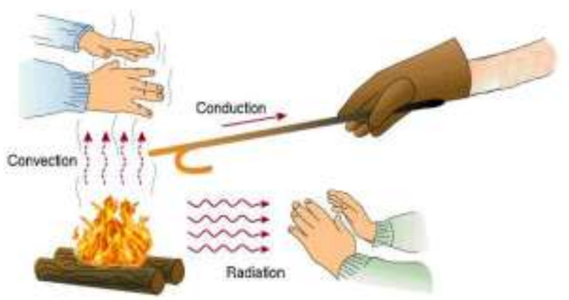
Chapter

# 15

# Transmission of Heat

Heat energy transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by one of the following modes.

**Conduction, Convection** and **Radiation**




## Conduction

The process of transmission of heat energy in which the heat is transferred from one particle to other particle without dislocation of the particle from their equilibrium position is called conduction.

(1) Heat flows from hot end to cold end. Particles of the medium simply oscillate but do not leave their place.

(2) Medium is necessary for conduction

Fig. 15.1

(3) It is a slow process

(4) The temperature of the medium increases through which heat flows

(5) Conduction is a process which is possible in all states of matter.

(6) When liquid and gases are heated from the top, they conduct heat from top to bottom.

(7) In solids only conduction takes place

(8) In non-metallic solids and fluids the conduction takes place only due to vibrations of molecules, therefore they are poor conductors.

(9) In metallic solids free electrons carry the heat energy, therefore they are good conductor of heat.

## Conduction in Metallic Rod

When one end of a metallic rod is heated, heat flows by conduction from the hot end to the cold end.

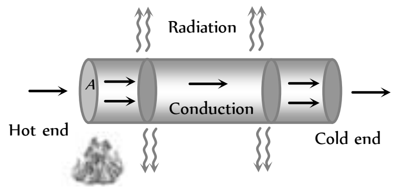



Fig. 15.2

(1) **Variable state :** In this state Temperature of every part of the rod increases

Heat received by each cross-section of the rod from hotter end used in three ways.

(i) A part increases temperature of itself.

(ii) Another part transferred to neighbouring cross-section.

(iii) Remaining part radiates.

- $\theta_1 > \theta_2 > \theta_3 > \theta_4 > \theta_5$
- $\theta \rightarrow$ Changing

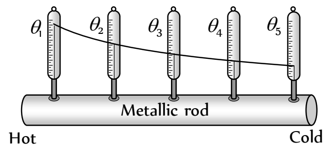



Fig. 15.3

(2) **Steady state :** After sometime, a state is reached when the temperature of every cross-section of the rod becomes constant. In this state, no heat is absorbed by the rod. The heat that reaches any cross-section is transmitted to the next except that a small part of heat is lost to surrounding from the sides by convection & radiation. This state of the rod is called steady state.

(3) **Isothermal surface :** Any surface (within a conductor) having its all points at the same temperature, is called isothermal surface. The direction

of flow of heat through a conductor at any point is perpendicular to the isothermal surface passing through that point.

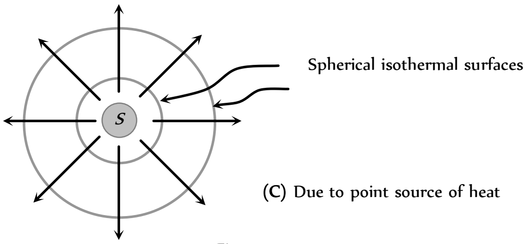


Fig. 15.4

(4) **Temperature gradient** (T.G.) **:** The rate of change of temperature with distance between two isothermal surfaces is called temperature gradient. Hence

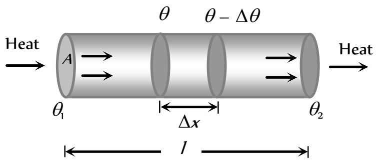


Fig. 15.5

(i) Temperature gradient = $\frac{-\Delta\theta}{\Delta x}$

(ii) The negative sign show that temperature $\theta$ decreases as the distance *x* increases in the direction of heat flow.

(iii) For uniform temperature fall $\frac{\theta_1-\theta_2}{l}=\frac{\Delta\theta}{\Delta x}$

(iv) Unit : *K/m* or *°C/m* (S.I.) and Dimensions $[L^{-1}\theta]$

(5) **Law of thermal conductivity :** Consider a rod of length *l* and area of cross-section *A* whose faces are maintained at temperature $\theta_1$ and $\theta_2$ respectively. The curved surface of rod is kept insulated from surrounding to avoid leakage of heat

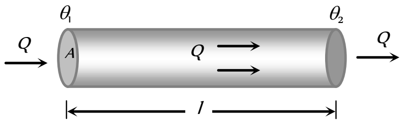


Fig. 15.6

(i) In steady state the amount of heat flowing from one face to the other face in time *t* is given by $Q=\frac{KA(\theta_1-\theta_2)t}{l}$

where *K* is coefficient of thermal conductivity of material of rod.

(ii) Rate of flow of heat *i.e.* heat current $\frac{Q}{t}=H=\frac{KA(\theta_1-\theta_2)}{l}$

(iii) In case of non-steady state or variable cross-section, a more general equation can be used to solve problems.

$$\frac{dQ}{dt}=-KA\frac{d\theta}{dx}$$

(6) **More about *K* :** It is the measure of the ability of a substance to conduct heat through it.

(i) Units : *Cal/cm-sec °C* (in C.G.S.), *kcal/m-sec-K* (in M.K.S.) and *W/m- K* (in S.I.). Dimension : $[MLT^{-3}\theta^{-1}]$

(ii) The magnitude of *K* depends only on nature of the material.

(iii) Substances in which heat flows quickly and easily are known as good conductor of heat. They possesses large thermal conductivity due to large number of free electrons *e.g.* Silver, brass *etc.* For perfect conductors, $K=\infty$.

(iv) Substances which do not permit easy flow of heat are called bad conductors. They possess low thermal conductivity due to very few free electrons *e.g.* Glass, wood *etc.* and for perfect insulators, $K=0$.

(v) The thermal conductivity of pure metals decreases with rise in temperature but for alloys thermal conductivity increases with increase of temperature.

(vi) Human body is a bad conductor of heat (but it is a good conductor of electricity).

