Maths (Standard) Delhi (Set 1)

General Instructions :

(i) This question paper comprises four sections – A, B, C and D. This question paper carries 40 questions. All questions are compulsory:
(ii) Section A : Q. No. 1 to 20 comprises of 20 questions of one mark each.
(iii) Section B : Q. No. 21 to 26 comprises of 6 questions of two marks each.
(iv) Section C: Q. No. 27 to 34 comprises of 8 questions of three marks each.
(v) Section D : Q. No. 35 to 40 comprises of 6 questions of four marks each.
(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of one mark each, 2 questions of two marks each, 3 questions of three marks each and 3 questions of four marks each. You have to **attempt only one of the choices** in such questions.
(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.
(viii) Use of calculators is not permitted.

Question 1

If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is
(a) 10
(b) −10
(c) −7
(d) −2

Solution:

Let the given polynomial be $p(x) = x^2 + 3x + k$
Since, one of the zeroes is 2.
Therefore, the value of $p(x)$ at $x = 2$ will be zero.
Therefore,

$$2^2 + 3 \times 2 + k = 0$$
$$\Rightarrow 4 + 6 + k = 0$$
$$\Rightarrow 10 + k = 0$$
$$\Rightarrow k = -10$$

Hence, the correct answer is option (b).

Question 2

The total number of factors of a prime number is
(a) 1
(b) 0
(c) 2
(d) 3

Solution:

The factors of a prime number are 1 and the number itself.
Therefore, the total number of factors of a prime number is 2.
Hence, the correct answer is option (c).

Question 3

The quadratic polynomial, the sum of whose zeroes is −5 and their product is 6, is
(a) $x^2 + 5x + 6$
(b) $x^2 - 5x + 6$
(c) $x^2 - 5x - 6$
(d) $-x^2 + 5x + 6$

Solution:

Let the zeroes be α and β respectively.
Therefore, $\alpha + \beta = -5$ and $\alpha\beta = 6$.
Hence, the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta$$
$$= x^2 - (-5)x + 6$$
$$= x^2 + 5x + 6$$

Hence, the correct answer is option A.

Question 4

The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is
(a) −2
(b) ≠2
(c) 3
(d) 2

Solution:

For a system of a quadratic equation to have no solution, the condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.
Given equations are $x + y - 4 = 0$ and $2x + ky - 3 = 0$, where
$a_1 = 1,\ b_1 = 1,\ c_1 = -4,\ a_2 = 2,\ b_2 = k,\ c_2 = -3$.
We have,
$\frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3}$
Now,
$\frac{1}{2} = \frac{1}{k}$
$\Rightarrow k = 2$
Hence, the correct answer is option (d).

Question 5

The HCF and the LCM of 12, 21, 15 respectively are
(a) 3, 140
(b) 12, 420
(c) 3, 420
(d) 420, 3

Solution:

Here,
$12 = 2^2 \times 3$
$21 = 3 \times 7$
$15 = 3 \times 5$
Therefore, HCF(12, 21, 15) = 3 and
LCM(12, 21, 15) = $2^2 \times 3 \times 5 \times 7 = 420$

Hence, the correct answer is option C.

Question 6

The value of x for which $2x$, $(x + 10)$ and $(3x + 2)$ are the three consecutive terms of an AP, is
(a) 6
(b) −6
(c) 18
(d) −18

Solution:

Given $2x,\ x+10,\ 3x+2$ are the consecutive terms of an AP.
Therefore, the common difference will be same.

$\Rightarrow (x+10)-2x=(3x+2)-(x+10)$

$\Rightarrow x+10-2x=3x+2-x-10$

$\Rightarrow 10-x=2x-8$

$\Rightarrow 3x=18$

$\Rightarrow x=6$

Hence, the correct answer is option (a).

Question 7

The first term of an AP is *p* and the common difference is *q*, then its 10^{th} term is
(a) $q + 9p$
(b) $p - 9p$
(c) $p + 9q$
(d) $2p + 9q$

Solution:

The nth term of an AP = a + (n − 1)d, where a and d are the first term and common difference respectively.

Therefore, 10^{th} term = p + (10 − 1)q = p + 9q.

Question 8

The distance between the points (a cos θ + b sin θ, 0) and (0, a sin θ − b cos θ), is
(a) a^2+b^2
(b) a^2-b^2
(c) $\sqrt{a^2+b^2}$
(d) $\sqrt{a^2-b^2}$

Solution:

The distance between two points $\mathrm{P}(x_1, y_1)$ and $\mathrm{Q}(x_2, y_2)$ is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Thus, the distance between the two given points is given by

$$= \sqrt{[0 - (a\cos\theta + b\sin\theta)]^2 + [(a\sin\theta - b\cos\theta) - 0]^2}$$

$$= \sqrt{(a\cos\theta + b\sin\theta)^2 + (a\mathrm{sin}\theta - b\mathrm{cos}\theta)^2}$$

$$= \sqrt{a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta}$$

$$= \sqrt{a^2\left(\cos^2\theta + \sin^2\theta\right) + b^2\left(\sin^2\theta + \cos^2\theta\right)}$$

$$= \sqrt{a^2 \times 1 + b^2 \times 1}$$

$$= \sqrt{a^2 + b^2}$$

Hence, the correct answer is option (c).

Question 9

If the point P(*k*, 0) divides the line segment joining the points A(2, −2) and B(−7, 4) in the ratio 1 : 2, then the value of *k* is

(a) 1
(b) 2
(c) −2
(d) −1

Solution:

Using the Section Formula, we have

$$k = \frac{1\times(-7)+2\times2}{1+2}$$

$$\Rightarrow \quad k = \frac{-7+4}{3}$$

$$\Rightarrow \quad k = \frac{-3}{3}$$

$$\Rightarrow \quad k = -1$$

Hence, the correct answer is option D.

Question 10

The value of *p*, for which the points A(3, 1), B(5, *p*) and C(7, −5) are collinear, is

(a) −2
(b) 2
(c) −1
(d) 1

Solution:

Given $A(3,\ 1),\ \ B(5,\ p)$ and $C(7,\ \ -5)$ are collinear.

