SAMPLE QUESTION PAPER (STANDARD) - 04

Class 10 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. $\triangle$ABC is a right triangle right-angled at A and $AD \perp BC$, Then $\frac{BD}{DC} =$ **[1]**

   a) $\frac{AB}{AD}$ b) $\frac{AB}{AC}$

   c) $\left(\frac{AB}{AD}\right)^2$ d) $\left(\frac{AB}{AC}\right)^2$

2. If α, β are the zeros of the polynomial $x^2 + 6x + 2$ then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ = ? **[1]**

   a) -3 b) -12

   c) 3 d) 12

3. The pair of equations 5x – 15y = 8 and 3x – 9y = $\frac{24}{5}$ has **[1]**

   a) infinitely many solutions b) no solution

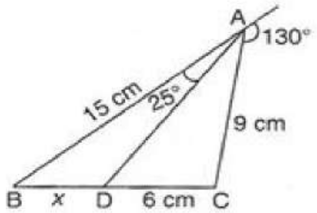
   c) two solutions d) one solution

4. The larger of the two supplementary angles exceeds the smaller by 18^o. The smaller angle is **[1]**

   a) 99^o b) 81^o

   c) 100^o d) 180^o

5. Find the value of a given figure. **[1]**



a) 12 cm.

b) 6 cm.

c) 10 cm.

d) 15 cm.

6. The probability of guessing the correct answer to a certain test questions is $\frac{x}{12}$. If the probability of not guessing the correct answer to this question is $\frac{2}{3}$, then x = **[1]**

a) 6

b) 4

c) 2

d) 3

7. $\frac{\sin\theta}{1+\cos\theta}$ is equal to **[1]**

a) $\frac{1-\sin\theta}{\cos\theta}$

b) $\frac{1-\cos\theta}{\cos\theta}$

c) $\frac{1-\cos\theta}{\sin\theta}$

d) $\frac{1+\cos\theta}{\sin\theta}$

8. If the mean of observations $x_1, x_2, ..., x_n$ is $\overline{x}$, then the mean of $x_1 + a, x_2 + a, ..., x_n + a$ is: **[1]**

a) $\overline{x}$ - a

b) $\frac{\overline{x}}{a}$

c) a$\overline{x}$

d) $\overline{x}$ + a

9. In △ABC and △DEF, it is given that ∠B = ∠E, ∠F = ∠C and AB = 3 DE, then the two triangles are **[1]**

a) congruent but not similar

b) similar but not congruent

c) neither congruent nor similar

d) similar as well as congruent

10. $(2+\sqrt{2})$ is **[1]**

a) none of these

b) an integer

c) a rational number

d) an irrational number

11. The discriminant of the quadratic equation $4x^2 - 6x + 3 = 0$ is: **[1]**

a) 12

b) -12

c) $2\sqrt{3}$

d) 84

12. The distance between the points (sinθ, cosθ) and (cosθ, – sinθ) is **[1]**

a) $\sqrt{2}$ units

b) 2 units

c) $2\sqrt{2}$ units

d) $\sqrt{\sin\theta+\cos\theta}$ units

13. If $\sum f_i u_i$ = -7, $\sum f_i$ = 25, a = 225 and h = 50, then the value of $\overline{x}$ is **[1]**

a) 211

b) 212

c) 214

d) 213

14. $\cot^2\theta - \frac{1}{\sin^2\theta}$ = **[1]**

a) $\tan^2\theta$

b) $\cot^2\theta$

c) -1

d) 1

15. A ladder 12 m long just reaches the top of a vertical wall. If the ladder makes an angle of 45^{o} with the wall, then the height of the wall is **[1]**

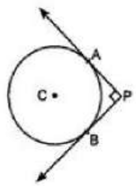
a) 6 m

b) $12\sqrt{2}m$

c) 12 m

d) $6\sqrt{2}m$

16. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA $\perp$ PB, then the length of each tangent is: **[1]**



a) 5 cm

b) 3 cm

c) 4 cm

17. If inΔABC and ΔDEF, $\frac{AB}{DE}=\frac{BC}{FD}$,then they will be similar, when **[1]**

a) $\angle B=\angle D.$

b) $\angle A=\angle D.$

c) $\angle A=\angle F$

d) $\angle B=\angle E.$

18. If x = 3 is a solution of the equation $3x^2 + (k - 1)x + 9 = 0$ then k = ? **[1]**

a) 13

b) -11

c) 11

d) -13

19. **Assertion:** $(2-\sqrt{3})$ is one zero of the quadratic polynomial then other zero will be $(2+\sqrt{3})$. **[1]**
Reason: Irrational zeros (roots) always occurs in pairs.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

20. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 300 cm^2. **[1]**
Reason (R): Total surface area of a cuboid is 2(lb + bh + lh)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the roots of the quadratic equation $x^2-3x - 10 = 0$ by factorization. **[2]**

22. In what ratio does the point (-4,6) divide the line segment joining the points A (-6, 10) and B(3, -8)? **[2]**

23. Can two numbers have 15 as their HCF and 175 as their LCM ? Give reasons. **[2]**

24. Prove: $\sqrt{\frac{1-\cos A}{1+\cos A}}=\csc A-\cot A$ **[2]**

OR

Evaluate : $\frac{\cos 45^\circ}{\sec 30^\circ} + \frac{1}{\sec 60^\circ}$.

25. D, E and F are the points on sides BC, CA and AB respectively of ΔABC such that AD bisects $\angle$A, BE bisects $\angle$B and CF bisects $\angle$C. If AB = 5 cm, BC = 8 cm and CA = 4 cm, determine AF, CE and BD. **[2]**

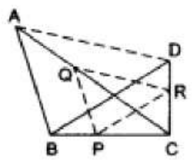
OR

ABC is an isosceles triangle, in which AB = AC, circumscribed about a circle. Show that BC is bisected at the point of contact.