(vii) **Decreasing order of conductivity :** For some special cases it is as follows

(a) $K_{Ag}>K_{Cu}>K_{Al}$

(b) $K_{Solid}>K_{Liquid}>K_{Gas}$

(c) $K_{Metals}>K_{Non-metals}$

**Table 15.1 : Thermal conductivity of some material**

| Substance | Thermal conductivity (*W/m-K*) | Substance | Thermal conductivity (*W/m-K*) |
|---|---|---|---|
| Aluminium | 240 | Concrete | 0.9 |
| Copper | 400 | Water | 0.6 |
| Gold | 300 | Glass wool | 0.04 |
| Iron | 80 | Air | 0.024 |
| Lead | 35 | Helium | 0.14 |
| Glass | 0.9 | Hydrogen | 0.17 |
| Wood | 0.1-0.2 | Oxygen | 0.024 |

(7) **Relation between temperature gradient and thermal conductivity :** In steady state, rate of flow of heat $\frac{dQ}{dt}=-KA\frac{d\theta}{dx}$ = – *KA* × (T.G.) ⇒

**(T.G.)** ∝ $\frac{1}{K}$ ($\frac{dQ}{dt}$ = constant)

Temperature difference between the hot end and the cold end in steady state is inversely proportional to *K*, *i.e.* in case of good conductors temperature of the cold end will be very near to hot end.

In ideal conductor where *K* = ∞, temperature difference in steady state will be zero.

(8) **Thermal resistance (*R*) :** The thermal resistance of a body is a measure of its opposition to the flow of heat through it.

It is defined as the ratio of temperature difference to the heat current (= Rate of flow of heat)

(i) Hence $R=\frac{\theta_1-\theta_2}{H}=\frac{\theta_1-\theta_2}{KA(\theta_1-\theta_2)/l}=\frac{l}{KA}$

(ii) **Unit :** $^{o}C\times sec/cal$ or $K\times sec/kcal$ and Dimension : $[M^{-1}L^{-2}T^{3}\theta]$

(9) **Wiedmann-Franz law :** At a given temperature *T*, the ratio of thermal conductivity to electrical conductivity is constant *i.e.*, $(K/\sigma T)$ = constant, *i.e.,* a substance which is a good conductor of heat (*e.g.,* silver) is also a good conductor of electricity. Mica is an exception to above law.

(10) **Thermometric conductivity or diffusivity :** It is a measure of rate of change of temperature (with time) when the body is not in steady state (*i.e.,* in variable state)

It is defined as the ratio of the coefficient of thermal conductivity to the thermal capacity per unit volume of the material. Thermal capacity per unit volume = $\frac{mc}{V}$ = $\rho c$

($\rho$ = density of substance) $\Rightarrow$ Diffusivity (*D*) = $\frac{K}{\rho c}$

**Unit :** *m/sec* and Dimension : $[L^{2}T^{-1}]$

**Table 15.2 : Electrical Analogy for Thermal Conduction**

| Electrical conduction | Thermal conduction |
|---|---|
| Electric charge flows from higher potential to lower potential | Heat flows from higher temperature to lower temperature |
| The rate of flow of charge is called the electric current, *i.e.* $I=\frac{dq}{dt}$ | The rate of flow of heat may be called as heat current *i.e.* $H=\frac{dQ}{dt}$ |
| The relation between the electric current and the potential difference is given by Ohm's law, that is $I=\frac{V_1-V_2}{R}$ where *R* is the electrical resistance of the conductor | Similarly, the heat current may be related with the temperature difference as $H=\frac{\theta_1-\theta_2}{R}$ where *R* is the thermal resistance of the conductor |
| The electrical resistance is defined as $R=\frac{\rho l}{A}=\frac{l}{\sigma A}$ where $\rho$ = Resistivity and $\sigma$ = Electrical conductivity | The thermal resistance may be defined as $R=\frac{l}{KA}$ where *K* = Thermal conductivity of conductor |
| $\frac{dq}{dt}=I=\frac{V_1-V_2}{R}=\frac{\sigma A}{l}(V_1-V_2)$ | $\frac{dQ}{dt}=H=\frac{\theta_1-\theta_2}{R}=\frac{KA}{l}(\theta_1-\theta_2)$ |

## Applications of Conductivity in Daily Life

(1) Cooking utensils are provided with wooden handles, because wood is a poor conductor of heat. The hot utensils can be easily handled from the wooden handles and our hands are saved from burning.


Fig. 15.7

(2) We feel warmer in a fur coat. The air enclosed in the fur coat being bad conductor heat does not allow the body heat to flow outside. Hence we feel warmer in a fur coat.

(3) Eskimos make double walled houses of the blocks of ice. Air enclosed in between the double walls prevents transmission of heat from the house to the cold surroundings.

Fig. 15.8

For exactly the same reason, two thin blankets are warmer than one blanket of their combined thickness. The layer of air enclosed in between the two blankets makes the difference.

(4) Wire gauze is placed over the flame of Bunsen burner while heating the flask or a beaker so that the flame does not go beyond the gauze and hence there is no direct contact between the flame and the flask. The wire gauze being a good conductor of heat, absorb the heat of the flame and transmit it to the flask.

Fig. 15.9

Davy's safety lamp has been designed on this principle. The gases in the mines burn inside the gauze placed around the flame of the lamp. The temperature outside the gauze is not high, so the gases outside the gauze do not catch fire.

(5) Birds often swell their feathers in winter. By doing so, they enclose more air between their bodies and the feathers. The air, being bad conductor of heat prevents the out flow of their body heat. Thus, birds feel warmer in winter by swelling their feathers.

## Combination of Metallic Rods

(1) **Series combination :** Let *n* slabs each of cross-sectional area *A*, lengths $l_1,l_2,l_3,......\ l_n$ and conductivities $K_1,K_2,K_3......\ K_n$ respectively be connected in the series

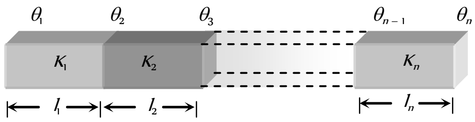



Fig. 15.10

(i) **Heat current :** Heat current is the same in all the conductors.*i.e.,*

$$\frac{Q}{t}=H_1=H_2=H_3.........=H_n$$

$$\frac{K_1A(\theta_1-\theta_2)}{l_1}=\frac{K_2A(\theta_2-\theta_3)}{l_2}=\frac{K_nA(\theta_{n-1}-\theta_n)}{l_n}$$

(ii) **Equivalent thermal resistance :** $R=R_1+R_2+.....R_n$

(iii) **Equivalent thermal conductivity :** It can be calculated as follows

From $R_S=R_1+R_2+R_3+...$

$$\frac{l_1+l_2+...l_n}{K_s}=\frac{l_1}{K_1A}+\frac{l_2}{K_2A}+....+\frac{l_n}{K_nA}$$

$$\Rightarrow K_s=\frac{l_1+l_2+......l_n}{\frac{l_1}{K_1}+\frac{l_2}{K_2}+........\frac{l_n}{K_n}}$$

(a) For *n* slabs of equal length $K_s=\frac{n}{\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}+.....\frac{1}{K_n}}$

(b) For two slabs of equal length, $K_s=\frac{2K_1K_2}{K_1+K_2}$

(iv) **Temperature of interface of composite bar** : Let the two bars are arranged in series as shown in the figure.

