 & 2+3 \\ 3+2 & 4+4 \end{bmatrix}$

$\Rightarrow A + A^T = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$

Thus, we get the matrix $\begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$.

40. Question

If $\begin{bmatrix} a+b & 2 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}$, then find a.

Answer

We are given that,

$\begin{bmatrix} a+b & 2 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}$

We need to find the value of x.

We know by the property of matrices,

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

So, if we have

$\begin{bmatrix} a+b & 2 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 4 \end{bmatrix}$

Corresponding elements of two elements are equal.

That is,

a + b = 6 …(i)

b = 4 …(ii)

To solve for a, we have equations (i) and (ii).

We can’t solve for a using only equation (i) as equation (i) contains a as well as b. We need to find the value of b from equation (ii) first.

From equation (ii),

b = 4

Substituting the value of b = 4 in equation (i),

a + b = 6

⇒ a + 4 = 6

⇒ a = 6 – 4

⇒ a = 2

Thus, we get a = 2.

41. Question

If A is a matrix of order 3 × 4 and B is a matrix of order 4 × 3, find the order of the matrix of AB.

Answer

We are given that,

Order of matrix A = 3 × 4

Order of matrix B = 4 × 3

We need to find the order of the matrix of AB.

We know that,

For matrices X and Y such that,

Order of X = m × n

Order of Y = r × s

In order to multiply the two matrices X and Y, the number of columns in X must be equal to the number of rows in Y. That is,

n = r

And order of the resulting matrix, XY is given as

Order of XY = m × s

Provided n = r.

So, we know

Order of A = 3 × 4

Here,

Number of rows = 3

Number of columns = 4

Order of B = 4 × 3

Here,

Number of rows = 4

Number of columns = 3

Note that,

Number of columns in A = Number of rows in B = 4

So,

Order of the resulting matrix, AB is given as

Order of AB = 3 × 3

Thus, order of AB = 3 × 3

42. Question

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ is identity matrix, then write the value of α.

Answer

We are given that,

$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ is an identity matrix.

We need to find the value of α.

We must understand what an identity matrix is.

An identity matrix is a square matrix in which all the elements of the principal diagonal are ones and all other elements are zeroes.

An identity matrix is denoted by

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

According to the question,

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = I$$

$$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

So, if we have

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The corresponding elements of matrices are equal.

That is,

cos α = 1

-sin α = 0

sin α = 0

cos α = 1

Since, the equations are repetitive, take

cos α = 1

⇒ α = $\cos^{-1} 1$

⇒ α = 0°

Thus, the value of α = 0°.

43. Question

If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then write the value of k.

Answer

We are given with

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$$

We need to find the value of k.

Take Left Hand Side (LHS) of the matrix equation.

$$LHS = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}$$

In multiplication of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

For c_{11}: dot multiply the matching members of 1st row of first matrix and 1st column of second matrix and then sum up.

$(a_{11}\ a_{12})(b_{11}\ b_{21}) = a_{11} \times b_{11} + a_{12} \times b_{21}$

Thus,

$(1\ 2)(3\ 2) = 1 \times 3 + 2 \times 2$

$\Rightarrow (1\ 2)(3\ 2) = 3 + 4$

$\Rightarrow (1\ 2)(3\ 2) = 7$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & \end{bmatrix}$$

For c_{12}: dot multiply the matching members of 1st row of first matrix and 2nd column of second matrix and then sum up.

$(a_{11}\ a_{12})(b_{12}\ b_{22}) = a$ �$_{11} \times b_{12} + a_{12} \times b_{22}$

Thus,

$(1\ 2)(1\ 5) = 1 \times 1 + 2 \times 5$

$\Rightarrow (1\ 2)(1\ 5) = 1 + 10$

$\Rightarrow (1\ 2)(1\ 5) = 11$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \end{bmatrix}$$

Similarly, do the same for other elements.

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ (3 \times 3) + (4 \times 2) & (3 \times 1) + (4 \times 5) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ 9 + 8 & 3 + 20 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ 17 & 23 \end{bmatrix}$$

Since,

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$$

Substituting the value of LHS,

$$\Rightarrow \begin{bmatrix} 7 & 11 \\ 17 & 23 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$$

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

Thus,

$7 = 7$

11 = 11

17 = k

23 = 23

Hence, k = 17.

44. Question

If I is the identity matrix and A is a square matrix such $A^2 = A$, then what is the value of $(I + A)^2 – 3A$?

Answer

We are given that,

I is the identity matrix.

A is a square matrix such that $A^2 = A$.

We need to find the value of $(I + A)^2 – 3A$.

We must understand what an identity matrix is.

An identity matrix is a square matrix in which all the elements of the principal diagonal are ones and all other elements are zeroes.

Take,

$(I + A)^2 – 3A = (I)^2 + (A)^2 + 2(I)(A) – 3A$

[∵, by algebraic identity,

$(x + y)^2 = x^2 + y^2 + 2xy$]

$\Rightarrow (I + A)^2 – 3A = (I)(I) + A^2 + 2(IA) – 3A$

By property of matrix,

(I)(I) = I

IA = A

$\Rightarrow (I + A)^2 – 3A = I + A^2 + 2A – 3A$

$\Rightarrow (I + A)^2 – 3A = I + A + 2A – 3A$ [∵, given in question, $A^2 = A$]

$\Rightarrow (I + A)^2 – 3A = I + 3A – 3A$

$\Rightarrow (I + A)^2 – 3A = I + 0$

$\Rightarrow (I + A)^2 – 3A = I$

Thus, the value of $(I + A)^2 – 3A = I$.

45. Question

If $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is written as B + C, where B is a symmetric matrix and C is a skew-symmetric matrix, then find B.

Answer

We are given that,

$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = B + C$

Where,

B = symmetric matrix

C = skew-symmetric matrix

We need to find B.

A symmetric matrix is a square matrix that is equal to its transpose.

A symmetric matrix ⇔ A = A^T

Now, let us understand what skew-symmetric matrix is.

A skew-symmetric matrix is a square matrix whose transpose equals its negative, that, it satisfies the condition

A skew symmetric matrix ⇔ A^T = -A

So, let the matrix B be

$$B = \frac{1}{2}(A + A^T)$$

Let us calculate A^T.

We know that the transpose of a matrix is a new matrix whose rows are the columns of the original.

We have,

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

Here,

1^{st} row of A = (1 2)

2^{nd} row of A = (0 3)

Transpose of this matrix A, A^T will be given as

1^{st} column of A^T = 1^{st} row of A = (1 2)

2^{nd} column of A^T = 2^{nd} row of A = (0 3)

Then,

$$\Rightarrow A^T = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$$

Substituting the matrix A and A^T in B,

$$B = \frac{1}{2}\left(\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}\right)$$

$$\Rightarrow B = \frac{1}{2}\begin{bmatrix} 1+1 & 2+0 \\ 0+2 & 3+3 \end{bmatrix}$$

$$\Rightarrow B = \frac{1}{2}\begin{bmatrix} 2 & 2 \\ 2 & 6 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} \frac{2}{2} & \frac{2}{2} \\ \frac{2}{2} & \frac{6}{2} \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

Taking transpose of B,

1^{st} row of B = (1 1)

2^{nd} row of B = (1 3)

Transpose of this matrix B, B^T will be given as

1^{st} column of B^T = 1^{st} row of B = (1 1)

2^{nd} column of A^T = 2^{nd} row of A = (1 3)

Then,

$$B^T = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

Since, B = B^T. Thus, B is symmetric.

Now, let the matrix C be

$$C = \frac{1}{2}(A - A^T)$$

Substituting the matrix A and A^T in C,

$$C = \frac{1}{2}\left(\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}\right)$$

$$\Rightarrow C = \frac{1}{2}\begin{bmatrix} 1-1 & 2-0 \\ 0-2 & 3-3 \end{bmatrix}$$

$$\Rightarrow C = \frac{1}{2}\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} \frac{0}{2} & \frac{2}{2} \\ -\frac{2}{2} & \frac{0}{2} \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Multiplying -1 on both sides,

$$\Rightarrow -C = -1 \times \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow -C = \begin{bmatrix} -1 \times 0 & -1 \times 1 \\ -1 \times -1 & -1 \times 0 \end{bmatrix}$$

$$\Rightarrow -C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Taking transpose of C,

1^{st} row of C = (0 1)

2^{nd} row of C = (-1 0)

Transpose of this matrix C, C^T will be given as

1^{st} column of C^T = 1^{st} row of C = (0 1)

2^{nd} column of C^T = 2^{nd} row of C = (-1 0)

Then,

$$C^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Since, C^T = -C. Thus, C is skew-symmetric.

