SAMPLE QUESTION PAPER (BASIC) - 04

Class 10 - Mathematics

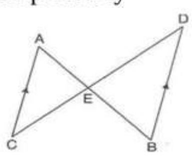
Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. In the adjoining figure $AC||BD$. If , EB = 4 cm, ED = 8 cm, AC = 6 cm, AE = 3 cm then CE and BD are respectively **[1]**



a) 5 cm, 7 cm. b) 7.5 cm, 9.5 cm.

c) 6 cm, 8 cm. d) 4 cm, 6 cm.

2. The zeros of the polynomial $x^2 - \sqrt{2}x - 12$ are **[1]**

a) 3, 1 b) $\sqrt{2}, -\sqrt{2}$

c) $3\sqrt{2}, -2\sqrt{2}$ d) 3, -1

3. If 29x + 37y = 103 and 37x + 29y = 95 then **[1]**

a) x = 3, y = 2 b) x = 2, y = 1

c) x = 2, y = 3 d) x = 1, y = 2

4. The pair of linear equations ax + by = c and px + qy = r has a unique solution then **[1]**

a) aq $\neq$ bp b) aq = bp

c) ap = bq d) ap $\neq$ bq

5. In a $\triangle$ ABC, perpendicular AD from A on BC meets BC at D. If BD = 8 cm, DC = 2 cm and AD = 4 cm, then: **[1]**

a) $\triangle$ABC is right - angled at A.
b) AC = 2 AB
c) $\triangle$ABC is isosceles
d) $\triangle$ABC is equilateral

6. In a lottery, there are 6 prizes and 24 blanks. What is the probability of not getting a prize? **[1]**

a) None of these
b) $\frac{3}{4}$
c) $\frac{3}{5}$
d) $\frac{4}{5}$

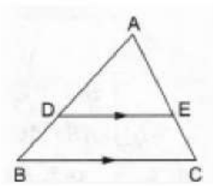
7. If cos A = $\frac{4}{5}$, then the value of tan A is? **[1]**

a) $\frac{4}{3}$
b) $\frac{3}{4}$
c) $\frac{3}{5}$
d) $\frac{5}{3}$

8. If $\sum f_i$ = 17, $\sum f_i x_i$ = 4p + 63 and the mean of the distribution is 7, then the value of **p** is **[1]**

a) 13
b) 12
c) 15
d) 14

9. In a $\triangle$ABC, if DE is drawn parallel to BC, cutting AB and AC at D and E respectively such that AB = 7.2 cm, AC = 6.4 cm and AD = 4.5 cm. Then, AE = ? **[1]**



a) 4 cm
b) 5.4 cm
c) 3.2 cm
d) 3.6 cm

10. If two positive integers ‘a’ and ‘b’ are written as $a = pq^2$ and $b = p^3q^2$, where ‘p’ and ‘q’ are prime numbers, then LCM(a, b) = **[1]**

a) pq
b) p^3q^2
c) p^2q^3
d) p^2q^2

11. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has **[1]**

a) two equal real roots
b) no real root
c) two distinct real roots
d) more than 2 real roots

12. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is **[1]**

a) 5
b) 11
c) $7 + \sqrt{5}$
d) 12

13. Mean of a certain number of observations is m. If each observation is divided by x($x \neq 0$) and increased by y, then the mean of new observation is: **[1]**

a) $m + \frac{y}{x}$
b) $m + \frac{x}{y}$
c) $\frac{m}{x}$ + y
d) $\frac{m}{y} + x$

14. The value of sin 45° + cos 45° is **[1]**

a) $\sqrt{2}$
b)

$\frac{1}{\sqrt{2}}$

c) 1

d) $\frac{1}{\sqrt{3}}$

15. The ________ is the line drawn from the eye of an observer to the point in the object viewed by the observer. **[1]**

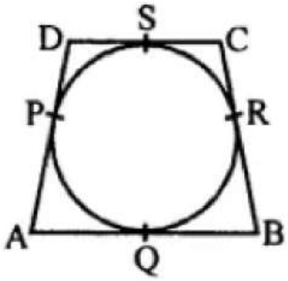
a) Horizontal line

b) line of sight

c) None of these

d) Vertical line

16. In the given figure, quad. ABCD is circumscribed, touching the circle at P, Q, R and S. If AP = 5 cm, BC = 7 cm and CS = 3 cm. Then, the length AB = ? **[1]**



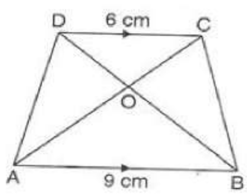
a) 8 cm

b) 9 cm

c) 10 cm

d) 12 cm

17. In trapezium ABCD, if $AB||DC$, AB = 9 cm, DC = 6 cm and BD = 12 cm, then BO is equal to **[1]**



a) 7 cm.

b) 7.2 cm.

c) 7.5 cm.

d) 7.4 cm.

18. If the equation $4x^2 - 3kx + 1 = 0$ has equal roots then k = ? **[1]**

a) $\pm\frac{3}{4}$

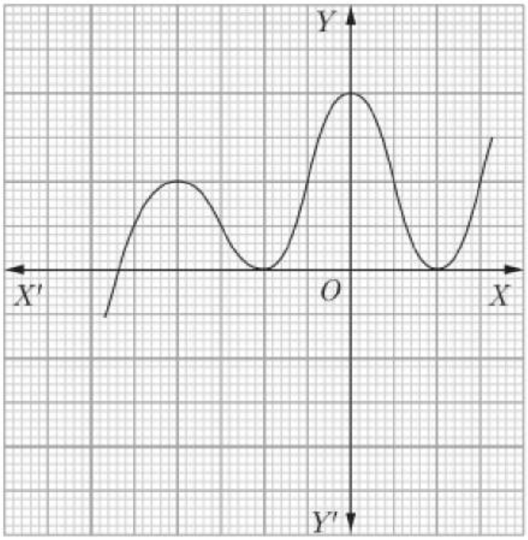
b) $\pm\frac{4}{3}$

c) $\pm\frac{2}{3}$

d) $\pm\frac{1}{3}$

19. **Assertion (A):** The graph y = f(x) is shown in figure, for the polynomial f(x). The number of zeros of f(x) is 4. **[1]**

Reason (R): The number of zero of the polynomial f(x) is the number of point of which f(x) cuts or touches the axes.