Section C

26. The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot by using the quadratic formula. **[3]**

27. $\triangle ABC$ and $\triangle DBC$ lie on the same side of BC, as shown in the figure. From a point P on BC, $PQ \| AB$ and $PR \| BD$ are drawn meeting AC at Q and CD at R respectively. Prove that $QR \| AD$. **[3]**



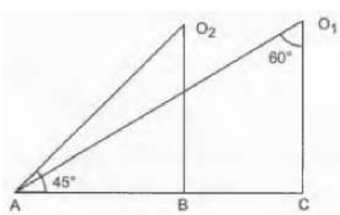
28. The vertices of a $\triangle$ABC are A (5,5), B (1,5) and C (9,1). A line is drawn to intersect sides AB and AC at P and Q respectively, such that $\frac{AP}{AB} = \frac{AQ}{AC} = \frac{3}{4}$. Find the length of the line segment PQ. **[3]**

OR

If (0, -3) and (0, 3) are the two vertices of an equilateral triangle, find the coordinates of its third vertex.

29. Prove that $(3 + \sqrt{2})$ is irrational. **[3]**

30. What are the angles of depression from the observing positions O_1 and O_2 of the object at A? **[3]**



OR

A tower is 50m high. Its shadow is x metres shorter when the sun's altitude is 45° than when it is 30°. Find the value of x. [Given $\sqrt{3}$ = 1.732.]

31. Weekly income of 600 families is given below : **[3]**

Income (in Rs)	0 -1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000
No. of Families	250	190	100	40	15	5

Find the median.

Section D

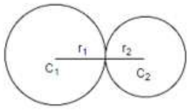
32. If twice the son's age in years is added to the father's age, the sum is 70. But if twice the father's age is added to the son's age, the sum is 95. Find the ages of father and son. **[5]**

OR

The student of the class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

33. O is the centre of a circle. PA and PB are tangents to touch the circle from a point P. Prove that (i) quadrilateral PAOB is a cyclic quadrilateral (ii) PO is the bisector of $\angle$APB (iii) $\angle$OAB = $\angle$OPA. **[5]**

34. Two farmers have circular plots. The plots are watered with the same water source placed in the point common to both the plots as shown in the figure. The sum of their areas is 130π and the distance between their centres is 14 m.Find the radii of the circles.What value is depicted by the farmers? **[5]**



OR

A chord of a circle of radius 10cm subtends a right angle at the center. Find the area of the corresponding: (Use π = 3.14)

i. minor sector
ii. major sector
iii. minor segment
iv. major segment

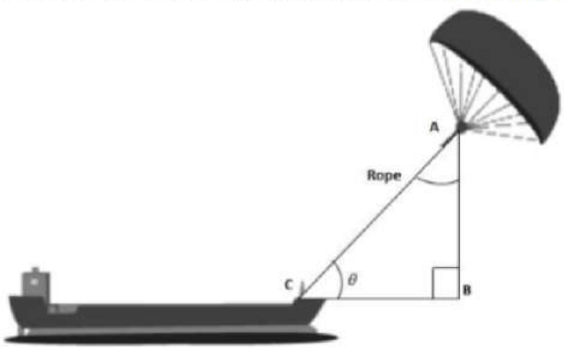
35. A bag contains 15 balls of which x are blue and the remaining are red. If the number of red balls are increased by 5, the probability of drawing **[5]**
the red balls doubles. Find :

i. P(red ball)
ii. P(blue ball)
iii. P(blue ball if of 5 extra red balls are actually added)

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.
Based on the following figure related to sky sailing, answer the following questions:



(i) In the given figure, if sin θ = cos(θ - 30°), where θ and θ - 30° are acute angles, then find the value of θ.

(ii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200m?

(iii) In the given figure, if sin θ = cos(3θ - 30°), where θ and 3θ - 30° are acute angles, then find the value of θ.

OR

What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 150m?

37. **Read the text carefully and answer the questions:** **[4]**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find an increase in the production of TV every year.

(ii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find in which year production of TV is 1000.

OR

They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the total production in first 7 years.

(iii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in the 10th year.

38. **Read the text carefully and answer the questions:** **[4]**

An ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day Sheetal and her brother came to his shop. Sheetal purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm. her brother purchased rectangular brick shaped ice cream with length 9 cm, width 4cm and thickness 2 cm.

(i) The volume of the ice-cream without hemispherical end.

(ii) The volume of the ice-cream with a hemispherical end.

OR

Whose quantity of ice cream is more and by how much?

(iii) Find the volume her brother ice cream?

Solution

SAMPLE QUESTION PAPER (STANDARD) - 04

Class 10 - Mathematics

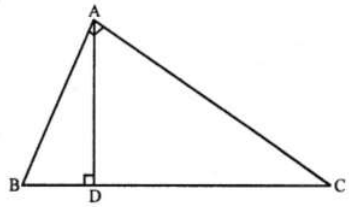
Section A

1. **(d)** $\left(\frac{AB}{AC}\right)^2$

Explanation: In right angled $\triangle$ABC, $\angle A = 90^\circ$

AD $\perp$ BC

$\therefore \triangle ABD \sim \triangle ABC$



$\frac{AB}{BC} = \frac{BD}{AB} \Rightarrow AB^2 = BD \times BC$ .....(i)

Similarly $\triangle$ACD $\sim$ ΔABC

DC$\times$BC = AC^2 ....(ii)

Dividing (ii) by (i)

$\frac{BD \times BC}{DC \times BC} = \frac{AB^2}{AC^2} \Rightarrow \frac{BD}{DC} = \frac{AB^2}{AC^2}$

Hence $\frac{BD}{DC} = \frac{AB^2}{AC^2}$

2. **(a)** -3

Explanation: $\alpha + \beta = -6$ and $\alpha\beta = 2$

$\therefore \quad \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \frac{(\alpha+\beta)}{\alpha\beta} = \frac{-6}{2} = -3$

3. **(a)** infinitely many solutions

Explanation: Given: $a_1 = 5, a_2 = 3, b_1 = -15, b_2 = -9, c_1 = 8$ and $c_2 = \frac{24}{5}$ Here

$\frac{a_1}{a_2} = \frac{5}{3}, \frac{b_1}{b_2} = \frac{-15}{-9} = \frac{5}{3}, \frac{c_1}{c_2} = \frac{8}{\frac{24}{5}} = \frac{5}{3} \quad \because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Since all have the same answer $\frac{5}{3}$.

