SAMPLE QUESTION PAPER (BASIC) - 06

Class 10 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. If the coordinates of one end of a diameter of a circle are (2, 3) and the coordinates of its centre are (-2, 5), then the coordinates of the other end of the diameter are **[1]**

   a) (0, 8) b) (0, 4)

   c) (6, – 7) d) (– 6, 7)

2. In a right triangle ABC, right angled at B, BC = 12 cm and AB = 5 cm. The radius of the circle inscribed in the triangle (in cm) is **[1]**

   a) 4 b) 1

   c) 2 d) 3

3. Two dice are thrown simultaneously. The probability that the sum of the numbers appearing on the dice is 1 is **[1]**

   a) 3 b) 0

   c) 2 d) 1

4. If A (2, 2), B (-4, - 4) and C (5, -8) are the vertices of a triangle, then the length of the median through vertex C is **[1]**

   a) $\sqrt{113}$ b) $\sqrt{65}$

   c) $\sqrt{85}$ d) $\sqrt{117}$

5. If 2x - 3 y = 7 and (a + b) x - (a + b - 3) y = 4a + b represent coincident lines, then a and b satisfy the equation **[1]**

a) a - 5b = 0
b) 5a - b = 0
c) a + 5b = 0
d) 5a + b = 0

6. The coordinates of the fourth vertex of the rectangle formed by the points (0, 0) (2, 0), (0, 3) are **[1]**

a) (2, 3)
b) (3, 0)
c) (3, 2)
d) (0, 2)

7. The probability of getting 2 heads, when two coins are tossed, is **[1]**

a) $\frac{1}{4}$
b) 1
c) $\frac{1}{2}$
d) $\frac{3}{4}$

8. A cubical block of side 7 cm is surmounted by a hemisphere. The greatest diameter of the hemisphere is **[1]**

a) 10.5cm
b) 7cm
c) 3.5cm
d) 14cm

9. If three coins are tossed simultaneously, then the probability of getting at least two heads, is **[1]**

a) $\frac{1}{2}$
b) $\frac{3}{8}$
c) $\frac{1}{4}$
d) $\frac{7}{4}$

10. If α and β are the roots of the equation $3x^2 + 8x + 2 = 0$ then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ = ? **[1]**

a) $\frac{2}{3}$
b) 4
c) -4
d) $\frac{-3}{8}$

11. If one root of the equation $3x^2 - 10x + 3 = 0$ is $\frac{1}{3}$ then the other root is **[1]**

a) $\frac{1}{3}$
b) 3
c) $\frac{-1}{3}$
d) -3

12. If x tan 45° cos 60° = sin 60° cot 60°, then x is equal to **[1]**

a) $\frac{1}{2}$
b) 1
c) $\frac{1}{\sqrt{2}}$
d) $\sqrt{3}$

13. HCF of 144 and 198 is: **[1]**

a) 18
b) 12
c) 9
d) 6

14. The distance between the points (a cos 25°, 0) and (0, a cos 65°) is **[1]**

a) None of these
b) 3a
c) a
d) 2a

15. If the angles of elevation of the top of a tower from two points distant a and b from the base and in the same straight line with it are complementary, then the height of the tower is **[1]**

a) ab
b) $\frac{a}{b}$
c) $\sqrt{ab}$
d) $\sqrt{\frac{a}{b}}$

16. If the mean of first n natural numbers is 15, then n = **[1]**

a) 30

b) 14

c) 15

d) 29

17. HCF of two numbers is 113, their LCM is 56952. If one number is 904, the second number is **[1]**

a) 7791

b) 7911

c) 7719

d) 7119

18. If a pair of linear equations in two variables is consistent, then the lines represented by two equations are **[1]**

a) parallel

b) always coincident

c) intersecting

d) intersecting or coincident

19. **Assertion (A):** No two positive numbers can have 18 as their H.C.F and 380 as their L.C.M. **[1]**
Reason (R): L.C.M. is always completely divisible by H.C.F.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If in a $\triangle$ABC, a line DE || BC, intersects AB in D and AC in E, then $\frac{AB}{AD} = \frac{AC}{AE}$ **[1]**
Reason (R): If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. In a musical chair game, the person playing the music has been advised to stop playing the music at any time within 2 minutes after she starts playing. What is the probability that the music will stop within the first half minute after starting? **[2]**

22. The sum of the digits of a two digit number is 13. The number obtained by interchanging the digits of the given number exceeds the number by 27. Find the number. **[2]**

OR

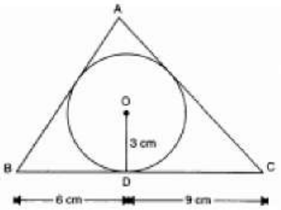
A two digit number is obtained by either multiplying sum of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2. Find the number.

23. Find the zeroes of a quadratic polynomial given as: $4u^2 + 8u$ and also verify the relationship between the zeroes and the coefficients. **[2]**

24. Find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also find the point of intersection. **[2]**

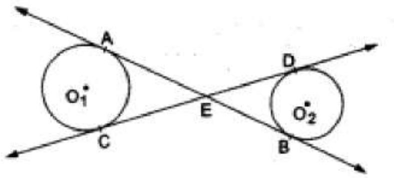
25. In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of $\triangle$ABC is 54 square centimeter, then find the lengths of **[2]**

sides AB and AC.



OR

In the given figure, common tangents AB and CD to the two circles with centres O_1 and O_2 intersect at E. Prove that AB = CD.



Section C

26. In $\triangle ABC$, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$. Find the value of cos A cos C - sin A sin C **[3]**

27. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h, it would have taken 3 hours more than the scheduled time. Find the distance covered by the train. **[3]**

28. Prove that 3 + 2 $\sqrt{5}$ is irrational. **[3]**

OR

Find the LCM of the following polynomials: $x\left(8x^3+27\right)$ and $2x^2\left(2x^2+9x+9\right)$

29. CD and GH are respectively the bisectors of $\angle$ACB and $\angle$EGF such that D and H lie on sides AB and FE of $\triangle$ABC and $\triangle$EFG respectively. If $\triangle$ABC $\sim$ $\triangle$FEG, show that: **[3]**
    i. $\frac{CD}{GH} = \frac{AC}{FG}$
    ii. $\triangle$DCB $\sim$ $\triangle$HGE
    iii. $\triangle$DCA $\sim$ $\triangle$HGF

30. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. **[3]**

OR

AB and AC are two equal chords of a circle. Prove that the bisector of the angle BAC passes through the centre of the circle.

