SAMPLE QUESTION PAPER (STANDARD) - 07

Class 10 - Mathematics

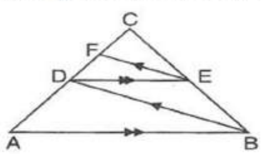
Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. In the given figure, $AB||DE$ and $BD||EF$. Then, **[1]**



a) BC^2 = AB.CE
b) $DC^2 = CF \times AC$
c) AB^2 = AC.DE
d) AC^2 = BC.DC

2. The sum and the product of the zeros of a quadratic polynomial are 3 and -10 respectively. The quadratic polynomial is **[1]**

a) x^2 - 3x + 10
b) x^2 - 3x -10
c) x^2 + 3x -10
d) x^2 + 3x + 10

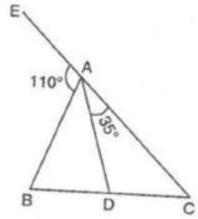
3. The solution of 217x + 131y = 913 and 131x + 217y = 827 is **[1]**

a) x = 2 and y =2
b) x = 2 and y = 3
c) x = 3 and y = 2
d) x = 3 and y = 3

4. A system of linear equations is said to be inconsistent if it has **[1]**

a) one solution
b) at least one solution
c) two solutions
d) no solution

5. In the adjoining figure if exterior $\angle EAB = 110^0, \angle CAD = 35^0$, AB = 5cm, AC = 7cm and BC = 3cm, then CD is equal to **[1]**



a) 2 cm.
b) 1.9 cm.
c) 1.75 cm.
d) 2.25 cm.

6. A box contains cards numbered 6 to 50. A card is drawn at random from the box. The probability that the drawn card has a number which is a perfect square is **[1]**

a) $\frac{1}{9}$
b) $\frac{4}{45}$
c) $\frac{2}{15}$
d) $\frac{1}{45}$

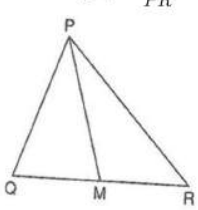
7. If x = a cos θ and y = b sin θ, then $b^2x^2 + a^2y^2 =$ **[1]**

a) $a^2 + b^2$
b) ab
c) a^4b^4
d) a^2b^2

8. The mean of first n odd natural number is: **[1]**

a) n
b) n^2
c) $\frac{n+1}{2}$
d) $\frac{n}{2}$

9. In $\triangle$ PQR, if $\frac{pQ}{PR} = \frac{QM}{MR}, \angle Q = 75^0$ and $\angle R = 45^0$, Then the measure of $\angle QPM$is **[1]**



a) 22.5^o.
b) 30^o.
c) 60^o.
d) 45^o.

10. If two positive integers m and n can be expressed as $m = x^2y^5$ and $n = x^3y^2$, where x and y are prime numbers, then HCF(m, n) = **[1]**

a) x^2y^2
b) x^2y^3
c) x^3y^2
d) x^3y^3

11. The roots of the equation $ax^2 + bx + c = 0$ will be reciprocal of each other if **[1]**

a) None of these
b) a = b
c) b = c
d) c = a

12. If the midpoint of the line segment joining the points (a, b - 2) and (-2, 4) is (2, -3), then the values of a and b are **[1]**

a) 6, 8

b) 6, -8

c) 4, -5

d) -6, 8

13. The marks obtained by 9 students in Mathematics are 59, 46, 30, 23, 27, 40, 52, 35 and 29. The median of the data is **[1]**

a) 29

b) 35

c) 40

d) 30

14. If $\sin\theta - \cos\theta = 0$ then the value of $\left(\sin^4\theta + \cos^4\theta\right)$ is **[1]**

a) $\frac{1}{2}$

b) 1

c) $\frac{3}{4}$

d) $\frac{1}{4}$

15. The ________ of an object is the angle formed by the line of sight with the horizontal when the object is above the horizontal level. **[1]**

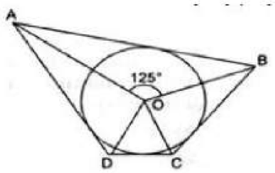
a) angle of projection

b) angle of depression

c) angle of elevation

d) none of these

16. In the given figure, if $\angle AOB = 125^o$, then $\angle COD$ is equal to : **[1]**



a) 45°

b) 62.5°

c) 55°

d) 35°

17. The line segments joining the midpoints of the adjacent sides of a quadrilateral form **[1]**

a) a rhombus

b) a square

c) a parallelogram

d) a rectangle

18. The sum of two numbers is 17 and the sum of their reciprocals is $\frac{17}{62}$. The quadratic representation of the above situation is **[1]**

a) $\frac{1}{x} + \frac{1}{x+17} = \frac{17}{62}$

b) $\frac{1}{x(17-x)} = \frac{17}{62}$

c) $\frac{1}{x} + \frac{1}{17-x} = \frac{17}{62}$

d) $\frac{1}{x} - \frac{1}{17-x} = \frac{17}{62}$

19. **Assertion (A):** Graph of a quadratic polynomial is always U shaped upward or downward. **[1]**

Reason (R): Curve of any quadratic polynomial is always symmetric about the fixed-line.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 350 cm^2. **[1]**

Reason (R): Total surface area of a cuboid is 2(lb + bh + hl)

a) Both A and R are true and R is the correct

b) Both A and R are true but R is not the explanation of A. correct explanation of A.

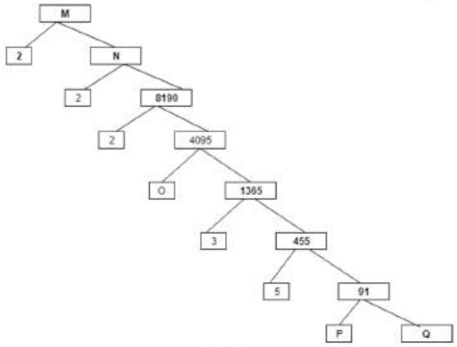
c) A is true but R is false. d) A is false but R is true.