$\Rightarrow \text{Area of } \Delta\,\text{ABC},\ A = 0$

$\Rightarrow \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right] = 0$

$\Rightarrow \left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right] = 0$

$\Rightarrow \left[3\left(p + 5\right) + 5\left(-5 - 1\right) + 7\left(1 - p\right)\right] = 0$

$\Rightarrow \left[3p + 15 - 30 + 7 - 7p\right] = 0$

$\Rightarrow -4p - 8 = 0$

$\Rightarrow 4p = -8$

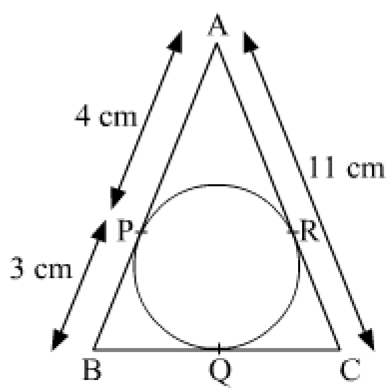
$\Rightarrow p = -2$

Hence, the correct answer is option A.

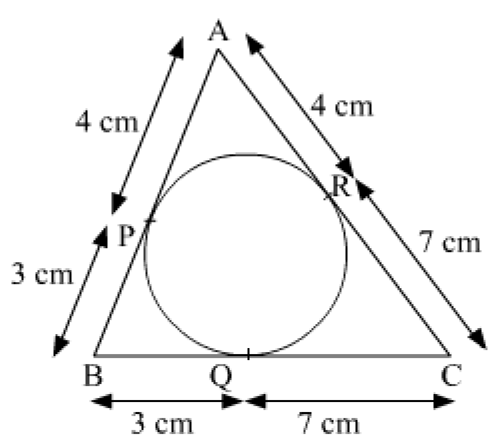
Question 11

Fill in the blank.

In the given figure ΔABC is circumscribing a circle, the length of BC is _____ cm.



Solution:



Since we know that the lengths of tangents drawn from an exterior point to a circle are equal.

Therefore, $AP = AR = 4$ cm, $BP = BQ = 3$ cm.
Therefore, $CR = AC - AR = 11 - 4 = 7$ cm.
Hence, $BC = BQ + CQ = BQ + CR = 3 + 7$ cm $= 10$ cm.

Question 12

Fill in the blank.

Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} =$ ___________.

Solution:

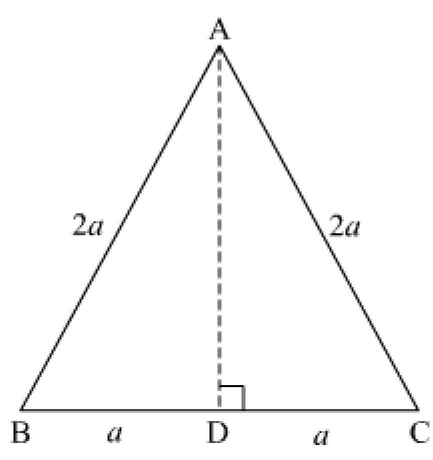
Given that $\triangle ABC \sim \triangle PQR$ and $\frac{AB}{PQ} = \frac{1}{3}$.

We know that $\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$.

Question 13

Fill in the blanks.
ABC is an equilateral triangle of side $2a$, then length of one of its altitude is ___________.

Solution:



We have the above equilateral triangle in which the length of each side is 2a units.
Drop a perpendicular from A on BC, intersecting it at D.

In $\triangle$ABD and $\triangle$ACD, we have
AB = AC (Sides of an equilateral triangle)
$\angle$ABD = $\angle$ACD (Angles of an equilateral triangle)
$\angle$ADB = $\angle$ADC = 90° (By construction)
Therefore, $\triangle ABD \cong \Delta ACD$ (By AAS rule)
$\Rightarrow$BD = CD = a (By CPCT)

Now, using Pythagoras Theorem in $\triangle$ABD, we have

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow (2a)^2 = AD^2 + a^2$$

$$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$$

$$\Rightarrow AD = \sqrt{3}a$$

This is the required length of the altitude.

Question 14

Fill in the blank.
$\frac{\cos\ 80^\circ}{\sin\ 10^\circ} + \cos\ 59^\circ\ \operatorname{cosec}\ 31^\circ =$ __________.

Solution:

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ$$

$$= \frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec}(90^\circ - 59^\circ)$$

$$= \frac{\sin 10^\circ}{\sin 10^\circ} + \cos 59^\circ \sec 59^\circ$$

$$= 1 + \cos 59^\circ \times \frac{1}{\cos 59^\circ}$$

$$= 1 + 1$$

$$= 2$$

Question 15

Fill in the blank.
The value of $\left(\sin^2\theta + \frac{1}{1+\tan^2\ \theta}\right)$ = ___________.

OR

Fill in the blank.
The value of $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta)$ = ____________.

Solution:

$$\sin^2\theta + \frac{1}{1+\tan^2\theta}$$
$$= \sin^2\theta + \frac{1}{\sec^2\theta} = \sin^2\theta + \cos^2\theta = 1$$

OR

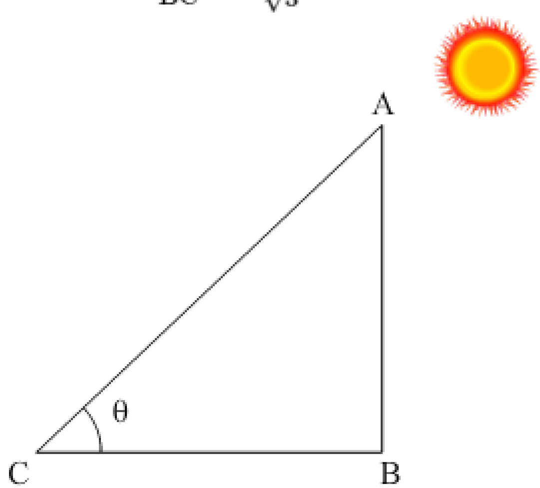
The given expression is
$$\left(1+\tan^2\theta\right)(1-\sin\theta)(1+\sin\theta)$$
$$= \left(1+\tan^2\theta\right)\left(1-\sin^2\theta\right)$$
$$= \sec^2\theta . \cos^2\theta$$
$$= \frac{1}{\cos^2\theta} . \cos^2\theta$$
$$= 1$$
Thus, the value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$ is 1.

Question 16

The ratio of the length of a vertical rod and the length of its shadow is 1: √3. Find the angle of elevation of the sun at that moment?

Solution:

Given that $\frac{AB}{BC} = \frac{1}{\sqrt{3}}$



From the figure, it is clear that △△ABC is a right-angled triangle in which AB is the vertical rod and BC is its shadow.

