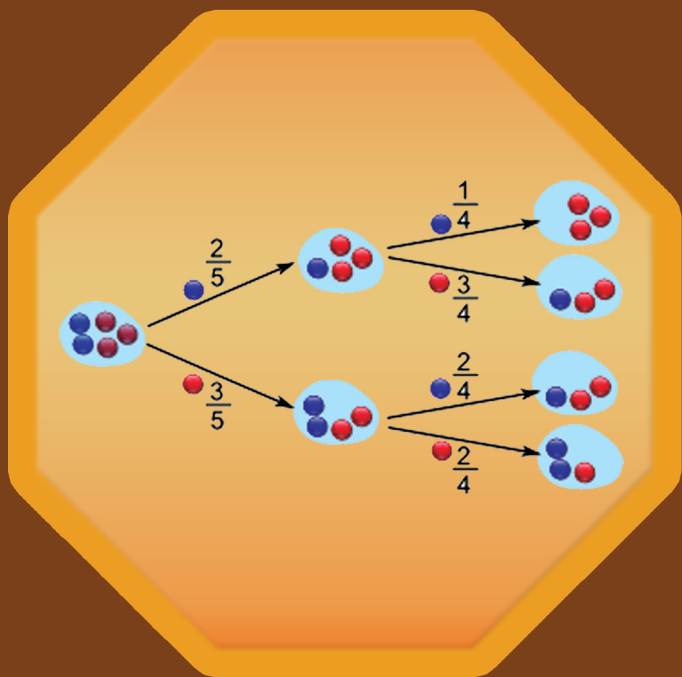
Mathematics

Grade 12

Teachers' Guide

(To be Implemented from 2017)



Department of Mathematics
Faculty of Science & Technology
National Institute of Education
Sri Lanka

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Message from the Director General …

With the primary objective of realizing the National Educational Goals recommended by the National Education Commission, the then prevalent content based curriculum was modernized, and the first phase of the new competency based curriculum was introduced to the eight year curriculum cycle of the primary and secondary education in Sri Lanka in the year 2007

The second phase of the curriculum cycle thus initiated was introduced to the education system in the year 2015 as a result of a curriculum rationalization process based on research findings and various proposals made by stake holders.

Within this rationalization process the concepts of vertical and horizontal integration have been employed in order to build up competencies of students, from foundation level to higher levels, and to avoid repetition of subject content in various subjects respectively and furthermore, to develop a curriculum that is implementable and student friendly.

The new Teachers' Guides have been introduced with the aim of providing the teachers with necessary guidance for planning lessons, engaging students effectively in the learning teaching process, and to make Teachers' Guides will help teachers to be more effective within the classroom. Further, the present Teachers' Guides have given the necessary freedom for the teachers to select quality inputs and activities in order to improve student competencies. Since the Teachers' Guides do not place greater emphasis on the subject content prescribed for the relevant grades, it is very much necessary to use these guides along with the text books compiled by the Educational Publications Department if, Guides are to be made more effective.

The primary objective of this rationalized new curriculum, the new Teachers' Guides, and the new prescribed texts is to transform the student population into a human resource replete with the skills and competencies required for the world of work, through embarking upon a pattern of education which is more student centered and activity based.

I wish to make use of this opportunity to thank and express my appreciation to the members of the Council and the Academic Affairs Board of the NIE the resource persons who contributed to the compiling of these Teachers' Guides and other parties for their dedication in this matter.

Dr. (Mrs.) Jayanthi Gunasekara
Director General
National Institute of Education

Message from the Director

Education from the past has been constantly changing and forging forward. In recent years, these changes have become quite rapid. The Past two decades have witnessed a high surge in teaching methodologies as well as in the use of technological tools and in the field of knowledge creation.

Accordingly, the National Institute of Education is in the process of taking appropriate and timely steps with regard to the education reforms of 2015.

It is with immense pleasure that this Teachers' Guide where the new curriculum has been planned based on a thorough study of the changes that have taken place in the global context adopted in terms of local needs based on a student-centered learning-teaching approach, is presented to you teachers who serve as the pilots of the schools system.

An instructional manual of this nature is provided to you with the confidence that, you will be able to make a greater contribution using this.

There is no doubt whatsoever that this Teachers' Guide will provide substantial support in the classroom teaching-learning process at the same time. Furthermore the teacher will have a better control of the classroom with a constructive approach in selecting modern resource materials and following the guide lines given in this book.

I trust that through the careful study of this Teachers Guide provided to you, you will act with commitment in the generation of a greatly creative set of students capable of helping Sri Lanka move socially as well as economically forward.

This Teachers' Guide is the outcome of the expertise and unflagging commitment of a team of subject teachers and academics in the field Education.

While expressing my sincere appreciation for this task performed for the development of the education system, my heartfelt thanks go to all of you who contributed your knowledge and skills in making this document such a landmark in the field.

Mr.K.R.Pathmasiri
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Common National Goals

The national system of education should assist individuals and groups to achieve major national goals that are relevant to the individual and society.

Over the years major education reports and documents in Sri Lanka have set goals that sought to meet individual and national needs. In the light of the weaknesses manifest in contemporary educational structures and processes, the National Education Commission has identified the following set of goals to be achieved through education within the conceptual framework of sustainable human development.

I Nation building and the establishment of a Sri Lankan identity through the promotion of national cohesion, national integrity, national unity, harmony and peace, and recognizing cultural diversity in Sri Lanka's plural society within a concept of respect for human dignity.

II Recognizing and conserving the best elements of the nation's heritage while responding to the challenges of a changing world.

III Creating and supporting an environment imbued with the norms of social justice and a democratic way of life that promotes respect for human rights, awareness of duties and obligations, and a deep and abiding concern for one another.

IV Promoting the mental and physical well-being of individuals and a sustainable life style based on respect for human values.

V Developing creativity, initiative, critical thinking, responsibility, accountability and other positive elements of a well-integrated and balance personality.

VI Human resource development by educating for productive work that enhances the quality of life of the individual and the nation and contributes to the economic development of Sri Lanka.

VII Preparing individuals to adapt to and manage change, and to develop capacity to cope with complex and unforeseen situations in a rapidly changing world.

VIII Fostering attitudes and skills that will contribute to securing an honourable place in the international community, based on justice, equality and mutual respect.

National Education Commision Report (2003) - December

Basic Competencies

The following Basic Competencies developed through education will contribute to achieving the above National Goals.

(i) Competencies in Communication

Competencies in Communication are based on four subjects: Literacy, Numeracy, Graphics and IT proficiency.

Literacy : Listen attentively, speck clearly, read for meaning, write accurately and lucidly and communicate ideas effectively.

Numeracy : Use numbers for things, space and time, count, calculate and measure systematically.

Graphics : Make sense of line and form, express and record details, instructions and ideas with line form and color.

IT proficiency : Computeracy and the use of information and communication technologies (ICT) in learning, in the work environment and in personal life.

(ii) Competencies relating to Personality Development

- General skills such as creativity, divergent thinking, initiative, decision making, problem solving, critical and analytical thinking, team work, inter-personal relations, discovering and exploring;
- Values such as integrity, tolerance and respect for human dignity;
- Emotional intelligence.

(iii) Competencies relating to the Environment

These competencies relate to the environment : social, biological and physical.

Social Environment :

Awareness of the national heritage, sensitivity and skills linked to being members of a plural society, concern for distributive justice, social relationships, personal conduct, general and legal conventions, rights, responsibilities, duties and obligations.

Biological Environment :

Awareness, sensitivity and skills linked to the living world, people and the ecosystem, the trees, forests, seas, water, air and life-plant, animal and human life.

Physical Environment :

Awareness, sensitivity and skills linked to space, energy, fuels, matter, materials and their links with human living, food, clothing, shelter, health, comfort, respiration, sleep, relaxation, rest, wastes and excretion.

Included here are skills in using tools and technologies for learning, working and living.

(iv) Competencies relating to Preparation for the World of Work.

Employment related skills to maximize their potential and to enhance their capacityto contribute to economic development, to discover their vocational interests ad aptitudes, to choose a job that suits their abilities, and to engage in a rewarding and sustainable livelihood.

(v) Competencies relating to Religion and Ethics

Assimilating and internalizing values, so that individuals may function in a manner consistent with the ethical, moral and religious modes of conduct in everyday living, selecting that which is most appropriate.

(vi) Competencies in Play and the Use of Leisure

Pleasure, joy, emotions and such human experiences as expressed through aesthetics, literature, play, sports and athletics, leisure pursuits and other creative modes of living.

(vii) Competencies relating to ‘ learning to learn’

Empowering individuals to learn independently and to be sensitive and successful in responding to and managing change through a transformative process, in a rapidly changing, complex and interdependent world.

Guidlines to use the Teacher's Guide

In the G.C.E (A/L) classes new education reforms introduced from the year 2017 in accordance with the new education reforms implemented in the interim classes in the year 2015. According to the reforms, Teacher's Guide for mathematics for grade 12 has been prepared.

The grade 12 Teacher's Guide has been organized under the titles competencies competency levels, content, learning out comes and number of periods. The proposed lesson sequence is given for the leaning teaching process. Furfur it is expected that this teacher's Guide will help to the teachers to prepare their lessons and lessons plans for the purpose of class room learning teaching process. Also it is expected that this Guide will help the teachers to take the responsibility to explains the subject matters more confidently. This teachers' Guide is divided into three parts each for a term.

In preparing lesson sequence, attention given to the sequential order of concepts, students ability of leaning and teachers ability of teaching. Therefore sequential order of subject matters in the syllabus and in the teachers Guide may differ. It is advice the teachers to follow the sequence as in the teachers' Guide.

To attain the learning outcomes mentioned in the teachers' Guide, teachers should consider the subject matters with extra attention. Further it is expecting to refer extra curricular materials and reference materials to improve their quality of teaching. Teachers should be able to understand the students, those who are entering grade 12 classes to learn mathematics as a subject. Since G.C.E (O/L) is designed for the general education, students will joined in the grade 12 stream will face some difficulties to learn mathematics. To over come this short coming an additional topics on basic algebra should be taught.For this purpose teachers can use their self prepared materials or Beginners course on mathematics book prepared by NIE.

Total number of periods to teach this mathematics syllabus is 600. Teachers can be flexible to change the number of periods according to their necessity. Teachers can use school based to assessment to assess the students.

The teacher has the freedom to make necessary amendments to the speciman lession plan given in the new teacher's manual which includes many new features, depanding on the classroom and the abilities of the students.

We would be greateful if you would send any amendments you make or any new lession you prepare to the Director, Department of Mathematiocs< National Institute of Education. The mathematics department is prepaired to incoporate any new suggestions that would advance mathematics education in the upper secondary school system.

S.Rajendran
Project Leader (Grade 12 - 13 Mathematics)
Dept. of Mathematics,
National Institute of Education.

Content

First Term

Mathematics - I

Competency 1 : **Analyses real number system**

Competency level 1.1 : Classifies the real number system

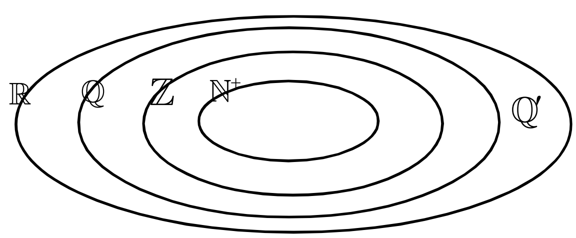
Number of periods : 04

Learning out comes :
1. Writes correct notations for sets of numbers
2. Represents real numbers geometrically

Guidelines to learning - teaching process :

1.
- Discuss the usage of number in the ancient time and evolution of number system.
- Recall the students knowledges, rational numbers irrational numbers. Introduce the following set notations for numbers.
- Set of positive integers $\mathbb{Z}^+ = \{1, 2, 3, 4........\}$ (natural numbers)
- Set of integers $\mathbb{Z} = \{......3, -2, -1, 0, 1, 2, 3.......\}$
- Set of rational numbers $Q = \left\{ \frac{p}{q}, q \neq 0,\ p, q \in \mathbb{Z} \right\}$
- Set of irrational numbers Q' or Q^c
- Set of real numbers R.

