SAMPLE QUESTION PAPER (STANDARD) - 08

Class 10 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

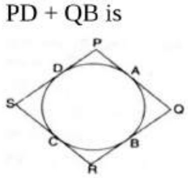
Section A

1. The line 2x + y - 4 = 0 divides the line segment joining A(2, -2) and B(3, 7) in the ratio **[1]**

a) 2 : 9 b) 2 : 7

c) 2 : 3 d) 2 : 5

2. Quadrilateral PQRS circumscribes a circle as shown in the figure. The side of the quadrilateral which is equal to PD + QB is **[1]**



a) PS b) PR

c) QR d) PQ

3. A coin is tossed thrice. The probability of getting at least two tails is **[1]**

a) $\frac{4}{5}$ b) $\frac{2}{3}$

c) $\frac{1}{4}$ d) $\frac{1}{2}$

4. If the endpoints of a diameter of a circle are (-4, -3) and (2, 7), then the coordinates of the centre are **[1]**

a) (1, -2) b) (0, 0)

c) (2, -1) d) (-1, 2)

5. If $\frac{2x+y+2}{5} = \frac{3x-y+1}{3} = \frac{3x+2y+1}{6}$ then [1]

a) x = 2, y = 1
b) x = -1, y = -1
c) x = 1, y = 1
d) x = 1, y = 2

6. The distance between the points $(\cos\theta, \sin\theta)$ and $(\sin\theta, -\cos\theta)$ is [1]

a) $\sqrt{3}$
b) $\sqrt{2}$
c) 2
d) 1

7. For the following frequency distribution: [1]

Class:	0-5	5-10	10-15	15-20	20-25
Frequency:	8	10	19	25	8

The upper limit of the median class is:

a) 15
b) 10
c) 20
d) 25

8. The radii of the base of a cylinder and a cone are in the ratio 3 :4. If they have their heights in the ratio 2 : 3, the ratio between their volumes is [1]

a) 9 : 8
b) 3 : 4
c) 8 : 9
d) 4 : 3

9. Two dice are thrown simultaneously. The probability that the product of the numbers appearing on the dice is 7 is [1]

a) 7
b) 2
c) 0
d) 1

10. The angry Arjun carried some arrows for fighting with Bheeshma. With half the arrows, he cut down the arrows thrown by Bheeshma on him and with six other arrows he killed the rath driver of Bheeshma. With one arrow each, he knocked down respectively the rath, flag and bow of Bheeshma. Finally, with one more than four times the square root of arrows, he laid Bheeshma unconscious on an arrow bed. The total number of arrows that Arjun had, is [1]

a) 100
b) 96
c) 80
d) 120

11. A rectangular field is 16m long and 10m wide. There is a path of uniform width all around it having an area of 120 sq.m, then the width of the path is [1]

a) 5 m
b) 3 m
c) 2m
d) 4 m

12. Find the value of $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ [1]

a) sin 30°
b) tan 60°
c) cos 60°
d) sin 60°

13. If two positive integers m and n are expressible in the form m pq^3 and n = p^3q^2, where p, q are prime numbers, then HCF (m, n) = [1]

a) pq
b) pq^2
c) p^2q^3
d) p^3q^3

14. A line segment is of length 10 units. If the coordinates of its one end are (2, - 3) and the abscissa of the other end is 10, then its ordinate is **[1]**

a) -3, 9
b) 9, -6
c) 9, 6
d) 3, -9

15. If a 1.5 m tall girl stands at a distance of 3 m from a lamp-post and casts a shadow of length 4.5 m on the ground, then the height of the lamp-post is **[1]**

a) 2.5 m
b) 2.8 m
c) 2 m
d) 1.5 m

16. While computing mean of grouped data, we assume that the frequencies are: **[1]**

a) centred at the class marks of the classes.
b) centred at the upper limit of the classes.
c) evenly distributed over all the classes.
d) centred at the lower limit of the classes.

17. $(1 + \sqrt{2}) + (1 - \sqrt{2})$ is **[1]**

a) a rational number
b) a non-terminating decimal
c) None of these
d) an irrational number

18. If the lines given by 3x + 2ky = 2 and 2x + 5y + 1 = 0 are parallel, then the value of k is **[1]**

a) $\frac{-5}{4}$
b) $\frac{3}{2}$
c) $\frac{15}{4}$
d) $\frac{2}{5}$

19. **Assertion (A):** If p is a prime number then H.C.F. of p, p^2 and p^3 is p. **[1]**
Reason (R): H.C.F. of 3 number is smallest number among them.

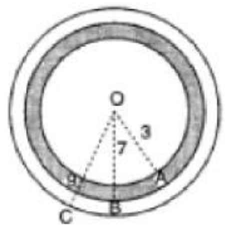
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** If two triangles are similar and have an equal area, then they are congruent. **[1]**
Reason (R): Corresponding sides of two triangles are equal, then triangles are congruent.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. A target shown in figure consists of three concentric circles of radii 3, 7 and 9 cm respectively. A dart is thrown and lands on the target. What is the probability that the dart will land on the shaded region? **[2]**



22. If three times the larger of the two numbers is divided by the smaller one, we get 4 as quotient and 3 as the remainder. Also, if seven times the smaller number is divided by the larger one, we get 5 as quotient and 1 as remainder. Find the numbers. **[2]**

23. Find the zeroes of a quadratic polynomial given as $t^2 - 15$ and verify the relationship between the zeroes and the coefficients. **[2]**

24. If the distance between points (x, 0) and (0,3) is 5, what are the values of x? **[2]**

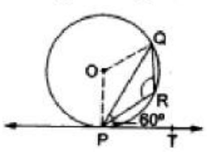
OR

Determine whether the given points are vertices of a right triangle: (8, 4),(5, 7) and (–1,1)

25. Prove that the perpendicular at the point of contact of the tangent to a circle passes through the centre. **[2]**

OR

In the given figure, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT$ = 60°, find $\angle PRQ$.