We have,

$$\tan\theta = \frac{\mathrm{AB}}{\mathrm{BC}} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \tan\theta = \tan 30^\circ$$
$$\Rightarrow \theta = 30^\circ$$
Hence, the required angle of elevation of the sun is 30°.

Question 17

Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

Solution:

Let the heights, radii and volumes of the two cones be $(h_1,\ r_1,\ V_1)$ and $(h_2,\ r_2,\ V_2)$.
Given: $\frac{h_1}{h_2} = \frac{1}{3}$ and $\frac{r_1}{r_2} = \frac{3}{1}$

The required ratio of their volumes = $\frac{V_1}{V_2}$
$$= \frac{\frac{1}{3}\pi {r_1}^2 h_1}{\frac{1}{3}\pi {r_2}^2 h_2}$$
$$= \frac{{r_1}^2}{{r_2}^2} \times \frac{h_1}{h_2}$$
$$= \frac{3^2}{1} \times \frac{1}{3}$$
$$= \frac{3}{1}$$
$$= 3 : 1$$
Hence, the required ratio of the volumes is 3 : 1.

Question 18

A letter of English alphabet is chosen at random. What is the probability that the chosen letter is a consonant.

Solution:

Total number of letters = 26
Total number of consonants = 21
Let E be the event of choosing a consonant.

Then, $\mathrm{P}(E) = \frac{\text{Total number of consonants}}{\text{Total number of letters}} = \frac{21}{26}.$

Question 19

A die is thrown once. What is the probability of getting a number less than 3?

OR

If the probability of winning a game is 0.07, what is the probability of losing it?

Solution:

When a die is thrown, all the outcomes are = {1, 2, 3, 4, 5, 6}
Total number of outcomes = 6
Favourable outcomes = {1, 2}
Favourable number of outcomes = 2

P(a number less than 3) = $\frac{2}{6} = \frac{1}{3}$

OR

P(winning) = 0.07
P(losing) = 1 — P(winning)
P(losing) = 1 — 0.07 = 0.93

Question 20

If the mean of the first *n* natural number is 15, then find *n*.

Solution:

Given: mean of the first *n* natural numbers is 15.

$\therefore \ \frac{1+2+3+...+n}{n} = 15$

$\Rightarrow 1 + 2 + 3 + \ldots + n = 15n$

$\Rightarrow \frac{n(n+1)}{2} = 15n$

$\Rightarrow n^2 + n = 30n$

$\Rightarrow n^2 - 29n = 0$

$\Rightarrow n\left(n - 29\right) = 0$

$\Rightarrow n = 0,\ 29$

So, *n* = 29 (Since *n* cannot be zero)

Question 21

Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in AP.

Solution:

We need to prove that $(a-b)^2,\ a^2+b^2,\ (a+b)^2$ are in A.P.
Let $a_1=(a-b)^2,\ a_2=a^2+b^2,\ a_3=(a+b)^2$
For these 3 three terms to be in A.P., the common difference would be same
i.e. $a_2-a_1=a_3-a_2$

$\Rightarrow \left(a^2+b^2\right)-(a-b)^2=(a+b)^2-\left(a^2+b^2\right)$

$\Rightarrow \left(a^2+b^2\right)-\left(a^2-2ab+b^2\right)=\left(a^2+2ab+b^2\right)-\left(a^2+b^2\right)$

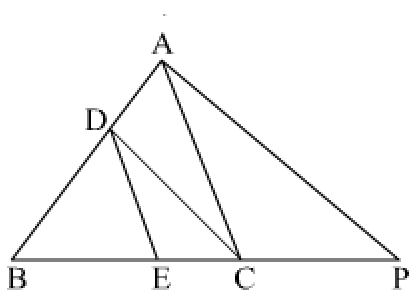
$\Rightarrow a^2+b^2-a^2+2ab-b^2=a^2+2ab+b^2-a^2-b^2$

$\Rightarrow 2ab=2ab$

Since common difference is same,
Hence, the given three terms are in A.P.

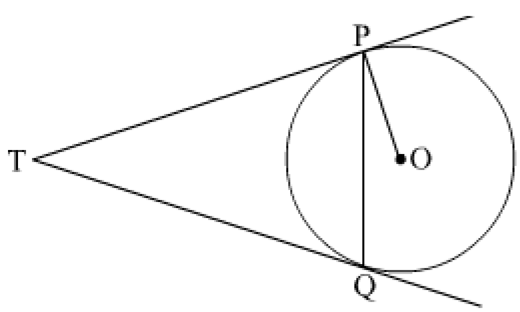
Question 22

In the given Figure, DE || AC and DC || AP. Prove that $\frac{\mathrm{BE}}{\mathrm{EC}}=\frac{\mathrm{BC}}{\mathrm{CP}}$



OR

In the given Figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2 ∠OPQ.



Solution:

In ΔABP, DC || AP

By Basic Proportionality theorem,

$$\frac{BD}{DA} = \frac{BC}{CP} \quad .....(i)$$

In ΔBAC, DE || AC

By Basic Proportionality theorem,

$$\frac{BD}{DA} = \frac{BE}{EC} \quad .....(ii)$$

Thus, from (i) and (ii) we have

$$\frac{BE}{EC} = \frac{BC}{CP}$$

Hence proved.

Hence proved.

OR

Given: PT and TQ are the tangents to the circle with centre O.

To prove: $\angle PTQ = 2\angle OPQ$

Proof:

In $\triangle$PTQ,

PT = PQ (Tangents from an external point to the circle are equal)

$\Rightarrow \angle TPQ = \angle TQP$ (Angles opposite to equal sides are equal)

Let $\angle PTQ = \theta$

So, in $\triangle$PTQ

$$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ$$

$$\Rightarrow \angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - \theta) = 90^\circ - \frac{1}{2}\theta$$

We know, angle made by the tangent with the radius is 90°.

So, $\angle OPT = 90^\circ$

Now,

$$\angle OPT = \angle OPQ + \angle TPQ$$

$$\Rightarrow 90^\circ = \angle OPQ + \left(90^\circ - \frac{1}{2}\theta\right)$$

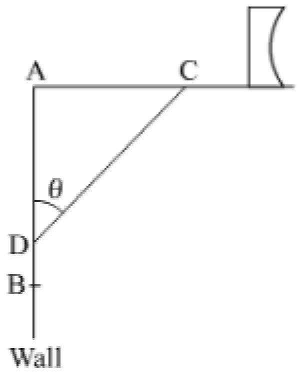
$$\Rightarrow \angle OPQ = \frac{1}{2}\theta = \frac{1}{2}\angle PTQ$$

$$\Rightarrow \angle PTQ = 2\angle OPQ$$

Hence Proved.

