Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper - 10

with Solution

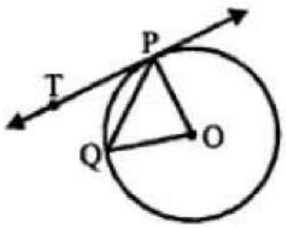
Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. In the given figure, O is the centre of a circle, PQ is a chord and PT is the tangent at P. If $\angle POQ = 70°$, $\angle TPQ$ is equal to **[1]**



a) 45° b) 70°

c) 55° d) 35°

2. The points P(0, 6), Q(-5, 3) and R(3,1) are the vertices of a triangle, which is **[1]**

a) scalene b) equilateral

c) isosceles d) right angled

3. The distance between $(at^2, 2at)$ and $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ is **[1]**

a) $a\left(t^2 + \frac{1}{t^2}\right)$ units b) $a\left(t - \frac{1}{t}\right)^2$ units

c) $a\left(t + \frac{1}{t}\right)^2$ d) $\left(t + \frac{1}{t}\right)^2$ units

4. A number x is chosen at random from the numbers -4, -3, -2, -1, 0, 1, 2, 3, 4, 5. The probability that $|x| < 3$ is **[1]**

a) 1

b) 0

c) $\frac{1}{2}$

d) $\frac{7}{10}$

5. The ratio in which the x-axis divides the segment joining (3, 6) and (12,-3) is **[1]**

a) 1 : -2

b) 2 : 1

c) 1 : 2

d) -2 : 1

6. A die is thrown once. The probability of getting an even number is **[1]**

a) $\frac{1}{3}$

b) $\frac{5}{6}$

c) $\frac{1}{6}$

d) $\frac{1}{2}$

7. The radii of the base of a cylinder and a cone are in the ratio 3 :4. If they have their heights in the ratio 2 : 3, the ratio between their volumes is **[1]**

a) 9 : 8

b) 3 : 4

c) 8 : 9

d) 4 : 3

8. If the system 6x – 2y = 3, kx – y = 2 has a unique solution, then **[1]**

a) k = 3

b) $k \neq 4$

c) $k \neq 3$

d) k = 4

9. If the sum of the roots of the equation $kx^2 + 2x + 3k = 0$ is equal to their product then the value of k is **[1]**

a) $\frac{1}{3}$

b) $\frac{-1}{3}$

c) $\frac{-2}{3}$

d) $\frac{2}{3}$

10. If x = 1 is a common root of $ax^2 + ax + 2 = 0$ and $x^2 + x + b = 0$ then, ab **[1]**

a) 2

b) 1

c) 3

d) 4

11. A bag contains 3 red balls, 5 white balls and 7 black balls. What is the probability that a ball drawn from the bag at random will be neither red nor black? **[1]**

a) $\frac{1}{3}$

b) $\frac{8}{15}$

c) $\frac{7}{15}$

d) $\frac{1}{5}$

12. The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, what is the other number? **[1]**

a) 9 b) 81

c) 45 d) 36

13. The point where the perpendicular bisector of the line segment joining the points A(2, 5) and B(4, 7) cuts is: **[1]**

a) (3, 6) b) (0, 0)

c) (2, 5) d) (6, 3)

14. $(\cos 0^{o} + \sin 30^{o} + \sin 45^{o})(\sin 90^{o} + \cos 60^{o} - \cos 45^{o}) = ?$ **[1]**

a) $\frac{5}{8}$ b) $\frac{7}{4}$

c) $\frac{5}{6}$ d) $\frac{3}{5}$

15. The relationship between mean, median and mode for a moderately skewed distribution is: **[1]**

a) Mode = 2 Median - 3 Mean b) Mode = 2 Median - Mean

c) Mode = Median - 2 Mean d) Mode = 3 Median - 2 mean

16. If a pole 12 m high casts a shadow $4\sqrt{3}$ m long on the ground then the sun's elevation is **[1]**

a) 30° b) 45°

c) 90° d) 60°

17. The pair of equations x = 2 and y = -3 has **[1]**

a) no solution b) one solution

c) infinitely many solutions d) two solutions

18. **Assertion (A):** 3 is a rational number. **[1]**
Reason (R): The square roots of all positive integers are irrationals.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

19. **Assertion (A):** If in a △ABC, a line DE || BC, intersects AB in D and AC in E, then $\frac{AB}{AD} = \frac{AC}{AE}$ **[1]**
Reason (R): If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.

a) Both A and R are true and R is b) Both A and R are true but R is

the correct explanation of A.	not the correct explanation of A.
c) A is true but R is false.	d) A is false but R is true.

20. The sum of a rational and an irrational number is **[1]**

a) Can be Rational or Irrational	b) Irrational
c) Always Rational	d) Rational

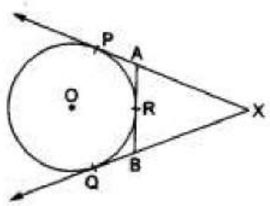
Section B

21. Ten students of class X took part in Mathematics quiz. If the number of girls is 4 more than the number of boys. Represent this situation algebraically and graphically. **[2]**

22. In a class of 40 students, there are 13 students who have 100% attendance, 15 students who do social work, 5 students participate in Adult Education and the remaining students participate in an educational cultural program. One student is selected from the class. What is the probability that he participates in a cultural program? **[2]**

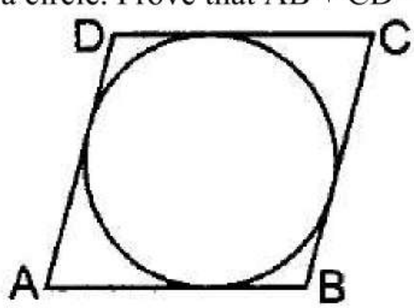
23. Find the zeroes of $x^2 - 2x - 8$ and verify the relationship between the zeros and the coefficients. **[2]**

24. In the given figure, XP and XQ are two tangents to the circle with centre O, drawn from an external point X. ARB is another tangent, touching the circle at R. Prove that XA +AR = XB + BR. **[2]**



OR

Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above prove the following: A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.