Section C

26. If $2\sin^2\theta - \cos^2\theta = 2$, then find the value of θ. **[3]**

27. Solve the pair of linear equations: **[3]**

$152x - 378y = -74; -378x + 152y = -604$.

28. 105 goats, 140 donkeys and 175 cows have to be taken across a river. There is only one boat which will have to make many trips in order to do so. The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same kind. He will naturally like to take the largest possible number each time. Can you tell how many animals went in each trip? **[3]**

OR

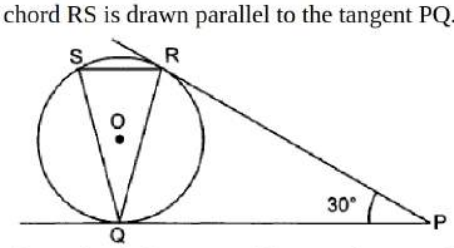
Prove that $2+\sqrt{5}$ is an irrational number.

29. Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle. **[3]**

30. If a hexagon ABCDEF circumscribe a circle, prove that AB + CD + EF = BC + DE + FA. **[3]**

OR

In figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle$RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find $\angle$RQS.



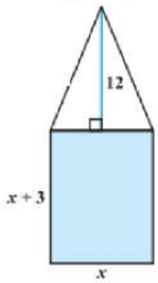
31. The angle of elevation of the top of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Calculate the height of the tower. **[3]**

Section D

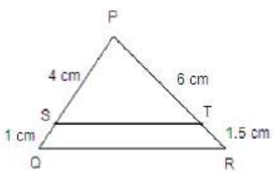
32. A shopkeeper buys a number of books for Rs.1200. If he had bought 10 more books for the same amount, each book would have cost him Rs.20 less. Find how many books did he buy? **[5]**

OR

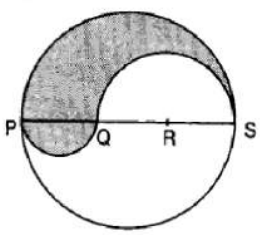
A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 sq m more than the area of park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and altitude 12 m. Find the length and breadth of the rectangular park.



33. In the given figure, PS, SQ,PT and TR are 4 cm,1 cm,6 cm and 1.5 cm,respectively. Prove that ST || QR. **[5]**



34. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. Find the perimeter and area of the shaded region **[5]**



OR

Two circular beads of different sizes are joined together such that the distance between their centres is 14 cm. The sum of their areas is 130π cm^2. Find the radius each bead.

35. The ages of employees in two factories A and B are given below: **[5]**

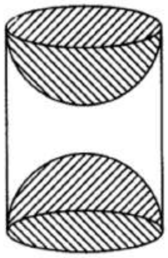
Age of Employees(in years)	**Number of employees in factories**	
	A	**B**
20 - 30	5	8
30 - 40	26	40
40 - 50	78	58
50 - 60	104	90
60 - 70	98	83

Find the modal age of employees in factory A and factory B.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A carpenter used to make different kinds and different shapes of a toy of wooden material. One day a man came to his shop to purchase an article that has values as per his requirement. He instructed the carpenter to make the toy by taking a wooden block of rectangular shape with height 12 cm and width 9 cm, then shaping this block as a solid cylinder and then scooping out a hemisphere from each end, as shown in the given figure. The difference between the length of rectangle and height of the cylinder is 2 cm (Rectangle length > Cylinder height), and the difference between the breadth of rectangle and the base of cylinder is also 2 cm (Rectangle breadth > Cylinder base(diameter)).



(i) Find the volume of the cylindrical block before the carpenter started scooping the hemisphere from it.

(ii) Find the volume of wood scooped out?

(iii) Find the total surface area of the article?

OR

Find the total surface area of cylinder before scooping out hemisphere?

37. **Read the text carefully and answer the questions:** **[4]**

Jaspal Singh is an auto driver. His autorickshaw was too old and he had to spend a lot of money on repair and maintenance every now and then. One day he got to know about the EV scheme of the Government of India where he can not only get a good exchange bonus but also avail heavy discounts on the purchase of an electric vehicle. So, he took a loan of ₹1,18,000 from a reputed bank and purchased a new autorickshaw.

Jaspal Singh repays his total loan of 118000 rupees by paying every month starting with the first instalment of 1000 rupees.

(i) If he increases the instalment by 100 rupees every month, then what amount will be paid by him in the 30th instalment?

(ii) If he increases the instalment by 100 rupees every month, then what amount of loan does he still have to pay after 30th instalment?

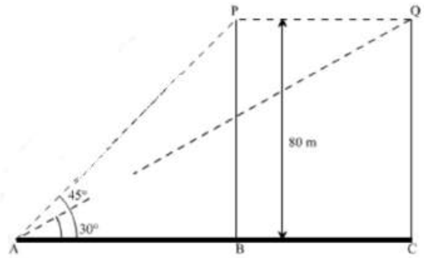
OR

If he increases the instalment by 200 rupees every month, then what amount would he pay in 40th instalment?

(iii) If he increases the instalment by 100 rupees every month, then what amount will be paid by him in the 100th instalment?

38. **Read the text carefully and answer the questions:** **[4]**

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45^o. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30^o. Find the speed of flying of the bird.



(i) Find the distance between observer and the bottom of the tree?

(ii) Find the speed of the bird?

OR

Find the distance between initial position of bird and observer?

