Class- X Session- 2022-23

Subject- Mathematics (Basic)

Sample Question Paper - 4

with Solution

Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. A polynomial of degree ________ is called a quadratic polynomial. **[1]**

a) 1 b) 3

c) 2 d) 0

2. In △ABC and △DEF, it is given that $\frac{AB}{DE} = \frac{BC}{FD}$ then **[1]**

a) ∠A = ∠D b) ∠B = ∠D

c) ∠B = ∠E d) ∠A = ∠F

3. If x = a, y = b is the solution of the equations x – y = 2 and x + y = 4, then the values of a and b are, respectively **[1]**

a) –1 and –3 b) 5 and 3

c) 3 and 5 d) 3 and 1

4. The sum of the digits of a two-digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. The number is **[1]**

a) 69 b) 87

c) 78 d) 96

5. A bag contains three green marbles, four blue marbles, and two orange marbles. If a marble is picked at random, then the probability that it is not an orange marble is **[1]**

a) $\frac{7}{9}$ b) $\frac{4}{9}$

c) $\frac{5}{4}$

d) $\frac{1}{4}$

6. In a △ ABC, perpendicular AD from A on BC meets BC at D. If BD = 8 cm, DC = 2 cm and AD = 4 cm, then: **[1]**

a) △ABC is right - angled at A.

b) AC = 2 AB

c) △ABC is isosceles

d) △ABC is equilateral

7. In the given data if n = 44, l = 400, cf = 8, h = 100, f = 20, then its median is **[1]**

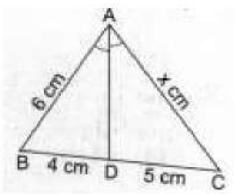
a) 400

b) 480

c) 470

d) 460

8. In a △ ABC it is given that AD is the internal bisector of ∠A. If BD = 4 cm, DC = 5 cm and AB = 6 cm, then AC = ? **[1]**



a) 9 cm

b) 7.5 cm

c) 4.5 cm

d) 8 cm

9. The HCF of 95 and 152, is **[1]**

a) 57

b) 19

c) 38

d) 1

10. The sum of the roots of the equation $x^2 - 6x + 2 = 0$ is **[1]**

a) 6

b) -2

c) -6

d) 2

11. The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by **[1]**

a) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

b) $\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right)$

c) $\left(\frac{x_1-y_1}{2}, \frac{x_2-y_2}{2}\right)$

d) $\left(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}\right)$

12. The mean of 2, 7, 6 and x is 5 and the mean of 18, 1, 6, x and y is 10. What is the value of y? **[1]**

a) 30

b) 10

c) 5

d) 20

13. If $2\sin 2\theta = \sqrt{3}$ then θ= ? **[1]**

a) 45°
b) 90°
c) 60°
d) 30°

14. The ________ is the angle between the horizontal and the line of sight to an object when the object is below the horizontal level. **[1]**

a) angle of projection
b) angle of elevation
c) None of these
d) angle of depression

15. sin 2A = 2 sin A is true when A = **[1]**

a) 60°
b) 30°
c) 0°
d) 45°

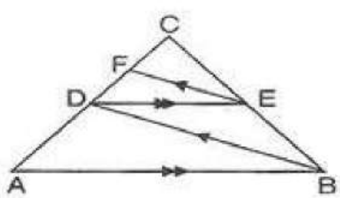
16. In a right triangle ABC, right angled at B, BC = 12 cm and AB = 5 cm. The radius of the circle inscribed in the triangle (in cm) is **[1]**

a) 4
b) 1
c) 2
d) 3

17. $5x^2 + 8x + 4 = 2x^2 + 4x + 6$ is a **[1]**

a) quadratic equation
b) cubic equation
c) constant
d) linear equation

18. We have, $AB||DE$ and $BD||EF$. Then, **[1]**



a) $BC^2 = AB\ .\ CE$
b) $AC^2 = BC\ .\ DC$
c) $AB^2 = AC\ .\ DE$
d) $DC^2 = CF \times AC$

19. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 300 cm^2. **[1]**
Reason (R): Total surface area of a cuboid is 2(lb + bh + lh)

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

[1]

20. **Assertion (A):** $x^2 + 7x + 12$ has no real zeros
Reason (R): A quadratic polynomial can have at the most two zeroes.

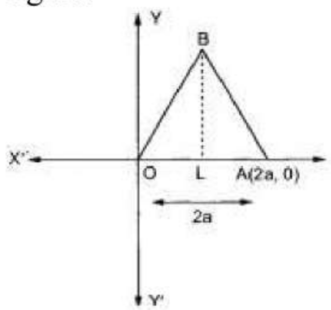
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Find the distance between the points (0, 0) and (36, 15). **[2]**

OR

Find the coordinates of the vertices of an equilateral triangle of side 2a as shown in the figure.



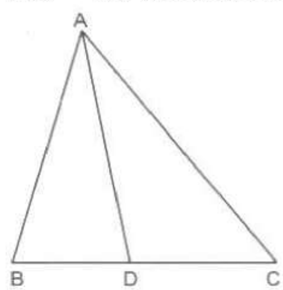
22. Find whether the equation has real roots. If real roots exist, find them: $5x^2 - 2x - 10 = 0$ **[2]**

23. Evaluate $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$ **[2]**

24. Determine the point which divides a given line segment internally in the ratio 3 : 4. **[2]**

OR

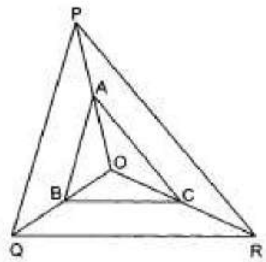
In Fig. check whether AD is the bisector of $\angle A$ of ΔABC if AB = 5 cm, AC = 10 cm, BD = 1.5 cm and CD = 3.5 cm



25. Write down the LCM of the following polynomials: (x–1) (x–2) and (x–2) (x–7) **[2]**

Section C

26. In Fig. A, B and C are points on OP, OQ and OR respectively such that $AB \| PQ$ and $BC \| QR$. Show that $AC \| PR$. **[3]**



27. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides. **[3]**

28. Find k so that the point P (-4, 6) lies on the line segment joining A (k, 10) and B (3, - 8). Also, find the ratio in which P divides AB. **[3]**

OR

Show that the points A(3, -1), B(5, -1) and C(3, -3) are the vertices of a right angled isosceles triangle.

