Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper -1

with Solution

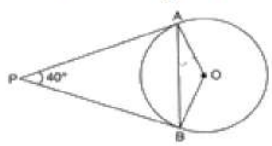
Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

Section A

1. If PA and PB are tangents to the circle with centre O such that $\angle APB = 40°$, then $\angle OAB$ is equal to **[1]**



a) 40° b) 30°

c) 20° d) 25°

2. If the coordinates of one end of a diameter of a circle are (2, 3) and the coordinates of its centre are (-2, 5), then the coordinates of the other end of the diameter are **[1]**

a) (0, 8) b) (0, 4)

c) (6, – 7) d) (– 6, 7)

3. The abscissa of any point on the x-axis is **[1]**

a) – 1 b) 1

c) x d) 0

4. Two dice are thrown simultaneously. The probability that the product of the numbers appearing on the dice is 7 is **[1]**

a) 7 b) 2

c) 0 d) 1

5. The coordinates of the fourth vertex of the rectangle formed by the points (0, 0) (2, 0), (0, 3) are **[1]**

a) (2, 3) b) (3, 0)

c) (3, 2) d) (0, 2)

6. If $2^{x+y} = 2^{x-y} = \sqrt{8}$ then the value of y is **[1]**

a) none of these b) 0

c) $\frac{3}{2}$ d) $\frac{1}{2}$

7. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged then the water level rises by **[1]**

a) 4 cm b) 5 cm

c) 3 cm d) 6 cm

8. One card is drawn at random from a well-shuffled deck of 52 cards. What is the probability of getting a black face card? **[1]**

a) $\frac{3}{13}$ b) $\frac{3}{14}$

c) $\frac{3}{26}$ d) $\frac{1}{26}$

9. If the equation $9x^2 + 6kx + 4 = 0$ has equal roots then k = ? **[1]**

a) -2 or 0 b) 0 only

c) 2 or 0 d) 2 or -2

10. A number x is chosen at random from the numbers -3, -2, -1, 0, 1, 2, 3 the probability that $|x| < 2$ is **[1]**

a) $\frac{1}{7}$ b) $\frac{2}{7}$

c) $\frac{3}{7}$ d) $\frac{5}{7}$

11. $\frac{1+\tan^2\theta}{\sec^2\theta} =$ **[1]**

a) $\sec^2\theta$ b) 1

c) $\frac{1}{\sin^2\theta-\cos^2\theta}$ d) $\frac{1}{3}$

12. The quadratic equation $ax^2 + 2x + a = 0$ has two distinct roots, if **[1]**

a) $a = \pm 1$ b) a = 0

c) a = 0, 1 d) 0 = - 1, 0

13. HCF of two numbers is 113, their LCM is 56952. If one number is 904, the second number is **[1]**

a) 7791

b) 7911

c) 7719

d) 7119

14. In what ratio does the x-axis divide the join of A(2, -3) and B(5, 6)? **[1]**

a) 1 : 2

b) 3 : 5

c) 2 : 1

d) 2 : 3

15. The angle of depression of a car, standing on the ground, from the top of a 75 m tower, is 30°. The distance of the car from the base of the tower (in metres) is **[1]**

a) $25\sqrt{3}$

b) $75\sqrt{3}$

c) 150

d) $50\sqrt{3}$

16. Consider the following distribution : **[1]**

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

the frequency of the class 30-40 is

a) 4

b) 48

c) 51

d) 3

17. Which of the following numbers is a prime number? **[1]**

a) 233

b) 377

c) 147

d) 253

18. The sum of the digits of a two digit number is 9. Nine times this number is twice the number obtained by reversing the digits, then the number is **[1]**

a) 72

b) 27

c) 18

d) 81

19. **Assertion (A):** L.C.M. and H.C.F. of a and 20 are 100 and 10 respectively, then a = 50. **[1]**

Reason (R): L.C.M × H.C.F. = First number × Second number

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If two triangles are similar and have an equal area, then they are congruent. **[1]**
Reason (R): Corresponding sides of two triangles are equal, then triangles are congruent.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

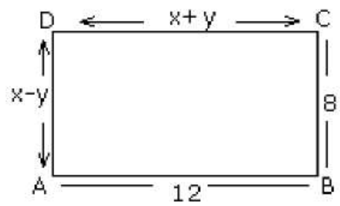
c) A is true but R is false.

d) A is false but R is true.

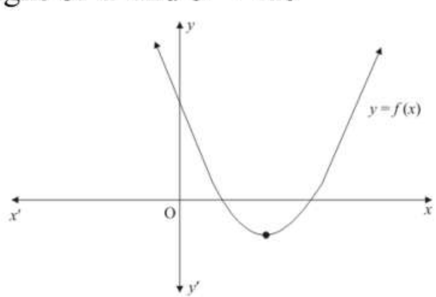
Section B

21. From a group of 3 boys and 2 girls we select two children. What is the set representing the event: **[2]**
 i. one girl is selected
 ii. at least one girl is selected?

22. ABCD is a rectangle if the value of AB = 12 is given, find the values of x and y. **[2]**



23. The graph of the polynomial $f(x) = ax^2 + bx + c$ is as shown below (Fig.). Write the signs of 'a' and $b^2 - 4ac$ **[2]**

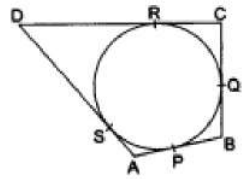


24. Two vertices of a triangle have coordinates (- 8,7) and (9,4). If the centroid of the triangle is at the origin, what are the coordinates of the third vertex? **[2]**

OR

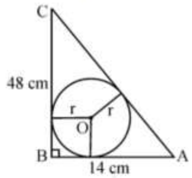
If A and B are (-2, -2) and (2, -4) respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB.