Section B

21. Find the roots of the equation, if they exist, by applying the quadratic formula: $3x^2 - 2x + 2 = 0$. **[2]**

22. If the point C(k,4) divides the join of points A(2,6) and B(5,1) in the ratio 2:3, find the value of k? **[2]**

23. Complete the factor-tree and find the composite number M. **[2]**

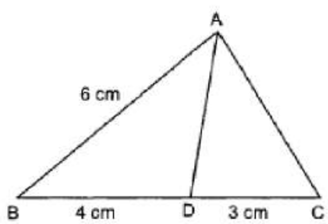


24. Given 15 cot A = 8 find sin A and sec A. **[2]**

OR

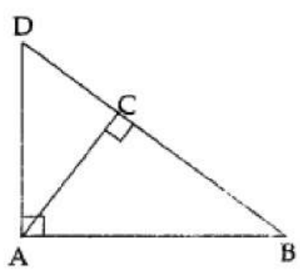
Prove the trigonometric identity : $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2 \operatorname{cosec}\theta$.

25. In Fig. AD is the bisector of $\angle$A. If BD = 4 cm, DC = 3 cm and AB = 6 cm, determine AC. **[2]**



OR

In the adjoining figure, ΔABD is a right triangle, right angled at A and AC $\perp$ BD. Prove that AB^2 = BC. BD.



Section C

26. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages. **[3]**

27. P and Q are points on the sides AB and AC respectively of a $\triangle ABC$. If AP = 2 cm, PB = 4 cm, AQ = 3 cm and QC = 6 cm, show that BC = 3PQ. **[3]**

28. Find the coordinates of points which trisect the line segment joining (1, -2) and (-3, 4). **[3]**

OR

Name the type of quadrilateral formed, if any, by the points (-3, 5), (3, 1), (0, 3), (-1, -4), and give a reason for your answer.

29. A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is **[3]**

10 ft. by 8 ft. What would be the size (in inches) of the tile required that has to be cut and how many such tiles are required?

30. From the top of a tower of height 50m,the angles of depression of the top and bottom of a pole are 30^o and 45^o,respectively. Find **[3]**

1. How far the pole is from the bottom of the tower?
2. The height of the pole.

OR

From a point on a ground, the angle of elevation of bottom and top of a transmission tower fixed on the top of a 20 m high building are 45^o and 60^o respectively. Find the height of the tower.

31. The distribution below gives the weights of 30 students of a class. Find the median weight of the students. **[3]**

Weight(in kg)	**Number of students**
40-45	2
45-50	3
50-55	8
55-60	6
60-65	6
65-70	3
70-75	2

Section D

32. Ved travels 600 km to his home partly by train and partly by car. He takes 8 hours if he travels 120 km by train and the rest by car. He takes 20 minutes longer if he travels 200 km by train and the rest by car. Find the speed of the train and the car. **[5]**

OR

Solve the following system of linear equations graphically:

$x - y = 1$

$2x + y = 8$

Shade the area bounded by these two lines and y-axis. Also, determine this area.

33. AB is a chord of length 24 cm of a circle of radius 13 cm. The tangents at A and B intersect at a point M. Find the length AM. **[5]**

34. Find the area of the segment of a circle of radius 12 cm whose corresponding sector central angle 60°. (Use π = 3.14). **[5]**

OR

Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circles is $\frac{24}{7}\text{cm}^2$. Find the radius of each circle.

35. From a deck of 52 playing cards, Jacks and kings of red colour and Queen and Aces of black colour are removed. The remaining cards are mixed and a card is drown at random. Find the probability that the drawn card is **[5]**

i. A black Queen
ii. A card of red colour

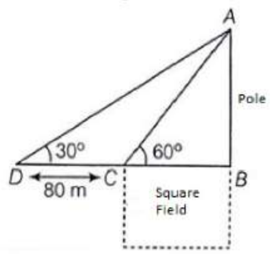
iii. A Jack of black colour

iv. A face card

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite to this corner is 60^{o}. When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30^{o}.



(i) Find the height of the pole too so that he can arrange a ladder accordingly to put an effigy on the pole.

(ii) Find the length of his square field so that he can buy material to do the fencing work accordingly.

(iii) Find the Distance from Farmer at position C and top of the pole?

OR

Find the Distance from Farmer at position D and top of the pole?

37. **Read the text carefully and answer the questions:** **[4]**

The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.

(i) How much distance did she cover in pacing 6 flags on either side of center point?

(ii) Represent above information in Arithmetic progression

OR

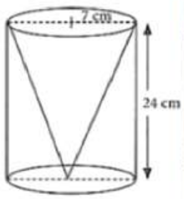
What is the maximum distance she travelled carrying a flag?

(iii) How much distance did she cover in completing this job and returning to collect her books?

38. **Read the text carefully and answer the questions:** **[4]**

One day Vinod was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is

7 cm. While watching this, some questions came into Vinod's mind.



(i) Find the slant height of the conical cavity so formed?

(ii) Find the curved surface area of the conical cavity so formed?

OR

Find the ratio of curved surface area of cone to curved surface area of cylinder?

(iii) Find the external curved surface area of the cylinder?

Solution

SAMPLE QUESTION PAPER (STANDARD) - 07

Class 10 - Mathematics

Section A

1. **(b)** $DC^2 = CF \times AC$

   Explanation: In triangle ABC, using Thales theorem,

   $\frac{DC}{AC} = \frac{CE}{BC}$ [Since AB||DE] ....(i)

   In triangle BCD, using Thales theorem,

   $\frac{CF}{DC} = \frac{CE}{BC}$ [Since BD||EF] ....(ii)

   From eq.(i) and (ii),

   $\frac{DC}{AC} = \frac{CF}{DC}$

   $\Rightarrow DC^2 = CF \times AC$

2. **(b)** $x^2 - 3x - 10$

   Explanation: Given, sum and the product of the zeros of a quadratic polynomial are 3 and -10 respectively

   i,e. $(\alpha + \beta) = 3$ and $\alpha\beta = -10$

   Therefore, required polynomial is

   $x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 3x - 10$

3. **(c)** x = 3 and y = 2

   Explanation: Firstly add up both eq.