29. Show that there are infinitely many positive primes. **[3]**

30. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent **[3]**

Number of Days	0-6	6-12	12-18	18-24	24-30	30-36	36-42
Number of students	10	11	7	4	4	3	1

31. A boy, flying a kite with a string of 90 m long, which is making an angle θ with the ground. Find the height of the kite. (Given tan $\theta = \frac{15}{8}$) **[3]**

OR

A rhombus of side 20 cm has two angles of 60° each. Find the length of the diagonals.

Section D

32. In a right triangle ABC in which $\angle B = 90^\circ$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at P bisect BC. **[5]**

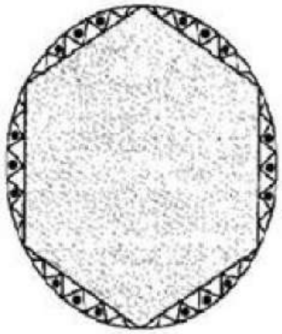
33. A number consists of two digits whose sum is five. When the digits are reversed, the number becomes greater by nine. Find the number. **[5]**

OR

Solve the following system of equation by elimination method 5x + 3y = 70; 3x - 7y = 60.

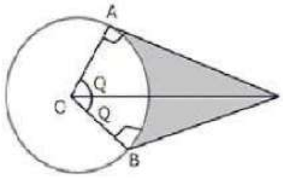
34. A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm ^{2}. **[5]**

(use $\sqrt{3} = 1.7$)



OR

An elastic belt is placed around therein of a pulley of radius 5cm. One point on the belt is pulled directly away from the center O of the pulley until it is at P, 10cm from O. Find the length of the best that is in contact with the rim of the pulley. Also, find the shaded area.



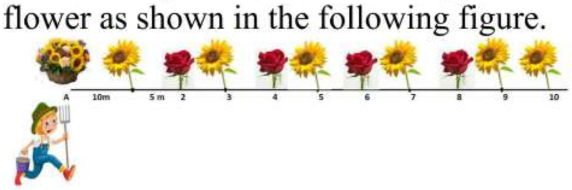
35. All the three face cards of spades are removed from a well-shuffled pack of 52 cards. A card is drawn at random from the remaining pack. Find the probability of getting **[5]**

i. a black face card
ii. a queen
iii. a black card
iv. a spade

Section E

36. **Read the text carefully and answer the questions:** **[4]**

In a school garden, Dinesh was given two types of plants viz. sunflower and rose flower as shown in the following figure.



The distance between two plants is to be 5m, a basket filled with plants is kept at point A which is 10 m from the first plant. Dinesh has to take one plant from the basket and then he will have to plant it in a row as shown in the figure and then he has to return to the basket to collect another plant. He continues in the same way until all the flower plants in the basket. Dinesh has to plant ten numbers of flower plants.

(i) Write the above information in the progression and find first term and common difference.

(ii) Find the distance covered by Dinesh to plant the first 5 plants and return to basket.

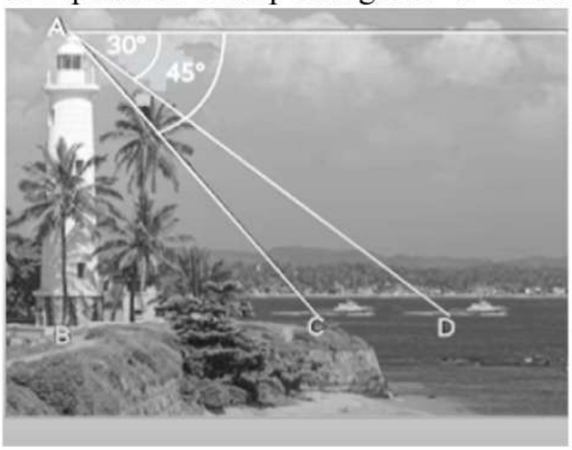
(iii) Find the distance covered by Dinesh to plant all 10 plants and return to basket.

OR

If the speed of Dinesh is 10 m/min and he takes 15 minutes to plant a flower plant then find the total time taken by Dinesh to plant 10 plants.

37. **Read the text carefully and answer the questions:** **[4]**

An observer on the top of a 40m tall light house (including height of the observer) observes a ship at an angle of depression 30° coming towards the base of the light house along straight line joining the ship and the base of the light house. The angle of depression of ship changes to 45° after 6 seconds.



(i) Find the distance of ship from the base of the light house after 6 seconds from the initial position when angle of depression is 45°.

(ii) Find the distance between two positions of ship after 6 seconds?

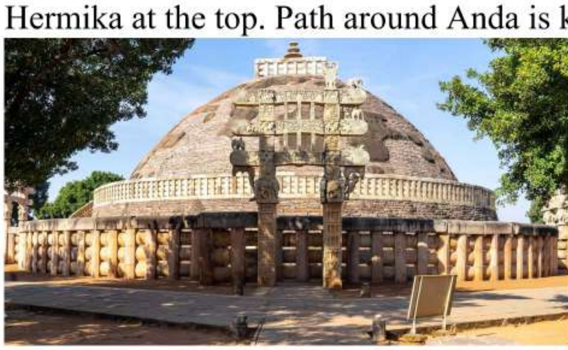
(iii) Find the speed of the ship?