a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 300 cm^2. **[1]**

Reason (R): Total surface area of a cuboid is 2(lb + bh + lh)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Two numbers differ by 4 and their product is 192. Find the numbers. **[2]**

22. If $\left(1, \frac{p}{3}\right)$ is the mid point of the line segment joining the points (2, 0) and $\left(0, \frac{2}{9}\right)$, then show that the line 5x + 3y + 2 = 0 passes through the point (-1, 3p). **[2]**

OR

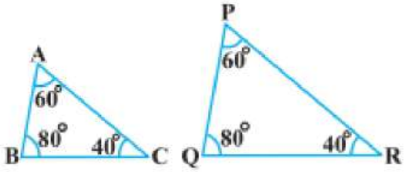
Find the distance between the points:

P(a sin α, a cos α,) and Q(a cos α, - a sin α)

23. Three pieces of timber 42 m, 49 m and 63 m long have to be divided into the planks of the same length. What is the greatest possible length of each plank ? Also find number of planks formed. **[2]**

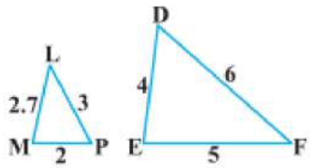
24. Given sec $\theta = \frac{13}{12}$, Calculate all other trigonometric ratios. **[2]**

25. State the pair of triangles in the the below fig, are similar. Write the similarity criterion used by you for answering the question and also write the pair of similar triangles in the symbolic form: **[2]**



OR

State the pair of triangles in the figure are similar. Write the similarity criterion used by you for answering the question and also write the pair of similar triangles in symbolic form:

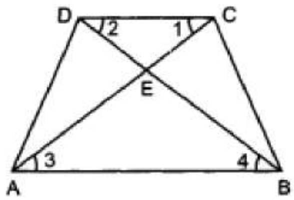


Section C

26. Solve the quadratic equation by factorization: **[3]**

25x (x + 1) = - 4

27. In fig ∠1 = ∠3, ∠2 = ∠4 **[3]**



DE = 4, CE = x + 1, AE = 2x + 4; BE = 4x - 2. Find x.

28. Show that A(1, 2), B(4, 3), C(6, 6) and D(3, 5) are the vertices of a parallelogram. Show that ABCD is not a rectangle. **[3]**

OR

Two vertices of an isosceles triangle are (2,0) and (2,5). Find the third vertex if the length of the equal sides is 3.

29. Let d be the HCF of 24 and 36. Find two numbers a and b, such that d = 24a + 36b. **[3]**

30. A kite is flying, attached to a thread which is 165 m long. The thread makes an angle of 30° with the ground. Find the height of the kite from the ground, assuming that there is no slack in the thread. **[3]**

OR

A ladder rests against a wall at an angle a to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle $\beta$with the horizontal. Show that

$\frac{a}{b}=\frac{\cos\alpha-\cos\beta}{\sin\beta-\sin\alpha}$

31. Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24. **[3]**

Age in years	0 - 10	10 -20	20 - 30	30 - 40	40 - 50
No. of persons	5	25	?	18	7

Section D

32. Solve the following pair of linear equations by substitution method. $\frac{3x}{2}-\frac{5y}{3}=-2; \frac{x}{3}+\frac{y}{2}=\frac{13}{6}$ **[5]**

OR

The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Find the fraction.

33. QR is the tangent to the circle whose centre is P. If QA || RP and AB is the diameter, prove that RB is a tangent to the circle. **[5]**

34. Two circular beads of different sizes are joined together such that the distance between their centres is 14 cm. The sum of their areas is 130π cm^2. Find the radius each bead. **[5]**

OR

Find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres (Use π = 22/7).

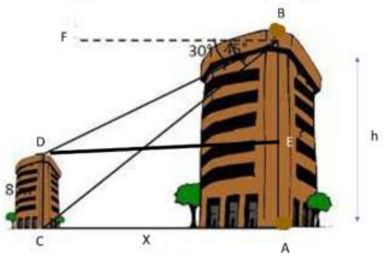
35. A number x is selected at random from the numbers 1 ,2 ,3 and 4. Another number y is selected at random from **[5]**

the numbers 1, 4, 9 and 16. Find the probability that product of x and y is less than 16.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30^{o} and 45^{o}, respectively.



(i) Now help Vinod and Basant to find the height of the multistoried building.

(ii) Also, find the distance between two buildings.

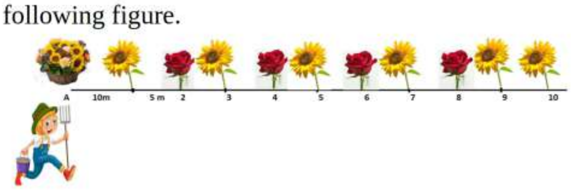
(iii) Find the distance between top of multistoried building and bottom of first building.

OR

Find the distance between top of multistoried building and top of first building.

37. **Read the text carefully and answer the questions:** **[4]**

In a school garden, Dinesh was given two types of plants viz. sunflower and rose flower as shown in the following figure.



The distance between two plants is to be 5m, a basket filled with plants is kept at point A which is 10 m from the first plant. Dinesh has to take one plant from the basket and then he will have to plant it in a row as shown in the figure and then he has to return to the basket to collect another plant. He continues in the same way until all the flower plants in the basket. Dinesh has to plant ten numbers of flower plants.

(i) Write the above information in the progression and find first term and common difference.

(ii) Find the distance covered by Dinesh to plant the first 5 plants and return to basket.

(iii) Find the distance covered by Dinesh to plant all 10 plants and return to basket.

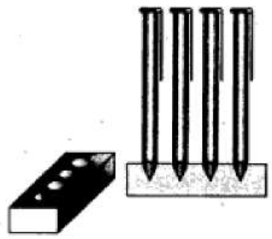
OR

If the speed of Dinesh is 10 m/min and he takes 15 minutes to plant a flower plant then find the total time taken by Dinesh to plant 10 plants.

38. **Read the text carefully and answer the questions:** **[4]**

A carpenter in the small town of Bareilly used to make and sell different kinds of wood items like a rectangular box, cylindrical pen stand, and cuboidal pen stand. One day a student came to his shop and asked him to make a pen stand with the dimensions as follows:

A pen stand should be in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid should be 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm.



(i) The volume of the cuboidal part.

(ii) The volume of wood in the entire stand.