Question 23

The rod AC of a TV disc antenna is fixed at right angles to the wall AB and a rod CD is supporting the disc as shown in the given figure. If AC = 1.5 m long and CD = 3 m, find (i) tanθ (ii) secθ + cosecθ



Solution:

In △ACD, we have
AC = 1.5 cm, CD = 3 cm.

Since △ACD is a right-angled triangle, so using Pythagoras Theorem, we have

$$\begin{aligned} AD^2 &= CD^2 - AC^2 \\ &= 3^2 - 1.5^2 \\ &= 6.75 \end{aligned}$$
$$\therefore AD = \sqrt{6.75} = 2.5 \text{ cm}$$

Consider
(i) $\tan\theta = \frac{AC}{AD} = \frac{1.5}{2.5} = \frac{3}{5}$
(ii) $\sec\theta + \operatorname{cosec}\theta = \frac{CD}{AD} + \frac{CD}{AC} = \frac{3}{2.5} + \frac{3}{1.5} = \frac{6}{5} + 2 = \frac{16}{5}$

Question 24

A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle.
$\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution:

Length of the wire = 22 cm
Angle subtended at the centre = θ = 60°
Length of the arc = $\frac{\theta}{360} \times 2\pi r$
$\Rightarrow \frac{60}{360} \times 2 \times \frac{22}{7} \times r = 22$
$\Rightarrow \frac{22}{7} r = 22 \times 3$
$\Rightarrow r = 21 \text{ cm}$

Question 25

If a number *x* is chosen at random from the numbers −3, −2, −1, 0, 1, 2, 3. What is probability that $x^2 \le 4$?

Solution:

The given numbers are $-3, -2, -1, 0, 1, 2, 3$.
Total number of possible outcomes = 7
Now, the favorable outcomes are given by $x^2 \le 4$
i.e. $-2 \le x \le 2$
i.e. $-2, -1, 0, 1, 2$
Total number of favorable outcomes = 5
Hence, the required probability = $\frac{5}{7}$.

Question 26

Find the mean of the following distribution:

Class:	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency:	5	10	10	7	8

OR

Find the mode of the following data :

Class:	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120 – 140
Frequency:	6	8	10	12	6	5	3

Solution:

Class	Frequency(f_i)	Class Mark(x_i)	$f_i x_i$
3 - 5	5	4	20
5 - 7	10	6	60
7 - 9	10	8	80
9 - 11	7	10	70
11 - 13	8	12	96
	$\sum f_i = 40$		$\sum f_i x_i$ = 326

Mean, $\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{326}{40} = 8.15$

Thus, mean = 8.15

OR

In the given data, the maximum class frequency is 12.
The class corresponding to the given class is 60 - 80, which is the modal class.

We have

Lower limit of modal class, l = 60
Frequency of modal class, f_1 = 12

Frequency of a class preceding to modal class, f_o = 10
Frequency of a class succeeding to modal class, f_2 = 6
Class size h = 20

$$\text{Mode} = l + \frac{(f_1 - f_0)}{(2f_1 - f_o - f_2)} \times h$$
$$= 60 + \frac{(12 - 10)}{(24 - 10 - 6)} \times 20$$
$$= 60 + \frac{2}{8} \times 20$$
$$= 60 + 5$$
$$= 65$$

Hence, the mode of the given data is 65.

Question 27

Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

OR

Divide the polynomial $f(x) = 3x^2 - x^3 - 3x + 5$ by the polynomial $g(x) = x - 1 - x^2$ and verify the division algorithm.

Solution:

The given quadratic polynomial is
$f(x) = ax^2 + bx + c,\ a \neq 0,\ c \neq 0$
Let α and β be the two zeroes of the given quadratic polynomial.
Then,
Sum of zeroes = $\alpha + \beta = \frac{-b}{a}$
Product of zeroes = $\alpha\beta = \frac{c}{a}$
$\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c}$ and $\frac{1}{\alpha}.\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{\frac{c}{a}} = \frac{a}{c}$
So, the required new quadratic polynomial is
$k\left[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}\right]$
$= k\left[x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha}.\frac{1}{\beta}\right]$
$= k\left[x^2 - \left(\frac{-b}{c}\right)x + \frac{a}{c}\right]$
where k is a real number.

OR

Given,
$f(x) = 3x^2 - x^3 - 3x + 5$
$g(x) = x - 1 - x^2$

$$
\begin{array}{r|l}
 & x - 2 \\
\hline
-x^2 + x - 1 & -x^3 + 3x^2 - 3x + 5 \\
 & \underset{+}{-}x^3 \underset{-}{+} x^2 \underset{+}{-} x \\
\hline
 & \quad 2x^2 - 2x + 5 \\
 & \quad \underset{-}{}2x^2 \underset{+}{-} 2x \underset{-}{+} 2 \\
\hline
 & \qquad\quad 3
\end{array}
$$

So,
$q(x) = (x-2)$ and $r(x) = 3$

To verify: $f(x) = g(x).q(x) + r(x)$
Verification:
$$
\begin{aligned}
g(x).q(x) + r(x) &= \left(-x^2 + x - 1\right)(x-2) + 3 \\
&= -x^2(x-2) + x(x-2) - 1(x-2) + 3 \\
&= -x^3 + 2x^2 + x^2 - 2x - x + 2 + 3 \\
&= -x^3 + 3x^2 - 3x + 5 \\
&= f(x)
\end{aligned}
$$
Hence verified.

Question 28

Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$.

OR

If 4 is a zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes.

