Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper - 9

with Solution

Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

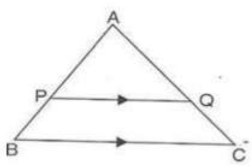
Section A

1. A quadratic polynomial whose product and sum of zeroes are $\frac{1}{3}$ and $\sqrt{2}$ respectively is **[1]**

a) $3x^2 - x + 3\sqrt{2}x$ b) $3x^2 - 3\sqrt{2}x + 1$

c) $3x^2 + x - 3\sqrt{2}x$ d) $3x^2 + 3\sqrt{2}x + 1$

2. In the given figure $PQ||BC$. $\frac{AP}{PB} = 4,$ then the value of $\frac{AQ}{AC}$ is **[1]**



a) 5 b) $\frac{4}{5}$

c) 4 d) $\frac{5}{4}$

3. If (x + 1) is a factor of $2x^3 + ax^2 + 2bx + 1$, then the values of a and b, given that 2a - 3b = 4 are **[1]**

a) a = -5 and b = -2 b) a = 5 and b = 2

c) a = -5 and b = 2 d) a = 5 and b = -2

4. It is given that $\triangle ABC \sim \triangle DFE$. If ∠A = 30°, ∠C = 50°, AB = 5 cm, AC = 8 cm and DF = 7.5 cm then which of the following is true? **[1]**

a) DE = 12 cm, $\angle F = 50°$

b) EF = 12 cm, $\angle F = 10°$

c) EF = 12 cm, $\angle D = 100°$

d) DE = 12 cm, $\angle F = 100°$

5. The graphs of the equations 5x - 15y = 8 and 3x - 9y = $\frac{24}{5}$ are two lines which are **[1]**

a) intersecting exactly at one point

b) coincident

c) perpendicular to each other

d) parallel

6. A bag contains 3 white, 4 red and 5 black balls. One ball is drawn at random. What is the probability that the ball drawn is neither black nor white? **[1]**

a) $\frac{1}{3}$

b) $\frac{1}{4}$

c) $\frac{1}{2}$

d) $\frac{4}{3}$

7. The arithmetic mean of a set of 40 values is 65. If each of the 40 values is increased by 5, what will be the mean of the set of new values: **[1]**

a) 65

b) 60

c) 70

d) 50

8. $\sqrt{\frac{1+\cos A}{1-\cos A}} = ?$ **[1]**

a) cosec A - cot A

b) None of these

c) cosec A + cot A

d) cosec A cot A

9. If in two triangles ABC and PQR, $\angle A = \angle Q$ and $\angle R = \angle B$, then which of the following is not true. **[1]**

a) $\frac{AB}{PQ} = \frac{BC}{RP}$

b) $\frac{BC}{RP} = \frac{AB}{QR}$

c) $\frac{BC}{PR} = \frac{AC}{PQ}$

d) $\frac{AB}{QR} = \frac{AC}{PQ}$

10. The roots of the quadratic equation $2x^2 - x - 6 = 0$ are **[1]**

a) $2, \frac{-3}{2}$

b) $2, \frac{3}{2}$

c) $-2, \frac{-3}{2}$

d) $-2, \frac{3}{2}$

11. The multiplicative inverse of zero **[1]**

a) 1

b) 0

c) does not exist

d) $\frac{1}{0}$

12. The distance between the points ($\cos\theta$, $\sin\theta$) and ($\sin\theta$, - $\cos\theta$) is **[1]**

a) $\sqrt{3}$

b) $\sqrt{2}$

c) 2

d) 1

13. One of the methods of determining mode is: **[1]**

a) Mode = 2 Median + 3 Mean

b) Mode = 3 Median + 2 Mean

c) Mode = 3 Median - 2 Mean

d) Mode = 2 Median - 3 Mean

14. The shadow of a 5 m long stick is 2 m long. At the same time, the length of the shadow of a 12.5 m high tree is **[1]**

a) 3 m

b) 4.5 m

c) 3.5 m

d) 5 m

15. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\theta = q$, then $q(p^2 - 1) =$ **[1]**

a) 2p

b) None of these

c) $\frac{q}{p^2}$

d) 2

16. In triangles ABC and DEF, $\angle A - \angle E = 40°$, AB : ED = AC : EF and $\angle F = 65°$, then $\angle B =$ **[1]**

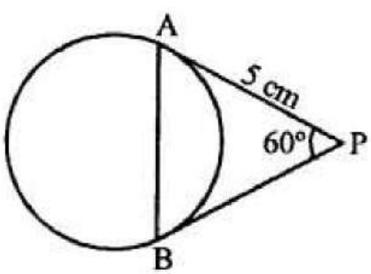
a) 75^o

b) 85^o

c) 35^o

d) 65^o

17. In the given figure, PA and PB are tangents to the given circle such that PA = 5 cm and $\angle APB = 60°$. The length of chord AB is **[1]**



a) $5\sqrt{2}$ cm

b) 5 cm

c) 7.5 cm

d) $5\sqrt{3}$ cm

18. In a cricket match, Kumble took three wickets less than twice the number of wickets taken by Srinath. The product of the number of wickets taken by these two is 20, then the number of wickets taken by Kumble is **[1]**

a) 4

b) 5

c) 10

d) 2

19. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 350 cm^2. **[1]**
Reason (R): Total surface area of a cuboid is 2(lb + bh + hl)

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion:** Degree of a zero polynomial is not defined. **[1]**
Reason: Degree of a non-zero constant polynomial is 0

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
c) Assertion is correct statement but reason is wrong statement.
d) Assertion is wrong statement but reason is correct statement.