(iii) Total volume of conical depression.

OR

If the cost of wood used is ₹10 per cm^3, then the total cost of making the pen stand.

Solution

SAMPLE QUESTION PAPER (BASIC) - 04

Class 10 - Mathematics

Section A

1. **(c)** 6 cm, 8 cm.

   Explanation: Given: $AC||BD.$ and AC = 6 cm, AE = 3 cm, EB = 4 cm, ED = 8 cm, In triangles ACE and DEB, $\angle$AEC = $\angle$DEB [Vertically opposite angles] $\angle$ECA = $\angle$EDB [Alternate angles as $AC||BD$]

   $\therefore \triangle ACE \sim \triangle DEB$ [AA similarity]

   $\therefore \frac{EB}{AE} = \frac{ED}{EC}$

   $\Rightarrow \frac{4}{3} = \frac{8}{EC}$

   $\Rightarrow$ EC = $\frac{8\times3}{4}$ = 6 cm

   Also $\frac{EB}{AE} = \frac{BD}{AC}$

   $\Rightarrow \frac{4}{3} = \frac{BD}{6}$

   $\Rightarrow$ BD = $\frac{4\times6}{3}$ = 8 cm

2. **(c)** $3\sqrt{2}, -2\sqrt{2}$

   Explanation: $x^2 - \sqrt{2}x - 12 = x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 12$

   $= x(x - 3\sqrt{2}) + 2\sqrt{2}(x - 3\sqrt{2}) = (x - 3\sqrt{2})(x + 2\sqrt{2})$

   $\therefore \quad x = 3\sqrt{2}$ or $x = -2\sqrt{2}$

3. **(d)** x = 1, y = 2

   Explanation: 29x + 37y=103 ....(i)

   37x+29y=95 ....(ii)

   Adding (i) and (ii), we get 66 (x + y) = 198 $\Rightarrow$ x + y = 3.

   Subtracting (ii) from (i), we get 8 (y - x) = 8 $\Rightarrow$ y - x = 1.

   Solve above equations we get

   x = 1, y = 2

4. **(a)** aq $\neq$ bp

   Explanation: Given: a_1 = a, a_2 = p, b_1 = b, b_2 = q, c_1 = c and c_2 = r.

   Since, the pair of given linear equations has a unique solution.

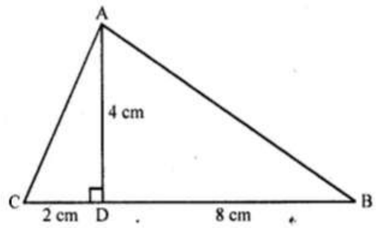
   $\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{a}{p} \neq \frac{b}{q}$

   $\Rightarrow$ aq $\neq$ bp

5. **(a)** $\triangle$ABC is right - angled at A.

   Explanation: In $\triangle$ABC , AD$\perp$ BC

   BD = 8 CM, DC = 2 CM, AD = 4 CM

   In right $\triangle$ACD,

   $AC^2 = AD^2 + CD^2$ (Pythagoras Theorem)

   $=(4)^2+(2)^2=16 + 4 = 20$

   and in right $\triangle$ABD,

   $AB^2 = AD^2 + DB^2$

   $= (4)^2 + (8)^2 = 16 + 64 = 80$

   and $BC^2 = (BD + DC)^2 = (8 + 2)^2 = (10)^2 = 100$

$AB^2 + AC^2 = 80 + 20 = 100 = BC^2$

$\triangle$ABC is a right triangle whose $\angle A = 90^\circ$

6. **(d)** $\frac{4}{5}$

Explanation: Total number of tickets = 6 + 24 = 30.

Number of blanks = 24.

$\therefore$P{not getting a prize} = $\frac{24}{30} = \frac{4}{5}$

7. **(b)** $\frac{3}{4}$

Explanation: Given: cos A = $\frac{4}{5}$ ...(i)

we know that tan A = $\frac{\sin A}{\cos A}$

Also we know that, sin A = $\sqrt{(1 - \cos^2 A)}$ ...(ii)

Thus,

Substituting eq. (i) in eq. (ii), we get

Sin A = $\sqrt{1 - \frac{16}{25}}$

$= \sqrt{(9/25)} = \frac{3}{5}$

Therefore, $\tan A = \frac{3}{5} \times \frac{5}{4} = \frac{3}{4}$

8. **(d)** 14

Explanation: Since, Mean = $\frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow$ 7 = $\frac{4p+63}{17}$

$\Rightarrow$ 4p + 63 = 119

$\Rightarrow$ 4p = 119 - 63

$\Rightarrow$ 4p = 56

$\Rightarrow$ p = 14

9. **(a)** 4 cm

Explanation: In $\triangle$ABC, DE || BC

AB = 7.2 cm, AC = 6.4 cm, AD = 4.5 cm

Let AE = x cm

DE || BC

$\triangle ADE \sim \triangle ABC$

$\frac{AD}{AB} = \frac{AE}{AC} \Rightarrow \frac{4.5}{7.2} = \frac{AE}{6.4}$

$\Rightarrow AE = \frac{4.5 \times 6.4}{7.2} = 4.0 = 4cm$

10. **(b)** p^3q^2

Explanation: We know that LCM = product of the highest powers of all the prime factors of the numbers pq^2, p^3q^2

LCM = p^3q^2

11. **(b)** no real root

Explanation: Discriminant = 5 - 4(2)(1) < 0

Therefore no real root.

12. **(d)** 12

Explanation: Given: the vertices of a triangle ABC, A(0, 4), B (0, 0) and C (3, 0).

$\therefore$ Perimeter of triangle ABC = AB + BC + AC

= $\sqrt{(0-0)^2 + (0-4)^2} + \sqrt{0-3)^2 + (0-0)^2} + \sqrt{0-3)^2 + (4-0)^2}$

= $\sqrt{0+16} + \sqrt{9+0} + \sqrt{9+16}$

= $\sqrt{16} + \sqrt{9} + \sqrt{25}$

= 4 + 3 + 5 = 12 units

13. **(c)** $\frac{m}{x}$ + y

Explanation: Let $x_1, x_2, x_3, .., x_n$ be 'n' observations.

