RATIONAL NUMBER

CONTENTS

➢ NUMBERS

1. **Natural Numbers : (N)** : The set of numbers 1, 2, 3, 4…… ∞ are called natural numbers and decimal numbers are not allowed in natural numbers. N = {1, 2, 3, …….. ∞}

   Addition and multiplication are closure, commutative and associative for natural numbers.

2. **Whole Numbers : (W) :** The set of natural numbers with '0' is called set of whole numbers W = {0, 1, 2, 3, ….∞}

   Addition and multiplication are closure, commutative and associative for whole numbers.

3. **Integers (I or z) :** The set of positive and negative without decimal numbers, is called integers.

   z = I = {–∞ …… –3, –2, –1, 0, 1, 2, 3, …..∞}

   Addition and multiplication are closure, commutative and associative for Integers.

4. **Even Numbers :** All integers which are divisible by 2 are called even numbers and denoted by $2n$ where n is integer.

   So E = { .... –4, –2, 0, 2, 4 ....}

5. **Odd Numbers :** All integers which are not divisible by 2 are called odd numbers and denoted by $2n + 1$, where n is integer.

   So O = {–7, –5, –3, –1, 1, 3, ...}

6. **Prime Numbers :** All natural numbers that have one & it self only as their factors are called prime numbers.

   So P = {2, 3, 5, 7, 11, 13, 17, 19, 23,......}

   ** 2 is only even prime number & it is smallest prime number.*

7. **Composite Numbers :** All natural numbers which are not prime called composite numbers.

   So C = {4, 6, 8, 9, 10, 12, 14........}

   ** 1 is neither prime nor composite number.*

8. **Co-prime Numbers :** If the H.C.F. (or G.C.D.) of the given numbers is 1 then they are known as co-prime numbers.

   Eg. 5, 8 are co-prime Θ Their HCF is 1.

   ** Any Two consecutive numbers are always co-prime.*

9. **Real Numbers :** Numbers which can represent actual physical quantities in a meaningful way are known as real numbers. These can represent on the number line. Number line is geometrical straight line with arbitrarily defined zero (origin).

10. **Rational Numbers :** The real numbers which are in form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$

    Eg. $\frac{5}{7}, \frac{-3}{8}, \frac{11}{1} = 11, 0, \frac{2}{71}$ ..... etc.

- All natural numbers, whole numbers and integers are rational.
- Rational numbers, includes all integers, terminitating fractions (if the decimal parts are terminating like 0.2, 0.5, –3.5 etc) and non terminating recurring decimals (like $0.\overline{6}$, –3.777 .... etc.)
- Rational number is in standard form or simplest form if H.C.F. of numerator and denominator is 1.

➤ ORDER OF RATIONAL NUMBERS

Ex.1 Arrange the following fractions in ascending order.

$\frac{3}{8}, \frac{4}{12}, \frac{-7}{16}, \frac{-2}{3}$.

Sol. LCM of denominators

8, 12, 16, and 3 = 2 × 2 × 2 × 2 × 3 = 48.

Then $\frac{3}{8} = \frac{3\times6}{8\times6} = \frac{18}{48}$;

$\frac{-7}{16} = \frac{-7\times3}{16\times3} = \frac{-21}{48}$;

$\frac{4}{12} = \frac{4\times4}{12\times4} = \frac{16}{48}$;

$-\frac{2}{3} = \frac{-2\times16}{3\times16} = \frac{-32}{48}$

2	8,	12,	16,	3
2	4,	6,	8,	3
2	2,	3,	4,	3
2	1,	3,	2,	3
3	1,	3,	1,	3
	1,	1,	1,	1

The equivalent rational numbers are

$\frac{18}{48}, \frac{16}{48}, \frac{-21}{48}$ and $\frac{-32}{48}$

Therefore, the smallest rational number is $\frac{-32}{48}$, then comes, $\frac{-21}{48}$, then comes $\frac{16}{48}$, and the greatest rational number is $\frac{18}{48}$. Hence, their ascending order is $\frac{-2}{3}, \frac{-7}{16}, \frac{4}{12}, \frac{3}{8}$.

➤ ADDITION OF RATIONAL NUMBERS

◈ When denominators are equal :

Ex.2 Add $\frac{5}{6}$ and $\frac{7}{6}$.

Sol. $\frac{5}{6} + \frac{7}{6} = \frac{5+7}{6} = \frac{12}{6}$

Ex.3 Add $\frac{7}{5}$ and $\frac{-13}{5}$.

Sol. $\frac{7}{5} + \left(\frac{-13}{5}\right) = \frac{7-13}{5} = \frac{-6}{5}$

◈ When one denominator is a multiple of the other denominator :

Ex.4 Solve $\frac{4}{3}$ and $\frac{5}{6}$.

Sol. We know that $\frac{4}{3} = \frac{4\times2}{3\times2} = \frac{8}{6}$

($\frac{8}{6}$ is equivalent rational number of $\frac{4}{3}$)

So, $\frac{4}{3} + \frac{5}{6} = \frac{8}{6} + \frac{5}{6} = \frac{13}{6}$

Ex.5 Solve $\frac{-3}{7} + \left(\frac{-5}{21}\right)$.

Sol. We know that

$\frac{-3}{7} = \frac{3\times3}{7\times3} = \frac{-9}{21}$

So, $\frac{-3}{7} + \left(\frac{-5}{21}\right) = \frac{-9}{21} - \frac{-5}{21}$

$= \frac{-9-5}{21} = \frac{-14}{21}$

◈ When denominator are co-prime :

Ex.6 Find the sum of $\frac{4}{5}$ and $\frac{-6}{7}$.

Sol. $\frac{4}{5} + \left(\frac{-6}{7}\right) = \frac{4\times7}{5\times7} - \frac{6\times5}{7\times5}$

(Multiplying and dividing each fraction by the denominator of the other fraction)

$= \frac{28}{35} - \frac{30}{35} = \frac{28-30}{35} = \frac{-2}{35}$

❖ When denominator have a common factor :

Ex.7 Solve $\frac{5}{12}+\frac{7}{8}$.

Sol. Since 12 and 8 have common factors, we will proceed by finding the LCM of 12 and 8. LCM of 12 and 8 is

$2 \times 2 \times 2 \times 3 = 24$

Now we will find equivalent fractions of the given numbers having 24 in the denominator.

Hence, $\frac{5}{12} = \frac{5\times 2}{12\times 2} = \frac{10}{24}$

and $\frac{7}{8} = \frac{7\times 3}{8\times 3} = \frac{21}{24}$

So, $\frac{5}{12} + \frac{7}{8} = \frac{10}{24} + \frac{21}{24} = \frac{10+21}{24} = \frac{31}{24}$

➤ PROPERTIES OF ADDITION OF RATIONAL NUMBERS

❖ Closure property :

When two rational numbers are added, the result is always a rational number, i.e., if $\frac{a}{b}$ and $\frac{c}{d}$ is always a rational number.

For example, $\frac{2}{5} + \frac{3}{6} = \frac{12+15}{30} = \frac{27}{30}$, which is also a rational number.