OR

Find the distance of ship from the base of the light house when angle of depression is 30°.

38. **Read the text carefully and answer the questions:** **[4]**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.



(i) Find the volume of the Hermika, if the side of cubical part is 10 m.

(ii) Find the volume of cylindrical base part whose diameter and height 48 m and 14 m.

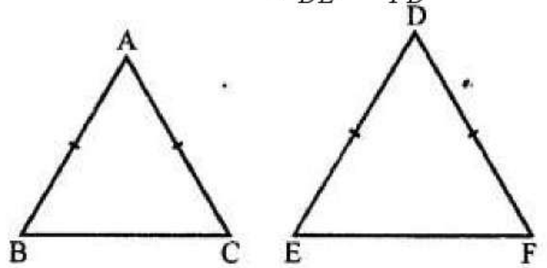
(iii) If the volume of each brick used is 0.01 m^3, then find the number of bricks used to make the cylindrical base.

OR

If the diameter of the Anda is 42 m, then find the volume of the Anda.

Solution

Section A

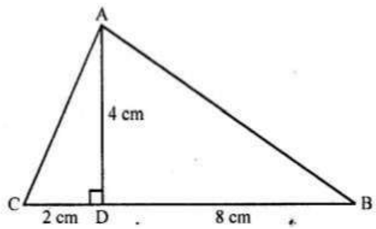
1. **(c)** 2
   Explanation: A polynomial of degree two is called a quadratic polynomial. An equation involving a quadratic polynomial is called a quadratic equation. A quadratic equation is an equation that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$.
2. **(b)** $\angle B = \angle D$
   Explanation:
   In $\triangle ABC$ and $\triangle DEF$, $\frac{AB}{DE} = \frac{BC}{FD}$
   

   For similarity,
   Here, included angles must be equal and these
   are $\angle B = \angle D$
3. **(d)** 3 and 1
   Explanation: Given equations are:
   x - y = 2 and
   x + y = 4;
   Adding them, we get
   2x = 6
   x = 3;
   Subtracting them, we get
   2y = 2
   y = 1;
   So, a = 3 and b = 1 is the solution of the equations.
4. **(c)** 78
   Explanation: Let us assume the tens and the unit digits of the required number be x and y respectively
   $\therefore$ Required number = (10x + y)
   According to the given condition in the question,
   we have
   x + y = 15 .....(i)
   By reversing the digits, we obtain the number = (10y + x)
   $\therefore$ (10y + x) = (10x + y) + 9
   10y + x - 10x - y = 9
   9y - 9x = 9
   y - x = 1 .....(ii)
   Now, on adding (i) and (ii) we get:
   2y = 16
   $\therefore y = \frac{16}{8} = 8$
   Putting the value of y in (i), we get:

x + 8 = 15
x = 15 - 8
x = 7
∴ Required number = (10x + y) = 10 × 7 + 8 = 70 + 8 = 78

5. **(a)** $\frac{7}{9}$
Explanation: In a bag, there are 3 green, 4 blue and 2 orange marbles
∴ Total marbles (n) = 3 + 4 + 2 = 9
No. of marbles which is not orange = 3 + 4 = 7
therefore m = 7
∴ Probability = $\frac{m}{n} = \frac{7}{9}$

6. **(a)** $\triangle$ABC is right - angled at A.
Explanation: In $\triangle$ABC , AD⊥ BC
BD = 8 CM, DC = 2 CM, AD = 4 CM

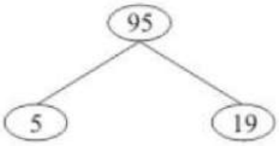


In right $\triangle$ACD,
$AC^2 = AD^2 + CD^2$ (Pythagoras Theorem)
$=(4)^2+(2)^2=16 + 4 = 20$
and in right $\triangle$ABD,
$AB^2 = AD^2 + DB^2$
$= (4)^2 + (8)^2 = 16 + 64 = 80$
and $BC^2 = (BD + DC)^2 = (8 + 2)^2 = (10)^2 = 100$
$AB^2 + AC^2 = 80 + 20 = 100 = BC^2$
$\triangle$ABC is a right triangle whose $\angle A = 90^\circ$

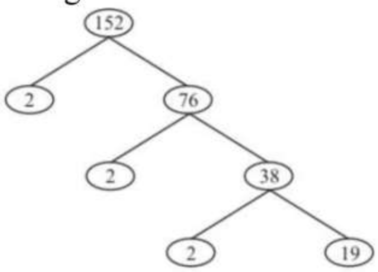
7. **(c)** 470
Explanation: Median = $l + \frac{\frac{n}{2} - c}{f} \times h$
$= 400 + \frac{\frac{44}{2} - 8}{20} \times 100$
$= 400 + \frac{14}{20} \times 100$
$= 400 + 14 \times 5$
$= 400 + 70$
$= 470$

8. **(b)** 7.5 cm
Explanation: It is given that AD bisects angle A.
Therefore, applying angle bisector theorem, we get:
$\frac{BD}{DC} = \frac{AB}{AC}$
$\Rightarrow \frac{4}{5} = \frac{6}{x}$
$\Rightarrow x = \frac{5 \times 6}{4} = 7.5$
Hence, AC = 7.5 cm

9. **(b)** 19
Explanation: Using the factor tree for 95, we have:



Using the factor tree for 152, we have:



Therefore,

$95 = 5 \times 19$

$152 = 2^3 \times 19$

HCF (95, 152) = 19

10. **(a)** 6

Explanation: Sum of the roots of the equation of $ax^2 + bx + c = 0$ is $\frac{-b}{a}$

Here, a = 1, b = -6, c = 2

By substitutlon of values we get

$\frac{-b}{a} = \frac{-(-6)}{1} = 6.$

11. **(a)** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Explanation: we know that the midpoint formula = $\frac{x_1+x_2}{2}$, $\frac{y_1+y_2}{2}$