25. If the mid-point of the line segment joining $A(\frac{x}{2}, \frac{y+1}{2})$ and B(x + 1, y - 3) is C(5, - 2), find y. **[2]**

OR

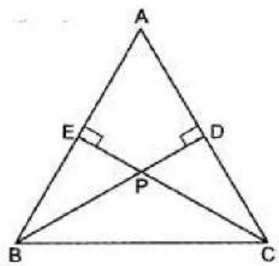
Find the distance between the points $A\left(at_1^2, 2at_1\right)$ and $B\left(at_2^2, 2at_2\right)$

Section C

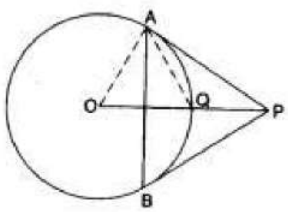
26. Form the pair of linear equations in the problem, and find its solution (if it exists) by the elimination method: **[3]**
If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes half if we only add 1 to the denominator. What is the fraction?

27. If $cosec\theta = \sqrt{10}$, find the value of all T-ratios of θ. **[3]**

28. In Fig. considering triangles BEP and CPD, prove that BP × PD = EP × PC. **[3]**

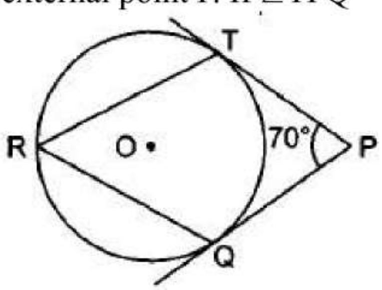


29. From a point P, two tangents PA and PB are drawn to a circle C(O r). If OP = 2r, show that $\triangle APB$ is equilateral. **[3]**



OR

In figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ∠TPQ = 70°, find ∠TRQ.



30. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60^o. At a point R, 40 m vertically above X, the angle of elevation of the top Q of tower is 45^o. Find the height of the tower PQ and the distance PX. **[3]**

31. Find the values of a and b so that the polynomials P(x) and Q(x) have $\left(x^2 - x - 12\right)$ as their HCF, where **[3]**
$P(x) = \left(x^2 - 5x + 4\right)\left(x^2 + 5x + a\right)$

$Q(x) = \left(x^2 + 5x + 6\right)\left(x^2 - 5x - 2b\right)$

OR

Show that $2 - \sqrt{3}$ is an irrational number.

Section D

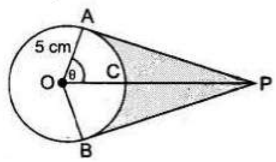
32. In trapezium ABCD, $AB \| DC$ and DC = 2AB. EF drawn parallel to AB cuts AD in F and BC in E such that $\frac{BE}{EC} = \frac{3}{4}$. Diagonal DB intersects EF at G. Prove that 7 FE = 10 AB. **[5]**

33. Solve the quadratic equation by factorization: **[5]**
$\frac{3}{x+1} - \frac{1}{2} = \frac{2}{3x-1}, x \neq -1, \frac{1}{3}$

OR

If the roots of the quadratic equation $\left(c^2 - ab\right)x^2 - 2\left(a^2 - bc\right)x + b^2 - ac = 0$ in x are equal then show that either a = 0 or $a^3 + b^3 + c^3 = 3abc$

34. An elastic belt is placed round the rim of a pulley of radius 5 cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from O. Find the length of the belt that is in contact with the rim of the pulley. Also, find the shaded area. **[5]**



OR

A chord of a circle of radius 10cm subtends a right angle at the center. Find the area of the corresponding: (Use $\pi = 3.14$)

i. minor sector
ii. major sector
iii. minor segment
iv. major segment

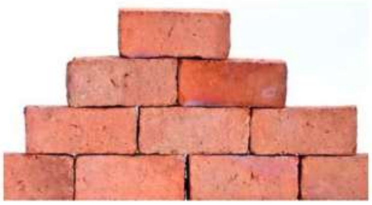
35. Find the mean from the following frequency distribution of marks at a test in statistics: **[5]**

Marks (x):	5	10	15	20	25	30	35	40	45	50
No. of students (f):	15	50	80	76	72	45	39	9	8	6

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Akshat's father is planning some construction work in his terrace area. He ordered 360 bricks and instructed the supplier to keep the bricks in such as way that the bottom row has 30 bricks and next is one less than that and so on.



The supplier stacked these 360 bricks in the following manner, 30 bricks in the bottom row, 29 bricks in the next row, 28 bricks in the row next to it, and so on.

(i) In how many rows, 360 bricks are placed?

(ii) How many bricks are there in the top row?

OR

If which row 26 bricks are there?

(iii) How many bricks are there in 10^{th} row?

37. **Read the text carefully and answer the questions:** **[4]**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.

(i) Find the volume of the Hermika, if the side of cubical part is 10 m.

(ii) Find the volume of cylindrical base part whose diameter and height 48 m and 14 m.

(iii) If the volume of each brick used is 0.01 m^3, then find the number of bricks used to make the cylindrical base.

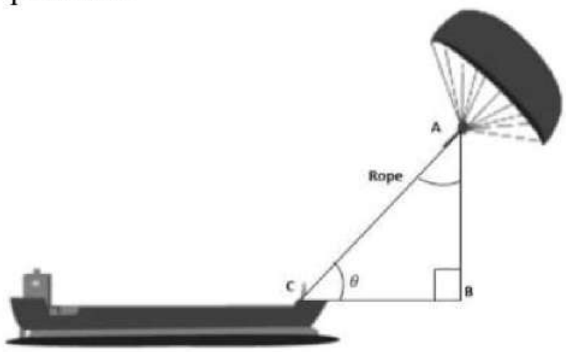
OR

If the diameter of the Anda is 42 m, then find the volume of the Anda.

38. **Read the text carefully and answer the questions:** **[4]**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to

be raised properly and effectively.
Based on the following figure related to sky sailing, answer the following questions:



(i) In the given figure, if $\sin\theta = \cos(\theta - 30^{o})$, where θ and $\theta - 30^{o}$ are acute angles, then find the value of θ.

(ii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200m?

OR

What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 150m?

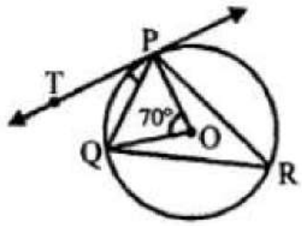
(iii) In the given figure, if $\sin\theta = \cos(3\theta - 30^{o})$, where θ and $3\theta - 30^{o}$ are acute angles, then find the value of θ.

Solution

Section A

1. **(d)** 35°

   Explanation: O is the centre of circle, PQ is a chord, PT is tangent.

   ∠POQ = 70°, then ∠TPQ = ?