$\Rightarrow$ m = $\frac{x_1+x_2+x_3+...+x_n}{n}$

Now , according to question, the observations become

$\frac{x_1}{x}$ + y, y + $\frac{x_2}{x}$ + y, ..., $\frac{x_n}{x}$ + y

Therefore, new mean is

$\Rightarrow m' = \frac{\frac{1}{x}(x_1+x_2+...+x_n)+ny}{n}$

$\Rightarrow m' = \frac{(x_1+x_2+...+x_n)}{nx} + \frac{ny}{n}$

$\Rightarrow m' = \frac{1}{x}\frac{(x_1+x_2+...+x_n)}{n}$ + y

$\Rightarrow m' = \frac{m}{x}$ + y

14. **(a)** $\sqrt{2}$

Explanation: Given: $\sin 45^\circ + \cos 45^\circ$

= $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

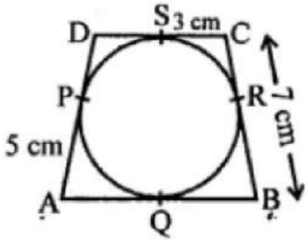
= $\frac{2}{\sqrt{2}} = \sqrt{2}$

15. **(b)** line of sight

Explanation: The line of sight is the imaginary line drawn from the eye of an observer to the point in the object viewed by the observer. The angle between the line of sight and the ground is called angle of elevation

16. **(b)** 9 cm

Explanation: In the figure, quadrilateral ABCD is circumscribed touches the circle at P, Q, R and S



AP = 5 cm, BC = 7 cm, CS = 3 cm AB = ?

Tangents drawn from the external point to the circle are equal

AQ = AP = 5 cm

CR = CS = 3 cm

BQ = BR

Now, BR = BC – CR = 7 – 3 = 4 cm

BQ = 4 cm

Now, AB = AQ + BQ = 5 + 4 = 9 cm

17. **(b)** 7.2 cm.

Explanation: In ΔCOD and ΔAOB,

$\angle$DOC = $\angle$AOB [Vertically opposite]

And $\angle$DCO = $\angle$OAB [Alternate angles]

$\Rightarrow \Delta COD \sim \Delta AOB$ [AA similarity]

Let OB = x cm

$\therefore \frac{AB}{CD} = \frac{OB}{OD}$

$\Rightarrow \frac{9}{6} = \frac{x}{12-x}$

$\Rightarrow$ 108 - 9x = 6x

$\Rightarrow$ 15x = 108

$\Rightarrow$ x = 7.2cm

18. **(b)** $\pm\frac{4}{3}$

Explanation: Since the roots are equal, we have D = 0.

$\therefore \quad 9k^2 - 16 = 0 \Rightarrow k^2 = \frac{16}{9} \Rightarrow k = \frac{4}{3}$ or $k = \frac{-4}{3}$.

19. **(c)** A is true but R is false.

Explanation: As the number zero of polynomial f(x) is the number of points at which f(x) cuts (intersects) the x-axis and the number of zero in the given figure is 4. So the assertion is correct but the reason is incorrect.

20. **(d)** A is false but R is true.

Explanation: A is false but R is true.

Section B

21. Let the required number be x and x + 4. Then,

x × (x + 4) = 192

$\Rightarrow x^2 + 4x = 192$

$\Rightarrow x^2 + 4x - 192 = 0$

$\Rightarrow x^2 + 16x - 12x - 192 = 0$

$\Rightarrow x(x + 16) - 12(x + 16) = 0$

$\Rightarrow (x + 16)(x - 12) = 0$

⇒ x + 16 = 0 or x - 12 = 0

⇒ x = -16 or x = 12

Case I: When x = -16

∴ x + 4 = -16 + 4 = -12

Case II: When x = 12

∴ x + 4 = 12 + 4 = 16

Hence, the numbers are -16, -12 or 12, 16.

22. Since $\left(1, \frac{p}{3}\right)$, is the mid point of the line segment joining the points (2,0) and $\left(0, \frac{2}{9}\right)$.

∴ $\frac{p}{3} = \frac{0+\frac{2}{9}}{2}$

$\frac{2p}{3} = \frac{2}{9}$

$p = \frac{1}{3}$

Now the given points are (-1, 3p). Let x = -1 , y = 3p

Hence, y = 3 × $\frac{1}{3}$ = 1

The equation to prove is: 5x + 3y + 2 = 0

L.H.S.

= 5x + 3y + 2

Put the values of x & y,

= 5(-1) + 3(1) + 2 = -5 + 3 + 2 = -2 + 2 = 0 = R.H.S.

Hence, the line 5x + 3y + 2 = 0 passes through the point (–1, 1) as 5(–1) + 3(1) + 2 = 0.

OR

The given points are P(a sin α, a cos α) and Q(a cos α, - a sin α)

$(x_1 = a\sin\alpha, y_1 = a\cos\alpha)$ and $(x_2 = a\cos\alpha, y_2 = -\text{a}\sin\alpha)$

Therefore,by using distance formula,we have,

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(a\cos\alpha - a\sin\alpha)^2 + (-a\sin\alpha - a\cos\alpha)^2}$

$= \sqrt{a^2\cos^2\alpha + a^2\sin^2\alpha - 2a^2\cos\alpha\sin\alpha + a^2\cos^2\alpha + a^2\sin^2\alpha + 2a^2\cos\alpha\sin\alpha}$

$= \sqrt{a^2\cos^2\alpha + a^2\sin^2\alpha + a^2\cos^2\alpha + a^2\sin^2\alpha}$

$= \sqrt{a^2\left(\cos^2\alpha + \sin^2\alpha\right) + a^2\left(\cos^2\alpha + \sin^2\alpha\right)}$

$= \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}a\,units$

23. The prime factorization of 42, 49 and 63 are:

42 = 2 × 3 × 7

49 = 7 × 7

63 = 3 × 3 × 7

HCF(42, 49, 63) = 7

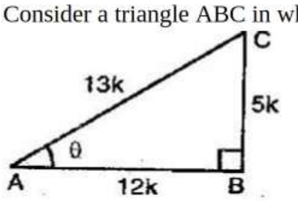
So the greatest possible length of each plank = 7m

Sum of of the length of three pieces of timber

= 42 + 49 + 63 = 154 m

Total no. of planks = $\frac{154}{7}$ = 22

24. Consider a triangle ABC in which $\angle A = \theta$ and $\angle B = 90^o$



Let AB = 12k and AC = 13k

Then, using Pythagoras theorem,

$BC = \sqrt{(\mathrm{AC})^2 - (\mathrm{AB})^2} = \sqrt{(13k)^2 - (12k)^2}$

$= \sqrt{169k^2 - 144k^2} = \sqrt{25k^2} = 5k$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13} \tan\theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$

$\cot\theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5} \cos ec\theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$

25. From the figure:

∠A = ∠P = 60°

∠B = ∠Q = 80°

∠C = ∠R = 40°

Therefore, $\triangle\mathrm{ABC} \sim \triangle\mathrm{PQR}$ [By AAA similarity]

Now corresponding sides of triangles will be propotional,

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$

OR

The given triangles are not similar because the corresponding sides are not proportional.