Section B

21. The line segment joining the points A (3, 2) and B (5,1) is divided at the point P in the ratio 1 : 2 and P lies on the line 3x - 18y + k = 0, Find the value of k. **[2]**

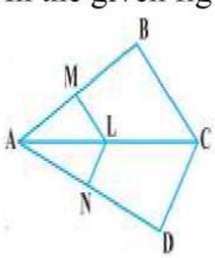
22. Solve the quadratic equation by factorization: **[2]**
$3x^2 - 14x - 5 = 0$

23. Find the HCF and LCM of 6, 72 and 120 using fundamental theorem of arithmetic. **[2]**

24. The perimeters of two similar triangles ABC and PQR are 32 cm and 24 cm respectively. If PQ = 12 cm, find AB. **[2]**

OR

In the given figure, $LM \| CB$ and LN$\|$CD. Prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



25. If tan θ + cot θ = 2, find the value of $\tan^2 \theta + \cot^2 \theta$. **[2]**

OR

Prove: $\sqrt{\frac{1-\cos A}{1+\cos A}} = \csc A - \cot A$

Section C

26. BO and CO are respectively the bisectors of $\angle B$ and $\angle C$ of ΔABC. AO produced meets BC at P. Show that **[3]**

i. $\frac{AB}{BP} = \frac{AO}{OP}$

ii. $\frac{AC}{CP} = \frac{AO}{OP}$

iii. $\frac{AB}{AC} = \frac{BP}{PC}$

iv. AP is the bisector of $\angle BAC$.

27. The sum of the squares of two consecutive multiples of 7 is 1225. Find the multiples. **[3]**

28. Find the ratio in which the line segment joining (-2, -3) and (5,6) is divided by (i) x-axis (ii) y-axis. Also, find the coordinates of the point of division in each case. **[3]**

OR

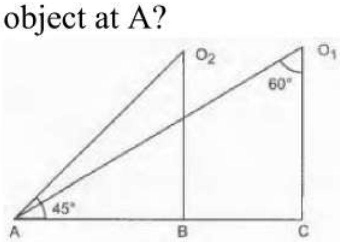
Let the opposite angular points of a square be (3, 4) and (1, -1). Find the coordinates of the remaining angular points.

29. Prove that $3 + 2\sqrt{5}$ is irrational. **[3]**

30. Find the mean of the following frequency distribution: **[3]**

Class Interval	Frequency
0 - 10	4
10 - 20	4
20 - 30	7
30 - 40	10
40 - 50	12
50 - 60	8
60 - 70	5
Total	**50**

31. What are the angles of depression from the observing positions O_1 and O_2 of the object at A? **[3]**



OR

A tower is 50m high. Its shadow is x metres shorter when the sun's altitude is 45° than when it is 30°. Find the value of x. [Given $\sqrt{3}$ = 1.732.]

Section D

32. Solve graphically system of linear equations. Also find the coordinates of the **[5]**

points where the lines meet y-axis.
$2x - y - 5 = 0$
$x - y - 3 = 0$

OR

One says, "Give me a hundred rupee, friend! I shall then become twice as rich as you are." The other replies, "If you give me ten rupees, I shall be six times as rich as you are." Tell me how much money both have initially?

33. A semicircular region and a square region have equal perimeters. The area of the square region exceeds that of the semicircular region by 4 cm^2. Find the perimeters and areas of the two regions. **[5]**

OR

Find the area of the segment of a circle of radius 12 cm whose corresponding sector central angle 60°. (Use $\pi = 3.14$).

34. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. **[5]**

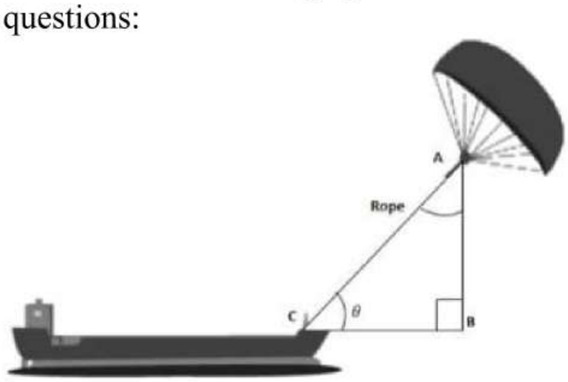
35. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is : **[5]**

i. a card of a spade or an ace.
ii. a black king.
iii. neither a jack nor a king.
iv. either a king or a queen

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.
Based on the following figure related to sky sailing, answer the following questions:



(i) In the given figure, if $\sin\theta = \cos(\theta - 30^\circ)$, where θ and $\theta - 30^\circ$ are acute angles, then find the value of θ.

(ii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200m?

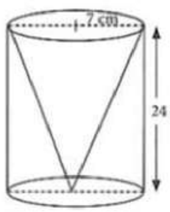
(iii) In the given figure, if $\sin\theta = \cos(3\theta - 30^{\circ})$, where θ and $3\theta - 30^{\circ}$ are acute angles, then find the value of θ.

OR

What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 150m?

37. **Read the text carefully and answer the questions:** **[4]**

One day Vinod was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Vinod's mind.



(i) Find the slant height of the conical cavity so formed?

(ii) Find the curved surface area of the conical cavity so formed?

OR

Find the ratio of curved surface area of cone to curved surface area of cylinder?

(iii) Find the external curved surface area of the cylinder?

38. **Read the text carefully and answer the questions:** **[4]**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.

In their garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row.

(i) How many rows are there of rose plants?

(ii) Also, find the total number of rose plants in the garden.

OR

If total number of plants are 80 in the garden, then find number of rows?

(iii) How many plants are there in 6th row.

Solution

Section A

1. **(b)** $3x^2 - 3\sqrt{2}x + 1$

Explanation: Given: $\alpha + \beta = \frac{\sqrt{2}}{1} = \frac{-\left(-\sqrt{2}\right)}{1} = \frac{-\left(-3\sqrt{2}\right)}{3}$

And $\alpha\beta = \frac{c}{a} = \frac{1}{3}$ On comparing, we get, a = 3, b = $-3\sqrt{2}$, c = 1

Putting these values in the general form of a quadratic polynomial $ax^2 + bx + c$,

we have $3x^2 - 3\sqrt{2} + 1$

2. **(b)** $\frac{4}{5}$

Explanation: Given: $\frac{AP}{PB} = \frac{4}{1}$

Let AP = 4x and PB = x, then AB = AP + PB = 4x + x = 5x

Since PQ ∥ BC, then

$\frac{AP}{AB} = \frac{AQ}{AC}$ [Using Thales theorem]

