Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper - 6

with Solution

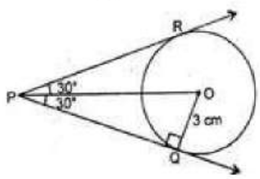
Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to: **[1]**



a) 6 cm b) $3\sqrt{3}$

c) 3 cm d) $\frac{3}{2}\sqrt{3}$ cm

2. The base of an equilateral triangle ABC lies on the y-axis. The coordinates of the point C is (0, – 3). If origin is the midpoint of BC, then the coordinates of B are **[1]**

a) (3, 0) b) (0, – 3)

c) (– 3, 0) d) (0, 3)

3. In a single throw of a die, the probability of getting a multiple of 3 is **[1]**

a) $\frac{1}{6}$ b) $\frac{2}{3}$

c) $\frac{1}{2}$ d) $\frac{1}{3}$

4. In what ratio does the y-axis divide the join of P(-4, 2) and Q(8, 3)? **[1]**

a) 1 : 3 b) 2 : 1

c) 3 : 1 d) 1 : 2

5. A system of linear equations is said to be consistent, if it has **[1]**

a) two solutions
b) one or many solutions
c) no solution
d) exactly one solution

6. If a digit is chosen at random from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, then the probability that it is odd, is **[1]**

a) $\frac{4}{9}$
b) $\frac{1}{9}$
c) $\frac{5}{9}$
d) $\frac{2}{3}$

7. A cubical block of side 7 cm is surmounted by a hemisphere. The greatest diameter of the hemisphere is **[1]**

a) 10.5cm
b) 7cm
c) 3.5cm
d) 14cm

8. If A(1, 3), B(-1, 2), C(2, 5) and D(x, 4) are the vertices of a ||gm ABCD then the value of x is **[1]**

a) 0
b) 3
c) $\frac{3}{2}$
d) 4

9. Two numbers 'a' and 'b' are selected successively without replacement in that order from the integers 1 to 10. The probability that $\frac{a}{b}$ is an integer, is **[1]**

a) $\frac{17}{45}$
b) $\frac{8}{45}$
c) $\frac{1}{5}$
d) $\frac{17}{90}$

10. The perimeter of a rectangle is 82 m and its area is 400 m^2. The breadth of the rectangle is **[1]**

a) 25 m
b) 9 m
c) 16 m
d) 20 m

11. One of the roots of the quadratic equation $a^2x^2 - 2abx + 2b^2 = 0$ is **[1]**

a) $\frac{-2b}{a}$
b) $\frac{-2a}{b}$
c) $\frac{2b}{a}$
d) $\frac{2a}{b}$

12. If $a = 2^3 \times 3, b = 2 \times 3 \times 5, c = 3^n \times 5$ and LCM (a, b, c) $= 2^3 \times 3^2 \times 5$, then n = **[1]**

a) 1
b) 4
c) 3
d) 2

13. $9\sec^2 A - 9\tan^2 A =$ **[1]**

a) 9

b) 1

c) 0

d) 99

14. The ________ of an object can be determined with the help of trigonometric ratios. **[1]**

a) height

b) shape

c) weight

d) None of these

15. If $\left(\frac{a}{2}, 4\right)$ is the midpoint of the line segment joining the points A (-6 ,5) and B(-2, 3) then the value of a is **[1]**

a) 3

b) 4

c) -8

d) -4

16. If the mean of observations $x_1, x_2, ..., x_n$ is $\overline{x}$, then the mean of $x_1 + a, x_2 + a, ..., x_n + a$ is: **[1]**

a) $\overline{x}$ - a

b) $\frac{\overline{x}}{a}$

c) $a\overline{x}$

d) $\overline{x} + a$

17. The area of the triangle formed by the lines $2x + 3y = 12$, $x - y = 1$ and $x = 0$ is **[1]**

a) 6.5 sq. units

b) 7 sq. units

c) 7.5 sq. units

d) 6 sq. units

18. The exponent of 2 in the prime factorisation of 144, is **[1]**

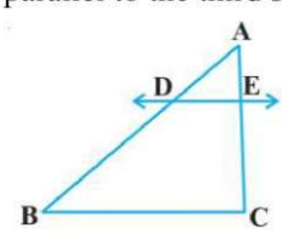
a) 4

b) 5

c) 6

d) 3

19. **Assertion (A):** D and E are points on the sides AB and AC respectively of a $\triangle ABC$ such that DE||BC then the value of x is 11, when AD = 4cm, DB = (x - 4)cm, AE = 8cm and EC = (3x - 19)cm. **[1]**
Reason (R): If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side.



a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** For any two positive integers a and b, HCF (a, b) $\times$ LCM (a, b) = a $\times$ b **[1]**
Reason (R): The HCF of the two numbers is 8 and their product is 280. Then their LCM is 40.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Do the equations $4x + 3y - 1 = 5$ and $12x + 9y = 15$ represent a pair of coincident lines? Justify your answer. **[2]**

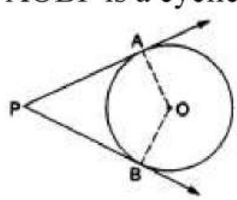
22. A carton consists of 100 shirts of which 88 are good and 8 have minor defects. Rohit, a trader, will only accept the shirts which are good. But, Kamal, an another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that it is acceptable to **[2]**
i. Rohit and
ii. Kamal?

23. If $x = \frac{2}{3}$ and -3 are the roots of the quadratic equation $ax^2 + 7x + b = 0$. then the values of a and b. **[2]**

24. Find the locus of centres of circles which touch two intersecting lines. **[2]**

OR

In the given figure, O is the centre of the circle. PA and PB are tangents. Show that AOBP is a cyclic quadrilateral.