Section C

26. We have the following equation,

25x (x + 1) = - 4

⇒ $25x^2 + 25x = -4$

⇒ $25x^2 + 25x + 4 = 0$

Factorise the equation,

⇒ $25x^2 + 20x + 5x + 4 = 0$

⇒ 5x(5x + 4) + 1(5x + 4) = 0

⇒ (5x + 4)(5x + 1) = 0

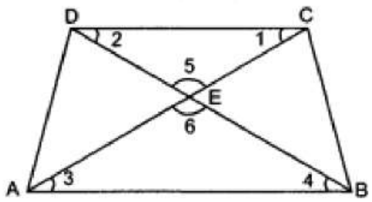
⇒ 5x + 4 = 0 or 5x + 1 = 0

$\Rightarrow x = -\frac{4}{5}$

or

$x = -\frac{1}{5}$

27. ∠1 = ∠3, ∠2 = ∠4 (Given)



$\Rightarrow \quad \angle 5 = \angle 6$ (Vertically opposite angles)

$\triangle\mathrm{CDE} \sim \triangle\mathrm{ABE}$ Using AAA similarity rule

$\Rightarrow \quad \frac{\mathrm{DE}}{\mathrm{BE}} = \frac{\mathrm{EC}}{\mathrm{EA}}$ (As sides of the similar triangles are in proportion to each other)

$\Rightarrow \frac{4}{4x-2} = \frac{x+1}{2x+4}$

⇒ $8x + 16 = 4x^2 - 2x + 4x - 2$

⇒ $8x + 16 = 4x^2 + 2x - 2$

⇒ $4x^2 + 2x - 8x - 2 - 16 = 0$

⇒ $4x^2 - 6x - 18 = 0$

⇒ $2x^2 - 3x - 9 = 0$

⇒ 2x(x - 3) + 3(x - 3) = 0

⇒ (x - 3)(2x - 3) = 0

Either x - 3 = 0 ⇒ x = 3 units

or 2x + 3 = 0

x = $\frac{-3}{2}$ units, which is not possible as side of a trapezium cannot be negative.

Hence, x = 3 units

28. Let A(1, 2), B(4, 3), C(6, 6) and D(3, 5) be the angular point of a quadrilateral ABCD.

Now,

$AB = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}\, units$

$BC = \sqrt{(6-4)^2 + (6-3)^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13}\, units$

$CD = \sqrt{(3-6)^2 + (5-6)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}\, units$

$DA = \sqrt{(3-1)^2 + (5-2)^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13}\, units$

∴ AB = CD and BC = DA

Thus, ABCD can be either a parallelogram or a rectangle.

$Diagonal\ AC = \sqrt{(6-1)^2 + (6-2)^2} = \sqrt{(5)^2 + (4)^2} = \sqrt{25+16} = \sqrt{41}\, units$

$Diagonal\ BD = \sqrt{(3-4)^2 + (5-3)^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}\, units$

∴ Diagonal AC $\neq$ Diagonal BD

Since the opposite sides are equal and the diagonals are not equal, hence ABCD is a parallelogram and it is not a rectangle.

OR

Two vertices of an isosceles triangle are A(2, 0) and B(2, 5), Let C(X, y) be the third vertex.

$AB = \sqrt{(2-2)^2 + (5-0)^2} = \sqrt{(0)^2 + (5)^2} = \sqrt{25} = 5$

$BC = \sqrt{(x-2)^2 + (y-5)^2} = \sqrt{x^2 + 4 - 4x + y^2 + 25 - 10y} = \sqrt{x^2 - 4x + y^2 - 10y + 29}$

$AC = \sqrt{(x-2)^2 + (y-0)^2} = \sqrt{x^2 + 4 - 4x + y^2}$

Also we are given that

AC = BC = 3

$\Rightarrow AC^2 = BC^2 = 9$

$\Rightarrow x^2 + 4 - 4x - y^2 = x^2 - 4x + y^2 - 10y + 29$

$\Rightarrow$ 10y = 25

$\Rightarrow \quad y = \frac{25}{10} = \frac{5}{2}$ =2.5

$AC^2 = 9$

$x^2 + 4 - 4x + y^2 = 9$

$x^2 + 4 - 4x + (2.5)^2 = 9$

$x^2 + 4 - 4x + 6.25 = 9$

$x^2 - 4x + 1.25 = 0$

$D = (-4)^2 - 4 \times 1 \times 1.25$

D = 16 - 5

D = 11

$x = \frac{-(-4)+\sqrt{11}}{2\times1} = \frac{4+3.31}{2} = \frac{7.31}{2} = 3.65$

or, $x = \frac{-(-4)-\sqrt{11}}{2} = \frac{4-\sqrt{11}}{2} = \frac{4-3.31}{2} = 0.35$

∴ The third vertex is (3.65, 2.5) or (0.35, 2.5)

29. Let d be the HCF of 24 and 36.

$24 = 2^3 \times 3$

$36 = 2^2 \times 3^2$

$HCF = 2^2 \times 3 = 12$

$\Rightarrow d = 12$

Now $d = 24a + 36b$

$12 = 24a + 36b$

When $a = -1$ and $b = 1$, we get

$12 = 24 \times (-1) + 36 \times (1)$

= -24 + 36

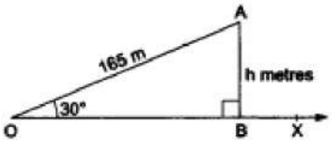
12 = 12

So, a = -1 , b = 1 satisfy the equation d = 24a + 36b

∴ One possible value of a and b is -1 and 1.

30. Let A be the position of the kite. Let O be the position of the observer and OA be the thread. Draw $AB \perp OX$

Then, $\angle BOA = 30^\circ$, OA = 165m and $\angle OBA = 90^\circ$.