25. A quadrilateral ABCD is drawn to circumscribe a circle, as shown in the figure. Prove that AB + CD = AD + BC. **[2]**



OR

In the given figure, ABC is a triangle in which $\angle B = 90^{o}$, BC = 48 cm and AB = 14 cm. A circle is inscribed in the triangle, whose centre is O. Find radius r of in-circle.



Section C

26. If $\sec\theta + \tan\theta = p$, obtain the values of $\sec\theta$, $\tan\theta$ and $\sin\theta$ in terms of p. **[3]**

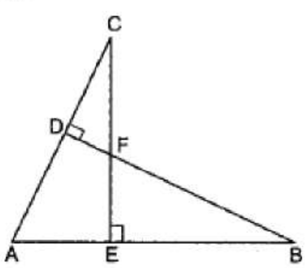
27. Two rails are represented by the linear equations x + 2y – 4 = 0 and 2x + 4y – 12 = 0. Represent this situation geometrically. **[3]**

28. Find the LCM and HCF of 404 and 96 and verify that LCM × HCF = product of the two numbers **[3]**

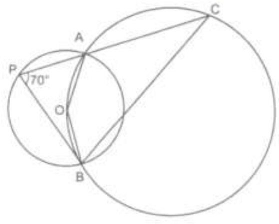
OR

Find the HCF of 506 and 1155 and express it as a linear combination of them.

29. In Fig. if $BD \perp AC$ and $CE \perp AB$, prove that **[3]**
   i. ΔAEC ~ ΔADB
   ii. $\frac{CA}{AB} = \frac{CE}{DB}$

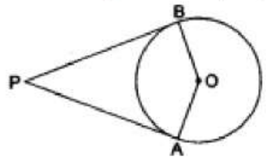


30. In a given figure, two circles intersect at A and B. The centre of the smaller circle is O and it lies on the circumference of the larger circle. If ∠APB = 70°, find ∠ACB. **[3]**



OR

In the given figure, PA and PB are the tangent segments to a circle with centre O. Show that the points A, O, B and P are concyclic.



31. From a point on a ground, the angle of elevation of bottom and top of a transmission tower fixed on the top of a 20 m high building are 45^{o} and 60^{o} respectively. Find the height of the tower. **[3]**

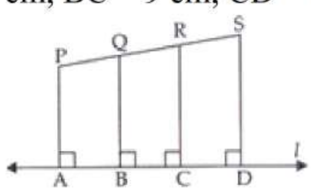
Section D

32. If (- 5) is a root of the quadratic equation $2x^2 + px + 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k. **[5]**

OR

The length of the hypotenuse of a right-angled triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle.

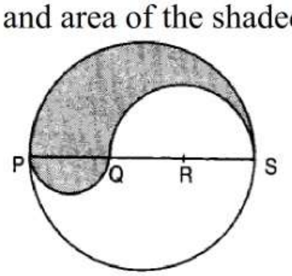
33. In the given figure, PA, QB, RC and SD are all perpendiculars to a line 'l', AB = 6 cm, BC = 9 cm, CD = 12 cm and SP = 36 cm. Find PQ, QR and RS. **[5]**



34. Find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres (Use $\pi = 22/7$). **[5]**

OR

PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. Find the perimeter and area of the shaded region



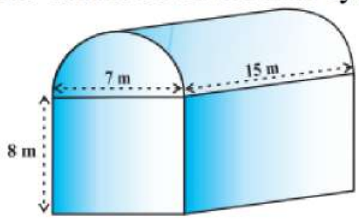
35. In a hospital, the ages of diabetic patients were recorded as follows. Find the median age. **[5]**

Age (in years)	0 - 15	15 - 30	30 - 45	45 - 60	60 - 75
Number of patients	5	20	40	50	25

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. The dimensions of the base were 15 m $\times$ 7 m $\times$ 8 m.
The diameter of the half cylinder was 7 m and length was 15 m.



(i) Find the volume of the air that the shed can hold.

(ii) If the industry requires machinery which would occupy a total space of 300 m^3 and there are 20 workers each of whom would occupy 0.08 m^3 space on an average, how much air would be in the shed when it is working?

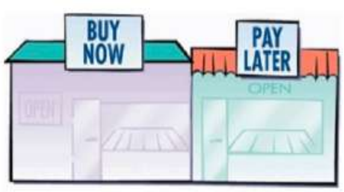
(iii) Find the surface area of the cuboidal part.

OR

Find the surface area of the cylindrical part.

37. **Read the text carefully and answer the questions:** **[4]**

Deepa has to buy a scooty. She can buy scooty either making cashdown payment of ₹ 25,000 or by making 15 monthly instalments as below.
Ist month - ₹ 3425, IInd month - ₹ 3225, IIIrd month - ₹ 3025, IVth month - ₹ 2825 and so on



(i) Find the amount of 6th instalment.

(ii) Total amount paid in 15 instalments.

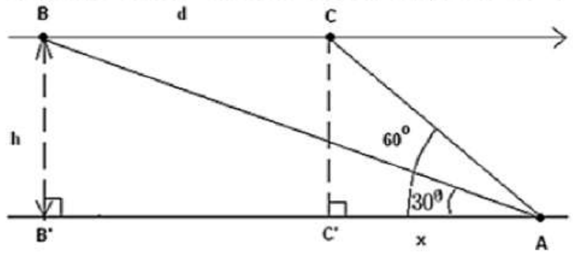
OR

If Deepa pays ₹2625 then find the number of instalment.