Therefore, the pair of given linear equations has infinitely many solutions.

4. **(b)** 81^o

Explanation: Let larger of the two supplementary angles be x and smaller be y

According to question, x + y = 180^o ... (i)

And x = y + 18^o

$\Rightarrow$ x - y = 18^o ... (ii)

Subtracting eq. (ii) from eq. (i),

we get 2y = 162^o

$\Rightarrow$ y = 81^o

Therefore, the smaller angle is 81^o

Putting the value of y in equation 1

x + 81^o = 180^o

x = 180^o - 81^o

x = 99^o, which is a larger angle.

5. **(c)** 10 cm.

Explanation: Here, $\angle$CAD = $180^\circ - (130^\circ + 25^\circ)$ = 25^o

Now, since $\angle$CAD = $\angle$DAB, therefore, the AD is the bisector of $\angle$BAC.

Since the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{\mathrm{BD}}{\mathrm{DC}}=\frac{\mathrm{AB}}{\mathrm{AC}} \Rightarrow \frac{x}{6}=\frac{15}{9}$

$\Rightarrow x=\frac{15\times 6}{9}$ = 10 cm

6. **(b)** 4

Explanation: Probability of guessing the correct answer

$=\frac{x}{12}$

and probability of not guessing the correct answer $=\frac{2}{3}$

$\frac{x}{12}+\frac{2}{3}=1 \because (A+\bar{A}=1)$

$\Rightarrow \frac{x}{12}=1-\frac{2}{3}=\frac{1}{3} \Rightarrow x=\frac{12}{3}=4$

$\therefore$ x = 4

7. **(c)** $\frac{1-\cos\theta}{\sin\theta}$

Explanation: We have, $\frac{\sin\theta}{1+\cos\theta}=\frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$=\frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}=\frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}$

$=\frac{1-\cos\theta}{\sin\theta}$

8. **(d)** $\overline{x}$ + a

Explanation: Mean of observations $x_1, x_2, ..., x_n$ is $\overline{x}$

i.e, $\bar{x}=\frac{x_1+x_2+x_3\cdots+x_n}{n}$

Now, $(x_1 + a) + (x_2 + a) + (x_3 + a) + ... +(x_n + a)$

$= x_1 + x_2 + x_3 + ... x_n + na$

$\therefore$ Mean of $(x_1 + a), (x_2 + a), (x_3 + a), ..., (x_n + a)$

$=\frac{(x_1+x_2+x_3\ldots+x_n)+na}{n}$

$=\frac{(x_1+x_2+x_3\ldots+x_n)}{n}+\frac{na}{n}$

$=\bar{x}+\frac{na}{n}=\bar{x}$ + a

9. **(b)** similar but not congruent

Explanation: In $\triangle$ABC and $\triangle$DEF,

$\angle$B = $\angle$E, $\angle$F = $\angle$C and AB = 3DE

The triangles are similar as two angles are equal but including sides are not proportional.

10. **(d)** an irrational number

Explanation: $(2+\sqrt{2})$ is an irrational number.

If it is rational, then the difference of two rational is rational.

$\therefore (2+\sqrt{2})-2=\sqrt{2}$ = irrational, which is a contradiction.

Hence, $(2+\sqrt{2})$, is an irrational number.

11. **(b)** -12

Explanation: $4x^2 - 6x + 3 = 0$

a = 4, b = -6, c = 3

$D = b^2 - 4ac$

$=(-6)^2 - 4(4)(3)$

= 36 - 48

= -12

12. **(a)** $\sqrt{2}$ units

Explanation: Distance between $(\sin\theta, \cos\theta)$ and $(\cos\theta, -\sin\theta)$

$=\sqrt{(\cos\theta-\sin\theta)^2+(-\sin\theta-\cos\theta)^2}$

$=\sqrt{\cos^2\theta+\sin^2\theta-2\cos\theta\sin\theta+\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta}$

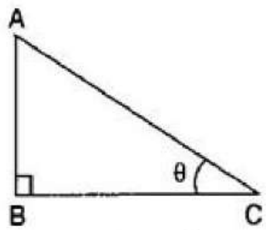
$=\sqrt{2\cos^2\theta+2\sin^2\theta}$

$=\sqrt{2\left(\cos^2\theta+\sin^2\theta\right)}$

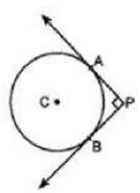
$[\because \cos^2\theta + \sin^2\theta = 1]$
= $\sqrt{2}$ units

13. **(a)** 211
Explanation: $\overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$
= 225 + $\frac{-7}{25} \times 50$
= 225 - 14
= 211

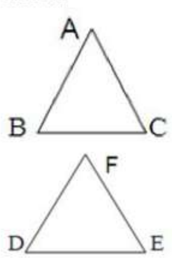
14. **(c)** -1
Explanation: Given: $\cot^2\theta$ - $\frac{1}{\sin^2\theta}$
= $\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$
= $\frac{\cos^2\theta - 1}{\sin^2\theta}$
= $\frac{-\sin^2\theta}{\sin^2\theta}$ = -1
[$\because$ 1 - $\cos^2\theta$ = $\sin^2\theta$]