$\therefore \frac{AQ}{AC} = \frac{AP}{AB} = \frac{4x}{5x} = \frac{4}{5}$

3. **(b)** a = 5 and b = 2

Explanation: Given that, (x + 1) is a factor of $f(x) = 2x^s + ax^2 + 2bx + 1$, then f(-1) = 0

[if $(x + \alpha)$ is factor of $f(x) = ax^2 = bx + c$, then f(-) then f(-) = 0]

$\Rightarrow 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$

$\Rightarrow$ -2 + a - 2ab + 1 = 0

$\Rightarrow$ a - 2b - 1 = 0 .... (i)

Also, 2a - 3b = 4

$\Rightarrow$ 3b = 2a - 4

$$\Rightarrow \quad b = \left(\frac{2a-4}{3}\right)$$

Now, put the value of b in Eq. (i), we get

$$a - 2\left(\frac{2a-4}{3}\right) - 1 = 0$$

$\Rightarrow$ 3a - 2(2a - 4) - 3 = 0

$\Rightarrow$ 3a - 4a + 8 - 3 = 0

$\Rightarrow$ -a + 5

Now, put the value of a in Eq. (i), we get

5 - 2b - 1 = 0

$\Rightarrow$ 2b = 4

$\Rightarrow$ b = 2

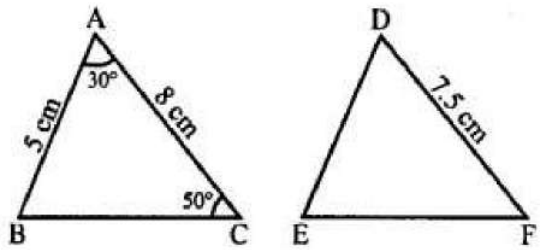
Hence, the required values of a and b are 5 and 2, respectively.

4. **(d)** DE = 12 cm, $\angle F = 100°$

Explanation:

$\triangle ABC \sim \triangle DFE$.

$\angle A = 30°$, $\angle C = 50°$, AB = 5cm, AC = 8 cm and DF = 7.5 cm



$\angle B = 180° - (\angle A + \angle C)$ ($\because$ Linear pair)

$= 180° - \left(30° + 50°\right)$

$\angle B = 180° - 80° = 100°$

$\because \triangle ABC \sim \Delta DFE$

$\therefore \angle D = 30°, \angle B = \angle F = 100°$

and $\angle C = \angle E = 50°$

and $\frac{AB}{DF} = \frac{AC}{DE} = \frac{BC}{EF}$

$\frac{5}{7.5} = \frac{8}{DE} \Rightarrow DE = \frac{8 \times 7.5}{5} = 12.0 = 12cm$

$\therefore DE = 12, \angle F = 100°$

5. **(b)** coincident

Explanation: We have,

5x - 15y - 8 = 0

And, $3x - 9y - \frac{24}{5} = 0$

Here, $a_1 = 5$, $b_1 = -15$ and $c_1 = -8$

And, $a_2 = 3$, $b_2 = -9$ and $c_2 = \frac{-24}{5}$

$\therefore \frac{a_1}{a_2} = \frac{5}{3}, \frac{b_1}{b_2} = \frac{-15}{-9} = \frac{5}{3}$ and $\frac{c_1}{c_2} = -8 \times \frac{5}{-24} = \frac{5}{3}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given system has a unique solution and the lines are coincident.

6. **(a)** $\frac{1}{3}$

Explanation: Total number of balls in the bag = 3 + 4 + 5 = 12.

Number of non-black and non-white balls = 4.

$\therefore$ P (getting a ball which is neither black nor white) = $\frac{4}{12} = \frac{1}{3}$

7. **(c)** 70

Explanation: Mean of 40 values = 65

Total of 40 values = 65 × 40 = 2600

When each value is increased by 5,

then the total of 40 values will be 40 × 5 = 200 more than 2600

$\therefore$ New total of 40 values = 2600 + 200 = 2800

Now, New mean of 40 values = $\frac{2800}{40}$ = 70

8. **(c)** cosec A + cot A

Explanation:

$$\sqrt{\frac{1+\cos A}{1-\cos A}} = \sqrt{\frac{(1+\cos A)}{(1-\cos A)} \times \frac{(1+\cos A)}{(1+\cos A)}} = \frac{(1+\cos A)}{\sqrt{1-\cos^2 A}} = \frac{(1+\cos A)}{\sqrt{\sin^2 A}}$$

$$= \frac{(1+\cos A)}{\sin A} = \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A}\right) = (cosec\, A + \cot A)$$

9. **(a)** $\frac{AB}{PQ} = \frac{BC}{RP}$

Explanation: In triangles ABC and PQR,

If ∠A = ∠Q and ∠R = ∠B, then∠C = ∠P

$\therefore$ △ABC ~ △QRP (By AA Similarity criteria)

$\therefore \frac{AB}{QR} = \frac{BC}{PR}, \frac{BC}{PR} = \frac{AC}{PQ}, \frac{AC}{PQ} = \frac{AB}{QR}$

Therefore, $\frac{AB}{PQ} \neq \frac{BC}{RP}$

10. **(a)** 2, $\frac{-3}{2}$

Explanation: The given quadratic equation is $2x^2 - x - 6 = 0$

$2x^2 - x - 6 = 0$

$\Rightarrow 2x^2 - 4x + 3x - 6 = 0$

$\Rightarrow 2x(x-2) + 3(x-2) = 0$

$\Rightarrow x - 2 = 0$ or $2x + 3 = 0$

$\Rightarrow x = 2$ or $x = -3/2$

Thus, the roots of the given equation are 2 and -3/2

11. **(c)** does not exist

Explanation: All numbers except zero have a multiplicative inverse because we cannot multiply any number by it to get 1.