25. Prove that the points A(2,4), B(2,6) and $C(2+\sqrt{3}, 5)$ are the vertices of an equilateral triangle. **[2]**

OR

In what ratio does the point C(4, 5) divide the join of A(2, 3) and B(7, 8)?

Section C

26. Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now? **[3]**

27. If $\tan\theta = \frac{12}{13}$, evaluate $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$. **[3]**

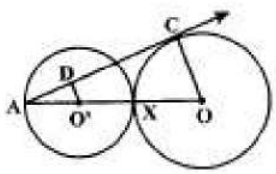
28. Define HCF of two positive integers and find the HCF of the pair of numbers: 475 **[3]**

and 495.

OR

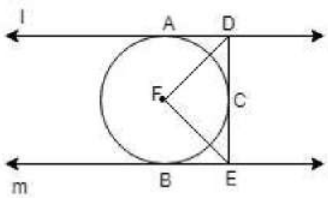
In a school there are two sections, namely A and B, of class X. There are 30 students in section A and 28 students in section B. Find the minimum number of books required for their class library so that they can be distributed equally among students of section A or section B.

29. Equal circles with centres O and O' touch each other at X. OO' produced to meet a circle with centre O', at A. AC is a tangent to the circle whose centre is O. O' D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$. **[3]**

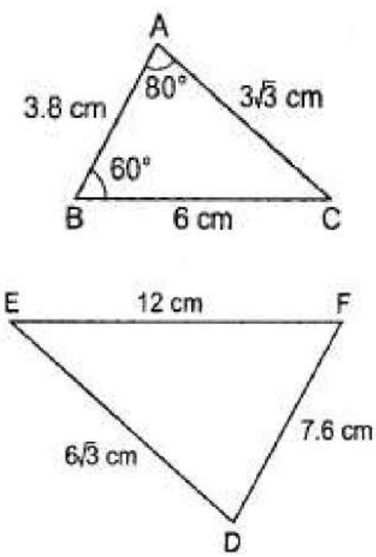


OR

In Fig. l and m are two parallel tangents at A and B. The tangent at C makes an intercept DE between l and m. Prove that $\angle DFE = 90^{\circ}$.



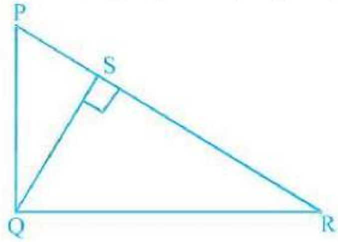
30. In Fig. find $\angle$F. **[3]**



31. The angle of elevation θ of the top of a lighthouse, as seen by a person on the ground, is such that $\tan\theta = \frac{5}{12}$ When the person moves a distance of 240 m towards the lighthouse, the angle of elevation become ϕ such that $\phi = \frac{3}{4}$. Find the height of the lighthouse. **[3]**

Section D

32. In the given figure, PQR is a right triangle right angled at Q and QS $\perp$ PR. If PQ = 6 cm and PS = 4 cm, find QS, RS and QR. **[5]**

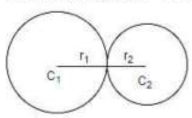


33. A piece of cloth costs 200 Rupees . If the piece was 5 m longer and each metre of cloth costs 2 Rupees less, the cost of the piece would have remain unchanged. How long is the piece and what is the original rate per metre? **[5]**

OR

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of a larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

34. Two farmers have circular plots. The plots are watered with the same water source placed in the point common to both the plots as shown in the figure. The sum of their areas is 130π and the distance between their centres is 14 m.Find the radii of the circles.What value is depicted by the farmers? **[5]**



OR

Find the area of the segment of a circle of radius 12 cm whose corresponding sector central angle 60°. (Use π = 3.14).

35. Find the mode, median and mean for the following data: **[5]**

Marks Obtained	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75	75 - 85
Number of students	7	31	33	17	11	1

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find an increase in the production of TV every year.

(ii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find in which year production of TV is 1000.

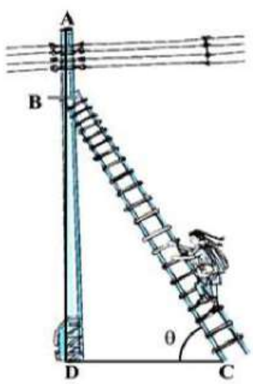
OR

They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the total production in first 7 years.

(iii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in the 10th year.

37. **Read the text carefully and answer the questions:** **[4]**

In a village, group of people complained about an electric fault in their area. On their complaint, an electrician reached village to repair an electric fault on a pole of height 10 m. She needs to reach a point 1.5 m below the top of the pole to undertake the repair work (see the adjoining figure). She used ladder, inclined at an angle of θ to the horizontal such that $\cos\theta = \frac{\sqrt{3}}{2}$, to reach the required position.



(i) Find the length BD?

(ii) Find the length of ladder.

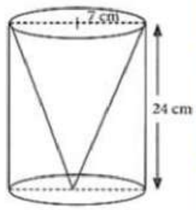
OR

If the height of pole and distance BD is doubled, then what will be the length of the ladder?

(iii) How far from the foot of the pole should she place the foot of the ladder?

38. **Read the text carefully and answer the questions:** **[4]**

One day Vinod was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Vinod's mind.



(i) Find the slant height of the conical cavity so formed?

(ii) Find the curved surface area of the conical cavity so formed?

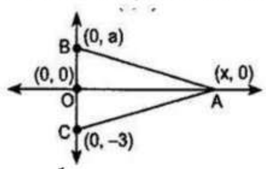
(iii) Find the external curved surface area of the cylinder?

OR

Find the ratio of curved surface area of cone to curved surface area of cylinder?