(iii) Find the distance between second position of bird and observer?

Solution

SAMPLE QUESTION PAPER (STANDARD) - 08

Class 10 - Mathematics

Section A

1. **(a)** 2 : 9

   Explanation: Let the required ratio be K : 1

   Then, the point of division is p $\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

   this point lies on the line $2x + y - 4 = 0$

   $= \frac{2(3k+2)}{k+1} + \frac{(7k-2)}{k+1} - 4 = 0 = 6k + 4 + 7k - 2 - 4k - 4 = 0$

   $\Leftrightarrow 9k = 2 \Rightarrow k = \frac{2}{9}$

   so, the required ratio is $\left(\frac{2}{9} : 1\right)$, i.e., $(2 : 9)$

2. **(d)** PQ

   Explanation: PD + QB = PA + QA [Tangents from an external point to a circle are equal]

   $\Rightarrow$ PD + QB = PQ

3. **(d)** $\frac{1}{2}$

   Explanation: Total outcomes = = {HHH, TTT, HHT, HTH, HTT, THH, THT, TTH} = 8

   Number of possible outcomes (at least two tails) = 4

   $\therefore$ Required Probability = $\frac{4}{8} = \frac{1}{2}$

4. **(d)** (-1, 2)

   Explanation: Let the coordinates of centre O be (x, y).

   The endpoints of a diameter of the circle are A(- 4, - 3) and B(2, 7).

   Since centre is the midpoint of diameter.

   $\therefore$ x = $\frac{x_1+x_2}{2}$ = $\frac{-4+2}{2} = \frac{-2}{2}$ = -1 and

   y = $\frac{y_1+y_2}{2}$ = $\frac{-3+7}{2} = \frac{4}{2}$ = 2

   Therefore, the coordinates of the centre O is (-1, 2)

5. **(c)** x = 1, y = 1

   Explanation: $\frac{2x+y+2}{5} = \frac{3x-y+1}{3}$

   $\Rightarrow$ 6x + 3y + 6 = 15x - 5y + 5 $\Rightarrow$ 9x - 8y = 1 .....(i)

   $\frac{3x-y+1}{3} = \frac{3x+2y+1}{6}$

   $\Rightarrow$ 18x - 6y + 6 = 9x + 6y + 3 $\Rightarrow$ 9x - 12y = - 3 .....(ii)

   Solve (i) and (ii) to get x = 1 and y = 1.

6. **(b)** $\sqrt{2}$

   Explanation: Distance between $(\cos\theta, \sin\theta)$ and $(\sin\theta, -\cos\theta)$

   $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

   $= \sqrt{(-\cos\theta - \sin\theta)^2 + (\sin\theta - \cos\theta)^2}$

   $= \sqrt{1+1} = \sqrt{2}\ \left\{\because \sin^2\theta + \cos^2\theta = 1\right\}$

7. **(a)** 15

   Explanation: 15

   Sum of frequencies = n =70

   $\frac{n}{2}$ = 35

   Hence median class = 10 - 15

   Therefore, upper limit of median class = 15

8. **(a)** 9 : 8

   Explanation: Let the radii of the base of the cylinder and cone be 3r and 4r and their heights be 2h and 3h, respectively.

   Then, ratio of their volumes = $\frac{\pi(3r)^2 \times (2h)}{\frac{1}{3}\pi(4r)^2 \times (3h)}$

   $= \frac{9r^2 \times 2 \times 3}{16r^2 \times 3}$

$= \frac{9}{8}$
= 9 : 8

9. **(c)** 0
Explanation: Elementary events are
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)
$\therefore$ Number of Total outcomes = 36
And Number of possible outcomes (product of numbers appearing on die is 7) = 0
$\therefore$ Required Probability = $\frac{0}{36} = 0$

10. **(a)** 100
Explanation: Let Arjun had x arrows.
According to question,
$\frac{x}{2} + 6 + 3 + 4\sqrt{x} + 1 = x$
$\Rightarrow 10 + 4\sqrt{x} = \frac{x}{2}$
$\Rightarrow 20 + 8\sqrt{x} = x$
$\Rightarrow 8\sqrt{x} = x - 20$
$\Rightarrow 64x = x^2 - 40x + 400$
$\Rightarrow x^2 - 104x + 400 = 0$
$\Rightarrow x^2 - 100x - 4x + 400 = 0$
$\Rightarrow x(x - 100) - 4(x - 100) = 0$
$\Rightarrow (x - 100)(x - 4) = 0$
$\Rightarrow x - 100 = 0$ and $x - 4 = 0$
$\Rightarrow x = 100$ and $x = 4$ [which is not possible]
Therefore, Arjun had 100 arrows.

11. **(c)** 2m
Explanation: Let the width of the path be x meter
$\therefore$ Area of path = Area of ABCD - Area of PQRS
$\Rightarrow 120 = (16 + 2x)(10 + 2x) - 16 \times 10$
$\Rightarrow 120 = 160 + 32x + 20x + 4x^2 - 160$
$\Rightarrow 4x^2 + 52x - 120 = 0$
$\Rightarrow x^2 + 13x - 30 = 0$
$\Rightarrow x^2 + 15x - 2x - 30 = 0$
$\Rightarrow x(x + 15) - 2(x + 15) = 0$
$\Rightarrow (x + 15)(x - 2) = 0$
$\Rightarrow x + 15 = 0$ and $x - 2 = 0$
$\Rightarrow x = -15$ and $x = 2$ ... [$x = -15$ is not possible]
Therefore, the width of the path is 2 m.