31. Two boats approach a light house in mid-sea from opposite directions. The angles of elevations of the top of the lighthouse from two boats are 30° and 45° respectively. If the distance between two boats is 100 m, find the height of the lighthouse. **[3]**

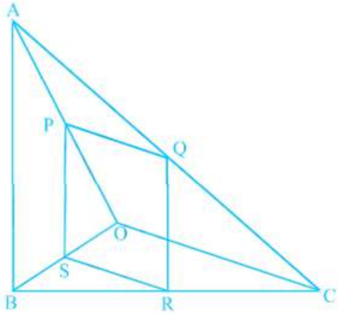
Section D

32. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages **[5]**

OR

Swati can row her boat at a speed of 5 km/hr in still water. If it takes her 1 hour more to row the boat 5.25 km upstream than to return downstream, find the speed of the stream.

33. In the figure, if PQRS is a parallelogram and AB || PS, then prove that OC || SR. **[5]**

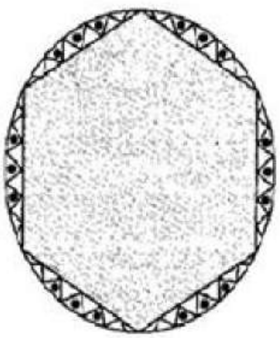


34. A chord of a circle of radius 10cm subtends a right angle at the center. Find the area of the corresponding: (Use $\pi = 3.14$) **[5]**

i. minor sector
ii. major sector
iii. minor segment
iv. major segment

OR

A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2. (use $\sqrt{3} = 1.7$)



35. The monthly income of 100 families are given as below: **[5]**

Income in (in ₹.)	**Number of families**
0-5000	8
5000-10000	26
10000-15000	41
15000-20000	16
20000-25000	3
25000-30000	3
30000-35000	2
35000-40000	1

Calculate the modal income.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called

Hermika at the top. Path around Anda is known as Pradakshina Path.

(i) Find the volume of the Hermika, if the side of cubical part is 10 m.

(ii) Find the volume of cylindrical base part whose diameter and height 48 m and 14 m.

(iii) If the volume of each brick used is 0.01 m^3, then find the number of bricks used to make the cylindrical base.

OR

If the diameter of the Anda is 42 m, then find the volume of the Anda.

37. **Read the text carefully and answer the questions:** **[4]**

Jaspal Singh is an auto driver. His autorickshaw was too old and he had to spend a lot of money on repair and maintenance every now and then. One day he got to know about the EV scheme of the Government of India where he can not only get a good exchange bonus but also avail heavy discounts on the purchase of an electric vehicle. So, he took a loan of ₹1,18,000 from a reputed bank and purchased a new autorickshaw.

Jaspal Singh repays his total loan of 118000 rupees by paying every month starting with the first instalment of 1000 rupees.

(i) If he increases the instalment by 100 rupees every month, then what amount will be paid by him in the 30th instalment?

(ii) If he increases the instalment by 100 rupees every month, then what amount of loan does he still have to pay after 30th instalment?

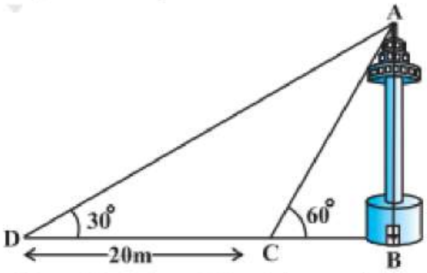
(iii) If he increases the instalment by 100 rupees every month, then what amount will be paid by him in the 100th instalment?

OR

If he increases the instalment by 200 rupees every month, then what amount would he pay in 40th instalment?

38. **Read the text carefully and answer the questions:** **[4]**

A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60^{o} from a point 20 m away from this point on the same bank the angle of elevation of the top of the tower is 30^{o}.



(i) Find the width of the canal.

(ii) Find the height of tower.

(iii) Find the distance between top of the tower and point D.

OR

Find the distance between top of tower and point C.

Solution

SAMPLE QUESTION PAPER (BASIC) - 06

Class 10 - Mathematics

Section A

1. **(d)** (– 6, 7)

   Explanation: Let the coordinates of the other end be B(x_2, y_2).

   One end of the diameter is A (2, 3) and the centre is O$(-2, 5)$.

   Since the centre is midpoint of the diameter of the circle.

   $\therefore x = \frac{x_1+x_2}{2}$

   $\Rightarrow -2 = \frac{2+x_2}{2}$

   $\Rightarrow x_2 = -6$

   And $y = \frac{y_1+y_2}{2}$

   $\Rightarrow 5 = \frac{3+y_2}{2}$

   $\Rightarrow y_2 = 7$

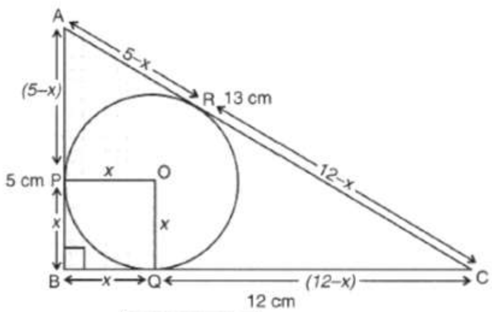
   Therefore, the coordinates of other ends of the diameter are $(-6, 7)$.