12. **(b)** $\sqrt{2}$

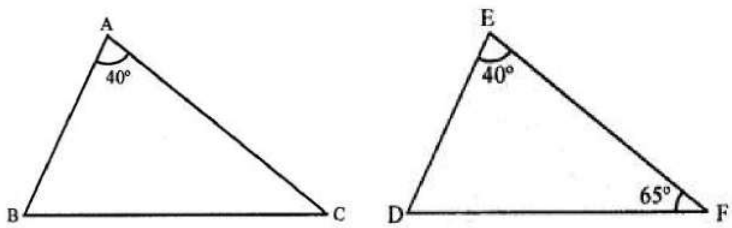
Explanation: Distance between $(\cos\theta, \sin\theta)$ and $(\sin\theta, -\cos\theta)$

$$= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$= \sqrt{(-\cos\theta - \sin\theta)^2 + (\sin\theta - \cos\theta)^2}$

$= \sqrt{1+1} = \sqrt{2}\ \left\{\because \sin^2\theta + \cos^2\theta = 1\right\}$

13. **(c)** Mode = 3 Median - 2 Mean
Explanation: It is well known fact that for moderately skewed matrix: Mode = 3 Median − 2 Mean
14. **(d)** 5 m
Explanation: Ratio of lengths of objects = ratio of lengths of their shadows.
Let the length of shadow of the tree be x m. Then,
$\frac{5}{12.5} = \frac{2}{x} \Rightarrow 5x = 2 \times 12.5 = 25$
$\Rightarrow x = 5$
15. **(a)** 2p
Explanation: Given: $\sin\theta + \cos\theta = p$
squaring both sides we get
$\sin^2\theta + \cos^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = p^2$
$1 + 2\sin\theta\cos\theta = p^2 (\sin^2\theta + \cos^2\theta = 1)$
$2\sin\theta\cos\theta = p^2 - 1$ ... (i)
and also $\sec\theta + \operatorname{cosec}\theta = q$ (given)
$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q$
$\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = q$
but $\sin\theta + \cos\theta = p$ ... (given)
$\frac{p}{\sin\theta\cos\theta} = q$ ... (ii)
from (i) and (ii) we get
$q(p^2 - 1) = 2p$
16. **(a)** 75°
Explanation:
In ΔABC and ΔDEF
$\angle A = \angle E = 40^{\circ}$
$AB : ED = AC : EF, \angle F = 65^{\circ}$



$\Rightarrow \frac{AB}{ED} = \frac{AC}{EF}$
$\because$ *In* △ABC *and* △EDF

$\angle A = \angle E \quad \left(\text{each } = 40^\circ\right)$

$\frac{AB}{ED} = \frac{AC}{EF}$

$\therefore \triangle ABC \sim \triangle EDF$ (SAS criterion)

$\therefore \angle C = \angle F = 65^\circ$

and $\angle B = \angle D$

But $\angle A + \angle B + \angle C = 180^\circ$ (Sum of angles of a triangle)

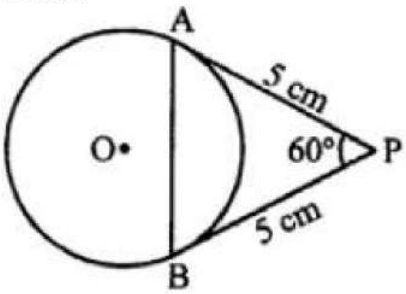
$\Rightarrow 40^\circ + 65^\circ + \angle C = 180^\circ$

$\Rightarrow 105^\circ + \angle C = 180^\circ$

$\therefore \angle C = 180^\circ - 105^\circ = 75^\circ$

17. **(b)** 5 cm

Explanation: In the given figure, PA and PB are the tangents to the circle with centre O from P



PA = 5 cm, $\angle APB = 60^\circ$

PA = PB = 5 cm

In $\triangle APB$, $\angle P = 60^\circ$ and PA = PB

PAB is an equilateral triangle

AB = AP = BP = 5 cm

18. **(b)** 5

Explanation: Let the number of wickets taken by Srinath be x

then, the number of wickets taken by Kumble will be 2x - 3

According to question, x(2x - 3) = 20

$\Rightarrow 2x^2 - 3x - 20 = 0$

$\Rightarrow 2x^2 - 8x + 5x - 20 = 0$

$\Rightarrow 2x(x - 4) + 5(x - 4) = 0$

$\Rightarrow (x - 4)(2x + 5) = 0$

$\Rightarrow x - 4 = 0$ and $2x + 5 = 0$

$\Rightarrow x = 4$ and $x = \frac{-5}{2}$ [$x = \frac{-5}{2}$ is not possible]

Therefore, the number of wickets taken by Srinath is 4.

Then, the number of wickets taken by Kumble = $2 \times 4 - 3 = 5$

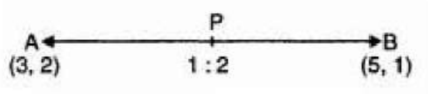
19. **(d)** A is false but R is true.

Explanation: A is false but R is true.

20. **(b)** Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Explanation: Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Section B

21. 

Since, the line segment joining the points A (3, 2) and B (5,1) is divided at the point P in the ratio 1:2

Therefore, according to the section formula,

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{1 \times 5 + 2 \times 3}{1+2} = \frac{11}{3}$$

$$and,\ y = \frac{my_2 + ny_1}{m+n} = \frac{1 \times 1 + 2 \times 2}{1+2} = \frac{5}{3}$$

$\Rightarrow (x, y) = \left(\frac{11}{3}, \frac{5}{3}\right)$ lies on $3x - 18y + k = 0$

Therefore, these points satisfy equation of given line.