❖ Commutative property :

When two rational numbers are added, the order of addition does not matter, i.e., if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then

$\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$

For example, $\frac{3}{4} + \frac{4}{5} = \frac{15+16}{20} = \frac{31}{20}$ and

$\frac{4}{5} + \frac{3}{4} = \frac{16+15}{20} = \frac{31}{20}$. Both results are equal.

❖ Associative property

If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ three rational numbers, then

$\left(\frac{a}{b}+\frac{c}{d}\right) + \frac{e}{f} = \frac{a}{b} + \left(\frac{c}{d}+\frac{e}{f}\right)$

Consider the fractions $\frac{2}{5}$, $\frac{1}{4}$ and $\frac{2}{3}$.

$\left(\frac{2}{5}+\frac{1}{4}\right)+\frac{2}{3}$	$\frac{2}{5}+\left(\frac{1}{4}+\frac{2}{3}\right)$
$=\left(\frac{8+5}{20}\right)+\frac{2}{3}$	$=\frac{2}{5}+\left(\frac{3+8}{12}\right)$
$=\frac{13}{20}+\frac{2}{3}$	$=\frac{2}{5}+\frac{11}{12}$
$=\frac{39+40}{60}$	$=\frac{24+55}{60}$
$=\frac{79}{60}$	$=\frac{79}{60}$

❖ Additive identity

If $\frac{a}{b}$ is a rational number, then there exists a rational number zero such that $\frac{a}{b} + 0 = \frac{a}{b}$. Zero is called the identity element of addition. Addition of zero does not change the value of the rational number.

❖ Additive inverse

If $\frac{a}{b}$ is a rational number, then there exists a rational number $\left(\frac{-a}{b}\right)$, called the additive inverse, such that $\frac{a}{b}+\left(\frac{-a}{b}\right)=0$

The additive inverse is also referred to as 'negative' of the given number.

Ex.8 $\frac{3}{4}+\left(\frac{-3}{4}\right) = 0$.

$\therefore$ $\left(\frac{-3}{4}\right)$ is the additive inverse of $\frac{3}{4}$.

Ex.9 $\frac{-5}{6} + \frac{5}{6} = 0.$

$\therefore \quad \frac{5}{6}$ is the additive inverse of $\left(\frac{-5}{6}\right)$.

➤ SUBTRACTION OF RATIONAL NUMBERS

When we have to subtract a rational number, say $\frac{5}{9}$ from $\frac{8}{9}$, we add the additive inverse of $\frac{5}{9}$, i.e., $\frac{-5}{9}$ to $\frac{8}{9}$. Thus, $\frac{8}{9} - \frac{5}{9} = \frac{8}{9} + \left(\frac{-5}{9}\right)$

$$= \frac{8-5}{9} = \frac{3}{9} = \frac{1}{3}$$

Ex.10 Subtract $\frac{3}{-7}$ from $\frac{3}{7}$.

Here, $\frac{4}{11} - \left(\frac{-3}{7}\right) = \frac{4}{11} + \left(\frac{+3}{7}\right)$

$$= \frac{4\times7}{11\times7} + \frac{3\times11}{7\times11}$$

$$= \frac{28}{77} + \frac{33}{77} = \frac{61}{77}$$

➤ MULTIPLICATION OF RATIONAL NUMBERS

Multiplication is the process of successive addition. Like 6 × 8 = 8 + 8 + 8 + 8 + 8 + 8 = 48.

Similarly, $6 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{6}{3} = 2$

Alternatively, $6 \times \frac{1}{3} = \frac{6}{1} \times \frac{1}{3} = \frac{6\times1}{1\times3} = \frac{6}{3} = \frac{2}{1} = 2$

So, when we multiply two rational numbers, we multiply the numerator with the numerator and the denominator with the denominator.

Thus, $\quad -5 \times (-7) = \frac{-5}{1} \times \left(\frac{-7}{1}\right) = \frac{(-5)(-7)}{1\times1} = 35$

and $\quad \frac{-2}{11} \times \frac{3}{5} = \frac{-2\times3}{11\times5} = \frac{-6}{55}$

➤ PROPERTIES OF MULTIPLICATION OF RATIONAL NUMBER

◈ Closure property :

The rational number are closed under multiplication. It means that the product of two rational numbers is always a rational number, i.e., if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers,

$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ is always a rational number.

For example, $\frac{-3}{7} \times \frac{5}{8} = -\frac{15}{56}$ which is rational number.

◈ Commutative property :

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then

$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$, i.e., $\frac{ac}{bd} = \frac{ca}{db}$

Ex.11

$\frac{4}{5} \times \left(\frac{-3}{7}\right)$	$\left(\frac{-3}{7}\right) \times \frac{4}{5}$
$= \frac{4\times(-3)}{5\times7}$	$= \frac{(-3)\times4}{7\times5}$
$= \frac{-12}{35}$	$= \frac{-12}{35}$

$$\frac{4}{5} \times \left(\frac{-3}{7}\right) = \left(\frac{-3}{7}\right) \times \frac{4}{5}$$

◈ Associative property :

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then

$$\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right)$$

i.e., $\frac{ac}{bd} \times \frac{e}{f} = \frac{a}{b} \times \frac{ce}{df}$ or $\frac{ace}{bdf} = \frac{ace}{bdf}$

Thus, rational numbers can be multiplied in any order.

Ex.12 $\left(\frac{-3}{7} \times \frac{4}{5}\right) \times \left(\frac{-5}{8}\right) = \left(\frac{-3}{7}\right) \times \left(\frac{4}{5} \times \frac{-5}{8}\right)$

$$\frac{(-3)\times4}{7\times5} \times \left(\frac{-5}{8}\right) = \left(\frac{-3}{7}\right) \times \frac{4\times(-5)}{5\times8}$$

$$\frac{-12}{35} \times \left(\frac{-5}{8}\right) = \left(\frac{-3}{7}\right) \times \left(\frac{-20}{40}\right)$$

$$\frac{60}{280} = \frac{60}{280}$$

$$\frac{3}{14} = \frac{3}{14}$$

◈ Multiplicative identity :

When any rational number, say $\frac{a}{b}$, is multiplied by the rational number 1, the product is always $\frac{a}{b}$.

$$\frac{a}{b} \times 1 = \frac{a \times 1}{b} = \frac{a}{b}$$

or $$1 \times \frac{a}{b} = \frac{1 \times a}{b} = \frac{a}{b}$$

Ex.13 $\frac{21}{35} \times 1 = \frac{21}{35} \times \frac{1}{1} = \frac{21 \times 1}{35 \times 1} = \frac{21}{35}$

Ex.14 $\frac{-3}{7} \times 1 = \frac{-3}{7} \times \frac{1}{1} = \frac{(-3) \times 1}{7 \times 1} = \frac{-3}{7}$

'One' is called the multiplicative identity or identity element of multiplication for rational numbers.

◈ Multiplicative inverse, or reciprocal :

For every non-zero rational number $\frac{a}{b}$, there exists a rational number $\frac{b}{a}$ such that $\frac{a}{b} \times \frac{b}{a} = 1$.