2. **(c)** 2

   Explanation:

   Here, AB = 5cm, BC = 12 and $\angle B = 90^\circ$

   Let the radius of circle be x cm



   $\therefore AC = \sqrt{(12)^2 + (5)^2}$

   $= \sqrt{144 + 25}$

   $= \sqrt{169} = 13\text{cm}$

   $\because$ AC = AR + RC

   $\therefore$ AC = (5 - x) + 12 - x

   $\Rightarrow$ 13 = 5 - x + 12 - x

   $\Rightarrow$ 2x = 17 - 13 = 4

   $\Rightarrow \quad x = \frac{4}{2} = 2\text{cm}$

   Hence, radius of the circle = 2cm.

3. **(b)** 0

   Explanation: Elementary events are

   (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

   (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

   (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

   (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

   (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

   (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

   $\therefore$ Number of Total outcomes = 36

   And Number of possible outcomes (sum of numbers appearing on die is 1) = 0

   $\therefore$ Required Probability = $\frac{0}{36} = 0$

4. **(c)** $\sqrt{85}$

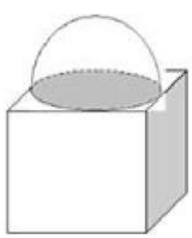
   Explanation: Let mid point of A(2, 2), B(-4, -4) be whose coordinates will be

$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2-4}{2}, \frac{2-4}{2}\right)$
or $\left(\frac{-2}{2}, \frac{-2}{2}\right) = (-1, -1)$
$\therefore$ Length of median CD
$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(5+1)^2 + (-8+1)^2}$
$= \sqrt{(6)^2 + (-7)^2} = \sqrt{36 + 49}$
$= \sqrt{85}$ units

5. **(a)** a - 5b = 0
Explanation: Given Equations are 2x - 3y = 7
and (a + b) x - (a + b - 3) y = 4 a + b represent coincident lines.
When lines are coincident then the condition of equations
$a_1x + b_1y = c_1$,
$a_2x + b_2y = c_2$
is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
On comparing, we get
$\frac{2}{a+b} = \frac{3}{a+b-3} = \frac{7}{4a+b}$
Now, we can equate any two equation. So, taking
$\frac{2}{a+b} = \frac{7}{4a+b}$
$\Rightarrow$ 2(4a + b) = 7(a + b)
$\Rightarrow$ 8a + 2b = 7a + 7b
$\Rightarrow$ 8a − 7a = 7b − 2b
$\Rightarrow$ a = 5b
$\Rightarrow$ a − 5b = 0
Therefore, The required equation satisfied by a and b is a - 5b = 0.

6. **(a)** (2, 3)
Explanation: We are given three vertices (0, 0), (2, 0) and (0, 3) of a rectangle.
We have to find the coordinates of the fourth vertex.
By plotting the given vertices on an XY plane, C (0, 3) are the consecutive vertices.
Consider D to represent the fourth vertex.
Since, AB = 2 units and BC = 3 units.
Thus, point D is at a horizontal distance of 3 units and a vertical distance of 2 units from the origin.
Thus, the coordinates of the fourth vertex of the rectangle are (2, 3).

7. **(a)** $\frac{1}{4}$
Explanation: All possible outcomes are HH, HT, TH, TT. Their number is 4.
Getting 2 heads, means getting HH. Its number is 1.
$\therefore$ P(getting 2 heads) = $\frac{1}{4}$

8. **(b)** 7cm
Explanation: 
It is clear that Maximum diameter of hemisphere can be the side of the cube.
$\therefore$ The greatest diameter of the hemisphere = 7 cm

9. **(a)** $\frac{1}{2}$
Explanation: Possible outcomes of tossing three coins are:
(HHH), (HHT), (HTH), (THH), (TTT), (TTH), (THT),(HTT)
here H and T are denoted for Head and Tail.

Total outcomes = 8

no. of outcomes with at least two heads = 4

$\therefore$ required probability $= \frac{4}{8} = \frac{1}{2}$

10. **(c)** -4

Explanation: We have $\alpha + \beta = \frac{-8}{3}$ and $\alpha\beta = \frac{2}{3}$.

$\therefore \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \frac{(\alpha+\beta)}{\alpha\beta} = \frac{-8}{3} \times \frac{3}{2} = -4$.

11. **(b)** 3

Explanation: Given:

$3x^2 - 10x + 3 = 0$

One root of the equation is 1/3.

Let the other root be α.

We know that:

Product of the roots $= \frac{c}{a}$

$\Rightarrow \frac{1}{3} \times \alpha = \frac{3}{3}$

$\Rightarrow \alpha = 3$

12. **(b)** 1

Explanation: We have, x tan 45° cos 60° = sin 60° cot 60°

$\Rightarrow x \times 1 \times \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} \Rightarrow \frac{x}{2} = \frac{1}{2}$

$\Rightarrow x = \frac{1}{2} \times 2 = 1$

13. **(a)** 18

Explanation:

We first factorise the two numbers:

2	144
2	72
2	36
2	18
3	9
3	3
	1

2	198
3	99
3	33
11	11
	1

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$

$198 = 2 \times 3 \times 3 \times 11 = 2 \times 3^2 \times 11$

Here, HCF $= 2 \times 3^2$

= 18

14. **(c)** a

Explanation: Distance between (a cos 25^o, 0) and (0, a cos65^o)

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0 - a\cos 25^\circ)^2 + (a\cos 65^\circ - 0)^2}$

$= \sqrt{a^2\cos^2 25^\circ + a^2\cos^2 65^\circ}$

$= \sqrt{a^2\left[\cos^2 25^\circ + \cos^2 65^\circ\right]}$

$= a\sqrt{\cos^2(90^\circ - 65^\circ) + \cos^2 65^\circ}$

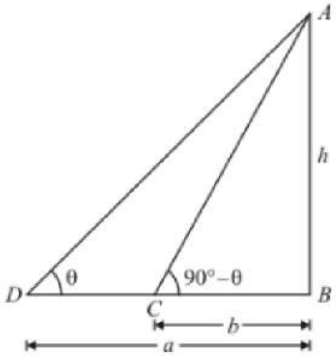
$= a\sqrt{\sin^2 65^\circ + \cos^2 65^\circ}$

$= a(\sqrt{1}) = a$

15. **(c)** $\sqrt{ab}$

Explanation:

Let h be the height of tower AB.