Stress that the above sets of numbers are subset of the set of real numbers and guide them to mark in a Venn diagram.



2. Guide students to mark real numbers in real number line.

Competency level 1.2 : Uses surds and decimals to represents real numbers

Number of periods : 04

Learning out comes :
1. Classifies decimal numbers
2. Classifies real numbers
3. Rationalities the denominator of expressions with surds
4. Uses arithmetical operation on surds

Guidelines to learning - teaching process :

1. Decimals
   - Finite Decimals
   - Infinite Decimals
     - Recurence Decimals
     - Non recurence Decimals

2. Real numbers
   - Rational Numbers
   - Irrational numbers

3. Introduce surds as a solutions of equations and rationalse the denominator.
   - Manipulates arithmetic operations on surds.
   - Introduce irrational numbers as a solution of an equation
   - Guide to rationalese the denominator of surds

   Eg: $\frac{1}{\sqrt{2-1}}$, $\frac{1}{\sqrt{3}+\sqrt{2}}$

4. Guide students to simplify problems involving surds.

Competency level 1.3 : Uses exponents (induces) and logarithms to communicate real numbers

Number of periods : 06

Learning outcomes :

1. Defines indicates and power.
2. Classifies positive integer exponents, negative integer, exponents, zero exponents and fractional exponents.
3. States laws of induces.
4. States laws of logarithams.
5. Applies laws of indices and laws of logarithams to simplify problems.

Guidelines to learning - teaching process :

1. Introduces indices and powers.
2. ***n^{th}* root of a real number**
   - Let a and b be real numbers and Len be a integer. if $a=b^n$ then b is an n^{th} root of a.

- It is a square root when $n=2$ and it is a cube root when $n=3$.
- There are two roots if and n is even. These roots are equal in magnitude and opposite in sign.

  Principle n^{th} root

  Let a be a real number that has at least one n^{th} root . The principle n^{th} root of a is the n^{th} root that has the same sign as a and it is denoted by $a^{\frac{1}{n}}$ *or* $\sqrt[n]{a}$

  When $n=2$ we omit the index n and write $\sqrt{a}$

- Let a and b are real numbers such that the indicated roots exist as real numbers, and Let $m, n \in \mathbb{Z}^+$ then
  - $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
  - $\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$
  - $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}} \quad for\ \ b \neq 0$
  - $\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$
  - $\left(\sqrt[n]{a}\right)^n = a$
  - $for\ \ n\ \ is\ \ even\ \ \left(\sqrt[n]{a^n} = |a|\right)$
  - $for\ \ n\ \ is\ \ odd\ \ \left(\sqrt[n]{a^n} = a\right)$

3. Remind the following as laws of indices, where $a, b \in \mathbb{R}$ and $m, n \in \mathbb{Z}$

- $a^m \times a^n = a^{m-n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $a^{-n} = \dfrac{1}{a^n} = \left(\dfrac{1}{a}\right)^n \quad for\ \ a \neq 0$
- $a^o = 1\ \ for\ \ a \neq 0$

- $\left(a^m\right)^n = a^{mn}$
- $\left(ab\right)^m = a^m \times b^m$
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \ for \ b \neq 0$

4. If $a^x = y, a > 0, y > 0$ then x is called the logarithm of the number $y(> 0)$ to the base a and is writen as $\log_a y = x$.

   ie : $a^x = y \Leftrightarrow \log_a y = x$, where $a, y > 0$ and $a \neq 1$ state the laws of logarithms

   Let $m, n, a > 0$ and $a \neq 1$

   $\log_a mn = \log_a m + \log_a n$

   $\log_a \frac{m}{n} = \log_a m - \log_a n$

   $\log a^m = m \log a$

   Change of base

   $\log_a b = \frac{1}{\log_b a}$

   $\log_a \frac{b}{c} = \frac{\log_c b}{\log_a c}$

5. - Explains the above results using examples.
   - Solves problems involving indices.

Competency 2 : **Manipulates Algebra of Sets**

Competency level 2.1 : Applies basicss concepts of sets to solve problems

Number of periods : 06

Learning out comes :

1. Explains set notations
2. Explains and writes the notations of universal set and null set
3. Explains finite sets and infinite sets
4. Defines, Cardinality of a set and writes its notation
5. Defines subsets, proper sub sets equality of two sets and power set

Guidelines to learning - teaching process :

1. Define the notation for sets
   - Introduce set and set notation.
   - Introduce elements of a set.

   Consider the following collections of objects:

   1. The collection of persons living in Colombo.
   2. The collection of beautiful flowers in the garden.
   3. The collection of honest boys in the school.
   4. The collection of vehicles in the country.
   5. The collection of best TV shows.
   6. The collection of vowels in the English alphabet.

   From the above (1), (4) and (6) are sets while (2),(3) and (5) are not sets becausebeautiful,honest and best are not well defined.

   In (1) each person living in Colombo is an element (object) of the set. In (4), each vehicle in the country is an element of the set. In (6) *a, e, i, o* and *u* are elements of the set.

   - A *set* is an unordered collection of distinct well defined objects. The objects that make up a set are called *elements* or *members* of the set.
   - Sets are often denoted by *capital letters* and elements are usually denoted by *lower caseletters*.
   - Describing Sets:

   There are several common ways to describe sets.

   - Explicit enumeration - list all elements in the set. Enclose the listing in curly braces, and separate the elements by commas.

For example, we write A = { *a, e, i, o, u*} to indicate that the set *A* contains the elements *a, e, i, o, u*, and no other elements. The notation { *a, e, i, o, u*} is read as "The set with the elements *a, e, i, o* and *u*.

- We write "*a*" is a member of "A" as : $a \in A$.
- We write "2" is not a member of "A" as : $2 \notin A$
- The order in which the elements are presented in a set is not important.

  For example, A = { *a, e, i, o, u*} and B = {*i, u, a, o, e*} both define the same set.
- In a set same member does not appear more than once.

  For example, C = { *a, e, i, o, u, i*} is incorrect since the element "*i*" repeats.
- Implicit enumeration - For larger sets, we cannot list all elements, so we indicate the pattern of the elements, and use an *ellipsis* (...) to say "and so on".

  For example A = { 1, 2, 3, 4, ..., 20} is the set of positive integers from 1 to 20.

  $\mathbb{Z} = \{1, 2, 3, \ldots\}$ is the set of all positive integers.
- Set Builder Notation - Describe a set by stating a property that all elements in the set have. For example A = {n|n is a positive even integer less tha 10} The curly braces tell us this is a set. The letter is a variable that stands in for any object that meets the criteria described after the vertical line. The vertical line serves as a divider and it is spoken "such that".The notation is read as "The set of all such that is a positive even integer less than 10".

2. - Introduce Null set and Universal set.

   Consider the following sets:

   1. The set of all integers satisfy the equation $2x = 3$.

      i.e. { x is an integer $2x = 3$ }
   2. A = { $x|x$ is an odd integer $2 < x < 3$}
   3. B = { $x|x$ is an integer $x^2 = 4$ }
   4. C = { $x|x$ is an integer $0 < x,\ 10$}
   5. D = { $x|x$ is an even positive integer $2 < x < 3$}

      Observe that the above sets 1), 2) and 3) contain no elements.

- A set which does not contain any element is called the empty set or the null set or the void set
- The empty set is denoted by the symbolor { }. When studying the sets in above 4), 5) and so forth, we are interested in the set of natural numbers.

- The members of all the investigated sets in a particular problem usually belong to some fixed large setis called the "Universal Set". The universal set is usually denoted by *U*.
- Universal Set*U*that includes all of the elements under consideration in a particular discussion.
- Universal Set depends on the context.

3. Introduce finite sets and infinite sets

Consider the following sets:

- P = { 1, 2, 3, 4, 5 }
- Q = { p, q, r, s, t, u, v, w }
- R = { People living presently in different parts of the world}

We observe that P contains 5 elements and Q contains 8 elements. How many elements does R contain? As it is, we do not know the number of elements in R, but it is somenatural number which may be quite a big number.

Consider the set of natural numbers. We see that the number of elements of this set is not finite since there are infinite numbers of natural numbers. We say that the set of natural numbers is an infinite set.

- A set which is empty or consists of a definite number of elements iscalled *finite*otherwise, the set is called *infinite*.

4. Introduce Cardinality of a set & notations for it

The above sets P, Q and R given above are finite sets and we observed the number of elements are 5, and some finite natural number.

By number of elements of a set A, we mean the number of distinct elements of the set.

- The number of elements in a set is called the *cardinality* of a set.
- Let 'A' be any set then its cardinality is denoted by n(A) or $|A|$.
- If n(A) is a natural number, then A is non-empty finite set.

5. Introduce subsets, proper subsets, equality of two sets, and power set Consider the setsis the set of all peoplesliving presently in our country and is the set of all peoples living presently in the world.

We note that every element of A is also an element of B; we say that A is a subset of B. The fact that A is subset of B is expressed in symbols as $A \subseteq B$. The symbol $\subseteq$ stands for 'is a subset of' or 'is contained in'.

- A set A is said to be a *subset* of a set B if every element of A is also anelement of B.In other words, if whenever $x \in A$, then $x \in B$.
- If A is not a subset of B, we write $A \not\subset B$.
- Every set A is a subset of itself, i.e. $A \subseteq A$.
- Since the empty set ϕ has no elements, we agree to say that ϕ is a subset of every set. i.e. $\phi \subseteq A$, for any set A.
- A set A is said to be apropersubset of a set B if $A \subseteq B$ and $B \not\subset A$.
- Sets A and B are said to be equal if $A \subseteq B$ and $B \subseteq A$.

  Then the *power set of* A is denoted by P(A) and is the set of all subset *A*.

  i.e. P(A) = $\{x | x$ is a set and $x \not\subset A\}$ and $n(P(A)) = 2^{n(A)}$.

Competency level 2.2 : Uses Venn diagrams and algebra of sets to solve problems

Number of periods : 06

Learning out comes :

1. States the set operations and illustrates with venn diagrams
2. Identify set identities
3 Solves problems involving set identities
4. Uses the cardiolity formula for two set and extend it to three sets.

Guidelines to learning - teaching process :

1. Introduce Set operations and Venn diagrams

- Identifies set operations (Intersection, Union, Complement and Relative Complement)

  Intersection:

  Consider the sets A = { 1, 2, 3, 4} and B = {2, 4, 6, 8}. Identifies all the elements which are common to both A and B. We see that 2, 4 are the only elements which are common to both A and B. The intersection of sets A and B is the set of all elements which are common to both A and B. Hence, the set {2, 4} is the intersection of sets A and B.

- The *intersection* of sets and is the set of all the elements which are common to both A and B. If A and B are sets, then the intersection of sets A and B is denoted by $(A \cap B)$, read "A intersection B". Its elements are those objects which are in A and in B i.e. those elements which are in both sets.

Union:

Consider the sets A = {1, 2, 3, 4} and B = {2, 4, 6, 8}. Identifies all the elements of and all the elements of the common elements being taken only once.We see that such elements are 1, 2,, 3, 4, 6, 8 Note that the common elements 2, 4 have been taken only once.

The union of sets A and B is the set which consists of all the elements of A and all the elements B of the common elements being taken only once. Hence, the set {1, 2, 3, 4, 6, 8}is the union of sets A and B.

- The *union* of sets A and B is the set which consists of all the elements of A and all the elements B of the common elements being taken only once.If A and B are sets, then the union of sets and is denoted by $A \cup B$, read "A union B". Its elements are those objects which are in A and in B the common elements being taken only once.

Complement:

Consider the set A = {1, 2, 3, 4} and the Universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} Identifies set of all elements of U which are not the elements of A. We see that such elements are 5, 6, 7, 8, 10. The complement of set A is the set of all elements of U which are not the elements of A.Thus the set {5, 6, 7, 10} is the complement of set A.

- If A and B are sets, then the complement of set is denoted by A' or A^c , read "complement of A". Its elements are those objects which are in U, not in A.

