SAMPLE QUESTION PAPER (BASIC) - 03

Class 10 - Mathematics

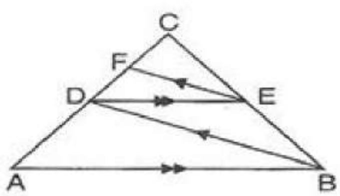
Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. We have, $AB||DE$ and $BD||EF$.Then, **[1]**



a) $BC^2 = AB\ .\ CE$

b) $AC^2 = BC\ .\ DC$

c) $AB^2 = AC\ .\ DE$

d) $DC^2 = CF \times AC$

2. If the product of zeros of the polynomial f(x) = $ax^3 - 6x^2 + 11x - 6$ is 4, then a = ? **[1]**

a) $\frac{2}{3}$

b) $-\frac{2}{3}$

c) $-\frac{3}{2}$

d) $\frac{3}{2}$

3. In △ABC, if ∠A = x^o, ∠B = $3x^o$ and ∠C = y^o. If 3y - 5x = 30, then ∠B = **[1]**

a) 60^o

b) 45^o

c) 30^o

d) 90^o

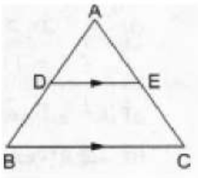
4. The graphs of the equations 6x - 2y + 9 = 0 and 3x - y + 12 = 0 are two lines which are **[1]**

a) perpendicular to each other

b) parallel

c) coincident | d) intersecting exactly at one point

5. In $\triangle$ABC, DE || BC so that AD = 2.4 cm, AE = 3.2 cm and EC = 4.8 cm. Then, AB = ? **[1]**



a) 6.4 cm
b) 7.2 cm
c) 3.6 cm
d) 6 cm

6. In a lottery, there are 8 prizes and 16 blanks. What is the probability of getting a prize? **[1]**

a) $\frac{1}{2}$
b) None of these
c) $\frac{1}{3}$
d) $\frac{2}{3}$

7. If $2\cos 3\theta = 1$ then θ= ? **[1]**

a) 30°
b) 10°
c) 15°
d) 20°

8. In a data, if l = 40, h = 15, f_1 = 7, f_0 = 3, f_2 = 6, then the mode is **[1]**

a) 82
b) 62
c) 52
d) 72

9. If in $\triangle$ ABC and $\triangle$ DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then $\triangle$ ABC $\sim$ $\triangle$DEF when **[1]**

a) ∠B = ∠E
b) ∠B = ∠D
c) ∠A = ∠F
d) ∠A = ∠D

10. The exponent of 2 in the prime factorisation of 144, is **[1]**

a) 4
b) 5
c) 6
d) 3

11. If $x^2 + 5kx + 16 = 0$, has equal roots, then the value of k is **[1]**

a) $\pm\frac{64}{25}$
b) $\pm\frac{8}{5}$
c) $\pm\frac{5}{8}$
d) $\pm\frac{25}{64}$

12. The point where the medians of a triangle meet is called the ________ of the triangle **[1]**

a) circumcentre
b) centroid
c) orthocentre
d) None of these

13. The mean of the first 10 prime numbers is **[1]**

a) 129
b) 1.29
c) 12.9
d) 11.9

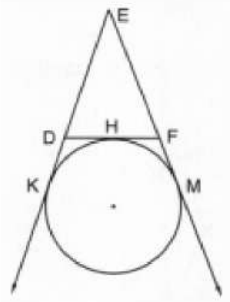
14. If $\sqrt{3}\tan\theta = 3\sin\theta$, then the value of $\sin^2\theta - \cos^2\theta$ is **[1]**

a) 1
b) $\frac{1}{2}$
c) 0
d) $\frac{1}{3}$

15. A pole casts a shadow of length $2\sqrt{3}$ m on the ground when the sun's elevation is 60°. The height of the pole is **[1]**

a) 12 m
b) 6 m
c) $4\sqrt{3}$ m
d) 3 m

16. In Figure, a circle touches the side DF of $\triangle$EDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then the perimeter of $\triangle$EDF is **[1]**



a) 18 cm
b) 13.5 cm
c) 9 cm
d) 12 cm

17. In $\triangle$DEF and $\triangle$PQR, it is given that $\angle$D = $\angle$Q and $\angle$R = $\angle$E, then which of the following is not true? **[1]**

a) $\frac{DE}{QR} = \frac{DF}{PQ}$
b) $\frac{EF}{RP} = \frac{DE}{QR}$
c) $\frac{EF}{PR} = \frac{DF}{PQ}$
d) $\frac{DE}{PQ} = \frac{EF}{RP}$

18. In the equation $ax^2 + bx + c = 0$, it is given that $D = (b^2 - 4ac) > 0$. Then, the roots of the equation are **[1]**

a) imaginary
b) real and equal
c) real and unequal
d) none of these

19. **Assertion:** $x^2 + 4x + 5$ has two zeroes. **[1]**
Reason: A quadratic polynomial can have at the most two zeroes.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
c) Assertion is correct statement but reason is wrong statement.
d) Assertion is wrong statement but reason is correct statement.

20. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 350 cm^2. **[1]**
Reason (R): Total surface area of a cuboid is 2(lb + bh + hl)

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

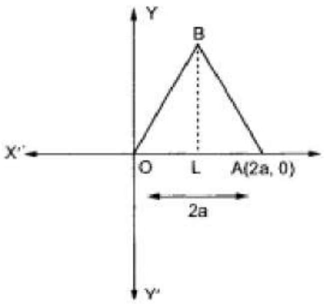
Section B

21. Find the non zero values of k for which the roots of the quadratic equation $9x^2 - 3kx + k = 0$ are real and equal. **[2]**

22. Prove that the points (7, 10), (-2, 5) and (3, - 4) are the vertices of an isosceles right triangle. **[2]**

OR

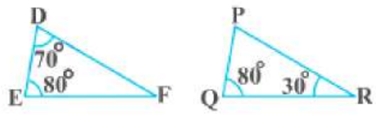
Find the coordinates of the vertices of an equilateral triangle of side 2a as shown in the figure.