Check:

$A=\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}=B+C$

Put the value of matrices B and C.

$\Rightarrow \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}=\begin{bmatrix}1 & 1\\1 & 3\end{bmatrix}+\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}=\begin{bmatrix}1+0 & 1+1\\1-1 & 3+0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}=\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$

Matrices B and C satisfies the equation.

Hence, $B=\begin{bmatrix}1 & 1\\1 & 3\end{bmatrix}$.

46. Question

If A is 2 × 3 matrix and B is a matrix such that A^TB and BA^T both are defined, then what is the order of B?

Answer

We are given that,

Order of matrix A = 2 × 3

A^TB and BA^T are defined matrices.

We need to find the order of matrix B.

We know that the transpose of a matrix is a new matrix whose rows are the columns of the original.

So, if the number of rows in matrix A = 2

And, number of columns in matrix A = 3

Then, the number of rows in matrix A^T = number of columns in matrix A = 3

Number of columns in matrix A^T = number of rows in matrix A = 2

So,

Order of matrix A^T can be written as

Order of matrix A^T = 3 × 2

Thus, we have

Number of rows of A^T = 3 …(i)

Number of columns of A^T = 2 …(ii)

If A^TB is defined, that is, it exists, then

Number of columns in A^T = Number of rows in B

⇒ 2 = Number of rows in B [from (ii)]

Or,

Number of rows in B = 2 …(iii)

If BA^T is defined, that is, it exists, then

Number of columns in B = Number of rows in A^T

Substituting value of number of rows in A^T from (i),

⇒ Number of columns in B = 3 …(iv)

From (iii) and (iv),

Order of B = Number of rows × Number of columns

⇒ Order of B = 2 × 3

Thus, order of B is 2 × 3.

47. Question

What is the total number of 2 × 2 matrices with each entry 0 or 1?

Answer

We are given with the information that,

Each element of the 2 × 2 matrix can be filled in 2 ways, either 0 or 1.

We need to find the number of total 2 × 2 matrices with each entry 0 or 1.

Let A be 2 × 2 matrix such that,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Note that, there are 4 elements in the matrix.

So, if 1 element can be filled in 2 ways, either 0 or 1.

That is,

Number of ways in which 1 element can be filled = 2^1

Then,

Number of ways in which 4 elements can be filled = 2^4

⇒ Number of ways in which 4 elements can be filled = 16

Thus, total number of 2 × 2 matrices with each entry 0 or 1 is 16.

48. Question

If $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$, then find the value of y.

Answer

We are given that,

$$\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$$

We need to find the value of y.

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

So, if we have

$$\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$$

Corresponding elements of two elements are equal.

That is,

x = 3 …(i)

x – y = 1 …(ii)

2x + y = 8 …(iii)

7 = 7

To solve for y, we have equations (i), (ii) and (iii).

From equation (i),

x = 3

Substituting the value of x = 3 in equation (ii),

x – y = 1

⇒ 3 – y = 1

⇒ y = 3 – 1

⇒ y = 2

Thus, we get y = 2.

49. Question

If a matrix has 5 elements, write all possible orders it can have.

Answer

We are given that,

A matrix has 5 elements.

We need to find all the possible orders.

We know that if there is a matrix A, of order m × n.

Then, there are mn elements.

Or,

If a matrix has mn elements, then

The order of the matrix = m × n or n × m

For example,

If a matrix is of order 1 × 2, then

There are 2 elements in the matrix.

$[a_{11} \quad a_{12}]_{1\times 2} = 2\ elements$

Or,

If a matrix is of order 2 × 1, then

There are 2 elements in the matrix.

$\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}_{2\times 1} = 2\ elements$

Similarly,

If a matrix has 5 elements, then

The order of this matrix are 1 × 5 or 5 × 1.

Thus, possible orders of a matrix having 5 elements are 1 × 5 and 5 × 1.

50. Question

For a 2 × 2 matrix A = [a_{ij}] whose elements are given by $a_{ij} = \frac{i}{j}$, write the value of a_{12}.

Answer

We are given with,

A matrix of order 2 × 2, A = [a_{ij}].

$$a_{ij} = \frac{i}{j}$$

We need to find the value of a_{12}.

Here, if A is of the order 2 × 2 then,

Number of rows of A = 2

Number of columns of A = 2

We can easily find the elements using the representation of element, $a_{ij} = \frac{i}{j}$.

Compare a_{ij} with a_{12}.

We get,

i = 1

j = 2

Putting these values in $a_{ij} = \frac{i}{j}$,

$$a_{12} = \frac{1}{2}$$

Thus, the value of $a_{12} = \frac{1}{2}$.

51. Question

If $x\begin{bmatrix}2\\3\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$, find the value of x.

Answer

We are given with,

$$x\begin{bmatrix}2\\3\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$$

We need to find the value of x.

By property of matrices,

$$r\begin{bmatrix}a_{11}\\a_{21}\end{bmatrix} = \begin{bmatrix}r \times a_{11}\\r \times a_{21}\end{bmatrix}$$

Similarly,

$$x\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}x \times 2\\x \times 3\end{bmatrix}$$

$$\Rightarrow x\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}2x\\3x\end{bmatrix} \text{ ...(i)}$$

And,

$y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}y\times -1\\y\times 1\end{bmatrix}$

$\Rightarrow y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}-y\\y\end{bmatrix}$ ...(ii)

Adding equations (i) and (ii),

$x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}2x\\3x\end{bmatrix}+\begin{bmatrix}-y\\y\end{bmatrix}$

Now,

Since $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

$\Rightarrow \begin{bmatrix}10\\5\end{bmatrix}=\begin{bmatrix}2x\\3x\end{bmatrix}+\begin{bmatrix}-y\\y\end{bmatrix}$

Adding the two matrices on the right hand side by simply adding the corresponding elements,

$\Rightarrow \begin{bmatrix}10\\5\end{bmatrix}=\begin{bmatrix}2x+(-y)\\3x+y\end{bmatrix}$

$\Rightarrow \begin{bmatrix}10\\5\end{bmatrix}=\begin{bmatrix}2x-y\\3x+y\end{bmatrix}$

We know by the property of matrices,

$\begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}=\begin{bmatrix}b_{11} & b_{12}\\b_{21} & b_{22}\end{bmatrix}$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

So, this means that we can get two equations,

10 = 2x – y ...(iii)

5 = 3x + y ...(iv)

We have two equations and two variables.

Solving equations (iii) and (iv),

$$\begin{array}{l}2x - y = 10\\3x + y = 5\\\hline 5x + 0 = 15\\\hline\end{array}$$

⇒ 5x = 15

$\Rightarrow x=\frac{15}{5}$

⇒ x = 3

Thus, we get x = 3.

52. Question

If $\begin{bmatrix}9 & -1 & 4\\-2 & 1 & 3\end{bmatrix}=A+\begin{bmatrix}1 & 2 & -1\\0 & 4 & 9\end{bmatrix}$, then find matrix A.

Answer

We are given that,

$$\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$$

We need to find the matrix A.

In order to find A, shift the matrix in addition with A to left hand side of the equation.

Just like in algebraic property,

X = A + Y

⇒ A = X – Y

Similarly,

$$\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$$

Subtraction in matrices is done by subtraction of corresponding elements in the matrices.

$$\Rightarrow A = \begin{bmatrix} 9-1 & -1-2 & 4-(-1) \\ -2-0 & 1-4 & 3-9 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$$

Thus, we get $A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$.

53. Question

If $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, find the value of b.

Answer

We are given that,

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

We need to find the value of b.

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

Similarly,

a – b = -1 …(i)

2a + c = 5 …(ii)

2a – b = 0 …(iii)

3c + d = 13 …(iv)

We have the equations (i), (ii), (iii) and (iv).

We need not solve equations (ii) and (iv). We will be able to solve for b from equations (i) and (iii).

Multiply equation (i) by 2.

a – b = -1 [× 2

⇒ 2a – 2b = -2 ...(v)

Subtracting equation (iii) from equation (v),

$$\begin{array}{r} 2a - 2b = -2 \\ 2a - b = 0 \\ (-)\ \ (+)\ \ \ (-) \\ \hline 0 - b = -2 \\ \hline \end{array}$$

⇒ -b = -2

⇒ b = 2

Thus, the value of b = 2.

54. Question

For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix?

Answer

We are given that,

$A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ is a skew-symmetric matrix.

We need to find the value of x.