   217x + 131y = 913,

   131x + 217y = 827,

   348x + 348y = 1740

   Dividing both side by 348

   We get x + y = 5 ... (i)

   Similarly Subtract given eqn 217x + 131y = 913 - (131x + 217y = 827)

   86x - 86y = 86

   Dividing both side by 86

   We get x - y = 1 ... (ii)equation

   Now, solve equation (i) and (ii)

   x + y = 5

   x - y = 1

   2x = 6

   $\Rightarrow$ x = 3

   Put x = 3 in equation (i)

   x + y = 5

   3 + y = 5

   y = 5 - 3

   $\Rightarrow$ y = 2

   Hence, x = 3 y = 2

4. **(d)** no solution

   Explanation: A system of linear equations is said to be inconsistent if it has no solution means two lines are running parallel and not cutting each other at any point.

5. **(c)** 1.75 cm.

   Explanation: Here, $\angle BAD = 180^o - (\angle EAB + \angle ADC) = 180^o - 110^o - 35^o = 35^o$

   Since, AD bisects $\angle A$.

   $\therefore \frac{AB}{AC} = \frac{BD}{CD}$ [Internal bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle]

   $\Rightarrow \frac{5}{7} = \frac{3-CD}{CD}$

   $\Rightarrow$ 5CD = 21 - 7CD $\Rightarrow$ 5CD + 7 CD = 21

   $\Rightarrow$ 12CD = 21 $\Rightarrow$ CD = 1.75 cm

6. **(a)** $\frac{1}{9}$

Explanation: Given numbers are 6, 7,8,9,..., 50

Number of these numbers = 50 - 5 = 45

Perfect square numbers from these are 3^2, 4^2 ,5^2 , 6^2, 7^2

Their number is 5.

$\therefore$ P(getting a perfect square number) = $\frac{5}{45} = \frac{1}{9}$

7. **(d)** a^2b^2

Explanation: x = a cos θ, y = b sin θ

bx = ab cos θ ...(i)

ay = ab sin θ ...(ii)

Squaring and adding (i) and (ii) we get,

$b^2x^2 + a^2y^2 = a^2b^2 \cos^2\theta + a^2b^2 \sin^2\theta$

$= a^2b^2 (\cos^2\theta + \sin^2\theta)$

$= a^2b^2 \times 1$

$= a^2b^2$

8. **(a)** n

Explanation: We know that the mean or average of observations, is the sum of the values of all the observations divided by the total number of observations.

and, we have first n odd natural numbers as

1, 3, ..., 2n - 1

Clearly the above series is an AP(Arithmetic progression) with first term, a = 1 and common difference, d = 2

And no of terms is clearly n.

And last term is (2n - 1)

We know, sum of terms of an AP if first and last terms are known is:

$S_n = \frac{n}{2}(a + a_n)$

Putting the values in above equation we have sum of series i.e.

$1 + 2 + 3 + ... + n = \frac{n}{2}(1 + 2n - 1) = \frac{n(2n)}{2} = n^2$ ...(i)

As,

Mean = $\frac{\text{Sum of all terms}}{\text{no of terms}}$

$\Rightarrow$ Mean = $\frac{n_2}{n}$ = n

9. **(b)** 30^o.

Explanation: In triangle PQR, $\angle P + \angle Q + \angle R = 180^\circ \Rightarrow \angle P + 75^\circ + 45^\circ = 180^\circ \Rightarrow \angle P = 60^\circ$

In triangle PQR, If $\frac{PQ}{PR} = \frac{QM}{MR}$, then $\frac{PQ}{PR} = \frac{QM}{MR} = \frac{\text{PM}}{\text{PM}}$ also as it is common.

$\therefore \Delta PQR \sim \Delta PMR$

$\Rightarrow \angle Q = \angle PMR$, $\angle PMQ = \angle R$ and $\angle QPM = \angle RPM$ Now, $\angle P = 60^\circ$

$\Rightarrow \angle QPM + \angle RPM = 60^\circ$

$\Rightarrow 2\angle QPM = 60^\circ$

$\Rightarrow \angle QPM = 30^\circ$

10. **(a)** x^2y^2

Explanation: $x^2y^5 = y^3(x^2y^2)$

$x^3y^3 = x(x^2y^2)$

Therefore HCF (m, n) is x^2y^2

11. **(d)** c = a

Explanation: Product of roots = $\frac{c}{a}$. Also $\left(\alpha \times \frac{1}{\alpha}\right) = 1$.

$\therefore \frac{c}{a} = 1 \Rightarrow c = a$.

12. **(b)** 6, -8

Explanation: Let the coordinates of midpoint O(2, -3) is equidistance from the points A(a, b - 2) and B(-2, 4).

$\therefore 2 = \frac{a-2}{2}$
$\Rightarrow$ a - 2 = 4
$\Rightarrow$ a = 6
Also -3 = $\frac{b-2+4}{2}$
$\Rightarrow$ b + 2 = -6
$\Rightarrow$ b = -8
Therefore, a = 6 and b = -8

13. **(b)** 35
Explanation: Arranging the given data in ascending order, we get
23, 27, 29, 30, 35, 40, 46, 52, 59
Here, n = 9, which is odd.
$\therefore$ Median = $\left(\frac{n+1}{2}\right)^{th}$ term
= $\left(\frac{9+1}{2}\right)^{th}$ term
= $\left(\frac{10}{2}\right)^{th}$ term
= 5th term
= 35

14. **(a)** $\frac{1}{2}$
Explanation: It is given that,
$\sin\theta - \cos\theta = 0$
$\Rightarrow \sin\theta = \cos\theta$
$\Rightarrow \frac{\sin\theta}{\cos\theta} = 1$
$\Rightarrow \tan\theta = 1$
$\Rightarrow \tan\theta = \tan 45^\circ$
$\Rightarrow \theta = 45^\circ$
$\therefore \sin^4\theta + \cos^4\theta$
$= \sin^4 45^\circ + \cos^4 45^\circ$
$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$
$= \frac{1}{4} + \frac{1}{4}$
$= \frac{1}{2}$

15. **(c)** angle of elevation
Explanation: The angle of elevation of an object is the angle formed by the line of sight with the horizontal when the object is above the horizontal level.