Given that: angle of elevation of top of the tower are $\angle$D = θ and $\angle$C = 90^o - θ.Distance BC = b and BD = a

Here, we have to find the height of tower.

So we use trigonometric ratios.

In a triangle ABC,

$\Rightarrow \tan C = \frac{AB}{BC}$

$\Rightarrow \tan(90^\circ - \theta) = \frac{h}{b}$

$\Rightarrow \cot \theta = \frac{h}{b}$

Again in a triangle *ABD*,

tan D = $\frac{AB}{BD}$

$\Rightarrow \tan \theta = \frac{h}{a}$

$\Rightarrow \frac{1}{\cot \theta} = \frac{h}{a}$

$\Rightarrow \frac{b}{h} = \frac{h}{a}$ [Put $\cot \theta = \frac{h}{b}$

$\Rightarrow$ h^2 = ab

$\Rightarrow$ h = $\sqrt{ab}$

16. **(d)** 29

Explanation: Mean of first n natural number = 15

$\frac{n(n+1)}{2n}$ = 15

$\frac{n+1}{2}$ = 15

$\Rightarrow$ n + 1 = 30

$\Rightarrow$n = 30 - 1 = 29

17. **(d)** 7119

Explanation: LCM $\times$ HCF = Product of two numbers

56952 $\times$ 113 = 904 $\times$ second number

$\frac{56952 \times 113}{904}$ = second number

Therefore, second number = 7119

18. **(d)** intersecting or coincident

Explanation: If a pair of linear equations in two variables is consistent, then its solution exists.

$\therefore$ The lines represented by the equations are either intersecting or coincident.

19. **(a)** Both A and R are true and R is the correct explanation of A.

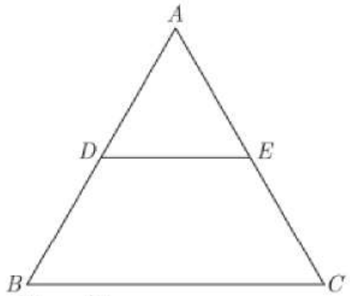
Explanation: 380 is not divisible by 18.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Reason is true: [This is Thale's Theorem]

For Assertion

Since, DE || BC by Thale's Theorem



$\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{DB}{AD} = \frac{EC}{AE}$

$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$

$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$

$\frac{AB}{AD} = \frac{AC}{AE}$

Assertion is true.

Since, reason gives Assertion.

Section B

21. The possible outcomes are all the numbers between $0\ and\ 2$.

Suppose A be the event 'music is stopped within the first half minute'.

$\therefore$ Outcomes favourable to the event A are all points on the number line from O to Q i.e. from $0\ to\ \frac{1}{2}$

Total number of outcomes are the points on the number line from O to P i.e. from 0 to 2.

$\therefore P(A) = \frac{\text{Length } OQ}{\text{Length } OP} = \frac{1/2}{2} = \frac{1}{4}$

22. Let the unit digit is b and ten's digit is a.

So, two digit number is 10a + b.

As per given condition

The sum of the digits of a two digit number is 13.

So, a + b = 13 .....(i)

and the number obtained by interchanging the digits of the given number exceeds the number by 27.

10b + a = (10a + b) + 27

⇒ 9b = 9a + 27

⇒b = a + 3 .....(ii)

Putting (ii) in (i), we get :

a + a + 3 = 13

⇒ 2a = 10

⇒ a = 5

Putting a = 5 in (ii), we get :

b = 5 + 3 = 8

Two digit number = 10a + b = 5(10) + 8 = 58

Therefore, the number is 58.

OR

Let the digit at units place be x and the digit at ten's place be y.

Then, Number = 10y + x

If a two digit number is obtained by either multiplying sum of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2.

According to the given conditions, we have

10y + x =8(x + y) + 1

⇒ 10y + x = 8x + 8y + 1

⇒ 10y - 8y + x - 8x - 1 = 0

⇒ 7x - 2y + 1=0

and, 10y + x =13(y - x) + 2

10y + x = 13y - 13x + 2

$\Rightarrow$14x - 3y -2=0

By using cross-multiplication, we hvae

$\frac{x}{-2\times-2-(-3)\times1} = \frac{-y}{7\times-2-14\times1} = \frac{1}{7\times-3-14\times-2}$

$\Rightarrow \quad \frac{x}{4+3} = \frac{-y}{-14-14} = \frac{1}{-21+28}$

$\Rightarrow \quad \frac{x}{7} = \frac{y}{28} = \frac{1}{7}$

$\Rightarrow \quad x = \frac{7}{7} = 1 \text{ and } y = \frac{28}{7} = 4$

Hence, the number =10y + x = 10 $\times$ 4 + 1 = 41.

23. The quadratic equation is given as: $4u^2 + 8u$

it can be written in the standard form as:

= $4u^2 + 8u + 0$

= 4u (u + 2)

The value of $4u^2 + 8u$ is zero when 4u = 0 or u + 2 = 0,

i.e., u = 0 or u = −2

Therefore, the zeroes of $4u^2 + 8u$ are 0 and −2

Sum of zeroes = $0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(\text{ coefficient of } u)}{\text{coefficient of } u^2}$

Product of zeroes = $0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{constant torm}}{\text{coefficient of } u^2}$

Hence verified

24. Let the point on y-axis be P(0, y) and AP: PB = K: 1

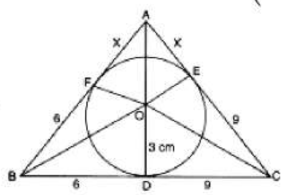
Therefore $\frac{5-k}{k+1} = 0$ gives k = 5

Hence required ratio is 5: 1

$y = \frac{-4(5)-6}{6} = \frac{-13}{3}$

Hence point on y-axis is $\left(0, \frac{-13}{3}\right)$.

25.