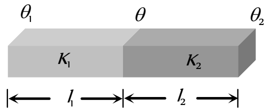



**Fig. 15.11**

Then heat current is same in the two conductors.

*i.e.,* $\frac{Q}{t} = \frac{K_1 A(\theta_1 - \theta)}{l_1} = \frac{K_2 A(\theta - \theta_2)}{l_2}$

By solving we get $\theta = \dfrac{\frac{K_1}{l_1}\theta_1 + \frac{K_2}{l_2}\theta_2}{\frac{K_1}{l_1} + \frac{K_2}{l_2}}$

(a) If $l_1 = l_2$ then $\theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$

(b) If $K_1 = K_2$ and $l_1 = l_2$ then $\theta = \frac{\theta_1 + \theta_2}{2}$

(2) **Parallel Combination :** Let $n$ slabs each of length $l$, areas $A_1, A_2, A_3, ..... A_n$ and thermal conductivities $K_1, K_2, K_3, ..... K_n$ are connected in parallel then

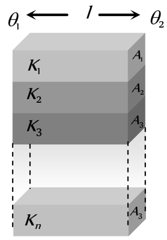



**Fig. 15.12**

(i) **Equivalent resistance :** $\frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ..... \frac{1}{R_n}$

For two slabs $R_s = \frac{R_1 R_2}{R_1 + R_2}$

(ii) **Temperature gradient :** Same across each slab.

(iii) **Heat current :** in each slab will be different. Net heat current will be the sum of heat currents through individual slabs. *i.e.,* $H = H_1 + H_2 + H_3 + .... H_n$

$$\frac{K(A_1 + A_2 + ..... + A_n)(\theta_1 - \theta_2)}{l}$$

$$= \frac{K_1 A_1(\theta_1 - \theta_2)}{l} + \frac{K_2 A_2(\theta_1 - \theta_2)}{l} + ... + \frac{K_n A_n(\theta_1 - \theta_2)}{l}$$

$$\Rightarrow K = \frac{K_1 A_1 + K_2 A_2 + K_3 A_3 + ..... K_n A_n}{A_1 + A_2 + A_3 + ..... A_n}$$

(a) For $n$ slabs of equal area $K = \frac{K_1 + K_2 + K_3 + ..... K_n}{n}$

(b) For two slabs of equal area $K = \frac{K_1 + K_2}{2}$.

## Ingen-Hauz Experiment

It is used to compare thermal conductivities of different materials. If $l_1$, $l_2$ and $l_3$ are the lengths of wax melted on rods as

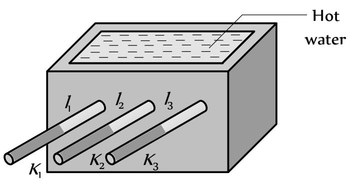



**Fig. 15.13**

shown in the figure, then the ratio of thermal conductivities is $K_1 : K_2 : K_3 = l_1^2 : l_2^2 : l_3^2$

$\Rightarrow$ Thermal conductivity $(K) \propto$ (Melted length $l$)$^2$

## Searle's Experiment

It is a method of determination of $K$ of a metallic rod.

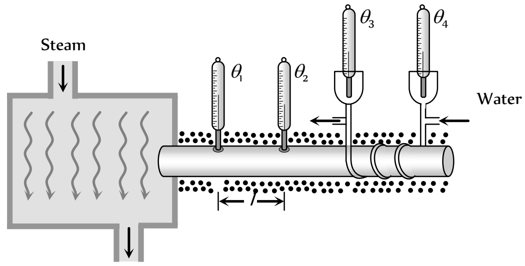



**Fig. 15.14**

(1) In this experiment a temperature difference $(\theta_1 - \theta_2)$ is maintained across a rod of length $l$ and area of cross section A. If the thermal conductivity of the material of the rod is $K$, then the amount of heat transmitted by the rod from the hot end to the cold end in time $t$ is given by, $Q = \frac{KA(\theta_1 - \theta_2)t}{l}$ ......(i)

(2) In Searle's experiment, this heat reaching the other end is utilized to raise the temperature of certain amount of water flowing through pipes circulating around the other end of the rod. If temperature of the water at the inlet is $\theta_3$ and at the outlet is $\theta_4$, then the amount of heat absorbed by water is given by, $Q = mc(\theta_4 - \theta_3)$

......(ii)

(3) Where, $m$ is the mass of the water which has absorbed this heat and temperature is raised and $c$ is the specific heat of the water

Equating (i) and (ii), $K$ can be determined *i.e.,* $K = \frac{mc(\theta_4 - \theta_3)l}{A(\theta_1 - \theta_2)t}$

(4) In numericals we may have the situation where the amount of heat travelling to the other end may be required to do some other work *e.g.,* it may be required to melt the given amount of ice. In that case equation (i) will have to be equated to $mL$.

*i.e.* $mL = \frac{KA(\theta_1 - \theta_2)t}{l}$

## Growth of Ice on Lake

(1) Water in a lake starts freezing if the atmospheric temperature drops below $0^o C$. Let $y$ be the thickness of ice layer in the lake at any instant $t$ and atmospheric temperature is $-\theta^o C$.

(2) The temperature of water in contact with lower surface of ice will be zero.

(3) If $A$ is the area of lake, heat escaping through ice in time $dt$ is $dQ_1 = \frac{KA[0 - (-\theta)]dt}{y}$

(4) Suppose the thickness of ice layer increases by $dy$ in time $dt$, due to escaping of above heat. Then $dQ_2 = mL = \rho(dy\, A)L$

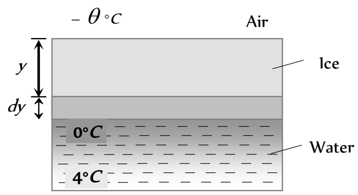



**Fig. 15.15**

(5) As $dQ_1 = dQ_2$, hence, rate of growth of ice will be $(dy/dt) = (K\theta/\rho L y)$

So, the time taken by ice to grow to a thickness $y$ is

$$t = \frac{\rho L}{K\theta}\int_0^y y\,dy = \frac{\rho L}{2K\theta}y^2$$

(6) If the thickness is increased from $y_1$ to $y_2$ then time taken $t = \frac{\rho L}{K\theta}\int_{y_1}^{y_2} y dy = \frac{\rho L}{2K\theta}(y_2^2 - y_1^2)$

(7) Take care and do not apply a negative sign for putting values of temperature in formula and also do not convert it to absolute scale.

(8) Ice is a poor conductor of heat, therefore the rate of increase of thickness of ice on ponds decreases with time.

(9) It follows from the above equation that time taken to double and triple the thickness, will be in the ratio of

$$t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2, \text{ i.e., } t_1 : t_2 : t_3 :: 1 : 4 : 9$$

(10) The time intervals to change the thickness from 0 to $y$, from $y$ to $2y$ and so on will be in the ratio

$$\Delta t_1 : \Delta t_2 : \Delta t_3 :: (1^2 - 0^2) : (2^2 - 1^2) : (3^2 : 2^2)$$

$$\Rightarrow \Delta t_1 : \Delta t_2 : \Delta t_3 :: 1 : 3 : 5$$

## Convection

Mode of transfer of heat by means of migration of material particles of medium is called convection. It is of two types.