   Take a point R on the major segment and join PR and QR

   arc PQ subtends ∠POQ at the centre and ∠PRQ at the remaining part of the circle

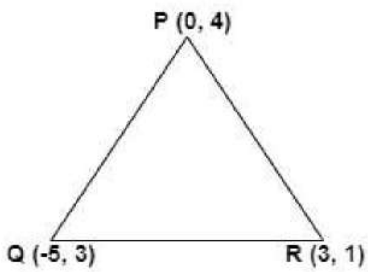
   $\angle PRQ = \frac{1}{2}\angle POQ = \frac{1}{2} \times 70° = 35°$

   But ∠TPQ = ∠PRQ (Angles in the alternate segment)

   ∠TPQ = 35°

2. **(d)** right angled

   Explanation:

   $PQ = \sqrt{(-5)^2 + (-3)^2} = \sqrt{25 + 3}$

   $= \sqrt{34}$

   $QR = \sqrt{R^2 + 2^2} = \sqrt{64 + 4} = \sqrt{68}$

   $PR = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34}$

   PQ = PR

   $QR^2 = PQ^2 + PR^2$

   $(\sqrt{68})^2 = (\sqrt{34})^2 + (\sqrt{34})^2$

   68 = 68

   △PQR is a Isosceles right angle triangle

3. **(c)** $a\left(t+\frac{1}{t}\right)^2$

Explanation: The distance between $(at^2, 2at)$ and $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$

$$= \sqrt{\left(\frac{a}{t^2} - at^2\right)^2 + \left(\frac{-2a}{t} - 2at\right)^2}$$

$$= a\sqrt{\frac{1}{t^4} + t^4 - 2 + \frac{4}{t^2} + 4t^2 + 8}$$

$$= a\sqrt{\frac{1}{t^4} + t^4 + \frac{4}{t^2} + 4t^2 + 6}$$

$$= a\sqrt{\frac{1}{t^4} + t^4 + 4 + 2 + \frac{4}{t^2} + 4t^2}$$

$$= a\sqrt{\left(t^2 + \frac{1}{t^2} + 2\right)^2}$$

$$= a\left(t^2 + \frac{1}{t^2} + 2\right)$$

$$= a\left(t + \frac{1}{t}\right)^2 \text{ units}$$

4. **(c)** $\frac{1}{2}$

Explanation: Number of total outcomes = 10
Number of possible outcomes = {-2, -1, 0, 1, 2} = 5

$\therefore$ Required Probability $= \frac{5}{10} = \frac{1}{2}$

5. **(b)** 2 : 1

Explanation: The point lies on x-axis
Its ordinate is zero
Let this point divides the line segment joining the points (3, 6) and (12, -3) in the ratio m : n

$\therefore 0 = \frac{my_2 + ny_1}{m+n} \Rightarrow 0 = \frac{m(-3) + n \times 6}{m+n}$

$\Rightarrow -3m + 6n = 0 \Rightarrow 6n = 3m$

$\Rightarrow \frac{m}{n} = \frac{6}{3} = \frac{2}{1}$

$\therefore$ Ratio = 2 : 1

6. **(d)** $\frac{1}{2}$

Explanation: Number of all possible outcomes = 6.
Even numbers are 2,4, 6. Their number is 3.

$\therefore$ P (getting an even number) = $\frac{3}{6} = \frac{1}{2}$

7. **(a)** 9 : 8

Explanation: Let the radii of the base of the cylinder and cone be 3r and 4r and their heights be 2h and 3h, respectively.

Then, ratio of their volumes = $\frac{\pi(3r)^2 \times (2h)}{\frac{1}{3}\pi(4r)^2 \times (3h)}$

$= \frac{9r^2 \times 2 \times 3}{16r^2 \times 3}$

$= \frac{9}{8}$

= 9 : 8

8. **(c)** $k \neq 3$

Explanation: If the system has a unique solution, then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Here $a_1 = 6, a_2 = k, b_1 = -2$
and $b_2 = -1$

$\therefore \frac{6}{k} \neq \frac{-2}{-1} \Rightarrow 3k \neq 6 \Rightarrow k \neq 3$

$2k \neq 6$
$k \neq 3$

9. **(c)** $\frac{-2}{3}$

Explanation: Sum of roots = $\frac{-2}{k}$ and products of roots = $\frac{74}{k} = 3$

$\therefore \quad \frac{-2}{k} = 3 \Rightarrow k = \frac{-2}{3}$.

10. **(a)** 2

Explanation: Here, $ax^2 + ax + 2 = 0$….. (1)
$x^2 + x + b = 0$….. (2)
Putting the value of x = 1 in equation (2) we get
$1^2 + 1 + b = 0$
$2 + b = 0$

b = −2
Now, putting the value of x = 1 in equation (1) we get
a + a + 2 = 0
2 a + 2 = 0

$$a = \frac{-2}{2}$$

= - 1
Then,
ab = (−1) × (−2) = 2

11. **(a)** $\frac{1}{3}$

Explanation: $P = \frac{5\,(Balls\ other\ than\ red\ and\ black)}{15\,(Total\ no\ of\ balls)}$

12. **(b)** 81

Explanation: Let the two numbers be x and y.
It is given that:
x = 54
HCF = 27
LCM = 162
We know,
x × y = HCF × LCM
⇒ 54 × y = 27 × 162
⇒ 54y = 4374

$$\Rightarrow \quad \therefore y = \frac{4374}{54} = 81$$

13. **(a)** (3, 6)

Explanation: Since, the point, where the perpendicular bisector of a line segment joining the points A(2, 5) and B(4, 7) cuts, is the mid-point of that line segment.

$$\therefore \text{ Coordinates of Mid-point of line segment AB} = \left(\frac{2+4}{2}, \frac{5+7}{2}\right) = (3, 6)$$

14. **(b)** $\frac{7}{4}$

Explanation: $(\cos 0^{o} + \sin 30^{o} + \sin 45^{o})(\sin 90^{o} + \cos 60^{o} - \cos 45^{o}) = ?$

$$= \left(1 + \frac{1}{2} + \frac{1}{\sqrt{2}}\right)\left(1 + \frac{1}{2} - \frac{1}{\sqrt{2}}\right) = \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right) = \left(\frac{9}{4} - \frac{1}{2}\right) = \frac{7}{4}$$

15. **(d)** Mode = 3 Median - 2 mean

Explanation: In case of a moderately skewed distribution, the difference between mean and mode is almost equal to three times the
difference between the mean and median. Thus, the empirical mean median mode relation is given as:
Mean – Mode = 3 (Mean – Median)
i.e, Mode = 3 Median - 2 Mean

16. **(d)** 60°

Explanation:

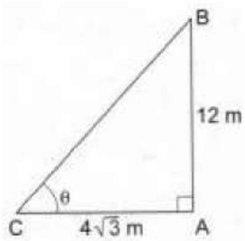
Let AB be the pole and AC be its shadow.