(iii) Deepa paid 10th and 11th instalment together find the amount paid that month.

38. **Read the text carefully and answer the questions:** **[4]**

Mr. Vinod is a pilot in Air India. During the Covid-19 pandemic, many Indian passengers were stuck at Dubai Airport. The government of India sent special aircraft to take them. Mr. Vinod was leading this operation. He is flying from Dubai to New Delhi with these passengers. His airplane is approaching point A along a straight line and at a constant altitude h. At 10:00 am, the angle of elevation of the airplane is 30^{o} and at 10:01 am, it is 60^{o}.



(i) What is the distance **d** is covered by the airplane from 10:00 am to 10:01 am if the speed of the airplane is constant and equal to 600 miles/hour?

(ii) What is the altitude **h** of the airplane? (round answer to 2 decimal places)

OR

Find the distance between passenger and airplane when the angle of elevation is 60^{o}.

(iii) Find the distance between passenger and airplane when the angle of elevation is 30^{o}.

Solution

Section A

1. **(c)** 20°
   Explanation: Let ∠OAB =∠OBA = x [Opposite angles of equal radii are equal]
   And ∠AOB = 180° - 40° = 140°
   Now, in triangle AOB,
   $\angle OAB + \angle OBA + \angle AOB = 180^{\circ}$
   ⇒x + x + 140° = 180°
   ⇒2x = 40°
   ⇒x = 20°
   ∴ ∠OAB = 20°
2. **(d)** (– 6, 7)
   Explanation: Let the coordinates of the other end be B(x_2, y_2).
   One end of the diameter is A (2, 3) and the centre is O$(-2, 5)$.
   Since the centre is midpoint of the diameter of the circle.
   $\therefore x = \frac{x_1+x_2}{2}$
   $\Rightarrow -2 = \frac{2+x_2}{2}$
   $\Rightarrow x_2 = -6$
   And $y = \frac{y_1+y_2}{2}$
   $\Rightarrow 5 = \frac{3+y_2}{2}$
   $\Rightarrow y_2 = 7$
   Therefore, the coordinates of other ends of the diameter are $(-6, 7)$.
3. **(c)** x
   Explanation: Since coordinates of any point on the x-axis is (x, 0)
   Therefore, abscissa is x.
4. **(c)** 0
   Explanation: Elementary events are
   (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
   (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
   (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
   (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
   (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
   (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)
   ∴ Number of Total outcomes = 36
   And Number of possible outcomes (product of numbers appearing on die is 7) = 0
   ∴ Required Probability = $\frac{0}{36} = 0$
5. **(a)** (2, 3)
   Explanation: We are given three vertices (0, 0), (2, 0) and (0, 3) of a rectangle.
   We have to find the coordinates of the fourth vertex.
   By plotting the given vertices on an XY plane, C (0, 3) are the consecutive vertices.
   Consider D to represent the fourth vertex.
   Since, AB = 2 units and BC = 3 units.
   Thus, point D is at a horizontal distance of 3 units and a vertical distance of 2 units from the origin.
   Thus, the coordinates of the fourth vertex of the rectangle are (2, 3).

6. **(b)** 0

Explanation: $2^{x+y} = 2^{x-y} = 2^{3/2} \Rightarrow x + y = \frac{3}{2}$ and $x - y = \frac{3}{2}$. So, by adding above two equations we get and x= y = 0

7. **(c)** 3 cm

Explanation: Increase in volume of water = volume of the sphere

$\Rightarrow \pi \times 18 \times 18 \times h = \frac{4}{3}\pi \times 9 \times 9 \times 9$

$\Rightarrow h = \left(\frac{4}{3} \times \frac{9\times9\times9}{18\times18}\right)$ cm = 3 cm

8. **(c)** $\frac{3}{26}$

Explanation: Total number of cards = 52.

Number of black face cards = 6

(2 kings + 2 queens + 2 jacks).

$\therefore$ P (getting a face card) = $\frac{6}{52} = \frac{3}{26}$

9. **(d)** 2 or -2

Explanation: Since the roots are equal, we have D = 0.

$\therefore 36k^2 - 4 \times 9 \times 4 = 0 \Rightarrow 36k^2 = 144 \Rightarrow k^2 = 4 \Rightarrow k = 2$ or -2.

10. **(c)** $\frac{3}{7}$

Explanation: Total possible number of events (n) = 7

Now $|x| < 2$

$x < 2$ or $-x < 2$

$\Rightarrow x > -2$

$\therefore$ x

$\Rightarrow x = 1, 0, -1, -2, -3$ or $x = -1, 0, 1, 2, 3$

$\therefore x = -1, 0, 1$

$\therefore m = 3$

$\therefore$ Probability = $\frac{m}{n} = \frac{3}{7}$

11. **(b)** 1

Explanation: Given: $\frac{1+\tan^2\theta}{\sec^2\theta}$

$= \frac{\sec^2\theta}{\sec^2\theta} = 1$

$[\because \sec^2\theta = 1 + \tan^2\theta]$

12. **(a)** $a = \pm 1$

Explanation: In the equation $ax^2 + 2x + a = 0$

$D = b^2 - 4ac = (2)^2 - 4 \times a \times a = 4 - 4a^2$

Roots are real and equal

D = 0

$\Rightarrow 4 - 4a^2 = 0$

$\Rightarrow 4 = 4a^2$

$\Rightarrow 1 = a^2$

$\Rightarrow a^2 = 1$

$\Rightarrow a^2 = (\pm 1)^2$

$\Rightarrow a = \pm 1$

13. **(d)** 7119

Explanation: LCM $\times$ HCF = Product of two numbers

$56952 \times 113 = 904 \times$ second number

$\frac{56952 \times 113}{904}$ = second number

Therefore, second number = 7119

14. **(a)** 1 : 2

Explanation: Let the x axis cut AB at P(x, 0) in the ratio K : 1

Then $\frac{6k-3}{k+1} = 0 \Rightarrow 6k - 3 - 0 \Rightarrow 6k = 3 \Rightarrow k = \frac{1}{2}$