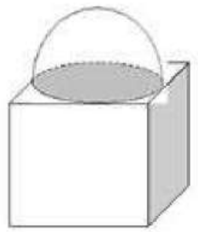
Solution

Section A

1. **(b)** $3\sqrt{3}$
Explanation: Refer fig
PQ and PR are two tangents to a circle
PQ = PR
PO bisects the angle between two tangents
therefore angle ∠OPQ = ∠OPR = 30°
In right angled triangle OPQ
$\tan 30^{\circ} = \frac{\mathrm{OQ}}{\mathrm{PQ}}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{\mathrm{PQ}}$
$\Rightarrow \mathrm{PQ} = 3\sqrt{3}$ cm = PR
2. **(d)** (0, 3)
Explanation: Let the coordinate of B be $(0, a)$. (0, a).
It is given that (0, 0) is the mid-point of BC.
Therefore 0 = (0 + 0) /2 , 0 =(a - 3) /2   a - 3 = 0 , a = 3 0 = $\frac{0+0}{2}$, 0 = $\frac{a-3}{2}$, a - 3 = 0, a = 3
Therefore, the coordinates of B are (0, 3).


3. **(d)** $\frac{1}{3}$
Explanation: A die is thrown, the possible number of events (n) = 6
Now multiple of 3 are 3, 6 which are 2
∴ m = 2
∴ Probability = $\frac{m}{n} = \frac{2}{6} = \frac{1}{3}$
4. **(d)** 1 : 2
Explanation: Let the y-axis cut AB at p (0, y) in the ratio K : 1 Then
$P\left(\frac{8k-4}{k+1} \cdot \frac{3k+2}{k+1}\right) = P(0, y) = \frac{8k-4}{k+1} = 0$
= 8k - 4= 0 =k = $\frac{1}{2}$
required ratio =$\left(\frac{1}{2}; 1\right) = 1 : 2$
5. **(b)** one or many solutions
Explanation: A system of linear equations is said to be consistent if it has at least one solution or can have many solutions. If a consistent system has an infinite number of solutions, it is dependent. When you graph the equations, both equations represent the same line. If a system has no solution, it is said to be inconsistent. The graphs of the lines do not intersect, so the graphs are parallel and there is no solution.
6. **(c)** $\frac{5}{9}$
Explanation: Total number of digits from 1 to 9(n) = 9
Numbers which are odd (m) = 1, 3, 5, 7, 9 = 5
∴ Probability = $\frac{m}{n} = \frac{5}{9}$

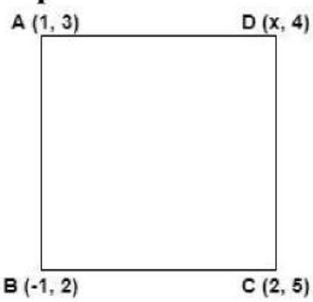
7. **(b)** 7cm

Explanation: 

It is clear that Maximum diameter of hemisphere can be the side of the cube.
$\therefore$ The greatest diameter of the hemisphere = 7 cm

8. **(d)** 4

Explanation:



Since ABCD is a ||gm, the diagonals bisect eachother. so
M is the mid- point of BD as well as AC.

$\frac{1+2}{2} = \frac{x-1}{2}$

$1 + 2 = x - 1$

$x = 4$

9. **(d)** $\frac{17}{90}$

Explanation: a and b are two number to be selected from the integers = 1 to 10 without replacement of a and b
i.e., 1 to 10 = 10
and 2 to 10 = 9
No. of ways = 10 $\times$ 9 = 90
Probability of $\frac{a}{b}$ where it is an integer
$\therefore$ Possible event will be
$= \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \frac{6}{1}, \frac{7}{1}, \frac{8}{1}, \frac{9}{1}, \frac{10}{1}, \frac{4}{2}, \frac{6}{2}, \frac{8}{2}, \frac{10}{2}, \frac{6}{3}, \frac{9}{3}, \frac{8}{4}, \frac{10}{5} = 17$

$P(E) = \frac{m}{n} = \frac{17}{90}$

10. **(c)** 16 m

Explanation: $2(l + b) = 82 \Rightarrow l + b = 41 \Rightarrow l = (41 - b)$.
And, $lb = 400 \Rightarrow (41 - b)b = 400$
$\Rightarrow b^2 - 41b + 400 = 0 \Rightarrow b^2 - 25b - 16b + 400 = 0$
$\Rightarrow b(b - 25) - 16(b - 25) = 0$
$\Rightarrow (b - 25)(b - 16) = 0$
$\therefore b = 25$ or $b = 16$.
But for $b = 25$ we have $l = (41 - 25) = 16 < b$.
$\therefore$ breadth = 16 m.

11. **(c)** $\frac{2b}{a}$

Explanation: $\Rightarrow a^2x^2 - 2abx + 2b^2 = 0$
$\Rightarrow ax(ax - 2b) - b(ax - 2b) = 0$
$\Rightarrow (ax - b)(ax - 2b) = 0$

$\Rightarrow$ ax - b = 0 and ax - 2b = 0
$\Rightarrow$ x = $\frac{b}{a}$ and x = $\frac{2b}{a}$