Let us understand what skew-symmetric matrix is.

A skew-symmetric matrix is a square matrix whose transpose equals its negative, that, it satisfies the condition

A skew symmetric matrix ⇔ A^T = -A

First, let us find –A.

$$-A = -1 \times \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$$

$$\Rightarrow -A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -x & 3 & 0 \end{bmatrix}$$

Let us find the transpose of A.

We know that the transpose of a matrix is a new matrix whose rows are the columns of the original.

In matrix A,

1^{st} row of A = (0 1 -2)

2^{nd} row of A = (-1 0 3)

3^{rd} row of A = (x -3 0)

In the formation of matrix A^T,

1^{st} column of A^T = 1^{st} row of A = (0 1 -2)

2^{nd} column of A^T = 2^{nd} row of A = (-1 0 3)

3^{rd} column of A^T = 3^{rd} row of A = (x -3 0)

So,

$$A^T = \begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$$

Substituting the matrices –A and A^T, we get

$-A = A^T$

$$\Rightarrow \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -x & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$$

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

By comparing the corresponding elements of the two matrices,

x = 2

Thus, the value of x = 2.

55. Question

If matrix $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and A^2 = pA, then write the value of p.

Answer

We are given that,

$$A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

A^2 = pA

We need to find the value of p.

First, let us find A^2.

We know that, A^2 = A.A

$$\Rightarrow A^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

In multiplication of matrices A and A, such that A^2 = Z(say):

$$A^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

For the calculation of z_{11}: Dot multiply the 1^{st} row of first matrix and the 1^{st} column of second matrix and then sum up.

(2 -2)(2 -2) = 2 × 2 + (-2) × (-2)

⇒ (2 -2)(2 -2) = 4 + 4

⇒ (2 -2)(2 -2) = 8

So,

$$\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

For the calculation of z_{12}: Dot multiply the 1st row of first matrix and the 2nd column of second matrix and then sum up.

(2 -2)(-2 2) = 2 × -2 + (-2) × 2

⇒ (2 -2)(-2 2) = -4 – 4

⇒ (2 -2)(-2 2) = -8

So,

$$\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ z_{21} & z_{22} \end{bmatrix}$$

Similarly,

$$\Rightarrow \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ (-2\times 2)+(2\times -2) & (-2\times -2)+(2\times 2) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -4-4 & 4+4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

So,

$$A^2 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \ldots\text{(i)}$$

Now, let us find pA.

Multiply p by matrix A,

$$pA = p \times \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$\Rightarrow pA = \begin{bmatrix} p\times 2 & p\times -2 \\ p\times -2 & p\times 2 \end{bmatrix}$$

$$\Rightarrow pA = \begin{bmatrix} 2p & -2p \\ -2p & 2p \end{bmatrix} \ldots\text{(ii)}$$

Substituting value of A^2 and pA from (i) and (ii) in

A^2 = pA

$$\Rightarrow \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 2p & -2p \\ -2p & 2p \end{bmatrix}$$

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

So,

2p = 8

-2p = -8

-2p = -8

2p = 8

Take equation,

2p = 8

$\Rightarrow p = \frac{8}{2}$

⇒ p = 4

Thus, the value of p = 4.

56. Question

If A is a square matrix such that $A^2 = A$, then write the value of $7A–(I + A)^3$, where I is the identity matrix.

Answer

We are given that,

A is a square matrix such that,

$A^2 = A$

I is an identity matrix.

We need to find the value of $7A – (I + A)^3$.

Take,

$7A – (I + A)^3 = 7A – (I^3 + A^3 + 3I^2A + 3IA^2)$

[∵, by algebraic identity, $(x + y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$]

$\Rightarrow 7A – (I + A)^3 = 7A – I^3 – A^3 – 3I^2A – 3IA^2$

$\Rightarrow 7A – (I + A)^3 = 7A – I – A^3 – 3I^2A – 3IA^2$

$\Rightarrow 7A – (I + A)^3 = 7A – I – A.A^2 – 3I^2A – 3IA^2$

$\Rightarrow 7A – (I + A)^3 = 7A – I – A.A^2 – 3A – 3A^2$

[∵, by property of identity matrix,

$I^2A = A$ & $IA^2 = A^2$]

$\Rightarrow 7A – (I + A)^3 = 7A – I – A.A – 3A – 3A$

[∵, it is given that, $A^2 = A$]

$\Rightarrow 7A – (I + A)^3 = 7A – I – A^2 – 6A$

[∵, $A.A = A^2$]

$\Rightarrow 7A – (I + A)^3 = 7A – I – A – 6A$

[∵, it is given that, $A^2 = A$]

$\Rightarrow 7A – (I + A)^3 = 7A – I – 7A$

$\Rightarrow 7A – (I + A)^3 = -I$

Thus, the value of $7A – (I + A)^3$ is –I.

57. Question

If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find x – y.

Answer

We are given that,

$$2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

We need to find the value of (x – y).

Take,

$$2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

Multiplying 2 by each element of the matrix,

$$\Rightarrow \begin{bmatrix} 2 \times 3 & 2 \times 4 \\ 2 \times 5 & 2 \times x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

In addition of matrices, we need to add the corresponding elements of the matrices.

So,

$$\Rightarrow \begin{bmatrix} 6+1 & 8+y \\ 10+0 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

So,

7 = 7

8 + y = 0 …(i)

10 = 10

2x + 1 = 5 …(ii)

Let us find x and y using the equations (i) and (ii).

From equation (i),

8 + y = 0

⇒ y = -8

From equation (ii),

2x + 1 = 5

⇒ 2x = 5 – 1

⇒ 2x = 4

$\Rightarrow x = \frac{4}{2}$

⇒ x = 2

So, x = 2 and y = -8.

Then,

x – y = 2 – (-8)

⇒ x – y = 2 + 8

⇒ x – y = 10

Thus, the value of (x – y) is 10.

58. Question

If $\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$, find x.

Answer

We are given that,

$[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$

We need to find the value of x.

Let matrices be,

$A = [x \quad 1]$

$B = \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$

Then,

Order of A = 1 × 2 [∵, Matrix A has 1 row and 2 columns]

Order of B = 2 × 2 [∵, Matrix B has 2 rows and 2 columns]

Since,

Number of columns in A = Number of rows in B = 2

∴, Order of resulting matrix AB will be 1 × 2.

Resulting matrix = O

O is zero-matrix, where every element of the matrix is zero.

Order of O = 1 × 2

That is,

$O = [0 \quad 0]$

So,

$[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = [0 \quad 0]$ ...(i)

Let,

$[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = [z_{11} \quad z_{12}]$

Let us solve the left hand side of the matrix equation.

In multiplication of matrices,

For z_{11}: Dot multiply 1st row of first matrix and 1st column of second matrix, and then sum up.

(x 1)(1 -2) = x × 1 + 1 × -2

⇒ (x 1)(1 -2) = x – 2

So,

$$[x \quad 1]\begin{bmatrix}1 & 0\\ -2 & 0\end{bmatrix} = [x-2 \quad z_{12}]$$

For z_{12}: Dot multiply 1st row of first matrix and 2nd column of second matrix, and then sum up.

(x 1)(0 0) = x × 0 + 1 × 0

⇒ (x 1)(0 0) = 0 + 0

⇒ (x 1)(0 0) = 0

So,

$$[x \quad 1]\begin{bmatrix}1 & 0\\ -2 & 0\end{bmatrix} = [x-2 \quad 0]$$

Substituting the resulting matrix in left hand side of (i),

⇒ $[x-2 \quad 0] = [0 \quad 0]$

We know by the property of matrices,

$$\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} = \begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

Therefore,

x – 2 = 0

⇒ x = 2

Thus, the value of x is 2.

59. Question

If $\begin{bmatrix}a+4 & 3b\\ 8 & -6\end{bmatrix} = \begin{bmatrix}2a+2 & b+2\\ 8 & a-8b\end{bmatrix}$, write the value of a – 2b.

Answer

We are given that,

$$\begin{bmatrix}a+4 & 3b\\ 8 & -6\end{bmatrix} = \begin{bmatrix}2a+2 & b+2\\ 8 & a-8b\end{bmatrix}$$

We need to find the value of a – 2b.

We know by the property of matrices,

$$\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} = \begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

Therefore,

a + 4 = 2a + 2 …(i)

3b = b + 2 …(ii)

8 = 8

-6 = a – 8b …(iii)

We have the equations (i), (ii) and (iii).