AB = 12 m and AC = $4\sqrt{3}$ m.

Let ∠ACB = θ. Then, $\tan\theta = \frac{AB}{AC} = \frac{12}{4\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{12}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3} = \tan 60^{\circ}$

$\Rightarrow \theta = 60^{\circ}$



17. **(b)** one solution

Explanation: Here, a unique solution of each variable of a pair of linear equations is given, therefore, it has one solution to a system of linear equations.

18. **(c)** A is true but R is false.

Explanation: Here, reason is not true.

$\sqrt{9} = \pm 3$, which is not an irrational number.

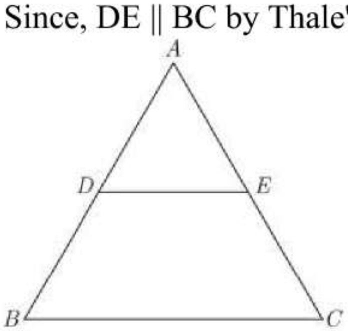
A is true but R is false.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Reason is true: [This is Thale's Theorem]

For Assertion

Since, DE || BC by Thale's Theorem



$\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{DB}{AD} = \frac{EC}{AE}$

$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$

$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$

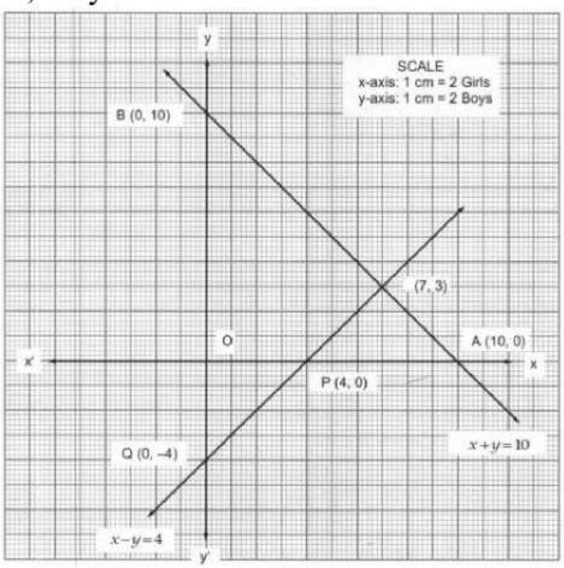
$\frac{AB}{AD} = \frac{AC}{AE}$

Assertion is true.
Since, reason gives Assertion.

20. **(b)** Irrational
Explanation: Let rational number + irrational number = rational number
And we know " rational number can be expressed in the form of PQ, where p, q are any integers,
So, we can express our assumption As :
PQ + x = ab (Here x is a irrational number)
x = ab - PQ
So,
x is a rational number, but that contradicts our starting assumption.
Hence rational number + irrational number = irrational number

Section B

21. Formulation: Let the number of girls be x and the number of boys be y.
It is given that total ten students took part in the quiz.
∴ Number of girls + Number of boys = 10
i.e. x + y = 10
It is also given that the number of girls is 4 more than the number of boys.
∴ Number of girls = Number of boys + 4
i.e. x = y + 4
or, x - y = 4



22. Total No of students = 40
Students who have 100% attendance = 13
Students who do social work = 15
Students participate in Adult Education = 5
So the remaining no. students who participate in educational cultural program = 40 - (13 + 15 + 5) = 7
Let K be the events of selecting the student who participates in an educational cultural program.
Then outcomes favoring K = 7

$$P(K) = \frac{\text{No. of favorable outcomes}}{\text{Total No of outcomes}} = \frac{7}{40}$$

23. Comparing polynomial $x^2 - 2x - 8$ with general form of quadratic polynomial $ax^2 + bx + c$,

We get a = 1, b = -2 and c = -8

We have, $x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4)$

$= (x - 4)(x + 2)$

Now, for zeroes of polynomial, we have;

$(x - 4)(x + 2) = 0$

$x - 4 = 0$ or $x + 2 = 0$

$x = 4$ or $x = -2$

$\Rightarrow x = 4, -2$ are two zeroes.

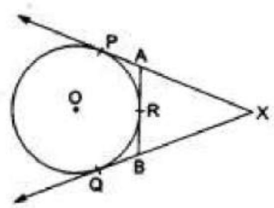
a = 1, b = -2 and c = -8

Sum of zeroes $= 4 + (-2) = 2$

$$\text{Sum of zeroes} = \frac{-(-2)}{1} = \frac{-b}{a} = \frac{-\text{ Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes $= 4 \times (-2) = -8$

$$\text{Product of zeroes} = \frac{-8}{1} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

24. 

We know that the lengths of tangents drawn from an exterior point to a circle are equal.

XP = XQ, ...(i) [tangents from X]

AP = AR, ... (ii) [tangents from A]

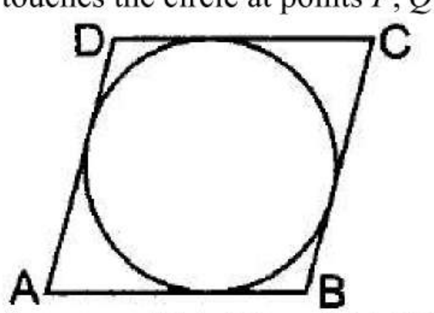
BR = BQ. ... (in) [tangents from B]

Now, XP = XQ $\Rightarrow$ XA + AP = XB + BQ

XA + AR = XB + BR [using (ii) and (iii)]

OR

Let ABCD be the quadrilateral circumscribing the circle with centre O. The quadrilateral touches the circle at points P, Q, R, S.