Relative Complement:

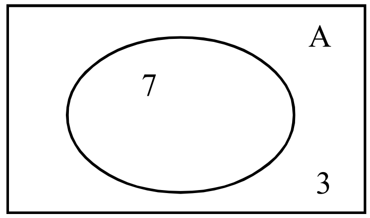
Consider the sets A = {1, 2, 3, 4} and B = {2, 4, 6, 8, 10}. Identifies all the elements which belong to A but not to B. We see that 1, 3.The complement of set B relative to set A is the set which consists of all the elements which belong to A but not to B. Hence, the set {1, 3} is the complement of set B relative to A.

- If A and B are sets, then the complement of set B relative to A set is denoted by $A \backslash B$. read "complement of B relative to A". Its elements are those objects which are in A, not in B.

- Introduce Venn diagrams:

  A Venn diagram is a pictorial representation of sets where sets are represented by enclosed areas in the plane. The universal set U is represented by a rectangle, and the other sets are represented by circles which are contained in the rectangle.

For example consider the following diagram.



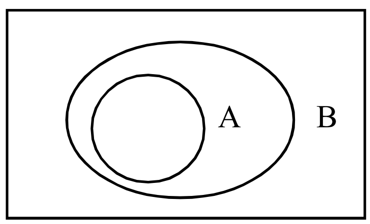
The above diagram consists of rectangles, closed curves usually circles and "x". Rectangle denotes the universal set U, circle denotes the set A subset of U and "x" represent the elements. Observe that $7 \in A$, but $3 \notin A$. In Venn diagrams, the elements of the sets are written in their respective areas,

Note that the null set can't be represent in the Venn diagram.

- Identifies Venn diagrams representation of subset, and set operations:
- Venn diagrams representation of a subset:

Let A and B be subset of the universal set U. Suppose $A \subseteq B$

Consider the following Venn diagram:

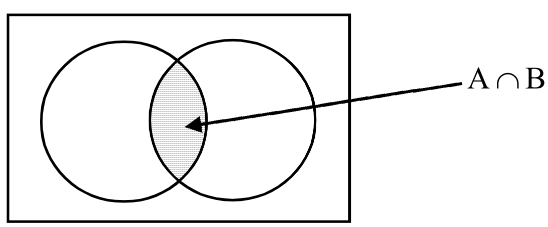


This Venn diagram represents the situation.

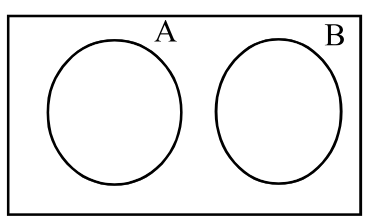
- Venn diagram representation of set operations:

Intersection:

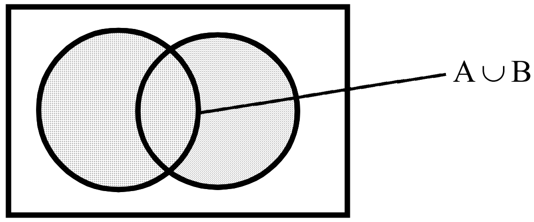
Let A and B be subset of the universal set U.



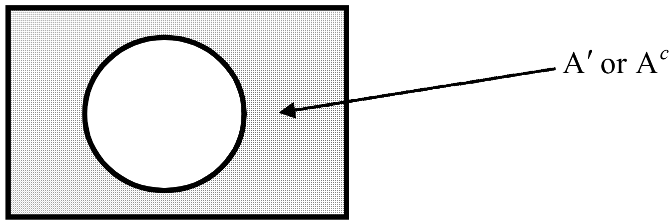
The following diagram represent the special situation.



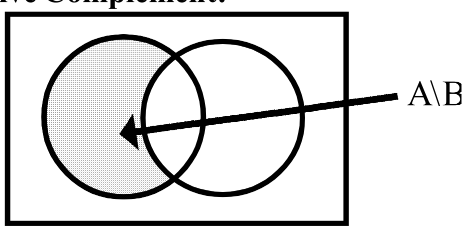
Union:



Complement:



Relative Complement:



2. Introduce set identities

- Identifies set identities by observing the areas of the Venn diagrams.

  For example, draw the Venn diagrams for the sets $(A \backslash B) \cup (B \backslash A)$ and $(A \cup B) \setminus (A \cap B)$ Hence, conclude that the areas corresponds $(A \backslash B) \cup (B \backslash A)$ and $(A \cup B) \setminus (A \cap B)$ are same.

3. Introduce problems involving set identities

- Verify set identities using Venn diagrams.
- Proves result involving set identities.

4.
- Derive the formula. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- Obtain formula for $n(A \cup B \cup C)$

Competency 3 : **Manipulates Mathematical Logic**

Competency level 3.1 : Identify statements

Number of periods : 10

Learning out comes :

1. Identifies statements.
2. Identifies the different types of statements.
3. Writes the definition of all types of statements.
4. Defines conditional statements.
5. Defines compound statements.
6. Constructs truth tables.
7. States the definitions of logical equivalence
8. Introduces predicates
9. Defines quantifiers.

Guidelines to learning - teaching process :

1. Identifies statements.
   - Gives sentences used in the environment.
   - Identify either true or false statements from the given sentences.

     Consider the following sentences :

     01. Colombo is the capital of Sri Lanka
     02. What is our name?
     03. Three plus seven equals ten
     04. Close the door
     05. Square root of 8
     06. $2x = 10$
     07. Are you a sleep
     08. He is an advance Level student
     09. I am hungry
     10. Every prime number is an odd number

   From the above 01, 03, 08, 10 are sentence others are not. 01 and 03 are true; 10 is no true (false).and 08 and 09 are neither true or false. 02 and 07 are questions. 04 is an order. 05 and 06 are incompleate.

2. Identifies the different types of statements.
   - Mathematical statements.
   - Non Mathematical statements
   - Simple statements
   - Complex statements

Mathematical statements / Non - Mathematical statement

From the above statement (3) and (10) are mathematical statements and (1) is Non mathematical statement

Simple statements / Complex statements
These all ((1), (3) and (10)) are simple statements. Simple statement is the basic building block of Logic. A Simple statement that does not have any other statement as a component.

The statement "one plus equals two or one is greater than zero" is not a simple statement, which has two simple statements 'one plus one equals two"and"one is greater than zero" as the components. This type statement is called a complex statement. A statement which is not a simple statement is called a complex statement.

3. Write the definition of all types of statements
   - Definition of a Statement
   - Truth values of a Statement
   - Symbols for Statement
   - Negation of a Statement

Definition of a Statement:
A statement is a declarative sentence that is true or false but not both.

For Example, from the above sentences (1), (3) and (10) are statements. Sentences (8) and (9) are neither true nor false, they are not statements. Non - statements can be further subdivided into questions ((2) and (7)), commands (4)), expressions ((5) and (6)), and so on.

Truth values of a statement:
Every statement is either true or false sentence. So, every statement has a truth value, namely true (denoted by T) or false (denoted by F).

For examples, truth values of the statements 'Colombo is the capital of Sri Lanka" is T, "Three plus seven equals ten" is T, and "Every prime number is an odd number" is F.

Symbols for statements:

We often use *p, q* and *r* to denote propositions, or perhaps $P_1, P_2, \ldots P_n$ if there are several propositions involved.

For example, We might write.
p : Colombo is the capital of Sri Lanka.
q : Three plus seen equals ten.

Negation of a statement:

- Identifies that negation also a statements.
- Identifies relation between the truth valves of statements and its negation.
- Introduce symbols for negation statements.
- Identifies that negation also a statements.

Consider the negation is also a statement:

(a) Colombo is not the capital of Sri Lanka
(b) Three plus seven not equal to ten
(c) Every prime number is not an odd number

Sentences (a) is false and it is the negation of the sentences (1) above. So, the sentence (a) is a statement. Sentence (b) is identified as a statement since, it is a false sentence. Sentence (c) is true since it is the negation of the sentence (10) above and hence, it is a statement.

- *Identifies relation between the truth valves of statements and its negation.*
  From the above example, we notice that te truth value of the statement negation of a statement is the opposite of the truth value of the original statement.

- *Introduce symbols for negation statements.*
  For a given statement "not p" or it is not the case that p by $\sim p$. In general, negating statement will always switch its truth value. Thus, if p is T, then ~p is F; if p is F, then ~p is T.

4. Constructs truth tables.
   - Identifies logical connectives (and, or, it them and it and only if)
   - Introduce Mathematical term, English expressions and symbolic form.
   - Introduce truth tables
   - Construct truth tables for negation
   - Construct truth tables corresponding to the connections.

Identifies logical connective ("and", "or", "if, then", if and only if").

In mathematics numbers combine with the familiar operations $+, \times, -, \div$.
For example consider, $(2 + 7 \times (11 - 5)) \div 8$

In the same way in logic, the simple statement can be combined by the connective 'and ', 'or', 'if, then', if and only if'. These connectives are called logical connectives. In logical notation they are written, ' $\wedge$ ', ' $\vee$ ', ' $\Rightarrow$ ', ' $\Leftrightarrow$ ' respectively.

For example, statement ; Two plus two equals five or three is a square root of nine" id formed by connecting two simple statements 'Two plus two equals five" and "three is a square root of nine" with the logical connective or".

Introduce Mathematical term, English expression and symbolic from.

Let p denotes a statement.

Mathematical Term	English Expression	Symbolic Form
Negation	Not p	$\sim p$
Conjunction	p and q	$p \wedge q$
Disjunction	p or q	$p \vee q$
Implication	If p, then q	$p \Rightarrow q$
Bi conditional Equivalence	P if and only if q	$p \Leftrightarrow q$

Introduce truth tables

A truth table displays the relationship between the truth values of the statement and its components (simple statements). A statement in general contains a number of components (simple statements).
For example, ($p \vee q$) contains components p and q each which represents an arbitrary simple statement. This relationship of the truth value of a statement and those of its constituent components
can be represented by a table. It tabulates the truth value of a statement for all possible truth values of its components and it is called a truth table.

Construct truth table for Negation

The truth value of the statement negation of a statement is the opposite of the truth value of the original statement. Thus, the following truth table relates the truth values of P and $\sim$P.

p	$\sim p$
T	F
F	T

Construct truth table corresponding to the logical connectives

Truth table of Conjunction

The following truth table relates the truth values of $p \wedge q$.

p	p	$P \wedge Q$
T	T	T
F	T	F
T	F	F
F	F	F

The conjunction $p \wedge q$ is true only if both p and q are true; otherwise, $p \wedge q$ is false.
For example, consider the statements

p : one is odd integer.

q : zero is greater than 2.

The conjunction of p and q, namely, $p \wedge q$: One is odd integer and zero is greater than 2 is a false statement since q is false (even though p is true).

Truth table of Disjunction

The following truth table relates the truth values of $p \vee q$.

p	p	$P \vee Q$
T	T	T
F	T	F
T	F	T
F	F	F

The disjunction $p \vee q$ is true if at least one of p and q is true; otherwise, $p \vee q$ is false. Therefore, $p \vee q$ is true if exactly one of p and q is true or if both p and q are true.

i.e. disjunction $p \vee q$ is false only if both p and q are false; otherwise, $p \vee q$ is true.

For example, consider the statements

p : The integer 2 is even.

q : 6 is less than 2.

The disjunction of p and q, namely, $p \vee q$: The integer 2 is even or 6 is less than 2 is a true statement since at least one of p and q is true (in this case, p is true).

5. Define conditional statement.

- Conditional statement
- Bi-conditional statement

Conditional statement:

A statement of the form "Tom is a dog, then it is an animal" is a conditional statement.

Truth table of conditional statement:

This statement is formed connecting two simple statements "Tom is a dog" and "Tom is an animal" with the connective "if, then". In general, for the statements p and q the conditional statement "if p, then q" is denoted by $p \Rightarrow q$.

The following truth table relates the truth values of $p \Rightarrow q$.

p	p	$P \Rightarrow Q$
T	T	T
F	T	F
T	F	F
F	F	T

If you find it difficult to understand, just remember that the $p \Rightarrow q$ means 'if p is true, then q is true'. If p is false, then we don't care about q, and by default, make $p \Rightarrow q$ evaluate to true in this case.

Following are some of the various English expressions that translate to $p \Rightarrow q$.