Height of the kite from the ground $= AB.$

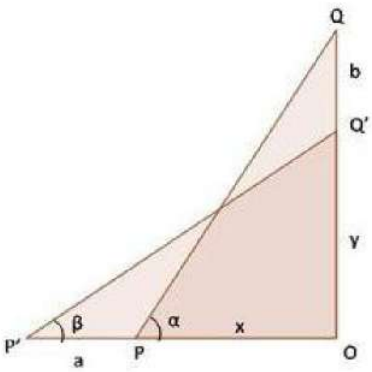
Let $AB = h\,m.$

From right ΔOBA, we have

$\frac{AB}{OA} = \sin 30^\circ = \frac{1}{2}$

$\Rightarrow \frac{h}{165} = \frac{1}{2} \Rightarrow h = \frac{165}{2} = 82.5$

OR



Let PQ be the ladder such that is top Q is on the wall OQ.

The ladder is pulled away from the wall through a distance a, so Q slides and takes position Q'.

Clearly, $PQ = P'Q'.$

In $\Delta's\ POQ\ and\ P'OQ'$,we have

$\sin\alpha = \frac{OQ}{PQ}, \cos\alpha = \frac{OP}{PQ}, \sin\beta = \frac{OQ'}{P'Q'}, \cos\beta = \frac{OP'}{P'Q'}$

$\Rightarrow \sin\alpha = \frac{b+y}{PQ}, \cos\alpha\frac{x}{PQ}, \sin\beta = \frac{y}{PQ}, \cos\beta = \frac{a+x}{PQ}$

$\Rightarrow \sin\alpha - \sin\beta = \frac{b+y}{PQ} - \frac{y}{PQ}$ and

$\cos\beta - \cos\alpha = \frac{a+x}{PQ} - \frac{x}{PQ}$

$\Rightarrow \sin\alpha - \sin\beta = \frac{b}{PQ}$ and

$\cos\beta - \cos\alpha = \frac{a}{PQ}$

$\Rightarrow \frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

31.

Class interval	**Frequency**	**Cumulative frequency**
0-10	5	5
10-20	25	30
20-30	x	30 + x
30-40	18	48 + x
40-50	7	55 + x
	N = 55 + x	

Let the missing frequency be x

Given, Median = 24 ...(1)

From table, total frequency N = 55 + x Or, $\left(\frac{N}{2}\right)$ = 27.5 + $\left(\frac{x}{2}\right)$

Hence, c.f. just greater than $\left(\frac{N}{2}\right)$ is (30 + x), which corresponding class is 20 - 30.

Then, median class = 20 - 30

$\therefore$ l = 20, h = 30 - 20 =10, f =x, F = 30

$\therefore$ Median= l + $\frac{\frac{N}{2} - F}{f} \times$ h

$\Rightarrow$ 24 = 20 + $\frac{\frac{55+x}{2} - 30}{x} \times 10$

$\Rightarrow 24 - 20 = \frac{\frac{55+x}{2}-30}{x} \times 10$

$\Rightarrow 4x = \left(\frac{55+x}{2} - 30\right) \times 10$

$\Rightarrow$ 4x = 5 (55 + x) - 300

$\Rightarrow$ 4x - 5x = -25

$\Rightarrow$ -x = -25

$\Rightarrow$ x = 25

$\therefore$ Missing frequency = 25

Section D

32. $\frac{3x}{2} - \frac{5y}{3} = -2; \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

The given system of linear equation is

$\frac{3x}{2} - \frac{5y}{3} = -2 \quad \ldots\ldots\ldots\ldots(1)$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ .... (2)

$\Rightarrow \quad 9x - 10y = -12$ .... (3)

2x + 3y = 13 ...... (4)

From equation (3)

9x - 10y = -12

9x = 10y - 12

$x = \frac{10y-12}{9}$

Substituting the value of y in equation (4), we get

$2\left(\frac{10y-12}{9}\right) + 3y = 13$

$20y - 24 + 27y = 117$

$47y = 117 + 24$

$y = \frac{141}{47}$

y = 3

Substituting the value of y in equation (4), we get

$2x + 3 \times 3 = 13$

$2x + 9 = 13$

$2x = 13 - 9$

$x = \frac{4}{2} = 2$

Therefore, the solution is

x = 2, y = 3

Verification, Substituting x = 2 and y = 3, we find that both the equations (1) and (2) are satisfied as shown below:

$\frac{3}{2}x - \frac{5y}{3} = \frac{3}{2}(2) - \frac{5}{3}(3) = 3 - 5 = -2$

$\frac{x}{3} + \frac{y}{2} = \frac{2}{3} + \frac{3}{2} = \frac{13}{6}$

This verifies the solution.

OR

Suppose the numerator of the fraction be x

Denominator of the fraction be y

$\therefore$ the fraction is $\frac{x}{y}$

According to the question,

The sum of the numerator and denominator of the fraction is 12.

$\Rightarrow x + y = 12$

$\Rightarrow x + y - 12 = 0$

If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$.

$\Rightarrow \frac{x}{y+3} = \frac{1}{2}$

$\Rightarrow 2x = (y + 3)$

$\Rightarrow 2x - y - 3 = 0$

So, we have two equations

$x + y - 12 = 0$

$2x - y - 3 = 0$

Here x and y are unknowns.

We have to solve the above equations for x and y.

By using cross-multiplication,

$\Rightarrow \frac{x}{(1)\times(-3)-(-1)\times-12} = \frac{-y}{1\times(-3)-2\times-12} = \frac{1}{1\times(-1)-2\times(1)}$

$\Rightarrow \frac{x}{-3-12} = \frac{-y}{-3+24} = \frac{1}{-1-2}$

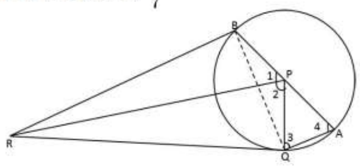
$\Rightarrow \frac{x}{-15} = \frac{-y}{21} = \frac{1}{-3}$

$\Rightarrow \frac{x}{15} = \frac{y}{21} = \frac{1}{3}$

$\Rightarrow x = \frac{15}{3},\ y = \frac{21}{3}$

$\Rightarrow x = 5, y = 7$

The fraction is $\frac{5}{7}$

33. 

Given: A circle with centre P, AB is the diameter.