This is so, because $\frac{a}{b} \times \frac{b}{a} = \frac{a \times b}{b \times a} = \frac{ab}{ba} = 1$

Ex.15 $\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} \times \frac{6}{6} = 1$. So $\frac{3}{2}$ is the multiplicative inverse of $\frac{2}{3}$ and $\frac{2}{3}$ is the multiplicative inverse of $\frac{3}{2}$.

Ex.16 $\left(-\frac{4}{7}\right) \times \left(-\frac{7}{4}\right) = \frac{(-4)(-7)}{7 \times 4} = \frac{28}{28} = 1$. So $-\frac{7}{4}$ is the multiplicative inverse of $-\frac{4}{7}$ and vice versa.

◈ Distributive property :

If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$.

Ex.17 $3(4 + 5) = 3 \times 4 + 3 \times 5$

$3 \times 9 = 12 + 15$

$27 = 27$

Ex.18 $\frac{4}{7}\left(\frac{2}{3} + \frac{3}{4}\right) = \frac{4}{7} \times \frac{2}{3} + \frac{4}{7} \times \frac{3}{4}$

$\frac{4}{7}\left(\frac{8+9}{12}\right) = \frac{8}{21} + \frac{12}{28}$

$\frac{4}{7} \times \frac{17}{12} = \frac{32+36}{84}$

$\frac{68}{84} = \frac{68}{84}$

Ex.19 $\frac{-3}{5}\left(\frac{3}{4} + \frac{-8}{9}\right) = \left(\frac{-3}{5}\right) \times \frac{3}{4} + \left(\frac{-3}{5}\right) \times \left(\frac{-8}{9}\right)$

$\frac{-3}{5}\left(\frac{27-32}{36}\right) = \frac{-9}{20} + \frac{24}{45}$

$\frac{-3}{5} \times \frac{-5}{36} = \frac{-81+96}{180}$

$\frac{15}{180} = \frac{15}{180}$

➢ MULTIPLICATION OF A RATIONAL NUMBER BY ZERO

When any rational number $\frac{a}{b}$ is multiplied by 0, the product is always zero.

$$\frac{a}{b} \times 0 = \frac{a \times 0}{b} = \frac{0}{b} = 0$$

Ex.20 $\frac{7}{8} \times 0 = \frac{7 \times 0}{8} = \frac{0}{8} = 0$

Ex.21 $\frac{-3}{4} \times 0 = \frac{-3 \times 0}{4} = \frac{0}{4} = 0$

➢ DIVISION OF RATIONAL NUMBERS

Division is the inverse process of multiplication.

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}.$$

Ex.22 $\frac{2}{7} \div \frac{5}{9} = \frac{2}{7} \times \frac{9}{5} = \frac{18}{35}$

Ex.23 $\frac{3}{8} \div \frac{-4}{9} = \frac{3}{8} \times \left(\frac{-9}{4}\right) = \frac{-27}{32}$

➤ PROPERTIES OF DIVISION OF RATIONAL NUMBERS CLOSURE PROPERTY

When a rational number is divided by another rational number, the quotient is always a rational number.

Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{c}{d} = \frac{ad}{bc}$, which is again a rational number since b, c, d are non-zero integers.

Ex.24 $\frac{3}{4} \div \left(\frac{-1}{3}\right) = \frac{3}{4} \times \left(\frac{-3}{1}\right) = \frac{-9}{4}$

❖ Division is not commutative :

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers in which b, c and $d \neq 0$, then

$\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$ because,

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ and $\frac{c}{d} \div \frac{a}{b} = \frac{c}{d} \times \frac{b}{a} = \frac{cb}{da}$

So $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$

Ex.25 $\frac{4}{7} \div \frac{1}{3}$ is not equal to $(\neq) \frac{1}{3} \div \frac{4}{7}$

$\frac{4}{7} \div \frac{1}{3} = \frac{4}{7} \times \frac{3}{1} = \frac{12}{7}$,

whereas $\frac{1}{3} \div \frac{4}{7} = \frac{1}{3} \times \frac{7}{4} = \frac{7}{12}$

So $\frac{4}{7} \div \frac{1}{3} \neq \frac{1}{3} \div \frac{4}{7}$

So Addition, Subtraction and Multiplication are closed for rationales. Addition, multiplication are commutative and associative for rationals.

Ex.26 $\frac{5}{7} + \frac{8}{13} = \frac{65+56}{91} = \frac{121}{91}$

Θ $\frac{5}{7}, \frac{8}{13}$ are rational no. and $\frac{121}{91}$ is also rational.

(Θ it is closured)

Ex.27 $\frac{1}{2} - \frac{3}{8}$

(Θ $\frac{1}{2}, \frac{3}{8}$ are rational & $\frac{1}{8}$ is also rational)

$= \frac{4-3}{8} = \frac{1}{8}$ (Θ Subtraction is closed)

Ex.28 Find $\frac{3}{7} + \left(\frac{-6}{11}\right) + \left(\frac{-8}{21}\right) + \left(\frac{5}{22}\right)$.

Sol. $\frac{3}{7} + \left(\frac{-6}{11}\right) + \left(\frac{-8}{21}\right) + \left(\frac{5}{22}\right)$

$= \frac{198}{462} + \left(\frac{-252}{462}\right) + \left(\frac{-176}{462}\right) + \left(\frac{105}{462}\right)$

(Note that 462 is the LCM of 7, 11, 21 and 22)

$= \frac{198-252-176+105}{462} = \frac{-125}{462}$

We can also solve it as.

$\frac{3}{7} + \left(\frac{-6}{11}\right) + \left(\frac{-8}{21}\right) + \frac{5}{22}$

$= \left[\frac{3}{7} + \left(\frac{-8}{21}\right)\right] + \left[\frac{-6}{11} + \frac{5}{22}\right]$

(by using commutative and associativity)

$= \left[\frac{9+(-8)}{21}\right] + \left[\frac{-12+5}{22}\right]$

(LCM of 7 and 21 is 21; LCM of 11 and 22 is 22)

$= \frac{1}{21} + \left(\frac{-7}{22}\right) = \frac{22-147}{462} = \frac{-125}{462}$

Ex.29 Find $\frac{-4}{5} \times \frac{3}{7} \times \frac{15}{16} \times \left(\frac{-14}{9}\right)$

Sol. We have

$\frac{-4}{5} \times \frac{3}{7} \times \frac{15}{16} \times \left(\frac{-14}{9}\right)$

$= \left(-\frac{4\times 3}{5\times 7}\right) \times \left(\frac{15\times(-14)}{16\times 9}\right)$

$= \frac{-12}{35} \times \left(\frac{-35}{24}\right) = \frac{-12\times(-35)}{35\times 24} = \frac{1}{2}$

We can also do it as.

$\frac{-4}{5} \times \frac{3}{7} \times \frac{15}{16} \times \left(\frac{-14}{9}\right)$

$= \left(\frac{-4}{5} \times \frac{15}{16}\right) \times \left[\frac{3}{7} \times \left(\frac{-14}{9}\right)\right]$

(Using commutativity and associativity)

$= \frac{-3}{4} \times \left(\frac{-2}{3}\right) = \frac{1}{2}$

THE ROLE OF ZERO (0) AND ONE (1)

Zero is called the identity for the addition of rational numbers. It is the additive identity for integers and whole numbers as well. 1 is the multiplicative identity for rational numbers.