If p, then q.

p implies q.

q if p.

q whenever p.

p only if q
p is sufficient for q
q is necessary for p

For example, consider the statements

p : 1+2 = 4

q : 3 is less than 4

The implication of p and q namely, $p \Rightarrow q$: If $1 + 2 = 4$, then 3 is less than 4 is a true statement since p is false. However, $q \Rightarrow p$: If 3 is less than 4, than $1 + 2 = 4$ is a false statement since q is true and p is false.

Bi conditional statements.

Truth table of Bi conditional:
A statement of the from "p if and only if q" is called Bi - conditional statement.
The following truth table relates the truth values of $p \Leftrightarrow q$.

p	p	$P \Leftrightarrow Q$
T	T	T
F	T	F
T	F	F
F	F	T

Following are some of the various English expressions that translate to $p \Leftrightarrow q$

- p if and only if q.
- p is equivalent to q.
- p is necessary and sufficient for q.

6. Defines compound statements
   - Introduce compound statements
   - Truth value of compound statements

Introduce Compound Statements :

A compound statement is a statement that has t least one simple statement as a component. From given statements, we can use ~, $\wedge$, $\vee$ and $\Rightarrow$ to form more complicated statements, called compound statements.

For example, for given statements p and q, the conjunction p^q is a compound statement.

For a slightly more complex example, consider the compound statement given by $((\sim p) \vee \sim(q \vee r)) \wedge (\sim (s \vee (q \ (\sim t))))$.

Truth value for compound statement:

Suppose the compound statement contains two propositions p and q. Then the truth table contains four rows. The number of columns is two plus number of connectives, Negation will be considered as a connective.

For example, consider the compound statement $\sim(p \wedge q) \Rightarrow (p \vee q)$.

This compound statement contains number of two propositions and four connectives. So, the truth table contains four rows and six columns.

p	q	$(p \wedge q)$	$\sim(p \wedge q)$	$(p \vee q)$	$\sim(p \wedge q) \Rightarrow (p \vee q)$
T	T	T	F	T	T
F	T	F	T	T	T
T	F	F	T	T	T
F	F	T	T	F	F

In the case of compound statement contains three propositions p, q and r. Then the truth table contains eight rows. The number of columns is three plus number of connectives.

For example, consider the compound statement $r \Rightarrow (p \wedge q)$.

This compound statement contains number of three propositions and two connectives. So, the truth table contains eight rows and five columns.

p	q	r	$(p \wedge q)$	$r \Rightarrow (p \wedge q)$
T	T	T	T	T
F	T	T	T	T
T	F	T	T	T
F	F	T	F	F
T	T	F	T	T
F	T	F	T	T
T	F	F	T	T
F	F	F	F	T

7. States the definition of logical equivalents and predicates of an event.

- Identify logical equivalent
- Define logical equivalent
- Verify logical equivalent using truth table
- Introduce predicates
- Define predicates

Identify logical equivalent :

Consider the following statement:

She has neither a cat nor a dog. In other words, the doesn't have a cat and she doesn't have a dog. So, these two statements has same meaning, and hence are logical equivalent.

In symbolically,

p : She has a cat.

q : She has a dog.

Then, $\sim(p \vee q)$ *and* $\sim p \wedge \sim q$ are Logical equivalent.

Define Logical equivalent :

Two propositions are said to be logical equivalent (or equal) if they have same identical truth values. We will denote Logical equivalence by the symbol ' a" '. For example, statements "$\sim(p \vee \sim q)$ " and "$\sim p \wedge \sim q$" are Logical equivalent. In symbolically, $\sim(p \vee q)$ *a"* $\sim p \wedge \sim q$.

Verify Logical equivalent using truth table :

For example consider the statements ~ (p ∨ q) and ~p ∧ ~q. Their truth table is:

p	q	$p \vee q$	$r(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
T	T	T	F	T	F	F
F	F	F	T	T	T	T

Here the statements ~(p $\vee$ q) and ~p $\wedge$ ~p have identical truth values for the possible ways of assigning truth values to the component simple statements p and q.
Hence, ~(p $\vee$ q) $\equiv$ ~p $\wedge$ ~q.

8. Introduce predicates :

Consider, for example, the sentence

p : x is an odd integer

This sentence is neither true nor false. The truth or falsity depends on the value of the variable x. For some values of x the sentence is true; for others it is false. Thus this sentence is not a statement.
However, let us denote this sentence by p(x), i.e. p(x) : x is an odd integer
Then, p(3) is true, while p(4) s false.
In tis example, the sentence 'p(x) : x is an odd integer" is a predicate with domain in the set of all integers since for each x, p(x) is a statements.
i.e. for each integer x, p(x) is true or false, but not both.

Define predicates :
A predicate is a sentence that contains a finite number of variable and becomes a statement when specific values are substituted for the variables. The domain of a predicate variable is the set of all values that may be substituted in place of the variables.

9. Define quantifiers.

- Introduce quantifiers

Introduce quantifiers :

- The universal quantifier

  Consider the statement $x^2 \geq 0$, it is clear that this is true real numbers. Thus, we can more precisely say that "For all real number x, $x^2 \geq 0$". This statement is called a universally quantified or for all statements. The phrase "for all" is denoted by the symbol "$\forall$" and it is called the universal quantifier.

- The existential quantifier

  Consider the statement $2x-6=0$, it is clear that this is only valid for one value of x. Thus, we can more precisely say that "There exists a real number x such that $2x-6=0$". This statement mean that there is at least one value of x for which the statement "$2x-6=0$" is true. The phrase "there exists" is dented by the symbol "$\exists$"" and it is called the existential quantifier.

- Write the symbolization of predicates.
  - Introduce symbols for predicates.
  - Symbolize statements involving quantifiers

Introduce symbols for predicates :

We will use $p(x)$ to denote the expression involving one variable x, and $p(x, y)$ to denote the expression involving two variables x, y.

For examples,

- $2 < x < 5$ is a predicate involving the variable x where x is a natural number.
- $x^2 + y^2 = 9$ is a predicate with two variable x and y, where x, y are real numbers.
- $1 + 2 + 3 + ... + n = \frac{n}{2}(n+1)$ where n is a natural number.

  In above (i), $P(x)\,;2 < x < 5$ i.e $P(x)$ denotes $2 < x < 5$

  In above (ii), $P(x, y)\,;\, x^2 + y^2 = 9$. i.e. $P(x, y)$ denotes $x^2 + y^2 = 9$.

  In above (iii), $P(n): 1 + 2 + 3 + ... + n = \frac{n}{2}(n+1)$. i.e.

$$P(n) \text{ denotes } 1 + 2 + 3 + ... + n = \frac{n}{2}(n+1)$$

Symbolize statements involving quantifiers:

Consider the two statements

(i) For all real number x, $x^2 \geq 0$.

(ii) There exists a real number x such that $2x - 6 = 0$.

For (i) above,

Let $P(x):\ x, x^2 \geq 0$, where x is a real number. Then, the symbolic form of the first statement is "$\forall x \in \mathbb{R}, P(x)$"

For (ii) above,

Let $Q(x): 2x - 6 = 0$ where x is a real number. Then, the symbolic form of the second statement is "$\exists x \in \mathbb{R},\ Q(x)$"

Competency 5 : Analyses functions of a real variable

Competency level 5.1 : Investigates functions

Number of periods : 10

Learning out comes :
1. Explains the defmition of a function
2. Explains domain and range of a function.
3. Describes vertical line test for a function.
4. Recognizes special functions.
5. Sketches graphs of special functions.
6. Sketches graphs of functions using translations (shifting).

Guidelines to learning - teaching process :

1. - Introduce relation, mapping, rule
   - Discuss the cases with examples where there are one-one, one - many, many to one, many to many relations
   - Define function.

Definition

A function f is a rule that assigns to each element x in a set A to exactly one element, called $f(x)$ (read 'f of x"), in a set B.

The set A is called the domain of the function and the set B is called the co domain of the function.

- Give examples on functions.

2. States the domain and range of functions. Give simple examples to find domain and range of a function.

3. - Describes vertical line test for functions.
   - Guide students to solve problems using vertical line test.

4. Introduce the following special functions
   - Constant functions
   - Modules functions
   - Piece wise functions
   - Give examples for each case

5. Guide students to sketches the graph of the following functions.
   - $f(x) = |x|$
   - $f(x) = x^2$

- $f(x)=\sqrt{x}, \quad x \geq 0$
- $f(x)=\frac{1}{x}, \quad x \neq 0$
- $f(x)=\frac{1}{x^2}, \quad x \neq 0$

6. Guide to deduce the graph of the function, when there is a horizontal translation.

Competency level 5.2 : Investigates operations on functions

Number of periods : 10

Learning out comes :
1. Performs basic operations on functions
2. Defines composite functions
3. Writes the notations for composite functions
4. Defines inverse functions and finds the inverse functions

Guidelines to learning - teaching process :

1. Basic operations on functions
   - $(f+g)(x)=f(x)+g(x)$
   - $(f-g)(x)=f(x)-g(x)$
   - $(f.g)(x)=f(x).g(x)$
   - $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$

2. Composite function:

   Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two functions. Then the composition of *f* and *g*, denoted by *g o f* is the function $g \, o \, f: X \rightarrow Y$ defined by $f(x)=g(f(x))$.

   Note that in order to define *g o f,* the co domain of *f* and the domain of *g* should be equal. In some definition of composition of functions, It is sufficient that the co domain of *f* is a subset of the domain of *g.* Also note that the order of writing *f* and *g* is important, *i.e. g o f* cannot be replaced by *f o g,* When *g o f* is defined, the composition *f o g* may not have been defined. Even if both *g o f* and *f o g* are defined, the two functions *f o g* and *g o f* may not be equal.

3. **Example 1:** Let $f: \mathbb{R} \rightarrow \mathbb{R}^+$ be the function defined by $f(x)=3x+2$ and let $g: \mathbb{R} \rightarrow \mathbb{R}^+$ be the function defined by $g(x)=x^2-1$. Then, both *g o f* and *f o g* are defined as.

$g\,o\,f : \mathbb{R} \to \mathbb{R}^+$

$g\ o\ f(x) = g(f(x)) = g(3x+2) = (3x+2)^2 - a = 9x^2 + 12x + 3$

and

$f\ o\ g(x) = f(g(x)) = f(x^2-1) = 3(x^2-1) + 2 = 3x^2 - 1$

Example 2: Let $f: \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = \sqrt{x} - x$ and *let* $g: \mathbb{R} \to \mathbb{R}$ be the function defined by $g(x) = 3x$. Then, $g\,o\,f$ is defined but $f\,o\,g$ is not defined because , $\mathbb{R} \neq \mathbb{R}^+$ .We have,

$g\,o\,f: \mathbb{R}^+ \to \mathbb{R}$

$g\ o\ f(x) = g(f(x)) = g(\sqrt{x} - x) = 3(\sqrt{x} - x)$

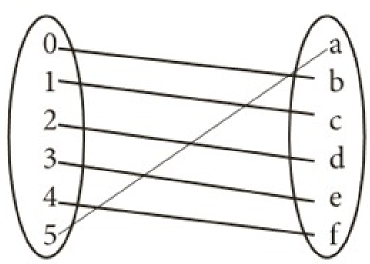
4. **One - to - one functions**

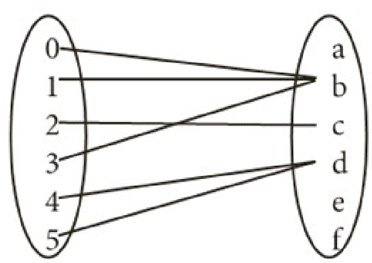
A function f: A $\to$ B A is said to be one - to - one for any $x_1, x_2 \in$ A

if and only if $f(x_1) = f(x_2)$ then $x_1 = x_2$

Ex: 1 Ex: 2

A one - to - one function : Not a one - to - one function :





Example:

Show that each of the functions is one - to - one:

(i) $f(x) = 3x + 5 \quad for\ x \in \mathbb{R}$

(ii) $f(x) = 3 - 2x^3 \quad for\ x \in \mathbb{R}$

(iii) $f(x) = x^2 - 2x \quad for\ x \in [-1, \infty]$

Horizontal Line Test:

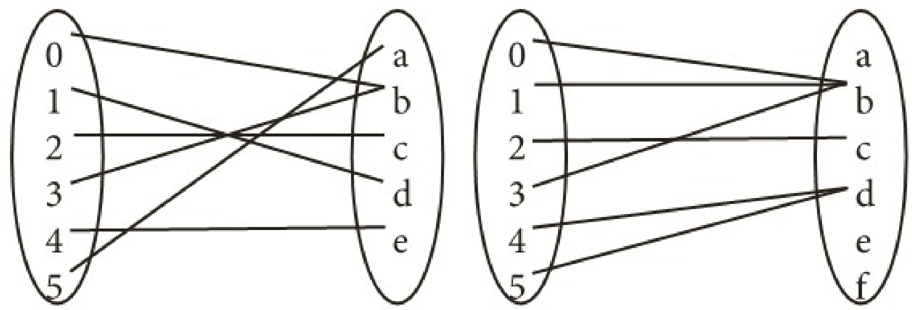
No horizontal line intersects the graph of a one - to - one function more than once.