16. **(c)** 55^o
Explanation: Since the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.
$\therefore \angle AOB + \angle COD = 180^\circ$
$\Rightarrow 125^\circ + \angle COD = 180^\circ \angle COD = 55^\circ$

17. **(c)** a parallelogram
Explanation: The line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.

18. **(c)** $\frac{1}{x} + \frac{1}{17-x} = \frac{17}{62}$
Explanation: Let one number be x, As the sum of the numbers is 17, then the other number will be (17 - x). Their reciprocals will be $\frac{1}{x}$ and $\frac{1}{17-x}$.
$\therefore$ According to question, $\frac{1}{x} + \frac{1}{17-x} = \frac{17}{62}$

19. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: Both A and R are true and R is the correct explanation of A.

20. **(d)** A is false but R is true.
Explanation: A is false but R is true.

Section B

21. The given equation is $3x^2 - 2x + 2 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

a = 3, b = -2 and c = 2

Now, discriminant $D = b^2 - 4ac = (-2)^2 - 4(3)(2)$

$= 4 - 24 = -20 < 0$ (roots are complex)

So, the given equation has no real roots.

22. Point C(k,4) divides the join of AB in the ratio 2:3 .

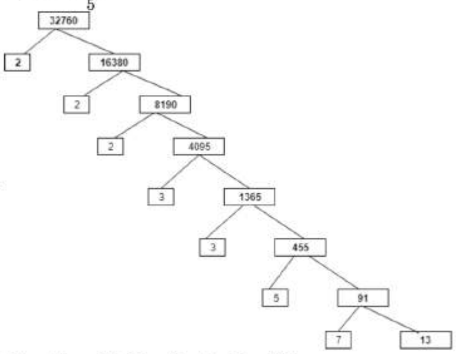
Using the section formula,

$(k, 4) = \left(\frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \frac{m_2y_1 + m_1y_2}{m_1 + m_2}\right) = (\frac{3\times2+2\times5}{2+3}, \frac{3\times6+2\times1}{2+3})$

$\Rightarrow k = \frac{6 + 10}{5}$

$\Rightarrow k = \frac{16}{5}$

23.



91 = 7 × 13, So, P = 7, Q = 13

$O = \frac{4095}{1365} = 3$

N = 2 × 8190 = 16380

The composite number =M = 16380 × 2 = 32760

24. Let us draw a right triangle ABC.

15 cot A = 8 ...... Given

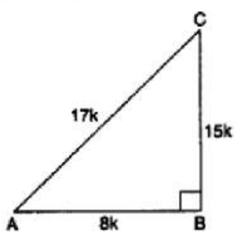
$\Rightarrow \cot A = \frac{8}{15} \Rightarrow \frac{AB}{BC} = \frac{8}{15}$

$\Rightarrow \frac{AB}{8} = \frac{BC}{15} = k(5ay)$

where k is a positive number

$\Rightarrow AB = 8k$

BC = 15k



By using the Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (8k)^2 + (15k)^2 \Rightarrow AC^2 = 64k^2 + 225k^2$

$\Rightarrow AC = \sqrt{289k^2} \Rightarrow AC = 17k$

Now, $\sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

and, $\sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

OR

LHS = $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{\sin\theta\times\sin\theta}{(1+\cos\theta)\times\sin\theta} + \frac{(1+\cos\theta)\times(1+\cos\theta)}{\sin\theta\times(1+\cos\theta)}$

$= \frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{\sin\theta\times(1+\cos\theta)}$

Using the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$

$= \frac{2+2\cos\theta}{\sin\theta\times(1+\cos\theta)}$

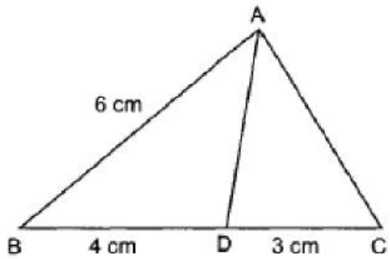
$= \frac{2(1+\cos\theta)}{\sin\theta \times (1+\cos\theta)}$

$= \frac{2}{sin\theta}$

Now $\frac{1}{sin\theta}$ = cosec θ

= 2 cosec $\boldsymbol{\theta}$.

25. In Δ ABC, AD is the bisector of $\angle$ A.



According to question it is given that

BD = 4 cm, CD = 3 cm and AB = 6 cm

We know that, the bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \quad \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{4}{3} = \frac{6}{AC}$

$\Rightarrow$ 4 AC = 18

$\Rightarrow AC = \frac{9}{2}\text{cm} = 4.5\text{cm}$.

OR

Given: ΔABD right anlged at A and $\mathrm{AB} \perp \mathrm{BD}$

To Prove = AB^2 = BC. BD

In ΔABD and ΔACD

$\angle$ABD = $\angle$ABC (common angle)

$\angle$DAB = $\angle$ACD (each 90°)

$\Delta\mathrm{BAD} \cong \Delta\mathrm{BCA}$

$\Rightarrow \frac{\mathrm{BA}}{\mathrm{BC}} = \frac{\mathrm{AD}}{\mathrm{CA}} = \frac{\mathrm{BD}}{\mathrm{BA}}$

$\Rightarrow \frac{\mathrm{BA}}{\mathrm{BC}} \times \frac{\mathrm{BD}}{\mathrm{BA}}$

$(BA)^2$ = BC.BD

AB^2 = BC.BD

Section C

26. Let the present age of girl's sister be 'x' years.

So, girl's present age = 2x years.

After 4 years;

Girl's age =(2x+4) years.

Sister's age =(x+4)years.