12. **(b)** $\tan 60^o$
Explanation: We have $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \frac{2\times\frac{1}{\sqrt{3}}}{1-(\frac{1}{\sqrt{3}})^2}$
$= \frac{2}{\sqrt{3}} \times \frac{3}{2}$
$= \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60^\circ$

13. **(b)** pq^2
Explanation: Two positive integers are expressed as follows:
$m = pq^3$

$n = p^3q^2$

p and q are prime numbers.

Then, taking the smallest powers of p and q in the values for m and n we get

HCF (m, n) = pq^2

14. **(d)** 3, -9

Explanation: Let the ordinate of other end = y

then distance between (2, -3) and (10, y) = 10 units

$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 10$

$\Rightarrow \sqrt{(10-2)^2 + (y+3)^2} = 10$

$\Rightarrow \sqrt{(8)^2 + (y+3)^2} = 10$

Squaring both sides

$(8)^2 + (y + 3)^2 = (10)^2 \Rightarrow 64 + (y + 3)^2 = 100$

$\Rightarrow (y + 3)^2 = 100 - 64 = 36 = (6)^2$

$\Rightarrow (y + 3)^2 - (6)^2 = 0 \Rightarrow (y + 3 + 6) (y + 3 - 6)$

$= 0$ $\{\because a^2 - b^2 = (a + b) (a - b)\}$

$\Rightarrow (y + 9) (y - 3) = 0$

Either y + 9 = 0, then y = -9

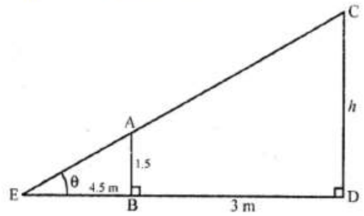
or y - 3 = 0, then y = 3

$\therefore$ y = 3, -9

15. **(a)** 2.5 m

Explanation: Let AB is girls and CD is lamp-post AB = 1.5

which casts her shadow EB



$\therefore$ EB = 4.5 m, BD = 3 m

Now in $\triangle$AEB

$\tan\theta = \frac{AB}{BE} = \frac{1.5}{4.5} = \frac{1}{3}$

and in $\triangle$CED

$\tan\theta = \frac{\mathrm{CD}}{\mathrm{ED}} \Rightarrow \frac{1}{3} = \frac{h}{4.5+3}$

$\Rightarrow \frac{1}{3} = \frac{h}{7.5} \Rightarrow h = \frac{7.5}{3}$ = - 2.5 m

$\therefore$ height of lamp-post = 2.5 m

16. **(a)** centred at the class marks of the classes.

Explanation: In computing the mean of grouped data, the frequencies are centred at the class marks of the classes.

17. **(a)** a rational number

Explanation: $(1 + \sqrt{2}) + (1 - \sqrt{2}) = 1 + \sqrt{2} + 1 - \sqrt{2} = 1 + 1 = 2$ And 2 is a rational number.

Therefore the given number is rational number.

18. **(c)** $\frac{15}{4}$

Explanation: Condition for parallel lines is

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \ldots (i)$

Given lines,

3x + 2ky - 2 = 0 and

2x + 5y - 1 = 0;

Comparing with standard form,

Here, $a_1 = 3$, $b_1 = 2k$, $c_1 = -2$

and $a_2 = 2$, $b_2 = 5$, $c_2 = -1$

From Eq. (i),

$\frac{3}{2} = \frac{2k}{5}$

$k = \frac{15}{4}$

19. **(c)** A is true but R is false.

Explanation: A is true but R is false.

20. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Two similar triangles of equal area are always congruent.

Section B

21. Suppose,

Radius of first circle $= 3cm$

Radius of second circle $= 7cm$

Radius of third circle $= 9cm$

Area of third circle = $\pi(9)^2 = 81\pi cm^2$

Area shaded region = area of second circle - area of first circle

$= \pi(7)^2 - \pi(3)^2$

$= 49\pi - 9\pi$

$= 40\pi\ cm^2$

$\therefore$ Probability that the dart will land on the shaded region.

$= \frac{\text{area of shaded region}}{\text{area of third circle}}$

$= \frac{40\pi}{81\pi} = \frac{40}{81}$.

22. Let the larger number is x and

smaller number is y.

We know that, $Dividend = (Divisor \times Quotient) + Remainder$ ....(i)

When 3x is divided by y, we get 4 as quotient and 3 as remainder.

$\therefore$ by using (i), we get

$3x = 4y + 3$

$\Rightarrow 3x - 4y = 3$ ....(ii)

When 7y is divided by x, we get 5 as quotient and 1 as remainder.

$\therefore$ by using (i). we get

$7y = 5x + 1$

$\Rightarrow 5x - 7y + 1 = 0$ ....(iii)

By Solving equations (ii) and (iii), we get

$\frac{x}{-4-21} = \frac{-y}{3+15} = \frac{1}{-21+20}$

$\Rightarrow \frac{x}{-25} = \frac{-y}{18} = \frac{1}{-1}$

$\Rightarrow x = 25$ and $y = 18$

23. We have quadratic polynomial as $t^2 - 15$

$= t^2 - (\sqrt{15})^2$

$= (t - \sqrt{15})(t + \sqrt{15})$ [As, $x^2 - y^2 = (x - y)(x + y)$]

The value of $t^2 - 15$ is zero when $(t - \sqrt{15}) = 0$ or $(t + \sqrt{15}) = 0$,

i.e., when $t = \sqrt{15}$ or $t = -\sqrt{15}$

therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes = $\sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{coefficient of } t)}{\text{coefficient of } t^2}$

Product of zeroes = $(\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{constant term}}{\text{coefficient of } t^2}$

Hence verified.