Let, AF = AE = x

ar $\triangle$ABC = ar$\triangle$AOB + ar$\triangle$BOC + ar$\triangle$AOC

ar $\triangle$ABC = $\frac{1}{2}(15)(3) + \frac{1}{2}(6+x)(3) + \frac{1}{2}(9+x)(3)$

$\frac{1}{2}[15 + 6 + x + 9 + x].3 = 54$

45 + 3x - 54

x = 3

$\therefore$ AB = 9 cm, AC = 12 cm

and BC = 15 cm.

OR

We know that tangent segments to a circle from the same external point are congruent.

So, $EA = EC$ for the circle having centre O_1

And, $ED = EB$ for the circle having centre O_2

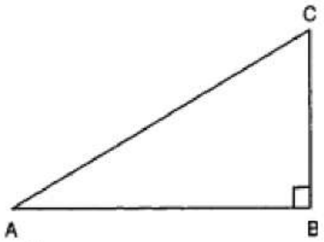
Now, Adding ED on both sides in $EA = EC$, we get

$EA + ED = EC + ED$

$\Rightarrow EA + EB = EC + ED$

$\Rightarrow AB = CD$

Section C

26. 

we have,

$\tan A = \frac{1}{\sqrt{3}} = tan 30^\circ$

$\therefore A = 30^\circ$

In $\triangle$ABC, we have

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 30^\circ + 90^\circ + \angle C = 180^\circ$

$\Rightarrow 120^\circ + \angle C = 180^\circ$

$\Rightarrow \angle C = 180^\circ - 120^\circ = 60^\circ$

So,

cos A.cos C - sin A.sin C

$= \cos 30^\circ . \cos 60^\circ - \sin 30^\circ . \sin 60^\circ$

$= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 0$

27. Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel be d km.

Since, Speed = $\frac{\text{Distance travelled}}{\text{Time taken to travel that distance}} \Rightarrow x = \frac{d}{t} \Rightarrow$ d = xt ....(1)

According to the question,

x + 10 = $\frac{d}{t-2} \Rightarrow$ (x + 10)(t - 2) = d

$\Rightarrow$ xt + 10t - 2x - 20 = d

$\Rightarrow$ -2x + 10t = 20 .....(2) [Using eq. (1)]

Again, x - 10 = $\frac{d}{t+3} \Rightarrow$ (x - 10)(t + 3) = d

$\Rightarrow$ xt - 10t + 3x - 30 = d

$\Rightarrow$ 3x - 10t = 30 .....(3) [Using eq. (1)]

Adding equations (2) and (3), we obtain:

x = 50

Substituting the value of x in equation (2), we obtain:

(-2) $\times$ (50) + 10t = 20 $\Rightarrow$-100 + 10t = 20

$\Rightarrow$10t = 120

t = 12

From equation (1), we obtain:

d = xt = 50 $\times$ 12 = 600

Thus, the distance covered by the train is 600 km.

28. Let us assume, to the contrary, that is $3 + 2\sqrt{5}$ rational.

That is, we can find coprime integers a and b $(b \neq 0)$ such that

$3 + 2\sqrt{5} = \frac{a}{b}$ Therefore, $\frac{a}{b} - 3 = 2\sqrt{5}$

$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$

$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2}$

Since a and b are integers,

We get $\frac{a}{2b} - \frac{3}{2}$ is rational, also so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction arose because of our incorrect assumption that $3 + 2\sqrt{5}$ is rational.

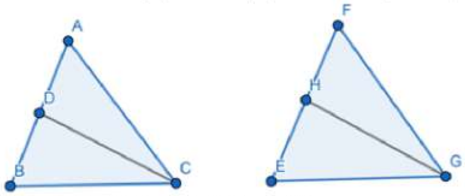
So, we conclude that $3 + 2\sqrt{5}$ is irrational.

OR

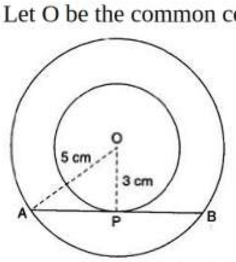
$P(x) = x\left(8x^3 + 27\right)$

$= x(2x+3)\left(4x^2 - 6x + 9\right)$ Using identity $a^3 + b^3 = (a+b)\left(a^2 + b^2 - ab\right)$

$Q(x)=2x^2\left(2x^2+6x+3x+9\right)$
$=2\times x^2[2x(x+3)+3(x+3)]$
$=2\times x^2\times(x+3)(2x+3)$
Common on factors: x, $(2x+3)$
Uncommon on factors: $\left(4x^2-6x+9\right)$ and $2, x, (x+3)$
$\therefore$ LCM of $P(x)$ and $Q(x)=2\times x^2(x+3)(2x+3)\left(4x^2-6x+9\right)$

29. 

Given, $\triangle$ABC $\sim$ $\triangle$FEG ….(1)
(i) Corresponding angles of similar triangles
$\Rightarrow$ ∠BAC = ∠EFG ….(2)
And ∠ABC = ∠FEG …(3)
$\Rightarrow$ ∠ACB = ∠FGE
$\Rightarrow \frac{1}{2}\angle ACB=\frac{1}{2}\angle FGE$
$\Rightarrow$ ∠ACD = ∠FGH and ∠BCD = ∠EGH ……(4)
Consider $\triangle$ACD and $\triangle$FGH
$\Rightarrow$ From (2) we have
$\Rightarrow$∠DAC = ∠HFG
From (4) we have
$\Rightarrow$ ∠ACD = ∠EGH
Also, ∠ADC = ∠FGH
If the $\angle A=\angle F$, then by angle sum property of triangle 3rd angle will also be equal.
By AAA similarity, in two triangles, if the angles are equal, then sides opposite to the equal angles are in the same ratio (or proportional) and hence the triangles are similar.
$\therefore$ $\triangle$ADC $\sim$ $\triangle$FHG
(ii) By Converse proportionality theorem
$\Rightarrow \frac{CD}{GH}=\frac{AC}{FG}$
(iii) Consider $\triangle$DCB and $\triangle$HGE
From eq(3) we have
$\Rightarrow$ ∠DBC = ∠HEG
From (4) we have
$\Rightarrow$ ∠BCD = ∠FGH
Also, ∠BDC = ∠EHG
$\therefore$ $\triangle$DCB $\sim$ $\triangle$HGE
Hence proved.
30. Let O be the common centre of the two concentric circles.