Solution:

The first given equation is $2y - x = 8$

x	0	−8
y	4	0

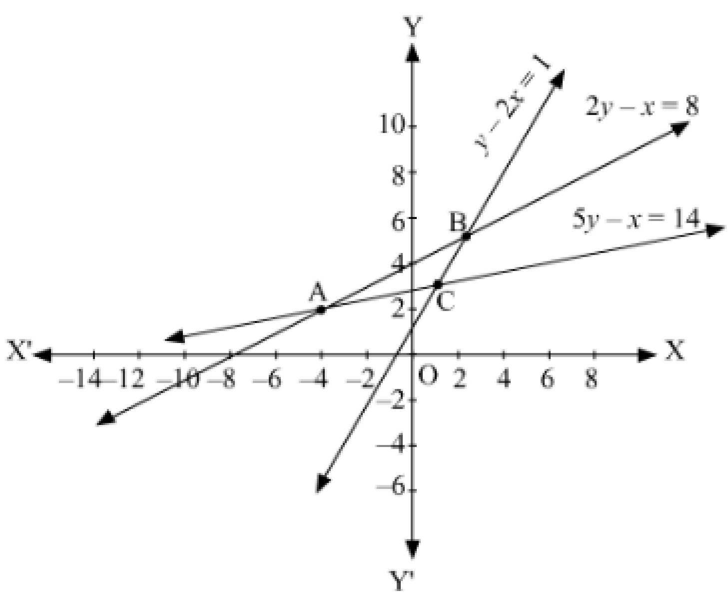
The second given equation is $5y - x = 14$

x	0	−14
y	2.8	0

The third given equation is $y - 2x = 1$

x	0	−0.5
y	1	0

Plotting the three given lines on the graph paper we get



The coordinates of the vertices of the triangle ABC are A(−-4, 2), B(2, 5) and C(1, 3).

OR

Given 4 is a zero of a cubic polynomial $x^3-3x^2-10x+24$
$\Rightarrow (x-4)$ is the factor of polynomial $x^3-3x^2-10x+24$

Therefore, we have

$$
\begin{array}{r|l}
 & x^2+x-6 \\
\hline
x-4 & x^3-3x^2-10x+24 \\
 & x^3-4x^2 \\
 & -\quad + \\
\hline
 & x^2-10x+24 \\
 & x^2-4x \\
 & -\quad + \\
\hline
 & -6x+24 \\
 & -6x+24 \\
 & +\quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

To find the other two zeroes of the given polynomial, we need to find the zeroes of the quotient $x^2 + x - 6$.

$$
\begin{aligned}
&\text{i.e. } x^2+x-6=0 \\
&\Rightarrow x^2+3x-2x-6=0 \\
&\Rightarrow x(x+3)-2(x+3)=0 \\
&\Rightarrow (x+3)(x-2)=0 \\
&\Rightarrow x+3=0 \text{ or } x-2=0 \\
&\Rightarrow x=-3 \text{ or } x=2
\end{aligned}
$$

Hence, the other two zeroes of the given polynomial are 2 and -3.

Question 29

In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/hr and time of flight increased by 30 minutes. Find the original duration of flight.

Solution:

Given: Distance = 600 km
Let average speed = x km/hr
$\Rightarrow$ time = $\frac{600}{x}$ hr
According to question,
New speed = $(x-200)$ km/hr and new time = $\left(\frac{600}{x}+\frac{1}{2}\right)$ hr
Now, distance = speed × time

$$\Rightarrow 600=(x-200)\left(\frac{600}{x}+\frac{1}{2}\right)$$
$$\Rightarrow 600=600+\frac{x}{2}-\frac{120000}{x}-100$$
$$\Rightarrow 100=\frac{x}{2}-\frac{120000}{x}$$
$$\Rightarrow 200x=x^2-240000$$
$$\Rightarrow x^2-200x-240000=0$$
$$\Rightarrow (x+400)(x-600)=0$$
$$\therefore\ x=-400,\ 600$$

Since speed cannot be negative, therefore x = 600 km/hr
Hence the original duration of flight = $\frac{600}{x}=\frac{600}{600}=$1 hr

Question 30

Find the area of triangle PQR formed by the points P(−5, 7), Q(−4, −5) and R (4, 5).

OR

If the point C(−1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio 3 : 4, find the coordinates of B.

Solution:

Given: Vertices of the triangle are $\mathrm{P}(-5,7),\ \mathrm{Q}(-4,-5)\ \text{and}\ \mathrm{R}(4,5)$.
Let A be the area of the triangle.
Using the formula to calculate the area of the triangle, we have

$$A=\frac{1}{2}\left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right]$$
$$=\frac{1}{2}\left[(-5)(-5-5)+(-4)(5-7)+(4)(7+5)\right]$$
$$=\frac{1}{2}[50+8+48]$$
$$=53$$

Hence, the area of the triangle is 53 square units.

OR

Since the point $\mathrm{C}(-1,\ 2)$ divides the line segment joining $\mathrm{A}(2,\ 5)\ \text{and}\ \mathrm{B}(x,\ y)$ in the ratio $3:4$.
Therefore using the section-formula of internal division, we get
For x- coordinate,
$-1=\frac{3\times x+4\times 2}{3+4}$

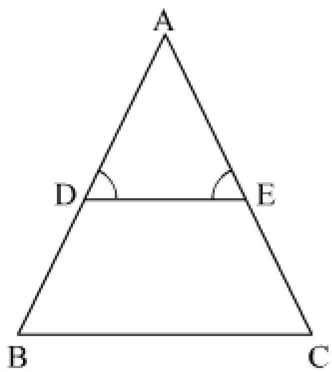
$\Rightarrow 3x+8=-7$
$\Rightarrow x=-5$

For y- coordinate,
$2=\frac{3\times y+4\times 5}{3+4}$

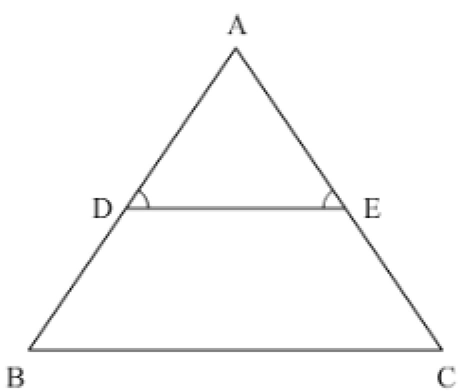
$\Rightarrow 3y+20=14$
$\Rightarrow y=-2$
Hence, the coordinates of B are $(-5,\ -2)$.

Question 31

In the given Figure 5, ∠D = ∠E and $\frac{\mathrm{AD}}{\mathrm{DB}}=\frac{\mathrm{AE}}{\mathrm{EC}}$, prove that BAC is an isosceles triangle.



Solution:



In the given triangle, we have
$\angle D = \angle E$ and
$\frac{AD}{DB} = \frac{AE}{EC}$
Therefore, $\triangle ADE \sim \triangle ABC$ (By SAS similarity criterion)
$\Rightarrow \angle D = \angle B$ and $\angle E = \angle C$ (By CPST)
But $\angle D = \angle E$
$\Rightarrow \angle B = \angle C$
Since it is known that if two angles of a triangle are equal then sides opposite to angles are also equal.
$\Rightarrow AB = AC$
Since, a triangle having two equal sides is called as isosceles triangle.
Hence, $\triangle ABC$ is an isosceles triangle.