Onto Functions

Let *A* and *B* be non-empty subset of $\mathbb{R}$. A function *f* from *A* to *B* is said to be onto if for each $b \in B$, there is at least one $a \in A$ such that *b = f (a).* (i.e. if the range of *f* is equal to *B*.)

Eg: 3 Eg: 2

An onto function : Not on to function :



Example:

Show that each of the functions is onto :

(i) $f(x) = 3x + 5$ from $\mathbb{R}$ to $\mathbb{R}$

(ii) $f(x) = 3 - 2x^3$ from $\mathbb{R}$ to $\mathbb{R}$

(iii) $f(x) = x^2 - 2x$ from $[-1, \infty)$ *to* $[-1, \infty)$

Inverse Functions

Let A and B be two non-empty subset of $\mathbb{R}$. Let f be a one - to - one and onto function from A to B. Then its inverse function f^{-1} is defined for $y \in B$ by $f^{-1}(y) = x \Leftrightarrow y = f(x)$

Example:

In each of the following, show that *f* is one - to - one and find f^{-1} *1* indicating its domain:

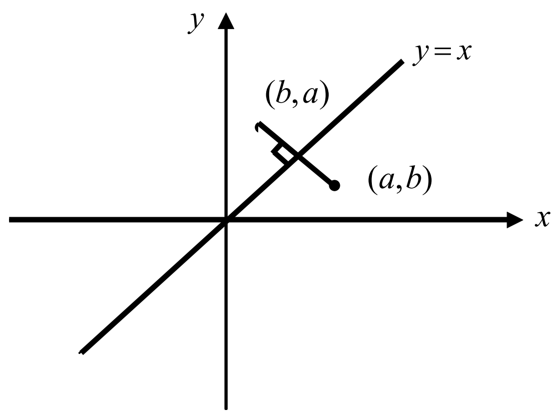
(i) $f(x) = 3x + 5$ $\quad for\ x \in \mathbb{R}$

(ii) $f(x) = \dfrac{2+x}{1-x}$ $\quad for\ x \in \mathbb{R} - \{1\}$

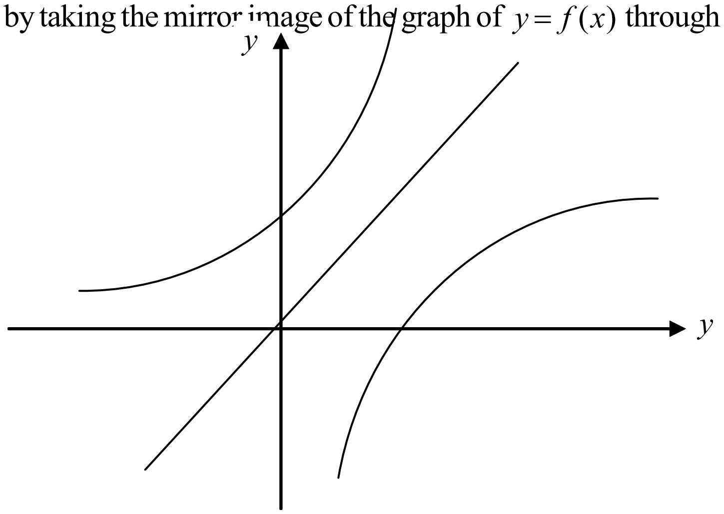
(iii) $f(x) = x^2 - 2x$ $\quad for\ x \in (-1, \infty)$

Graph of $y = f^{-1}(x)$

First note that the point *(b,a)* is the mirror image of point (*a, b*) through the line $y = x$.



Since $y = f^{-1}(x)$ if and only if $x = f(y)$, we can obtain the graph of $y = f^{-1}(x)$ by taking the mirror image of the graph of $y = f(x)$ through the line $y = x$.



Competency 6 : **Analyses the polynomial with one variables.**

Competency level 6.1 : Investigates polynomials.

Number of periods : 02

Learning out comes :
1. Defines a polynomial in a single variable
2. Defines the terms degree, leading term and leading efficient.
3. States the condition for two polynomials to be equal

Guidelines to learning - teaching process :

1. Any functions like as $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2}........ + a_0$ called a polynomial with variable *x*. Here $a_n,\ a_{n-1},\ a_{n-2},\ ...., a_0 \in \mathbb{R}$ and there are constant and $r \in \mathbb{Z}^+$

2. For a given polynomial leading power is defined as the degree of the polynomial and. the coofficient of the term consist the leading power it defined as the leading coefficient of the polynomial.

   Eg: $3x^3 + 5x^2 + 2x + 1$
   degree = 3
   leading term = $3x^3$
   leading coefficient = 3

3. If two polynomials are equal then their co efficients of the corresponding terms are equal.

   Eg: $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_0$
   $g(x) = b_n x^n + b_{n-1}x^{n-1} + b_{n-2}x^{n-2} + ... + b_0$
   if $f(x) = g(x)$
   Then $a_n = b_n,\ a_{n-1} = b_{n-1},\ a_{n-2} = b_{n-2},\ ..., a_1 = b_1,\ a_0 = b_0$

Competency level 6.2 : Manipulate the mathematical operation polynomials

Number of periods : 10

Learning out comes :
1. Manipulates the basic mathematical operations on polynomials
2. Divides a polynomial by another polynomial
3. States synthetic division
4. States the remainder theorem
5. Proves the remainder theorem

6. States the factor theorem
7. Expresses converse of the factor theorem
8. Solves problems using remainder theorem and factor theorem.
9. Solves polynomial equations (up to 4th order)
10. Defines zeros of a polynomial

Guidelines to learning - teaching process :

1. Addition, substraction, multiplication, of polynomials.

   Eg. Let $P(x) = x^3 + 3x^2 + 2x + 1$

   $Q(x) = 2x^3 - 8x^2 + 3$

   Guide students to find $\lambda P(x) + \mu Q(x)$ for different value of λ and μ

   Guide students to find $\lambda P(x).\mu Q(x)$ for different value of λ and μ

2. Divide the polynomial by another polynomial using long division.

   Eg : 1

$$
\begin{array}{rl}
 & x^3 - x^2 + 4x - 2 \leftarrow \text{Quotient} \\
\text{diviser} \rightarrow (x+1) & \overline{\big) x^4 + 3x^2 + 2x - 1} \leftarrow \text{divident} \\
 & x^4 + x^2 \\
\hline
 & -x^2 + 3x + 2x - 1 \\
 & -x^2 - x^2 \\
\hline
 & 4x^2 + 2x - 1 \\
 & 4x^2 + 4x \\
\hline
 & -2x - 1 \\
 & -2x - 2 \\
\hline
 & 1 - \text{Remainder}
\end{array}
$$

Eg : 2

$$
\begin{array}{rl}
 & x + 3 \leftarrow \text{Quotient} \\
(x^2 - 1) & \overline{\big) x^3 + 3x^2 + 1} \\
 & x^3 - x \\
\hline
 & 3x^2 + x + 1 \\
 & 3x^2 - 3 \\
\hline
 & x + 4 \text{ - Remainder}
\end{array}
$$

3. Introduce synthetic division.

4. When a polynomial $f(x)$ is divided by a linear expression $(x-a)$ then the reminder is given by $f(a)$.

5. Let the remainder when the polynomial $f(x)$ divide by $(x-a)$ *is R* and quotient is $\phi(x)$

$f(x)=\phi(x)(x-a)+\mathrm{R}$

If $x=a$

$f(a)=Q(a).0+R$

$f(a)=R$

Hence the remainder theorem.

Eg: 1. Find the remainder. when the polynomial x^3+x^2+1 is divided by $x-1$.

solution : $f(x)=x^3+x^2+1$

By remainder theorem , the remainder when it is divided by $x-1$ is given by $f(1)=1^3+1^2+1=3$ a

Hence the remainder is 3

Eg: 2. Find the remainder. When the polynomial $\mathrm{g}(x)=x^4+x^3+2x+1$ is divide by $x-2$

solution : Let $\mathrm{g}(x)=x^4+x^3+2x+1$

by remainder theorem the remaimder when $\mathrm{g}(x)$ is is divided by $x-2$ is $\mathrm{g}(2)$

$$\mathrm{g}(2)=2^4+2^3+2(2)+1$$
$$=16+8+4+1=29$$

Hence the remainder is $\mathrm{g}(2)=29$

6. If $(x-a)$ is factor of a polynomial $f(x)$ then. $f(a)=0$ this is said to be factor theorem

Ex: 1. Show that $(x-1)$ is a factor of $f(x)=x^3-2x+1$

$f(x)=x^3-2x+1$

$f(1)=1^3-2\times1+1=0$

Therefore by factor theorem $(x-1)$ is a factor of $f(x)$.

7. Let $f(x)$ is a polynomial of x, and $f(a)=0$ where $a \in \mathbb{R}$ then $(x-a)$ is factor of the polynomial $f(x)$.

8. Guide students to solve problems involving remainder theorem and factor theorem.

9. Guide students to solve polynomials equation upto fourth (4) order.

10. Discuss the zero point to the given polynomial.

Competency level 6.3 : Investigates quadratic functions and their properties

Number of periods : 10

Learning out comes :
1. Introduces linear functions
2. Explains quadratic functions
3. Explains the properties of a quadratic function
4. Sketches the graph of a quadratic function
5. Describes different types of graph of quadratic functions
6. Solves problems involving quadratic functions.

Guidelines to learning - teaching process :

1. Discuss about linear functions and their graphs.

2. Introduce quadratic function

$y = ax^2 + bx + c,$ where $a, b, c \in \mathbb{R},\ a \neq o, x \in \mathbb{R}$

$$y = a\left\{x^2 + \left(\frac{b}{a}\right)x + \frac{c}{a}\right\}$$

$$y = a\left\{x + \left(\frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c\right\}$$ Transform to the above term.

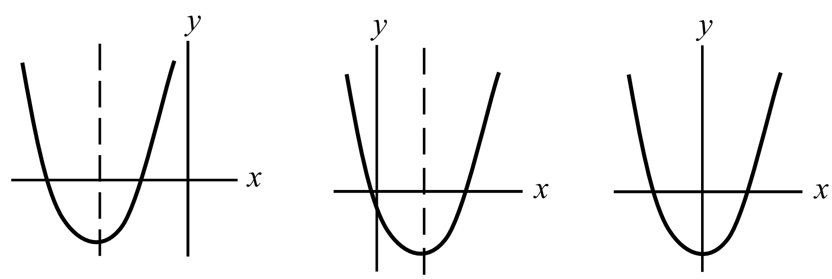
3. Discuss about the axis of symmetry and state with reasons as $x+\frac{b}{2a}=0$ is the axis of symmetry.