15. **(d)** $6\sqrt{2}m$
Explanation: 

Let the height of the top of the ladder reaches to a vertical wall = AB
The length of the ladder = AC = 12 m
The angle of elevation = $\theta = 45^\circ$
$\therefore \sin 45^\circ = \frac{\text{AB}}{\text{AC}}$
$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\text{AB}}{12}$
$\Rightarrow \text{AB} = \frac{12}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$\Rightarrow \text{AB} = 6\sqrt{2}$ m

16. **(c)** 4 cm
Explanation: 

Construction: Joined AC and BC. Here CA $\perp$ AP and CB $\perp$ BP and PA $\perp$PB Also AP = PB
Therefore, BPAC is a square. $\Rightarrow$ AP = PB = BC = 4 cm

17. **(a)** $\angle B = \angle D$.
Explanation: In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then if, $\angle B = \angle D$ (the included angles) are equal then the triangles are similar


18. **(b)** -11
Explanation: $3x^2 + (k-1)x + 9 = 0$
x = 3 is a solution of the equation means it satisfies the equation

Put x = 3, we get

$3(3)^2 + (k - 1) 3 + 9 = 0$

$27 + 3k - 3 + 9 = 0$

$27 + 3k + 6 = 0$

$3k = -33$

$k = -11$

19. **(a)** Assertion and reason both are correct statements and reason is correct explanation for assertion.

Explanation: As irrational roots/zeros always occurs in pairs, therefore, when one zero is $(2-\sqrt{3})$ then other will be $(2+\sqrt{3})$. So, both Assertion and Reason are correct and Reason explains Assertion.

20. **(d)** A is false but R is true.

Explanation: A is false but R is true.

Section B

21. $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x-5) + 2(x-5) = 0$

$\Rightarrow (x-5)(x+2) = 0$

$\Rightarrow x = 5, -2$

22. Let (-4, 6) divide AB internally in the ratio k:1. Using the section formula, we get

$(-4,6)=\left(\frac{3k-6}{k+1},\frac{-8k+10}{k+1}\right)$

So, $-4=\frac{3k-6}{k+1}$

i.e., $-4k - 4 = 3k - 6$

i.e., $7k = 2$

i.e., k:1 = 2:7

The same can be checked for the y-coordinate also.

Therefore, the ratio in which the point (-4,6) divides the line segment AB is 2: 7.

23. $\frac{175}{15}=11.667$

Hence 175 is not divisible by 15

But LCM of two numbers should be divisible by their HCF.

$\therefore$ Two numbers cannot have their HCF as 15 and LCM as 175.

24. $\sqrt{\frac{1-\cos A}{1+\cos A}}=cosecA-\cot A$

L.H.S. $=\sqrt{\frac{1-\cos A}{1+\cos A}}$

$=\sqrt{\frac{(1-\cos A)\times(1-\cos A)}{(1+\cos A)\times(1-\cos A)}}$ [Multiplying and dividing by (1 - cos A)]

$=\frac{1-\cos A}{\sqrt{1-\cos^2 A}}=\frac{1-\cos A}{\sin A}$ Since, $\sqrt{1-\cos^2 A}=\sin A$

$=cosecA-\cot A\left[\because \frac{1}{\sin A}=cosecA, \frac{\cos A}{\sin A}=\cot A\right]$

= R.H.S. proved

OR

$\frac{\cos 45^\circ}{\sec 30^\circ}+\frac{1}{\sec 60^\circ}$

$=\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}}+\frac{1}{2}$

$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{2}$

$=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2}$

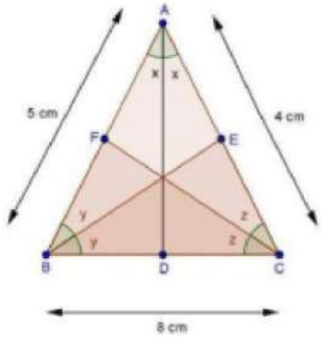
Rationalise the denominator,

$=\frac{\sqrt{3}}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}+\frac{1}{2}$

$=\frac{\sqrt{6}}{4}+\frac{1}{2}$

25. It is given that D, E and F are the points on sides BC, CA and AB respectively of ΔABC such that AD bisects $\angle$A, BE bisects $\angle$B and CF bisects $\angle$C.

Also AB = 5 cm, BC = 8 cm and CA = 4 cm



We know that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{AF}{FB} = \frac{AC}{BC}$

$\Rightarrow \frac{AF}{5-AF} = \frac{4}{8}$ [∵ FB = AB - AF = 5 - AF]

$\Rightarrow \frac{AF}{5-AF} = \frac{1}{2}$

⇒ 2AF = 5 - AF

⇒ 2AF + AF = 5

⇒ 3AF = 5

$\Rightarrow AF = \frac{5}{3} cm$

Again, in $\triangle ABC$, BE bisects $\angle B$

$\therefore \frac{AE}{EC} = \frac{AB}{BC}$

$\Rightarrow \frac{4-CE}{CE} = \frac{5}{8}$ [∴ AE = AC - CE = 4 - CE]

⇒ 8(4 - CE) = 5 × CE

⇒ 32 - 8CE = 5CE

⇒ 32 = 13CE

$\Rightarrow CE = \frac{32}{13}$cm

Similarly,

$\frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{BD}{8-BD} = \frac{5}{4}$ [∴ DC = BC - BD = 8 - BD]

⇒ 48D = 5(8 - 8D)

⇒ 48D = 40 - 58D

⇒ 98D = 40

$\Rightarrow BD = \frac{40}{9}$cm

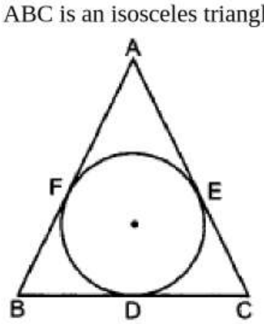
Hence, $AF = \frac{5}{3} cm$,

$CE = \frac{32}{13}$cm

and $BD = \frac{40}{9} cm$

OR

ABC is an isosceles triangle.