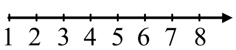
$a + 0 = 0 + a = a$

& $a \times 1 = 1 \times a = a$

REPRESENTATION OF RATIONAL NUMBERS ON THE NUMBER LINE

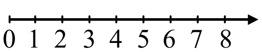
You have learnt to represent natural numbers, whole numbers, integers and rational numbers on a number line. Let us revise them.

Natural Numbers

(i) 

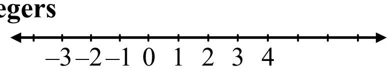
Note :– The line extends indefinitely only to the right side of 1.

Whole Numbers

(ii) 

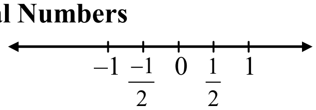
Note :– The line extends indefinitely to the right, but from 0. There are no numbers to the left of 0.

Integers

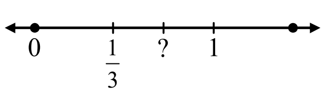
(iii) 

Note :– The line extends indefinitely on both sides.

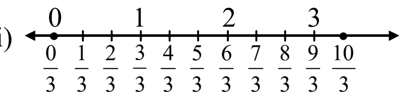
Rational Numbers

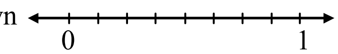
(iv) 

Note :– The line extends indefinitely on both sides. You can see numbers between –1, 0; 0, 1 etc.

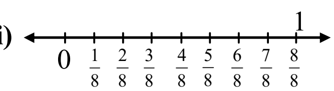
(v) 

The point to be labeled is twice as far from and to the right of 0 as the point labeled $\frac{1}{3}$. So it is two times $\frac{1}{3}$, i.e., $\frac{2}{3}$. The next marking is 1. You can see that 1 is the same as $\frac{3}{3}$. Then comes $\frac{4}{3}$, $\frac{5}{3}$, $\frac{6}{3}$ (or 2), $\frac{7}{3}$ and so on as shown on the number line (vi)

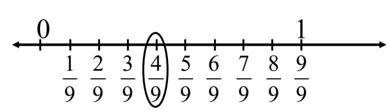
(vi) 

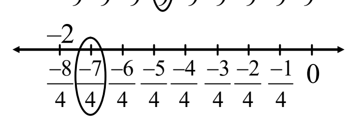
Similarly, to represent $\frac{1}{8}$, the number line may be divided into eight equal parts as shown 

We use the number $\frac{1}{8}$ to name the first point of this division. The second point of division will be labeled $\frac{2}{8}$, the third point $\frac{3}{8}$, and so on as shown on number line (vii)

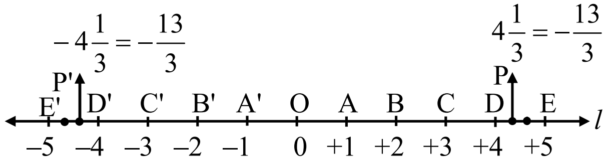
(vii) 

Any rational number can be represented on the number line in this way. In a rational number, the numeral below the bar, i.e., the denominator, tells the number of equal parts into which the first unit has been divided. The numeral above the bar i.e., the numerator, tells 'how many' of these parts are considered. So, a rational number such as $\frac{4}{9}$ means four of nine equal parts on the right of 0 (number line viii) and for $\frac{-7}{4}$, we make 7 marking of distance $\frac{1}{4}$ each on the left of zero and starting from 0. The seventh marking is $\frac{-7}{4}$ [number line (ix)].

(viii) 

(ix) 

Ex.30 Represent $\frac{13}{3}$ and $-\frac{13}{3}$ on number line.



Sol. Then $\Theta\ \frac{13}{3} = 4\frac{1}{3} = 4 + \frac{1}{3}$

Draw a line *l* and mark zero on it

$$\frac{13}{3} = 4\frac{1}{3} = 4 + \frac{1}{3} \text{ and } \frac{-13}{3} = -\left(4 + \frac{1}{3}\right)$$

Therefore, from O mark OA, AB, BC, CD and DE to the right of O. Such that OA = AB = BC = CD = DE = 1 unit.

Clearly,

Point A represents the Rational number = 1

Point B represents the Rational number = 2

Point C represents the Rational number = 3

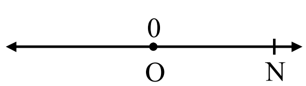
Point D represents the Rational number = 4

Point E represents the Rational number = 5

Since we have to consider 4 complete units and a part of the fifth unit, therefore divide the fifth unit DE into 3 equal parts. Take 1 part out of these 3 parts. Then point P is the representation of number $\frac{13}{3}$ on the number line. Similarly, take 4 full unit lengths to the left of 0 and divide the fifth unit D'E' into 3 equal parts. Take 1 part out of these three equal parts. Thus, P' represents the rational number $-\frac{13}{3}$.

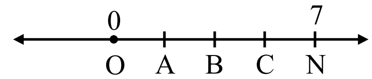
Ex.31 Represent the rational number $\frac{7}{4}$ on the number line.

Sol. In order to represent $\frac{7}{4}$ on the number line, we first draw a number line and mark a point O on it which represent '0' as shown in the figure.



Now we have to find a point, say, **N on the number line which represents the numerator 7 of the rational number $\frac{7}{4}$**.

So, N is the point that represents the integer 7 on the number line and is on the right hand side of the point O. Divide the segment ON into **four** (Denominator of $\frac{7}{4}$) equal parts (with the help of a ruler). Let A, B, C be the points of division as shown in the figure.



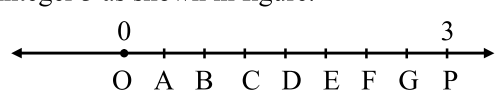
Then OA = AB = BC = CN.

By construction, each segment OA, AB, BC and CN represents $\frac{1}{4}$th of segment ON. Therefore, the point A represents the rational number $\frac{7}{4}$. Similarly, $\frac{-7}{4}$ can be represented on the number line on the left hand side of 'O'.

Ex.32 Draw the number line and represent the following rational numbers on it.

(i) $\frac{3}{8}$ (ii) $-\frac{5}{3}$

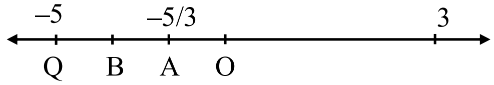
Sol. (i) In order to represent $\frac{3}{8}$ on number line, we first draw a number line and mark the point O on it representing '0' (zero), we find the point P on the number line representing the positive integer 3 as shown in figure.



Now divide the segment OP into 8 equal parts. Let A and B be points of division so that
OA = AB = BC = CD = DE = EF = FG = GP.

By construction, OA is $\frac{1}{8}$ of OP. Therefore, A represents the rational number $\frac{3}{8}$.

(ii) In order to represent $-\frac{5}{3}$ on number line, we first draw a number line and mark a point O on it representing zero. We find the point Q on the number line representing the integer –5 on the left side of O, as shown in figure.



Now divide OQ into 3 equal parts (with the help of ruler). Let A and B be the points of division as shown in figure. Then OA = AB = BQ.