(1) **Natural convection :** This arise due to difference of densities at two places and is a consequence of gravity because on account of gravity the hot light particles rise up and cold heavy particles try setting down. It mostly occurs on heating a liquid/fluid.

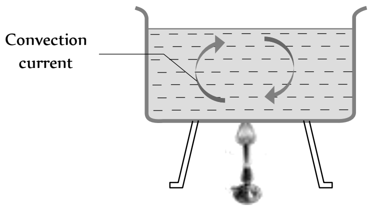



**Fig. 15.16**

(2) **Forced convection :** If a fluid is forced to move to take up heat from a hot body then the convection process is called forced convection. In this case Newton's law of cooling holds good. According to which rate of loss of heat from a hot body due to moving fluid is directly proportional to the surface area of body and excess temperature of body over its surroundings *i.e.*

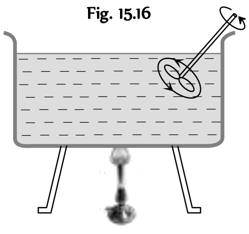


**Fig. 15.17**

$$\frac{Q}{t} \propto A(T - T_0) \Rightarrow \frac{Q}{t} = h\,A(T - T_0)$$

where $h$ = Constant of proportionality called convection coefficient, $T$ = Temperature of body and $T_0$ = Temperature of surrounding

Convection coefficient ($h$) depends on properties of fluid such as density, viscosity, specific heat and thermal conductivity.

(3) Natural convection takes place from bottom to top while forced convection in any direction.

(4) In case of natural convection, convection currents move warm air upwards and cool air downwards. That is why heating is done from base, while cooling from the top.

(5) Natural convection plays an important role in ventilation, in changing climate and weather and in forming land and sea breezes and trade winds.

(6) Natural convection is not possible in a gravity free region such as a free falling lift or an orbiting satellite.

(7) The force of blood in our body by heart helps in keeping the temperature of body constant.

(8) If liquids and gases are heated from the top (so that convection is not possible) they transfer heat (from top to bottom) by conduction.

(9) Mercury though a liquid is heated by conduction and not by convection.

## Radiation

(1) The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation.

**Fig. 15.18**

(2) Precisely it is electromagnetic energy transfer in the form of electromagnetic wave through any medium. It is possible even in vacuum *e.g.* the heat from the sun reaches the earth through radiation.

(3) The wavelength of thermal radiations ranges from $7.8 \times 10^{-7}\, m$ to $4 \times 10^{-4}\, m$. They belong to *infra-red* region of the electromagnetic spectrum. That is why thermal radiations are also called *infra-red* radiations.

(4) Medium is not required for the propagation of these radiations.

(5) They produce sensation of warmth in us but we can't see them.

(6) Every body whose temperature is above zero Kelvin emits thermal radiation.

(7) Their speed is equal to that of light *i.e.* $(= 3 \times 10^8\, m/s)$.

(8) Their intensity is inversely proportional to the square of distance of point of observation from the source (*i.e.* $I \propto 1/d^2$).

(9) Just as light waves, they follow laws of reflection, refraction, interference, diffraction and polarisation.

(10) When these radiations fall on a surface then exert pressure on that surface which is known as radiation pressure.

(11) While travelling these radiations travel just like photons of other electromagnetic waves. They manifest themselves as heat only when they are absorbed by a substance.

(12) Spectrum of these radiations can not be obtained with the help of glass prism because it absorbs heat radiations. It is obtained by quartz or rock salt prism because these materials do not have free electrons and interatomic vibrational frequency is greater than the radiation frequency, hence they do not absorb heat radiations.

(13) **Diathermanous Medium :** A medium which allows heat radiations to pass through it without absorbing them is called diathermanous medium. Thus the temperature of a diathermanous medium does not increase irrespective of the amount of the thermal radiations passing through it *e.g.*, dry air, $SO_2$, rock salt (*NaCl*).

(i) Dry air does not get heated in summers by absorbing heat radiations from sun. It gets heated through convection by receiving heat from the surface of earth.

(ii) In winters heat from sun is directly absorbed by human flesh while the surrounding air being diathermanous is still cool. This is the reason that sun's warmth in winter season appears very satisfying to us.

(14) **Athermanous medium :** A medium which partly absorbs heat rays is called a thermous medium As a result temperature of an athermanous medium increases when heat radiations pass through it *e.g.*, wood, metal, moist air, simple glass, human flesh *etc*.

## Colour of Heated Object

When a body is heated, all radiations having wavelengths from zero to infinity are emitted.

(1) Radiations of longer wavelengths are predominant at lower temperature.

(2) The wavelength corresponding to maximum emission of radiations shifts from longer wavelength to shorter wavelength as the temperature increases. Due to this the colour of a body appears to be changing.

(3) A blue flame is at a higher temperature than a yellow flame

**Table 15.3 : Variation of colour of a body on heating**

| Temperature | Colour |
|---|---|
| 525°$C$ | Dull red |
| 900°$C$ | Cherry red |
| 1100°$C$ | Orange red |
| 1200°$C$ | Yellow |
| 1600°$C$ | White |

## Interaction of Radiation with Matter

When thermal radiations ($Q$) fall on a body, they are partly reflected, partly absorbed and partly transmitted.

(1) $Q = Q_a + Q_r + Q_t$

(2) $\frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = a + r + t = 1$

(3) $a = \frac{Q_a}{Q}$ = Absorptance or absorbing power

$r = \frac{Q_r}{Q}$ = Reflectance or reflecting power

$t = \frac{Q_t}{Q}$ = Transmittance or transmitting power

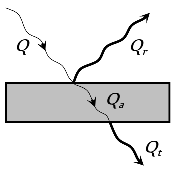



**Fig. 15.19**

(4) $r$, $a$ and $t$ all are the pure ratios so they have no unit and dimension.

(5) **Different bodies**

(i) If $a = t = 0$ and $r = 1 \rightarrow$ body is perfect reflector

(ii) If $r = t = 0$ and $a = 1 \rightarrow$ body is perfectly black body

(iii) If, $a = r = 0$ and $t = 1 \rightarrow$ body is perfect transmitter

(iv) If $t = 0 \Rightarrow r + a = 1$ or $a = 1 - r$ *i.e.* good reflectors are bad absorbers.

## Emissive Power, Absorptive Power and Emissivity

If temperature of a body is more than it's surrounding then body emits thermal radiation

(1) **Monochromatic Emittance or Spectral emissive power ($e_\lambda$) :** For a given surface it is defined as the radiant energy emitted per sec per unit area of the surface with in a unit wavelength around $\lambda$ *i.e.* lying between $\left(\lambda - \frac{1}{2}\right)$ to $\left(\lambda + \frac{1}{2}\right)$.