12. **(d)** 2
Explanation: LCM (a, b, c) $= 2^3 \times 3^2 \times 5$ .... (I)
we have to find the value of n
Also we have
$a = 2^3 \times 3$
$b = 2 \times 3 \times 5$
$c = 3^n \times 5$
We know that the while evaluating LCM, we take greater exponent of the prime numbers in the factorisation of the number.
Therefore, by applying this rule and taking $n \geq 1$ we get the LCM as
LCM (a, b, c) = $2^3 \times 3^n \times 5$ ..... (II)
On comparing (I) and (II) sides, we get:
$2^3 \times 3^2 \times 5 = 2^3 \times 3^n \times 5$
n = 2
13. **(a)** 9
Explanation: Given: 9 $\sec^2$A - 9 $\tan^2$A
= 9($\sec^2$A - $\tan^2$A)
= 9 $\times$ 1 = 9 ... [$\because$ $\sec^2\theta$ - $\tan^2\theta$ = 1]
14. **(a)** height
Explanation: The height of an object can be determined with the help of trigonometric ratios if angle of elevation /depression and one side is known.
15. **(c)** -8
Explanation: $\frac{a}{2} = \frac{(-6-2)}{3} = -4 \Rightarrow a = -8$
16. **(d)** $\overline{x}$ + a
Explanation: Mean of observations x_1, x_2, ..., x_n is $\overline{x}$
i.e, $\bar{x} = \frac{x_1+x_2+x_3\cdots+x_n}{n}$
Now, (x_1 + a) + (x_2 + a) + (x_3 + a) + ... +(x_n + a)
= x_1 + x_2 + x_3 + ... x_n + na
$\therefore$ Mean of (x_1 + a), (x_2 + a), (x_3 + a), ..., (x_n + a)
$=\frac{(x_1+x_2+x_3\ldots+x_n)+na}{n}$
$= \frac{(x_1+x_2+x_3\ldots+x_n)}{n} + \frac{na}{n}$
$= \bar{x} + \frac{na}{n} = \bar{x}$ + a
17. **(c)** 7.5 sq. units
Explanation:
Graph of the equation 2x + 3y - 12 = 0
We have
2x + 3y = 12
2x = 12 - 3y
$x = \frac{12-3y}{2}$
Putting y = 4
We get $x = \frac{12-3\times4}{2} = 0$
Putting y = 2,

We get $x = \frac{12-3\times2}{2} = 3$

Thus, we have the following table for the points:

x	0	3
y	4	2

Plotting point A(0, 4), B(3, 2) on the graph paper and drawing a line passing through them we obtain a graph of the equation.

Graph of the equation x - y - 1

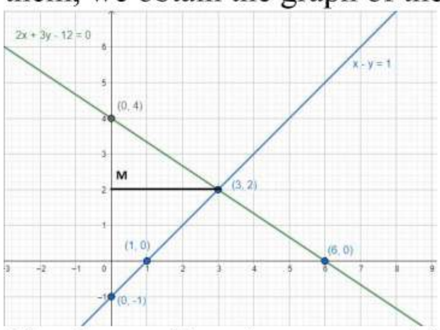
We have x - y = 1

x = 1 + y

Thus, we have the following table for the points for the line x - y = 1

x	1	0
y	0	-1

Plotting point C(1, 0) and D(0, -1) on the same graph paper drawing a line passing through them, we obtain the graph of the line represented by the equation x - y = 1



Clearly two lines intersect at A(3, 2).

The graph of line 2x + 3y = 12 intersect with y-axis at B(0, 4) and the graph of the line x - y = 1 intersect with y-axis at C(0, -1)

So, the vertices of the triangle formed by the two straight lines and y-axis are A(3, 2) and B(0, 4) and C(0, -1)

Now,

Area of $\Delta ABC = \frac{1}{2}$ [Base × Height]

$= \frac{1}{2}(BC \times AB)$

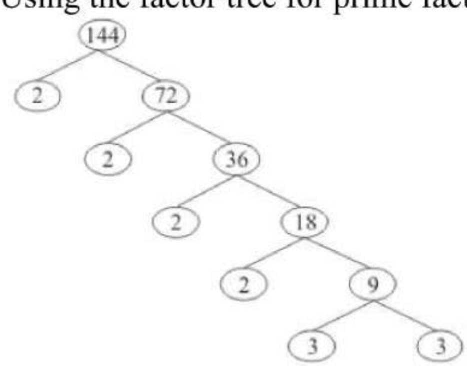
$= \frac{1}{2}(5 \times 3)$

$= \frac{15}{2} sq.units = 7.5 sq.units$

18. **(a)** 4

Explanation:

Using the factor tree for prime factorisation, we have:



Therefore, 144 = 2 × 2 × 2 × 2 × 3 × 3

$\Rightarrow 144 = 2^4 \times 3^2$

Thus, the exponent of 2 in 144 is 4.

19. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side. This is the Converse of the Basic Proportionality theorem.

So, the Reason is correct.

By Basic Proportionality theorem, we have $\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{4}{x-4} = \frac{8}{3x-19}$

$\Rightarrow$ 4(3x - 19) = 8(x - 4)

$\Rightarrow$ 12x - 76 = 8x - 32

$\Rightarrow$ 4x = 44 $\Rightarrow$ x = 11 cm

So, Assertion is correct.

But reason (R) is not the correct explanation of assertion (A).

20. **(c)** A is true but R is false.

Explanation: HCF (a, b) $\times$ LCM (a, b) = a $\times$ b

$\Rightarrow$ 8 $\times$ LCM = 280

$\Rightarrow$ LCM = $\frac{280}{8}$ = 35

A is true but R is false.

Section B

21. No

We may rewrite the equations as

4x + 3y = 6

12x + 9y = 15

Here, $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{2}{5}$

As $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the given equations do not represent a pair of coincident lines.

22. Total numbers of shirts = 100.

The number of good shirts = 88.

The number of shirts with minor defects = 8.

Number of shirts with major defects = 100 – 88 – 8 = 4.

i. P(the drawn shirt is acceptable to Rohit) = $\frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}} = \frac{88}{100}$ $= \frac{22}{25}$

Thus, the probability that the drawn shirt is acceptable to Rohit is $\frac{22}{25}$.

ii. P(the drawn shirt is acceptable to Kamal) = $\frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}} =$ $\frac{88+8}{100} = \frac{96}{100} = \frac{24}{25}$

Thus, the probability that the drawn shirt is acceptable to Kamal is $\frac{24}{25}$.