23. Find the largest number which divides 245 and 1037, leaving remainder 5 in each case. **[2]**

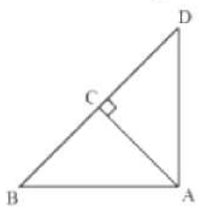
24. Verify: sin 60° cos 30° - cos 60° sin 30° = sin 30° **[2]**

25. State the pair of triangles in Fig, are similar. Write the similarity criterion used by you for answering the question and also write the pair of similar triangles in the symbolic form: **[2]**



OR

$\triangle$ABD is a right triangle right-angled at A and AC$\perp$ BD. Show that AB^2 =BC $\times$ BD



Section C

26. Solve: $\frac{2}{(x+1)} + \frac{3}{2(x-2)} = \frac{23}{5x}, x \neq$ 0, -1, 2. **[3]**

27. In the given figure, E is a point on side CB produced of an isosceles $\triangle ABC$ with AB = AC. If $AD \perp BC$ and $EF \perp AC$, Prove that $\triangle ABD \sim \triangle ECF$. **[3]**

28. Find the point on the x–axis which are at a distance of $2\sqrt{5}$ from point (7, –4). How many such points are there? **[3]**

OR

The line segment joining the points A(3, - 4) and B(1, 2) is trisected at the points P and Q. Find the coordinates of P.

29. In a school there are two sections, namely A and B, of class X. There are 30 students in section A and 28 students in section B. Find the minimum number of books required for their class library so that they can be distributed equally among students of section A or section B. **[3]**

30. The angle of elevation of an aeroplane from a point on the ground is 45^o. After flying for 15 s, the angle of elevation changes to 30^o. If the aeroplane if flying at a constant height of 2500 m, then find the average speed of the aeroplane. **[3]**

OR

Two men standing on opposite sides of a tower measure the angles of elevation of the top of the tower as 30^o and 60^o respectively. If the height of the tower is 20 m, then find the distance between the two men.

31. If the median of the following frequency distribution is 32.5, find the values of f_1 and f_2. **[3]**

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	**Total**
Frequency	f_1	5	9	12	f_2	3	2	40

Section D

32. When 3 is added to the denominator and 2 is subtracted from the numerator a fraction becomes $\frac{1}{4}$. When 6 is **[5]**

added to numerator and denominator is multiplied by 3, fraction becomes $\frac{2}{3}$. Find the fraction.

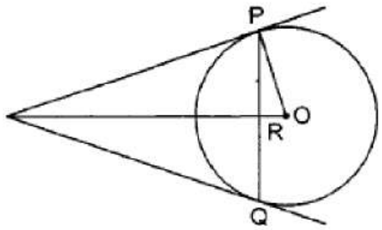
OR

Solve system of equations:

$11x + 15y + 23 = 0$

$7x - 2y - 20 = 0$.

33. In figure, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T. Find the length of TP. **[5]**



34. Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending angle of 90° at the centre. **[5]**

OR

A chord of a circle of radius 10cm subtends a right angle at the center. Find the area of the corresponding: (Use π = 3.14)

i. minor sector
ii. major sector
iii. minor segment
iv. major segment

35. From a deck of 52 playing cards, Jacks and kings of red colour and Queen and Aces of black colour are removed. The remaining cards are mixed and a card is drown at random. Find the probability that the drawn card is **[5]**

i. A black Queen
ii. A card of red colour
iii. A Jack of black colour
iv. A face card

Section E

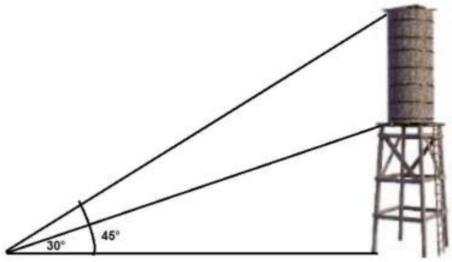
36. **Read the text carefully and answer the questions:** **[4]**

In a society, there are many multistory buildings. The RWA of the society wants to install a tower and a water tank so that all the households can get water without using water pumps.

For this they have measured the height of the tallest building in the society and now they want to install a tower that will be taller than that so that the level of water must be higher than the tallest building in their society. Here is one solution they have found and now they want to check if it will work or not.

From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is

300. the angle of elevation of the top of the water tank is 45^{o}.



(i) What is the height of the tower?

(ii) What is the height of the water tank?

(iii) At what distance from the bottom of the tower the angle of elevation of the top of the tower is 45^{o}.

OR

What will be the angle of elevation of the top of the water tank from the place at $\frac{40}{\sqrt{3}}$m from the bottom of the tower.

37. **Read the text carefully and answer the questions:** **[4]**

Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves ₹27.5 On each succeeding day he increases his saving by ₹2.5.

(i) Find the amount saved by Sehaj on 10^{th} day.

(ii) Find the amount saved by Sehaj on 25^{th} day.

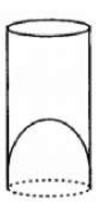
(iii) Find the total amount saved by Sehaj in 30 days.

OR

Find in how many days Sehaj saves ₹1400.

38. **Read the text carefully and answer the questions:** **[4]**

A juice seller is serving his customers using cylindrical container with radius 20cm and height 50cm. He serves juice into a glass as shown in Fig. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass.



(i) If the height of a glass was 10 cm, find the apparent capacity of the glass.

(ii) Also, find its actual capacity. (Use π = 3.14)

(iii) Find the capacity of the container in liter?

OR

How many glasses he serves if the container is full?

Solution

SAMPLE QUESTION PAPER (BASIC) - 03

Class 10 - Mathematics

Section A

1. **(d)** $DC^2 = CF \times AC$

   Explanation: In $\triangle$ABC, using Thales theorem,
   $\frac{\text{DC}}{\text{AC}} = \frac{\text{CE}}{\text{BC}}$ $[AB||DE]$ ....(i)
   And in triangle BCD, using Thales theorem,
   $\frac{\text{CF}}{\text{DC}} = \frac{\text{CE}}{\text{BC}}$ $[BD||EF]$ ....(ii)
   From eq. (i) and (ii), we have
   $\frac{\text{DC}}{\text{AC}} = \frac{\text{CF}}{\text{DC}}$
   $\Rightarrow DC^2 = CF \times AC$

2. **(d)** $\frac{3}{2}$

   Explanation: Since α and β are the zeros of quadratic polynomial f(x) = $ax^3 - 6x^2 + 11x - 6$
   $\alpha\beta = \frac{-\text{ Constant term}}{\text{Coefficient of } x^2}$
   So we have
   $4 = -\left(\frac{-6}{a}\right)$
   $4 = \frac{6}{a}$
   $a = \frac{3}{2}$
   Therefore value of a is $\frac{3}{2}$.