Spectral emissive power $(e_\lambda) = \frac{\text{Energy}}{\text{Area} \times \text{time} \times \text{wavelength}}$

Unit : $\frac{Joule}{m^2 \times sec \times Å}$ and Dimension : $[ML^{-1}T^{-3}]$

(2) **Total emittance or total emissive power ($e$) :** It is defined as the total amount of thermal energy emitted per unit time, per unit area of the body for all possible wavelengths.

$$e = \int_0^\infty e_\lambda d\lambda$$

Unit : $\frac{Joule}{m^2 \times \sec}$ or $\frac{Watt}{m^2}$ and Dimension : $[MT^{-3}]$

(3) **Monochromatic absorptance or spectral absorptive power ($a_\lambda$) :** It is defined as the ratio of the amount of the energy absorbed in a certain time to the total heat energy incident upon it in the same time, both in the unit wavelength interval. It is dimensionless and unit less quantity. It is represented by $a_\lambda$.

(4) **Total absorptance or total absorpting power ($a$):** It is defined as the total amount of thermal energy absorbed per unit time, per unit area of the body for all possible wavelengths.

$$a = \int_0^\infty a_\lambda d\lambda$$

(5) **Emissivity ($\varepsilon$) :** Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the body ($e$) to the total emissive power of a perfect black body ($E$) at that temperature *i.e.* $\varepsilon = \frac{e}{E}$

($\varepsilon \rightarrow$ read as epsilon)

(i) For perfectly black body $\varepsilon = 1$

(ii) For highly polished body $\varepsilon = 0$

(iii) But for practical bodies emissivity ($\varepsilon$) lies between zero and one ($0 < \varepsilon < 1$).

## Perfectly Black Body

(1) A perfectly black body is that which absorbs completely the radiations of all wavelengths incident on it.

(2) As a perfectly black body neither reflects nor transmits any radiation, therefore the absorptance of a perfectly black body is unity *i.e.* $t = 0$ and $r = 0 \Rightarrow a = 1$.

(3) We know that the colour of an opaque body is the colour (wavelength) of radiation reflected by it. As a black body reflects no wavelength so, it appears black, whatever be the colour of radiations incident on it.

(4) When perfectly black body is heated to a suitable high temperature, it emits radiation of all possible wavelengths. For example, temperature of the sun is very high (6000 *K* approx.) it emits all possible radiation so it is an example of black body.

(5) **Ferry's black body :** A perfectly black body can't be realised in practice. The nearest example of an ideal black body is the Ferry's black body. It is a doubled walled evacuated spherical cavity whose inner wall is blackened. The space between the wall is evacuated to prevent the loss of heat by conduction and radiation. There is a fine hole in it. All the radiations incident upon this hole are absorbed by this black body. If this black body is heated to high temperature then it emits radiations of all wavelengths. It is the hole which is to be regarded as a black body and not the total enclosure

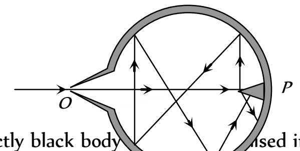
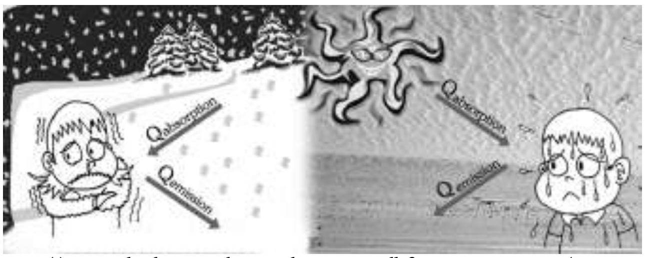



Fig. 15.20

(6) A perfectly black body can't be realised in practice but materials like Platinum black or Lamp black come close to being ideal black bodies. Such materials absorbs 96% to 85% of the incident radiations.

## Prevost Theory of Heat Exchange




(1) Every body emits heat radiations at all finite temperature (Except 0 *K*) as well as it absorbs radiations from the surroundings.

(2) Exchange of energy along various bodies takes place via radiation.

(3) The process of heat exchange among various bodies is a continuous phenomenon.

(4) At absolute zero temperature (0 *K* or – 273°*C*) this law is not applicable because at this temperature the heat exchange among various bodies ceases.

(5) If $Q_{emission} > Q_{absorbed} \rightarrow$ temperature of body decreases and consequently the body appears colder.

If $Q_{emission} < Q_{absorbed} \rightarrow$ temperature of body increases and it appears hotter.

If $Q_{emission} = Q_{absorbed} \rightarrow$ temperature of body remains constant (thermal equilibrium)

## Kirchoff's Law

According to this law the ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature. Hence $\frac{e_1}{a_1} = \frac{e_2}{a_2} = ... \left(\frac{E}{A}\right)_{\text{Perfectly black body}}$

But for perfectly black body $A = 1$ *i.e.* $\frac{e}{a} = E$

If emissive and absorptive powers are considered for a particular wavelength $\lambda$, $\left(\frac{e_\lambda}{a_\lambda}\right) = (E_\lambda)_{\text{black}}$

Now since $(E_\lambda)_{\text{black}}$ is constant at a given temperature, according to this law if a surface is a good absorber of a particular wavelength it is also a good emitter of that wavelength.

This in turn implies that a good absorber is a good emitter (or radiator)

## Applications of Kirchoff's Law

(1) Sand is rough black, so it is a good absorber and hence in deserts, days (when radiation from the sun is incident on sand) will be very hot. Now in accordance with Kirchoff's law, good absorber is a good emitter so nights (when sand emits radiation) will be cold. This is why days are hot and nights are cold in desert.

(2) Sodium vapours, on heating, emit two bright yellow lines. These are called $D_1$, $D_2$ lines of sodium. When continuos white light from an arc lamp is made to pass through sodium vapours at low temperature, the continuous spectrum is intercepted by two dark lines exactly in the same places as $D_1$ and $D_2$ lines. Hence sodium vapours when cold, absorbs the same wavelength, as they emit while hot. This is in accordance with Kirchoff's law.

(3) When a shining metal ball having some black spots on its surface is heated to a high temperature and is seen in dark, the black spots shine brightly and the shining ball becomes dull or invisible. The reason is that the black spots on heating absorb radiation and so emit these in dark while the polished shining part reflects radiations and absorb nothing and so does not emit radiations and becomes invisible in the dark.

(4) When a green glass is heated in furnace and taken out, it is found to glow with red light. This is because red and green are complimentary colours. At ordinary temperatures, a green glass appears green, because it transmits green colour and absorb red colour strongly. According to Kirchoff's law, this green glass, on heating must emit the red colour, which is absorbed strongly. Similarly when a red glass is heated to a high temperature it will glow with green light.