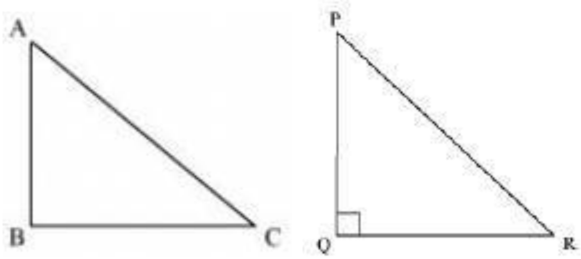
Question 32

In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first side is a right angle.

Solution:

Given: In ΔABC, $AC^2 = AB^2 + BC^2$
To prove: $\angle B = 90°$

Construction: ΔPQR right-angled at Q such that PQ = AB and QR = BC



In ΔPQR,
$PR^2 = PQ^2 + QR^2$ (By Pythagoras Theorem, as $\angle Q = 90°$)

$\Rightarrow PR^2 = AB^2 + BC^2$ ..... (1) (By construction)
However, $AC^2 = AB^2 + BC^2$ ..... (2) (Given)

From (1) and (2), we obtain
AC = PR ..... (3)

Now, In ΔABC and ΔPQR, we obtain
AB = PQ (By construction)
BC = QR (By construction)
AC = PR [From (3)]

Therefore, $\Delta ABC \cong \Delta PQR$ (by SSS congruency criterion)

$\Rightarrow \angle B = \angle Q$ (By CPCT)
However, $\angle Q = 90°$ (By construction)

$\therefore \angle B = 90°$
Hence proved.

Question 33

If $\sin\ \theta + \cos\ \theta = \sqrt{3}$, then prove that tan θ + cot θ = 1.

Solution:

Given that $\sin\theta + \cos\theta = \sqrt{3}$
Squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = \left(\sqrt{3}\right)^2$$
$$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$
$$\Rightarrow 1 + 2\sin\theta\cos\theta = 3 \qquad \left(\because\ \sin^2\theta + \cos^2\theta = 1\right)$$
$$\Rightarrow 2\sin\theta\cos\theta = 2$$
$$\Rightarrow \sin\theta\cos\theta = 1 \qquad .....(1)$$

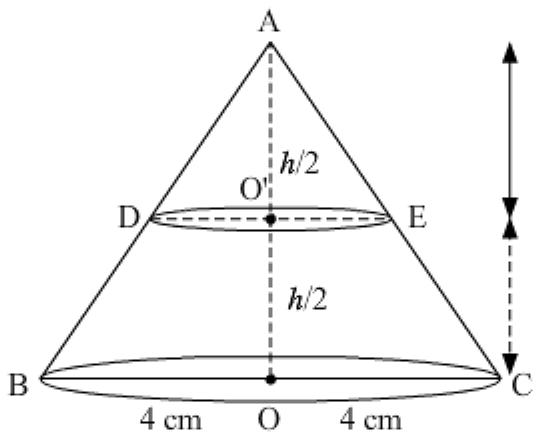
Consider the LHS:

$$\tan\theta + \cot\theta$$
$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta\cos\theta}$$
$$= \frac{1}{1} \qquad [\text{Using}\ \ (1)]$$
$$= 1$$

Question 34

A cone of base radius 4 cm is divided into two parts by drawing a plane through the mid-points of its height and parallel to its base. Compare the volume of the two parts.

Solution:



Given: $\mathrm{OC}=4\,\mathrm{cm}, \quad \mathrm{AO'}=\mathrm{OO'}$
Let $\mathrm{AO}=h$
$\Rightarrow \mathrm{AO'}=\mathrm{OO'}=\frac{h}{2}$

In $\triangle\ \mathrm{AO'C}$ and $\triangle\ \mathrm{AOC}$
$\angle\mathrm{E}=\angle\mathrm{C}$ [Corresponding angles]
$\angle\mathrm{A}=\angle\mathrm{A}$ [Common angle]
$\Rightarrow\triangle\ \mathrm{AO'E}\cong\triangle\ \mathrm{AOC}$ [By SS similarity criterion]

Therefore, $\frac{\mathrm{O'E}}{\mathrm{OC}}=\frac{\mathrm{AO'}}{\mathrm{AO}}=\frac{1}{2}$
$\Rightarrow \frac{\mathrm{O'E}}{\mathrm{OC}}=\frac{1}{2}$
Let $V_1,\ V_2$ are the volumes of the cone ADE and cone ABC respectively.
$\frac{V_1}{V_2}=\frac{\left[\frac{1}{3}\pi(\mathrm{O'E})^2\ \mathrm{AO'}\right]}{\left[\frac{1}{3}\pi(\mathrm{OC})^2\ \mathrm{AO}\right]}$
$=\left(\frac{\mathrm{O'E}}{\mathrm{OC}}\right)^2\left(\frac{\mathrm{AO'}}{\mathrm{AO}}\right)$
$=\frac{1}{4}\times\frac{1}{2}$
$=\frac{1}{8}$
$\frac{\text{Volume of the upper part of the cone}}{\text{Volume of the lower part of the cone}}=\frac{V_1}{V_2-V_1}$
$=\frac{\left(\frac{V_1}{V_2}\right)}{1-\left(\frac{V_1}{V_2}\right)}$
$=\frac{1}{7} \qquad \left(\because \frac{V_1}{V_2}=\frac{1}{8}\right)$

Question 35

Show that the square of any positive integer cannot be of the form $(5q + 2)$ or $(5q + 3)$ for any integer q.

OR

Prove that one of every three consecutive positive integers is divisible by 3.

Solution:

Let b be an arbitrary positive integer.
By Euclid's division lemma,
$b = aq + r$, where $0 \le r < a$

Now, if we divide b by 5, then b can be written in the form of $5m$, $5m+1$, $5m+2$, $5m+3$ or $5m+4$.
This implies that we have five possible cases.

Case I:
If $b = 5m$

Squaring both sides, we get

$$b^2 = (5m)^2 = 25m^2 = 5\left(5m^2\right)$$
$$\Rightarrow b^2 = 5q$$
where $q = 5m^2$ is an integer.

Case II:
If $b = 5m + 1$,
Squaring both sides, we get
$$b^2 = (5m+1)^2 = 25m^2 + 1 + 10m$$
$$\Rightarrow b^2 = 5\left(5m^2 + 2m\right) + 1$$
$$\Rightarrow b^2 = 5q + 1$$
where $q = 5m^2 + 2m$ is an integer.