According to the question ;

(x + 4)(2x + 4) = 160 ($\because$ product of their ages 4 years hence is 160

$\Rightarrow 2x^2 + 12x - 144 = 0 \Rightarrow x^2 + 6x - 72 = 0$

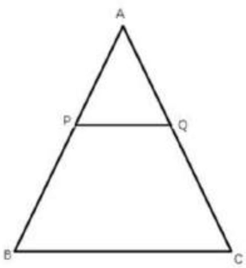
$\Rightarrow x^2 + 12x - 6x - 72 = 0 \Rightarrow x(x + 12) - 6(x + 12) = 0$

$\Rightarrow (x + 12)(x - 6) = 0 \Rightarrow x + 12 = 0$ or $x - 6 = 0$

$\Rightarrow$ x = -12 or x = 6

$\Rightarrow$ x = 6 [$\because$ age cannot be negative]

Hence, sister's present age =x= 6 years and girl's present age = 2x = 12 years

27. 

It is given that, AP = 2 cm, PB = 4 cm, AQ = 3 cm, and QC = 6 cm.

To Prove: BC = 3PQ

Proof: From the given figure, we have AB = AP + PB = 2 + 4 = 6 cm and AC = AQ + QC = 3 + 6 = 9 cm.

Now, $\frac{AP}{PB} = \frac{2}{4} = \frac{1}{2}$

and $\frac{AQ}{QC} = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \frac{AP}{PB} = \frac{AQ}{QC}$

$\Rightarrow \angle A = \angle A$

Therefore, by SAS criteria of similarity, we obtain

$\triangle \mathrm{APQ} \sim \triangle \mathrm{ABC}$

$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$

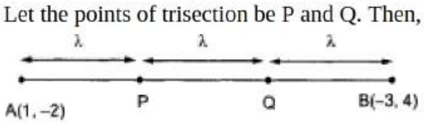
$\Rightarrow \frac{PQ}{BC} = \frac{AQ}{AC}$

$\Rightarrow \frac{PQ}{BC} = \frac{3}{9}$

$\Rightarrow$ BC = 3PQ

28. Let A (1, -2) and B (-3,4) be the given points.

Let the points of trisection be P and Q. Then, AP = PQ = QB = X(say)



PB = PQ + QB = 2λ and AQ = AP + PQ = 2λ

$\Rightarrow$ AP : PB = $\lambda : 2\lambda$ = 1 : 2 and AQ : QB = $2\lambda : \lambda$ = 2 : 1

So, P divides AB internally in the ratio 1 : 2 while Q divides internally in the ratio 2: 1. Thus, the coordinates of P and Q are

$P\left(\frac{1\times-3+2\times1}{1+2}, \frac{1\times4+2\times-2}{1+2}\right) = P\left(\frac{-1}{3}, 0\right)$

$Q\left(\frac{2\times-3+1\times1}{2+1}, \frac{2\times4+1\times(-2)}{2+1}\right) = Q\left(\frac{-5}{3}, 2\right)$

Hence, the two points of trisection are (-1/3, 0) and (-5/3, 2)

OR

(-3, 5), (3, 1), (0, 3), (-1, -4)

Let A $\rightarrow$ (-3, 5), B $\rightarrow$ (3, 1), C $\rightarrow$ (0, 3) and D $\rightarrow$ (-1, -4)

Then, $AB = \sqrt{(3-(-3))^2 + (1-5)^2} = \sqrt{(6)^2 + (-4)^2}$

$= \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$BC = \sqrt{(0-3)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$

$CD = \sqrt{(-1-0)^2 + (-4-3)^2} = \sqrt{1+49} = \sqrt{50}$

$DA = \sqrt{[(-3)-(-1)]^2 + [5-(-4)]^2} = \sqrt{4+81} = \sqrt{85}$

$AC = \sqrt{[0-(-3)]^2 + (3-5)^2} = \sqrt{13}$

$BD = \sqrt{(-1-3)^2 + (-4-1)^2} = \sqrt{41}$

We see that BC + AC = AB

Hence, the points A, B and C are collinear.

So, ABCD is not a quadrilateral.

29. **Given:** Size of bathroom = 10 ft by 8 ft.

= (10×12) inch by (8×12) inch

= 120 inch by 96 inch

Area of bathroom = 120 inch by 96 inch

To find the largest size of tile required , we find HCF of 120 and 96.

By applying Euclid's division lemma

$120 = 96 \times 1 + 24$

$96 = 24 \times 4 + 0$

Therefore, HCF = 24

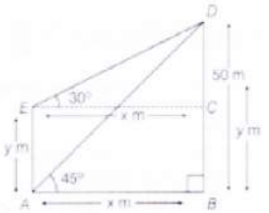
Therefore, Largest size of tile required = 24 inches

no.of tiles required $= \frac{\text{area of bathroom}}{\text{area of a tile}} = \frac{120 \times 96}{24 \times 24} = 5 \times 4$ = 20 tiles

Hence number of tiles required is 20 and size of tiles is 24 inches.

30. Let distance of the pole , say AE, from the bottom of the tower, say BD, is x m. and let height of the pole, AE = y m

Now,draw EC || AB.



Then, $\angle DEC = 30^o, \angle DAB = 45^o$

and DC = DB - BC = DE - AE[$\because BC = AE$]

$\Rightarrow DC = (50 - y)\ m$

i. In right-angled triangle ABD, $\tan 45^o = \frac{P}{B} = \frac{BD}{AB}$

$1\ = \frac{50}{AB}$ [$\because \tan 45^o = 1$

AB = 50 m]

The pole is 50 m away from bottom of the tower.

ii. In right-angled triangle ECD, $\tan 30^o = \frac{P}{B} = \frac{DC}{EC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50-y}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50-y}{50}$

$\Rightarrow (50 - y)\sqrt{3} = 50$

$\Rightarrow 50 - y = \frac{50}{\sqrt{3}}$

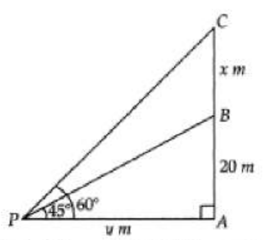
$\Rightarrow 50 - y = 28.86$

$\Rightarrow y = 50 - 28.87$

$\Rightarrow y = 21.13\text{m}$

Height of the pole is 21.13 m.