(5) A person with black skin experiences more heat and more cold as compared to a person of white skin because when the outside temperature is greater, the person with black skin absorbs more heat and when the outside temperature is less the person with black skin radiates more energy.

(6) **Kirchoff' law also explains the Fraunhoffer lines :**

(i) Sun's inner most part (photosphere) emits radiation of all wavelength at high temperature.

(ii) When these radiation enters in outer part (chromosphere) of sun, few wavelength are absorbed by some terrestrial elements (present in vapour form at lower temperature)

(iii) These absorbed wavelengths, which are missing appear as dark lines in the spectrum of the sun called **Fraunhoffer lines.**

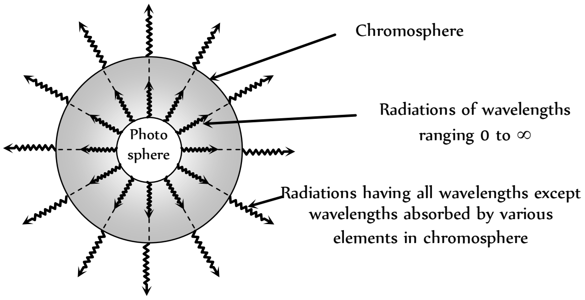



Fig. 15.21

(iv) During total solar eclipse these lines appear bright because the gases and vapour present in the chromosphere start emitting those radiation which they had absorbed.

## Stefan's Law

According to it the radiant energy emitted by a perfectly black body per unit area per sec (*i.e.* emissive power of black body) is directly proportional to the fourth power of its absolute temperature, *i.e.* $E \propto T^4$ $\Rightarrow$ ***E* = σT**

where $\sigma$ is a constant called Stefan's constant having dimension $[MT^{-3}\theta^{-4}]$ and value $5.67 \times 10^{-8} W/m^2K^4$.

(i) For ordinary body : $e = \varepsilon E = \varepsilon\sigma T^4$

(ii) Radiant energy : If $Q$ is the total energy radiated by the ordinary body then $e = \dfrac{Q}{A \times t} = \varepsilon\sigma T^4 \Rightarrow Q = A\varepsilon\sigma T^4 t$

(iii) Radiant power ($P$) : It is defined as energy radiated per unit area *i.e.* $P = \dfrac{Q}{t} = A\varepsilon\sigma T^4$.

(iv) If an ordinary body at temperature $T$ is surrounded by a body at temperature $T_0$ then Stefan's law may be put as

$$e = \varepsilon\,\sigma(T^4 - T_0^4)$$

## Rate of Loss of Heat ($R_H$) and Rate of Cooling ($R_C$)

(1) **Rate of loss of heat (or initial rate of loss of heat) :** If an ordinary body at temperature $T$ is placed in an environment of temperature $T_0$ ($T_0 < T$) then heat loss by radiation is given by

$$\Delta Q = Q_{\text{emission}} - Q_{\text{absorption}} = A\varepsilon\,\sigma(T^4 - T_0^4)$$

(2) Rate of loss of heat $(R_H) = \dfrac{dQ}{dt} = A\varepsilon\,\sigma(T^4 - T_0^4)$

(i) If two bodies are made of same material, have same surface finish and are at the same initial temperature then $\dfrac{dQ}{dt} \propto A \Rightarrow \dfrac{\left(\dfrac{dQ}{dt}\right)_1}{\left(\dfrac{dQ}{dt}\right)_2} = \dfrac{A_1}{A_2}$

(3) **Initial rate of fall in temperature (Rate of cooling):** If $m$ is the body and $c$ is the specific heat then

$$\frac{dQ}{dt} = mc.\frac{dT}{dt} = mc\frac{d\theta}{dt} \qquad (\because Q = mc\,\Delta T \text{ and } dT = d\theta)$$

(i) Rate of cooling $(R_c) = \dfrac{d\theta}{dt} = \dfrac{(dQ/dt)}{mc} = \dfrac{A\varepsilon\,\sigma}{mc}(T^4 - T_0^4)$

$= \dfrac{A\varepsilon\,\sigma}{V\rho\,c}(T^4 - T_0^4)$ ; where $m$ = density ($\rho$) × volume ($V$)

(ii) for two bodies of the same material under identical environments, the ratio of their rate of cooling is $\dfrac{(R_c)_1}{(R_c)_2} = \dfrac{A_1}{A_2} . \dfrac{V_2}{V_1}$

(4) **Dependence of rate of cooling :** When a body cools by radiation the rate of cooling depends on

(i) Nature of radiating surface *i.e.* greater the emissivity, faster will be the cooling.

(ii) Area of radiating surface, *i.e.* greater the area of radiating surface, faster will be the cooling.

(iii) Mass of radiating body *i.e.* greater the mass of radiating body slower will be the cooling.

(iv) Specific heat of radiating body *i.e.* greater the specific heat of radiating body slower will be cooling.

(v) Temperature of radiating body *i.e.* greater the temperature of body faster will be cooling.

(vi) Temperature of surrounding *i.e.* greater the temperature of surrounding slower will be cooling.

**Table 15.4 : Comparison of rate of heat loss ($R_H$) and rate of cooling ($R_c$) for different bodies**

| Body | Condition | Rate of heat loss $R_H = \dfrac{dQ}{dt}$ | Rate of cooling $R_c = \dfrac{dT}{dt}$ or $\dfrac{d\theta}{dt}$ |
|---|---|---|---|
| Two solid sphere | $T$, $T_0$ , $c$, $\rho$ are same | $R_H \propto A \propto r^2$ $\Rightarrow \dfrac{(R_H)_1}{(R_H)_2} = \dfrac{r_1^2}{r_2^2}$ | $R_c \propto \dfrac{A}{V} \propto \dfrac{r^2}{r^3} \propto \dfrac{1}{r}$ |
| Two solid sphere of diff. material | $T$, $T_0$ – same | $R_H \propto A \propto r^2$ | $R_c \propto \dfrac{A}{V\rho c} \propto \dfrac{1}{r\rho c}$ |
| Different shape bodies like cube, sphere plate | $T$, $T_0$, $c$, $\rho$ - same | $R_H \propto A$ $A_{\max} \to$ Plate $A_{\min} \to$ sphere | $R_c \propto \dfrac{A}{V}$ |
| Bodies of different materials | $T$, $T_0$, $m$, $A$ are same but $c$ diff. | $R_H \to$ same for all. bodies | $R_c \propto \dfrac{1}{c}$ |

## Newton's Law of Cooling

When the temperature difference between the body and its surrounding is not very large *i.e.* $T - T_0 = \Delta T$ then $T^4 - T_0^4$ may be approximated as $4T_0^3\Delta T$

By Stefan's law, $\dfrac{dT}{dt} = \dfrac{A\varepsilon\sigma}{mc}[T^4 - T_0^4]$

Hence $\dfrac{dT}{dt} = \dfrac{A\varepsilon\sigma}{mc}4T_0^3\Delta T \Rightarrow \dfrac{dT}{dt} \propto \Delta T$ or $\dfrac{d\theta}{dt} \propto \theta - \theta_0$

*i.e.,* if the temperature of body is not very different from surrounding, **rate of cooling is proportional to temperature difference** between the body and its surrounding. This law is called Newton's law of cooling.