QA||RP, where RQ is the tangent to the circle.

To prove: RB is tangent to the circle i.e. ∠RBP = 90°.

Construction: Join BQ and PQ.

Proof: RQ is tangent to the circle and PQ is the radius at point of contact Q.

∴ PQ⊥RQ (radius of a circle is perpendicular to the tangent at point of contact)

⇒ ∠PQR = 90°....(1)

QA || RP and PQ is the transversal.

⇒ ∠2 = ∠3.....(2) (Alternate interior angles)

But, PQ = PA (radii of the circle)

∴ In ΔPQA, ∠3 = ∠4..... (3) (Angles opposite to equal sides are equal)

From (2) and (3) ⇒ ∠2 = ∠3 = ∠4

∠BPQ and ∠BAQ are the angles made by the arc BQ at the centre P and on the remaining part of the circle respectively.

∴ ∠BPQ = 2 ∠BAQ

i.e., ∠1 + ∠2 = 2∠4

⇒ ∠1 + ∠2 = ∠4 + ∠4

⇒ ∠1 = ∠4 (as ∠2 = ∠4)

So, ∠1 = ∠2 = ∠3 = ∠4

⇒ ∠1 = ∠2

In ΔBPR and ΔRPQ,

BP = PQ (radii of the circle)

∠1 = ∠2 (proved)

RP = RP (common)

∴ ΔBPR ≅ ΔQPR (SAS congruency)

⇒ ∠PBR = ∠PQR (corresponding angles)

But ∠PBR = 90° from equation (1)

i.e., PB ⊥ BR

Therefore, RB is a tangent to the circle at point B.

34. Let the radii of the circles are r_1 cm and r_2 cm

$\therefore r_1 + r_2 = 14$ ...(i)

And, sum of their areas = $\pi r_1^2 + \pi r_2^2$

$130\pi = \pi\left(r_1^2 + r_2^2\right)$

or, $130\pi = \pi\left(r_1^2 + r_2^2\right)$

$\therefore \quad r_1^2 + r_2^2 = 130$ ...(ii)

$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$

or, $(14)^2 = 130 + 2r_1r_2$

or, $2r_1r_2 = 196 - 130$

0r, $2r_1r_2$= 66

$(r_1-r_2)^2=r_1^2+r_2^2-2r_1r_2$

$(r_1-r_2)^2$= 130 - 66

$(r_1-r_2)^2$= 64

or,$r_1-r_2=8$ ...(iii)

From (i) and (iii), $2r_1=22$

or, $r_1=11$ cm

$r_2=14-11$

$r_2=3\ cm$.

OR

We have to find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres.

For the first triangle, we have a = 35, b = 53 and c = 66.

$\therefore \quad s=\frac{a+b+c}{2}=\frac{35+53+66}{2}=77\text{cm}$

Let Δ_1 be the area of the first triangle. Then,

$\Delta_1=\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_1=\sqrt{77(77-35)(77-53)(77-66)}=\sqrt{77\times 42\times 24\times 11}$

$\Rightarrow \quad \Delta_1=\sqrt{7\times 11\times 7\times 6\times 6\times 4\times 11}$= $\sqrt{7^2\times 11^2\times 6^2\times 2^2}=7\times 11\times 6\times 2=924\text{cm}^2$ ...(i)

For the second triangle, we have a = 33,b = 56,c = 65

$\therefore \quad s=\frac{a+b+c}{2}=\frac{33+56+65}{2}=77\text{cm}$

Let Δ_2 be the area of the second triangle. Then,

$\Delta_2=\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_2=\sqrt{77(77-33)(77-56)(77-65)}$

$\Rightarrow \quad \Delta_2=\sqrt{77\times 44\times 21\times 12}$= $\sqrt{7\times 11\times 4\times 11\times 3\times 7\times 3\times 4}$= $\sqrt{7^2\times 11^2\times 4^2\times 3^2}$

$\Rightarrow \quad \Delta_2=7\times 11\times 4\times 3=924\text{cm}^2$

Let r be the radius of the circle. Then,

Area of the circle = Sum of the areas of two triangles

$\Rightarrow \quad \pi r^2=\Delta_1+\Delta_2$

$\Rightarrow \quad \pi r^2$= 924 + 924

$\Rightarrow \quad \frac{22}{7}\times r^2=1848$

$\Rightarrow \quad r^2=1848\times\frac{7}{22}=3\times 4\times 7\times 7\Rightarrow\ r=\sqrt{3\times 2^2\times 7^2}=2\times 7\times\sqrt{3}=14\sqrt{3}\text{cm}$

35. Two number x(1,2,3,4)and y(1,4,9,16) can be selected as pairs in the following 16 ways :

(1,1),(1,4),(1,9),(1,16),(2,1),(2,4),(2,9),(2,16),(3,1),(3,4),(3,9),(3,16),(4,1),(4,4),(4,9),(4,16)

$\therefore$ Total number of possible outcomes = 16

The pairs with the product less than 16 are:

$(1,1),(1,4),(1,9),(2,1),(2,4),(3,1),(3,4),(4,1)$

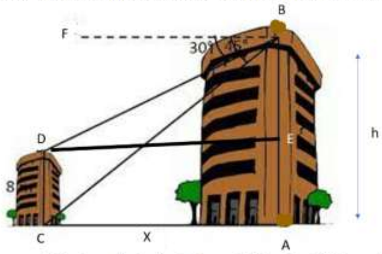
$\therefore$ Number of favorable outcomes = 8

Hence , required probability = $\frac{8}{16}=\frac{1}{2}$

Section E

36. **Read the text carefully and answer the questions:**

Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30° and 45°, respectively.



(i) Let h is height of big building, here as per the diagram.

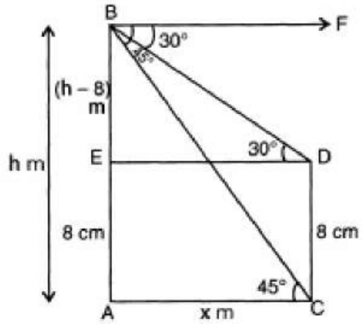
AE = CD = 8 m (Given)

BE = AB - AE = (h - 8) m

Let AC = DE = x

Also, $\angle FBD = \angle BDE = 30^\circ$

$\angle FBC = \angle BCA = 45^\circ$



In $\triangle$ACB, $\angle A = 90^\circ$

$\tan 45^\circ = \frac{AB}{AC}$

$\Rightarrow$ x = h, ...(i)

In $\triangle$BDE, $\angle E = 90^\circ$

$\tan 30^\circ = \frac{BE}{ED}$

$\Rightarrow x = \sqrt{3}(h - 8)$.(ii)

From (i) and (ii), we get

$h = \sqrt{3}h - 8\sqrt{3}$

h($\sqrt{3}$ - 1) = 8$\sqrt{3}$

h = $\frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

= $\frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84)$ = 18.92 m

(ii) Let h is height of big building, here as per the diagram.