To prove: $AB + CD = AD + BC$

proof: lengths of tangents drawn from an external point are equal

Hence, $AP = AS$ ...(i)

$BP = BQ$ ...(ii)

$CR = CQ$...(iii)

$DR = DS$...(iv)

Adding (i) + (ii) + (iii) + (iv), we get

$AB + BP + CR + DR = AS + BQ + CQ + DSAB + CD = AD + BC$

Hence proved

25. At mid-point of AB = $\left(\dfrac{\frac{x}{2}+x+1}{2}\right) = 5$

or, x = 6

$$\left(\dfrac{\frac{y+1}{2}+y-3}{2}\right) = -2$$

or, y + 1 + 2y - 6 = -8

y = -1

OR

Using distance formula, we obtain

$$AB = \sqrt{\left(at_2^2 - at_1^2\right)^2 + \left(2at_2 - 2at_1\right)^2}$$

$$\Rightarrow AB = \sqrt{a^2\left(t_2 - t_1\right)^2\left(t_2 + t_1\right)^2 + 4a^2\left(t_2 - t_1\right)^2}$$

$$\Rightarrow AB = a\left(t_2 - t_1\right)\sqrt{\left(t_2 + t_1\right)^2 + 4}$$

Section C

26. Let the fraction be $\frac{x}{y}$

Then, according to the question,

$\dfrac{x+1}{y-1} = 1$......(1)

$\dfrac{x}{y+1} = \dfrac{1}{2}$.........(2)

$\Rightarrow$ x + 1 = y - 1 ....(3)

2x = y + 1................(4)

$\Rightarrow$ x - y = - 2................(5)

2x - y = 1......................(^)

Substituting equation (5) from equation (6), we get x =3

Substituting this value of x in equation (5), we get

3 - y = -2

$\Rightarrow$ y = 3 + 2

$\Rightarrow$ y = 5

Hence, the required fraction is $\frac{3}{5}$

Verification: Substituting the value of x = 3 and y = 5,
we find that both the equations(1) and (2) are satisfied as shown below:

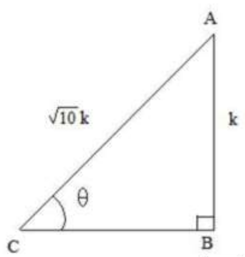
$$\frac{x+1}{y-1} = \frac{3+1}{5-1} = \frac{4}{4} = 1$$

$$\frac{x}{y+1} = \frac{3}{5+1} = \frac{3}{6} = \frac{1}{2}$$

Hence, the solution is correct.

27. Let us first draw a right △ABC, right angled at B and $\angle C = \theta$.

Now, we know that $cosec\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{AB} = \frac{\sqrt{10}}{1}$.



So, if AC = $(\sqrt{10})$k, then AB = k, where k is a positive number.

Now, by using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2 = 10k^2 - k^2$

$\Rightarrow BC^2 = 9k^2$

$\Rightarrow$ BC = 3k

Now, finding the other T-ratios using their definitions, we get:

$$\tan\theta = \frac{AB}{BC} = \frac{k}{3k} = \frac{1}{3}$$

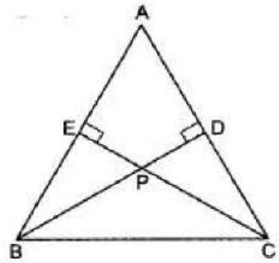
$$\cos\theta = \frac{BC}{AC} = \frac{3k}{\sqrt{10k}} = \frac{3}{\sqrt{10}}$$

$$\therefore \sin\theta = \frac{1}{\cos ec\theta} = \frac{1}{\sqrt{10}}, \cot\theta = \frac{1}{\tan\theta} = 3 \text{ and } \sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{10}}{3}$$

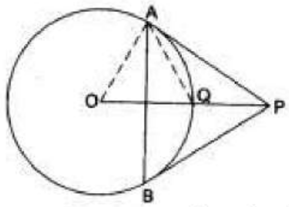
28. **GIVEN** A ΔABC in which $BD \perp AC$ and $CE \perp AB$ and BD and CE intersect at P.

TO PROVE BP × PD = EP × PC

PROOF In ΔEPB and ΔDPC, we have

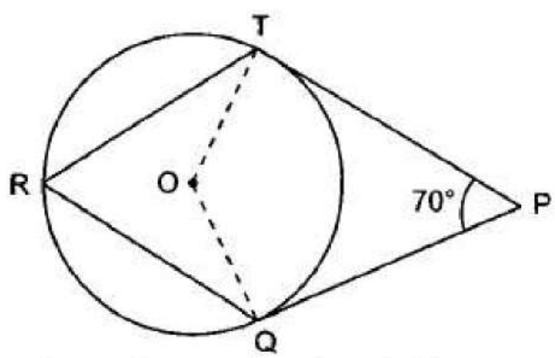


$\angle PEB = \angle PDC$ [Each equal to 90°]
$\angle EPB = \angle DPC$ [Vertically opposite angles]
Thus, by AA-criterion of similarity, we obtain
$\Delta EPB \sim \Delta DPC$
$$\frac{EP}{DP} = \frac{PB}{PC}$$
$\Rightarrow$ BP $\times$ PD = EP $\times$ PC

29. 

Let OP meet the circle at Q.
Join OA and AQ.
Clearly, $OA \perp AP$
$\Rightarrow \angle OAP = 90°$ [radius through the point of contact is perpendicular to the tangent].
Now, OQ = QP = r.
Thus, Q is the midpoint of the hypotenuse OP of $\triangle OAP$
So Q is equidistant from O, A and P.
$\therefore$ QA = OQ = QP = r
$\Rightarrow$ OA = OQ = QA = r
$\Rightarrow$ ΔAOQ is equilateral
$\Rightarrow \angle AOQ = 60°$ [$\because$ each angle of an equilateral triangle is 60^{O}]
$\Rightarrow \angle AOP = 60°$
$\Rightarrow \angle APO = 30° \left[\because \angle AOP + \angle OAP + \angle APO = 180^{\circ} \right]$
$\Rightarrow \angle APB = 2$
$\angle APO = 60°$
Also, PA = PB
$\Rightarrow \angle PAB = \angle PBA = 60°$.
Hence, $\triangle PAB$ is an equilateral triangle.

OR



we know that , angle subtended by an arc at centre of the circle is twice the angle subtended by it in alternate segment.