3. **(d)** 90^o

   Explanation: Here, A = x, B = 3x, C = y.
   180 = 4x + y ... (i) (Sum of angles of a triangle = x + 3x + y)
   180 - 4x = y ... (ii)
   Also, 3y - 5x = 30 ... (iii)
   Substituting the value of (ii) in (iii)
   3(180 - 4x) - 5x = 30
   540 - 12x - 5x = 30
   -17x = 30 - 540
   17x = 510
   x = 30 ... (iv)
   But, angle B = 3x ... (Given)
   Therefore angle B = 3 $\times$ 30 = 90^o

4. **(b)** parallel

   Explanation: We have,
   6x - 2y + 9 = 0
   And, 3x - y + 12 = 0
   Here, $a_1 = 6$, $b_1 = -2$ and $c_1 = 9$
   $a_2 = 3$, $b_2 = -1$ and $c_2 = 12$
   $\frac{a1}{a2} = \frac{6}{3} = \frac{2}{1}, \frac{b1}{b2} = \frac{-2}{-1} = \frac{2}{1}$ and $\frac{c_1}{c_2} = \frac{9}{12} = \frac{3}{4}$
   Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
   Hence, the given system has no solution and the lines are parallel.

5. **(d)** 6 cm

   Explanation: In $\triangle$ABC, DE || BC
   AD = 2.4 cm, AE = 3.2 cm, EC = 4.8 cm
   Let AD = x cm
   DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$
$\Rightarrow \frac{2.4}{x} = \frac{3.2}{4.8} \Rightarrow x = \frac{2.4 \times 4.8}{3.2}$
$\Rightarrow x = \frac{24 \times 48 \times 10}{32 \times 10 \times 10} = \frac{36}{10} = 3.6$
$\therefore AB = AD + DB = 2.4 + 3.6 = 6.0\text{cm}$

6. **(c)** $\frac{1}{3}$

Explanation: Total number of lottery tickets = 8 + 16 = 24.

Number of prizes = 8.

$\therefore$P (getting a prize) = $\frac{8}{24} = \frac{1}{3}$

7. **(d)** 20°

Explanation: $2\cos 3\theta = 1 \Rightarrow \cos 3\theta = \frac{1}{2} = \cos 60^\circ \quad \Rightarrow 3\theta = 60^\circ \Rightarrow \theta = 20^\circ$

8. **(c)** 52

Explanation: Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

= 40 + $\frac{7-3}{7 \times 2 - 3 - 6} \times 15$

= 40 + $\frac{4}{5} \times 15$

= 40 + 12

= 52

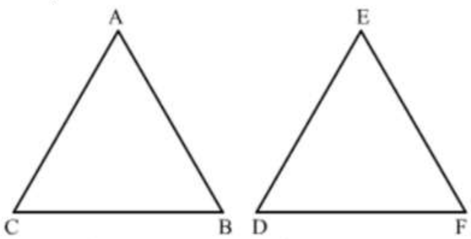
9. **(b)** ∠B = ∠D

Explanation:

Given: In △ABC and △DEF, $\frac{AB}{DE} = \frac{BC}{FD}$.

We know that if in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.

Then, ∠B = ∠D

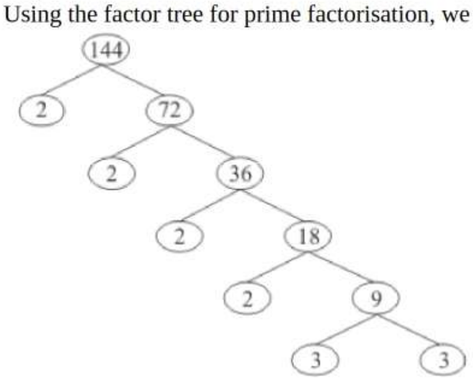


Hence, △ABC is similar to △DEF, we should have ∠B = ∠D.

10. **(a)** 4

Explanation:

Using the factor tree for prime factorisation, we have:



Therefore, 144 = $2 \times 2 \times 2 \times 2 \times 3 \times 3$

$\Rightarrow 144 = 2^4 \times 3^2$

Thus, the exponent of 2 in 144 is 4.

11. **(b)** $\pm\frac{8}{5}$

Explanation: Here, a = 1, b = 5k, c = 16

If $x^2 + 5kx + 16 = 0$ has equal roots,

then, $b^2 - 4ac = 0$

$\Rightarrow (5k)^2 - 4 \times 1 \times 16 = 0$

$\Rightarrow 25k^2 - 64 = 0$

$\Rightarrow 25k^2 = 64$

$\Rightarrow k^2 = \frac{64}{25}$

$\Rightarrow k = \pm\frac{8}{5}$

12. **(b)** centroid

Explanation: The point where three medians of a triangle meet is called the centroid of the triangle. It is the centre of gravity of the triangle. It divides the median in the ratio 2 :1

13. **(c)** 12.9

Explanation: The first 10 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

$\therefore$ Mean = $\frac{\text{Sum of first 10 prime numbers}}{10}$

= $\frac{2+3+5+7+11+13+17+19+23+29}{10}$

= $\frac{129}{10}$

= 12.9

14. **(d)** $\frac{1}{3}$

Explanation: Given: $\sqrt{3}\tan\theta = 3\sin\theta$

$\Rightarrow \sqrt{3}\frac{\sin\theta}{\cos\theta} = 3\sin\theta$

$\Rightarrow \frac{\sqrt{3}}{3} = \cos\theta$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$

And $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{1}{3}} = \sqrt{\frac{2}{3}}$

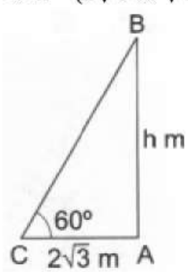
$\therefore \sin^2\theta - \cos^2\theta = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

15. **(b)** 6 m

Explanation: Let the height of the pole be h metres.