24. Distance between (x, 0) and (0, 3) = 5

$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5$

$\Rightarrow \sqrt{(0 - x)^2 + (3 - 0)^2} = 5$

$\Rightarrow \sqrt{x^2 + 9} = 5$

Squaring,

$x^2 + 9 = 25 \Rightarrow x^2 = 25 - 9 = 16$

$\Rightarrow x^2 - 16 = 0 \Rightarrow (x + 4)(x - 4) = 0$

Either $x + 4 = 0$, then $x = -4$

or $x - 4 = 0$, then $x = 4$

Hence $x = 4, -4$

OR

Let the given points (8, 4), (5, 7) and (–1, 1) be denoted by A, B and C respectively.

$AB = \sqrt{(5-8)^2 + (7-4)^2} = \sqrt{9+9} = \sqrt{18}$

$BC = \sqrt{(-1-5)^2 + (1-7)^2} = \sqrt{36+36} = \sqrt{72}$

$AC = \sqrt{(-1-8)^2 + (1-4)^2} = \sqrt{81+9} = \sqrt{90}$

Since $AB^2 + BC^2 = 18 + 72 = 90 = AC^2$

$\therefore \Delta ABC$ is right angled at B.

Hence the given points are the vertices of a right triangle.

25. Let O be the centre of the given circle.

AB is the tangent drawn touching the circle at A.

Draw $AC \perp AB$ at point A, such that point C lies on the given circle.

$\angle OAB = 90°$ (Radius of the circle is perpendicular to the tangent)

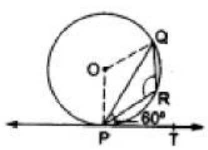
Given $\angle CAB = 90°$

$\therefore \angle OAB = \angle CAB$

This is possible only when centre O lies on the line AC.

Hence, perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

OR



In the given figure it is given that PQ is a chord of a circle with center O and $\angle QPT = 60°$.

let X be the point on the tangent PT of the circle.

Now from the given figure we have, $\angle QPT + \angle QPX = 180°$ (Linear pair)

$\Rightarrow \angle QPX = 180° - \angle QPT = 180° - 60° = 120°$

and $\angle QPX = \angle PRQ$ (Alternate segment theorem)

$\Rightarrow \angle PRQ = 120°$

Section C

26. Given ,

$2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow 2(1 - \cos^2\theta) - \cos^2\theta = 2$

$\Rightarrow 2 - 2\cos^2\theta - \cos^2\theta = 2$

$\Rightarrow 2 - 3\cos^2\theta = 2$

$\Rightarrow 3\cos^2\theta = 0$

$\Rightarrow \cos^2\theta = 0$

$\Rightarrow \cos^2\theta = \cos^2 90^\circ$

$\Rightarrow \theta = 90^\circ$

27. The given pair of linear equations is

$152x - 378y = -74$ ...(1)

$-378x + 152y = -604$ ....(2)

Adding equation (1) and equation (2), we get

$-226x - 226y = -678$

$\Rightarrow x + y = 3$ ....(3) .....Dividing throughout by -226

Subtracting equation (2) from equation (1), we get $530x - 530y = 530$

$\Rightarrow x - y = 1$ .....(4) ....Dividing throughout by 530

Adding equation (3) and equation (4), we get $2x = 4$

$\Rightarrow\ x=\frac{4}{2}=2$

Subtracting equation (4) from equation (3), we get 2y = 2

$\Rightarrow\ y=\frac{2}{2}=1$

Hence, the solution of the given pair of linear equations is x = 2, y = 1.

Verification: Substituting x = 2, y = 1,

We find that both the equations (1) and (2) are satisfied as shown below:

152x - 378y = (152)(2) - (378)(1)

= 304 - 378 = -74

-378x + 152y = (-378)(2) + (152)(1)

= -756 + 152 = -604

28. **Given:** Number of goats for trip = 105

Number of donkey for trip = 140

Number of cows for trip = 175

Therefore, The largest number of animals in one trip = HCF of 105. 140 and 175.

First consider 105 and 140

By applying Euclid's division lemma, we get

140 = 105 × 1 + 35

105 = 35 × 3 + 0

Therefore, HCF of 105 and 140 = 35

Now consider 35 and 175

Again applying Euclid's division lemma, we get

175 = 35 × 5 +0

HCF of 105, 140 and 175 is 35.

So 35 animals of same kind can go for trip in a single trip and number of trip is 105/35 +140/35 + 175/35 = 12

OR

We assume that $2+\sqrt{5}$ is a rational number.

So it can be written as $\frac{a}{b}$, where a, b are co-prime integers and b is not zero.

The new equation will be as below:

$\implies 2+\sqrt{5}=\frac{a}{b}$

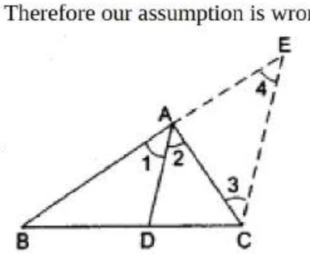
So, we will get $\sqrt{5}=\frac{a}{b}-2$

As we know that 2 and $\frac{a}{b}$ are rational numbers, their difference will be rational number only.

on the other hand, $\sqrt{5}$ is an irrational number and it can not be written as $\frac{a}{b}$.

So, this contradicts the fact that $\sqrt{5}$ is a rational number.

Therefore our assumption is wrong and $2+\sqrt{5}$ is irrational.

29. 

It is given that in $\triangle ABC$, AD, the bisector of $\angle A$ meets BC in D.

To Prove $\frac{BD}{DC}=\frac{AB}{AC}$

Construction Draw $CE \| DA$, meeting BA produced at E.