By construction, each segment OA, AB and BQ represents $\frac{1}{3}$ of OQ. Therefore, the point A represents the rational number $-\frac{5}{3}$.

Similarly, B represents the rational number $-\frac{5}{3}\times 2$ and Q represents the rational number $-\frac{5}{3}\times 3=-5$.

Note : There are countless rational number between any two given rational numbers.

Ex.33 Write any 3 rational numbers between –2 and 0.

Sol. –2 can be written as $\frac{-20}{10}$ and 0 as $\frac{0}{10}$.

Thus we have $\frac{-19}{10}$, $\frac{-18}{10}$, $\frac{-17}{10}$, $\frac{-16}{10}$, $\frac{-15}{10}$,....... $\frac{-1}{10}$ between – 2 and 0.

Ex.34 Find any ten rational numbers between $\frac{-5}{6}$ and $\frac{5}{8}$.

Sol. We first convert $\frac{-5}{6}$ and $\frac{5}{8}$ to rational numbers with the same denominators.

$$\frac{-5\times 4}{6\times 4}=\frac{-20}{24} \text{ and } \frac{5\times 3}{8\times 3}=\frac{15}{24}.$$

Thus we have, $\frac{-19}{24}$, $\frac{-18}{24}$, $\frac{-17}{24}$, ... $\frac{14}{24}$ as the rational numbers between $\frac{-20}{44}$ and $\frac{15}{24}$

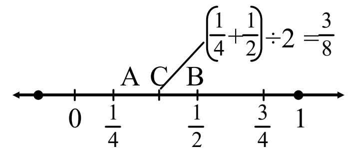
Ex.35 Find a rational number between $\frac{1}{4}$ and $\frac{1}{2}$.

Sol. We find the mean of the given rational numbers.

$$\left(\frac{1}{4}+\frac{1}{2}\right)\div 2=\left(\frac{1+2}{4}\right)\div 2=\frac{3}{4}\times\frac{1}{2}=\frac{3}{8}$$

$\frac{3}{8}$ lies between $\frac{1}{4}$ and $\frac{1}{2}$.

This can be seen on the number line also.



We find mid point of AB which is C, represented by $\left(\frac{1}{4}+\frac{1}{2}\right)\div 2=\frac{3}{8}$.

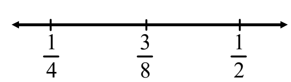
We find that $\frac{1}{4}<\frac{3}{8}<\frac{1}{2}$.

If a and b are two rational numbers, then $\frac{a+b}{2}$ is a rational number between a and b such that $a<\frac{a+b}{2}<b$.

This again shows that there are countless number of rational numbers between any two given rational numbers.

Ex.36 Find three rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

Sol. We find the mean of the given rational number.

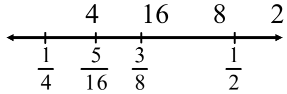


As given in the above example, the mean is $\frac{3}{8}$ and $\frac{1}{4}<\frac{3}{8}<\frac{1}{2}$.

Now we find another rational number between $\frac{1}{4}$ and $\frac{3}{8}$. For this, we again find the mean of $\frac{1}{4}$ and $\frac{3}{8}$. That is,

$$\left(\frac{1}{4}+\frac{3}{8}\right)\div 2 = \frac{5}{8}\times\frac{1}{2} = \frac{5}{16}$$

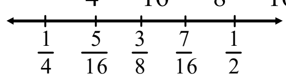
$$\frac{1}{4} < \frac{5}{16} < \frac{3}{8} < \frac{1}{2}$$



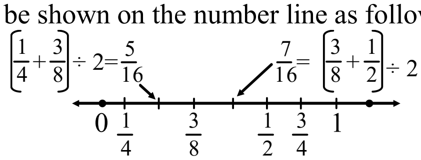
Now find the mean of $\frac{3}{8}$ and $\frac{1}{2}$. We have,

$$\left(\frac{3}{8}+\frac{1}{2}\right)\div 2 = \frac{7}{8}\times\frac{1}{2} = \frac{7}{16}$$

Thus we get $\frac{1}{4} < \frac{5}{16} < \frac{3}{8} < \frac{7}{16} < \frac{1}{2}$.



Thus, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{7}{16}$ are the three rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$. This can clearly be shown on the number line as follows :



In the same way we can obtain as many rational numbers as we want between two given rational numbers. You have noticed that there are countless rational numbers between any two given rational numbers.

POWERS

Exponential Notation and Rational Numbers :

Exponential notation can be extended to rational numbers. For example: $\left(\frac{4}{5}\right)\times\left(\frac{4}{5}\right)\times\left(\frac{4}{5}\right)$ can be written as $\left(\frac{4}{5}\right)^3$ which is read as $\frac{4}{5}$ raised to the power 3.

(i) $\left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)\times\left(\frac{3}{4}\right)\times\left(\frac{3}{4}\right) = \frac{3^3}{4^3} = \frac{27}{64}$

(ii) $\left(\frac{-5}{6}\right)^2 = \left(\frac{-5}{6}\right)\times\left(\frac{-5}{6}\right) = \frac{(-5)^2}{6^2} = \frac{25}{36}$

(ii) $\left(\frac{-2}{3}\right)^3 = \left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right) = \frac{(-2)^3}{3^3} = \frac{-8}{27}$

In general, if $\frac{x}{y}$ is a rational number and a is a positive integer, then

$$\boxed{\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}}$$

Ex.37 Evaluate $\left(-\frac{4}{5}\right)^3$.

Sol. $\left(-\frac{4}{5}\right)^3 = \left(-\frac{4}{5}\right)\times\left(-\frac{4}{5}\right)\times\left(-\frac{4}{5}\right) = \frac{(-4)^3}{5^3}$

$= \frac{-64}{125}$

Ex.38 Express $\frac{27}{64}$ and $\frac{-8}{27}$ as the powers of rational numbers.

Sol. $\frac{27}{64} = \frac{3\times3\times3}{4\times4\times4} = \frac{3^3}{4^3} = \left(\frac{3}{4}\right)^3$

and $\frac{-8}{27} = \frac{(-2)\times(-2)\times(-2)}{3\times3\times3} = \frac{(-2)^3}{3^3} = \left(\frac{-2}{3}\right)^3$

Reciprocals with Positive Integral Exponents:

The reciprocal of 2 is $\frac{1}{2}$, reciprocal of 2^3 is $\frac{1}{2^3}$.

Reciprocal of $\left(\frac{2}{3}\right)^4 = \frac{1}{\left(\frac{2}{3}\right)^4} = \frac{1}{\frac{2^4}{3^4}} = \frac{3^4}{2^4} = \left(\frac{3}{2}\right)^4$

Reciprocal of $\left(\frac{-4}{5}\right)^4 = \left(\frac{-5}{4}\right)^4$ and

Reciprocal of $\left(\frac{1}{3}\right)^5 = \left(\frac{3}{1}\right)^5 = 3^5$

Reciprocals with Negative Integral Exponents

Reciprocal of $2 = \frac{1}{2} = \frac{1}{2^1}$.

Therefore, the reciprocal of 2 is 2^{-1}. The reciprocal of $3^2 = \frac{1}{3^2} = 3^{-2}$.