Case III:
If $b = 5m + 2$
Squaring both sides, we get
$$b^2 = (5m+2)^2 = 25m^2 + 4 + 20m$$
$$\Rightarrow b^2 = 5\left(5m^2 + 4m\right) + 4$$
$$\Rightarrow b^2 = 5q + 4$$

where $q = 5m^2 + 4m$ is an integer.

Case IV:
If $b = 5m + 3$
Squaring both sides, we get
$$b^2 = (5m+3)^2 = 25m^2 + 9 + 30m$$
$$\Rightarrow b^2 = 25m^2 + 5 + 4 + 30m$$
$$\Rightarrow b^2 = 5\left(5m^2 + 1 + 6m\right) + 4$$
$$\Rightarrow b^2 = 5q + 4$$
where $q = 5m^2 + 1 + 6m$ is an integer.

Case V:
If $b = 5m + 4$
Squaring both sides, we get
$$b^2 = (5m+4)^2 = 25m^2 + 16 + 40m$$
$$\Rightarrow b^2 = 25m^2 + 15 + 1 + 40m$$
$$\Rightarrow b^2 = 5\left(5m^2 + 3 + 8m\right) + 1$$
$$\Rightarrow b^2 = 5q + 1$$
where $q = 5m^2 + 3 + 8m$ is an integer.

Hence, we can conclude that the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer.

OR

Let n, $n + 1$, $n + 2$ be three consecutive positive integers, where n is any natural number.
By Euclid's division lemma,
$n = aq + r$, where $0 \le r < a$.

Now, if we divide n by 3, then n can be written in the form of $3q$, $3q+1$ or $3q+2$.
This implies that we have three possible cases.

Case I:
If $n = 3q$, then n is divisible by 3.
However, $n + 1$ and $n + 2$ are not divisible by 3.

Case II:
If $n = 3q + 1$, then $n + 2 = 3q + 3 = 3(q + 1)$, which is divisible by 3.
However, n and $n + 1$ are not divisible by 3.

Case III:
If $n = 3q + 2$, then $n + 1 = 3q + 3 = 3(q + 1)$, which is divisible by 3.
However, n and $n + 2$ are not divisible by 3.

Hence, we conclude that one of any three consecutive positive integers must be divisible by 3.

Question 36

The sum of four consecutive numbers in AP is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7 : 15. Find the numbers.

OR

Solve : 1 + 4 + 7 + 10 + ... + x = 287

Solution:

Let the four terms of the AP be $a - 3d$, $a - d$, $a + d$ and $a + 3d$.
Given:
$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$
$\Rightarrow 4a = 32$
$\Rightarrow a = 8$

Also,

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$
$$\Rightarrow \frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$$
$$\Rightarrow \frac{(8)^2-9d^2}{(8)^2-d^2} = \frac{7}{15}$$
$$\Rightarrow \frac{64-9d^2}{64-d^2} = \frac{7}{15}$$

$$\Rightarrow 960 - 135d^2 = 448 - 7d^2$$
$$\Rightarrow 512 = 128d^2$$
$$\Rightarrow d^2 = 4$$
$$\Rightarrow d = \pm 2$$

When a = 8 and d = 2, then the terms are 2, 6, 10, 14.
When a = 8 and d = −2, then the terms are 14, 10, 6, 2.

OR

In the given AP, we have

$a = 1,\ d = 3,\ S_n = 287$

The formula for sum of n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$.

This implies

$\frac{n}{2}[2(1) + (n-1)(3)] = 287$

$\Rightarrow n(2 + 3n - 3) = 574$

$\Rightarrow 3n^2 - n - 574 = 0$

$\Rightarrow 3n^2 - 42n + 41n - 574 = 0$

$\Rightarrow 3n(n-14) + 41(n-14) = 0$

$\Rightarrow (n-14) = 0 \text{ or } 3n + 41 = 0$

$\Rightarrow n = 14 \text{ or } n = -\frac{41}{3}$

$\therefore\ n = 14$

$x = a_{14} = a + (14-1)d = 1 + 13(3) = 1 + 39 = 40$

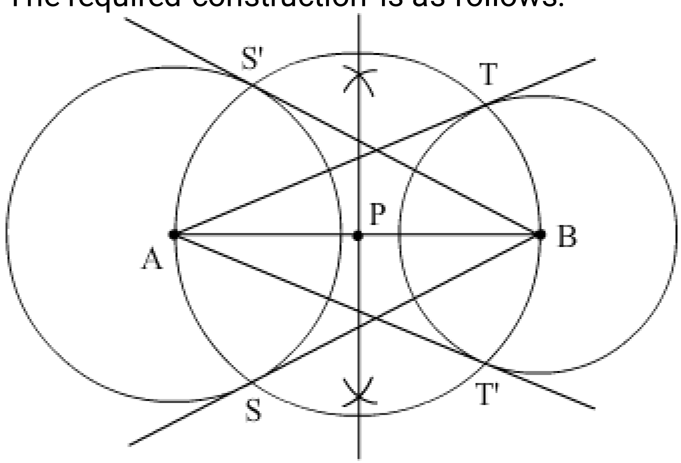
Thus, the value of x is 40.

Question 37

Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

Solution:

The required construction is as follows:



Steps of construction:

Step I: First of all, we draw a line AB = 7 cm.

Step II: Taking A as a centre, draw a circle of radius 3 cm. Similarly, taking B as a centre and draw a circle of radius 2 cm.

Step III: Draw the perpendicular bisector of AB which intersects AB at P.

Step IV: Draw another circle with P as the centre and PA = PB as the radius which intersects the circle with centre A at S and S'. BS and BS' are the required tangents at it. Similarly, AT and AT' are the required tangents at the circle with centre B.

Question 38

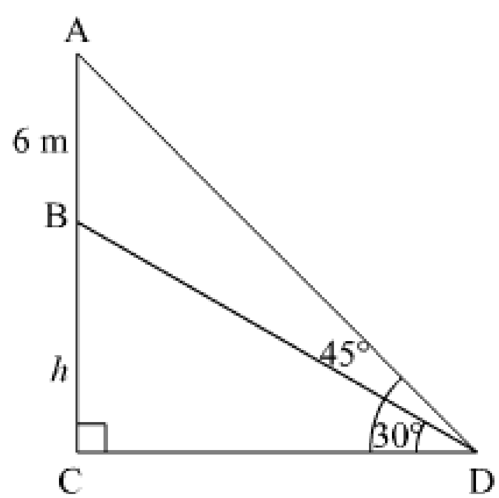
A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower.
(Take √3 = 1.73)

Solution:

Let BC be the tower of height *h* m, AB be the flag staff of height 7 m on tower and D be the point on the plane making an angle of elevation of the top of the flag staff as 45° and angle of elevation of the bottom of the flag staff as 30°.