Let AB be a chord of the larger circle which touches the smaller circle at P.
Join OP and OA
Then, ∠OPA = 90^0 [∵The tangent at any point of a circle is perpendicular to th radius through the point of contact]
$\therefore$ $OA^2 = OP^2 + AP^2$ .... By Pythagoras theorem
$\Rightarrow (5)^2 = (3)^2 + AP^2$
$\Rightarrow 25 = 9 + AP^2$

$\Rightarrow P^2 = 25 - 9$

$\Rightarrow AP^2 = 16$

$\Rightarrow AP = \sqrt{16} = 4$ cm

SInce the perpendicular from the centre of a circle to a chord bisects the chord, therfore,

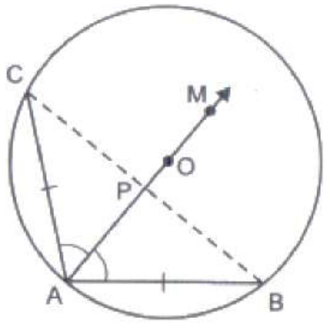
AP = BP = 4 cm

$\therefore$ AB = AP + BP = AP + AP = 2AP = 2(4) = 8 cm

Hence, the required length is 8 cm.

OR

Given: AB =AC and AM is the bisector of $\angle BAC$.



To prove: AM passes through O.

Construction: Join BC. Let AM intersect BC at P.

Proof: In ΔBAP and ΔCAP

AB = AC [Given]

$\angle BAP = \angle CAP$ [Given]

And AP = BP [Common side]

$\therefore \Delta BAP \cong \Delta CAP$ [By SAS congruency]

$\therefore \angle BPA = \angle CPA$ [By C.P.C.T.]

And CP = PB

But $\angle BPA + \angle CPA = 180^\circ$ [Linear pair $\angle s$]

$\therefore \angle BPA = \angle CPA = 90^\circ$

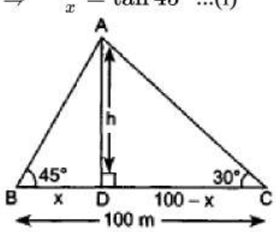
$\therefore$ AP is perpendicular bisector of the chord BC, which will pass through the centre O on being produced.

Hence, AM passes through O.

31. In right $\triangle$ADB,

h = x

$\Rightarrow \quad \frac{h}{x} = \tan 45^\circ$ ...(i)



Now in rt. $\triangle$ADC

$\frac{h}{100-x} = \tan 30^\circ$

Solve for h and x.

$\Rightarrow \frac{h}{100-x} = \frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}h = 100 - x$

$\Rightarrow \quad \sqrt{3}x = 100 - x$ [Using eq.(i)]

$\Rightarrow (\sqrt{3}+1)x = 100 \Rightarrow x = \frac{100}{\sqrt{3}+1}$

$\Rightarrow \quad x = \frac{100(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$\Rightarrow \quad x = \frac{100(\sqrt{3}-1)}{2} = 50(\sqrt{3}-1)$m

$\therefore$ h = height of lighthouse = $50(\sqrt{3}-1)$m

Section D

32. If the present age of sister be x,then ,by the first condition of the question,we have,

present age of the girl = 2x

By the second condition of the question,we have,

(2x + 4)(x + 4) = 160

$2x^2 + 8x + 4x + 16 = 160$

$2x^2 + 12x - 144 = 0$

$2x^2 + (24 - 12)x - 144 = 0$

2x(x + 12) - 12(x + 12) = 0

(2x - 12)(x +12) = 0

$\therefore$ x = 6; x = -12

Since age can't be negative,therefore

x=6

So, Age of sister = 6 and Age of girl = 2(6)=12

OR

Let the speed of the stream be x km/hr.

Speed of boat upstream = (5 - x) km/hr.

Speed of boat downstream = (5 + x) km/hr.

Time taken to go upstream = $\frac{5.25}{5-x}$ hours.

Time taken to go downstream = $\frac{5.25}{5+x}$ hours.

According to question,

$\therefore \quad \frac{5.25}{5-x} - \frac{5.25}{5+x} = 1$

$\Rightarrow \quad 5.25[\frac{1}{5-x} - \frac{1}{5+x}] = 1$

$\Rightarrow \quad \frac{21}{4}[\frac{5+x-5+x}{(5-x)(5+x)}] = 1$

$\Rightarrow \quad \frac{21}{4} \times \frac{2x}{25-x^2} = 1$

$\Rightarrow \quad 21x = 50 - 2x^2$

$\Rightarrow 2x^2 + 21x - 50 = 0$

$\Rightarrow 2x^2 + 25x - 4x - 50 = 0$

$\Rightarrow \quad x(2x+25) - 2(2x+25) = 0$

$\Rightarrow$ (2x + 25) (x - 2) = 0

$\Rightarrow$ x - 2 = 0, 2x + 25 = 0

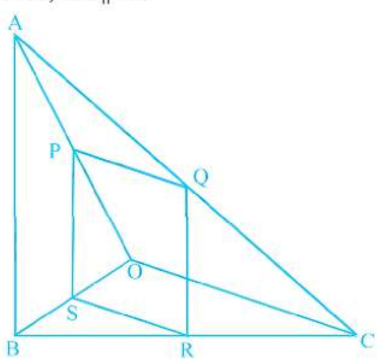
$\Rightarrow$ x = 2 $\left[\because x \neq -\frac{25}{2} \text{ as } x > 0\right]$

Hence, the speed of the stream is 2 km/hr.

33. It is given that PQRS is a parallelogram,

So, PQ || SR and PS || QR.

Also, AB || PS.