According to the question, $AB = AC$ ...(i)

$AF = AE$ (Tangents from A) ...(ii)

$AB - AF = AC - AE$

$\Rightarrow BF = CE$ ...(iii)

Now, $BF = BD$ (Tangents from B)

Also,$CE = CD$ (Tangents from C)

$\Rightarrow BD = CD$

So, BC is bisected at the point of contact.

Section C

26. Let the breadth of the plot be x metres.

Then the length is (2x + 1) metres

Then we are given that x(2x + 1) = 528, i.e., $2x^2 + x - 528 = 0$.

This is of the form $ax^2 + bx + c = 0$, where a = 2, b = 1, c = – 528.

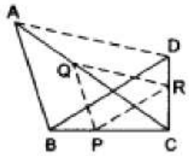
So, the quadratic formula gives us the solution as

$x = \frac{-1 \pm \sqrt{1+4(2)(528)}}{4} = \frac{-1 \pm \sqrt{4225}}{4} = \frac{-1 \pm 65}{4}$

i.e., $x = \frac{64}{4}$ or $x = \frac{-66}{4}$

i.e., x = 16 or x = $-\frac{33}{2}$

Since x cannot be negative, being a dimension, the breadth of the plot is 16 metres and hence, the length of the plot is 33m.

27. 

It is given that In $\triangle$ CAB, $\mathrm{PQ} \parallel \mathrm{AB}$.

By Applying the Thales' theorem to $\triangle$ CAB , we obtain

$\frac{CP}{PB} = \frac{CQ}{QA}$ ....(i)

Similarly, by applying Thales theorem in $\triangle BDC$, where $\mathrm{PR} \parallel \mathrm{DM}$ we get:

$\frac{CP}{PB} = \frac{CR}{RD}$ ....(ii)

Hence, from (i) and (ii) we have

$\frac{CQ}{QA} = \frac{CR}{RD}$

Applying the converse of Thales' theorem, we conclude that $\mathrm{QR} \parallel \mathrm{AD}$ in $\triangle ADC$.

28. Given: The vertices of a $\triangle$ABC are A (5,5), B (1,5) and C (9,1)

We have,

$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{3}{4}$

$\Rightarrow \quad \frac{AP}{AP+PB} = \frac{AQ}{AQ+QC} = \frac{3}{4}$

$\Rightarrow \frac{AP}{AP+PB} = \frac{3}{4}, \frac{AQ}{AQ+QC} = \frac{3}{4}$

$\Rightarrow$ 4AP = 3AP + 3PB and 4AQ = 3AQ + 3 QC

$\Rightarrow$AP = 3PB and AQ = 3QC

$\Rightarrow \quad \frac{AP}{PB} = \frac{3}{1}$ and $\frac{AQ}{QC} = \frac{3}{1}$

$\Rightarrow$P and Q divide AB and AC respectively in the same ratio 3 : 1

Thus, the coordinates of P and Q are

$\left(\frac{3\times1+1\times5}{3+1}, \frac{3\times5+1\times5}{3+1}\right)$=(2, 5) and $\left(\frac{3\times9+1\times5}{3+1}, \frac{3\times1+1\times5}{3+1}\right)$ = (8, 2)

$\therefore \quad PQ = \sqrt{(2-8)^2 + (5-2)^2} = \sqrt{45} = 3\sqrt{5}$ units

OR

Let the coordinates of third vertex be (x,y)

Each length of equilateral triangle = $\sqrt{(0-0)^2 + (3+3)^2} = \sqrt{6^2} = 6$

Since the triangle is equilateral, therefore length of each side = 6.

Thus, Distance between (x, y) and (0, 3) = 6

$\sqrt{(x-0)^2 + (y-3)^2} = 6$

$(x - 0)^2 + (y - 3)^2 = 36$

$x^2 + (y - 3)^2 = 36$ .... (i)

Also, Distance between (x, y) and (0, -3) = 6

$\sqrt{(x-0)^2 + (y+3)^2} = 6$

$x^2 + (y + 3)^2 = 36$ .... (ii)

From (i) and (ii), we get,

$x^2 + (y - 3)^2 = x^2 + (y + 3)^2$

$\Rightarrow 12y = 0$

$\Rightarrow y = 0$

From (i), $x^2 + (0 - 3)^2 = 36$

$x^2 = 36 - 9 = 27$

$x = \pm 3\sqrt{3}$

Thus, the third vertex is $(\pm 3\sqrt{3}, 0)$

29. If possible let $3 + \sqrt{2}$ is rational number, and we take another rational number 3 for our calculation.

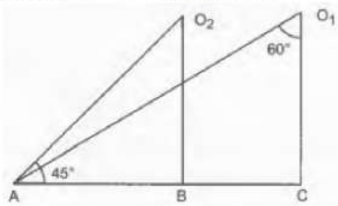
$\Rightarrow (3 + \sqrt{2}) - 3 = \sqrt{2}$ (difference of two rational number is a rational number)

$\therefore \sqrt{2}$ is rational

This contradicts the fact that $\sqrt{2}$ is irrational

Since the contradiction arises by assuming that $3 + \sqrt{2}$ is rational.

Hence, $3 + \sqrt{2}$ is irrational.

30. 