The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

12. **(d)** 20

Explanation: Mean of 2, 7, 6 and x = 5

$\Rightarrow \frac{2+7+6+x}{4} = 5$

$\Rightarrow$ 15 + x = 20

$\Rightarrow$ x = 5

Also, Mean of 18, 1, 6, x and y = 10

$\Rightarrow \frac{18+1+6+x+y}{5} = 10$

$\Rightarrow \frac{18+1+6+5+y}{5} = 10$

$\Rightarrow$ 30 + y = 50

$\Rightarrow$ y = 20

13. **(d)** 30°

Explanation: We have, $2\sin 2\theta = \sqrt{3} \Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^\circ$

$\Rightarrow 2\theta = 60^\circ$

$\Rightarrow \theta = 30^\circ$

14. **(d)** angle of depression

Explanation: The angle of depression is the angle between the horizontal and line of sight to an object when the object is below the horizontal level.

The angle of depression is formed when the observer is higher than the object he is looking at. It is the angle between the horizontal line and the line joining the observer 's eye and the object. It plays a very important role in determining the heights and distances.

15. **(c)** 0°

Explanation: sin 2A = 2 sin A is true when A =0°

$\because \sin 2A = 2 \sin A$

$\Rightarrow \sin(2 \times 0^\circ) = \sin 0^\circ$

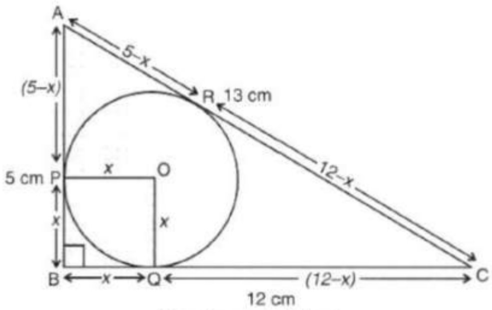
$\Rightarrow \sin 0^\circ = \sin 0^\circ$

16. **(c)** 2

Explanation:

Here, AB = 5cm, BC = 12 and $\angle B = 90^\circ$

Let the radius of circle be x cm



$\therefore AC = \sqrt{(12)^2 + (5)^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169} = 13\text{cm}$

$\because$ AC = AR + RC

$\therefore$ AC = (5 - x) + 12 - x

$\Rightarrow$ 13 = 5 - x + 12 - x

$\Rightarrow$ 2x = 17 - 13 = 4

$\Rightarrow \quad x = \frac{4}{2} = 2\text{cm}$

Hence, radius of the circle = 2cm.

17. **(a)** quadratic equation

Explanation: Given: $5x^2 + 8x + 4 = 2x^2 + 4x + 6$

$\Rightarrow 5x^2 - 2x^2 + 8x - 4x + 4 - 6$

$\Rightarrow 3x^2 + 4x - 2 = 0$

Here, the degree is 2, therefore it is a quadratic equation.

18. **(d)** $DC^2 = CF \times AC$

Explanation: In $\triangle$ABC, using Thales theorem,

$\frac{DC}{AC} = \frac{CE}{BC}$ $[AB||DE]$ ……..(i)

And in triangle BCD, using Thales theorem,

$\frac{CF}{DC} = \frac{CE}{BC}$ $[BD||EF]$ ……..(ii)

From eq. (i) and (ii), we have

$\frac{DC}{AC} = \frac{CF}{DC}$

$\Rightarrow DC^2 = CF \times AC$

19. **(d)** A is false but R is true.

Explanation: A is false but R is true.

20. **(d)** A is false but R is true.

Explanation: $x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 4x + 3x + 12 = 0$

$\Rightarrow x(x + 4) + 3(x + 4) = 0$

$\Rightarrow (x + 4)(x + 3) = 0$

$\Rightarrow (x + 4) = 0$ or $(x + 3) = 0$

$\Rightarrow x = -4$ or $x = -3$

Therefore, $x^2 + 7x + 12$ has two real zeroes.

Section B

21. Required distance

$= \sqrt{(36-0)^2 + (15-0)^2}$

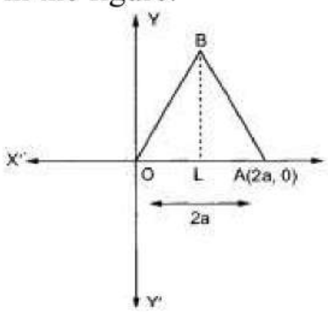
$= \sqrt{(36)^2 + (15)^2}$

$= \sqrt{1296 + 225} = \sqrt{1521}$

$= 39$

OR

We have to find the coordinates of the vertices of an equilateral triangle of side 2a as shown in the figure.



Since, OAB is an equilateral triangle of side 2a. Therefore,

OA = AB = OB = 2a

Let BL perpendicular from B on OA.Then

OL = LA = a

In $\triangle$OLB, we have

$OB^2 = OL^2 + LB^2$

$\Rightarrow (2a)^2 = a^2 + LB^2$

$\Rightarrow LB^2 = 3a^2$

$\Rightarrow \quad LB = \sqrt{3}a$

Clearly, coordinates of O are (0,0) and that of A are (2a, 0). Since, OL = a and LB = $\sqrt{3}$a.

So, the coordinates of B are (a, $\sqrt{3}$a)

22. To check whether the quadratic equation has real roots or not, we need to check the discriminant value i.e.,

$D = b^2 - 4ac$

Given, $5x^2 - 2x - 10 = 0$

$\therefore D = (-2)^2 - 4(5)(-10)$

$\Rightarrow D = 4 + 200 > 0$

Hence, the roots are real and distinct.