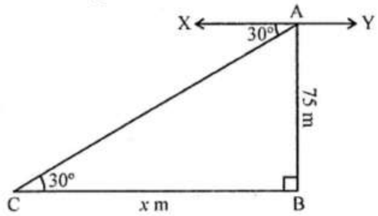
required ratio = $\left(\frac{1}{2} : 1\right) = 1 : 2$

15. **(b)** $75\sqrt{3}$

Explanation: AB is as tower and AB = 75 m

From A, the angle of depression of a car C

on the ground is 30^{o}



Let distane BC = x

Now in right $\triangle$ACB,

$\tan\theta = \frac{AB}{BC} \Rightarrow \tan 30^{\text{o}} = \frac{75}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x} \Rightarrow$ x = 75 $\sqrt{3}$ m

$\therefore$ BC = $75\sqrt{3}$ m

16. **(d)** 3

Explanation:

Marks Obtained	Number of students	f
0-10	(63-58)=5	5
10-20	(58-55)=3	3
20-30	(55-51)=4	4
30-40	(51-48)=3	3
40-50	(48-42)=6	6
50...	42=42	42

Hence, frequency in the class interval 30 - 40 is 3.

17. **(a)** 233

Explanation: We know that A prime number is a whole number greater than 1 whose only two whole-number factors which are 1 and itself. e.g. 2, 23, 29 ...

233 has only two factors = 1 $\times$ 233

Because 233 has only two factors 1 and itself i.e 233, so it's a prime number.

18. **(c)** 18

Explanation: Let unit digit = x , Tens digit = y , therefore original no will be 10y + x

Sum of digits are 9 So that x + y = 9 ... (i)

nine times this number is twice the number obtained by reversing the order of the digits

9(10y + x) = 2(10x + y)

90y + 9x = 20 x + 2y

88y - 11x = 0

Divide by 11 we get 8y - x = 0 ... (ii)

Adding equations (i) and (ii), we get
9y = 9
y = $\frac{9}{9}$ = 1
Putting this value in equation 1 we get
x + y = 9
x + 1 = 9
x = 8
Therefore the number is 10(1) + 8 = 18

19. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: It is a result.

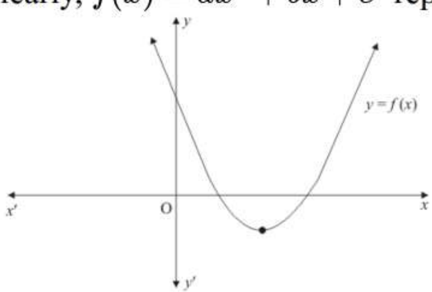
20. **(b)** Both A and R are true but R is not the correct explanation of A.
Explanation: Two similar triangles of equal area are always congruent.

Section B

21. Let boys be B_1 B_2, B_3 (3 Boys)
Let girls be G_1, G_2(2 girls)
Therefore, the set which represents, one girl is selected and at least one girl is selected are respectively as,
 i. {B_1G_1, B_2G_1, B_3G_1, B_1G_2, B_2G_2, B_3G_2}
 ii. {B_1G_1, B_2G_1, B_3G_1 B_1G_2, B_2G_2, B_3G_2, G_1G_2]

22. x + y = 12 ...(i)
x - y = 8 ...(ii)
On adding (i) and (ii),
2x = 20
⇒ x = 10
∴ 10 + y = 12
⇒ y = 2

23. Clearly, $f(x) = ax^2 + bx + c$ represent a parabola opening upwards. Therefore, a>0



Since the parabola cuts the x-axis at two points, this means that the polynomial will have two real solutions.
Hence $b^2 - 4ac > 0$
Hence a>0 and $b^2 - 4ac > 0$

24. Two vertices of a triangle are (-8, 7) and (9, 4)
Let the third vertex be (x, y)
If $A(x_1, y_1),\ B(x_2, y_2)\ and\ C(x_3, y_3)$ are the three vertices of the ΔABC, then
Coordinates of the centroid are ($\frac{x_1+x_2+x_3}{2}$, $\frac{y_1+y_2+y_3}{2}$)
Here, Centroid of the triangle is given to be (0, 0).
$\therefore \frac{x_1+x_2+x_3}{3} = 0 \Rightarrow \frac{-8+9+x}{3} = 0$
$\Rightarrow 1 + x = 0 \Rightarrow x = -1$
and $\frac{y_1+y_2+y_3}{3} = 0 \Rightarrow \frac{7+4+y}{3} = 0$

$\Rightarrow 11 + y = 0 \Rightarrow y = -11$

$\therefore$ Third vertex will be (-1, -11)

OR

A = (-2, -2) and B=(2, -4)

It is given that AP= $\frac{3}{7}$ AB

PB = AB - AP = AB $-\frac{3}{7}$AB = $\frac{4}{7}$AB

So, we have AP:PB = 3:4

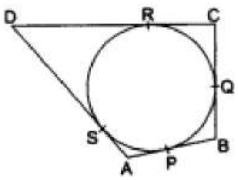
Let coordinates of P be (x, y)

Using Section formula to find coordinates of P, we get

$x = \frac{(-2)\times4+2\times3}{3+4} = \frac{6-8}{7} = \frac{-2}{7}$

$y = \frac{(-2)\times4+(-4)\times3}{3+4} = \frac{-8-12}{7} = \frac{-20}{7}$

Therefore, Coordinates of point P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$.