(1) Greater the temperature difference between body and its surrounding greater will be the rate of cooling.

(2) If $\theta = \theta_0$, $\dfrac{d\theta}{dt} = 0$ *i.e.* a body can never be cooled to a temperature lesser than its surrounding by radiation.

(3) If a body cools by radiation from $\theta_1^o C$ to $\theta_2^o C$ in time *t*, then $\dfrac{d\theta}{dt} = \dfrac{\theta_1 - \theta_2}{t}$ and $\theta = \theta_{av} = \dfrac{\theta_1 + \theta_2}{2}$. The Newton's law of cooling becomes $\left[\dfrac{\theta_1 - \theta_2}{t}\right] = K\left[\dfrac{\theta_1 + \theta_2}{2} - \theta_0\right]$.

This form of law helps in solving numericals.

(4) Practical examples

(i) Hot water loses heat in smaller duration as compared to moderate warm water.

(ii) Adding milk in hot tea reduces the rate of cooling.

## Cooling Curves

(1) **Curve between log($\theta$ – $\theta_0$) and time**

As $\dfrac{d\theta}{dt} \propto -(\theta - \theta_0) \Rightarrow \dfrac{d\theta}{(\theta - \theta_0)} = -Kdt$

Integrating $\log_e(\theta - \theta_0) = -Kt + C$

$\log_e(\theta - \theta_0) = -Kt + \log_e A$

This is a straight line with negative slope

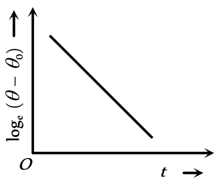


Fig. 15.22

(2) **Curve between temperature of body and time**

As $\log_e(\theta - \theta_0) = -Kt + \log_e A \Rightarrow \log_e \dfrac{\theta - \theta_0}{A} = -Kt$

$\Rightarrow \theta - \theta_0 = Ae^{-kt}$

which indicates temperature decreases exponentially with increasing time.

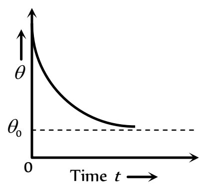


Fig. 15.23

(3) **Curve between the rate of cooling** ($R$) and body temperature ($\theta$).

$R = K(\theta - \theta_0) = K\theta - K\theta_0$

This is a straight line intercept $R$-axis at $-K\theta_0$

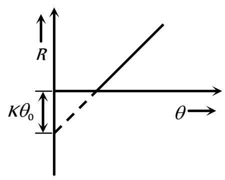


Fig. 15.24

(4) **Curve between rate of cooling ($R$)** and temperature difference between body ($\theta$) and surrounding ($\theta_0$)

$R \propto (\theta - \theta_0)$. This is a straight line passing through origin.

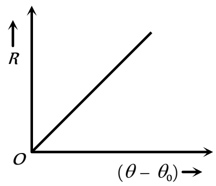


Fig. 15.25

## Determination of Specific Heat of Liquid

If volume, radiating surface area, nature of surface, initial temperature and surrounding of water and given liquid are equal and they are allowed to cool down (by radiation) then rate of loss of heat and fall in temperature of both will be same.

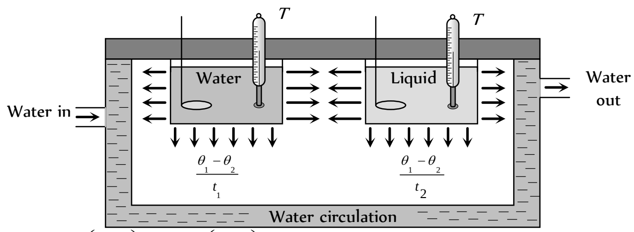


Fig. 15.26

*i.e.* $\left(\dfrac{dQ}{dt}\right)_{\text{water}} = \left(\dfrac{dQ}{dt}\right)_{\text{liquid}}$

$$(m_W c_W + W)\frac{(\theta_1 - \theta_2)}{t_1} = (m_l c_l + W)\frac{(\theta_1 - \theta_2)}{t_2}$$

or $$\left[\frac{m_W c_W + W}{t_1}\right] = \left[\frac{m_l c_l + W}{t_2}\right]$$

$W = m_c c_c$ = Water equivalent of calorimeter, where $m_c$ and $c_c$ are mass and specific heat of calorimeter.

If density of water and liquid is $\rho$ and $\rho'$ respectively then $m_W = V\rho_W$ and $m_l = V\rho_l$

Specific heat of liquid $c_l = \dfrac{1}{m_l}\left[\dfrac{t_l}{t_W}(m_W c_W + W) - W\right]$

## Distribution of Energy in the Spectrum of Black Body

A perfectly black body emits radiation of all possible wavelength.

Langley and later on Lummer and Pringsheim investigated the distribution of energy amongst the different wavelengths in the thermal spectrum of a black body radiation. The results obtained are shown in figure. From these curves it is clear that

(1) At a given temperature energy is not uniformly distributed among different wavelengths.

(2) At a given temperature intensity of heat radiation increases with wavelength, reaches a maximum at a particular wavelength and with further increase in wavelength it decreases.

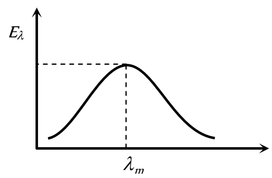


Fig. 15.27

(3) For all wavelengths an increase in temperature causes an increase in intensity.

(4) The area under the curve will represent the total intensity of radiation at a particular temperature *i.e.* Area = $E = \int E_\lambda d\lambda$

From Stefan's law $E = \sigma T \Rightarrow$ Area under $E_\lambda$ - $\lambda$ curve $(A) \propto T$

(5) The energy $(E_m)$ emitted corresponding to the wavelength of maximum emission $(\lambda_m)$ increases with fifth power of the absolute temperature of the black body *i.e.*, $E_{\max} \propto T^5$

## Wien's Displacement Law

According to Wien's law the product of wavelength corresponding to maximum intensity of radiation and temperature of body (in Kelvin) is constant, *i.e.* $\lambda_m T = b = \text{constant}$

where $b$ is Wien's constant and has value $2.89 \times 10^{-3}\, m\text{-}K$.

As the temperature of the body increases, the wavelength at which the spectral intensity $(E_\lambda)$ is maximum shifts towards left. Therefore it is also called Wien's displacement law.