$\angle TOQ + \angle TPQ = 180^{\circ}$
$\Rightarrow \angle TOQ = 110^{\circ}$

$\angle TOQ = 2\angle TRQ$

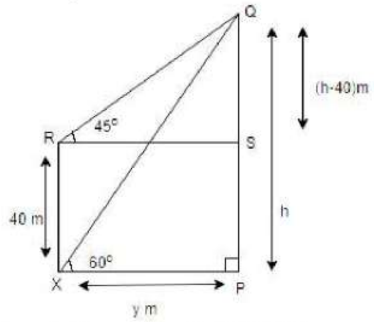
$\Rightarrow 110^\circ = 2\angle TRQ \Rightarrow \angle TRQ = 55^\circ$

30. Let h be the height of the tower.

i.e, PQ = h m and let PX = y m

Now, draw $RS \parallel XP$,

Then, we have RX =SP = 40 m, $\angle QXP = 60^o$ and $\angle QRS = 45^o$



In right angled $\triangle XPQ$,

$$\tan 60^o = \frac{P}{B} = \frac{PQ}{XP}$$

$$\Rightarrow \frac{\sqrt{3}}{1} = \frac{h}{y} \; [\because \tan 60^o = \sqrt{3}]$$

$$\Rightarrow y = \frac{h}{\sqrt{3}} ..(i)$$

In right angled $\triangle RSQ$,

$$\tan 45^o = \frac{P}{B} = \frac{QS}{RS}$$

$$\Rightarrow \tan 45^o = \frac{PQ - SP}{XP}$$

$$\Rightarrow 1 = \frac{h-40}{y}$$

$$\Rightarrow y = h - 40 ....(ii)$$

Now,solve Eq(i) and Eq(ii) , to find h and y.

$$\frac{h}{\sqrt{3}} = h - 40$$

$(\sqrt{3} - 1)\, h = 40\sqrt{3}$

$$h = \frac{40\sqrt{3}}{\sqrt{3} - 1} = \frac{40(1.732)}{1.732 - 1} = \frac{68.28}{0.732} = 94.64$$

$$\Rightarrow y = 94.64 - 40$$

$$\Rightarrow y = 54.64$$

$$\Rightarrow PQ = 94.64 \; m \; and \; PX = 54.64 \; m$$

31. HCF $= \left(x^2 - x - 12\right) = (x + 3)(x - 4)$

$$P(x) = \left(x^2 - 5x + 4\right)\left(x^2 + 5x + a\right)$$

$= (x-4)(x-1)\left(x^2+5x+a\right)$

Since, $(x+3)(x-4)$ is the HCF of $P(x)$ and $Q(x)$therefore, (x+3) and (x-4) are factors of p(x), As (x-4) is already seen in p(x) and (x+3) is also a factor of p(x).

Thus, by factor theorem, x + 3 = 0 $\Rightarrow x = -3i, e \cdot P(-3) = 0$

Hence, $P(-3) = (-7)(-4)(9-15+a) = 0$

$\Rightarrow 28(-6+a) = 0 \quad \Rightarrow a = 6$

Again, Q(x) = $\left(x^2+5x+6\right)\left(x^2-5x-2b\right)$

$= (x+2)(x+3)\left(x^2-5x-2b\right)$

Since, $x-4$ is a factor of $Q(x)$

$x-4=0 \Rightarrow x=4$, by factor theorem Q(4) must equal to 0.

$Q(4) = (6)(7)(16-20-2b) = 0$

$\Rightarrow 42(-4-2b) = 0 \Rightarrow 2b = -4 \Rightarrow b = -2$

Hence, a = 6, b = -2

OR

Let us assume that $2-\sqrt{3}$ is rational.

Then, there exist positive co-primes a and b such that

$2-\sqrt{3} = \frac{a}{b}$

$\sqrt{3} = 2-\frac{a}{b}$

As 2 and $\frac{a}{b}$ are rational number .

So, $\sqrt{3}$ is also rational number .

But $\sqrt{3}$ is not rational number .

Since a rational number cannot be equal to an irrational number. Our assumption that $2-\sqrt{3}$ is rational wrong .

Hence $2-\sqrt{3}$ is irrational

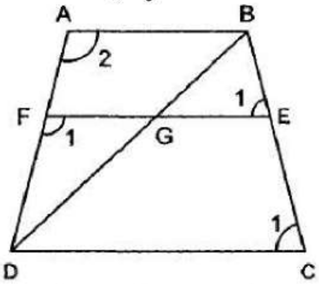
Section D

32. In ΔDFG and Δ DAB, we have

$\angle 1 = \angle 2$ [$\because AB \parallel DC \parallel EF \therefore \angle 1$ and $\angle 2$ are corresponding angles]

∠FDG = ∠ADB [Common]

Therefore, by AA-criterion of similarity, we have



$\therefore \quad \Delta DFG \sim \Delta DAB$

$$\Rightarrow \quad \frac{DF}{DA} = \frac{FG}{AB} \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

In trapezium ABCD, we have

$EF \parallel AB \parallel DC$

$$\therefore \quad \frac{AF}{DF} = \frac{BE}{EC}$$

$$\Rightarrow \quad \frac{AF}{DF} = \frac{3}{4} \quad \left[\because \frac{BE}{EC} = \frac{3}{4}(\text{ given })\right]$$

$$\Rightarrow \quad \frac{AF}{DF} + 1 = \frac{3}{4} + 1 \text{ [Adding 1 on both sides]}$$

$$\Rightarrow \quad \frac{AF+DF}{DF} = \frac{7}{4}$$

$$\Rightarrow \quad \frac{AD}{DF} = \frac{7}{4} \Rightarrow \frac{DF}{AD} = \frac{4}{7} \ldots\ldots\ldots\ldots(ii)$$

From (i) and (ii), we get

$$\frac{FG}{AB} = \frac{4}{7} \Rightarrow FG = \frac{4}{7}AB \ldots\ldots\ldots\ldots\ldots\ldots(iii)$$

So far as the given figure is concerned , in ΔBEG and ΔBCD, we have

∠BEG = ∠BCD [Corresponding angles]

∠B = ∠B [Common]

$\therefore \quad \Delta BEG \sim \Delta BCD$ [By AA-criterion of similarity]

$$\Rightarrow \quad \frac{BE}{BC} = \frac{EG}{CD}$$

$$\Rightarrow \quad \frac{3}{7} = \frac{EG}{CD} \quad \left[\because \frac{BE}{EC} = \frac{3}{4} \Rightarrow \frac{EC}{BE} = \frac{4}{3} \Rightarrow \frac{EC}{BE} + 1 = \frac{4}{3} + 1 \Rightarrow \frac{BC}{BE} = \frac{7}{3}\right]$$

$$\Rightarrow \quad EG = \frac{3}{7}\text{CD}$$

$$\Rightarrow \quad EG = \frac{3}{7} \times 2AB \; [\because \; \text{CD} = 2 \text{ AB (given)}]$$

$$\Rightarrow \quad EG = \frac{6}{7}AB \ldots\ldots\ldots\ldots\ldots\ldots(iv)$$

Adding (iii) and (iv), we get

$$FG + EG = \frac{4}{7}AB + \frac{6}{7}AB \Rightarrow EF = \frac{10}{7}AB \Rightarrow 7FE = 10AB$$