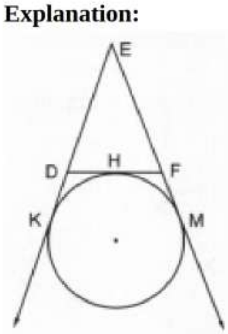
Then, $\frac{h}{2\sqrt{3}}$ = tan 60^o = $\sqrt{3}$

$\Rightarrow$ h = $(2\sqrt{3} \times \sqrt{3})$ = 6.



16. **(a)** 18 cm

Explanation:



In $\triangle DEF$

DF touches the circle at H

and circle touches ED and EF Produced at K and M respectively

EK = 9 cm

EK and EM are the tangents to the circle

EM = EK = 9 cm

Similarly DH and DK are the tangent

DH = DK and FH and FM are tangents

FH = FM

Now, perimeter of ΔDEF

= ED + DF + EF

= ED + DH + FH + EF

= ED + DK + FM + EF

= EK + EM

= 9 + 9

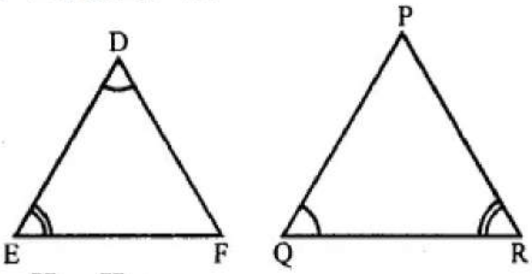
= 18 cm

17. **(d)** $\frac{DE}{PQ} = \frac{EF}{RP}$

Explanation:

In $\triangle$DEF and $\triangle$PQR,

$\angle$D = $\angle$Q and $\angle$R = $\angle$E



Then $\frac{\text{DE}}{\text{PQ}} = \frac{\text{EF}}{\text{RP}}$ is not true.

($\because$ There are not corresponding sides)

18. **(c)** real and unequal

Explanation: When D > 0, the roots of the given quadratic equation are real and unequal.

19. **(d)** Assertion is wrong statement but reason is correct statement.

Explanation: Assertion is wrong statement but reason is correct statement.

20. **(d)** A is false but R is true.

Explanation: A is false but R is true.

Section B

21. Here, a = 9, b = - 3 k, c = k

Since roots of the equation are equal

So, $b^2 - 4ac = 0$

$(-3k)^2 - (4 \times 9 \times k) = 0$

$9k^2 - 36k = 0$

$k^2 - 4k = 0$

k(k - 4) = 0

k = 0 or k = 4

But given k is non zero hence k = 4 for which roots of the quadratic equation are real and equal.

22. Let A(7, 10), B(-2, 5) and C(3, -4) be the vertices of given isosceles triangle.

Then, $AB = \sqrt{(7+2)^2 + (10-5)^2} = \sqrt{106}$

$AC = \sqrt{(7-3)^2 + (10+4)^2} = \sqrt{212}$

$BC = \sqrt{(3+2)^2 + (-4-5)^2} = \sqrt{106}$

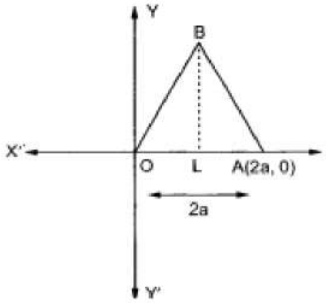
Thus, AB = AC

Also $AB^2 + BC^2 = 106 + 106 = 212 = AC^2$ (pythagoras theorem)

Hence, given vertices are coordinates of an isosceles right triangle.

OR

We have to find the coordinates of the vertices of an equilateral triangle of side 2a as shown in the figure.



Since, OAB is an equilateral triangle of side 2a. Therefore,

OA = AB = OB = 2a

Let BL perpendicular from B on OA.Then

OL = LA = a

In △OLB, we have

$OB^2 = OL^2 + LB^2$

$\Rightarrow (2a)^2 = a^2 + LB^2$

$\Rightarrow LB^2 = 3a^2$

$\Rightarrow \quad LB = \sqrt{3}a$

Clearly, coordinates of O are (0,0) and that of A are (2a, 0). Since, OL = a and LB = $\sqrt{3}$a. So, the coordinates of B are (a, $\sqrt{3}$a)

23. Clearly, the required number divides (245 - 5) = 240 and (1037 - 5) = 1032 exactly.

So, the required number is HCF of (240, 1032).

2	240
2	120
2	60
2	30
3	15
	5

2	1032
2	516
2	258
3	129
	43

Now $240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = \left(2^4 \times 3 \times 5\right)$

and $1032 = 2 \times 2 \times 2 \times 3 \times 43 = \left(2^3 \times 3 \times 43\right)$

$\therefore$ HCF (240, 1032)= $\left(2^3 \times 3\right)$

HCF (240, 1032) = 24

Hence, the largest number which divides 245 and 1037, leaving remainder 5 is 24.

24. Consider L.H.S. = sin 60° cos 30° - cos 60° sin 30°

= ($\sqrt{3}$/2) × ($\sqrt{3}$/2) – (1/2)(1/2)

= (3/4) – (1/4)

= 2/4

= 1/2

Consider R.H.S. = sin 30° = 1/2

L.H.S. = R.H.S.

Hence, verified.

25. In triangle DEF, we have

$\angle D + \angle E + \angle F = 180^\circ$ (Sum of angles of triangle)

$70^\circ + 80^\circ + \angle F = 180^\circ$

$\angle F = 30^\circ$

In PQR, we have

$\angle P + \angle Q + \angle R = 180^\circ$

$\angle P + 80^\circ + 30^\circ = 180^\circ$

$\angle P = 70^\circ$

In triangle DEF and PQR, we have

$\angle D = \angle P = 70^\circ$

$\angle F = \angle Q = 80^\circ$

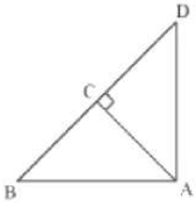
$\angle F = \angle R = 30^\circ$

Hence, ΔDEF ~ ΔPQR (AAA similarity)

OR

Given: $\triangle$ABD is a right triangle right-angled at A and AC$\perp$ BD.