Hence, $3 \times \frac{11}{3} - 18 \times \frac{5}{3} + k = 0$

$\Rightarrow 11 - 30 + k = 0$

$\Rightarrow$ k = 19,

Hence required value of k =19

22. We have,

$3x^2 - 14x - 5 = 0$

So, $3x^2 - 14x - 5 = 0$

$\Rightarrow 3x^2 - 15x + 1x - 5 = 0$

$\Rightarrow 3x(x - 5) + 1(x - 5) = 0$

$\Rightarrow (x - 5)(3x + 1) = 0$

$\Rightarrow x - 5 = 0$ or $3x + 1 = 0$

$\Rightarrow x = 5$ or $x = -\frac{1}{3}$. Hence the roots are 5 and $-\frac{1}{3}$

23. $6 = 2 \times 3$

$72 = 8 \times 9 = 2^3 \times 3^2$

$120 = 8 \times 15 = 2^3 \times 3 \times 5$

HCF(6,72,120) $= 2 \times 3 = 6$

LCM (6,12,120)$= 2^3 \times 3^2 \times 5 = 360$

24. According to question it is given that triangles ABC and PQR are similar.

Also, perimeter of Δ ABC = 32 & Perimeter of Δ PQR = 24

Therefore,

$$\frac{\text{Perimeter } (\triangle ABC)}{\text{Perimeter } (\triangle PQR)} = \frac{AB}{PQ}$$

$$\Rightarrow \frac{32}{24} = \frac{AB}{12}$$

$\Rightarrow AB = \frac{32 \times 12}{24}$

$= 16$ cm

OR

According to question it is given that In $\triangle ALM$, $LM \parallel CB$

$\therefore \quad \frac{AB}{AM} = \frac{AC}{AL}$

Therefore, by Thales' theorem

$\Rightarrow \frac{AM}{AB} = \frac{AL}{AC}$ ....(i)

In $\triangle ALN$, $LN \parallel CD$

$\therefore \quad \frac{AC}{AL} = \frac{AD}{AN}$

Therefore by Thales theorem

$\Rightarrow \frac{AL}{AC} = \frac{AN}{AD}$ ....(ii)

From (i) and (ii) we get

$\frac{AM}{AB} = \frac{AN}{AD}$

25. We have,.

$\tan\theta + \cot\theta = 2$

$\Rightarrow (\tan\theta + \cot\theta)^2 = 4$ [On squaring both sides]

$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 4$

$\Rightarrow \tan^2\theta + \cot^2\theta + 2 = 4$ [$\because \tan\theta\cot\theta = 1$]

$\Rightarrow \tan^2\theta + \cot^2\theta = 2$

OR

$\sqrt{\frac{1-\cos A}{1+\cos A}} = cosecA - \cot A$

L.H.S. $= \sqrt{\frac{1-\cos A}{1+\cos A}}$

$= \sqrt{\frac{(1-\cos A)\times(1-\cos A)}{(1+\cos A)\times(1-\cos A)}}$ [Multiplying and dividing by (1 - cos A)]

$= \frac{1-\cos A}{\sqrt{1-\cos^2 A}} = \frac{1-\cos A}{\sin A}$ Since, $\sqrt{1-\cos^2 A} = \sin A$

$= cosecA - \cot A\left[\because \frac{1}{\sin A} = cosecA, \frac{\cos A}{\sin A} = \cot A\right]$

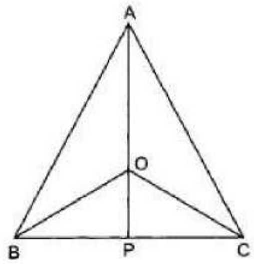
= R.H.S. proved

Section C

26. we shall use the following theorm:

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

i. In ΔABP, BO is the bisector of ∠B



$\therefore \quad \frac{AB}{BP} = \frac{AO}{OP}$

ii. In ΔACP, OC is the bisector of ∠C

$\therefore \quad \frac{AC}{CP} = \frac{AO}{OP}$

iii. We have, proved that

$\frac{AB}{BP} = \frac{AO}{OP}$ and $\frac{AC}{CP} = \frac{AO}{OP}$

$\Rightarrow \quad \frac{AB}{BP} = \frac{AC}{CP}$

$\Rightarrow \quad \frac{AB}{AC} = \frac{BP}{PC}$

iv. As proved above that in ΔABC, we have

$\frac{AB}{AC} = \frac{BP}{CP} \Rightarrow$ AP is the bisector of ∠BAC.

27. Let the consecutive multiples of 7 be x and (x + 7).

According to the question ;

$x^2 + (x + 7)^2 = 1225$

$\Rightarrow x^2 + x^2 + 14x + 49 = 1225$

$\Rightarrow 2x^2 + 14x - 1176 = 0$

$\Rightarrow x^2 + 7x - 588 = 0$ (dividing both sides by 2)

$\Rightarrow x^2 + 28x - 21x - 588 = 0$

$\Rightarrow x(x + 28) - 21(x + 28) = 0$

$\Rightarrow x + 28 = 0$ or $x - 21 = 0$

$\Rightarrow x = -28$ or $x = 21$ (both values are accepted as both are multiples of 7)

When x = -28

x + 7 = -28 + 7 = -21

When x = 21

x + 7 = 21 + 7 = 28

Hence, the required numbers are 21, 28 or -21, -28

28. Let A(-2, -3) and B(5, 6) be the given points.

i. Suppose x-axis divides AB in the ratio k:1 at point P

Then, the coordinates of the point of division are

$P\left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1}\right]$

Since P lies on x-axis, and y-coordinates of every point on x-axis is zero.

$\therefore \quad \frac{6k-3}{k+1} = 0$

$\Rightarrow$ 6k - 3=0

$\Rightarrow$ 6k = 3

$\Rightarrow k = \frac{3}{6} \Rightarrow k = \frac{1}{2}$

Hence, the required ratio is 1:2.

Putting $K = \frac{1}{2}$ in the coordinates of P.

We find that its coordinates are $\left(\frac{1}{3}, 0\right)$.

ii. Suppose y-axis divides AB in the ratio k:1 at point Q.
Then, the coordinates of the point of division are

$Q\left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1}\right]$

Since, Q lies on y-axis, and x-coordinates of every point on y-axis is zero.

$\therefore \quad \frac{5k-2}{k+1} = 0$

$\Rightarrow$ 5k - 2 = 0

$\Rightarrow \quad k = \frac{2}{5}$

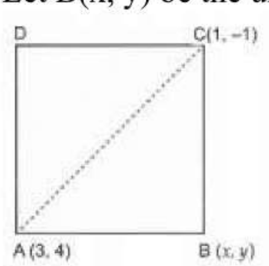
Hence, the required ratio is $\frac{2}{5} : 1 = 2:5$

Putting $K = \frac{2}{5}$ in the coordinates of Q.