Distinguishes algebrecaly then explains that
if $a>0$ then there is a local minimum
if $a<0$ then there is a local maximum

4. Guide students to sketches the graph of quadratic functions.

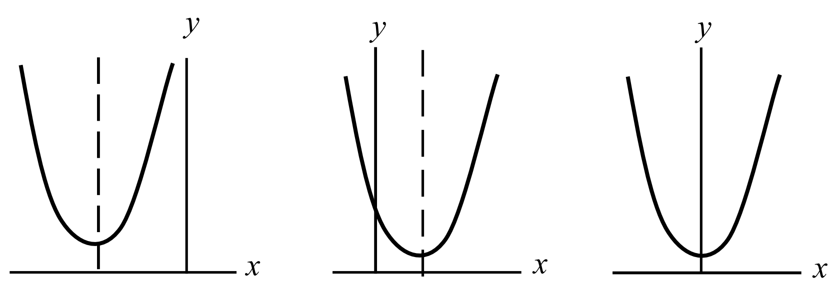
5. Guide students to obtain the following graphs and discuss about them. In each case give attention to the turning points, axis of symmetry and points where its crosses the axis.

- $a>0,\quad b^2-4ac>0$



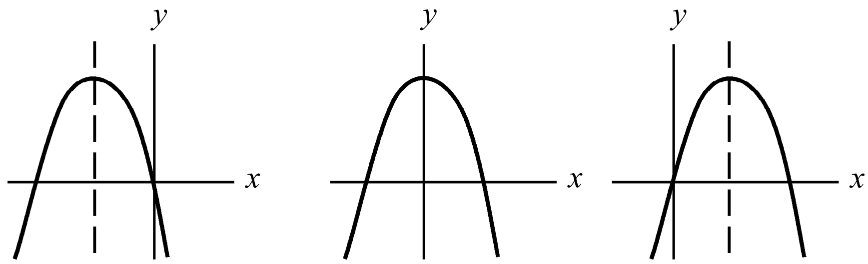
$b>0$ $\qquad$ $b<0$ $\qquad$ $b=0$

- $a<0,\quad b^2-4ac<0$



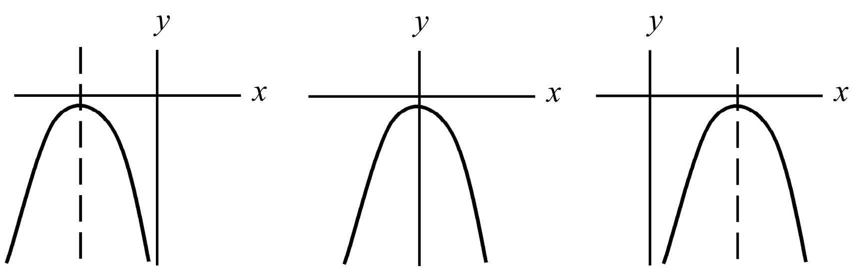
$b>0$ $\qquad$ $b<0$ $\qquad$ $b=0$

- $a < 0, \quad b^2 - 4ac > 0$



$b > 0$ $\qquad b = 0$ $\qquad b < 0$

- $a < 0, \quad b^2 - 4ac < 0$



$b > 0$ $\qquad b = 0$ $\qquad b < 0$

- Guide students to discus the other cases

6. Guide students to solve problems involving quadratic functions.

Competency level 6.4 : Investigates quadratic equations

Number of periods : 16

Learning out comes :

1. Explains the roots of a quadratic equation
2. Finds the roots of a quadratic equation
3. Describes the nature of the roots of a quadratic equation
4. Express the sum and products of the roots of a quadratic equations in terms of its coefficients
5. Constructs quadratic equations where roots are symmetric expression of root of another quadratic equations
6. Solves problems involving quadratic functions and quadratic equations

Guidelines to learning -teaching process :

1. States that $y = ax^2 + bx + c$ is a quadratic function and the equation that gives the zero's of this quadratic function is called quadratic equation. where $a \neq 0, x \in \mathbb{R}$
   Show that a quadratic equation can have there only two distinct roots .

2. Find the roots of a quadratic equation using completing square method.

3. States that since the nature of the roots will vary according to the sign of $b^2 - 4ac$, it is said to be discriminant and denoted by Δ.
   - It $b^2 - 4ac > 0$ then roots are real and distinct
   - It $b^2 - 4ac = 0$ then roots are real and coincident.
   - It $b^2 - 4ac < 0$ then roots are not real but distinct.

   Discuss the converse of the above statements.

4. Show that sum of the roots of a quadratic equation $ax^2 + bx + c = 0$ is $\frac{-b}{a}$ and product is $\frac{c}{a}$ Also obtain the value for $|\alpha - \beta|$

5. Obtain quadratic equations which roots are symmetric expressions of α, β
   - Find the roots of a quadratic equation in term of α and β where the equation which roots are α, β is given.
   - When the equation which roots are α, β is given, guide the students to deduce the quadratic equation which roots are symmetric expression of α and β .

6. Guide students to solves problems involving quadratic functions and quadratic equations

Mathematics - II

Competency 1 : **Interprets the basics of statistics**

Competency level 1.1 : Investigates the nature of statistics

Number of periods : 03

Learning out comes :
1. Explains statistics and its nature
2. Explains probability and distribution theory
3. Distinguish between descriptive and inferential
4. Identify the role of probability in (inferential) statistics
5. Identify some application of statistics

Guidelines to learning - teaching process :

1.
   - Get some tables and figures probability from current news papers and ask the students, how they were constructed.
   - Takes some marks of the students and ask them to categories into grades.
   - Based on the above task explain the nature of statistics as the science of data which involves data collection, summarization, presentation and generating information to make decision.

     Note that the above only examples you may used student sensitive examples.

2. Discuss situation like the chance of getting train tonight based on that, explains the probability as a measure of uncertainty.

3. Gives some examples where sampling is used to make decisions such as blood test, checking rice before buying based on the above examples introduce inferential statistics as obtaining conclusion to a giber set (population) based on the data collected from a samples.

   Explains that descriptive statistics will the conclusion to the set where data were collected from all items.

   Establish the distinction between descriptive statistics and inferential statistics.

4. Show that exact conclusion cannot be obtained through inferential statistics, therefore there is an uncertainty in the conclusion given to quantity the uncertainty using probability.
5. gives some examples of applications of statistics in different subject areas such as,
   - Education, Agriculture, Medicine.

Competency level 1.2 : Manipulates data to obtain information.

Number of periods : 03

Learning out comes :
1. Explains data and information.
2. Explains controlled experiments censuses and surveys.
3. Explains the types of data.

Guidelines to learning - teaching process :

1. Ask the students to summarize the data collected to achieve learning outcome 1.
   - Show that original data cannot be used to make decision and summaries data can be used.
   - Explains that summarized data or analyses data know as information.
   - States that data known as information.
   - States that data analysis is the process of generating information from data.
2. Explains Explains controlled experiments censuses and surveys.

3. Ask the students to collect data on some variables covering all types of data.
   - Qualitative data
     - Nominal, Ordinal
   - Quantitative data
     - Discrete and continuous.

Competency 2 : **Presents data and information systematically**

Competency level 2.1 : Classifies data

Number of periods : 02

Learning out comes : 1. Classifies data.

2. States aims and basis of classification of data.

Guidelines to learning - teaching process :

1.
- Ask students to calculate their BMI (Body Mass Index) and ask them to classify BMI values under the following categories.

  Under weight, normal weight, over weight

  $$\text{Body mass index} = \frac{\text{weight (in kg)}}{(\text{height (in m)})^2}$$

- Gives guidelines for under weight normal weight and over weight

2. Show that data classification its done to get information about different groups.
- Tell to students that round figures are used as class unit and be used classes with equal width more frequently due to convince in drawing histogram etc.
- Stress that the range of the values is divided into classes such a way that each value should go into one but not to two classes (Mutually exclusive and collectively exhaustive)

Competency level 2.2 : Tabulates data

Number of periods : 02

Learning out comes : 1. Prepares ungrouped frequency distribution.

2. Prepares grouped frequency distribution.
3. Constructs two way tables.
4. Explains importance of tabulation.

Guidelines to learning - teaching process :

1.
   - Ask the students to construct ungrounded frequency distribution for normal, ordinal and quantitative data with small number of values.
   - Ask the students to construct group frequency distribution for quantitative variable having large number of values.
2.
   - In addition to the frequency ask the students to includes relative frequency (percentage for easy understanding of a table)
   - Ask the students to interpret the table.
3. Explains two way tables.
4. Guide students to explains the importance of two way tables.

Competency level 2.3 : Represents data and information using charts.

Number of periods : 03

Learning out comes :
1. Identifies the significant of using charts.
2. Uses different types of charts to represent data.
3. Uses maps to represent data.

Guidelines to learning -teaching process :

1.
   - Show a chart and a table constructed for the same data, and ask the students that which one its more sensitive to them.

     (To get an idea about the data quickly)
   - Stress that the tables are to give exact figures and charts are to give a rough idea quickly (More sensitively)
2.
   - Introduce the following charts with examples for sibyl based on the collected by students.
   - Simple bar charts (Horizontal, Vertical) to represent ungrounded frequency distribution) stress that histogram is the most suitable presentation for grouped frequency distribution.
   - Compound bar chart (Stack bar chart) to show give more details about the composition of a simple bar chart.
   - Multiple bar charts (Cluster, bar charts) to compare individual components of an entity
   - Pie charts to show the relative size of a component of an entity.

3. • Maps and graphs

Show that maps also can be used to represent data

Eg : Rain fall of an area, different color for different range of rain fall.

Graphs to represent the relationship between two variables.

Eg :The relationship between cost of advertising and sales.

Competency level 2.4 : Represents data and information graphically

Number of periods : 03

Learning out comes :

1. Describes line graphs.
2. Draws histogram.
3. Draws frequency polygon.
4. Draws frequency curves.
5. Smooth frequency curves.
6. Draws cumulative frequency curves.
7. Solves problems by extracting information from graphs.

Guidelines to learning - teaching process :

1. Discuss the following graphical techniques
   - Line graphs
   - Line graphs for more than one variables
2. Describes Histogram and its properties
3. Explains Frequency polygons through examples
4. Explains Frequency curves through examples
5. Guide stiudents to Smooth frequency curves
6. Smooth frequency curves
6. Ogive or cumulative frequency curves
7. Guide students to solves problems involving above graphs

Second Term

Mathematics - I

Competency 12 : **Investigates straight lines in terms of Cartesian coordinates**

Competency level 12.1 : Describes the rectangular Cartesian coordinate systems

Number of periods : 01

Learning out come : 1. Plots points on a Cartesian plane

Guidelines to learning - teaching process :

1. • Introduce the Cartesian plane
   - Explains the horizontal and vertical axes as X,Y axis.
   - Explain the origin and sign changes along the X and Y axis.
   - Explain the X - coordinate and Y coordinate of a point as *x, y*

Competency level 12.2 : Finds the distance between two points and the area of a triangle

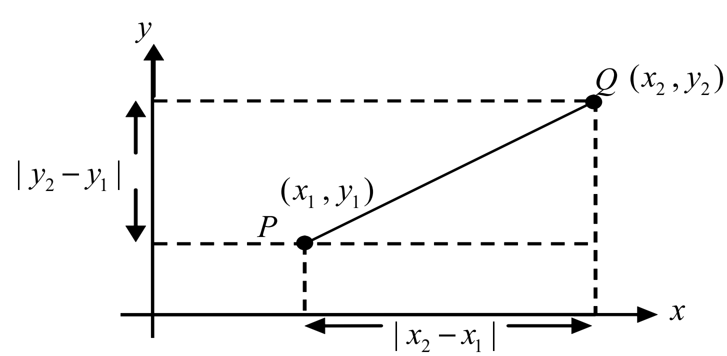
Number of periods : 06

Learning out comes :

1. Writes the formula for the distance in between two points in a Cartesian plane
2. Writes the co-ordinaries of a point which divides a joining two given points at a given ratio
3. Finds the area of a triangle when the vertices are given

Guidelines to learning - teaching process :

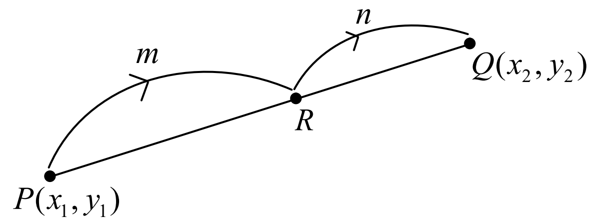
1. Distance between two points.