To Prove: AB2 =BC × BD



Proof: In $\triangle$ADB and $\triangle$CAB

$\angle DAB = \angle ACB = 90^\circ$

$\angle ABD = \angle CBA$ [common angle]

$\angle ADB = \angle CAB$ [remaining angle]

So, $\triangle$ADB $\sim$ $\triangle$CAB (by AAA similarity)

Therefore $\frac{AB}{CB} = \frac{BD}{AB}$

$\Rightarrow$ AB^2 = CB × BD

Section C

26. Given,

$\frac{2}{(x+1)} + \frac{3}{2(x-2)} = \frac{23}{5x}$

Taking LCM, we get

$\Rightarrow \frac{4(x-2)+3(x+1)}{2(x+1)(x-2)} = \frac{23}{5x} \Rightarrow \frac{7x-5}{2\left(x^2-x-2\right)} = \frac{23}{5x}$

By cross multiplication

$\Rightarrow$ 5x(7x - 5) = 46(x^2 - x - 2)

$\Rightarrow$ 35x^2 -25x = 46x^2 - 46x - 92

$\Rightarrow 46x^2 - 35x^2 - 46x + 25x - 92 = 0$

$\Rightarrow$ 11x^2 - 21x - 92 = 0

$\Rightarrow$ 11x^2 - 44x + 23x - 92 = 0

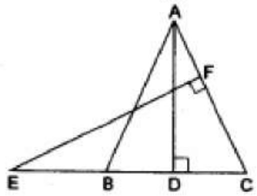
$\Rightarrow$ 11x(x - 4) + 23(x - 4) = 0

$\Rightarrow$ (x - 4)(11x + 23) = 0

$\Rightarrow$ x - 4 = 0 or 11x + 23 = 0

$\Rightarrow$ x = 4 or x = $\frac{-23}{11}$

Therefore, 4 or $\frac{-23}{11}$ are the roots of the given equation.

27. 

Given A $\triangle ABC$ in which AB = AC and $AD \perp BC$. Side CB is produced to E and $EF \perp AC$.

To Prove $\triangle ABD \sim \triangle ECF$.

Proof: We know that the angles opposite to equal sides of a triangle are equal.

$\therefore \quad \angle B = \angle C$ [$\because$ AB = AC].

Now, in $\triangle ABD$ and $\triangle ECF$, we have

$\angle B = \angle C$ [proved above]

$\angle ADB = \angle EFC = 90^\circ$

$\therefore \quad \triangle ABD \sim \triangle ECF$ [by AA-similarity],

28. Let point P(x, 0) be a point on x–axis, and A be the point (7, –4).

So, AP = $2\sqrt{5}$ [Given]

⇒ AP^2 = 4 × 5 = 20

⇒ (x – 7)^2 + [0 – (–4)]^2 = 20

$\Rightarrow x^2 + 49 - 14x + 16 = 20$

$\Rightarrow x^2 - 14x - 20 + 65 = 0$

$\Rightarrow x^2 - 14x + 45 = 0$

$\Rightarrow x^2 - 9x - 5x + 45 = 0$

$\Rightarrow x(x - 9) - 5(x - 9) = 0$

$\Rightarrow (x - 9)(x - 5) = 0$

$\Rightarrow x - 9 = 0$ or $x - 5 = 0$

$\Rightarrow x = 9$ or $x = 5$

Hence, there are two such points on x–axis whose distance from (7, –4) is $2\sqrt{5}$. Hence, required points are (9, 0), (5, 0).

OR

Given: points A(3, - 4) and B(1, 2) trisected at the points P and Q

Let the co-ordinates of P be (x, y)

Since AP = PQ = QB

∴AP: PB = 1: 2

Using intersection formula

$x = \frac{x_2(m_1)+x_1(m_2)}{m_1+m_2}$, $y = \frac{y_2(m_1)+y_1(m_2)}{m_1+m_2}$

$x = \frac{1\times1+2\times3}{1+2} = \frac{7}{3}$

$y = \frac{1\times2+2\times-4}{1+2} = -2$

Hence point P is $\left(\frac{7}{3}, -2\right)$.

29. As per question, the required number of books are to be distributed equally among the students of section A or B.

There are 30 students in section A and 28 students in section B.

So, the number of these books must be a multiple of 30 as well as that of 28.

Consequently, the required number is LCM(30, 28).

Now, $30 = 2 \times 3 \times 5$

and $28 = 2^2 \times 7$.

∴ LCM(30, 28) = product of prime factors with highest power

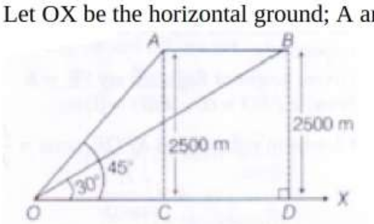
$= 2^2 \times 3 \times 5 \times 7$

$= 4 \times 3 \times 5 \times 7$

= 420

Hence, the required number of books = 420.

30. Let OX be the horizontal ground; A and B be the two positions of the plane and O be the point of observation.



Here, AC = BD = 2500 m,

$\angle AOC = 45^o$ *and* $\angle BOD = 30^o$

In right angled $\triangle OCA$, $\cot 45^o = \frac{B}{P} = \frac{OC}{AC}$

$\Rightarrow 1 = \frac{OC}{AC}$

$\Rightarrow OC = AC = 2500\ m$

In right angled $\triangle ODB$, $\cot 30^o = \frac{B}{P} = \frac{OD}{BD}$

$\Rightarrow \sqrt{3} = \frac{OD}{2500}$

$\Rightarrow OD = 2500\sqrt{3}\ m$

Now, CD = OD - OC $= 2500\sqrt{3} - 2500$

⇒ CD = 2500(1.732 - 1)

⇒ CD = 2500(0.732)

⇒ CD = 1830 m

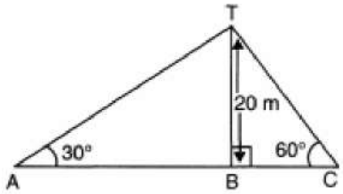
Thus, distance covered by plane in 15 s is 1830 m.