We find that the coordinates are $\left(0, \frac{-3}{7}\right)$

OR

Let ABCD be a square and let A (3,4) and C (1, -1) be the given angular points.
Let B(x, y) be the unknown vertex



Then, AB = BC

$\Rightarrow AB^2 = BC^2$

$\Rightarrow (x - 3)^2 + (y - 4)^2 = (x - 1)^2 + (y + 1)^2$

$\Rightarrow 4x + 10y - 23 = 0$

$\Rightarrow \quad x = \dfrac{23-10y}{4}$ .... (i)

In right-angled triangle ABC, we have

$AB^2 + BC^2 = AC^2$

$\Rightarrow (x - 3)^2 + (y - 4)^2 + (x - 1)^2 + (y + 1)^2 = (3 - 1)^2 + (4 + 1)^2$

$\Rightarrow x^2 + y^2 - 4x - 3y - 1 = 0$ ...(ii)

Substituting the value of x from (i) into (ii), we get

$$\left(\frac{23-10y}{4}\right)^2 + y^2 - (23 - 10y) - 3y - 1 = 0$$

$\Rightarrow 4y^2 - 12y + 5 = 0 \Rightarrow (2y - 1)(2y - 5) = 0 \Rightarrow y = \dfrac{1}{2}$ or $\dfrac{5}{2}$

Putting $y = \dfrac{1}{2}$ and $y = \dfrac{5}{2}$ respectively in (i), we get $x = \dfrac{9}{2}$ and $x = \dfrac{-1}{2}$ respectively.

Hence, the required vertices of the square are (9/2,1/2) and (-1 /2 ,5 /2).

29. Let us assume, to the contrary, that is $3 + 2\sqrt{5}$ rational.

That is, we can find coprime integers a and b ($b \neq 0$) such that

$3 + 2\sqrt{5} = \dfrac{a}{b}$ Therefore, $\dfrac{a}{b} - 3 = 2\sqrt{5}$

$\Rightarrow \dfrac{a-3b}{b} = 2\sqrt{5}$

$\Rightarrow \dfrac{a-3b}{2b} = \sqrt{5} \Rightarrow \dfrac{a}{2b} - \dfrac{3}{2}$

Since a and b are integers,

We get $\dfrac{a}{2b} - \dfrac{3}{2}$ is rational, also so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction arose because of our incorrect assumption that $3 + 2\sqrt{5}$ is rational.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

30. Calculation of Mean:

C.I.	**Class mark(x_i)**	**f_i**	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$
0 - 10	5	4	-3	-12
10 - 20	15	4	-2	-8
20 - 30	25	7	-1	-7

C.I.	Class mark(x_i)	f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
30 - 40	35	10	0	0
40 - 50	45	12	1	12
50 - 60	55	8	2	16
60 - 70	65	5	3	15
Total		$\Sigma f_i = 50$		$\Sigma f_i u_i = 16$

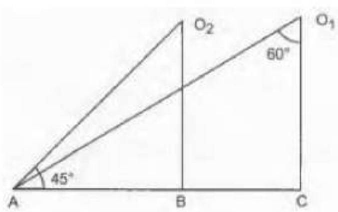
$a = 35, h = 10$

we know that,

Mean = a + $\frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$= 35 + \frac{16}{50} \times 10$

$= 35 + 3.2 = 38.2$

31. 

In triangle O_1AC,

$\Rightarrow \angle A = 180^o - (90^o + 60^o)$

$\Rightarrow \angle A = 180^o - 150^o$ We know that $[\angle A + \angle B + \angle C = 180^o]$

$\Rightarrow \angle A = 30^o$

Again,

In a triangle O_2AB,

$\Rightarrow \angle O_2 = 180^o - (90^o + 45^o)$

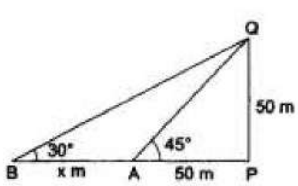
$\Rightarrow \angle O_2 = 180^o - 135^o$

$\Rightarrow \angle O_2 = 45^o$

Hence the required angles are $30^o, 45^o$.

OR

Let PQ be the tower and let PA and PB be its shadows when the altitudes of the sun are 45° and 30° respectively. Then,



$\angle PAQ = 45^\circ, \angle PBQ = 30^\circ, \angle BPQ = 90^\circ, PQ = 50$m.

Let $AB = x\ m$.

From right ΔAPQ, we have

$$\frac{AP}{PQ} = \cot 45^{\circ} = 1$$

$$\Rightarrow \frac{AP}{50\text{m}} = 1 \Rightarrow AP = 50\text{m}.$$

From right ΔBPQ, we have

$$\frac{BP}{PQ} = \cot 30^{\circ} = \sqrt{3} \Rightarrow \frac{x+50}{50} = \sqrt{3} \Rightarrow \quad x = 50(\sqrt{3} - 1).$$

$$\Rightarrow x = 50(1.732 - 1) = (50 \times 0.732) = 36.6$$

Hence, $x = 36.6$

Section D

32. The given system of linear equation is

2x - y - 5 = 0 ... (1)

x - y - 3 = 0 ...(2)

let us draw the graphs of equations (1) and (2) by finding two solutions for each of the equations. These two solutions of the equations (1) and (2) are given below in table 1 and table 2 respectively.

For equation (1) 2x - y - 5 = 0 $\Rightarrow$ y = 2x - 5

Table 1 of solutions

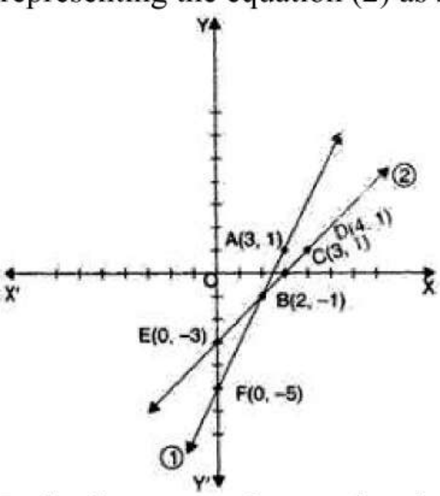
x	3	2
y	1	-1

For equation (2) x - y - 3 = 0 $\Rightarrow$ y = x - 3

Table 2 of solutions

x	0	4
y	-3	1

We plot the points A(3, 1) and B(2, -1) on a graph paper and join these points to form the line AB representing the equation (1) as shown in the figure. Also, we plot the points C(0, -3) and D(4, 1) on the same graph paper and join these points to form the line CD representing the equation (2) as shown in the same figure.