Reciprocal of $\left(\frac{4}{5}\right)^2 = \left(\frac{4}{5}\right)^{-2}$

Reciprocal of $\left(\frac{-2}{3}\right)^{3} = \left(\frac{-2}{3}\right)^{-3}$, etc.

In general, if x is any rational number other than zero and a is any positive integer, then:

$$\boxed{x^{-a} = \frac{1}{x^{a}}}$$

Ex.39 Simplify $\left(\frac{2}{3}\right)^{-3} \div \left(\frac{4}{3}\right)^{-2}$.

Sol. $\left(\frac{2}{3}\right)^{-3} \div \left(\frac{4}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{3} \div \left(\frac{3}{4}\right)^{2}$

$= \frac{3\times3\times3}{2\times2\times2} \div \frac{3\times3}{4\times4}$

$= \frac{27}{8} \div \frac{9}{16} = \frac{27}{8} \times \frac{16}{9} = 6$

◈ Laws of Exponents :

1. Consider the following.

(i) $3^3 \times 3^4 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 3^7 = 3^{3+4}$

(ii) $\left(\frac{5}{2}\right)^{2} \times \left(\frac{5}{2}\right)^{3} = \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}$

$= \left(\frac{5}{2}\right)^{5} = \left(\frac{5}{2}\right)^{2+3}$

$\therefore$ $\boxed{x^a \times x^b = x^{a+b}}$

2. (i) $2^5 \div 2^2 = \frac{2\times2\times2\times2\times2}{2\times2} = 2 \times 2 \times 2$
$= 2^3 = 2^{5-2}$

(ii) $\left(\frac{2}{3}\right)^{6} \div \left(\frac{2}{3}\right)^{2} = \frac{\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}}{\frac{2}{3}\times\frac{2}{3}}$

$= \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \left(\frac{2}{3}\right)^{4} = \left(\frac{2}{3}\right)^{6-2}$

$\therefore$ $\boxed{x^a \div x^b = x^{a-b}}$

3. (i) $(2^3)^2 = (2 \times 2 \times 2)^2$
$= (2 \times 2 \times 2) \times (2 \times 2 \times 2)$
$= 2^6 = 2^3 \times 2^3$

(ii) $\left\{\left(\frac{2}{3}\right)^{3}\right\}^{2} = \left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right)^{2}$

$= \left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right) \times \left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right) = \left(\frac{2}{3}\right)^{6} = \left(\frac{2}{3}\right)^{3\times2}$

$\therefore$ $\boxed{(x^a)^b = x^{ab}}$

4. (i) $2^4 \times 3^4 = (2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3)$
$= (2\times3)\times(2\times3)\times(2\times3)\times(2\times3)$
$= (2 \times 3)^4$

(ii) $\left(\frac{3}{5}\right)^{4} \times \left(\frac{1}{2}\right)^{4}$

$= \left(\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\right) \times \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$

$= \left(\frac{3}{5}\times\frac{1}{2}\right) \times \left(\frac{3}{5}\times\frac{1}{2}\right) \times \left(\frac{3}{5}\times\frac{1}{2}\right) \times \left(\frac{3}{5}\times\frac{1}{2}\right)$

$= \left(\frac{3}{5}\times\frac{1}{2}\right)^{4}$

$\therefore$ $\boxed{x^a \times y^a = (x \times y)^a}$

5. (i) $2^4 \div 3^4 = \frac{2\times2\times2\times2}{3\times3\times3\times3} = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \left(\frac{2}{3}\right)^{4}$

(ii) $\left(\frac{3}{5}\right)^{4} \div \left(\frac{1}{2}\right)^{4} = \frac{\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}}{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}}$

$= \left(\frac{\frac{3}{5}}{\frac{1}{2}}\right) \times \left(\frac{\frac{3}{5}}{\frac{1}{2}}\right) \times \left(\frac{\frac{3}{5}}{\frac{1}{2}}\right) \times \left(\frac{\frac{3}{5}}{\frac{1}{2}}\right) = \left(\frac{\frac{3}{5}}{\frac{1}{2}}\right)^{4}$

$\therefore$ $\boxed{x^a \div y^a = \left(\frac{x}{y}\right)^{a}}$

Ex.40 Simplify $\left[\left(\frac{2}{3}\right)^{2}\right]^{3} \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$

Sol. $\left[\left(\frac{2}{3}\right)^{2}\right]^{3} \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$

$= \left(\frac{2}{3}\right)^{6} \times 3^4 \times \frac{1}{3} \times \frac{1}{6}$

$= \frac{2^6}{3^6} \times 3^4 \times \frac{1}{3} \times \frac{1}{6}$

$= 2^6 \times 3^{-6} \times 3^4 \times 3^{-1} \times 6^{-1}$

$= 2^6 \times 3^{-6} \times 3^4 \times 3^{-1} \times (2 \times 3)^{-1}$

$= 2^6 \times 3^{-6} \times 3^4 \times 3^{-1} \times 2^{-1} \times 3^{-1}$

$= 2^{6+(-1)} \times 3^{-1+4+(-1)+(-6)}$

$= 2^{6-1} \times 3^{-1+4-1-6}$

$= 2^5 \times 3^{-4} = \frac{2^5}{3^4} = \frac{32}{81}$

Ex.41 Find x so that $\left(\frac{2}{3}\right)^{-5} \times \left(\frac{2}{3}\right)^{-11} \times \left(\frac{2}{3}\right)^{8x}$

Sol. $\left(\frac{2}{3}\right)^{-5} \times \left(\frac{2}{3}\right)^{-11} \times \left(\frac{2}{3}\right)^{8x}$

$\Rightarrow \left(\frac{2}{3}\right)^{(-5)+(-11)} = \left(\frac{2}{3}\right)^{8x}$

$\Rightarrow \left(\frac{2}{3}\right)^{-5-11} = \left(\frac{2}{3}\right)^{8x} \Rightarrow \left(\frac{2}{3}\right)^{-16} = \left(\frac{2}{3}\right)^{8x}$

$\Rightarrow 8x = -16 \qquad \therefore \quad x = -2$

So, If x is any rational number different from zero and a, b are any integers, then,

Law I: $\boxed{x^a \times x^b = x^{a+b}}$

Law II: $\boxed{x^a \div x^b = x^{a-b}}$

Law III: $\boxed{(x^a)^b = x^{ab}}$

Law IV: $\boxed{x^a \times y^a = (x \times y)^a}$

(where y is also a non-zero rational number)

Law V: $\boxed{x^a \div y^a = \left(\frac{x}{y}\right)^a}$

(where y is also a non-zero rational number)

Ex.42 Evaluate $\left(\frac{2}{3}\right)^{4} \div \left(\frac{2}{3}\right)^{4}$

Sol. $\left(\frac{2}{3}\right)^{4} \div \left(\frac{2}{3}\right)^{4} = \left(\frac{2}{3}\right)^{4-4} \times \left(\frac{2}{3}\right)^{0}$

but $\left(\frac{2}{3}\right)^{4} \div \left(\frac{2}{3}\right)^{4} = \frac{\left(\frac{2}{3}\right)^{4}}{\left(\frac{2}{3}\right)^{4}} = 1$