$\therefore$ Speed of the plane= $\frac{Distance}{Time} = \frac{1830}{15} \times \frac{60\times60}{1000} = 439.2\ km/h$

OR

Let two men are standing at A and C.

BT is the tower



In rt $\triangle$ABT,

$\tan 30^\circ = \frac{BT}{AB}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{20}{AB}$

$\Rightarrow \quad AB = 20\sqrt{3}$

In rt. $\triangle$BTC,

$\tan 60^\circ = \frac{BT}{BC}$

$\Rightarrow \quad \sqrt{3} = \frac{20}{BC}$

$\Rightarrow \quad BC = \frac{20}{\sqrt{3}}$

$\therefore$ Distance between two men

= AB + BC

$= 20\sqrt{3} + \frac{20}{\sqrt{3}}$

$= \frac{60+20}{\sqrt{3}} = \frac{80\sqrt{3}}{3}\text{m}$

Hence, distance between the two men is $\frac{80\sqrt{3}}{3}\text{m}$

31. Let f_1 and f_2 be the frequencies of class intervals 0 - 10 and 40 - 50.

$f_1 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$

$\Rightarrow f_1 + f_2 = 9$

Median is 32.5 which lies in 30 - 40, so the median class is 30 - 40.

l = 30, h = 10, f = 12, N = 40 and c = $f_1 + 5 + 9 = (f_1 + 14)$

Now, median $= l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f}\right]$

$\Rightarrow \quad 32.5 = \left[30 + \left(10 \times \frac{20 - f_1 - 14}{12}\right)\right]$

$= \left[30 + \left(10 \times \frac{6 - f_1}{12}\right)\right]$

$= \left[30 + \left(\frac{30 - 5f_1}{6}\right)\right]$

$\frac{30 - 5f_1}{6} = 2.5$

$30 - 5f_1 = 15$

$5f_1 = 15 \Rightarrow f_1 = 3$

$f_1 = 3$ and $f_2 = (9 - 3) = 6$

Section D

32. Let the numerator and the denominator be x & y respectively. Hence, the fraction is $\frac{x}{y}$.

Now, according to the question, we have

$\frac{x-2}{y+3} = \frac{1}{4}$

$\Rightarrow 4(x - 2) = y + 3$

$\Rightarrow 4x - 8 = y + 3$

$\Rightarrow 4x - y = 3 + 8$

$\Rightarrow 4x - y = 11 \ldots (i)$

Also according to the question, $\frac{x+6}{3y} = \frac{2}{3}$

$\Rightarrow \frac{3(x+6)}{3y} = 2$

$\Rightarrow x+6=2y$

$\Rightarrow x-2y=-6$ ...(ii)

Multiplying equation (i) by 2 & then subtracting equation (ii) from it, we get

⇒ 8x - x = 22 + 6

⇒ 7x = 28

$\Rightarrow x=\frac{28}{7}=4$

Putting x = 4 in equation (ii), we get

⇒4 - 2y = -6

⇒ -2y = -6 - 4

⇒ -2y = -10

$\Rightarrow y=\frac{-10}{-2}=5$

Hence, the fraction is $\frac{4}{5}$.

OR

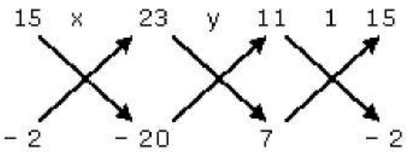
The given system of equations is

11x + 15y + 23 = 0 ....(1)

7x - 2y - 20 = 0 ....(2)

To solve the equations (1) and (2) by cross multiplication method,

we draw the diagram below:



Then,

$\Rightarrow \frac{x}{(15)(-20)-(-2)(23)}=\frac{y}{(23)(7)-(-20)(11)}=\frac{1}{(11)(-2)-(7)(15)}$

$\Rightarrow \frac{x}{-300+46}=\frac{y}{161+220}=\frac{1}{-22-105}$

$\Rightarrow \frac{x}{-254}=\frac{y}{381}=\frac{1}{-127}$

$\Rightarrow x=\frac{-254}{-127}=2$ and $y=\frac{381}{-127}=-3$

Hence, the required solution of the given pair of equations is

x = 2, y = -3

Verification: Substituting x = 2, y = -3,

We find that both the equations (1) and (2) are satisfied as shown below:

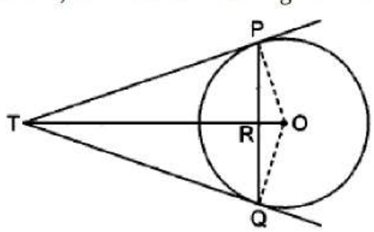
11x + 15y + 23 = 11(2) + 15(-3) + 23

= 22 - 45 + 23 = 0

7x - 2y - 20 = 7(2) - 2(-3) - 20

= 14 + 6 - 20 = 0

Hence, the solution we have got is correct.

33. 

Given, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T.

Construction: Join OP and OQ.

In triangles $OTP\ and\ OTQ$, OT is common

$OP=OQ$ (radii)

$TP=TQ$

$\therefore \Delta OPT\cong\Delta OQT$

$\therefore \angle\mathrm{POT}=\angle\mathrm{QOT}$

Consider, triangles OPR and OQR;

$OP=OQ$ (radii); OP is common

$\angle POR=\angle QOR$ [from(i)]

$\therefore \Delta OPR \cong \Delta OQR$ (SAS cong. rule)

$\therefore \text{PR} = \text{RQ} = \frac{1}{2} \times 16 = 8\text{cm}$ ...(ii)

$\angle\text{ORP} = \angle\text{ORQ} = 90^\circ$ ....(iii)

In right angled triangle TRP,

$\text{TR}^2 = \text{TP}^2 - (8)^2 = \text{TP}^2 - 64$ [from (iii)] ,..(iv)]