$PQ = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$ $\qquad \therefore \; PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. Co - ordinates of the point which divides the line coining two given points.

Internally

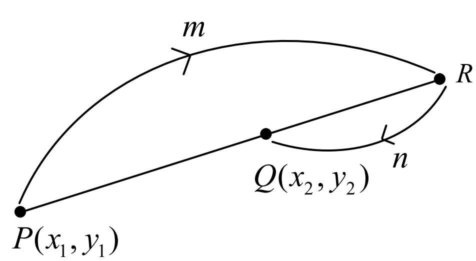


$$\frac{PR}{PQ} = \frac{m}{n}$$

$$R \equiv \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

Guide students to prove the above result.

Externally



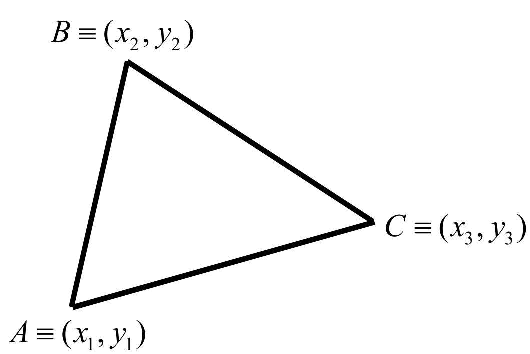
$$Q \equiv \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right)$$

Guide students to prove the above results.

Discuss the case when $m < n$.

Guide students to find mid point and centroid of a triangle.

3. Area of a triangle



Guide the students to obtain the area of the triangle ABC by using the area of trapeziums.

$$\text{area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\{x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2)\}$$

$$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

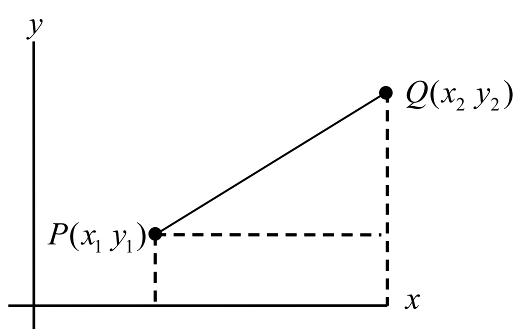
Competency level 12.3 : Describes the equation of a straight line

Number of periods : 06

Learning out comes :
1. Finds the gradient of a straight line
2. Finds x-intercept and y-intercept of a straight line

Guidelines to learning - teaching process :

1. Introduce the gradient of a line connecting two points



Introduce gradient as a slope gradient

$$m = \frac{\text{difference in } y \text{ co-ordinate}}{\text{difference of } x \text{ co-ordinate}}$$

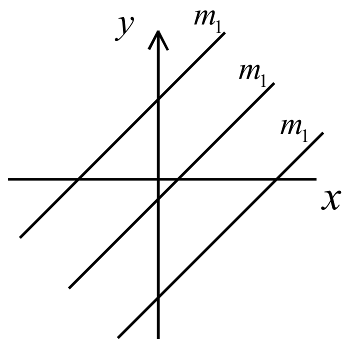
Where difference in x

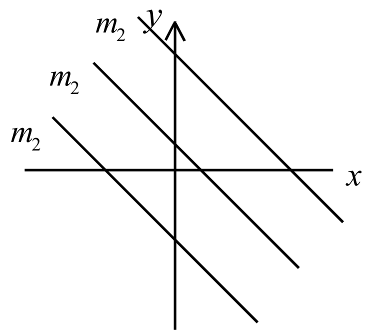
$$M = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Discuss the cases (i) m is positive
(ii) m is Negative

Parallel line:

States that gradient of parallel line are equal



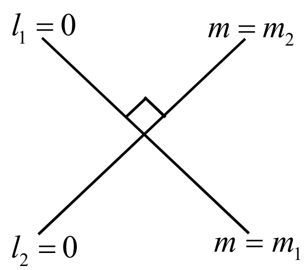


Gradientsare equal but positive

Gradientsare equal but negative

Perpendicular lines :

States that product of the gradients of two perpendicular lines is equal to -1.

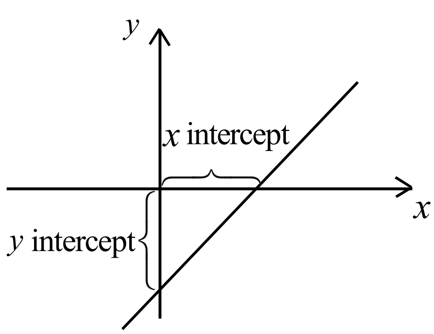


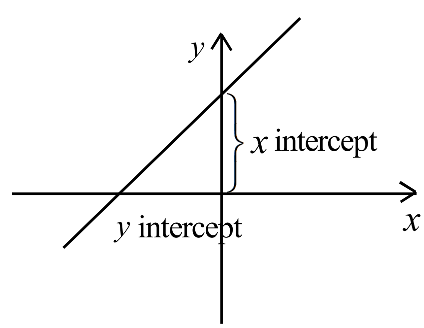
If the lines $l_1 = 0$ and $l_2 = 0$ are perpendicular each other then $m_1 m_2 = -1$.

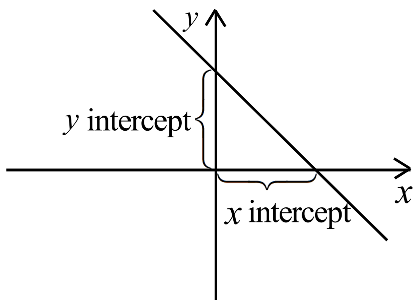
Discuss the following cases.

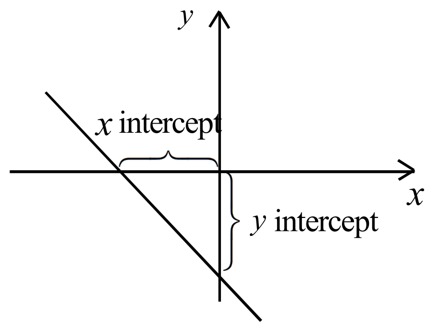
- one line parallel to *x* axis.
- one line parallel to *y* axis.

2. Introduce the *x* intercept, *y* intercept of a lines.









Competency level 12.4 : Interprets the equation of a straight line

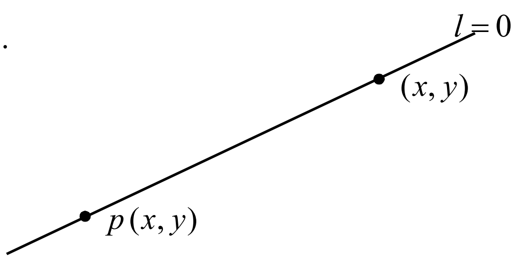
Number of periods : 12

Learning out comes :

1. Obtains equation of straight lines in Point - Gradient form
2. Obtains equation of straight lines in gradient intercept form
3. Obtains equation of straight lines in two point form
4. Obtains equation of straight lines in intercept form
5. Obtains equation of straight lines in General form
6. Obtains equation of straight lines according to the data given

Guidelines to learning - teaching process :

1.



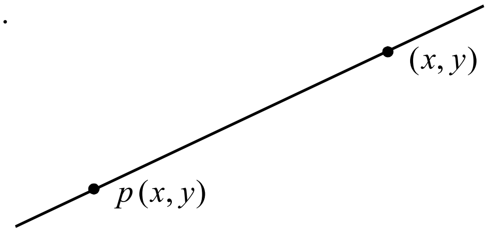
Gradient is m

$$m = \frac{y - y_1}{x - x_1}, \; x \neq x_1$$

$$\therefore \; y - y_1 = m(x - x_1)$$

equation of the line is $y - y_1 = m(x - x_1)$

2.

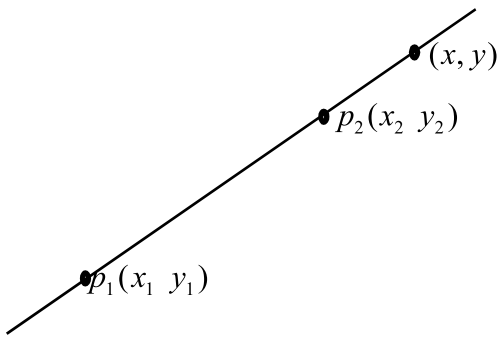


Gradient is m

$$y - y_1 = m(x - x_1)$$
$$y = mx + (y_1 - mx_1)$$
$$y = mx + c$$

equation of the line is $y - y_1 = m(x - x_1)$
Where m - gradient of the line
c - intercept on y axis and $c = (y_1 - mx_1)$

3.

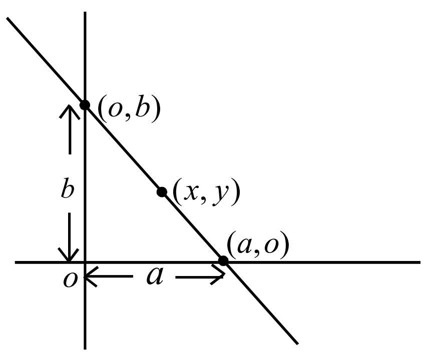


Gradient of the line

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

4.



Gradient m

$$\frac{b - o}{o - a} = \frac{y - b}{x - o}$$
$$-\frac{b}{a} = \frac{y - b}{a}$$
$$-\frac{x}{a} = \frac{y - b}{b}$$
$$\frac{x}{a} + \frac{y}{b} = 1$$

equation of the line is $\frac{x}{a} + \frac{y}{b} = 1$

5. Guide students to obtain the general equation $ax + by + c = 0$ from all of the above.

- $y - y_1 = m(x - x_1)$

  $mx - y + (y_1 + mx_1) = 0$
  $ax + by + c = 0$

  equation of the line is $ax + by + c = 0$, where $a = m$, $b = -1$, $c = y_1 - mx_1$

- $y = mx + c$

  $mx - y + c = 0$

  $ax + by + c = 0$

  $a = -m \; , \; b = 1 \; , \; c = -c$

  equation of the line is $ax + by + c = 0$ where $a = -m \; , \; b = 1 \; , \; c = -c$

- $y - y_1 = \left(\dfrac{y_2 - y_1}{x_2 - x_1} \right)(x - x_1)$

  $\left(\dfrac{y_2 - y_1}{x_2 - x_1} \right) x - y + \left(y_1 - \dfrac{y_2 - y_1}{x_2 - x_1} \right) x_1 = 0$

  $ax + by + c = 0$

  equation of the line is $ax + by + c = 0$

  where $a = \dfrac{y_2 - y_1}{x_2 - x_1}, b = -1, c = \left(y_1 - \dfrac{y_2 - y_1}{x_2 - x_1} \right) x_1$

6. $\dfrac{x}{a} + \dfrac{y}{b} = 0$

   $x + \left(\dfrac{a}{b} \right) y + c = 0$

   $a_1 x + b_1 y + c_1 = 0$

   where $a_1 = 1, \; b_1 = \dfrac{a}{b}, \quad c = 0$

   Discuss the equation of line when

   - line parallal to x axis
   - line parallal to x axis

   Guide students to solve various problems involving above theories.

Competency level 12.5 : Derives the equation of a straight line passing through the point of intersection of two given straight lines

Number of periods : 05

Learning out comes :

1. Finds the coordinate of the point of intersection of two non parallel lines
2. Interprets and uses the equation

Guidelines to learning - teaching process :

1. Stress that the point of intersection of two non parallel line can be obtained by solving equation of the given line.
   - Guide students to solve problems involving intersection of two non parallel lines.
2. Obtains the equation of lines passing through the intersection of two parallal lines $u = 0$ and $v = 0$ as $u + \lambda v = 0$
   - Give suitable examples to make the students comfortable in this regards

Competency 7 : **Investigates rational functions, exponential functions and lorarithms functions.**

Competency level 7.1 : Resolution of a rational function into partial fractions

Number of periods : 15

Learning out comes :
1. Defines rational functions
2. Defines proper rational functions and in proper rational functions
3. Finds patial fractions of proper rational functions.
4. Finds partial fractions of proper rational functions. (not more than 4 unknowns are expected)

Guidelines to learning - teaching process :

1. A function of the from $\dfrac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials in x with $Q(x) = 0$ is called a rational function. Its domain is the set of values which $Q(x) \neq 0$.