To find the roots, use the formula,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

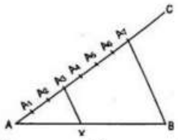
$\therefore x = \frac{2 \pm \sqrt{204}}{2(5)}$

$= \frac{2 \pm 2\sqrt{51}}{10} = \frac{1+\sqrt{51}}{5}, \frac{1-\sqrt{51}}{5}$

23. We have,

$\cos 60^\circ = \frac{1}{2}$, $\cos\ 30^\circ = \frac{\sqrt{3}}{2}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$ and $\sin 30^\circ = \frac{1}{2}$

therefore,
$\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$
$= \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = 2\left(\frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{2}$

24. 

Steps of construction:

i. Draw AB of given length.
ii. Draw any line AC that $\angle$ CAB is acute.
iii. Using compass and any radius, mark points $A_1, A_2, A_3, ...A_7 (3 + 4 = 7)$ such that $AA_1 = A_1A_2$
$A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$
iv. Join A_7B.
v. From A_3(Since the ratio is 3:4) draw a line parallel to A_73 which meets AB in X. X is the required point such that AX : XB = 3 : 4.

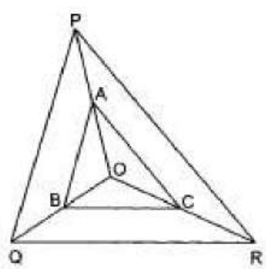
OR

It is given that, AB = 5 cm, AC = 10 cm, BD = 1.5 cm and CD = 3.5 cm
We have to check whether AD is bisector of $\angle A$
First we will check proportional ratio between sides
So,
$\frac{AB}{AC} = \frac{5}{10} = \frac{1}{2}$
$\frac{BD}{CD} = \frac{1.5}{3.5} = \frac{3}{7}$
Since $\frac{AB}{AC} \neq \frac{BD}{CD}$
Hence, AD is not the bisector of $\angle A$

25. Let p(x) = $(x-1)(x-2)$
and q(x) = $(x-2)(x-7)$
$\therefore$ LCM = $(x-1)(x-2)(x-7)$

Section C

26. In ΔOPQ, we have



$AB \| PQ$, By using BPT we can write,
$\Rightarrow \quad \frac{OA}{AP} = \frac{OB}{BQ}$ ...(1)
In ΔOQR, we have
$BC \| QR$, By using BPT we can write,
$\Rightarrow \quad \frac{OB}{BQ} = \frac{OC}{CR}$ ...(2)
From (i) and (ii), we get
$\frac{OA}{AP} = \frac{OC}{CR}$
Thus, A and C are points on sides OP and OR respectively of ΔOPR, such that

$\frac{OA}{AP} = \frac{OC}{CR}$

$\Rightarrow \quad AC \| PR$ [Using the converse of BPT]

Hence proved.

27. Let the base of the right triangle be x cm.

Then altitude = (x - 7) cm

Hypotenuse = 13 cm

By Pythagoras theorem

$(\text{Base})^2 + (\text{Altitude})^2 = (\text{Hypotenuse})^2$

$\implies x^2 + (x-7)^2 = (13)^2$

$\implies x^2 + x^2 - 14x + 49 = 169$

$\implies 2x^2 - 14x - 120 = 0$

$\implies 2(x^2 - 7x - 60) = 0$ or $x^2 - 7x - 60 = 0$

$\implies x^2 - 12x + 5x - 60 = 0$

$\implies x(x - 12) + 5(x - 12) = 0$

$\implies (x + 5)(x - 12) = 0$

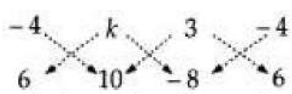
Either x + 5 = 0 or x - 12 = 0

$\implies x = -5, 12$

Since side of the triangle cannot be negative. So, x = 12 cm and x = - 5 is rejected.

Hence, length of the other two sides are 12cm, (12 - 7) = 5cm.

28. If P (-4, 6) lies on the line segment joining A (k, 10) and B (3, -8), then P, A and B are collinear.



$\therefore (-4 \times 10 + k \times -8 + 3 \times 6) - (6k + 30 + -4 \times -8) = 0$

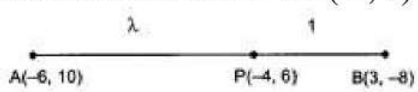
$\Rightarrow (-40 - 8k + 18) - (6k + 30 + 32) = 0$

$\Rightarrow (-22 - 8k) - (6k + 62) = 0$

$\Rightarrow -14k - 84 = 0$

$\Rightarrow k = -6$

Suppose P divides AB in the ratio λ: 1. Then, the coordinates of Pare $\left(\frac{3\lambda-6}{\lambda+1}, \frac{-8\lambda+10}{\lambda+1}\right)$.But, the coordinates of P are (-4, 6).



$\therefore \quad \frac{3\lambda-6}{\lambda+1} = -4$ and $\frac{-8\lambda+10}{\lambda+1} = 6$

$\Rightarrow \quad \lambda = \frac{2}{7}$

Hence, P divides AB in the ratio $\frac{2}{7}$: 1 or 2: 7.

OR

Given: A(3, -1), B(5, -1) and C(3, -3)

$AB = \sqrt{(5-3)^2 + (-1+1)^2} = \sqrt{2^2 + 0^2} = 2$

$BC = \sqrt{(5-3)^2 + (-1+3)^2} = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

$AC = \sqrt{(3-3)^2 + (-1+3)^2} = \sqrt{2^2} = 2$

Clearly, AB = AC

Therefore, $\triangle$ABC is an isosceles triangle.

Now, $AB^2 = 4$, $BC^2 = 8$ and $AC^2 = 4$

Therefore, $BC^2 = AB^2 + AC^2$

Therefore, △ABC is right angled also.

Hence, △ABC is right isosceles triangle.