33. The given equation is:

$$\frac{3}{x+1} - \frac{1}{2} = \frac{2}{3x-1}$$

$$\Rightarrow \frac{3}{x+1} - \frac{2}{3x-1} = \frac{1}{2}$$

$$\Rightarrow \frac{3(3x-1)-2(x+1)}{(x+1)(3x-1)} = \frac{1}{2} \text{ (By cross multiplication method)}$$

$$\Rightarrow \frac{9x-3-2x-2}{3x^2-x+3x-1} = \frac{1}{2}$$

$$\Rightarrow \frac{7x-5}{3x^2+2x-1} = \frac{1}{2}$$

$\Rightarrow$ $14x - 10 = 3x^2 + 2x - 1$

$\Rightarrow$ $3x^2 + 2x - 1 - 14x + 10 = 0$

$\Rightarrow$ $3x^2 - 12x + 9 = 0$

$\Rightarrow$ $x^2 - 4x + 3 = 0$

Now by factorization method we have,

$x^2 - 3x - x + 3 = 0$

$\Rightarrow$ $x(x - 3) -1(x - 3) = 0$

$\Rightarrow$ $(x - 3)(x - 1) = 0$

$\Rightarrow$ $x - 3 = 0$ or $x - 1 = 0$

Therefore either $x = 3$ or $x = 1$

OR

We, $A = \left(c^2 - ab\right)$, $B = -2(a^2 - bc)$, $C = b^2 - ac$

For real equal roots, $D = B^2 - 4AC = 0$

$$\Rightarrow \left[-2\left(a^2 - bc\right)\right]^2 - 4\left(c^2 - ab\right)\left(b^2 - ac\right) = 0$$

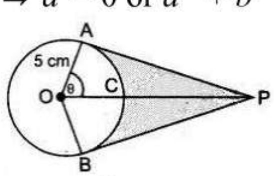
$$\Rightarrow 4\left(a^4 + b^2c^2 - 2a^2bc\right) - 4\left(b^2c^2 - c^3a - ab^3 - a^2bc\right) = 0$$

$$\Rightarrow 4\left[a^4 + b^2c^2 - 2x^2bc - b^2c^2 + c^3a + ab^3 - a^2bc\right) = 0$$

$$\Rightarrow 4\left[a^4 + ac^3 + ab^3 - 3a^2bc\right] = 0$$

$$\Rightarrow a\left(a^3 + c^3 + b^3 - 3abc\right) = 0$$

$$\Rightarrow a = 0 \text{ or } a^3 + b^3 + c^3 = 3abc$$

34. 

$\cos\theta = \frac{1}{2}$ or, $\theta = 60°$

Reflex $\angle AOB = 120°$

$$\therefore \text{ ADB} = \frac{2 \times 3.14 \times 5 \times 240}{360} = 20.93 \text{ cm}$$

Hence length of elastic in contact = 20.93 cm

Now, AP = $5\sqrt{3}$cm

a (△OAP) = $\frac{1}{2} \times$ base $\times$ height = $\frac{1}{2} \times 5 \times 5\sqrt{3} = \frac{25\sqrt{3}}{2}$

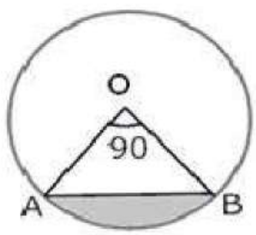
Area (△OAP + △OBP) = $2 \times \frac{25\sqrt{3}}{2} = 25\sqrt{3} = 43.25\ \text{cm}^2$

Area of sector OACB = $\frac{\theta}{360} \times \pi r^2$

$= \frac{25 \times 3.14 \times 120}{360} = 26.16\ \text{cm}^2$

Shaded Area = 43.25 - 26.16 = 17.09 cm^2

OR



i. Area of minor sector = $\frac{\theta}{360}\pi r^2$

$= \frac{90}{360}(3.14)(10)^2$

$= \frac{1}{4} \times 3.14 \times 100$

$= \frac{314}{4}$

= 78.50 = 78.5 cm^2

ii. Area of major sector = Area of circle - Area of minor sector

$= \pi(10)^2 - \frac{90}{360}\pi(10)^2 = 3.14\ (100) - \frac{1}{4}(3.14)\ (100)$

= 314 - 78.50 = 235.5 cm^2

iii. We know that area of minor segment

= Area of minor sector OAB - Area of ΔOAB

$\because$ area of △OAB = $\frac{1}{2}(OA)(OB)\sin\angle AOB$

$= \frac{1}{2}(OA)(OB)\left(\because \angle AOB = 90^\circ\right)$

Area of sector = $\frac{\theta}{360}\pi r^2$

$= \frac{1}{4}(3.14)\ (100) - 50 = 25(3.14) - 50 = 78.50 - 50 = 28.5\ \text{cm}^2$

iv. Area of major segment = Area of the circle - Area of minor segment

$= \pi(10)^2 - 28.5$

$= 100(3.14) - 28.5$

$= 314 - 28.5 = 285.5\ cm^2$

35. Let the assumed mean be A = 25 and h = 5.

marks (x_1):	no. of students (f_1):	$d_1 = x_1 = A = x_1 - 25$	$u_1 = \frac{1}{h}(d_1)$	f_1u_1
5	15	-20	-4	-60
10	50	-15	-3	-150
15	80	-10	-2	-160
20	76	-5	-1	-76
25	72	0	0	0
30	45	5	1	45
35	39	10	2	78
30	9	15	3	27
45	8	20	4	32
50	6	25	5	30
	$\sum f_1 = 400$			$\sum f_1u_1 = -234$

We know that mean, $\bar{X} = A + h\left(\frac{1}{N}\sum_{i=1}^{n} f_iu_i\right)$

Now, we have $N = \sum f_1 = 400$, = -234, h = 5 and A = - 234, h = 5 and A = 25.

Putting the values in the above formula, we get

$$\bar{X} = A + h\left(\frac{1}{N}\sum_{i=1}^{n} f_iu_i\right)$$

$$= 25 + 5\left(\frac{1}{400} \times (-234)\right)$$

$$= 25 - \frac{234}{80}$$

$= 25 - 2.925$

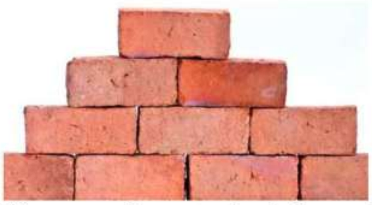
$= 22.075$

Hence, the mean marks is 22.075

Section E

36. **Read the text carefully and answer the questions:**

Akshat's father is planning some construction work in his terrace area. He ordered 360 bricks and instructed the supplier to keep the bricks in such as way that the bottom row has 30 bricks and next is one less than that and so on.