From equation (i),

a + 4 = 2a + 2

⇒ 2a – a = 4 – 2

⇒ a = 2

From equation (ii),

3b = b + 2

⇒ 3b – b = 2

⇒ 2b = 2

$\Rightarrow b = \frac{2}{2}$

⇒ b = 1

We have a = 2 and b = 1.

Substituting the values of a and b in

a – 2b = 2 – 2(1)

⇒ a – 2b = 2 – 2

⇒ a – 2b = 0

Thus, the value of a – 2b is 0.

60. Question

Write a 2 × 2 matrix which is both symmetric and skew-symmetric.

Answer

We need to find a matrix of order 2 × 2 which is both symmetric and skew-symmetric.

We must understand what symmetric matrix is.

A symmetric matrix is a square matrix that is equal to its transpose.

A symmetric matrix ⇔ $A = A^T$

Now, let us understand what skew-symmetric matrix is.

A skew-symmetric matrix is a square matrix whose transpose equals its negative, that, it satisfies the condition

A skew symmetric matrix ⇔ $A^T = -A$

And,

A square matrix is a matrix with the same number of rows and columns. An n-by-n matrix is known as a square matrix of order n.

Take a 2 × 2 null matrix.

Say,

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Let us take transpose of the matrix A.

We know that, the transpose of a matrix is a new matrix whose rows are the columns of the original.

So,

$A^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Since, A = A^T.

∴, A is symmetric.

Take the same matrix and multiply it with -1.

$-A = -1 \times \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow -A = -\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow -A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Let us take transpose of the matrix –A.

So,

$-A^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Since,

A^T = -A

∴, A is skew-symmetric.

Thus, A (a null matrix) of order 2 × 2 is both symmetric as well as skew-symmetric.

61. Question

If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, write the value of (x + y + z).

Answer

We are given that,

$\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

We need to find the value of (x + y + z).

We know by the property of matrices,

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

We have,

$\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

Therefore,

xy = 8 ...(i)

4 = w ...(ii)

z + 6 = 0 ...(iii)

x + y = 6 ...(iv)

We have the equations (i), (ii), (iii) and (iv).

We just need to find the values of x, y and z. So,

From equation (iii),

z + 6 = 0

⇒ z = -6

Now, let us find (x + y + z).

Substituting z = -6 and the value of (x + y) from equation (iv),

x + y + z = (x + y) + z

⇒ x + y + z = 6 + (-6)

⇒ x + y + z = 6 – 6

⇒ x + y + z = 0

Thus, the value of (x + y + z) is 0.

62. Question

Construct a 2 × 2 matrix A = [a_{ij}] whose elements a_{ij} are given by $a_{ij} = \begin{cases} \dfrac{|-3i + j|}{2}, & \text{if } i \neq j \\ (I + j)^2, & \text{if } i = j \end{cases}$.

Answer

We are given that a_{ij} is given as

$$a_{ij} = \begin{cases} \dfrac{|-3i + j|}{2}, if\ i \neq j \\ (i + j)^2, if\ i = j \end{cases}$$

We need to construct a 2 × 2 matrix A defined as A = [a_{ij}].

Since, this is a 2 × 2 matrix where A = [a_{ij}], we know

Number of rows = 2

Number of columns = 2

∴, i = 1, 2

& j = 1, 2

First, put i = 1 and j = 1 in a_{ij}, here i = j.

For i = j,

$a_{ij} = (i + j)^2$

So,

$a_{11} = (1 + 1)^2$

$\Rightarrow a_{11} = 2^2$

$\Rightarrow a$ �$_{11} = 4$

Put i = 1 and j = 2 in a_{ij}, here $i \neq j$.

For $i \neq j$,

$$a_{ij} = \frac{|-3i + j|}{2}$$

So,

$$a_{12} = \frac{|-3(1) + (2)|}{2}$$

$$\Rightarrow a_{12} = \frac{|-3 + 2|}{2}$$

$$\Rightarrow a_{12} = \frac{|-1|}{2}$$

$$\Rightarrow a_{12} = \frac{1}{2}$$

Put i = 2 and j = 1 in a_{ij}, here $i \neq j$.

For $i \neq j$,

$$a_{ij} = \frac{|-3i + j|}{2}$$

So,

$$a_{21} = \frac{|-3(2) + 1|}{2}$$

$$\Rightarrow a_{21} = \frac{|-6 + 1|}{2}$$

$$\Rightarrow a_{21} = \frac{|-5|}{2}$$

$$\Rightarrow a_{21} = \frac{5}{2}$$

Put i = 2 and j = 2 in a_{ij}, here i = j.

For i = j,

$a_{ij} = (i + j)^2$

So,

$a_{22} = (2 + 2)^2$

$\Rightarrow a_{22} = 4^2$

$\Rightarrow a_{22} = 16$

Thus, we get

$$A = \begin{bmatrix} 4 & \frac{1}{2} \\ \frac{5}{2} & 16 \end{bmatrix}$$

63. Question

If $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then write the value of (x, y).

Answer

We are given that,

$$\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

We need to find the value of (x, y).

Multiply the matrices on the right hand side of the equation,

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} z_{11} \\ z_{21} \end{bmatrix} \ (say)$$

For z_{11}: Dot multiply the 1st row of first matrix and 1st column of second matrix, then sum up.

(2 1)(1 -2) = 2 × 1 + 1 × -2

⇒ (2 1)(1 -2) = 2 – 2

⇒ (2 1)(1 -2) = 0

So,

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ z_{21} \end{bmatrix}$$

For z_{21}: Dot multiply the 2nd row of first matrix and 1st column of second matrix, then sum up.

(4 3)(1 -2) = 4 × 1 + 3 × (-2)

⇒ (4 3)(1 -2) = 4 – 6

⇒ (4 3)(1 -2) = -2

So,

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

Equate the resulting matrix to the given matrix equation.

$$\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

Therefore,

x + y = 0

x – y = -2

Adding these two equations, we get

(x + y) + (x – y) = 0 + (-2)

⇒ x + y + x – y = -2

⇒ x + x + y – y = -2

⇒ 2x + 0 = -2

⇒ 2x = -2

⇒ $x = -\frac{2}{2}$

⇒ x = -1

Putting x = -1 in

x + y = 0

⇒ (-1) + y = 0

⇒ -1 + y = 0

⇒ y = 1

So, putting values of x and y from above in (x, y), we get

(x, y) = (-1, 1)

Thus, (x, y) is (-1, 1).

64. Question

Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find the values of a and b.

Answer

We are given that,

$A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is symmetric matrix.

We need to find the values of a and b.

We must understand what symmetric matrix is.

A symmetric matrix is a square matrix that is equal to its transpose.

A symmetric matrix ⇔ A = A^T

This means, we need to find the transpose of matrix A.

Let us take transpose of the matrix A.

We know that, the transpose of a matrix is a new matrix whose rows are the columns of the original.

We have,

1^{st} row of matrix A = (0 2b -2)

2^{nd} row of matrix A = (3 1 3)

3^{rd} row of matrix A = (3a 3 -1)

For matrix A^T, it will become

1^{st} column of A^T = 1^{st} row of A = (0 2b -2)

2^{nd} column of A^T = 2^{nd} row of A = (3 1 3)

3^{rd} column of A^T = 3^{rd} row of A = (3a 3 -1)

$$\therefore, A^T = \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix}$$

Now, as A = A^T.

Substituting the matrices A and A^T, we get

$$\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix}$$

We know by the property of matrices,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

This implies,

$a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

Applying this property, we can write

2b = 3 …(i)

-2 = 3a …(ii)

3 = 2b

3a = -2

We can find a and b from equations (i) and (ii).

From equation (i),

2b = 3

$$\Rightarrow b = \frac{3}{2}$$

From equation (ii),

-2 = 3a

$$\Rightarrow a = -\frac{2}{3}$$

Thus, we get $a = -\frac{2}{3}$ and $b = \frac{3}{2}$.

65. Question

Write the number of all possible matrices of order 2 × 2 with each entry 1, 2 or 3.

Answer

We are given with the information that,

Each element of the 2 × 2 matrix can be filled in 3 ways, either 1, 2 or 3.

We need to find the number of total 2 × 2 matrices with each entry 1, 2 or 3.

Let A be 2 × 2 matrix such that,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Note that, there are 4 elements in the matrix.

So, if 1 element can be filled in 3 ways, either 1, 2 or 3.

That is,

Number of ways in which 1 element can be filled = 3^1

Then,

Number of ways in which 4 elements can be filled = 3^4

⇒ Number of ways in which 4 elements can be filled = 81

Thus, total number of 2 × 2 matrices with each entry 1, 2 or 3 is 81.