In the figure, we observe that the two lines intersect at the point B (2, -1).

So, x = 2 and y = -7 is the required solution of the given pair of linear equations.

Also, we observe that the line (1) and (2) meet the y-axis in the points E(0, -3) and F(0, -5) respectively.

OR

Suppose initially, they had Rs x and Rs. y with them respectively.

as per condition given in the question, we obtain

$x + 100 = 2(y - 100)$

$\Rightarrow x + 100 = 2y - 200$

$\Rightarrow x - 2y = -300$ ...(i)

and $6(x - 10) = (y + 10)$

$6x - 60 = y + 10$

$\Rightarrow 6x - y = 70$ .....(ii)

Multiplying equation (ii) by 2 & then subtracting eqeation (i) from it, we obtain:-

$(12x - 2y)-(x - 2y) = 140 - (- 300)$

$\Rightarrow 11x = 140 + 300$

$\Rightarrow 11x = 440$

$\Rightarrow x = 40$

Putting x = 40 in equation (i), we obtain

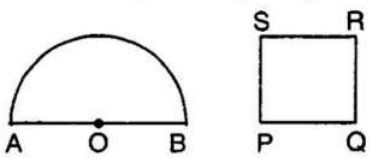
$40 - 2y = -300$

$\Rightarrow 40 + 300 = 2y$

$\Rightarrow 2y = 340$

$\Rightarrow y = 170$

Therefore, initially they had Rs 40 and Rs 170 with them respectively.

33. 

Let radius of semicircular region be r units.

Perimeter = $2r + \pi r$

Let side of square be x units

Perimeter = 4x units.

A.T.Q, $4x = 2r + \pi r \Rightarrow x = \frac{2r+\pi r}{4}$

Area of semicircle = $\frac{1}{2}\pi r^2$

Area of square = x^2

A.T.Q, $x^2 = \frac{1}{2}\pi r^2 + 4$

$$\Rightarrow \left(\frac{2r+\pi r}{4}\right)^2 = \frac{1}{2}\pi r^2 + 4$$

$$\Rightarrow \frac{1}{16}\left(4r^2 + \pi^2 r^2 + 4\pi r^2\right) = \frac{1}{2}\pi r^2 + 4$$

$$\Rightarrow 4r^2 + \pi^2 r^2 + 4\pi r^2 = 8\pi r^2 + 64$$

$$\Rightarrow 4r^2 + \pi^2 r^2 - 4\pi r^2 = 64$$

$$\Rightarrow r^2\left(4 + \pi^2 - 4\pi\right) = 64$$

$$\Rightarrow r^2(\pi - 2)^2 = 64$$

$$\Rightarrow r = \sqrt{\frac{64}{(\pi-2)^2}}$$

$$\Rightarrow r = \frac{8}{\pi-2} = \frac{8}{\frac{22}{7}-2} = 7 \text{ cm}$$

Perimeter of semicircle $= 2 \times 7 + \frac{22}{7} \times 7 = 36$ cm

Perimeter of square = 36 cm

Side of square $= \frac{36}{4} = 9$ cm

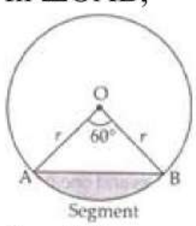
Area of square $= 9 \times 9 = 81 \text{ cm}^2$

Area of semicircle $= \frac{\pi r^2}{2} = \frac{22}{2\times 7} \times 7 \times 7 = 77 \text{ cm}^2$

OR

Area of minor segment = Area of sector – Area of △OAB

In △OAB,



$\theta = 60°$

OA = OB = r = 12 cm

∠B = ∠A = x [∠s opp. to equal sides are equal]

$\Rightarrow \angle A + \angle B + \angle O = 180°$

$\Rightarrow x + x + 60° = 180°$

$\Rightarrow 2x = 180° - 60°$

$\Rightarrow x = \frac{120°}{2} = 60°$

∴ △OAB is equilateral △ with each side (a) = 12 cm

Area of the equilateral △ $= \frac{\sqrt{3}}{4}a^2$

Area of minor segment = Area of the sector – Area of △OAB

$$= \frac{\pi r^2 \theta}{360°} - \frac{\sqrt{3}}{4}a^2$$

$$= \frac{3.14 \times 12 \times 12 \times 60°}{360°} - \frac{\sqrt{3}}{4} \times 12 \times 12$$

$= 6.28 \times 12 - 36\sqrt{3}$

∴ Area of minor segment $= (75.36 - 36\sqrt{3}) \text{ cm}^2$.

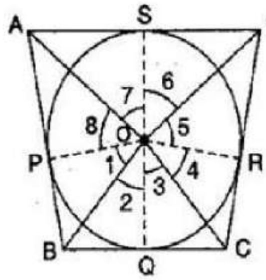
34. Given: ABCD is a quadrilateral circumscribing a circle whose centre is O.

To prove:

i. $\angle AOB + \angle COD = 180^{o}$

ii. $\angle BOC + \angle AOD = 180^{o}$

Construction: Join OP, OQ, OR and OS.



Proof: Since tangents from an external point to a circle are equal.

$\therefore$ AP = AS,

BP = BQ .... (i)

CQ = CR

DR = DS

In $\triangle$OBP and $\triangle$OBQ,

OP = OQ [Radii of the same circle]

OB = OB [Common]

BP = BQ [From eq. (i)]

$\therefore$ $\triangle OPB \cong \triangle OBQ$ [By SSS congruence criterion]

$\therefore$ $\angle 1 = \angle 2$ [By C.P.C.T.]

Similarly, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$

Since, the sum of all the angles round a point is equal to 360^{o}.