25. 

We know that the lengths of tangents drawn from an exterior point to a circle are equal.

AP = AS, ... (i) [tangents from A]

BP = BQ, ... (ii) [tangents from B]

CR = CQ, ... (iii) [tangents from C]

DR = DS. ... (iv) [tangents from D]

AB + CD = (AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS) [using (i), (ii), (iii), (iv)]

= (AS + DS) + (BQ + CQ)

= AD + BC.

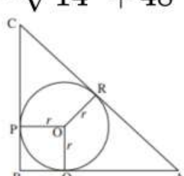
Hence, AB + CD = AD + BC.

OR

Here,

AC = $\sqrt{AB^2 + BC^2}$

=$\sqrt{14^2 + 48^2}$ = $\sqrt{2500}$ = 50 cm



$\angle OQB = 90^{o}$

$\Rightarrow$ OPBQ is a square

$\Rightarrow$ BQ = r

QA = 14 - r = AR (tangents from a external point are equal in length)

Again PB = r,

PC = 48 - r

$\Rightarrow$ RC = 48 - r (tangents from a external point are equal in length)

AR + RC = AC

$\Rightarrow$ 14 - r + 48 - r = 50

$\Rightarrow$ r = 6 cm.

Section C

26. Given, $\sec\theta + \tan\theta = p$ ...(1)

We know that, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow p(\sec\theta - \tan\theta) = 1$ from (1)

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$ ...(2)

Adding (1) and (2), we get;

$\Rightarrow (\sec\theta + \tan\theta) + (\sec\theta - \tan\theta) = p + \frac{1}{p}$

and on subtracting (2) from (1), we get;

$(\sec\theta + \tan\theta) - (\sec\theta - \tan\theta) = p - \frac{1}{p}$

$\Rightarrow 2\sec\theta = p + \frac{1}{p}$

and, $2\tan\theta = p - \frac{1}{p}$

$\Rightarrow \sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$ and, $\tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$

Also, $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \sin\theta = \tan\theta . \cos\theta$

$\therefore \sin\theta = \frac{\tan\theta}{\sec\theta}$

$\Rightarrow \sin\theta = \frac{\frac{1}{2}\left(p - \frac{1}{p}\right)}{\frac{1}{2}\left(p + \frac{1}{p}\right)} = \frac{p^2-1}{p^2+1}$

27. Two solutions of each of the linear equations :

$x + 2y - 4 = 0$ ...(1)

$2x + 4y - 12 = 0$ ...(2)

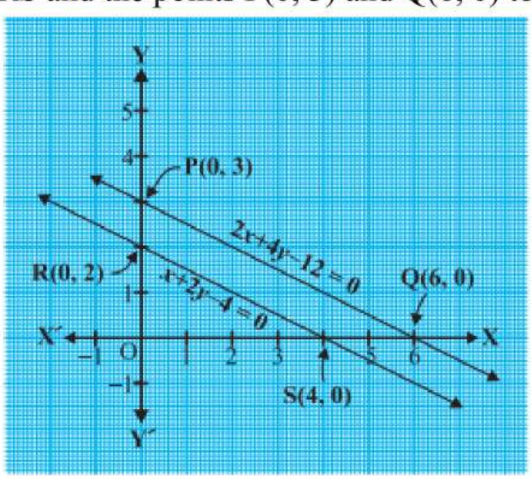
are given in Table

x	0	4
$y = \frac{4-x}{2}$	2	0

and

x	0	6
$y = \frac{12-2x}{4}$	3	0

To represent the equations graphically, we plot the points R(0, 2) and S(4, 0), to get the line RS and the points P(0, 3) and Q(6, 0) to get the line PQ.



Here, the lines do not intersect anywhere, i.e., they are parallel.

28. Prime factorisation of 404 and 96 is:

$404 = 2 \times 2 \times 101$

or $404 = 2^2 \times 101$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

or $96 = 2^5 \times 3$

$\therefore$ HCF(404,96)= $2^2 = 4$

LCM(404, 96) = $101 \times 2^5 \times 3$

LCM(404, 96) = 9696

Now we have to verify that,

HCF(404, 96) $\times$LCM(404, 96) = 404$\times$96

Hence, LHS = HCF $\times$ LC M = 4 $\times$ 9696 = 38784

RHS = Product of numbers = 404 $\times$ 96 = 38784

Since, LHS = RHS

$\therefore$ HCF $\times$ LCM = Product of 404 and 96.

Hence verified.

OR

We need to find the H.C.F. of 506 and 1155 and express it as a linear combination of 506 and 1155.

By applying Euclid's division lemma

1155 = 506 $\times$ 2 + 143.

506 = 143 $\times$ 3 + 77.

143 = 77 $\times$ 1 + 66.

77 = 66 $\times$ 1 + 11.

66 = 11 $\times$ 6 + 0.

Therefore, H.C.F. = 11.

Now, 11 = 77 - 66 $\times$ 1 = 77 - [143 - 77 $\times$ 1] $\times$ 1 {$\because$ 143 = 77 $\times$ 1 + 66}

= 77 - 143 $\times$ 1 + 77 $\times$ 1

= 77 $\times$ 2 - 143 $\times$ 1

= [506 - 143 $\times$ 3] $\times$ 2 - 143 $\times$ 1 {$\because$ 506 = 143 $\times$ 3 + 77 }

= 506 $\times$ 2 - 143 $\times$ 6 - 143 $\times$ 1

= 506 $\times$ 2 - 143 $\times$ 7

= 506 $\times$ 2 - [1155 - 506 $\times$ 2] $\times$ 7 {$\because$1155 = 506 $\times$ 2 + 143 }

= 506 $\times$ 2 - 1155 $\times$ 7 + 506 $\times$ 14

= 506 $\times$ 16 - 1155 $\times$ 7

Hence obtained.