23. Given, $ax^2 + 7x + b = 0$

it's roots are $\frac{2}{3}$ and -3

Let A and B are coefficients of x^2, and x and C be the constant term.

So A = a, B = 7 and C = b

Now, Sum or roots $= \frac{2}{3} + (-3) = \frac{2-9}{3} = \frac{-7}{3}$

Hence $-\frac{7}{3} = -\frac{\mathrm{B}}{\mathrm{A}} = -\frac{7}{\mathrm{a}}$

$\Rightarrow$ a = 3

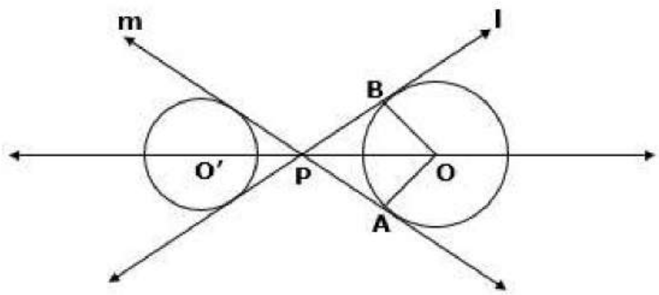
Product of roots $= \frac{2}{3} \times (-3) = -2$

So $\frac{\mathrm{C}}{\mathrm{A}} = \frac{\mathrm{b}}{\mathrm{a}} = -2$

$\Rightarrow \frac{b}{3} = -2$

$\Rightarrow$ b = -6

Hence, a = 3 and b = -6

24. 

Let l and m be two intersecting lines which intersect at point P. Let O be the center of the circle which touches both l and m

From given figure in triangles OAP and OBP, we get

OA = OB (both are radius)

PA = PB (tangents from an external point are equal)

and OP = OP (Common)

$\Rightarrow \quad \Delta OAP \cong \Delta OBP$ (by SSS congruence criteria)

$\Rightarrow \quad \angle APO = \angle BPO$

$\Rightarrow$ OP is the bisector of $\angle APB$

$\Rightarrow$ O lies on the bisector of the angle between two intersecting lines l and m. Thus, the locus of centres of circles which touches two intersecting lines is the angle bisector between the two lines.

OR

We know that the radius and tangent are perpendicular at their point of contact

$\because \angle OBP = \angle OAP = 90^\circ$

Now, In quadrilateral AOBP

$\angle APB + \angle AOB + \angle OBP + \angle OAP = 360^\circ$

$\Rightarrow \angle APB + \angle AOB + 90^\circ + 90^\circ = 360^\circ$

$\Rightarrow \angle APB + \angle AOB = 180^\circ$

Since, the sum of the opposite angles of the quadrilateral is 180°

Hence, AOBP is a cyclic quadrilateral.

25. Given : A(2, 4), B(2, 6) and C(2 + $\sqrt{3}$, 5)

$AB = \sqrt{(2-2)^2 + (6-4)^2} = \sqrt{0+2^2} = \sqrt{4} = 2\ units$

$BC = \sqrt{(2+\sqrt{3}-2)^2 + (5-6)^2} = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = 2\ units$

$AC = \sqrt{(2+\sqrt{3}-2)^2 + (5-4)^2} = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2\ units$

We find that AB = BC = AC

Hence, the given points are the vertices of an equilateral triangle.

OR

Let the point C(4, 5) divides the join of A(2, 3) and B(7, 8) in the ratio k:1

The point C is $\left(\frac{7k+2}{k+1}, \frac{8k+3}{k+1}\right)$

But C is (4, 5)

$\Rightarrow \frac{7k+2}{k+1} = 4$

or 7k + 2 = 4k + 4

or 3k = 2

$\therefore k = \frac{2}{3}$

Thus, C divides AB in the ratio 2:3

Section C

26. Let Salim's present age be x years and his daughter's age be y years.

Two years ago,

Salim's age = (x - 2) years
Daughter's age = (y - 2) years
As per given condition
Two years ago, Salim was thrice as old as his daughter .
x - 2 = 3(y - 2)
x - 2 = 3y - 6
$\Rightarrow$ x - 3y = -4 ....(i)
Six years hence,
Salim's age = (x + 6) years
Daughter's age = (y + 6) years
As per second condition
Six years later, he will be four years older than twice her age.
x + 6 = 2(y + 6) + 4
x + 6 = 2y + 12 + 4
$\Rightarrow$ x - 2y = 10 ....(ii)
Subtracting (i) from (ii), we have
x - 2y - x + 3y =10 - (-4)
y = 14
Put y = 14 in (i)
$\Rightarrow$ x - 3(14) = -4
$\Rightarrow$ x - 42 = - 4
$\Rightarrow$ x = 38
Therefore, the present age of Salim is 38 years and that of his daughter is 14 years.

27. We have, $\tan\theta = \frac{12}{13}$

Now, $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{\frac{2\sin\theta\cos\theta}{\cos^2\theta}}{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}$ [dividing numerator and denominator by $\cos^2\theta$]

$= \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\times\frac{12}{13}}{1-\left(\frac{12}{13}\right)^2} = \frac{\frac{24}{13}}{1-\frac{144}{169}} = \frac{\frac{24}{13}}{\frac{25}{169}} = \frac{24}{13} \times \frac{169}{25} = \frac{312}{25}$

Hence, $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{312}{25}$

28. **HCF (highest common factor) :** The largest positive integer that divides given two positive integers is called the Highest Common Factor of these positive integers.
We need to find H.C.F of 475 and 495.
By applying Euclid's Division lemma, we have
495 = 475 $\times$ 1 + 20.
Since remainder $\neq$ 0, apply division lemma on 475 and remainder 20
475 = 20 $\times$ 23 + 15.
Since remainder $\neq$ 0, apply division lemma on 20 and remainder 15
20 = 15 $\times$ 1 + 5.
Since remainder $\neq$ 0, apply division lemma on 15 and remainder 5
15 = 5 $\times$ 3+ 0.
Therefore, H.C.F. of 475 and 495 = 5

OR

As per question, the required number of books are to be distributed equally among the students of section A or B.
There are 30 students in section A and 28 students in section B.
So, the number of these books must be a multiple of 30 as well as that of 28.
Consequently, the required number is LCM(30, 28).
Now, 30 = 2 $\times$3 $\times$5

and $28 = 2^2 \times 7$.