In triangle O_1AC,

$\Rightarrow \angle A = 180^o - (90^o + 60^o)$

$\Rightarrow \angle A = 180^o - 150^o$ We know that $[\angle A + \angle B + \angle C = 180^o]$

$\Rightarrow \angle A = 30^o$

Again,

In a triangle O_2AB,

$\Rightarrow \angle O_2 = 180^o - (90^o + 45^o)$

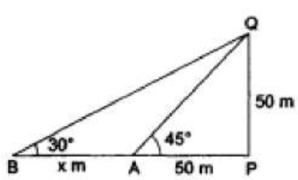
$\Rightarrow \angle O_2 = 180^o - 135^o$

$\Rightarrow \angle O_2 = 45^o$

Hence the required angles are 30^o, 45^o.

OR

Let PQ be the tower and let PA and PB be its shadows when the altitudes of the sun are 45° and 30° respectively. Then,



$\angle PAQ = 45^\circ, \angle PBQ = 30^\circ, \angle BPQ = 90^\circ, PQ = 50\text{m}.$

Let $AB = x\ m.$

From right ΔAPQ, we have

$\frac{AP}{PQ} = \cot 45^\circ = 1$

$\Rightarrow \frac{AP}{50\text{m}} = 1 \Rightarrow AP = 50\text{m}.$

From right ΔBPQ, we have

$\frac{BP}{PQ} = \cot 30^\circ = \sqrt{3} \Rightarrow \frac{x+50}{50} = \sqrt{3} \Rightarrow \quad x = 50(\sqrt{3} - 1).$

$\Rightarrow x = 50(1.732 - 1) = (50 \times 0.732) = 36.6$

Hence, $x = 36.6$

31.

Income	No. of Families	c.f.
$0 - 1000$	250	250
$1000 - 2000$	190	$250 + 190 = 440$
$2000 - 3000$	100	$440 + 100 = 540$

$3000-4000$	40	$540+40=580$
$4000-5000$	15	$580+15=595$
$5000-6000$	5	$595+5=600$

Here, N = 600

$\Rightarrow$ Median $=\frac{N}{2}$ th term

$=\frac{600}{2}=$ 300th term

So, Median class = 1000 - 2000

$l=1000,\ h=1000,\ c.f.=250,\ f=190$

Median = $l+\left(\frac{\frac{N}{2}-c.f}{f}\right)\times h$

Median = $1000+\left(\frac{300-250}{190}\right)\times 1000$

= 1000 + $\frac{50}{190}\times 1000$

= 1000 + $\frac{5000}{19}$

$=1000+263.16$

$=1263.16$

Median = Rs 1263.16

Section D

32. Let father's age (in years) be x and that of son's be y.

$\Rightarrow x+2y=70$ (by first condition)

$2x+y=95$ (by second condition)

This system of equations may be written as

$x+2y-70=0$

$2x+y-95=0$

By cross-multiplication, we get

$\frac{x}{2\times-95-(-70)}=\frac{-y}{1\times-95-2\times-70}=\frac{1}{1\times1-2\times2}$

$\Rightarrow \frac{x}{-190+70}=\frac{-y}{-95+140}=\frac{1}{-3}$

$\Rightarrow \frac{x}{-120}=\frac{y}{-45}=\frac{1}{-3}\Rightarrow x=\frac{-120}{-3}=40$ and $y=\frac{-45}{-3}=15$

$\therefore$ father's age is 40 years and the son's age is 15 years.

OR

Let the number of students in the class is x and the number of rows is y.

Then, the number of students in each row.= $\frac{x}{y}$

According to the question-

If 3 students are extra in a row. Then there would be 1 row less, i.e., when each row has $\left(\frac{x}{y}+3\right)$ students. Then the number of rows is (y - 1).

$\therefore$ Total number of students = number of rows × Number of students in each row

$\Rightarrow x=\left(\frac{x}{y}+3\right)(y-1)\Rightarrow x=x-\frac{x}{y}+3y-3$

$\Rightarrow \frac{x}{y}-3y+3=0$ ...(1)

And, if three students are less in a row, then there would be 2 rows more, i.e., when each row has $\left(\frac{x}{y}-3\right)$ students, then the number of rows is (y + 2).

$\therefore$ Total number of students = Number of rows × Number of students in each row

$\Rightarrow x=\left(\frac{x}{y}-3\right)(y+2)\Rightarrow x=x+\frac{2x}{y}-3y-6$

$\Rightarrow \frac{2x}{y}-3y-6=0$ ...(2)

Put $\frac{x}{y}=u$ ...(3)

Then equation (1) and (2) can be rewritten as

u - 3y + 3 = 0 ...(4)

2u - 3v - 6 = 0 ...(5)

Subtracting equation (4) from equation (5),

we get u - 9 = 0

$\Rightarrow$ u = 9 ...(6)

Substituting this value of x in equation (4),

We get 9 - 3y + 3 = 0

$\Rightarrow$ -3y + 12 = 0 $\Rightarrow$ 3y = 12

$\Rightarrow y = \frac{12}{3} = 4$ ...(7)

From equation (3) and equation (6), we get $\frac{x}{y} = 9$

$\Rightarrow \frac{x}{4} = 9$ [using (7)] $\Rightarrow$ x = 36 ...(8)

So, the solution of the equations (1) and (2)

is x = 36 and y = 4.

Hence, the number of students in the class

= xy = (36)(4) = 144.

Verification :

Substituting x = 36, y = 4,

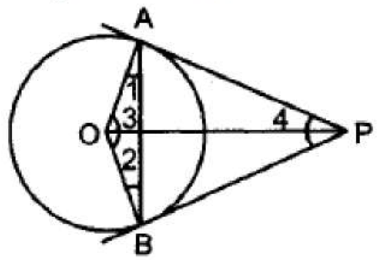
we find that both the equations (1) an (2)

are satisfied as shown below:

$\frac{x}{y} - 3y + 3 = \frac{36}{4} - 3(4) + 3$ = 9 - 12 + 3 = 0

$\frac{2x}{y} - 3y - 6 = \frac{2(36)}{4} - 3(4) - 6$ = 18 - 12 - 6 = 0

Hence, the solution is correct.