29. i. Resorting to the given figure we observe that In Δ's AEC and ADB,

$\angle$AEC = $\angle$ADB = 90° [$\because CE \perp AB$ and $BD \perp AC$]

and, $\angle$EAC = $\angle$DAB [Each equal to $\angle$ A]

Therefore, by AA-criterion of similarity, we obtain

ΔAEC~ ΔADB

ii. We have,

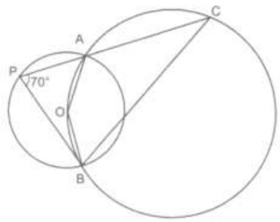
ΔAEC ~ ΔADB [As proved above]

$\Rightarrow \quad \frac{CA}{BA} = \frac{EC}{DB}$ {For similar triangles corresponding sides are proportional}

$\Rightarrow \quad \frac{CA}{AB} = \frac{CE}{DB}$

30. Consider the smaller circle whose centre is given as O.

The angle subtended by an arc at the centre of the circle is double the angle subtended by the arc in the remaining part of the circle.



Therefore,we have,
∠AOB = 2∠APB

= 2(70^{o})

∠AOB = 140^{o}

Now consider the larger circle and the points A,C, B and O along its circumference. AOBC from a cycle quadrilateral.

In a cyclic quadrilateral , the opposite angles are supplementary, meaning that the opposite angles add up to 180^{o}.

∠AOB + ∠ACB = 180^{o}

∠ACB = 180^{o} - ∠AOB

= 180^{o} - 140^{o}

∠ ACB = 40^{o}

Therefore, the measure of angle ACB is 40^{o}.

OR

Here, OA = OB

And OA ⊥ AP, OB ⊥ BP (Since tangent is perpendicular to the radius at the point of contact)

$\therefore \angle OAP = 90°$,

$\angle OBP = 90°$

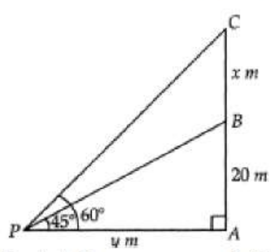
$\therefore \angle OAP + \angle OBP = 90° + 90° = 180°$

$\therefore \angle AOB + \angle APB = 180°$

(Since, $\angle AOB + \angle OAP + \angle OBP + \angle APB = 360°$)

Thus, sum of opposite angle of a quadrilateral is 180°.

Hence, A, O, B and P are concyclic.

31. 

Let AP = y m and BC = xm.

$\therefore$ In $\triangle BAP = \frac{BA}{PA} = \tan 45°$

$\Rightarrow \quad \frac{20}{y} = 1$

$\Rightarrow$ y = 20

In $\triangle$CAP $\frac{CA}{PA} = \tan 60°$

$\Rightarrow \quad \frac{20+x}{y} = \sqrt{3}$

$\Rightarrow \quad 20 + x = y\sqrt{3}$ (using y = 20)

$\Rightarrow \quad 20 + x = 20\sqrt{3}$

$x = 20\sqrt{3} - 20$

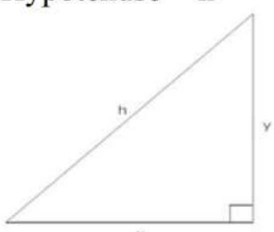
$= 20(\sqrt{3} - 1)$
$= 20 \times (1 \cdot 73 - 1)$
Hence, height of the tower = 14.64 m

Section D

32. Since (-5) is a root of given quadratic equation $2x^2 + px + 15 = 0$,then,
$2(-5)^2 + p(-5) - 15 = 0$
$50 - 5p - 15 = 0$
$5p = 35 \Rightarrow p = 7$
Now $p\left(x^2 + x\right) + k = 0$ has equal roots
$px^2 + px + k = 0$
So $(b)^2 - 4ac = 0$
$(p)^2 - 4p \times k = 0$
$(7)^2 - 4 \times 7 \times k = 0$
28k = 49
$k = \frac{49}{28} = \frac{7}{4}$
hence p = 7 and $k = \frac{7}{4}$

OR

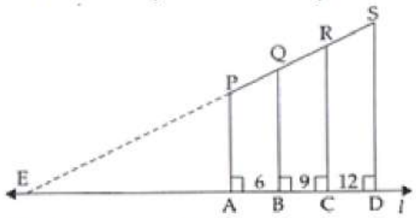
Let base = x
Altitude = y
Hypotenuse = h



According to question,
h = x + 2
h = 2y + 1
$\Rightarrow$ x + 2 = 2y + 1
$\Rightarrow$ x + 2 - 1 = 2y
$\Rightarrow$ x - 1 = 2y
$\Rightarrow \frac{x-1}{2} = y$
And $x^2 + y^2 = h^2$
$\Rightarrow x^2 + \left(\frac{x-1}{2}\right)^2 = (x+2)^2$
$\Rightarrow x^2 - 15x + x - 15 = 0$
$\Rightarrow x^2 - 15x + x - 15 = 0$
$\Rightarrow (x - 15)(x + 1) = 0$
$\Rightarrow$ x = 15 or x = -1
Base = 15 cm
Altitude = $\frac{x+1}{2} = 8$cm
Height, $h = 2 \times 8 + 1 = 17$cm

33. **Given:** PA, QB, RC and SD are perpendicular on line l.