OR



Let AP = y m and BC = xm.

$\therefore$ In $\triangle BAP = \frac{BA}{PA} = \tan 45^\circ$

$\Rightarrow \quad \frac{20}{y} = 1$

$\Rightarrow$ y = 20

In $\triangle$CAP $\frac{CA}{PA} = \tan 60^\circ$

$\Rightarrow \quad \frac{20+x}{y} = \sqrt{3}$

$\Rightarrow \quad 20 + x = y\sqrt{3}$ (using y = 20)

$\Rightarrow \quad 20 + x = 20\sqrt{3}$

$x = 20\sqrt{3} - 20$

$= 20(\sqrt{3} - 1)$

$= 20 \times (1 \cdot 73 - 1)$

Hence, height of the tower = 14.64 m

31.

Weight (in kg)	Number of students	Cumulative frequency
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30

Now, n = 30

So, $\frac{n}{2} = \frac{30}{2} = 15$

This observation lies in the class 55-60,

So, 55-60 is the median class.

Therefore,

l = 55

h = 5

f = 6

cf = 13

$\therefore$ Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 55 + \left(\frac{15-13}{6}\right) \times 5$

$= 55 + \frac{10}{6} = 55 + \frac{5}{3}$

= 55 + 1.67 = 56.67

Hence, the median weight of the students is 56.67 kg.

Section D

32. Suppose the speed of the train be x km/hr and the speed of the car be y km/hr.

CASE I

Distance covered by car is $(600 - 120)km$ = $480km.$

Now, Time taken to cover 480 km by train $\frac{120}{x}$hrs $[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}]$

Time taken to cover 480 km by car =$\frac{480}{y}$ hrs

$\therefore \frac{120}{x} + \frac{480}{y} = 8$

$\Rightarrow 8\left(\frac{15}{x} + \frac{60}{y}\right) = 8$

$\Rightarrow \frac{15}{x} + \frac{60}{y} = 1$

$\Rightarrow \frac{15}{x} + \frac{60}{y} - 1 = 0$ ....(i)

CASE II

Distance travelled by car is $(600 - 200)km$ = $400km$

Now, Time taken to cover $200km$ by train = $\frac{200}{x}$hrs

Time taken to cover $400km$ by train =$\frac{400}{y}$hrs

In this case the total time of journey is 8 hour 20 minutes

$\therefore \frac{200}{x} + \frac{400}{y} = 8 \text{ hrs } 20\ minutes$

$\Rightarrow \frac{200}{x} + \frac{400}{y} = 8\frac{1}{3}$ $\left[\because 8 \text{ hrs } 20 \text{ minutes } = 8\frac{20}{60}\text{hrs} = 8\frac{1}{3}\text{hrs}\right]$

$\Rightarrow \frac{200}{x} + \frac{400}{y} = \frac{25}{3}$

$\Rightarrow \quad 25\left(\frac{8}{x} + \frac{16}{y}\right) = \frac{25}{3}$

$\Rightarrow \quad \frac{8}{x} + \frac{16}{y} = \frac{1}{3}$

$\Rightarrow \quad \frac{24}{x} + \frac{48}{y} = 1$

$\Rightarrow \quad \frac{24}{x} + \frac{48}{y} - 1 = 0$ ....(ii)

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in equations (i) and (ii), we get

15 u+ 60v -1=0 ....(iii)

24u+ 48v -1=0 ....(iv)

By using cross-multiplication, we have

$\frac{u}{60\times-1-48\times-1}=\frac{-v}{15\times-1-24\times-1}=\frac{1}{15\times48-24\times60}$

$\Rightarrow \frac{u}{-60+48}=\frac{-v}{-15+24}=\frac{1}{720-1440}$

$\Rightarrow \quad \frac{u}{-12}=\frac{v}{-9}=\frac{1}{-720}$

$\Rightarrow \quad u=\frac{-12}{-720}=\frac{1}{60}$ and $v=\frac{-9}{-720}=\frac{1}{80}$

Now, $u=\frac{1}{x}\Rightarrow\frac{1}{60}=\frac{1}{x}\Rightarrow x=60$

and, $v=\frac{1}{y}\Rightarrow\frac{1}{80}=\frac{1}{y}\Rightarrow y=80$

Speed of train = $60km/hr$

Speed of car=$80km/hr$.

OR

Given system of equations are:

$x-y=1$

$2x+y=8$

Graph of the equation $x-y=1$:

We have,

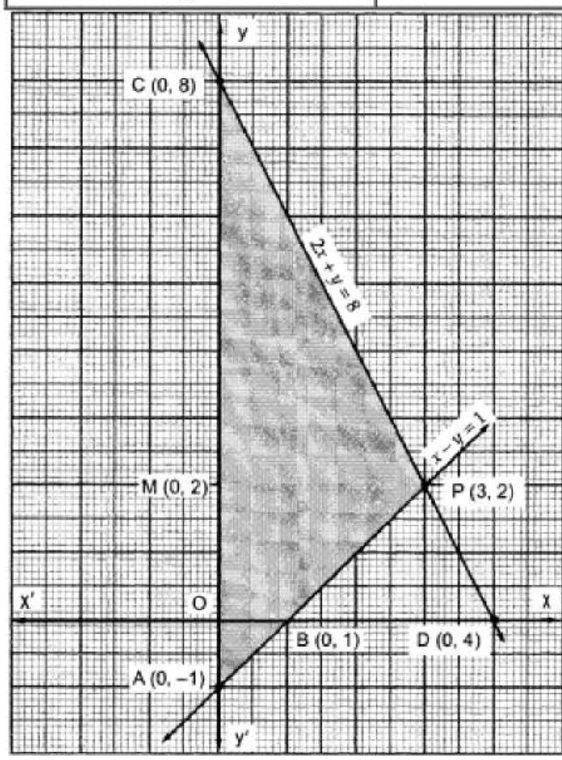
x - y = 1 ⇒ y = x - 1 and x = y + 1

Putting x = 0, we get y = -1

Putting y = 0, we get x = 1

Thus, we have the following table for the points on the line $x-y=1$:

x	0	1
y	-1	0



Graph of the equation $2x+y=8$:

We have,

2x + y = 8 ⇒ y = 8 - 2x and $x=\frac{8-y}{2}$

Putting x = 0, we get y = 8

Putting y = 0, we get x = 4

Thus, we have the following table giving two points on the line represented by the equation 2x + y = 8.

x	0	4
y	8	0

Clearly, the two lines intersect at P (3,2). The area enclosed by the lines represented by the given equations and the y-axis is shaded in Fig.