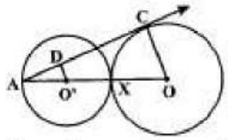
$\therefore$ LCM(30, 28) = product of prime factors with highest power

$= 2^2 \times 3 \times 5 \times 7$

$= 4 \times 3 \times 5 \times 7$

$= 420$

Hence, the required number of books = 420.

29. 

Let the radius of both the circles is r.

In the fig, O'D⊥AC and AC is tangent of circle (O,r)

So, OC ⊥ AC (as line joining center to tangent is ⊥ to the tangent)

Now in ΔAO'D and ΔAOC,

∠O'DA = ∠OCA = 90°

∠A = ∠A (common)

Therefore, ΔAO'D∼ΔAOC [by AA rule]

So, $\frac{DO'}{CO} = \frac{AO'}{AO}$ -----------(1)

Now, AO= r + r + r = 3r

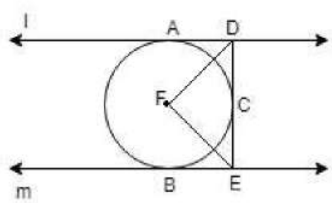
and O'A=r

Putting the value of AO and AO' in equation (1), we get

$\frac{DO'}{CO} = \frac{r}{3r} = \frac{1}{3}$

Therefore, DO':CO = 1:3

OR



Since tangents drawn from an external point to a circle are equal. Therefore, DA = DC.

Thus, in triangles ADF and DFC, we have

DA = DC

DF = DF Common]

AF = CF (radii of the circle)

So, by SSS-criterion of congruence, we obtain

$\Delta ADF \cong \Delta DFC$

$\Rightarrow \quad \angle ADF = \angle CDF$

$\Rightarrow \quad \angle ADC = 2\angle CDF$ ...(i)

Similarly, we can prove that

$\angle BEF = \angle CEF$

$\Rightarrow \quad \angle CEB = 2\angle CEF$ .....(ii)

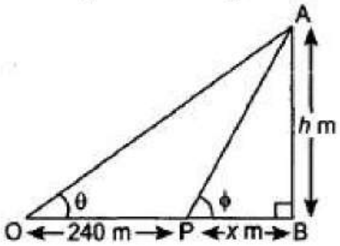
Now, $\angle ADC + \angle CEB = 180°$ (Sum of the interior angles on the same side of transversal is 180^o)

$\Rightarrow 2\angle CDF + 2\angle CEF = 180°$ [Using equations (i) and (ii)]

$\Rightarrow \angle CDF + \angle CEF = 90°$

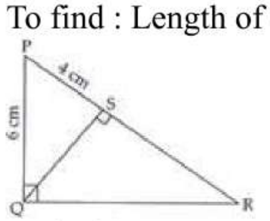
$\Rightarrow 180° - \angle DFE = 90° \quad \left[\begin{array}{l}\because \angle CDF, \angle CEF \text{ and } \angle DFE \text{ are angles of a triangle} \\ \therefore \angle CDF + \angle CEF + \angle DFE = 180°\end{array}\right.$

$\Rightarrow \quad \angle DFE = 90°$

30. In triangles ABC and DEF, we have
$\frac{AB}{DF}=\frac{BC}{FE}=\frac{CA}{ED}=\frac{1}{2}$
Therefore, by SSS-criterion of similarity, we have
$\Delta ABC \sim \Delta DFE$
$\Rightarrow \angle A = \angle D$, $\angle B = \angle F$ and $\angle C = \angle E$
$\Rightarrow \angle D = 80°$, $\angle F = 60°$
Hence, $\angle F = 60°$.
31. From point O angle of elevation is θ and from point P it is ϕ, OP =240 m.


Let PB = x m.
$\tan\theta=\frac{5}{12};\tan\phi=\frac{3}{4}$
In right angled $\triangle$OBA,
$\frac{\mathrm{AB}}{\mathrm{OB}}=\tan\theta$
$\Rightarrow \frac{h}{240+x}=\frac{5}{12}$ .....(i)
In right-angled $\triangle$PBA,
$\frac{AB}{PB}=\tan\phi$
$\Rightarrow \quad \frac{h}{x}=\frac{3}{4}$ ....(ii)
Dividing (i) by (ii), we get
$\frac{h}{240+x}\times\frac{x}{h}=\frac{5}{12}\times\frac{4}{3}$
$\Rightarrow \frac{x}{240+x}=\frac{5}{9}\Rightarrow 9x=1200+5x$
$\Rightarrow \quad 4x=1200\Rightarrow x=300$
Putting x= 300 in (ii) we get, $h=\frac{3}{4}\times 300=225\mathrm{m}$
Hence, height of lighthouse is 225 metres.

Section D

32. Given: According to the question, PQR is a right triangle right angled at Q and QS $\perp$ PR.
PQ = 6 cm and PS = 4 cm
To find : Length of QS, RS and QR.