29. For any finite set of primes $\{p_1, p_2, p_3, \ldots\ldots, p_n\}$, Euclid considered the number

$n = 1 + p_1 \times p_2 \times p_3 \times \ldots\ldots \times p_n$

n has a prime divisor p (every integer has at least one prime divisor). But p is not equal to any of the p_i .(If p were equal to any of the p_i , then p would have to divide 1, which is impossible).

So for any finite set of prime numbers, it is possible to find another prime that is not in that set.

In other words, a finite set of primes cannot be the collection of all prime numbers.

Hence, there are infinitely many positive primes.

30. Calculation of mean:

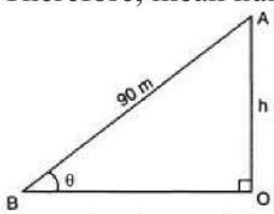
Number of Days	**Number of students (f_i)**	x_i	f_ix_i
0-6	10	3	30
6-12	11	9	99
12-18	7	15	105
18-24	4	21	84
24-30	4	27	108
30-36	3	33	99
36-42	1	39	39
	$\Sigma f_i = 40$		$\Sigma f_ix_i =$ **564**

We know that, Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{564}{40}$

= 14.1

Therefore, mean number of days a student was absent is 14.1

31. 

Let A be the position of kite and AB be the string.

Since it is given that

$\tan\theta = \frac{15}{8} = \frac{AO}{BO}$

AO = 15 km

BO = 8 km

Since we know that in an right angled triangle sum of square of hypotenuse is equal to sum of squares of perpendicular and base.

$AB = \sqrt{(15k)^2 + (8k)^2}$

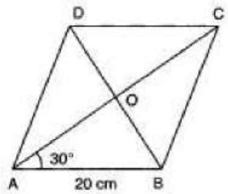
$AB = 17k$

$\therefore \quad \sin\theta = \frac{15}{17}$

In △ ABO, $\frac{AO}{AB} = \sin\theta$

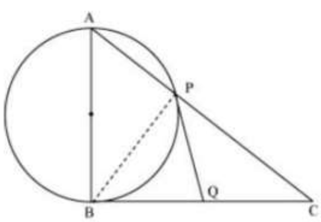
$\Rightarrow \quad \frac{h}{90} = \frac{15}{17}$
$\Rightarrow \quad h = \frac{15 \times 90}{17}$
= 79.41 m
Hence, height of kite is 79.41 m.

OR



According to question, we first draw a rhombus ABCD of side 20 cm and $\angle BAD = \angle BCD = 60^\circ$ .Then we will find length of diagonals.
Note that the diagonals of a rhombus are perpendicular bisector of each other and diagonals AC and BD are bisectors of $\angle BAD$ and $\angle ABC$ respectively.
So, ΔAOB is a right triangle such that,
$\angle BAO = 30^\circ, \angle AOB = 90^\circ$
and AB = 20 cm.
$\therefore \cos \angle BAO = \frac{OA}{AB}$
$\Rightarrow \cos 30^\circ = \frac{OA}{20}$
$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OA}{20}$
$\Rightarrow OA = \frac{\sqrt{3}}{2} \times 20 = 10\sqrt{3}$
Also we have, $\sin \angle BAO = \frac{OB}{AB}$
$\Rightarrow \sin 30^\circ = \frac{BO}{20}$
$\Rightarrow \frac{1}{2} = \frac{BO}{20}$
$\Rightarrow BO = \frac{20}{2} = 10$
$\therefore AC = 2 \times 10\sqrt{3} = 20\sqrt{3}\text{cm}, BD = 2\text{BO} = 2 \times 10 = 20\text{cm}$

Section D

32. 

ΔABC is a right angled triangle.
∠ABC = 90°.
A circle is drawn with AB as diameter intersecting AC in P, PQ is the tangent to the circle which intersects BC at Q.
Join BP.
PQ and BQ are tangents drawn from an external point Q.
∴ PQ = BQ -------------- (1) (Length of tangents drawn from an external point to the circle are equal)
⇒ ∠PBQ = ∠BPQ (In a triangle, angles opposite to equal sides are equal)
Given that, AB is the diameter of the circle.
∴ ∠APB = 90° (Angle in a semi-circle is a right angle)
∠APB + ∠BPC = 180° (Linear pair)
∴ ∠BPC = 180° – ∠APB = 180° – 90° = 90°
Consider ΔBPC,

$\angle BPC + \angle PBC + \angle PCB = 180°$ (Angle sum property of a triangle)
$\therefore \angle PBC + \angle PCB = 180° - \angle BPC = 180° - 90° = 90°$................(2)
$\angle BPC = 90°$
$\therefore \angle BPQ + \angle CPQ = 90°$ ...(3)
From equations (2) and (3), we get
$\angle PBC + \angle PCB = \angle BPQ + \angle CPQ$
$\Rightarrow \angle PCQ = \angle CPQ$ (Since, $\angle BPQ = \angle PBQ$)
Consider ΔPQC,
$\angle PCQ = \angle CPQ$
$\therefore PQ = QC$.................(4)
From equations (1) and (4), we get
BQ = QC
Therefore, tangent at P bisects the side BC.