Let CD = *x*, AB = 6m and $\angle BDC = 30°$ and $\angle ADC = 45°$.

We need to find the height of the tower i.e. *h*.
We have the corresponding figure as follows:



So we use trigonometric ratios.

In a triangle BCD :

$$\Rightarrow \quad \tan D = \frac{BC}{CD}$$

$$\Rightarrow \quad \tan 30^\circ = \frac{h}{x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad x = \sqrt{3}h$$

Again in a triangle ADC:

$$\tan\ D = \frac{AB+BC}{CD}$$

$$\Rightarrow\ \tan\ 45^\circ = \frac{h+6}{x}$$

$$\Rightarrow 1 = \frac{h+6}{x}$$

$$\Rightarrow x = h + 6$$

$$\Rightarrow \sqrt{3}h = h + 6$$

$$\Rightarrow h\left(\sqrt{3}\ -\ 1\right) = 6$$

$$\Rightarrow h = \frac{6}{\sqrt{3}-1} = \frac{6(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{6(\sqrt{3}+1)}{(\sqrt{3})^2-1^2} = \frac{6(\sqrt{3}+1)}{3-1} = 3\left(\sqrt{3}+1\right)$$

$$\Rightarrow h = 3\left(\sqrt{3}+1\right) = 3\,(1.732+1) = 3 \times 2.732 = 8.196\mathrm{m}$$

Hence the height of the tower is 8.196 m.

Question 39

A bucket in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm, respectively. Find the capacity of the bucket. Also find the cost of milk which can completely fill the bucket at the rate of Rs. 40 per litre. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Solution:

It is given that radius of upper end of the bucket = r_1 = 20 cm
Radius of lower end of the bucket = r_2 = 10 cm

Height of the bucket, h = 30 cm
The bucket is in the shape of frustum of a cone.

Therefore, volume of the bucket

$= \frac{\pi}{3} h \left(r^2{}_1 + r^2{}_2 + r_1 r_2 \right)$

$= \frac{22}{7 \times 3} \times 30 \left(20^2 + 10^2 + 20 \times 10 \right)$

$= \frac{220}{7} (700)$

$= 22000 \text{ cm}^3$

$\text{Amount of milk the bucket can hold } = \frac{22000}{1000} = 22 \text{ L}$

$\text{Total cost of milk} = 22 \times 40 = 880 \text{ Rs}$

Question 40

The following table gives production yield per hectare (in quintals) of wheat of 100 farms of a village :

Production yield/hect.	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70
No. of farms	4	6	16	20	30	24

Change the distribution to 'a more than' type distribution and draw its ogive.

OR

The median of the following data is 525. Find the values of *x* and *y*, if total frequency is 100:

Class :	**Frequency:**
0 – 100	2
100 – 200	5
200 – 300	*x*
300 – 400	12
400 – 500	17
500 – 600	20
600 – 700	*y*
700 – 800	9
800 – 900	7
900 – 1000	4

Solution:

Given:

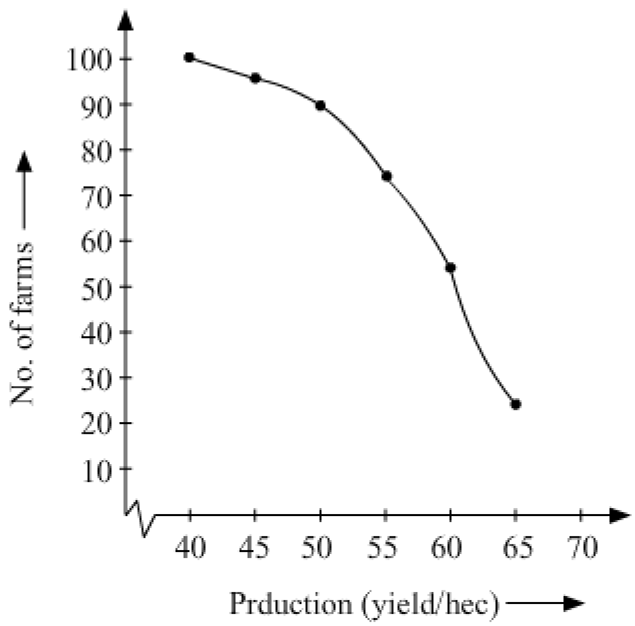
Production yield/hect.	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70
No. of farms	4	6	16	20	30	24

"more than type" distribution table is as follows-

Production (yield/hec)	No. of farms
More than 40	100
More than 45	96
More than 50	90
More than 55	74
More than 60	54
More than 65	24

To draw the ogive, we have the following points-
(40, 100), (45, 96), (50, 90), (55, 74), (60, 54), (65, 24)

Plotting these points, we get the following ogive-



OR

Given, median = 525

We prepare the cumulative frequency table, as given below.

Class interval:	Frequency: (f_i)	Cumulative frequency $(c.f.)$
0-100	2	2
100-200	5	7
200-300	f_1	$7+f_1$
300-400	12	$19+f_1$
400-500	17	$36+f_1$
500-600	20	$56+f_1$
600-700	f_2	$56+f_1+f_2$
700-800	9	$65+f_1+f_2$
800-900	7	$72+f_1+f_2$
900-1000	4	$76+f_1+f_2$
	$N=100=76+f_1+f_2$	

$$N=100$$
$$76+f_1+f_2=100$$
$$f_2=24-f_1 \qquad .....(1)$$
$$\frac{N}{2}=50$$

Since median = 525,
So, the median class is $500-600$.

Here, $l=500, f=20, F=36+f_1$ and $h=100$

We know that

$$\text{Median}=l+\left\{\frac{\frac{N}{2}-F}{f}\right\}\times h$$
$$525=500+\left\{\frac{50-(36+f_1)}{20}\right\}\times 100$$
$$25=\frac{(14-f_1)\times 100}{20}$$

$$25 \times 20 = 1400 - 100 f_1$$
$$100 f_1 = 1400 - 500$$
$$f_1 = \frac{900}{100}$$
$$= 9$$

Putting the value of f_1 in (1), we get

$$f_2 = 24 - 9$$
$$= 15$$

Hence, the missing frequencies are 9 and 15.