In ΔPQR, $\angle PQR=90$
and QS $\perp$ PR
So, $\triangle$PSQ ~ $\triangle$PQR (By AA similarity)
Thus, $\frac{PS}{QS}=\frac{QS}{SR}$
$\therefore$ QS^2 = PS.SR..........(i)
In ΔPQS,
$QS^2=PQ^2-PS^2$ [By Pythagoras theorem]
$= 6^2 - 4^2 = 36 - 16$
$\Rightarrow QS^2 = 20$

$\Rightarrow$ QS = $2\sqrt{5}$ cm

Now QS^2 = PS.SR [From eqn(i)]
$\Rightarrow (2\sqrt{5})^2 = 4 \times$ SR
$\Rightarrow \frac{20}{4}$ = SR
$\Rightarrow$ SR = 5 cm
Now, $QS \perp SR$
$\therefore \angle QSR = 90°$
$\Rightarrow QR^2 = QS^2 + SR^2$ [By Pythagoras theorem]
$\Rightarrow QR^2 = (2\sqrt{5})^2 + 5^2$
$\Rightarrow QR^2 = 20 + 25$
$\Rightarrow QR^2 = 45$
$\Rightarrow$ QR = $3\sqrt{5}$ cm
Hence, QS = $2\sqrt{5}$ cm, RS = 5 cm and QR = $3\sqrt{5}$ cm.

33. Let the length of piece be x m
Then , rate = $\frac{200}{x}$ per m
Now , new length = (x + 5)m
Since, the cost remains same .
$\therefore$ New rate = $\frac{200}{x+5}$ per m .
Then, $\frac{200}{x+5} = \frac{200}{x} - 2$
$\frac{200}{x+5} = \frac{200-2x}{x}$
$\Rightarrow 200x = (x + 5)(200 - 2x)$
$\Rightarrow 200x = 200x - 2x^2 + 1000 - 10x$
$\Rightarrow 2x^2 + 10x - 1000 = 0$
$\Rightarrow x^2 + 5x - 500 = 0$
$\Rightarrow x^2 + 25x - 20x - 500 = 0$
$\Rightarrow x(x + 25) - 20(x + 25) = 0$
$\Rightarrow (x - 20)(x + 25) = 0$
Therefore, x = 20 or x = -25
But length cannot be negative, therefore x = 20 m
Therefore , length of the piece = 20m

OR

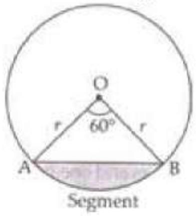
Let the time taken by the smaller pipe to fill the tank be x hr.
Time taken by the larger pipe = (x - 10) hr
Part of the tank filled by a smaller pipe in 1 hour $= \frac{1}{x}$
Part of the tank filled by the larger pipe in 1 hour $= \frac{1}{x-10}$
It is given that the tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together. So $\frac{75}{8}$ hours, multiplied by the sum of parts filled with both pipes in one hour equal to complete work i.e 1.
$\frac{75}{8}\left(\frac{1}{x} + \frac{1}{x-10}\right) = 1$
$\Rightarrow \frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$
$\Rightarrow \frac{x-10+x}{x(x-10)} = \frac{8}{75}$

$\Rightarrow \frac{2x-10}{x(x-10)} = \frac{8}{75}$
$\Rightarrow 75(2x-10) = 8x^2 - 80x$
$\Rightarrow 150x - 750 = 8x^2 - 80x$
$\Rightarrow 8x^2 - 230x + 750 = 0$
Now for factorizing the above quadratic equation, two numbers are to be found such that their product is equal to 750×8 and their sum is equal to 230
$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$
$\Rightarrow 8x(x-25) - 30(x-25) = 0$
$\Rightarrow (x-25)(8x-30) = 0$
$\Rightarrow x = 25, \frac{30}{8}$
Time taken by the smaller pipe cannot be $\frac{30}{8} = 3.75$ hours.
As in this case, the time taken by the larger pipe will be negative, which is logically not possible.
Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 - 10 = 15 hours respectively.

34. Let the radii of the two circlular plots be r_1 and r_2, respectively.
Then, $r_1 + r_2 = 14$ [$\because$ Distance between the centres of two circlular plots = 14 cm,given].... (i)
Also, Sum of Areas of the plots $= 130\pi$
$\therefore \pi r_1^2 + \pi r_2^2 = 130\pi \Rightarrow r_1^2 + r_2^2 = 130$ ...(ii)
Now, from equation (i) and equation (ii),
$\Rightarrow (14 - r_2)^2 + r_2^2 = 130$
$\Rightarrow 196 - 2r_2 + 2r_2^2 = 130$
$\Rightarrow 66 - 2r_2 + 2r_2^2 = 0$
Solving the quadratic equation we get,
$r_2 = 3$ or $r_2 = 11$,
but from figure it is clear that, $r_1 > r_2$
$\therefore r_1 = 11$ cm and $r_2 = 3$ cm
The value depicted by the farmers are of cooperative nature and mutual understanding.

OR

Area of minor segment = Area of sector – Area of $\triangle$OAB
In $\triangle$OAB,



$\theta = 60°$
OA = OB = r = 12 cm
$\angle$B = $\angle$A = x [$\angle$s opp. to equal sides are equal]
$\Rightarrow \angle A + \angle B + \angle O = 180°$
$\Rightarrow x + x + 60° = 180°$
$\Rightarrow 2x = 180° - 60°$
$\Rightarrow x = \frac{120°}{2} = 60°$
$\therefore \triangle$OAB is equilateral $\triangle$ with each side (a) = 12 cm
Area of the equilateral $\triangle = \frac{\sqrt{3}}{4}a^2$

Area of minor segment = Area of the sector – Area of $\triangle OAB$

$= \frac{\pi r^2 \theta}{360^\circ} - \frac{\sqrt{3}}{4} a^2$

$= \frac{3.14 \times 12 \times 12 \times 60^\circ}{360^\circ} - \frac{\sqrt{3}}{4} \times 12 \times 12$

$= 6.28 \times 12 - 36\sqrt{3}$

$\therefore$ Area of minor segment = $(75.36 - 36\sqrt{3})$ cm^2.