The supplier stacked these 360 bricks in the following manner, 30 bricks in the bottom row, 29 bricks in the next row, 28 bricks in the row next to it, and so on.

(i) Number of bricks in the bottom row = 30. in the next row = 29, and so on.
Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ...,
which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1
Suppose number of rows is n, then sum of number of bricks in n rows should be 360.
i.e. $S_n = 360$

$\Rightarrow \frac{n}{2}[2 \times 30 + (n-1)(-1)] = 360 \; \{S_n = \frac{n}{2}(2a + (n-1)d)\}$

$\Rightarrow 720 = n(60 - n + 1)$

$\Rightarrow 720 = 60n - n^2 + n$

$\Rightarrow n^2 - 61n + 720 = 0$

$\Rightarrow n^2 - 16n - 45n + 720 = 0$ [by factorization]

$\Rightarrow n(n-16) - 45(n-16) = 0$

$\Rightarrow (n-16)(n-45) = 0$

$\Rightarrow (n-16) = 0$ or $(n-45) = 0$

$\Rightarrow n = 16$ or $n = 45$

Hence, number of rows is either 45 or 16.
n = 45 not possible so n = 16
$a_{45} = 30 + (45-1)(-1) \; \{a_n = a + (n-1)d\}$
$= 30 - 44 = -14$ [$\because$ The number of logs cannot be negative]
Hence the number of rows is 16.

(ii) Number of bricks in the bottom row = 30. in the next row = 29, and so on.
Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27,...,
which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1
Suppose number of rows is n, then sum of number of bricks in n rows should be 360.
Number of bricks on top row are $n = 16$,
$a_{16} = 30 + (16 - 1)(-1) \; \{a_n = a + (n - 1)d\}$
= 30 - 15 = 15
Hence, and number of bricks in the top row is 15.

OR

Number of bricks in the bottom row = 30. in the next row = 29, and so on.
Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27,...,
which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1.
Suppose number of rows is n, then sum of number of bricks in n rows should be 360.
$a_n = 26$, a = 30, d = -1
$a_n = a + (n - 1)d$

$\Rightarrow 26 = 30 + (n - 1) \times -1$

$\Rightarrow 26 - 30 = -n + 1$

$\Rightarrow$ n = 5

Hence 26 bricks are in 5th row.

(iii) Number of bricks in the bottom row = 30. in the next row = 29, and so on.

therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ...,
which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1.
Suppose number of rows is n, then sum of number of bricks in n rows should be 360
Number of bricks in 10th row a = 30, d = -1, n = 10

$a_n = a + (n - 1)d$

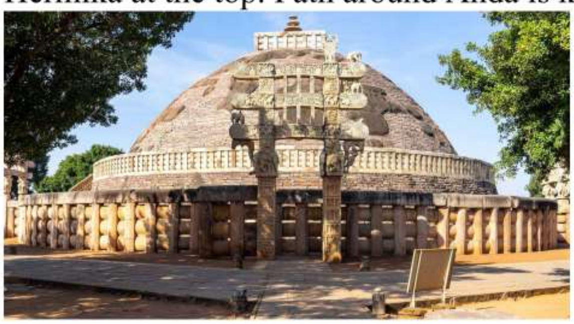
$\Rightarrow a_{10} = 30 + 9 \times -1$

$\Rightarrow a_{10} = 30 - 9 = 21$

Therefore, number of bricks in 10th row are 21.

37. **Read the text carefully and answer the questions:**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.



(i) Volume of Hermika = side^3 = $10 \times 10 \times 10 = 1000$ m^3

(ii) r = radius of cylinder = 24, h = height = 16

Volume of cylinder = $\pi r^2 h$

$\Rightarrow V = \frac{22}{7} \times 24 \times 24 \times 14 = 25344$ m^3

(iii) Volume of brick = 0.01 m^3

$\Rightarrow$ n = Number of bricks used for making cylindrical base = $\frac{\text{Volume of cylinder}}{\text{Volume of one brick}}$

$\Rightarrow n = \frac{25344}{0.01} = 2534400$

OR

Since Anda is hespherical in shape r = radius = 21

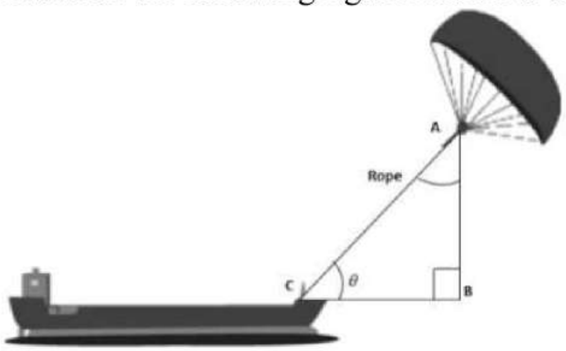
V = Volume of Anda = $\frac{2}{3} \times \pi \times r^3$

$\Rightarrow V = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$\Rightarrow V = 44 \times 21 \times 21 = 19404$ m^3

38. **Read the text carefully and answer the questions:**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively. Based on the following figure related to sky sailing, answer the following questions:



(i) $\sin\theta = \cos(\theta - 30^{o})$

$\cos(90^{o} - \theta) = \cos(\theta - 30^{o})$

$\Rightarrow 90^\circ - \theta = \theta - 30^\circ$

$\Rightarrow \theta = 60^\circ$

(ii) $\frac{AB}{AC} = \sin 60^\circ$

$\therefore$ Length of rope, AC $= \frac{AB}{sin60^{o}} = \frac{200}{\frac{\sqrt{3}}{2}} = \frac{200 \times 2}{\sqrt{3}} = 230.94$ m

OR

$\frac{AB}{AC} = \sin 30^\circ$

$\therefore$ Length of rope, AC $= \frac{AB}{sin30^{o}} = \frac{150}{\frac{1}{2}} = 150 \times 2 = 300$ m

(iii) $\sin\theta = \cos(3\theta - 30^{o})$

$\cos(90^{o} - \theta) = \cos(3\theta - 30^{o})$

$\Rightarrow 90^\circ - \theta = 3\theta - 30^\circ \Rightarrow \theta = 30^\circ$