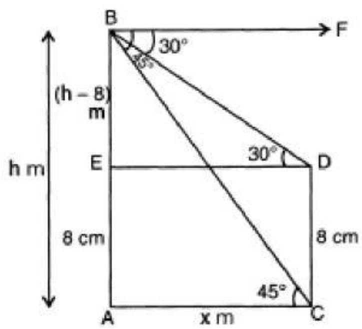
AE = CD = 8 m (Given)

BE = AB - AE = (h - 8) m

Let AC = DE = x

Also, $\angle FBD = \angle BDE = 30^\circ$

$\angle FBC = \angle BCA = 45^\circ$



In $\triangle$ACB, $\angle A = 90^\circ$

$\tan 45^\circ = \frac{AB}{AC}$

$\Rightarrow$ x = h, ...(i)

In $\triangle$BDE, $\angle E = 90^\circ$

$\tan 30^\circ = \frac{BE}{ED}$

$\Rightarrow x = \sqrt{3}(h-8)$.(ii)

From (i) and (ii), we get

$h = \sqrt{3}h - 8\sqrt{3}$

h($\sqrt{3}$ - 1) = 8$\sqrt{3}$

h = $\frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

= $\frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84)$ = 18.92 m

Hence height of the multistory building is 18.92 m and the distance between two buildings is 18.92 m.

(iii) In $\triangle$ABC

$\sin 45^\circ = \frac{AB}{BC}$

$\Rightarrow BC = \frac{AB}{\sin 45^0}$

$\Rightarrow BC = \frac{18.92}{\frac{1}{\sqrt{2}}}$

$\Rightarrow$ BC = 26.76 m

Hence the distance between top of multistoried building and bottom of first building is 26.76 m.

OR

In $\triangle$BDE

cos 30^0 = $\frac{ED}{BD}$

$\Rightarrow BD = \frac{ED}{\cos 30^0}$

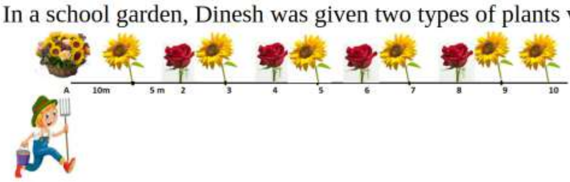
$\Rightarrow BD = \frac{\frac{8\sqrt{3}}{\sqrt{3}-1}}{\frac{\sqrt{3}}{2}} = \frac{16}{\sqrt{3}-1}$

$\Rightarrow BD = 8\left(\sqrt{3}+1\right)$ = 21.86 m

Hence, the distance between top of multistoried building and top of first building is 21.86 m.

37. **Read the text carefully and answer the questions:**

In a school garden, Dinesh was given two types of plants viz. sunflower and rose flower as shown in the following figure.



The distance between two plants is to be 5m, a basket filled with plants is kept at point A which is 10 m from the first plant. Dinesh has to take one plant from the basket and then he will have to plant it in a row as shown in the figure and then he has to return to the basket to collect another plant. He continues in the same way until all the flower plants in the basket. Dinesh has to plant ten numbers of flower plants.

(i) The distance covered by Dinesh to pick up the first flower plant and the second flower plant,

= 2 $\times$ 10 + 2 $\times$ (10 + 5) = 20 + 30

therefore, the distance covered for planting the first 5 plants

= 20 + 30 + 40 + ... 5 terms

This is in AP where the first term a = 20

and common difference d = 30 - 20 = 10

(ii) We know that a = 20, d = 10 and number of terms = n = 5 so,

$S_n = \frac{n}{2}[2a + (n-1)d]$

So, the sum of 5 terms

$S_5 = \frac{5}{2}[2 \times 20 + 4 \times 10] = \frac{5}{2} \times$ 80 = 200 m

Hence, Dinesh will cover 200 m to plant the first 5 plants.

(iii) As a = 20, d = 10 and here n = 10

So, S_{10} = $\frac{10}{2}[2 \times 20 + 9 \times 10] = 5 \times 130$ = 650 m

So, hence Ramesh will cover 650 m to plant all 10 plants.

OR

Total distance covered by Ramesh 650 m

Time = $\frac{\text{distance}}{\text{speed}} = \frac{650}{10}$ = 65 minutes

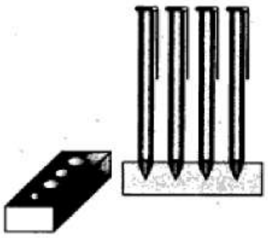
Time taken to plant all 10 plants = 15 × 10 = 150 minutes

Total time = 65 + 150 = 215 minutes = 3 hrs 35 minutes

38. **Read the text carefully and answer the questions:**

A carpenter in the small town of Bareilly used to make and sell different kinds of wood items like a rectangular box, cylindrical pen stand, and cuboidal pen stand. One day a student came to his shop and asked him to make a pen stand with the dimensions as follows:

A pen stand should be in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid should be 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm.



(i) Volume of the cuboid

$= 15 \times 10 \times 3.5 = 525\text{cm}^3$

(ii) Volume of a conical depression

$= \frac{1}{3}\pi(0.5)^2(1.4)$

$= \frac{1}{3} \times \frac{22}{7} \times 0.25 \times \frac{14}{10} = \frac{11}{30}\text{cm}^3$

$\therefore$ Volume of four conical depressions

$= 4 \times \frac{11}{30}\text{cm}^3 = \frac{22}{15}\text{cm}^3 = 1.47\text{cm}^3$

(iii) $\because$ Volume of the wood in the entire stand = volume of cuboid - volume of 4 conical depressions

$= 525 - 1.47 = 523.53\text{cm}^3$

OR

Cost of wood per cm^3 = ₹10

Total cost of making pen stand = 10 × 523.53 = ₹5235.3