35. Table:

Class	Frequency	Mid value x_i	$f_i x_i$	Cumulative frequency
25 - 35	7	30	210	7
35 - 45	31	40	1240	38
45 - 55	33	50	1650	71
55 - 65	17	60	1020	88
65 - 75	11	70	770	99
75 - 85	1	80	80	100
	N = 100		$\sum f_i x_i = 4970$	

i. Mean

$\frac{\sum f_i x_i}{\sum f_i} = \frac{4970}{100} = 49.70$

ii. N = 100, $\frac{N}{2} = 50$

Median Class is 45 - 55

$l = 45, h = 10, N = 100, c = 38, f = 33$

$\therefore$Median = $l + h\left(\frac{\frac{N}{2} - c}{f}\right)$

$= 45 + \left\{10 \times \frac{50-38}{33}\right\}$

$= 45 + 3.64 = 48.64$

iii. we know that, Mode = $3 \times$ median $- 2 \times$ mean

$= 3 \times 48.64 - 2 \times 49.70$

$= 145.92 - 99.4 = 46.52$

Section E

36. **Read the text carefully and answer the questions:**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We have, $a_3 = 600$ and

$a_3 = 600$

$\Rightarrow 600 = a + 2d$

$\Rightarrow a = 600 - 2d$ ...(i)

$\Rightarrow a_7 = 700$

$\Rightarrow a_7 = 700$

$\Rightarrow 700 = a + 6d$

$\Rightarrow a = 700 - 6d$ ...(ii)

From (i) and (ii)

$600 - 2d = 700 - 6d$

$\Rightarrow 4d = 100$

$\Rightarrow d = 25$

(ii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We know that first term = a = 550 and common difference = d = 25

$a_n = 1000$

$\Rightarrow 1000 = a + (n - 1)d$

$\Rightarrow 1000 = 550 + 25n - 25$

$\Rightarrow 1000 - 550 + 25 = 25n$

$\Rightarrow 475 = 25n$

$\Rightarrow n = \frac{475}{25} = 19$

OR

Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

Total production in 7 years = Sum of 7 terms of the A.P. with first term a (= 550) and d (= 25).

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (7-1)25]$

$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (6) \times 25]$

$\Rightarrow S_7 = \frac{7}{2}[1100 + 150]$

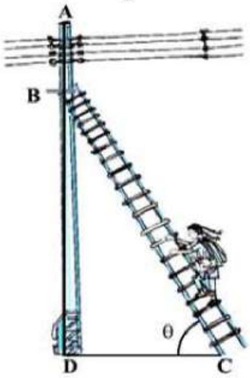
$\Rightarrow S_7 = 4375$

(iii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

The production in the 10th term is given by a_{10}. Therefore, production in the 10th year $= a_{10} = a + 9d = 550 + 9 \times 25 = 775$. So, production in 10th year is of 775 TV sets.

37. **Read the text carefully and answer the questions:**

In a village, group of people complained about an electric fault in their area. On their complaint, an electrician reached village to repair an electric fault on a pole of height 10 m. She needs to reach a point 1.5 m below the top of the pole to undertake the repair work (see the adjoining figure). She used ladder, inclined at an angle of θ to the horizontal such that $\cos\theta = \frac{\sqrt{3}}{2}$, to reach the required position.



(i) Length BD = AD - AB = 10 - 2.5 = 8.5

(ii) The length of ladder BC

In $\triangle$BDC

$\cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 30^{\circ}$

$\sin 30^{\circ} = \frac{BD}{BC}$

$\Rightarrow \frac{1}{2} = \frac{8.5}{BC}$

$\Rightarrow BC = 2 \times 8.5$ = 17 m

OR

If the height of pole and distance BD is doubled, then the length of the ladder is

$\sin 30^{\circ} = \frac{BD}{BC}$

$\Rightarrow \frac{1}{2} = \frac{17}{BC}$

$\Rightarrow BC = 2 \times 17$ = 34 m

(iii) Distance between foot of ladder and foot of wall CD

In $\triangle$BDC

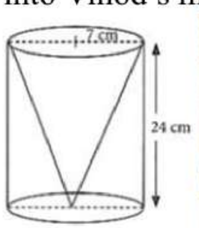
$\cos 30^{\circ} = \frac{CD}{BC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{CD}{17}$

$\Rightarrow CD = 8.5\sqrt{3}$ m

38. **Read the text carefully and answer the questions:**

One day Vinod was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Vinod's mind.



(i) Given height of cone = 24cm and radius of base = r = 7cm
Slant height of conical cavity,
l = $\sqrt{h^2 + r^2}$
$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625}$ = 25 cm

(ii) we know that r = 7cm, l = 25 cm
Curved surface area of conical cavity = πrl
= $\frac{22}{7} \times 7 \times 25$ = 550 cm^2

(iii) For cylinder height = h = 24cm, radius of base = r = 7cm
External curved surface area of cylinder
= 2πrh = 2 $\times \frac{22}{7} \times 7 \times 24$ = 1056 cm^2

OR

Curved surface area of conical cavity = πrl
= $\frac{22}{7} \times 7 \times 25$ = 550 cm^2
External curved surface area of cylinder
= 2πrh = 2 $\times \frac{22}{7} \times 7 \times 24$ = 1056 cm^2
$\frac{curved\ surface\ area\ of\ cone}{curved\ surface\ area\ of\ cylinder} = \frac{550}{1056} = \frac{275}{528}$
hence required ratio = 275:528