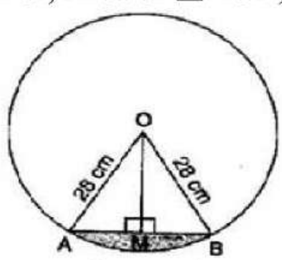
33. Let the digits at the units and at the tens place of the given number be x and y respectively.
Thus, the number is 10y + x.
Given, the sum of the digits of the number is 5.
Hence, x + y = 5...........(1)
After interchanging the digits, the number becomes 10x + y.
Also given, the number obtained by interchanging the digits is greater by 9 from the original number.
Hence, 10x + y = (10y + x) +9
$\Rightarrow 10x + y - 10y - x= 9$
$\Rightarrow 9x - 9y =9$
$\Rightarrow 9(x - y) = 9$
$\Rightarrow x - y = 1$................(2)
Adding equation (1) & equation (2), we get ;
$(x + y) + (x - y) = 5 + 1$
$\Rightarrow x + y + x - y = 5 +1$
$\Rightarrow 2x = 6$
$\Rightarrow x = \frac{6}{2}$
$\Rightarrow x = 3$
Substituting the value of x in the equation (1), we get
$3 + y = 5$
$\Rightarrow y = 5 - 3$
$\Rightarrow y = 2$
Hence, the number is $10y + x = 10 \times 2 + 3 = 23$

OR

The given system of equation is
$5x + 3y = 70$ ...(1)
$3x - 7y = 60$ ...(2)
Multiplying equation (1) by 3 and equation (2) by 5, we get
$15x + 9y = 210$ ...(3)
$15x - 35y = 300$ ...(4)
Subtracting equation (4) from equation (3), we get
$44y = -90$
$\Rightarrow y = \frac{-90}{44} = \frac{-45}{22}$
Substituting this value of y in equation (1), we get
$5x + 3\left(\frac{-45}{22}\right) = 70$

$\Rightarrow 5x - \frac{135}{22} = 70$
$\Rightarrow 5x = 70 + \frac{135}{22}$
$\Rightarrow 5x = \frac{1540+135}{22} = \frac{1675}{22}$
$\Rightarrow x = \frac{335}{22},\ y = \frac{-45}{22}.$
Verification: Substituting $x = \frac{335}{22},\ y = \frac{-45}{22}.$
We find that both the equation (1) and (2) are satisfied as shown below.
$5x + 3y = 5\left(\frac{335}{22}\right) + 3\left(\frac{-45}{22}\right) = \frac{1675}{22} - \frac{135}{22}$
$= \frac{1675}{22} = \frac{1540}{22} = 70$
$3x - 7y = 3\left(\frac{335}{22}\right) - \left(\frac{-45}{22}\right)$
$= \frac{1005}{22} + \frac{315}{22} = \frac{1005+315}{22} = \frac{1320}{20} = 60$
Hence, the solution is correct.

34. r = 28 cm and θ = $\frac{360}{6} = 60^\circ$
Area of minor sector = $\frac{\theta}{360}\pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 28 \times 28 = \frac{1232}{3}$
= 410.67 cm^2
For, Area of $\triangle$AOB,

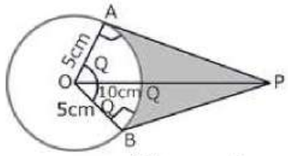


Draw OM $\perp$ AB.
In right triangles OMA and OMB,
OA = OB [Radii of same circle]
OM = OM [Common]
$\therefore \triangle$OMA $\cong$ OMB [RHS congruency]
$\therefore$ AM = BM [By CPCT]
$\Rightarrow$ AM = BM = $\frac{1}{2}$AB and $\angle$AOM = $\angle$BOM = $\frac{1}{2}$AOB = $\frac{1}{2} \times 60^o = 30^o$
In right angled triangle OMA, $\cos 30^o = \frac{OM}{OA}$
$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{28}$
$\Rightarrow$ OM = $14\sqrt{3}$ cm
Also, $\sin 30^\circ = \frac{AM}{OA}$
$\Rightarrow \frac{1}{2} = \frac{AM}{28}$
$\Rightarrow$ AM = 14 cm
$\Rightarrow$ 2 AM = 2 $\times$ 14 = 28 cm
$\Rightarrow$ AB = 28 cm
$\therefore$ Area of $\triangle$AOB = $\frac{1}{2} \times$ AB $\times$ OM = $\frac{1}{2} \times 28 \times 14\sqrt{3} = 196\sqrt{3} = 196 \times 1.7 = 333.2$ cm
2
$\therefore$ Area of minor segment = Area of minor sector - Area of $\triangle$AOB
= 410.67 - 333.2 = 77.47 cm^2
$\therefore$ Area of one design = 77.47 cm^2

$\therefore$ Area of six designs = 77.47 $\times$ 6 = 464.82 cm^2

Cost of making designs = 464.82 $\times$ 0.35 = Rs. 162.68

OR



$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \theta = 60^o$

$\Rightarrow \angle AOB = 2 \times \theta = 120^o$

$\therefore$ ARC AB = $\frac{120\times\ 2\times\pi\times5}{360} cm = \frac{10\pi}{3} cm\ \left[\because l = \frac{\theta}{360} \times 2\pi r\right]$

Length of the belt that is in contact with the rim of the pulley

= Circumference of the rim - length of arc AB

= $2\pi \times 5$ cm - $\frac{10\pi}{3}$cm

= $\frac{20\pi}{3}$cm

Now, the area of sector OAQB = $\frac{120\times\pi\times5\times5}{360} cm^2 = \frac{25\pi}{3} cm^2\ \left[\because Area = \frac{\theta}{360} \times \pi r^2\right]$

Area of quadrilateral OAPB = 2(Area of $\triangle$OAP) = $25\sqrt{3}$ cm^2

$\left[\because AP = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}\ cm\right]$

Hence, shaded area = $25\sqrt{3} - \frac{25\pi}{3} = \frac{25}{3}[3\sqrt{3} - \pi]\ cm^2$