To prove OC || SR

In $\triangle$OPS and OAB,

PS || AB

$\angle$POS = $\angle$AOB [common angle]

$\angle$OSP = $\angle$OBA [corresponding angles]

$\therefore$OPS $\sim$ $\triangle$OAB [by AAA similarity criteria]

Then,

$\frac{PS}{AB} = \frac{OS}{OB}$ ...(i) [by basic proportionality theorem]

In $\triangle$CQR and ΔCAB,

QR || PS || AB

$\angle$QCR = $\angle$ACB [common angle]

$\angle$CRQ = $\angle$CBA [corresponding angles]

$\therefore \triangle CQR \sim \triangle CAB$

Then, by basic proportionality theorem

$= \frac{QR}{AB} = \frac{CR}{CB}$

$\Rightarrow \frac{PC}{AB} = \frac{CR}{CB}$ .....(ii)

[PS $\cong$ QR Since, PQRS is a parallelogram,]

From Equation (i) and (ii),

$\frac{\mathrm{OS}}{\mathrm{OB}} = \frac{\mathrm{CR}}{\mathrm{CB}}$

or $\frac{\mathrm{OB}}{\mathrm{OS}} = \frac{\mathrm{CB}}{\mathrm{CR}}$

On subtracting from both sides, we get,

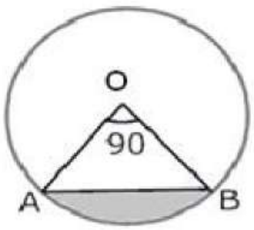
$\frac{\mathrm{OB}}{\mathrm{OS}} - 1 = \frac{\mathrm{CB}}{\mathrm{CR}} - 1$

$\Rightarrow \frac{\mathrm{OB-OS}}{\mathrm{OS}} = \frac{(\mathrm{CB-CR})}{\mathrm{CR}}$

$\Rightarrow \frac{\mathrm{BS}}{\mathrm{OS}} = \frac{\mathrm{BR}}{\mathrm{CR}}$

By converse of basic proportionality theorem, SR || OC

Hence proved.

34. 

i. Area of minor sector = $\frac{\theta}{360}\pi r^2$

$= \frac{90}{360}(3.14)(10)^2$

$= \frac{1}{4} \times 3.14 \times 100$

$= \frac{314}{4}$

= 78.50 = 78.5 cm^2

ii. Area of major sector = Area of circle - Area of minor sector

= $\pi(10)^2$ - $\frac{90}{360}$ $\pi(10)^2$ = 3.14 (100) - $\frac{1}{4}$(3.14) (100)

= 314 - 78.50 = 235.5 cm^2

iii. We know that area of minor segment

= Area of minor sector OAB - Area of ΔOAB

$\because$ area of $\triangle \mathrm{OAB} = \frac{1}{2}(OA)(OB)\sin\angle AOB$

$= \frac{1}{2}(OA)(OB) \, (\because \angle AOB = 90^\circ)$

Area of sector = $\frac{\theta}{360}\pi r^2$

= $\frac{1}{4}$(3.14) (100) - 50 = 25(3.14) - 50 = 78.50 - 50 = 28.5 cm^2

iv. Area of major segment = Area of the circle - Area of minor segment

= $\pi(10)^2$ - 28.5

= 100(3.14) - 28.5

= 314 - 28. 5 = 285.5 cm^2

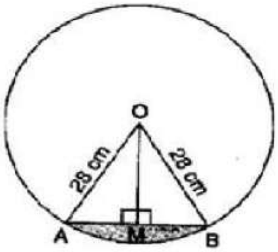
OR

r = 28 cm and θ = $\frac{360}{6}$ $= 60^\circ$

Area of minor sector = $\frac{\theta}{360}\pi r^2$ $= \frac{60}{360} \times \frac{22}{7} \times 28 \times 28 = \frac{1232}{3}$

= 410.67 cm^2

For, Area of $\triangle$AOB,



Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore$ $\triangle$OMA $\cong$ OMB [RHS congruency]

$\therefore$ AM = BM [By CPCT]

$\Rightarrow$ AM = BM = $\frac{1}{2}$AB and $\angle$AOM = $\angle$BOM = $\frac{1}{2}$AOB = $\frac{1}{2}$ $\times$ 60^o = 30^o

In right angled triangle OMA, cos30^o = $\frac{OM}{OA}$

$\Rightarrow$ $\frac{\sqrt{3}}{2} = \frac{OM}{28}$

$\Rightarrow$ OM = 14$\sqrt{3}$ cm

Also, sin30° = $\frac{AM}{OA}$

$\Rightarrow$ $\frac{1}{2}$ = $\frac{AM}{28}$

$\Rightarrow$ AM = 14 cm

$\Rightarrow$ 2 AM = 2 $\times$ 14 = 28 cm

$\Rightarrow$ AB = 28 cm

$\therefore$ Area of $\triangle$AOB = $\frac{1}{2}$$\times$ AB $\times$ OM = $\frac{1}{2}$ $\times$ 28 $\times$ 14$\sqrt{3}$ = 196 $\sqrt{3}$ = 196 $\times$ 1.7 = 333.2 cm^2

$\therefore$ Area of minor segment = Area of minor sector - Area of $\triangle$AOB

= 410.67 - 333.2 = 77.47 cm^2

$\therefore$ Area of one design = 77.47 cm^2

$\therefore$ Area of six designs = 77.47 $\times$ 6 = 464.82 cm^2

Cost of making designs = 464.82 $\times$ 0.35 = Rs. 162.68

35. class 10000 - 15000 has the maximum frequency,

so it is the modal class.

$\therefore$ l = 10000, h = 5000, f = 41, f_1 = 26 and f_2 = 16

Mode = l + $\frac{f-f_1}{2f-f_1-f_2}$$\times$ h

= 10000 + $\frac{41-26}{2(41)-26-16}$$\times$ 5000

= 10000 + $\frac{15}{40}$$\times$ 5000

= 10000 + 1875

= 11875

Section E

36. **Read the text carefully and answer the questions:**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.