Also $\text{OT}^2 = \text{TP}^2 + (10)^2$

$(TR+6)^2 = TP^2 + 100$ [$\because \text{OR} = \sqrt{100-64} = 6$]

$\text{TR}^2 + 12\text{TR} + 36 = \text{TP}^2 + 100$

$\text{TP}^2 - 64 + 12\text{TR} + 36 = \text{TP}^2 + 100$

$12\text{TR} = 128 \Rightarrow \text{TR} = \frac{32}{3}\text{cm}$

From (iv)

$\left(\frac{32}{3}\right)^2 = TP^2 - 64$

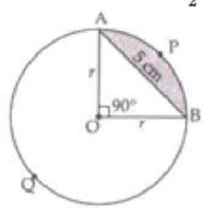
$\Rightarrow \text{TP}^2 = \frac{1024}{9} + 64 = \frac{1024+576}{9} = \frac{1600}{9}$

$\Rightarrow \quad TP = \frac{40}{3}\text{cm}$

34. Chord AB = 5 cm divides the circle into two segments minor segment APB and major segment AQB. We have to find out the difference in area of major and minor segment.

Here, we are given that $\theta = 90°$

Area of $\triangle$OAB = $\frac{1}{2}$Base $\times$ Altitude = $\frac{1}{2}$r $\times$ r = $\frac{1}{2}r^2$



Area of minor segment APB

$= \frac{\pi r^2 \theta}{360^\circ}$ – Area of $\triangle$AOB

$= \frac{\pi r^2 90^\circ}{360^\circ} - \frac{1}{2}r^2$

$\Rightarrow$ Area of minor segment = $\left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$ …(i)

Area of major segment AQB = Area of circle – Area of minor segment

$= \pi r^2 - \left[\frac{\pi r^2}{4} - \frac{r^2}{2}\right]$

$\Rightarrow$ Area of major segment AQB $= \left[\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right]$ …(ii)

Difference between areas of major and minor segment

$= \left(\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right) - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$

$= \frac{3}{4}\pi r^2 + \frac{r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2}$

$\Rightarrow$ Required area $= \frac{2}{4}\pi r^2 + r^2 = \frac{1}{2}\pi r^2 + r^2$

In right $\triangle$OAB,

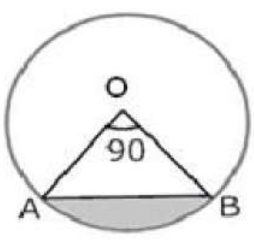
$r^2 + r^2 = AB^2$

$\Rightarrow 2r^2 = 5^2$

$\Rightarrow r^2 = \frac{25}{2}$

Therefore, required area $= \left[\frac{1}{2}\pi \times \frac{25}{2} + \frac{25}{2}\right] = \left[\frac{25}{4}\pi + \frac{25}{2}\right]$ cm^2

OR



i. Area of minor sector = $\frac{\theta}{360}\pi r^2$

$= \frac{90}{360}(3.14)(10)^2$

$= \frac{1}{4} \times 3.14 \times 100$

$= \frac{314}{4}$

= 78.50 = 78.5 cm^2

ii. Area of major sector = Area of circle - Area of minor sector

= $\pi(10)^2$ - $\frac{90}{360}$ $\pi(10)^2$ = 3.14 (100) - $\frac{1}{4}$(3.14) (100)

= 314 - 78.50 = 235.5 cm^2

iii. We know that area of minor segment

= Area of minor sector OAB - Area of ΔOAB

$\because$ area of $\triangle$OAB $= \frac{1}{2}(OA)(OB)\sin\angle AOB$

$= \frac{1}{2}(OA)(OB)\,(\because \angle AOB = 90^\circ)$

Area of sector = $\frac{\theta}{360}\pi r^2$

= $\frac{1}{4}$(3.14) (100) - 50 = 25(3.14) - 50 = 78.50 - 50 = 28.5 cm^2

iv. Area of major segment = Area of the circle - Area of minor segment

= $\pi(10)^2$ - 28.5

= 100(3.14) - 28.5

= 314 - 28. 5 = 285.5 cm^2

35. No. of cards removed= 2 jacks+2 queens +2 kings +2 aces=8

No. of all possible outcomes $n = 52 - 8 = 44 - - - - - - - - - - - - -(1)$

i. No. of black Queens in the deck = 0

$\therefore$ P(getting a black Queen) = $\frac{0}{44} = 0$

Hence it is an impossible event

ii. No. of red cards = 26 - 4 = 22

P(getting a red card) = $\frac{m}{n} = \frac{22}{44} = \frac{1}{2}$

iii. No. of Jacks (black) = 2 so m=2

$\therefore$ P(getting a black colored Jack) = $\frac{m}{n} = \frac{2}{44} = \frac{1}{22}$

iv. No. of face cards in the deck = 12 - 6 = 6 so m=6

$\therefore$P(getting a face card) = $\frac{m}{n} = \frac{6}{44} = \frac{3}{22}$

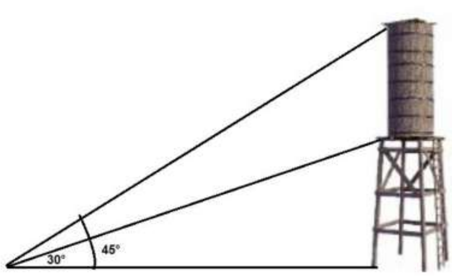
Section E

36. **Read the text carefully and answer the questions:**

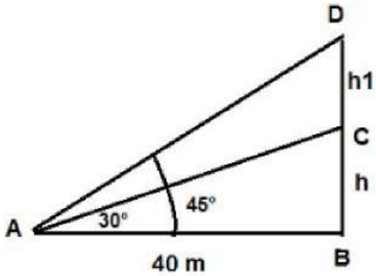
In a society, there are many multistory buildings. The RWA of the society wants to install a tower and a water tank so that all the households can get water without using water pumps.

For this they have measured the height of the tallest building in the society and now they want to install a tower that will be taller than that so that the level of water must be higher than the tallest building in their society. Here is one solution they have found and now they want to check if it will work or not.

From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is 300. the angle of elevation of the top of the water tank is 45^o.