35. Here, all face cards of spades are removed from a deck of 52 playing cards.

So, remaining cards in deck = 52 - 3 = 49

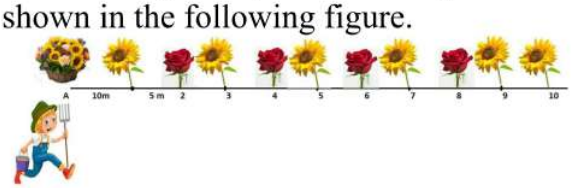
$\therefore$ Total number of outcomes n = 49

i. We know that there are 6 black face cards in a deck of cards. Afte removing face cards of spades only 3 face cards of club are left.
   so number of favorable outcomes m= 3
   $\therefore$ Required Probability = $P(E) = \frac{m}{n} = \frac{3}{49}$
ii. There are 4 queens in a deck . After removing a queen of spade, we are left with 3 queens.
   Then, number of favorable outcomes m= 3
   $\therefore$ Required Probability = P(E)=$\frac{m}{n} = \frac{3}{49}$
iii. There are 26 black cards in a regular deck of cards.
   After removing 3 face cards of spades , there are only 23 black cards.
   Then, number of favorable outcomes m= 23
   $\therefore$ Required Probability = P(E)=$\frac{m}{n} = \frac{23}{49}$
iv. There are 13 cards of spade in a deck.After removing 3 face cards of spade only 10 spades cards are left .
   So number of favorable outcomes m= 10
   $\therefore$ Required Probability = P(E) =$\frac{m}{n} = \frac{10}{49}$

Section E

36. **Read the text carefully and answer the questions:**

In a school garden, Dinesh was given two types of plants viz. sunflower and rose flower as shown in the following figure.



The distance between two plants is to be 5m, a basket filled with plants is kept at point A which is 10 m from the first plant. Dinesh has to take one plant from the basket and then he will have to plant it in a row as shown in the figure and then he has to return to the basket to collect another plant. He continues in the same way until all the flower plants in the basket. Dinesh has to plant ten numbers of flower plants.

(i) The distance covered by Dinesh to pick up the first flower plant and the second flower plant,
$= 2 \times 10 + 2 \times (10 + 5) = 20 + 30$
therefore, the distance covered for planting the first 5 plants
$= 20 + 30 + 40 + ...$ 5 terms
This is in AP where the first term a = 20
and common difference d = 30 - 20 = 10

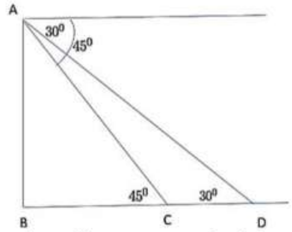
(ii) We know that a = 20, d = 10 and number of terms = n = 5 so,
$S_n = \frac{n}{2}[2a + (n-1)d]$
So, the sum of 5 terms
$S_5 = \frac{5}{2}[2 \times 20 + 4 \times 10] = \frac{5}{2} \times \ 80 = 200$ m
Hence, Dinesh will cover 200 m to plant the first 5 plants.

(iii) As a = 20, d = 10 and here n = 10
So, $S_{10} = \frac{10}{2}[2 \times 20 + 9 \times 10] = 5 \times 130 = 650$ m
So, hence Ramesh will cover 650 m to plant all 10 plants.

OR

Total distance covered by Ramesh 650 m
Time $= \frac{\text{distance}}{\text{speed}} = \frac{650}{10} = 65$ minutes
Time taken to plant all 10 plants = $15 \times 10 = 150$ minutes
Total time = 65 + 150 = 215 minutes = 3 hrs 35 minutes

37. **Read the text carefully and answer the questions:**
An observer on the top of a 40m tall light house (including height of the observer) observes a ship at an angle of depression 30° coming towards the base of the light house along straight line joining the ship and the base of the light house. The angle of depression of ship changes to 45° after 6 seconds.

(i) 

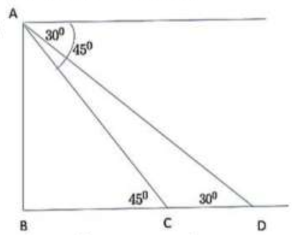
The distance of ship from the base of the light house after 6 seconds from the initial position when angle of depression is 45^{o}.

In $\triangle$ABC

$\tan 45^{o} = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{40}{BC}$

$\Rightarrow$ BC = 40 m

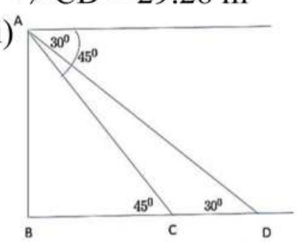
(ii) 

The distance between two positions of ship after 6 seconds

CD = BD - BC

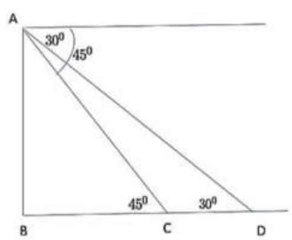
$\Rightarrow$ CD = $40\sqrt{3}$ - 40 = $40(\sqrt{3}$ - 1)

$\Rightarrow$ CD = 29.28 m

(iii) 

Speed of ship = $\frac{Distance}{Time} = \frac{29.28}{6}$ = 4.88 m/sec

OR



The distance of ship from the base of the light house when angle of depression is 30^{o}.

In $\triangle$ABD

$\tan 30^{o} = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{BD}$

$\Rightarrow$ BD = $40\sqrt{3}$ m

38. **Read the text carefully and answer the questions:**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.

(i) Volume of Hermika = side^3 = 10 × 10 × 10 = 1000 m^3

(ii) r = radius of cylinder = 24, h = height = 16

Volume of cylinder = $\pi r^2 h$

$\Rightarrow$ V = $\frac{22}{7} \times 24 \times 24 \times 14$ = 25344 m^3

(iii) Volume of brick = 0.01 m^3

$\Rightarrow$ n = Number of bricks used for making cylindrical base = $\frac{\text{Volume of cylinder}}{\text{Volume of one brick}}$

$\Rightarrow n = \frac{25344}{0.01}$ = 2534400

OR

Since Anda is hespherical in shape r = radius = 21

V = Volume of Anda = $\frac{2}{3} \times \pi \times r^3$

$\Rightarrow$ V = $\frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$\Rightarrow$ V = 44 × 21 × 21 = 19404 m^3