AB = 6 cm, BC = 9 cm, CD = 12 cm, SP=36 cm



To find: PQ, QR and RS.

Construction: we produce SP so that it joins l at E.

Proof: In ΔEDS,

AP || BQ || CR || SD [Given]

∴ PQ : QR : RS = AB : BC : CD

PQ : QR : RS = 6 : 9 : 12

Let PQ = 6x

then QR = 9x

and RS = 12x

Now, PQ + QR + RS = 36 cm (given)

⇒ 6x + 9x + 12x = 36

⇒ 27x = 36

$\Rightarrow x = \frac{36}{27} = \frac{4}{3}$

Therefore, PQ = $6 \times \frac{4}{3}$ = 8 cm

QR = $9 \times \frac{4}{3}$ = 12 cm

RS = $12 \times \frac{4}{3}$ = 16 cm

34. We have to find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres.

For the first triangle, we have a = 35, b = 53 and c = 66.

$\therefore \quad s = \frac{a+b+c}{2} = \frac{35+53+66}{2} = 77\text{cm}$

Let Δ_1 be the area of the first triangle. Then,

$\Delta_1 = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_1 = \sqrt{77(77-35)(77-53)(77-66)} = \sqrt{77 \times 42 \times 24 \times 11}$

$\Rightarrow \quad \Delta_1 = \sqrt{7 \times 11 \times 7 \times 6 \times 6 \times 4 \times 11} =$

$\sqrt{7^2 \times 11^2 \times 6^2 \times 2^2} = 7 \times 11 \times 6 \times 2 = 924\text{cm}^2$ ...(i)

For the second triangle, we have a = 33,b = 56,c = 65

$\therefore \quad s = \frac{a+b+c}{2} = \frac{33+56+65}{2} = 77\text{cm}$

Let Δ_2 be the area of the second triangle. Then,

$\Delta_2 = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_2 = \sqrt{77(77-33)(77-56)(77-65)}$

$\Rightarrow \quad \Delta_2 = \sqrt{77 \times 44 \times 21 \times 12} =$

$\sqrt{7 \times 11 \times 4 \times 11 \times 3 \times 7 \times 3 \times 4} = \sqrt{7^2 \times 11^2 \times 4^2 \times 3^2}$

$\Rightarrow \quad \Delta_2 = 7 \times 11 \times 4 \times 3 = 924\text{cm}^2$

Let r be the radius of the circle. Then,

Area of the circle = Sum of the areas of two triangles

$\Rightarrow \quad \pi r^2 = \Delta_1 + \Delta_2$

$\Rightarrow \quad \pi r^2 = 924 + 924$

$\Rightarrow \quad \frac{22}{7} \times r^2 = 1848$

$\Rightarrow \quad r^2 = 1848 \times \frac{7}{22} = 3 \times 4 \times 7 \times 7 \Rightarrow \quad r = \sqrt{3 \times 2^2 \times 7^2} = 2 \times 7 \times \sqrt{3} = 14\sqrt{3}\text{cm}$

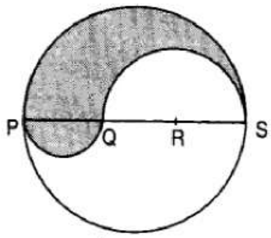
OR

PS = Diameter of a circle of radius 6 cm = 12 cm

$\therefore$ PQ = QR = RS = $\frac{12}{3}$ = 4cm, QS = QR + RS = (4 + 4)cm = 8 cm

Let P be the perimeter and A be the area of the shaded region.

P = Arc of semi-circle of radius 6 cm + Arc of semi-circle of radius 4 cm + Arc of semi-circle of radius 2 cm



$\Rightarrow P = (\pi \times 6 + \pi \times 4 + \pi \times 2)\text{cm} = 12\pi\text{cm}$

and, A = Area of semi-circle with PS as diameter + Area of semi-circle with PQ as diameter - Area of semi-circle with QS as diameter.

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7}\left(6^2 + 2^2 - 4^2\right) = \frac{1}{2} \times \frac{22}{7} \times 24 = \frac{264}{7}\text{cm}^2$ = 37.71 cm^2

35. Calculation of median:

Class interval	Frequency(f_i)	Cumulative frequency
0 - 15	5	5
15 - 30	20	25
30 - 45	40	65
45 - 60	50	115
60 - 75	25	140

N = 140$\Rightarrow$ $\frac{N}{2}$ = 70.

The median class is 45 -60.

$\therefore l = 45, h = 15, f = 50, c.f. = 65$

Median, M =$l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

$= 45 + \left\{15 \times \frac{(70-65)}{50}\right\}$

$= 45 + \left\{15 \times \frac{5}{50}\right\}$

$= 45 + 1.5 = 46.5$

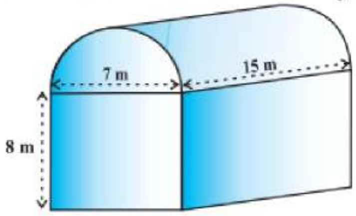
Hence, the median age of diabetic patients is 46.5 years.

Section E

36. **Read the text carefully and answer the questions:**

Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. The dimensions of the base were 15 m × 7 m × 8 m.

The diameter of the half cylinder was 7 m and length was 15 m.