Now, Required area= Area of the shaded region

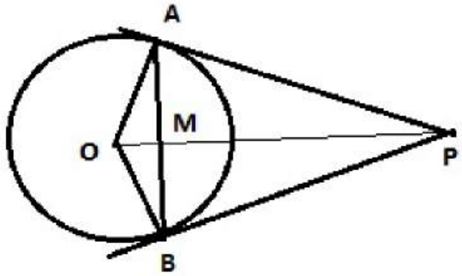
$\Rightarrow$ Required area = Area of ΔPAC

$\Rightarrow \quad$ Required area $= \frac{1}{2}(\text{ Base } \times \text{ Height })$

$\Rightarrow \quad$ Required area $= \frac{1}{2}(AC \times PM)$

$\Rightarrow \quad$ Required area $= \frac{1}{2}(9 \times 3)$ sq.units [$\because$ PM = x-coordinate of P = 3]

= 13.5 sq. units.

33. 

According to the question,

Given: Chord AB = 24 cm, radius $OB = OA = 13cm.$

Construction: Draw OP $\perp$ AB.

In $\triangle$OPB, OP $\perp$ AB

$\Rightarrow$ AP = PB [Perpendicular from centre on chord bisect the chord]

$= \frac{1}{2}AB = 12$

$OB^2 = OP^2 + PB^2$

$\Rightarrow (13)^2 = OP^2 + PB^2 \Rightarrow 169 = OP^2 + (12)^2$

$OP^2 = 169 - 144 = 25 \Rightarrow OP = 5$ cm

In $\triangle$BPM ,

By Pythagoras theorem , we get

$BM^2 = x^2 + k^2$

$BM^2 = x^2 + 144$ ...(i)

In $\triangle$OBM,

$OM^2 = OB^2 + BM^2$

$(x + 5)^2 = (13)^2 + BM^2$

$\Rightarrow x^2 + 25 + 10x = 169 + BM^2$ ...(ii)

$\Rightarrow x^2 + 25 + 10x = 169 + x^2 + 144$

$\Rightarrow 25 + 10x = 169 + 144$

$\Rightarrow 10x = 169 + 144 - 25 = 288$

$\Rightarrow x = \frac{288}{10} = 28.8\text{cm}$

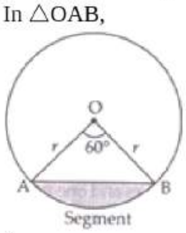
Put value of x in (i) $BM^2 = x^2 + 144 = \frac{(144)^2}{25} + 144 = 144\left(\frac{144}{25} + 1\right) = \frac{144(169)}{25}$

$BM = \frac{12\times13}{5} = \frac{156}{5} = 31.2$

$AM = BM = 31.2cm$

34. Area of minor segment = Area of sector – Area of $\triangle$OAB

In $\triangle$OAB,



$\theta = 60°$

OA = OB = r = 12 cm

$\angle$B = $\angle$A = x [$\angle$s opp. to equal sides are equal]

$\Rightarrow \angle A + \angle B + \angle O = 180°$

$\Rightarrow x + x + 60° = 180°$

$\Rightarrow 2x = 180° - 60°$

$\Rightarrow x = \frac{120°}{2} = 60°$

$\therefore \triangle OAB$ is equilateral $\triangle$ with each side (a) = 12 cm

Area of the equilateral $\triangle = \frac{\sqrt{3}}{4}a^2$

Area of minor segment = Area of the sector – Area of $\triangle OAB$

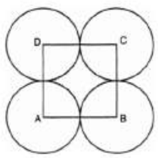
$= \frac{\pi r^2\theta}{360°} - \frac{\sqrt{3}}{4}a^2$

$= \frac{3.14 \times 12 \times 12 \times 60°}{360°} - \frac{\sqrt{3}}{4} \times 12 \times 12$

$= 6.28 \times 12 - 36\sqrt{3}$

$\therefore$ Area of minor segment = $(75.36 - 36\sqrt{3})$ cm^2.

OR



Let r cm be the radius of each circle.

Area of square - Area of 4 sectors = $\frac{24}{7}\text{cm}^2$

$(\text{side})^2 - 4\left[\frac{\theta}{360}\pi r^2\right] = \frac{24}{7}\text{cm}^2$

or, $(2r)^2 - 4\left(\frac{90°}{360°} \times \pi r^2\right) = \frac{24}{7}$

or, $(2r)^2 - 4\left(\frac{1}{4°} \times \pi r^2\right) = \frac{24}{7}$

or, $(2r)^2 - \left(\pi r^2\right) = \frac{24}{7}$

or, $4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$

or, $\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$

or, $6r^2 = 24$

or, $r^2 = 4$

or, $r = \pm 2$

or, Radius of each circle is 2 cm (r cannot be negative)

35. No. of cards removed= 2 jacks+2 queens +2 kings +2 aces=8

No. of all possible outcomes $n = 52 - 8 = 44 - - - - - - - - - - - - (1)$

i. No. of black Queens in the deck = 0

$\therefore$ P(getting a black Queen) = $\frac{0}{44} = 0$

Hence it is an impossible event

ii. No. of red cards = 26 - 4 = 22

P(getting a red card) = $\frac{m}{n} = \frac{22}{44} = \frac{1}{2}$

iii. No. of Jacks (black) = 2 so m=2

$\therefore$ P(getting a black colored Jack) = $\frac{m}{n} = \frac{2}{44} = \frac{1}{22}$