<u>**Proof:**</u> Since $DA \| CE$, we have

$\angle 2=\angle 3$ ....(alternative interior angles)

and $\angle 1=\angle 4$ ..... (corresponding angles)

But, $\angle 1=\angle 2$ ....(Because AD is bisector of $\angle A$)

$\therefore\ \angle 3=\angle 4$

So, AE = AC [since sides opposite to equal sides of a triangle are equal].

Now, in $\triangle BCE,$ we have $DA \| CE$.

Therefore by basic proportionality theorem, we have

$\frac{BD}{DC} = \frac{AB}{AE}$

$\Rightarrow \quad \frac{BD}{DC} = \frac{AB}{AC}$ [∵ AE = AC]

or $\frac{BD}{DC} = \frac{AB}{AC}$

Hence proved

30. Hexagon ABCDEF touches a circle at G, H, I, J, K, L. So, from the external point tangents drawn on the circle are equal in length.

If A is external point and AG and AL are tangents, so

AG = AL ...(i)

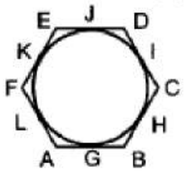
Similarly for B, BG = BH ...(ii)

Similarly for C, Cl = CH ... (iii)

Similarly for D, DI = DJ ... (iv)

EK = EJ ... (v)

and FK = FL ... (vi)



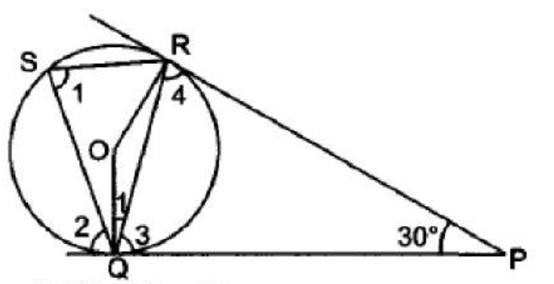
Adding (i), (ii), (iii), (iv), (v) and (vi), we get

$AG + BG + CI + ID + EK + FK = BH + CH + DJ + EJ + FL + AL$

$\Rightarrow (AG + BG) + (CI + ID) + (EK + FK) = (BH + CH) + (JD + EJ) + (FL + AL)$

$\Rightarrow AB + CD + EF = BC + DE + FA.$

OR



In $\triangle RQP$, $QP = RP$

$\therefore \angle 3 = \angle 4$

Now $\angle 3 + \angle 4 + 30^\circ = 180^\circ$

$\Rightarrow 2\angle 3 = 150^\circ \Rightarrow \angle 3 = 75^\circ$

Now $\angle \mathrm{QOR} + \angle \mathrm{QPR} = 180^\circ$

$\Rightarrow \angle \mathrm{QOR} = 150^\circ$

Now, $\angle 1 = \frac{1}{2}\angle \mathrm{QOR} \Rightarrow \angle 1 = 75^\circ$

Also $SR \,||\, QP$

$\therefore \quad \angle 1 = \angle 2$ [Alternate interior angles]

$\Rightarrow \angle 2 = 75^\circ$

$\angle 2 + \angle RQS + \angle 3 = 180^\circ$

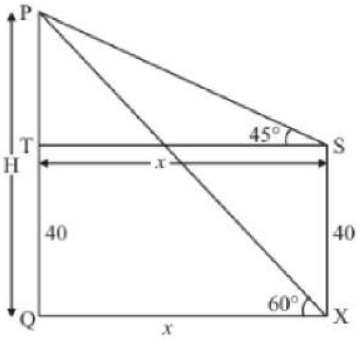
$\Rightarrow \angle \mathrm{ROS} = 180^\circ - 150^\circ = 30^\circ$

31. Let PQ be the tower of height H m and an angle of elevation of the top of tower PQ from point X is 60°. The angle of elevation at 40 m vertical from point X is 45°.

Let PQ = H m and SX = 40m. QX = x, $\angle$PST = 45$^{\mathrm{o}}$, $\angle$PXQ = 60$^{\mathrm{o}}$,

Here we have to find height of tower

The corresponding figure is as follows



We use trigonometric ratios.

In $\triangle$PST

$\Rightarrow \tan 45^o = \frac{h}{x}$

$\Rightarrow 1 = \frac{h}{x}$

$\Rightarrow$ x = h

Again in $\triangle$PXQ

$\Rightarrow \tan 60^o = \frac{h+40}{x}$

$\Rightarrow \sqrt{3} = \frac{h+40}{x}$

$\Rightarrow$ h + 40 = $\sqrt{3}h$

$\Rightarrow$ h $(\sqrt{3}-1)$ = 40

$\Rightarrow$ h = $\frac{40}{\sqrt{3}-1}$

$\Rightarrow$ h = 54.64

Therefore H = 54.64 + 40

$\Rightarrow$ H = 94.64

Hence the height of tower is 94.64 m.

Section D

32. Let number of books the shopkeeper buys = x

Price of each book = Rs $\frac{1200}{x}$

cost of each book when x +10 books are bought = RS $\frac{1200}{x+10}$

According to given question,

$\frac{1200}{x} - \frac{1200}{x+10} = 20$

$1200(\frac{1}{x} - \frac{1}{x+10}) = 20$

$(\frac{1}{x} - \frac{1}{x+10}) = \frac{20}{1200}$

$\frac{(x+10)-x}{x(x+10)} = \frac{1}{60}$

x +10- x = $\frac{x^2+10x}{60}$

600 = x^2 + $10x$

x^2+ $10x$ - 600 = 0

Here, it is quadratic equation

x^2 + $30x$ - $20x$ - 600 = 0

$x(x+30)$ -20(x+30) = 0

(x+30) (x-20) = 0

either

(x+30) =0 or (x-20) = 0

x = -30 or x= 20

x= -30, is not possible because the number of books can't be negative.

so, number of books = x = 20.