$\therefore$ the expression $= \left(\frac{2}{3}\right)^{0} = 1$

EXERCISE # 1

By Using property find value of **(Q.1 to Q.3)**

Q.1 $-\frac{2}{3} \times \frac{3}{5} + \frac{5}{2} - \frac{3}{5} \times \frac{1}{6}$

Q.2 $\frac{2}{5} \times \left(-\frac{3}{7}\right) - \frac{1}{6} \times \frac{3}{2} + \frac{1}{14} \times \frac{2}{5}$

Q.3 $\frac{5}{7} + \frac{1}{3} + \frac{8}{9} + \frac{1}{14}$

Q.4 Subtract the first rational number from the second in each of the following:

(i) $\frac{3}{8}, \frac{5}{8}$ (ii) $\frac{-7}{9}, \frac{4}{9}$

(iii) $\frac{-2}{11}, \frac{-9}{11}$ (iv) $\frac{11}{13}, \frac{-4}{13}$

(v) $\frac{1}{4}, \frac{-3}{8}$ (vi) $\frac{-2}{3}, \frac{5}{6}$

(vii) $\frac{-6}{7}, \frac{-13}{14}$ (viii) $\frac{-8}{33}, \frac{-7}{22}$

Q.5 The sum of the two numbers is $\frac{5}{9}$. If one of the numbers is $\frac{1}{3}$, find the other.

Q.6 The sum of two numbers is $\frac{-1}{3}$. If one of the numbers is $\frac{-12}{3}$, find the other.

Q.7 The sum of two numbers is $\frac{-4}{3}$. If one of the numbers is –5, find the other.

Q.8 The sum of two rational numbers is–8. If one of the numbers is $\frac{-15}{7}$, find the other.

Q.9 What should be added to $\frac{-7}{8}$ so as to get $\frac{5}{9}$?

Q.10 What number should be added to $\frac{-5}{11}$ so as to get $\frac{26}{33}$?

Q.11 What number should be added to $\frac{-5}{7}$ to get $\frac{-2}{3}$?

Q.12 What number should be subtracted from $\frac{-5}{3}$ to get $\frac{5}{6}$?

Q.13 What number should be subtracted from $\frac{3}{7}$ to get $\frac{5}{4}$?

Q.14 What should be added to $\left(\frac{2}{3} + \frac{3}{5}\right)$ to get $\frac{-2}{15}$?

Q.15 What should be added to $\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5}\right)$ to get 3 ?

Q.16 What should be subtracted from $\left(\frac{3}{4} - \frac{2}{3}\right)$ to get $\frac{-1}{6}$?

Q.17 Simply each of the following and write as a rational number of the from $\frac{p}{q}$:

(i) $\frac{3}{4} + \frac{5}{6} + \frac{-7}{8}$ (ii) $\frac{2}{3} + \frac{-5}{6} + \frac{-7}{9}$

(iii) $\frac{-11}{2} + \frac{7}{6} + \frac{-5}{8}$ (iv) $\frac{-4}{5} + \frac{-7}{10} + \frac{-8}{15}$

(v) $\frac{-9}{10} + \frac{22}{15} + \frac{13}{-20}$ (vi) $\frac{5}{3} + \frac{3}{-2} + \frac{-7}{3} + 3$

Q.18 Express each of the following as a rational number of the form $\frac{p}{q}$:

(i) $\frac{-8}{3} + \frac{-1}{4} + \frac{-11}{6} + \frac{3}{8} - 3$

(ii) $\frac{6}{7} + 1 + \frac{-7}{9} + \frac{19}{21} + \frac{-12}{7}$

(iii) $\frac{15}{2} + \frac{9}{8} + \frac{-11}{3} + 6 + \frac{-7}{6}$

(iv) $\frac{-7}{4} + 0 + \frac{-9}{5} + \frac{19}{10} + \frac{11}{14}$

(v) $\frac{-7}{4} + \frac{5}{3} + \frac{-1}{2} + \frac{-5}{6} + 2$

Q.19 Simplify:

(i) $\frac{-3}{2}+\frac{5}{4}-\frac{7}{4}$ (ii) $\frac{5}{3}-\frac{7}{6}+\frac{-2}{3}$

(iii) $\frac{5}{4}-\frac{7}{6}-\frac{-2}{3}$ (iv) $\frac{-2}{5}-\frac{-3}{10}-\frac{-4}{7}$

(v) $\frac{5}{6}+\frac{-2}{5}-\frac{-2}{15}$ (vi) $\frac{3}{8}-\frac{-2}{9}+\frac{-5}{36}$

Q.20 Multiply:

(i) $\frac{7}{11}$ by $\frac{5}{4}$ (ii) $\frac{5}{7}$ by $\frac{-3}{4}$

(iii) $\frac{-2}{9}$ by $\frac{5}{11}$ (iv) $\frac{-3}{17}$ by $\frac{-5}{-4}$

(v) $\frac{9}{-7}$ by $\frac{36}{-11}$ (vi) $\frac{-11}{13}$ by $\frac{-21}{7}$

(vii) $-\frac{3}{5}$ by $-\frac{4}{7}$ (viii) $-\frac{15}{11}$ by 7

Q.21 Multiply:

(i) $\frac{-5}{17}$ by $\frac{51}{-60}$ (ii) $\frac{-6}{11}$ by $\frac{-55}{36}$

(iii) $\frac{-8}{25}$ by $\frac{-5}{16}$ (iv) $\frac{6}{7}$ by $\frac{-49}{36}$

(v) $\frac{8}{-9}$ by $\frac{-7}{-16}$ (vi) $\frac{-8}{9}$ by $\frac{3}{64}$

Q.22 Simplify each of the following and express the result as a rational number in standard form:

(i) $\frac{-16}{21}\times\frac{14}{5}$ (ii) $\frac{7}{6}\times\frac{-3}{28}$

(iii) $\frac{-19}{36}\times16$ (iv) $\frac{-13}{9}\times\frac{27}{-26}$

(v) $\frac{-9}{16}\times\frac{-64}{-27}$ (vi) $\frac{-50}{7}\times\frac{14}{3}$

(vii) $\frac{-11}{9}\times\frac{-81}{-88}$ (viii) $\frac{-5}{9}\times\frac{72}{-25}$

Q.23 Simplify:

(i) $\left(\frac{25}{8}\times\frac{2}{5}\right)-\left(\frac{3}{5}\times\frac{-10}{9}\right)$

(ii) $\left(\frac{1}{2}\times\frac{1}{4}\right)+\left(\frac{1}{2}\times6\right)$

(iii) $\left(-5\times\frac{2}{15}\right)-\left(-6\times\frac{2}{9}\right)$

(iv) $\left(\frac{-9}{4}\times\frac{5}{3}\right)+\left(\frac{13}{2}\times\frac{5}{6}\right)$

(v) $\left(\frac{-4}{3}\times\frac{12}{-5}\right)+\left(\frac{3}{7}\times\frac{21}{15}\right)$

(vi) $\left(\frac{13}{5}\times\frac{8}{3}\right)-\left(\frac{-5}{2}\times\frac{11}{3}\right)$