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^{o}$

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360^{o}$

$\Rightarrow 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360^{o}$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180^{o}$

$\Rightarrow (\angle 1 + \angle 5) + (\angle 4 + \angle 8) = 180^{o}$

$\Rightarrow \angle AOB + \angle COD = 180^{o}$

Similarly we can prove that

$\angle BOC + \angle AOD = 180^{o}$

35. i. Cards of spade or an ace = 13 + 3 = 16 Hence m=16

Total no. of cards = 52 So n=52

P (spade or an ace) = $\frac{m}{n} = \frac{16}{52} = \frac{4}{13}$

ii. Black kings = 2 so m=2

P(a black king) = $\frac{m}{n} = \frac{2}{52} = \frac{1}{26}$

iii. Jack or king = 4 + 4 = 8 so for neither jack nor a king =52-8=44 hence m=44

P(neither jack nor a king) = $\frac{m}{n} = \frac{44}{52} = \frac{11}{13}$

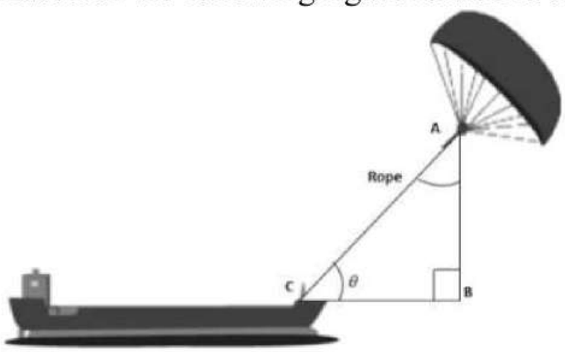
iv. King or queen = 4 + 4 = 8,So m=8

P (either a king or a queen) = $\frac{m}{n} = \frac{8}{52} = \frac{2}{13}$

Section E

36. **Read the text carefully and answer the questions:**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively. Based on the following figure related to sky sailing, answer the following questions:



(i) $\sin\theta = \cos(\theta - 30^{o})$

$\cos(90^{o} - \theta) = \cos(\theta - 30^{o})$

$\Rightarrow 90^{\circ} - \theta = \theta - 30^{\circ}$

$\Rightarrow \theta = 60^{\circ}$

(ii) $\frac{AB}{AC} = \sin 60^{\circ}$

$\therefore$ Length of rope, AC = $\frac{AB}{sin60^{o}} = \frac{200}{\frac{\sqrt{3}}{2}} = \frac{200\times 2}{\sqrt{3}}$ = 230.94 m

(iii) $\sin\theta = \cos(3\theta - 30^{o})$

$\cos(90^{o} - \theta) = \cos(3\theta - 30^{o})$

$\Rightarrow 90^{\circ} - \theta = 3\theta - 30^{\circ} \Rightarrow \theta = 30^{\circ}$

OR

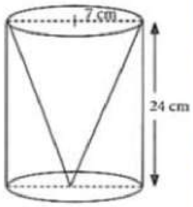
$\frac{AB}{AC} = \sin 30^{\circ}$

$\therefore$ Length of rope, AC = $\frac{AB}{sin30^{o}} = \frac{150}{\frac{1}{2}} = 150 \times 2$ = 300 m

37. **Read the text carefully and answer the questions:**

One day Vinod was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder.

The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Vinod's mind.

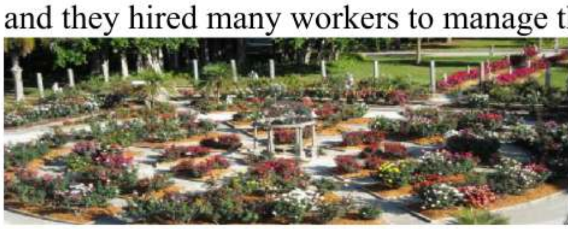


(i) Given height of cone = 24cm and radius of base = r = 7cm
Slant height of conical cavity,
$$l = \sqrt{h^2 + r^2}$$
$$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625} = 25 \text{ cm}$$
(ii) we know that r = 7cm, l = 25 cm
Curved surface area of conical cavity = πrl
$$= \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$$

OR

Curved surface area of conical cavity = πrl
$$= \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$$
External curved surface area of cylinder
$$= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 24 = 1056 \text{ cm}^2$$
$$\frac{\textit{curved surface area of cone}}{\textit{curved surface area of cylinder}} = \frac{550}{1056} = \frac{275}{528}$$
hence required ratio = 275:528
(iii) For cylinder height = h = 24cm, radius of base = r = 7cm
External curved surface area of cylinder
$$= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 24 = 1056 \text{ cm}^2$$

38. **Read the text carefully and answer the questions:**
Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.



In their garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row.

(i) The number of rose plants in the 1^{st}, 2^{nd}, ... are 23, 21, 19, ... 5

$a = 23$, $d = 21 - 23 = -2$, $a_n = 5$

$\therefore\ a_n = a + (n - 1)d$

or, $5 = 23 + (n - 1)(-2)$

or, $5 = 23 - 2n + 2$

or, $5 = 25 - 2n$

or, $2n = 20$

or, $n = 10$

(ii) Total number of rose plants in the flower bed,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{10} = \frac{10}{2}[2(23) + (10 - 1)(-2)]$$

$S_{10} = 5[46 - 20 + 2]$

$S_{10} = 5(46 - 18)$

$S_{10} = 5(28)$

$S_{10} = 140$

OR

$S_n = 80$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow\ 80 = \frac{n}{2}[2 \times 23 + (n - 1) \times -2]$$

$\Rightarrow\ 80 = 23n - n^2 + n$

$\Rightarrow\ n^2 - 24n + 80 = 0$

$\Rightarrow\ (n - 4)(n - 20) = 0$

$\Rightarrow\ n = 4$ or $n = 20$

$n = 20$ not possible

$a_{20} = 23 + 19 \times (-2) = -15$

Number of plants cannot be negative.

$n = 4$

(iii) $a_n = a + (n - 1)d$

$\Rightarrow\ a_6 = 23 + 5 \times (-2)$

$\Rightarrow\ a_6 = 13$