33. 

Given,O is the centre of a circle. PA and PB are tangents to touch the circle from a point P.

i. PA and PB are tangents to the circle at the points A and B respectively.

$\Rightarrow \angle OAP = \angle OBP = 90^\circ$ ..(i)

$\Rightarrow \angle OAP + \angle OBP = 90^\circ + 90^\circ = 180^\circ$ ...(ii)

In quadrilateral OAPB,

$\angle AOB + \angle OAP + \angle OBP + \angle APB = 360^\circ$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle AOB + \angle APB + 180^\circ = 360^\circ$ [From (ii)]

$\Rightarrow \angle APB + \angle AOB$

= 360°-180° = 180° ...(iii)

From (ii) and (iii), we have

$\angle OAP + \angle OBP = 180^\circ$

and $\angle APB + \angle AOB = 180^\circ$

ii. In $\triangle$OAP and $\triangle$OBP, r

AP = BP [Tangents from the same external point are equal]

OP = OP [Common]

OA = OB [Radii of the same circle]

$\Rightarrow \Delta OAP \cong \triangle OBP$ [Using SSS cong.]

$\Rightarrow \angle APO = \angle BPO$ [CPCT]

$\Rightarrow$ PO bisects $\angle$APB

iii. $\angle 3 + \angle 4 = 180^\circ$

$\angle 1 + \angle 2 + \angle 3 = 180^\circ$ [Angle sum property] ...(iv)

In $\triangle$OAB,

OA =OB [Radii]

$\angle 1 = \angle 2$ [Angles opposite to equal sides of a A are equal] ...(v)

From (iii) and (iv),

$\angle 3 + \angle 4 = \angle 1 + \angle 2 + \angle 3$

$\Rightarrow \angle 4 = \angle 1 + \angle 2$
$= \angle 1 + \angle 1$
$= \angle 2 + \angle 2$
$\Rightarrow \angle 4 = 2\angle 1$
$= 2\angle 2$
$\Rightarrow \frac{1}{2}\angle 4 = \angle 1$
$\Rightarrow \angle OPA = \angle OAB$
Hence proved

34. Let the radii of the two circlular plots be r_1 and r_2, respectively.

Then, $r_1 + r_2 = 14$ [$\because$ Distance between the centres of two circlular plots = 14 cm,given]....(i)

Also, Sum of Areas of the plots = 130π

$\therefore \pi r_1^2 + \pi r_2^2 = 130\pi \Rightarrow r_1^2 + r_2^2 = 130$ ...(ii)

Now, from equation (i) and equation (ii),

$\Rightarrow (14 - r_2)^2 + r_2^2 = 130$

$\Rightarrow 196 - 2r_2 + 2r_2^2 = 130$

$\Rightarrow 66 - 2r_2 + 2r_2^2 = 0$

Solving the quadratic equation we get,

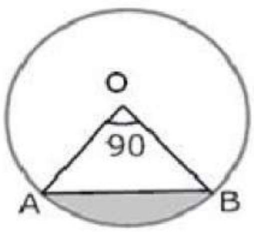
$r_2 = 3$ or $r_2 = 11$,

but from figure it is clear that, $r_1 > r_2$

$\therefore r_1 = 11$ cm and $r_2 = 3$ cm

The value depicted by the farmers are of cooperative nature and mutual understanding.

OR



i. Area of minor sector = $\frac{\theta}{360}\pi r^2$
$= \frac{90}{360}(3.14)(10)^2$
$= \frac{1}{4} \times 3.14 \times 100$
$= \frac{314}{4}$
$= 78.50 = 78.5$ cm^2

ii. Area of major sector = Area of circle - Area of minor sector
$= \pi(10)^2 - \frac{90}{360}\pi(10)^2 = 3.14 (100) - \frac{1}{4}(3.14)(100)$
$= 314 - 78.50 = 235.5$ cm^2

iii. We know that area of minor segment
= Area of minor sector OAB - Area of ΔOAB
$\because$ area of $\triangle OAB = \frac{1}{2}(OA)(OB)\sin\angle AOB$
$= \frac{1}{2}(OA)(OB)\ (\because \angle AOB = 90^\circ)$
Area of sector = $\frac{\theta}{360}\pi r^2$
$= \frac{1}{4}(3.14)(100) - 50 = 25(3.14) - 50 = 78.50 - 50 = 28.5$ cm^2

iv. Area of major segment = Area of the circle - Area of minor segment
$= \pi(10)^2 - 28.5$
$= 100(3.14) - 28.5$
$= 314 - 28.5 = 285.5$ cm^2

35. initially P(red ball)=$\frac{15-x}{15}$

and after adding 5 red balls P(red ball)=$\frac{20-x}{20}$ So as per the question

$\frac{20-x}{20} = 2 \times (\frac{15-x}{15})$

$\frac{20-x}{4} = 2 \times (\frac{15-x}{3})$

$60 - 3x = 120 - 8x$

$5x = 60$

$x = 12$

So Blue balls = 12 and red balls =15-12= 3

i. P(red ball) = $\frac{3}{15} = \frac{1}{5}$

ii. P(blue ball) = $\frac{12}{15} = \frac{4}{5}$

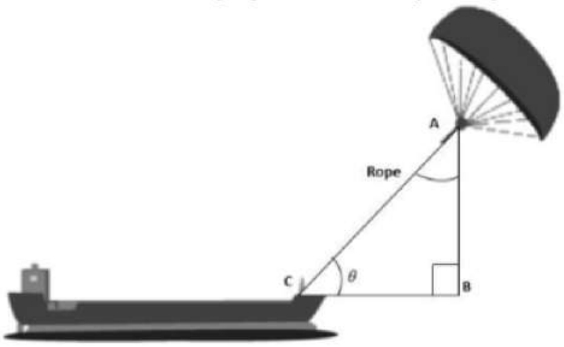
iii. P(blue ball if 5 red balls are added) =$\frac{12}{20} = \frac{3}{5}$