(i)



Let BC be the tower of height h and CD be the water tank of height h_1

In ΔABD, we have

$\tan 45^\circ = \frac{BD}{AB}$

$\Rightarrow 1 = \frac{h+h_1}{40}$

$\Rightarrow h + h_1 = 40$ ...(1)

In ΔABC, we have

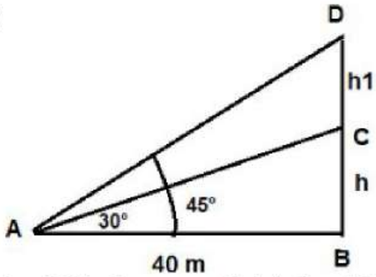
$\tan 30^\circ = \frac{BC}{AB}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{40}$

$\Rightarrow h = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3}$ = 23.1 m

Thus height of the tower is 23.1 m.

(ii)



Let BC be the tower of height h and CD be the water tank of height h_1

In ΔABD, we have

$\tan 45^\circ = \frac{BD}{AB}$

$\Rightarrow 1 = \frac{h+h_1}{40}$

$\Rightarrow h + h_1 = 40$ ...(1)

In ΔABC, we have

$\tan 30^\circ = \frac{BC}{AB}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{40}$

$\Rightarrow h = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3}$ = 23.1 m

Thus height of the tower is 23.1 m.

Substituting the value of h in (1), we have

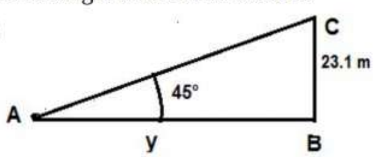
$23.1 + h_1 = 40$

$\Rightarrow h_1 = 40 - 23.1$

= 6.9 m

Thus height of the tank is 6.9 m.

(iii)



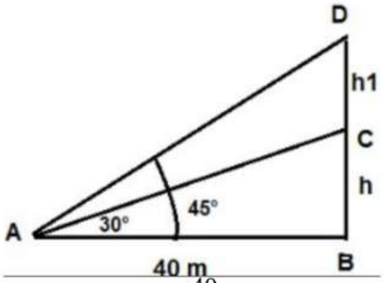
In the $\triangle ABC$ if $\angle CAB = 45^\circ$ then

$\cot 45^\circ = \frac{y}{23.1} = 1$

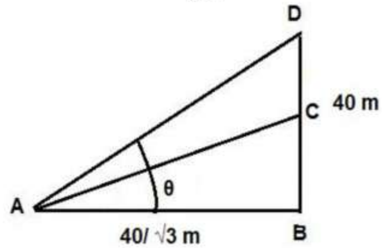
y = 23.1 m

Thus the angle of elevation will be 45^0 at 23.1 m.

OR



Given that $AB = \frac{40}{\sqrt{3}}$



In the $\triangle ABD$

$\cot\theta = \frac{AB}{BD} = \frac{\frac{40}{\sqrt{3}}}{40}$

$\cot\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 60^\circ$

Hence the angle of elevation would be 60^o.

37. **Read the text carefully and answer the questions:**

Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves ₹27.5 On each succeeding day he increases his saving by ₹2.5.

(i) Money saved on 1st day = ₹27.5

∵ Sehaj increases his saving by a fixed amount of ₹2.5

∴ His saving form an AP with a = 27.5 and d = 2.5

∴ Money saved on 10th day,

a_{10} = a + 9d = 27.5 + 9(2.5)

= 27.5 + 22.5 = ₹50

(ii) a_{25} = a = 24d

= 27.5 + 24(2.5)

= 27.5 + 60 = ₹ 87.5

(iii) Total amount saved by Sehaj in 30 days.

= $\frac{30}{2}[2 \times 27.5 + (30 - 1) \times 2.5]$

= 15(55 + 29(2.5)

= ₹1912.5

OR

Let S_n = 387.5, a = 27.5 and d = 2.5

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 387.5 = \frac{n}{2}[2 \times 27.5 + (n-1)2.5]$

$\Rightarrow 387.5 = \frac{n}{2}[55 + (n-1) \times 2.5]$

$\Rightarrow$ 775 = 55n + $2.5n^2$ - 2.5n

$\Rightarrow$ $25n^2$ + 525n = 7750 = 0

$\Rightarrow n^2 + 21n - 310 = 0$

$\Rightarrow (n + 31)(n - 10) = 0$

$\Rightarrow n = -31$ reject $n = 10$ accept

So in 10 years Sehaj saves ₹ 387.5.

38. **Read the text carefully and answer the questions:**

A juice seller is serving his customers using cylindrical container with radius 20cm and height 50cm. He serves juice into a glass as shown in Fig. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass.

(i) We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

The apparent capacity of the glass = Volume of cylinder

$= \pi r^2 h$

$= 3.14 \times \left(\frac{5}{2}\right)^2 \times 10$

$= 3.14 \times \frac{25}{4} \times 10 = 196.25 \text{ cm}^3$

(ii) We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

The actual capacity of glass = Apparent capacity of glass - Volume of hemispherical part of the glass

The volume of hemispherical part = $\frac{2}{3}\pi r^2 h = \frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3 = 32.71 \text{ cm}^3$

Actual capacity of glass = $196.25 - 32.71 = 163.54 \text{ cm}^3$

(iii) We have, inner diameter of the glass, d = 5 cm, height of the glass = 10 cm

Volume of container = $V = \pi r^2 h$

$\Rightarrow V = 3.14 \times 20 \times 20 \times 50 = 62800 \text{ cm}^3$

$\Rightarrow V = 62.8$ litre

OR

We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

Number of glasses = $\frac{\text{Volume of container}}{\text{Actual volume of one glass}}$

$\Rightarrow$ Number of glasses = $\frac{20 \times 3.14 \times 20 \times 50}{3.14 \times \frac{2\pi}{4} \times 10 - \frac{2}{3} \times 3.14 \times \frac{125}{8}}$

$\Rightarrow$ Number of glasses = $\frac{20000}{\frac{250}{4} - \frac{125}{12}} = \frac{20000 \times 12}{750 - 125} = \frac{240000}{625} = 384$

$\Rightarrow$ Number of glasses = 384