66. Question

If $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$, then write the order of matrix A.

Answer

We are given that,

$$\begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$$

We need to find the order of the matrix A.

Let the matrices be,

$$X = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$$

$$Y = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Let us find the order of X.

Number of rows of matrix X = 1

Number of columns of matrix X = 3

So, order of matrix X = 1 × 3 …(i)

Now, let us find the order of Y.

Number of rows of matrix Y = 3

Number of columns of matrix Y = 3

So, order of matrix Y = 3 × 3 …(ii)

From (i) and (ii),

Order of resulting XY = 1 × 3 [∵, Number of columns of X = Number of rows of Y] …(iii)

Let us find the order of Z.

Number of rows of matrix Z = 3

Number of columns of matrix Z = 1

So, order of matrix Z = 3 × 1 …(iv)

Order of resulting XYZ = 1 × 1 [∵, Number of columns of XY = Number of rows of Z]

Thus, the order of matrix A = 1 × 1

67. Question

If $A = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$ is written as A = P + Q, where as A = P + Q, where P is symmetric and Q is skew-symmetric matrix, then write the matrix P.

Answer

We are given that,

$$A = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix} = P + Q$$

Where,

P = symmetric matrix

Q = skew-symmetric matrix

We need to find P.

A symmetric matrix is a square matrix that is equal to its transpose.

A symmetric matrix ⇔ $P = P^T$

Now, let us understand what skew-symmetric matrix is.

A skew-symmetric matrix is a square matrix whose transpose equals its negative, that, it satisfies the condition

A skew symmetric matrix ⇔ $Q^T = -Q$

So, let the matrix P be

$$P = \frac{1}{2}(A + A^T)$$

Let us calculate A^T.

We know that the transpose of a matrix is a new matrix whose rows are the columns of the original.

We have,

$$A = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$$

Here,

1^{st} row of A = (3 5)

2^{nd} row of A = (7 9)

Transpose of this matrix A, A^T will be given as

1^{st} column of A^T = 1^{st} row of A = (3 5)

2^{nd} column of A^T = 2^{nd} row of A = (7 9)

Then,

$$\Rightarrow A^T = \begin{bmatrix} 3 & 7 \\ 5 & 9 \end{bmatrix}$$

Substituting the matrix A and A^T in P,

$$P = \frac{1}{2}\left(\begin{bmatrix}3 & 5\\7 & 9\end{bmatrix} + \begin{bmatrix}3 & 7\\5 & 9\end{bmatrix}\right)$$

$$\Rightarrow P = \frac{1}{2}\begin{bmatrix}3+3 & 5+7\\7+5 & 9+9\end{bmatrix}$$

$$\Rightarrow P = \frac{1}{2}\begin{bmatrix}6 & 12\\12 & 18\end{bmatrix}$$

$$\Rightarrow P = \begin{bmatrix}\frac{6}{2} & \frac{12}{2}\\\frac{12}{2} & \frac{18}{2}\end{bmatrix}$$

$$\Rightarrow P = \begin{bmatrix}3 & 6\\6 & 9\end{bmatrix}$$

Taking transpose of P,

1^{st} row of P = (3 6)

2^{nd} row of P = (6 9)

Transpose of this matrix P, P^T will be given as

1^{st} column of P^T = 1^{st} row of P = (3 6)

2^{nd} column of P^T = 2^{nd} row of P = (6 9)

Then,

$$P^T = \begin{bmatrix}3 & 6\\6 & 9\end{bmatrix}$$

Since, P = P^T. Thus, P is symmetric.

Now, let the matrix Q be

$$Q = \frac{1}{2}(A - A^T)$$

Substituting the matrix A and A^T in Q,

$$Q = \frac{1}{2}\left(\begin{bmatrix}3 & 5\\7 & 9\end{bmatrix} - \begin{bmatrix}3 & 7\\5 & 9\end{bmatrix}\right)$$

$$\Rightarrow Q = \frac{1}{2}\begin{bmatrix}3-3 & 5-7\\7-5 & 9-9\end{bmatrix}$$

$$\Rightarrow Q = \frac{1}{2}\begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}$$

$$\Rightarrow Q = \begin{bmatrix}\frac{0}{2} & -\frac{2}{2}\\\frac{2}{2} & \frac{0}{2}\end{bmatrix}$$

$$\Rightarrow Q = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$$

Multiplying -1 on both sides,

$$\Rightarrow -Q = -1 \times \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$$

$$\Rightarrow -Q = \begin{bmatrix}-1\times 0 & -1\times -1\\-1\times 1 & -1\times 0\end{bmatrix}$$

$\Rightarrow -Q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Taking transpose of Q,

1st row of Q = (0 -1)

2nd row of Q = (1 0)

Transpose of this matrix Q, Q^T will be given as

1st column of Q^T = 1st row of Q = (0 -1)

2nd column of Q^T = 2nd row of Q = (1 0)

Then,

$Q^T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Since, $Q^T = -Q$. Thus, Q is skew-symmetric.

Check:

$A = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix} = P + Q$

Put the value of matrices P and Q.

$\Rightarrow \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 3+0 & 6-1 \\ 6+1 & 9+0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$

Matrices P and Q satisfies the equation.

Hence, $P = \begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix}$.

68. Question

Let A and B be matrices of orders 3 × 2 and 2 × 4 respectively. Write the order of matrix AB.

Answer

We are given that,

Order of matrix A = 3 × 2

Order of matrix B = 2 × 4

We need to find the order of matrix AB.

We know that,

Matrix A × Matrix B = Matrix AB

If order of matrix A is (m × n) and order of matrix B is (r × s), then matrices A and B can be multiplied if and only if n = r.

That is,

Number of columns in A = Number of rows in B

Also, the order of resulting matrix AB comes out to be m × s.

Applying it,

Number of columns in A = 2

Number of rows in B = 2

This means,

Matrices A and B can be multiplied, and its order will be given as:

Order of matrix AB = 3 × 4

Thus, order of matrix AB = 3 × 4

MCQ

1. Question

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to

A. a null matrix

B. a unit matrix

C. –A

D. A

Answer

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ a+0-a & 0+b-b & 0+0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, the Option (B) is correct, as the main diagonal elements are 1 except other which are 0.

2. Question

If $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, $n \in N$, the A^{4n} equals

A. $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

Answer

$A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

$A^{4n} = A^4 = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \times \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \times \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \times \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ {n = 1, so the exponent comes out to be 4 and if n = 2, which will turn the exponent to 8, and the same cycle will repeat.}

$= \begin{bmatrix} i^4 & 0 \\ 0 & i^4 \end{bmatrix}$

$= \begin{bmatrix} (-1)^2 & 0 \\ 0 & (-1)^2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Option (C) is the answer.

3. Question

If A and B are two matrices such that AB = A and BA = B, then B^2 is equal to

A. B

B. A

C. 1

D. 0

Answer

AB = A ----- (i)

BA = B ------(ii)

From equation (ii)

B × (AB) = B

$B^2A = B$

From equation (ii)

$B^2A = BA$

$B^2 = B$

Option (A) is the answer.

4. Question

If AB = A and BA = B, where A and B are square matrices, then

A. $B^2 = B$ and $A^2 = A$

B. $B^2 \neq B$ and $A^2 = A$

C. $A^2 \neq A$, $B^2 = B$

D. $A^2 \neq A$, $B^2 \neq B$

Answer

AB = A ----- (i)

BA = B ------(ii)

From equation (ii)

B × (AB) = B

$B^2A = B$

From equation (ii)

$B^2A = BA$

$B^2 = B$

From equation (i)

$A \times (BA) = A$

$A^2B = A$

From equation (i)

$A^2B = AB$

$A^2 = A$

Hence, $A^2 = A$ & $B^2 = B$.

Option (A) is the correct answer.

5. Question

If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ is equal to

A. 2AB

B. 2BA

C. A + B

D. AB

Answer

AB = A ----- (i)

BA = B ------(ii)

From equation (ii)

$B \times (AB) = B$

$B^2A = B$

From equation (ii)

$B^2A = BA$

$B^2 = B$

From equation (i)

$A \times (BA) = A$

$A^2B = A$

From equation (i)

$A^2B = AB$

$A^2 = A$

Hence, $A^2 + B^2 = A + B$.

Option (C) is the correct answer.