Section E

36. **Read the text carefully and answer the questions:**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the following questions:



(i) $\sin \theta = \cos(\theta - 30^o)$

$\cos (90^o - \theta) = \cos (\theta - 30^o)$

$\Rightarrow 90^\circ - \theta = \theta - 30^\circ$

$\Rightarrow \theta = 60^\circ$

(ii) $\frac{AB}{AC} = \sin 60^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin 60^o}$ = $\frac{200}{\frac{\sqrt{3}}{2}}$ = $\frac{200 \times 2}{\sqrt{3}}$ = 230.94 m

(iii) $\sin \theta = \cos(3\theta - 30^o)$

$\cos(90^o - \theta) = \cos(3\theta - 30^o)$

$\Rightarrow 90^\circ - \theta = 3\theta - 30^\circ \Rightarrow \theta = 30^\circ$

OR

$\frac{AB}{AC} = \sin 30^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin 30^o}$ = $\frac{150}{\frac{1}{2}}$ = 150 × 2 = 300 m

37. **Read the text carefully and answer the questions:**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a'

denotes the production in the first year and d denotes the number of units by which the production increases every year.
We have, a_3 = 600 and

a_3 = 600

⇒ 600 = a + 2d

⇒ a = 600 - 2d ...(i)

⇒ a_7 = 700

⇒ a_7 = 700

⇒ 700 = a + 6d

⇒ a = 700 - 6d ...(ii)

From (i) and (ii)

600 - 2d = 700 - 6d

⇒ 4d = 100

⇒ d = 25

(ii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
We know that first term = a = 550 and common difference = d = 25

a_n = 1000

⇒ 1000 = a + (n - 1)d

⇒ 1000 = 550 + 25n - 25

⇒ 1000 - 550 + 25 = 25n

⇒ 475 = 25n

⇒ n = $\frac{475}{25}$ = 19

OR

Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
Total production in 7 years = Sum of 7 terms of the A.P. with first term a (= 550) and d (= 25).

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (7-1)25]$

$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (6) \times 25]$

$\Rightarrow S_7 = \frac{7}{2}[1100 + 150]$

⇒ S_7 = 4375

(iii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
The production in the 10th term is given by a_{10}. Therefore, production in the 10th year = a_{10} = a + 9d = 550 + 9 × 25 = 775. So, production in 10th year is of 775 TV sets.

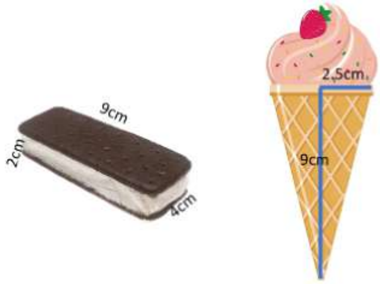
38. **Read the text carefully and answer the questions:**

An ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day Sheetal and her brother came to his shop. Sheetal purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm. her brother purchased rectangular brick shaped ice cream with length 9 cm, width 4cm and thickness 2 cm.


(i) For cone, radius of the base (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

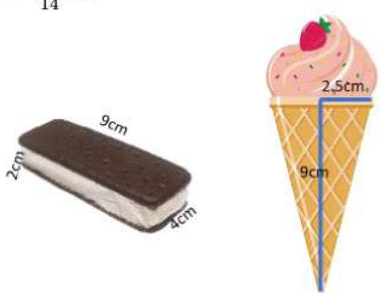
Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$
$= \frac{825}{14}\text{cm}^3$



For hemisphere,
Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$
$\therefore$ Volume = $\frac{2}{3}\pi r^3$
$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$
The volume of the ice-cream without hemispherical end = Volume of the cone
$= \frac{825}{14}\text{cm}^3$

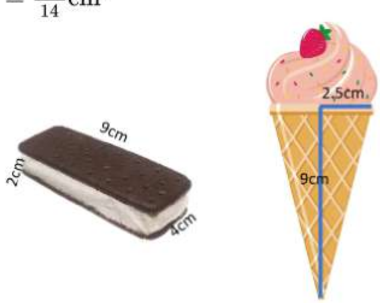
(ii) For cone, radius of the base (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$
Height (h) = 9 cm
$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$
$= \frac{825}{14}\text{cm}^3$



For hemisphere,
Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$
$\therefore$ Volume = $\frac{2}{3}\pi r^3$
$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$
Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere
$= \frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42}$
$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3}\text{cm}^3$

OR

For cone, Radius of the base (r)
$= 2.5\text{cm} = \frac{5}{2}\text{cm}$
Height (h) = 9 cm
$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$
$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

Sheetal ice cream quantity is more than her brother

Volume of Sheeta's ice cream - Volume her brother's ice cream

= 91.66 - 72 = 19.66 cm^3

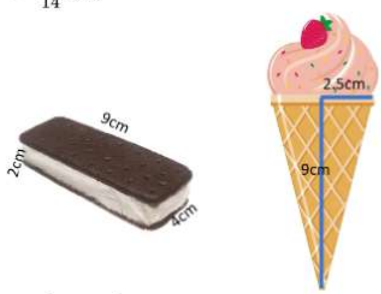
(iii) For cone, Radius of the base (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

Volume of rectangular brick shaped ice cream = $9 \times 4 \times 2$ = 72 cm^3