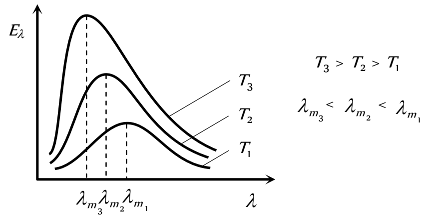


Fig. 15.28

This law is of great importance in 'Astrophysics' as through the analysis of radiations coming from a distant star, by finding $\lambda_m$ the temperature of the star $T (= b / \lambda_m)$ is determined.

## Law of Distribution of Energy (Plank's Hypothesis)

(1) The theoretical explanation of black body radiation was done by Planck.

(2) According to Plank's atoms of the walls of a uniform temperature enclosure behave as oscillators, each with a characteristic frequency of oscillation.

(3) These oscillations emits electromagnetic radiations in the form of photons (The radiation coming out from a small hole in the enclosure are called black body radiation). The energy of each photon is $h\nu$. Where $\nu$ is the frequency of oscillator and $h$ is the Plank's constant. Thus emitted energies may be $h\nu$, $2h\nu$, $3h\nu$ ... $nh\nu$ but not in between.

According to Planck's law $E_\lambda d\lambda = \dfrac{8\pi hc}{\lambda^5} \dfrac{1}{[e^{hc/\lambda KT} - 1]} d\lambda$

where $c$ = speed of light and $k$ = Boltzmann's constant. This equation is known as Plank's radiation law. It is correct and complete law of radiation

(4) This law is valid for radiations of all wavelengths ranging from zero to infinite.

(5) For radiations of short wavelength $\left(\lambda << \dfrac{hc}{KT}\right)$ Planck's law reduces to *Wien's energy distribution law* $E_\lambda d\lambda = \dfrac{A}{\lambda^5} e^{-B/\lambda T} d\lambda$

(6) For radiations of long wavelength $\left(\lambda >> \dfrac{hc}{KT}\right)$ Planck's law reduces to *Rayleigh-Jeans energy distribution law* $E_\lambda d\lambda = \dfrac{8\pi KT}{\lambda^4} d\lambda$

## Temperature of the Sun and Solar Constant

If $R$ is the radius of the sun and $T$ its temperature, then the energy emitted by the sun per sec through radiation in accordance with Stefan's law will be given by

$$P = A\sigma T^4 = 4\pi R^2 \sigma T^4$$

In reaching earth this energy will spread over a sphere of radius $r$ (= average distance between sun and earth); so the intensity of solar radiation at the surface of earth (called solar constant $S$) will be given by

$$S = \frac{P}{4\pi r^2} = \frac{4\pi R^2 \sigma T^4}{4\pi r^2}$$

$$i.e.\ T = \left[\left(\frac{r}{R}\right)^2 \frac{S}{\sigma}\right]^{1/4}$$

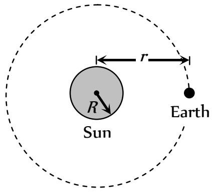


Fig. 15.29

$$= \left[\left(\frac{1.5 \times 10^8}{7 \times 10^5}\right)^2 \times \frac{1.4 \times 10^3}{5.67 \times 10^{-8}}\right]^{1/4} \simeq 5800\, K$$

As $r = 1.5 \times 10^8$ *km*, $R = 7 \times 10^5$ *km*,

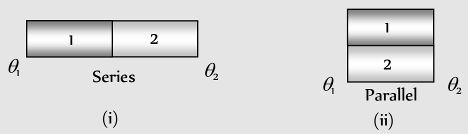
$$S = 2\frac{cal}{cm^2 min} = 1.4\frac{kW}{m^2} \text{ and } \sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

This result is in good agreement with the experimental value of temperature of sun, *i.e.*, 6000 *K*.

## Tips & Tricks

✍ Glass and water vapours transmit shorter wavelengths through them but reflects longer wavelengths. This concept is utilised in Green house effect. Glass transmits those waves which are emitted by a source at a temperature greater than 100°*C*. So, heat rays emitted from sun are able to enter through glass enclosure but heat emitted by small plants growing in the nursery gets trapped inside the enclosure.

✍ Suppose two metallic rods are first connected in series then in parallel.




If $Q_s$ heat flows in time $t_s$ in series combination and $Q_p$ heat flows in time $t_p$ in parallel combine, then $\dfrac{Q_p}{Q_s} = \dfrac{t_p}{t_s} \times \dfrac{R_s}{R_p}$

If Rods are identical then $R_S = \frac{R}{2}$ and $R_p = 2R \Rightarrow \frac{Q_p}{Q_s} = 4\left(\frac{t_p}{t_s}\right)$

✍ If temperature of a body becomes $\theta_1$ to $\theta_2$ in $t$ time and it becomes $\theta_2$ to $\theta_3$ in next time then use

$$\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0} = \frac{\theta_3 - \theta_0}{\theta_2 - \theta_0}$$ ($\theta_0$ = temperature of enviroment)

✍ Newton's law of cooling can be used to compare the specific heat of the two liquids.

If equal masses of two liquids having same surface are and finish cools from same initial temperature to same final temperature with same surrounding then $\frac{t_1}{t_2} = \frac{K_2}{K_1} = \frac{C_1}{C_2}$

✍ Radiations from sun take 8 *min* and 20 *sec* to reach earth.

✍ Suppose temperature of a body decreases $\theta_1°C$ to $\theta_2°C$ in time $t_1$ and $\theta_2°C$ to $\theta_3°C$ in time $t_2$ in the same invirment

If $(\theta_1 - \theta_2) \geq (\theta_2 - \theta_3)$ then $t_2 > t_1$

✍ Green glass is a good absorber of red light and a good reflector of green light. Consequently at lower temperature it is a good emitter of red light.

Hence Green $\rightleftharpoons$ Red

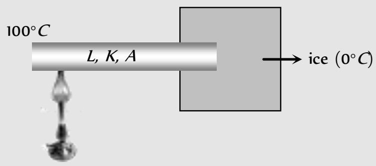
Also Yellow $\rightleftharpoons$ Blue

✍ While solving the problems of heat flow, remember the following equation

*e.g.* If we are interested in finding the mass of ice which transfoms into water in unit time. For this we will take

$$\frac{T.D.}{R} = L_f . \frac{dm}{dt}$$

$$\Rightarrow \frac{dm}{dt} = \frac{T.D.}{(L_f)(R)}$$




✍ **Confusion**

The rate of cooling has been used in many books, with double meanings. At some places. Rate of cooling $= \frac{dQ}{dt}$ and at other places, rate of cooling $= \frac{d\theta}{dt}$. Our suggestion is that look for the units, if the rate of cooling is in *cal/m* in or *J/sec etc.*, then it is $\frac{dQ}{dt}$. But if rate of cooling is in °*C/min* it means $\frac{d\theta}{dt}$.