6. Question

If $\begin{bmatrix} \cos\frac{2\pi}{7} & -\sin\frac{2\pi}{7} \\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the least positive integral value of k is

A. 3

B. 4

C. 6

D. 7

Answer

$$\begin{bmatrix} \mathrm{Cos}\frac{2\pi}{7} & -\mathrm{Sin}\frac{2\pi}{7} \\ \mathrm{Sin}\frac{2\pi}{7} & \mathrm{Cos}\frac{2\pi}{7} \end{bmatrix}^k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$|A| = \mathrm{Cos}\frac{2\pi}{7} - \mathrm{Sin}\frac{2\pi}{7}\left(-\mathrm{Sin}\frac{2\pi}{7}\right)$

$= \mathrm{Cos}^2\frac{2\pi}{7} + \mathrm{Sin}^2\frac{2\pi}{7}$

$\mathrm{I} = 1$

I^k = I {K can be anything}

Let $\theta = \frac{2\pi}{7}$

$A^2 = \begin{bmatrix} \mathrm{Cos}\theta & -\mathrm{Sin}\theta \\ \mathrm{Sin}\theta & \mathrm{Cos}\theta \end{bmatrix} \times \begin{bmatrix} \mathrm{Cos}\theta & -\mathrm{Sin}\theta \\ \mathrm{Sin}\theta & \mathrm{Cos}\theta \end{bmatrix}$

$= \begin{bmatrix} \mathrm{Cos}^2\theta - \mathrm{Sin}^2\theta & -\mathrm{Sin}\theta\mathrm{Cos}\theta - \mathrm{Sin}\theta\mathrm{Cos}\theta \\ \mathrm{Sin}\theta\mathrm{Cos}\theta + \mathrm{Sin}\theta\mathrm{Cos}\theta & \mathrm{Cos}^2\theta - \mathrm{Sin}^2\theta \end{bmatrix}$

As $\{\mathrm{Cos}^2\theta - \mathrm{Sin}^2\theta = \mathrm{Cos}2\theta \;\&\; 2\mathrm{Sin}\theta\mathrm{Cos}\theta = \mathrm{Sin}2\theta\}$

$= \begin{bmatrix} \mathrm{Cos}2\theta & -2\mathrm{Sin}\theta\mathrm{Cos}\theta \\ 2\mathrm{Sin}\theta\mathrm{Cos}\theta & \mathrm{Cos}2\theta \end{bmatrix}$

$= \begin{bmatrix} \mathrm{Cos}2\theta & -\mathrm{Sin}2\theta \\ \mathrm{Sin}2\theta & \mathrm{Cos}2\theta \end{bmatrix}$

$A^4 = \begin{bmatrix} \mathrm{Cos}2\theta & -\mathrm{Sin}2\theta \\ \mathrm{Sin}2\theta & \mathrm{Cos}2\theta \end{bmatrix} \times \begin{bmatrix} \mathrm{Cos}2\theta & -\mathrm{Sin}2\theta \\ \mathrm{Sin}2\theta & \mathrm{Cos}2\theta \end{bmatrix}$

$= \begin{bmatrix} \mathrm{Cos}4\theta & -\mathrm{Sin}4\theta \\ \mathrm{Sin}4\theta & \mathrm{Cos}4\theta \end{bmatrix}$

Similarly, $A^7 = \begin{bmatrix} \mathrm{Cos}7\theta & -\mathrm{Sin}7\theta \\ \mathrm{Sin}7\theta & \mathrm{Cos}7\theta \end{bmatrix}$

Hence, $\theta = \frac{2\pi}{7}$

$7\theta = 2\pi$

Multiplying Cos & Sin, to LHS & RHS,

$\mathrm{Cos}7\theta = \mathrm{Cos}2\pi = 1$

$\mathrm{Sin}7\theta = \mathrm{Sin}2\theta = 0$

$$\begin{bmatrix} \cos 7\theta & -\sin 7\theta \\ \sin 7\theta & \cos 7\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, k = 7

$A^7 = I$

Hence, k = 7.

Option (D) is the correct answer.

7. Question

If the matrix AB is zero, then

A. It is not necessary that either A = O or, B = O

B. A = O or B = O

C. A = O and B = O

D. all the above statements are wrong

Answer

If the matrix AB is zero, then, it is not necessary that either A = 0 or, B = 0

Option (A) is the correct answer

8. Question

Let $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then A^n is equal to

A. $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a \end{bmatrix}$

B. $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

C. $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$

D. $\begin{bmatrix} na & 0 & 0 \\ 0 & na & 0 \\ 0 & 0 & na \end{bmatrix}$

Answer

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

$$A^n = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \times \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \times$$

{n times, (where n ∈ N)}

$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \times \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \dots\dots\dots$$

$$A^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$$

Option (C) is the answer.

9. Question

If A, B and are square matrices or order 3, A is non-singular and AB = O, then B is a

A. null matrix

B. singular matrix

C. unit matrix

D. non-singular matrix

Answer

As AB = 0

And Order of the matrices A & B is 3,

Matrix B has to be a null matrix.

Option (A) is the answer.

10. Question

If $A = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n \end{bmatrix}$ and $B = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & c_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, then AB is equal to

A. B

B. nB

C. B^n

D. A + B

Answer

$$AB = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n \end{bmatrix} \times \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$= \begin{bmatrix} na_1 + 0 + 0 & na_2 + 0 + 0 & na_3 + 0 + 0 \\ 0 + nb_1 + 0 & 0 + nb_2 + 0 & 0 + nb_3 + 0 \\ 0 + 0 + nc_1 & 0 + 0 + nc_2 & 0 + 0 + nc_3 \end{bmatrix}$$

$$= \begin{bmatrix} na_1 & na_2 & na_3 \\ nb_1 & nb_2 & nb_3 \\ nc_1 & nc_2 & nc_3 \end{bmatrix}$$

$$= n \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$= nB$

Option (B) is the answer.

11. Question

If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then A^n (where n ∈ N) equals

A. $\begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & n^2a \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & na \\ 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} n & na \\ 0 & n \end{bmatrix}$

Answer

$A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ {n times, (where n ∈ N)}

$A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$

Option (A) is the answer.

12. Question

If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and AB = I_3, then x + y equals

A. 0

B. –1

C. 2

D. none of these

Answer

$AB = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0+0 & -2+2+0 & y+0+x \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & x+y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$I_3 = \begin{bmatrix} 1 & 0 & x+y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence x+y=0

Option (A) is the answer.

13. Question

If $A = \begin{bmatrix} 1 & -1 \\ 2 & - \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then values of a and b are

A. a = 4, b = 1

B. a = 1, b = 4

C. a = 0, b = 4

D. a = 2, b = 4

Answer

$$A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$$

$$= \begin{bmatrix} a+1 & 0 \\ b+2 & -2 \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} a+1 & 0 \\ b+2 & -2 \end{bmatrix} \times \begin{bmatrix} a+1 & 0 \\ b+2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (a+1)^2 + 0 & 0+0 \\ (b+2)(a+1) - 4 - b & 0+4 \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + 2a + 1 & 0 \\ 2 + 2a + b + ab - 4 - b & 4 \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + 2a + 1 & 0 \\ 2a + ab - 2 & 4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1-2 & -1+1 \\ 2-2 & -2+1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \times \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} a^2 + b & a^2 - 1 \\ ab - b & b+1 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} a^2 + b & a^2 - 1 \\ ab - b & b+1 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} a^2 + b - 1 & a^2 - 1 \\ ab - b & b \end{bmatrix}$$

As, $A^2 + B^2 = (A+B)^2$

$$\therefore \begin{bmatrix} a^2 + b - 1 & a^2 - 1 \\ ab - b & b \end{bmatrix} = \begin{bmatrix} a^2 + 2a + 1 & 0 \\ 2 + 2a + b + ab - 4 - b & 4 \end{bmatrix}$$

$a^2 = 1$ & $b = 4$

$a = \pm 1$

Hence Option (B) is the correct answer.

14. Question

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

A. $1 + \alpha^2 + \beta\gamma = 0$

B. $1 - \alpha^2 + \beta\gamma = 0$

C. $1 - \alpha^2 - \beta\gamma = 0$

D. $1 + \alpha^2 - \beta\gamma = 0$

Answer

$$A^2 = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \times \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \gamma\beta + \alpha^2 \end{bmatrix}$$

As $A^2 = I$,

$$A^2 = \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \gamma\beta + \alpha^2 \end{bmatrix} = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, $\alpha^2 + \beta\gamma = 1$

$\alpha^2 + \beta\gamma - 1 = 0$

$1 - \alpha^2 - \beta\gamma$.

Hence, Option (C) is the correct answer.