(vii) $\left(\frac{13}{7}\times\frac{11}{26}\right)-\left(\frac{-4}{3}\times\frac{5}{6}\right)$

(viii) $\left(\frac{8}{5}\times\frac{-3}{2}\right)+\left(\frac{-3}{10}\times\frac{11}{16}\right)$

ANSWER KEY

EXERCISE # 1

1. 2 **2.** $-\frac{11}{28}$ **3.** $\frac{253}{126}$

4. (i) $\frac{1}{4}$ (ii) $\frac{11}{9}$ (iii) $-\frac{7}{11}$ (iv) $-\frac{15}{13}$ (v) $-\frac{5}{8}$ (vi) $\frac{3}{2}$ (vii) $-\frac{1}{14}$ (viii) $-\frac{5}{66}$ **5.** $\frac{2}{9}$ **6.** $\frac{11}{3}$ **7.** $\frac{11}{3}$

8. $-\frac{41}{7}$ **9.** $\frac{103}{72}$ **10.** $\frac{41}{32}$ **11.** $\frac{1}{21}$ **12.** $\frac{-5}{2}$ **13.** $\frac{-23}{28}$

14. $\frac{-7}{5}$ **15.** $\frac{59}{30}$ **16.** $\frac{1}{4}$

EXERCISE # 2

Q.1 Give examples of
(a) The rational number that does not have a reciprocal.
(b) The rational numbers that are equal to their reciprocals.
(c) The rational number that is equal to its negative.

Q.2 Fill in the blanks.
(a) Zero has ______ reciprocal.
(b) The numbers______ and _____ are their own reciprocals.
(c) The reciprocal of – 5 is _______.
(d) Reciprocal of $\frac{1}{x}$, where $x \neq 0$ is______.
(e) The product of two rational numbers is always a______.
(f) The reciprocal of a positive rational number is ________.

Q.3 Represent these numbers on the number line.
(i) $\frac{7}{4}$ (ii) $\frac{-5}{6}$

Q.4 Represent $\frac{-2}{11}, \frac{-5}{11}, \frac{-9}{11}$ on the number line.

Q.5 Write five rational numbers which are smaller than 2.

Q.6 Find ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.

Q.7 Find five rational numbers between.
(i) $\frac{2}{3}$ and $\frac{4}{5}$ (ii) $\frac{-3}{2}$ and $\frac{5}{3}$
(iii) $\frac{1}{4}$ and $\frac{1}{2}$

Q.8 Write five rational numbers greater than –2.

Q.9 Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$.

Q.10 What expression to be added to $(5x^2 - 7x + 2)$ to produce $(7x^2 - 1)$.
(A) $2x^2 + 7x + 3$ (B) $2x^2 + 7x - 3$
(C) $12x^2 - 7x + 1$ (D) $2x^2 - 3$

Q.11 What must be added to $1 - x + x^2 - 2x^3$ to obtain x^3 ?
(A) $x^3 - x^2 + x - 1$
(B) $-1 + x + x^2 - 3x^3$
(C) $3x^3 - x^2 + x - 1$
(D) None of these

Q.12 What must be added to the sum of $4x^2 + 3x - 7$ and $3x^2 + 6x + 5$ to get : 1 ?
(A) $7x^2 + 9x - 3$
(B) $3 - 9x - 7x^2$
(C) $7x^2 + 9x - 2$
(D) None of these

Q.13 By what number should $\left(\frac{1}{-15}\right)$ be divided so that the quotient equal to $\left(\frac{1}{-5}\right)$.

Q.14 Simplify each of the following :
(i) $\left[\left\{\left(\frac{-1}{5}\right)^{-2}\right\}^{2}\right]^{-1}$
(ii) $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

Q.15 Simplify :
(i) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5}$ (ii) $\left(\frac{-2}{3}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3}$
(iii) $\left(\frac{3}{4}\right)^{-4} \div \left(\frac{3}{2}\right)^{-3}$ (iv) $\left(\frac{3}{7}\right)^{-2} \times \left(\frac{7}{6}\right)^{-3}$

Q.16 Evaluate : $\frac{8^{-1} \times 5^{3}}{2^{-4}}$

Q.17 Simplify:
(i) $\frac{25 \times a^{-4}}{5^{-3} \times 10 \times a^{-8}}$ (ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Q.18 By what number should $(-4)^{-2}$ be multiplied so that the product may be equal to 10^{-2} ?

Q.19 By what number should $(-12)^{-1}$ be divided so that the quotient may be $\left(\frac{2}{3}\right)^{-1}$?

Q.20 By what number should $\left(\frac{-3}{2}\right)^{-3}$ be divided so that the quotient may be $\left(\frac{4}{27}\right)^{-2}$?

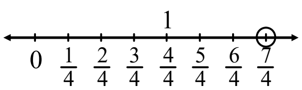
Q.21 Find m so that $\left(\frac{2}{9}\right)^{3} \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2m-1}$

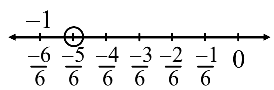
ANSWER KEY

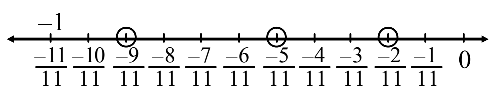
EXERCISE # 2

1. (a) 0 ; (b) 1 and (– 1) ; (c) 0

2. (a) No ; (b) 1, – 1; (c) $\frac{-1}{5}$; (d) x; (e) Rational Number; (f) positive

3. (i) 

; (ii) 

4. 

5. Some of these are 1, $\frac{1}{2}$, 0, –1, $\frac{-1}{2}$

6. $\frac{-7}{20}, \frac{-6}{20}, \frac{-5}{20}, \frac{-4}{20}, \frac{-3}{20}, \frac{-2}{20}, \frac{-1}{20}, 0, ..., \frac{1}{20}, \frac{2}{20}$ (These can be many more such rational numbers)

7. (i) $\frac{41}{60}, \frac{42}{60}, \frac{43}{60}, \frac{44}{60}, \frac{45}{60}$; (ii) $\frac{-8}{6}, \frac{-7}{6}, 0, \frac{1}{6}, \frac{2}{6}$; (iii) $\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}$
(There can be many more such rational numbers)

8. $\frac{-3}{2}, -1, \frac{-1}{2}, 0, \frac{1}{2}$ (There can be many more such rational numbers)

9. $\frac{97}{160}, \frac{98}{160}, \frac{99}{160}, \frac{100}{160}, \frac{101}{160}, \frac{102}{160}, \frac{103}{160}, \frac{104}{160}, \frac{105}{160}, \frac{106}{160}$
(There can be many more such rational numbers)

13. $\frac{1}{3}$

14. (i) $\frac{1}{625}$; (ii) $\frac{1}{16}$

15. (i) $\frac{64}{25}$; (ii) $\frac{1125}{256}$; (iii) $\frac{32}{3}$; $\frac{24}{7}$

16. 250

17. (i) $\frac{625}{2}a^4$; (ii) 5^5

18. $\frac{4}{25}$

19. $\frac{-1}{18}$

20. $-2 \times \left(\frac{4}{27}\right)^{3}$

21. m = – 1