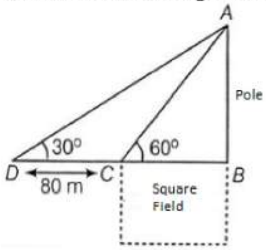
iv. No. of face cards in the deck = 12 - 6 = 6 so m=6

$\therefore$P(getting a face card) = $\frac{m}{n} = \frac{6}{44} = \frac{3}{22}$

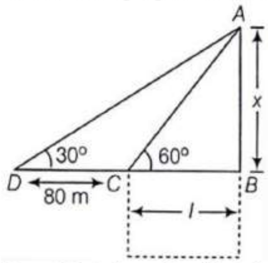
Section E

36. **Read the text carefully and answer the questions:**

Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite to this corner is 60^{o}. When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30^o.



(i) The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

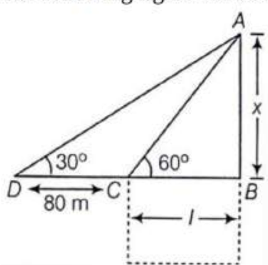
Now, l = 40 metres

We get,

$x = \sqrt{3}\,l = 40\sqrt{3} = 69.28$

Thus, height of the pole is 69.28 metres.

(ii) The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

In $\triangle ABC$,

$\tan 60^o = \frac{x}{l}$

$\sqrt{3} = \frac{x}{l}$

$x = \sqrt{3}\,l$ ...(i)

Now, in $\triangle ABD$,

$\tan 30^o = \frac{x}{80+l}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}l}{80+l}$ (From eq(i))

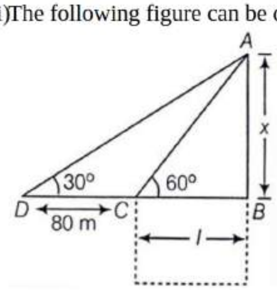
80 + l = 3l

2l = 80

l = 40

Thus, length of the field is 40 metres.

(iii)The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

Distance from Farmer at position C and top of the pole is AC.

In △ABC

$\cos 60^0 = \frac{CB}{AC}$

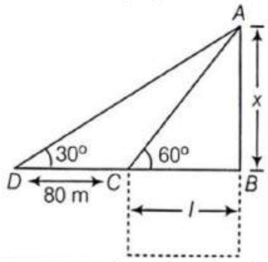
$\Rightarrow AC = \frac{CB}{\cos 60^0}$

$\Rightarrow AC = \frac{40}{\frac{1}{2}}$

$\Rightarrow$ AC = 80 m

OR

The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

Distance from Farmer at position D and top of the pole is AD

In △ABC

$\cos 30^0 = \frac{DB}{AD}$

$\Rightarrow AD = \frac{DB}{\cos 30^0}$

$\Rightarrow AD = \frac{120}{\frac{\sqrt{2}}{2}} = \frac{240}{\sqrt{3}}$

$\Rightarrow$ AC = 138.56 m

37. **Read the text carefully and answer the questions:**

The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.

(i) Distance covered in placing 6 flags on either side of center point is 84 + 84 = 168 m

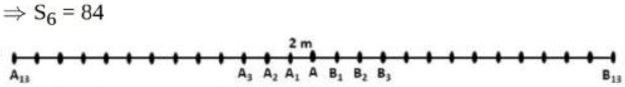
$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_6 = \frac{6}{2}[2 \times 4 + (6 - 1) \times 4]$

$\Rightarrow S_6 = 3[8 + 20]$

$\Rightarrow S_6 = 84$

(ii)



Let A be the position of the middle-most flag.

Now, there are 13 flags ($A_1, A_2 ... A_{12}$) to the left of A and 13 flags ($B_1, B_2, B_3 ... B_{13}$) to the right of A.

Distance covered in fixing flag to A_1 = 2 + 2 = 4 m

Distance covered in fixing flag to A_2 = 4 + 4 = 8 m

Distance covered in fixing flag to A_3 = 6 + 6 = 12 m

...

Distance covered in fixing flag to A_{13} = 26 + 26 = 52 m

This forms an A.P. with,

First term, a = 4

Common difference, d = 4

and n = 13

OR

Maximum distance travelled by Ruchi in carrying a flag

= Distance from A_{13} to A or B_{13} to A = 26 m

(iii) $\because$ Distance covered in fixing 13 flags to the left of A = S_{13}

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{13} = \frac{13}{2}[2 \times 4 + 12 \times 4]$

$= \frac{13}{2} \times [8 + 48]$

$= \frac{13}{2} \times 56$

= 364

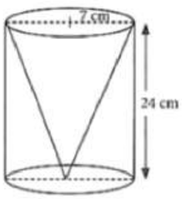
Similarly, distance covered in fixing 13 flags to the right of A = 364

Total distance covered by Ruchi in completing the task

= 364 + 364 = 728 m

38. **Read the text carefully and answer the questions:**

One day Vinod was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Vinod's mind.



(i) Given height of cone = 24cm and radius of base = r = 7cm

Slant height of conical cavity,

$l = \sqrt{h^2 + r^2}$

$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625}$ = 25 cm

(ii) we know that r = 7cm, l = 25 cm

Curved surface area of conical cavity = πrl

$= \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$

OR

Curved surface area of conical cavity = πrl

$= \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$

External curved surface area of cylinder

$= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 24 = 1056 \text{ cm}^2$

$\frac{curved\ surface\ area\ of\ cone}{curved\ surface\ area\ of\ cylinder} = \frac{550}{1056} = \frac{275}{528}$

hence required ratio = 275:528

(iii) For cylinder height = h = 24cm, radius of base = r = 7cm

External curved surface area of cylinder

$= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 24 = 1056 \text{ cm}^2$