OR

Let breadth of the rectangular park = x m

Then, length of the rectangular park = $(x+3)$ m

Now, area of the rectangular park is = $x(x+3) = (x^2+3x)m^2$ $[\because area = length \times breadth]$

Given, base of the triangular park = Breadth of the rectangular park

Therefore base of triangular park is = x m

and it is given that altitude of triangular park is = 12 m

Therefore, area of the triangular park will be = $\frac{1}{2} \times x \times 12 = 6x \ m^2$ $[\because area(Triangle) = \frac{1}{2} \times base \times altitude]$

As per the question area of rectangular park is = $4+$ Area of triangular park

$\Rightarrow x^2 + 3x = 4 + 6x$

$\Rightarrow x^2 + 3x - 6x - 4 = 0$

$\Rightarrow x^2 - 3x - 4 = 0$

$\Rightarrow x^2 - 4x + x - 4 = 0$ [by factorization]

$\Rightarrow x(x-4) + 1(x-4) = 0$

$\Rightarrow (x-4)(x+1) = 0$

$\Rightarrow x - 4 = 0$ or $x + 1 = 0$

$\Rightarrow$ x = 4 or x = - 1

Since, breadth cannot be negative , so we will neglect $x = -1$ and choose $x = 4$

Hence, breadth of the rectangular park will be = 4 m

and length of the rectangular park will be = $x + 3 = 4 + 3 = 7m$

Verification:

Area of rectangular park is = 28 m^2

Area of triangular park is = 24 m^2 = (28 – 4) m^2

33. In $\triangle PQR$, $\frac{PS}{SQ} = \frac{4}{1} = 4$

and $\frac{PT}{TR} = \frac{6}{1.5} = 4$

Thus, $\frac{PS}{SQ} = \frac{PT}{TR}$

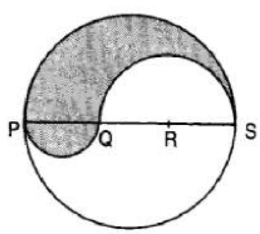
Hence, ST $\parallel$ QR [by converse of basic proportionality theorem]

34. PS = Diameter of a circle of radius 6 cm = 12 cm

$\therefore$ PQ = QR = RS = $\frac{12}{3}$ = 4cm, QS = QR + RS = (4 + 4)cm = 8 cm

Let P be the perimeter and A be the area of the shaded region.

P = Arc of semi-circle of radius 6 cm + Arc of semi-circle of radius 4 cm + Arc of semi-circle of radius 2 cm



$\Rightarrow P = (\pi \times 6 + \pi \times 4 + \pi \times 2)\text{cm} = 12\pi\text{cm}$

and, A = Area of semi-circle with PS as diameter + Area of semi-circle with PQ as diameter - Area of semi-circle with QS as diameter.

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7}\left(6^2 + 2^2 - 4^2\right) = \frac{1}{2} \times \frac{22}{7} \times 24 = \frac{264}{7}\text{cm}^2$ = 37.71 cm^2

OR

Let the radii of the circles are r_1 cm and r_2 cm

$\therefore r_1 + r_2$ = 14 ...(i)

And, sum of their areas = $\pi r_1^2 + \pi r_2^2$

$130\pi = \pi\left(r_1^2 + r_2^2\right)$

or, $130\pi = \pi\left(r_1^2 + r_2^2\right)$

$\therefore \quad r_1^2 + r_2^2 = 130$ ...(ii)

$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$

or, $(14)^2 = 130 + 2r_1r_2$

or, $2r_1r_2$ = 196 -130

0r, $2r_1r_2$= 66

$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$

$(r_1 - r_2)^2$= 130 - 66

$(r_1 - r_2)^2$= 64

or, $r_1 - r_2 = 8$ ...(iii)
From (i) and (iii), $2r_1 = 22$
or, $r_1 = 11$ cm
$r_2 = 14 - 11$
$r_2 = 3\ cm$.

35.

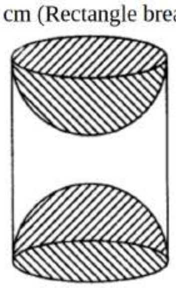
Age of Employees(in years)	**Number of employees in factories**	
	A	**B**
20 - 30	5	8
30 - 40	26	40
40 - 50	78	58
50 - 60	104	90
60 - 70	98	83

For factory A, max. frequency = 104.
Modal Class = 50 - 60
Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$
$f_1 = 100$
$f_0 = 78$
$f_2 = 98$
h = 10
l = 50
Mode = 50 + $\left[\frac{104-78}{2\times104-78-98}\right] \times 10$
= 50 + $\frac{26}{32} \times 10$
$= 50 + 8.125 = 58.125$
For factory B, max. frequency = 90.
Modal class = 50 - 60.
Mode = 50 + $\left(\frac{90-58}{2\times90-58-83}\right) \times 10$
= 50 + $\frac{32\times10}{39}$
$= 50 + 8.205 = 58.205$

Section E

36. **Read the text carefully and answer the questions:**

A carpenter used to make different kinds and different shapes of a toy of wooden material. One day a man came to his shop to purchase an article that has values as per his requirement. He instructed the carpenter to make the toy by taking a wooden block of rectangular shape with height 12 cm and width 9 cm, then shaping this block as a solid cylinder and then scooping out a hemisphere from each end, as shown in the given figure. The difference between the length of rectangle and height of the cylinder is 2 cm (Rectangle length > Cylinder height), and the difference between the breadth of rectangle and the base of cylinder is also 2 cm (Rectangle breadth > Cylinder base(diameter)).