15. Question

If $S = [s_{ij}]$ is a scalar matrix such that $s_{ij} = k$ and A is a square matrix of the same order, then AS = SA = ?

A. A^k

B. $k + A$

C. kA

D. kS

Answer

$$S = [S_{ij}]$$

$$S = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

As, $S_{ij} = k$

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ {Square Matrix}

$$AS = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$= \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

$$= k \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= kA$$

$SA = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$= \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$

$= k \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$= kA$

Hence, AS = SA = kA

Option (C) is the answer.

16. Question

If A is a square matrix such that $A^2 = A$, then $(I + A)^3 – 7A$ is equal to

A. A

B. I – A

C. I

D. 3A

Answer

$(I + A)^3 = I^3 + A^3 + 3A^2I + 3AI^2$ (Using the identity of $(a + b)^3 = a^3 + b^3 + 3ab (a + b)$)

$(I + A)^3 = I + A^2(A) + 3AI + 3A$ [I stands for Identity Matrix]

$(I + A)^3 = I + A^2 + 3A + 3A$

$(I + A)^3 = 7A + I$

$(I + A)^3 – 7A$

7A + I – 7A

= I

Option (C) is the answer.

17. Question

If a matrix A is both symmetric and skew-symmetric, then

A. A is a diagonal matrix

B. A is a zero matrix

C. A is a scalar matrix

D. A is a square matrix

Answer

If a matrix A is both symmetric and skew-symmetric,

A’ = A & A’ = -A

Comparing both the equations,

A = -A

A + A = 0

2A = 0

A = 0

then A is a zero matrix.

Option (B) is the answer.

18. Question

The matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is

A. a skew-symmetric matrix

B. a symmetric matrix

C. a diagonal matrix

D. an upper triangular matrix

Answer

$$A = \begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & -5 & 7 \\ 5 & 0 & -11 \\ -7 & 11 & 0 \end{bmatrix}$$

$$-A = \begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$$

$\because A^T = -A$

Then, the given matrix is a skew – symmetric matrix.

Option (A) is the answer.

19. Question

If A is a square matrix, then AA is a

A. skew-symmetric matrix

B. symmetric matrix

C. diagonal matrix

D. none of these

Answer

If A is a square matrix,

Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

$$AA = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$$

then AA is neither of the matrices given in the options of the question.

Option (D) is the answer.

20. Question

If A and B are symmetric matrices, then ABA is

A. symmetric matrix

B. skew-symmetric matrix

C. diagonal matrix

D. scalar matrix

Answer

A’ = A & B’ = B

(ABA)’ = A’ (AB)’

= A’B’A’

= ABA

Symmetric Matrix

Option (A) is the answer.

21. Question

If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and A = A^T, then

A. x = 0, y = 5

B. x + y = 5

C. x = y

D. none of these

Answer

A = A^T

$$\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$$

x = y

Option (C) is the answer.

22. Question

If A is 3 × 4 matrix and B is a matrix such that A^TB and BA^T are both defined. Then, B is of the type

A. 3 × 4

B. 3 × 3

C. 4 × 4

D. 4 × 3

Answer

Order of A = 3 × 4

Order of A’ = 4 × 3

As A^TB and BA^T are both defined, so the number of columns in B should be equal to the number of rows in A’ for BA’ and also the number of columns in A’ should be equal to the number of rows in A’ for BA’.

So the order of matrix B = 3 × 4.

Option (A) is the answer.

23. Question

If A = [a_{ij}] is a square matrix of even order such that $a_{ij} = i^2 - j^2$, then

A. A is a skew-symmetric matrix and |A| = 0

B. A is symmetric matrix and |A| is a square

C. A is symmetric matrix and |A| = 0

D. none of these

Answer

$a_{ij} = i^2 - j^2$

a11 = 12 – 12 = 0

a12 = 12 – 22 = -3

a21 = 22 – 12 = 3

a22 = 22 – 22 = 0

$\therefore A = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$-A = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

So, $A^T = -A$

|A| = 0(0) – (-3)(3) = 0 + 9 = 9 ≠ 0

So, none of these.

Option (D) is the answer.

24. Question

If $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then $A^T + A = I_2$, if

A. $\theta = n\pi,\ n \in Z$

B. $\theta = (2n+1)\frac{\pi}{2},\ n \in Z$

C. $\theta = 2n\pi + \frac{\pi}{3}, n \in Z$

D. none of these

Answer

$A = \begin{bmatrix} \mathrm{Cos}\theta & -\mathrm{Sin}\theta \\ \mathrm{Sin}\theta & \mathrm{Cos}\theta \end{bmatrix}$

$A^T = \begin{bmatrix} \mathrm{Cos}\theta & \mathrm{Sin}\theta \\ -\mathrm{Sin}\theta & \mathrm{Cos}\theta \end{bmatrix}$

$A + A^T = \begin{bmatrix} \mathrm{Cos}\theta & -\mathrm{Sin}\theta \\ \mathrm{Sin}\theta & \mathrm{Cos}\theta \end{bmatrix} + \begin{bmatrix} \mathrm{Cos}\theta & \mathrm{Sin}\theta \\ -\mathrm{Sin}\theta & \mathrm{Cos}\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$2\mathrm{Cos}\theta = 1$

$\text{Cos}\theta = \frac{1}{2}$

$\theta = 2n\pi + \frac{\pi}{3}$ {n ∈ Z}

Option (C) is the answer.

25. Question

If $A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$ is expressed as the sum of a symmetric and skew-symmetric matrix, then the symmetric matrix is

A. $\begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Answer

$A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$ & $A' = \begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}$

As, sum is expressed as

$B = \frac{1}{2}(A + A')$ {Newly formed Symmetric Matrix}

$$\frac{1}{2}(A + A') = \frac{1}{2}\left[\begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}\right]$$

$$= \frac{1}{2}\begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -4 \\ 4 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$$

Option (A) is the answer.

26. Question

Out of the following matrices, choose that matrix which is a scalar matrix:

A. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Answer

∵ Scalar Matrix is a matrix whose all off-diagonal elements are zero and all on-diagonal elements are equal.

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Option (A) is the answer.

27. Question

The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is

A. 27

B. 18

C. 81

D. 512

Answer

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Elements = 9 Order = 3 × 3

Every item in this matrix can be filled in two ways either by 0 or by 1.

Possible Matrices = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

= 512

Option (D) is the answer.

28. Question

Which of the given values of x and y make the following pairs of matrices equal?

$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$ and, $\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

A. $x = -\frac{1}{3}, y = 7$

B. $y = 7, x = -\frac{2}{3}$

C. $x = -\frac{1}{3}, 4 = -\frac{2}{5}$

D. Not possible to find

Answer

As the given matrices are equal,

3x + 7 = 0

$x = -\frac{7}{3}$

y – 2 = 5

y = 7

y + 1 = 8

y = 7

2 – 3x = 4

3x = -2

$x = -\frac{2}{3}$

These values of x are not equal to each other, so it is not possible to find.

Option (D) is the answer.

29. Question

If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a, b, are respectively

A. –6, –12, –18

B. –6, 4, 9

C. –6, –4, –9

D. –6, 12, 18

Answer

$A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ & $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix} = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$

Comparing the equations,

-4k = 24

k = -6

3k = 2b

3(-6) = 2b

2b = -18

b= -9

3a = 2k

3a = 2(-6)

3a = -12

a = -4

Values are

k = -6, a = -4 & b = -9

Option (C) is the answer.

30. Question

If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then B equals

A. I cos θ + J sin θ

B. I sin θ + J cos θ

C. I cos θ – J sin θ

D. – I cos θ + J sin θ

Answer

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$I\text{Cos}\theta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{Cos}\theta = \begin{bmatrix} \text{Cos}\theta & 0 \\ 0 & \text{Cos}\theta \end{bmatrix}$

$J\text{Sin}\theta = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{Sin}\theta = \begin{bmatrix} 0 & \text{Sin}\theta \\ -\text{Sin}\theta & 0 \end{bmatrix}$

$I\text{Cos}\theta + J\text{Sin}\theta = \begin{bmatrix} \text{Cos}\theta & 0 \\ 0 & \text{Cos}\theta \end{bmatrix} + \begin{bmatrix} 0 & \text{Sin}\theta \\ -\text{Sin}\theta & 0 \end{bmatrix}$

$= \begin{bmatrix} \text{Cos}\theta & \text{Sin}\theta \\ -\text{Sin}\theta & \text{Cos}\theta \end{bmatrix}$

So, $B = I\text{Cos}\theta + J\text{Sin}\theta$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ & $B = \begin{bmatrix} \text{Cos}\theta & \text{Sin}\theta \\ -\text{Sin}\theta & \text{Cos}\theta \end{bmatrix}$

$I\text{Cos}\theta + J\text{Sin}\theta$

Option (A) is the answer.