(i) Volume of Hermika = side^3 = 10 × 10 × 10 = 1000 m^3

(ii) r = radius of cylinder = 24, h = height = 16

Volume of cylinder = $\pi r^2 h$

$\Rightarrow$ V = $\frac{22}{7} \times 24 \times 24 \times 14$ = 25344 m^3

(iii) Volume of brick = 0.01 m^3

$\Rightarrow$ n = Number of bricks used for making cylindrical base = $\frac{\text{Volume of cylinder}}{\text{Volume of one brick}}$

$\Rightarrow n = \frac{25344}{0.01}$ = 2534400

OR

Since Anda is hespherical in shape r = radius = 21

V = Volume of Anda = $\frac{2}{3} \times \pi \times r^3$

$\Rightarrow$ V = $\frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$\Rightarrow$ V = 44 × 21 × 21 = 19404 m^3

37. **Read the text carefully and answer the questions:**

Jaspal Singh is an auto driver. His autorickshaw was too old and he had to spend a lot of money on repair and maintenance every now and then. One day he got to know about the EV scheme of the Government of India where he can not only get a good exchange bonus but also avail heavy discounts on the purchase of an electric vehicle. So, he took a loan of ₹1,18,000 from a reputed bank and purchased a new autorickshaw.

Jaspal Singh repays his total loan of 118000 rupees by paying every month starting with the first instalment of 1000 rupees.

(i) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100

Now, amount paid in the 30th installment,

a_{30} = 1000 + (30 - 1)100 = 3900 {$a_n = a + (n - 1)d$}

(ii) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month=1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100

Amount paid in 30 instalments,

$S_{30} = \frac{30}{2}[2 \times 1000 + (30 - 1)100]$ = 73500

Hence, remaining amount of loan that he has to pay = 118000 - 73500 = 44500 Rupees

(iii) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100

Amount paid in 100 instalments

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_n = \frac{100}{2}[2 \times 1000 + (100 - 1)100]$

$\Rightarrow S_n$ = 100000 + 9900

$\Rightarrow$ 109900

OR

Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month therefore, installment amount in second month = ₹ 1100, third month = ₹1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100

If he increases the instalment by 200 rupees every month, amount would he pay in 40th instalment

Then a = 1000, d = 200 and n = 40

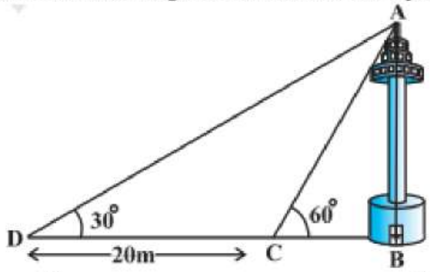
$a_{40} = a + (n - 1)d$

$\Rightarrow a_{40} = 1000 + (40 - 1)200$

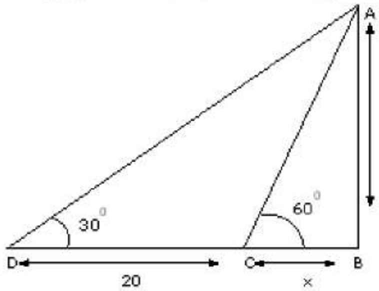
$\Rightarrow a_{40} = 880$

38. **Read the text carefully and answer the questions:**

A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° from a point 20 m away from this point on the same bank the angle of elevation of the top of the tower is 30°.



(i)



Let 'h' (AB) be the height of tower and x be the width of the river.

In $\triangle ABC, \frac{h}{x} = \tan 60^{\circ}$

$\Rightarrow h = \sqrt{3}x$ ...(i)

In $\Delta ABD, \frac{h}{x+20} = \tan 30^{\circ}$

$\Rightarrow h = \frac{x+20}{\sqrt{3}}$ ...(ii)

Equating (i) and (ii),

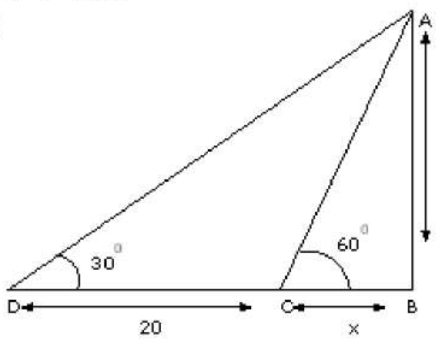
$\sqrt{3}x = \frac{x+20}{\sqrt{3}}$

$\Rightarrow 3x = x + 20$

$\Rightarrow 2x = 20$

$\Rightarrow x = 10$ m

(ii)



Let 'h' (AB) be the height of tower and x be the width of the river.

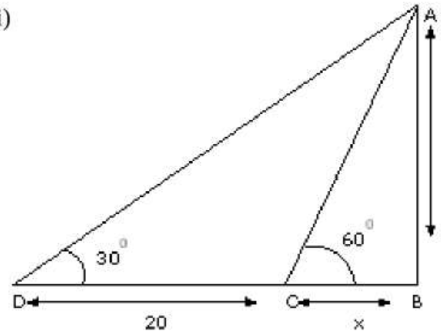
In $\triangle ABC, \frac{h}{x} = \tan 60^{\circ}$

$\Rightarrow h = \sqrt{3}x$ ...(i)

Put x = 10 in (i), $h = \sqrt{3}x$

$\Rightarrow h = 10\sqrt{3}\text{m}$

(iii)



In $\triangle$ABD

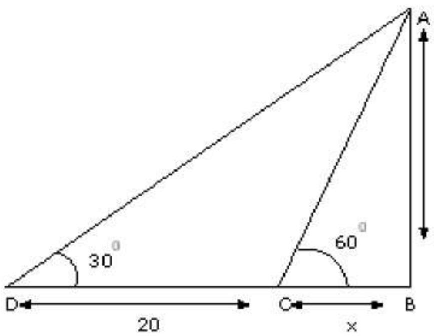
sin $30^o = \frac{AB}{AD}$

$\Rightarrow$ AD = $\frac{AB}{\sin 30^\circ}$

$\Rightarrow$ AD = $\frac{10\sqrt{3}}{\frac{1}{2}}$

$\Rightarrow$ AD = $20\sqrt{3}$ m

OR



In $\triangle$ABC

sin $60^o = \frac{AB}{AC}$

$\Rightarrow$ AC = $\frac{AB}{\sin 30^0}$

$\Rightarrow$ AC = $\frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AC = 20 m