(i) Given:
Length of rectangle = 12 cm
Width of rectangle = 9 cm
After scratching the rectangle into a cylinder,
Height of cylinder = 10 cm

Diameter of base = 7 cm

$\Rightarrow$ Radius of base = 3.5 cm

Volume of cylinder = $\pi r^2 h$

= $\frac{22}{7} \times 3.5^2 \times 10$ = 385 cm^3

(ii) Given:

length of rectangle = 12 cm

width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

height of cylinder = 10 cm

diameter of base = 7 cm

$\Rightarrow$ radius of base = 3.5 cm

Volume of wood scooped out = 2 $\times$ volume of hemisphere

$\Rightarrow$ Volume of wood scooped-out = $2 \times \frac{2}{3} \times \pi \times r^3$

$\Rightarrow$ Volume of wood scooped out = $\frac{4}{3} \times \frac{22}{7} \times (3.5)^3 = 179.66 \text{ cm}^3$

(iii)Given:

length of rectangle = 12 cm

width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

height of cylinder = 10 cm

diameter of base = 7 cm

$\Rightarrow$ radius of base = 3.5 cm

Total surface area of the article

$= 2\pi(3.5)(10) + 2\left[2\pi(3.5)^2\right]$

$= 70\pi + 49\pi = 119\pi$

$= 119 \times \frac{22}{7} = 17 \times 22$

= 374 cm^2

OR

Given:

length of rectangle = 12 cm

width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

height of cylinder = 10 cm

diameter of base = 7 cm

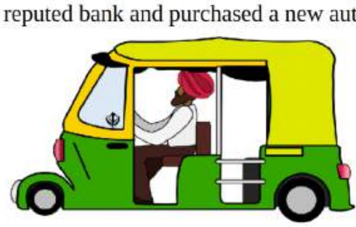
$\Rightarrow$ radius of base = 3.5 cm

T.S.A of cylinder = $2\pi r(r + h)$

$\Rightarrow$ T.S.A of cylinder = $2 \times \frac{22}{7} \times 3.5(3.5 + 10)$ = 99 cm^2

37. **Read the text carefully and answer the questions:**

Jaspal Singh is an auto driver. His autorickshaw was too old and he had to spend a lot of money on repair and maintenance every now and then. One day he got to know about the EV scheme of the Government of India where he can not only get a good exchange bonus but also avail heavy discounts on the purchase of an electric vehicle. So, he took a loan of ₹1,18,000 from a reputed bank and purchased a new autorickshaw.



Jaspal Singh repays his total loan of 118000 rupees by paying every month starting with the first instalment of 1000 rupees.

(i) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100

Now, amount paid in the 30th installment,

a_{30} = 1000 + (30 - 1)100 = 3900 {a_n = a + (n - 1)d}

(ii) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month=1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100

Amount paid in 30 instalments,

$S_{30} = \frac{30}{2}[2 \times 1000 + (30 - 1)100]$ = 73500

Hence, remaining amount of loan that he has to pay = 118000 - 73500 = 44500 Rupees

OR

Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month therefore, installment amount in second month = ₹ 1100, third month = ₹1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100

If he increases the instalment by 200 rupees every month, amount would he pay in 40th instalment

Then a = 1000, d = 200 and n = 40

a_{40} = a + (n - 1)d

$\Rightarrow a_{40}$ = 1000 + (40 - 1)200

$\Rightarrow a_{40}$ = 880

(iii) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100

Amount paid in 100 instalments

$S_n = \frac{n}{2}[2a + (n - 1)d]$

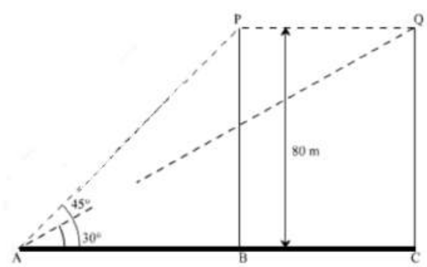
$S_n = \frac{100}{2}[2 \times 1000 + (100 - 1)100]$

$\Rightarrow S_n$ = 100000 + 9900

$\Rightarrow$ 109900

38. **Read the text carefully and answer the questions:**

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45^o. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30^o. Find the speed of flying of the bird.



(i) Given height of tree = 80m, P is the initial position of bird and Q is position of bird after 2 sec the distance between observer and the bottom of the tree

In $\triangle$ABP

tan $45^o = \frac{BP}{AB}$

$\Rightarrow 1 = \frac{80}{AB}$

$\Rightarrow$ AB = 80 m

(ii) The speed of the bird

In $\triangle$AQC

tan $30^o = \frac{QC}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{AC}$

$\Rightarrow$ AC = $80\sqrt{3}$ m

AC - AB = BC

$\Rightarrow$ BC = $80\sqrt{3} - 80 = 80(\sqrt{3} - 1)$m

Speed of bird = $\frac{\text{Distance}}{\text{Time}}$

$\Rightarrow \frac{BC}{2} = \frac{80(\sqrt{3}-1)}{2}$ = $40(\sqrt{3}-1)$

$\Rightarrow$ Speed of the bird = 29.28 m/sec

OR

The distance between initial position of bird and observer.

In $\triangle$ABP

sin 45° = $\frac{BP}{AP}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{80}{AP}$

$\Rightarrow$ AP = $80\sqrt{2}m$

(iii) The distance between second position of bird and observer.

In $\triangle$AQC

sin 30° = $\frac{QC}{AQ}$

$\Rightarrow \frac{1}{2} = \frac{80}{AQ}$

$\Rightarrow$ AQ = 160 m