(i) Total volume = volume of cuboid + $\frac{1}{2}$ × volume of cylinder.
For cuboidal part we have
length = 15 m, breadth = 7 m and height = 8 m
$\therefore$ Volume of cuboidal part = l × b × h $= 15 \times 7 \times 8\text{m}^2 = 840\text{m}^3$
Clearly,
r = Radius of half-cylinder = $\frac{1}{2}$ (Width of the cuboid) = $\frac{7}{2}$m
and, h = Height (length) of half-cylinder = Length of cuboid = 15 m
$\therefore$ Volume of half-cylinder = $\frac{1}{2}\pi r^2 h = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 15\text{m}^3 = \frac{1155}{4}\text{m}^3 = 288.75\text{m}^3$
Thus the volume of the air that the shed can hold = (840 + 288.75) m^3 = 1128.75 m^3

(ii) Total space occupied by 20 workers = $20 \times 0.08\text{m}^3$ = = 1.6 m^3
Total space occupied by the machinery = 300 m^3
$\therefore$ Volume of the air inside the shed when there are machine and workers inside it
= (1128.75 - 1.6 - 300) m^3
= (1128.75 - 301.6) m^3 = 827.15 m^3
Hence, volume of air when there are machinery and workers is 827.15

(iii) Given for the cuboidal part
length L = 15 m, Width B = 7 m, Height = 8 m
Surface area of the cuboidal part
= 2(LB + BH + HL)
= 2(15 × 7 + 7 × 8 + 8 × 15)
= 2(105 + 56 + 120) = 2 × 281 = 562 m^2

OR

For the cylindrical part r = 3.5 cm and l = 15 m
Thus the surface area of the cylindrical part
$= \frac{1}{2}(2\pi r l) = 3.14 \times 3.5 \times 15$
= 164.85 m^2

37. **Read the text carefully and answer the questions:**
Deepa has to buy a scooty. She can buy scooty either making cashdown payment of ₹ 25,000 or by making 15 monthly instalments as below.
Ist month - ₹ 3425, IInd month - ₹ 3225, IIIrd month - ₹ 3025, IVth month - ₹ 2825 and so on

(i) 1st installment = ₹3425
2nd installment = ₹3225
3rd installment = ₹3025
and so on
Now, 3425, 3225, 3025, ... are in AP, with

a = 3425, d = 3225 - 3425 = -200

Now 6th installment = a_n = a + 5d = 3425 + 5(-200) = ₹2425

(ii) Total amount paid = $\frac{15}{2}$(2a + 14d)

= $\frac{15}{2}$[2(3425) + 14(-200)] = $\frac{15}{2}$(6850 - 2800)

= $\frac{15}{2}$(4050) = ₹30375

OR

$a_n = a + (n - 1)d$ given $a_n = 2625$

2625 = 3425 + (n - 1) × - 200

⇒ - 800 = (n - 1) × - 200

⇒ 4 = n - 1

⇒ n = 5

So, in 5th installment, she pays ₹2625.

(iii) $a_n = a + (n - 1)d$

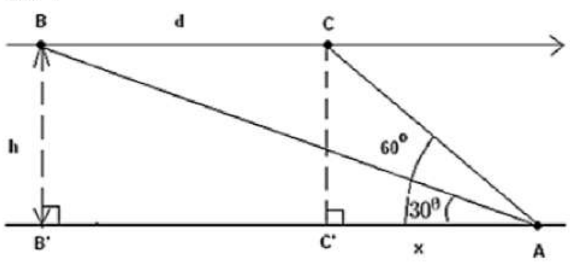
$\Rightarrow a_{10} = 3425 + 9 \times (-200) \ = 1625$

⇒ a_{11} = 3425 + 10 × (-200) = 1425

$a_{10} + a_{11}$ = 1625 + 1425 = 3050

38. **Read the text carefully and answer the questions:**

Mr. Vinod is a pilot in Air India. During the Covid-19 pandemic, many Indian passengers were stuck at Dubai Airport. The government of India sent special aircraft to take them. Mr. Vinod was leading this operation. He is flying from Dubai to New Delhi with these passengers. His airplane is approaching point A along a straight line and at a constant altitude h. At 10:00 am, the angle of elevation of the airplane is 30^o and at 10:01 am, it is 60^o.



(i) Time covered 10.00 am to 10.01 am = 1 minute = $\frac{1}{60}$ hour

Given: Speed = 600 miles/hour

Thus, distance d = 600 × $\frac{1}{60}$ = 10 miles

(ii) Now, tan $30^o = \frac{BB'}{B'A} = \frac{h}{10+x}$ ...(i)

And tan $60^o = \frac{CC'}{C'A} = \frac{BB'}{C'A} = \frac{h}{x}$

x = $\frac{h}{\tan 60^o} = \frac{h}{\sqrt{3}}$

Putting the value of x in eq(1), we get,

tan $30^o = \frac{h}{10+\frac{h}{\sqrt{3}}} = \frac{\sqrt{3}h}{10\sqrt{3}+h}$

$$\tan 30^0 = \frac{\sqrt{3}h}{10\sqrt{3}+h}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}h}{10\sqrt{3}+h}$$

$\Rightarrow 3h = 10\sqrt{3} + h$
$\Rightarrow 2h = 10\sqrt{3}$
$\Rightarrow h = 5\sqrt{3}$ = 8.66 miles
Thus, the altitude 'h' of the airplane is 8.66 miles.

OR

The distance between passenger and airplane when the angle of elevation is 60^o.
In $\triangle$ACC'
$\sin 60^o = \frac{CC'}{AC}$
$\Rightarrow \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{AC}$
$\Rightarrow$ AC = 10 miles

(iii) The distance between passenger and airplane when the angle of elevation is 30^o.
In $\triangle$ABB'
$\sin 30^o = \frac{BB'}{AB}$
$\Rightarrow \frac{1}{2} = \frac{8.66}{AB}$
$\Rightarrow$ AB = 17.32 miles