31. Question

The trace of the matrix $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$ is

A. 17

B. 25

C. 3

D. 12

Answer

As the trace of a matrix is the sum of on – diagonal elements, So, 1 + 7 + 9 = 17

Trace = 17

Option (A) is the answer.

32. Question

If A = $[a_{ij}]$ is scalar matrix of order n × n such that a_{ij} = k for all i, then trace of A is equal to

A. nk

B. n + k

C. $\frac{n}{k}$

D. none of these

Answer

$\because A = [a_{ij}]_{n\times n}$

Trace of A, i.e., tr (A) = $\sum a_{ij}^{n} i = 1 = a_{11} + a_{22} + \cdots \ldots \ldots \ldots \ldots \ldots . + a_{nn}$

= k + k + k + k + k + ... (n times)

= k(n)

= nk

Option (A) is the answer.

33. Question

The matrix $A = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a

A. square matrix

B. diagonal matrix

C. unit matrix

D. none of these

Answer

None of these

Option (D) is the answer.

34. Question

The number of possible matrices of order 3 × 3 with each entry 2 or 0 is

A. 9

B. 27

C. 81

D. none of these

Answer

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Elements = 9 Order = 3 × 3

Every item in this matrix can be filled in two ways either by 0 or by 2.

Possible Matrices = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

= 512

Option (D) is the answer.

35. Question

If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of x + y is

A. x = 3, y = 1

B. x = 2, y = 3

C. x = 2, y = 4

D. x = 3, y = 3

Answer

Comparing the equations,

2x + y = 7 & 4x = x + 6

3x = 6, x = 2

2(2) + y = 7

y = 7 – 4 = 3

x = 2 & y = 3

Option (B) is the answer.

36. Question

If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

A. A

B. I – A

C. I + A

D. 3A

Answer

Expansion of the given expression,

$A^3 - I^3 + 3AI^2 - 3A^2I + A^3 + I^3 + 3AI^2 + 3A^2I - 7A$

$2A^3 - 7A + 6AI^2$

$2A^2A - 7A + 6A$

$2AI - A$ $\{A^2 = I\}$

2A – A

A

Option (A) is the answer.

37. Question

If A and B are two matrices of order 3 × m and 3 × n respectively and m = n, then the order of 5A – 2B is

A. m × 3

B. 3 × 3

C. m × n

D. 3 × n

Answer

m = n

If A and B are two matrices of order 3 × m and 3 × n respectively and m = n

Then, A & B have same orders as 3 × n each,

So the order of (5A – 2B) should be same as 3 × n.

Option (D) is the answer.

38. Question

If A is a matrix of order m × n and B is a matrix such that AB^T and B^TA are both defined, then the order of matrix B is

A. m × n

B. n × n

C. n × m

D. m × n

Answer

Let $A = [a_{ij}]m \times n$ & $B = [b_{ij}]p \times q$

$B' = [b_{ij}]p \times q$

As AB’, is a defined matrix, {Given}

So n = q

BA’ is also a defined matrix, {Given}

So, p = m

Hence order of B is m × n.

Option (D) is the answer.

39. Question

If A and B are matrices of the same order, then $AB^T – B^TA$ is a

A. skew-symmetric matrix

B. null matrix

C. unit matrix

D. symmetric matrix

Answer

A & B are matrices of same order,

Let K = $(AB^T - BA^T)$

= $(AB^T)^T - (BA^T)^T$

= $(B^T)^T(A)^T - (A^T)^TB^T$

= $BA^T - AB^T$

= $-(AB^T - BA^T)$

= -K

Hence, $(AB^T – B^TA)$ is a skew - symmetric matrix .

Option (A) is the answer.

40. Question

If matrix $A = \left[a_{ij}\right]_{2\times 2}$, where $a_{ij} = \begin{cases} 1, & \text{if } i \neq j \\ 0, & \text{if } i + j \end{cases}$, then A^2 is equal to

A. I

B. A

C. O

D. – I

Answer

As per the given conditions,

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= I, which is an Identity Matrix.

Option (A) is the answer.

41. Question

If $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(\pi x) & \tan^{-1}\left(\frac{\pi}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$, $B = \frac{1}{\pi}\begin{bmatrix} -\cot^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$, then A – B is equal to

A. I

B. 0

C. 2I

D. $\frac{1}{2}I$

Answer

$$A-B=\frac{1}{\pi}\left[\begin{bmatrix}\operatorname{Sin}^{-1}\pi x & \tan^{-1}\frac{x}{\pi}\\ \operatorname{Sin}^{-1}\frac{x}{\pi} & \operatorname{Cot}^{-1}\pi x\end{bmatrix}-\begin{bmatrix}-\operatorname{Cos}^{-1}\pi x & \tan^{-1}\frac{x}{\pi}\\ \operatorname{Sin}^{-1}\frac{x}{\pi} & -\tan^{-1}\pi x\end{bmatrix}\right]$$

$$A-B=\frac{1}{\pi}\begin{bmatrix}\operatorname{Sin}^{-1}\pi x+\operatorname{Cos}^{-1}\pi x & \tan^{-1}\frac{x}{\pi}-\tan^{-1}\frac{x}{\pi}\\ \operatorname{Sin}^{-1}\frac{x}{\pi}-\operatorname{Sin}^{-1}\frac{x}{\pi} & \operatorname{Cot}^{-1}\pi x+\ -\tan^{-1}\pi x\end{bmatrix}$$

$$A-B=\frac{1}{\pi}\begin{bmatrix}\operatorname{Sin}^{-1}\pi x+\operatorname{Cos}^{-1}\pi x & 0\\ 0 & \operatorname{Cot}^{-1}\pi x+\ -\tan^{-1}\pi x\end{bmatrix}$$

$$\because \operatorname{Sin}^{-1}x+\operatorname{Cos}^{-1}x=\frac{\pi}{2}\ \&\ \operatorname{Cot}^{-1}x+\tan^{-1}\pi x=\frac{\pi}{2}$$

$$A-B=\frac{1}{\pi}\begin{bmatrix}\frac{\pi}{2} & 0\\ 0 & \frac{\pi}{2}\end{bmatrix}$$

$$A-B=\begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{bmatrix}$$

$$A-B=\frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

As $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$$A-B=\frac{1}{2}I$$

Option (D) is the answer.

42. Question

If A and B are square matrices of the same order, then (A + B)(A – B) is equal to

A. $A^2 - B^2$

B. $A^2 - BA - AB - B^2$

C. $A^2 - B^2 + BA - AB$

D. $A^2 - BA + B^2 + AB$

Answer

(A + B)(A – B) = A (A – B) + B (A – B)

= A.A – A.B + B.A – B.B

$=A^2$ – A.B + B.A – B.B

= A^2 – AB + BA – BB

Matrix multiplication does not have a commutative property i.e.., A.B ≠ B.A

Hence, Option (C) is the answer.

43. Question

If $A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$, then

A. only AB is defined

B. only BA is defined

C. AB and BA both are defined

D. AB and BA both are not defined

Answer

As the matrices, A & B, both are defined, having orders as 2 × 3 & 3 × 2,

So multiplication of matrices is defined. (AB ≠ BA)

Matrix multiplication is defined only if

$[A]_{m\times n}$ & $[B]_{n\times o}$

$AB = [A]_{2\times 3}$ & $[B]_{3\times 2}$

$BA = [B]_{3\times 2}$ & $[A]_{2\times 3}$

Hence, AB and BA both are defined.

Option (C) is the answer.

44. Question

The matrix $A = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a

A. diagonal matrix

B. symmetric matrix

C. skew-symmetric matrix

D. scalar matrix

Answer

$$A = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 5 & -8 \\ -5 & 0 & -12 \\ 8 & 12 & 0 \end{bmatrix}$$

$$-A = \begin{bmatrix} 0 & 5 & -8 \\ -5 & 0 & -12 \\ 8 & 12 & 0 \end{bmatrix}$$

$\because A^T = -A$

Skew-Symmetric Matrix

Option (C) is the answer.

45. Question

The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

A. identify matrix

B. symmetric matrix

C. skew-symmetric matrix

D. diagonal matrix

Answer

As the elements off – diagonal are not zero unlike the on – diagonal elements, so it is a diagonal matrix.

Option (D) is the answer.