Ex. 1. $\dfrac{x^2+1}{x^3+x^2+1}$

where $P(x) = x^2 + 1$

$Q(x) = x^2 + x + 1$

2. $\dfrac{1}{x^2+2x+1}$

where $P(x) = 1$

$Q(x) = x^2 + 2x + 1$

3. $\dfrac{x^4+x^2+1}{x^2+1}$

where $P(x) = x^4 + x^2 + 1$

$Q(x) = x^2 + 1$

4. $\dfrac{3x^2+2x+1}{x^2+1}$

where $P(x) = 3x^2 + 2x + 1$

$Q(x) = x^2 + 1$

2. If the degree of the polynomial in the numerator is less than the degree of the polynomial in the denomination then the rational function is said to be a proper rational function.

Eg. 1 $\dfrac{x-1}{x^2+5x+6}$

where $P(x) = x+1$

$Q(x) = x^2 + 5x + 6$

degree of $P(x) = 1$ degree of $Q(x) = 3$ degree of $P(x) <$ Q(x)

Eg. 2 $\dfrac{x^2+x+1}{(x+1)(x+2)(x+3)}$

where $P(x) = x^2 + x + 1$

$Q(x) = (x+)(x+2)(x+3)$

degree of $P(x) = 2$ degree of $Q(x) = 3$ $\therefore$ degree of $P(x) < Q(x)$

- If the degree of the polynomial in the numerator is greater then the degree of the polynomial in the denomination then the rational function is said to be improper rational function.

Eg: 1. $\dfrac{x^3}{x^2-1}$

where $P(x) = x^3$

$Q(x) = x^2 - 1$

degree of $P(x) = 3$ degree of $Q(x) = 2$ $\therefore$ degree of $P(x) > Q(x)$

Eg: 2. $\dfrac{x^4-x+1}{(x^2+1)(x-1)}$

where $P(x) = x^4 - x + 1$

$Q(x) = (x^2+1)(x-1)$

degree of $P(x) = 4$ degree of $Q(x) = 3$ $\therefore$ degree of $P(x) > Q(x)$

Eg: 3. $\dfrac{x^2-4}{x^2-5x+6}$

where $P(x)=x^2-4$

$Q(x)=x^2-5x+6$

degree of $P(x)=2$

degree of $Q(x)=2$

$\therefore$ degree of $P(x)=Q(x)$

3. Guide students to resolves rational function into partial functions (Maximum 4 unknowns)

4. - when $Q(x)$ can be expressed linear factor

   $$\frac{P(x)}{(x-\alpha)(x-\beta)}=\frac{A}{(x-\alpha)}+\frac{B}{(x-\beta)}$$

   (Maximum 4 unknowns)

   - when $Q(x)$ can included repelled linear factor

   $$\frac{P(x)}{(x-\alpha)^2(x-\beta)}=\frac{A}{(x-\alpha)}+\frac{B}{(x-\alpha)^2}+\frac{C}{(x-\beta)}$$

   - when $Q(x)$ can be expressed one or two quadratic factor

   Ex: 1. $\dfrac{P(x)^2 2x+r}{(x^2-\alpha)(x-\beta)}=\dfrac{Px^2+Qx+r}{(x^2-\alpha)(x-\beta)}+\dfrac{Ax+B}{(x-\alpha)^2}+\dfrac{C}{(x-\beta)}$

   Ex: 2. $\dfrac{P(x)^3 2x+r}{(x^2+\alpha)(x^2+\beta)}=\dfrac{Px^2+Qx+r}{(x^2+\alpha)(x^2+\beta)}+\dfrac{Ax+B}{(x^2+\alpha)}+\dfrac{C}{(x^2+\beta)}$

5. Guide students to resolves improper rational functions into partial functions (Maximum 4 unknowns)

   - If degree *of* $P(x)$ = degree of $Q(x)$ then $\dfrac{P(x)}{Q(x)}$ csn be written in the form

   $\dfrac{P(x)}{Q(x)}=k+\dfrac{R(x)}{Q(x)}$ Where degree of $R(x)<$ degree of $Q(x)$ and k is a constant.

Eg: 1. $\dfrac{2x^2+1}{x^2-5x+6} = k + \dfrac{R(x)}{Q(x)}$

Where *k, R(x)* to be found

Where

$P(x) = 2x^2+1$

$Q(x) = x^2-5x+6$

degree *of* $P(x)$ = degree of $Q(x)$

degree of $R(x) <$ degree of $Q(x)$

- If degree *of* $P(x)$ > degree of $Q(x)$ then $\dfrac{P(x)}{Q(x)}$ csn be written in the form

$\dfrac{P(x)}{Q(x)} = h(x) + \dfrac{R(x)}{Q(x)}$ and $h(x)$ in a polynomial called the quotient when $P(x)$ divided by $Q(x)$

We have to find $h(x)$ and express $\dfrac{R(x)}{Q(x)}$ into partial functions.

Where

$P(x) = x^2-2x+1$

$R(x) = x-1$

Here degree of $R(x)$ C degree of $Q(x)$.

Stress the following when $\dfrac{P(x)}{Q(x)}$ is improper rational function and

- degree *of* $P(x)$ - degree of $Q(x)$ = 2 then

  $\dfrac{P(x)}{Q(x)} = A_1x + B_1 + \dfrac{R(x)}{Q(x)}$

- degree *of* $P(x)$ - degree of $Q(x)$ = 1 then

  $\dfrac{P(x)}{Q(x)} = Ax^2 + Bx + C + \dfrac{R(x)}{Q(x)}$

  also stress that maximum unknowns is 4

Competency level 7.2 : Analyses the Exponential and Logarithmic functions

Number of periods : 15

Learning out comes :
1. States the properties of exponential functions
2. Sketches graph of exponential functions
3. States the properties of and draws its graph
4. States the properties of lnx
5. Writes change of base of a logarithmic function
6. Draws the graph of $\ln x$
7. Compares the relations between $\ln x$ and e^x
8. Finds the compound Interest, population growth using proper equation.

Guidelines to learning - teaching process :

1. State that the sum of the infinite series,

   $1+\frac{x^1}{1!}+\frac{x^2}{2!}+....+\frac{x^n}{n!}+...$ is denoted by e^x, and that it is known as the exponential function.

2.
   - Implosive since x is an exponent, e^x is refered as the exponential function. State if
   - $f_{(x)}=e^x$, $D_f=\mathbb{R}$, $R_f=\mathbb{R}^+$,
   - Guide students to sketches the graph of exponential functions
   - e is the sum of the above infinite series when $x = 1$ and is a positive irrational number.
   - $1+\frac{1}{1!}+\frac{2}{2!}+....+\frac{3}{n!}+...$

3. States the following
   - $e^0=1$
   - $e^{x_1+x_2}=e^{x_1}\,.\,e^{x_2}$
   - $e^{x_1-x_2}=\frac{e^{x_1}}{e^{x_2}}$
   - For rational values of r, $\left(e^x\right)^r=e^{rx}$

4. Explain that $\ln x$ defined as is known as $y = \ln x \Leftrightarrow x = e^y$, is known as the natural logarithmic function.

If then main of $g_{(x)} = \ln(x)$ is $x > 0$ and the range is then $D_g = \mathbb{R}^+, R_g = \mathbb{R}$

- $\ln x y = \ln x + \ln y$ for $x > 0$, and $y > 0$
- $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$ for $x > 0$, and $y > 0$
- $\ln(x^p) = P \ln x$; $x > 0$

5. Guide sstudents to prove the chang of base in logarithms.
6. Guide the students to draw the graph of lnx.
7. Guide the students to compares the relations between $\ln x$, lnx and e^x,
8. By using suitable examples guide student to Finds the compound Interest, population growth using proper equation.

Competency 4 : **Manipulates the methods of proofs to prove the mathematical results**

Competency level 4.1 : Proves Mathematical results by using direct proof, proof by contradiction and proof by mathematical induction

Number of periods : 12

Learning out comes :
1. States methods of proofs
2. Describes direct proof, proof by contradiction ,proof by contrapositive and proof by principle of mathematical induction
3. Solves problems involving different type of proofs

Guidelines to learning - teaching process :

1. Describe the methods of proofs and for given the above mathematical statement.

2. • Describe the direct proof

Consider an implication $P \Rightarrow q$.

It P is false, then the implication is always true.

True, show that if P is true, them q is true.

To perform a direct proof, assume that P is true, and show that q must therefore be true.

Eg: 1. Show that if n is even n^2 is even, where n is a natural number.

Proof

Assume n is even.

Then $n=2x$, for some integer x

Therefore $n2=(2x)^2= 4x^2=2(2x)^2$

Therefore n^2 is 2 times an integer

Hence n^2 is even.

Therefore if n is even, then n^2 is even.

Eg: 2. Show that if n is an odd integer, then $5n+3$ is an even integer.

Proof

Assume n is an odd integer.

Then $n = 2x+1$, for some integer x

$$\begin{aligned} \text{Now, } 5n+3 &= 5(2x+1)+3 \\ &= 10x+5+3 \\ &= 10x+8 \\ &= 2(5x+4) \\ &= 2m \quad \text{where} \quad m = 5x+4 \end{aligned}$$

Since x is an integer, we must have m is an integer hence $5x+3=2m$ for soma integer m

Thus $5n+3$ is an even integer.

Therefore if n is an odd integer, then $5n+3$ is an even integer.

- Describe the proof by contradiction.

The method of "proof by contradiction" relies in the fact that, If not P is false, then P is true. hence, to prove that P is true, we attempt to show that not P is false. Here, we add not P to the list of given statements and attempt to show that this argument list of statement leads to a contradiction. When the contradiction is reached, we know hat not P is not consist as with our given true statements as hence that it is false. Hence P is true.

Given a statement P

Assume it is false.

i.e assume ~P is true.

Prove that ~P leads to false.

A contradiction exists.

Eg: 1. Show that if n^2 is an odd integer, then n is an odd integer.

Proof

Assume " n is an odd integer, then n is an odd integer" is false.

i.e n^2 is an odd integer and n is an even integer since n is an even $n=2m$ for some integer m

Therefore, $n^2 = (2m)^2 = 4m^2 = 2\times 2m^2$. There $x=2m^2$ is an integer. Hence n^2 is even, This is contradiction.

Therefore if n^2 is an odd integer, then n is an odd integer.

Eg: 2. Prove that if $n^2 \neq 25$ then $n \neq 5$

Proof

Assume " $n^2 \neq 25$ then $n \neq 5$ " is false.

Then $n^2 \neq 25$ and $n \neq 5$

Therefore $n^2 \neq 25$ and $n^2 = 25$. This is contradiction.

Hence if $n^2 \neq 25$ then $n \neq 5$

- **Proof by contrapositive**

  when direct proof is hard we can use contrapositive method. That is "If A then B" $(A \Rightarrow B)$ is logically equivalent to if $\sim B$ then $\sim A$ $(\sim B \Rightarrow \sim A)$.

  To prove a statement ,form the contrpositive of the statement and prove it.

Eg: Prove that

When $n \in \mathbb{Z}$,

if $n^2 - 6n + 5$ is even then n is odd.

Contrpositive when $n \in \mathbb{Z}$,

if n is even then $n^2 - 6n + 5$ is odd.

Let n is even

$\therefore\ n = 2m$ for some m.

$$n - 6n + 5 = (2m)^2 - 6(2m) + 5$$

$$= 4m^2 - 12m + 5$$

$$= (2(2m^2 - 6m + 2) + 1$$

$\therefore\ n^2 - 6n + 5$ *is odd.*

Hence the prove.

- **Proof by mathematical induction.**

  The way used in mathematical induction is to prove the first statement in the sequence, and then. Prove that if any particular statement is true, then the one after it is also true. This enables us to conclude hat all the statements are true.

Let $P(n)$ be predicate that is defined for all positive integers n

If (1) $P(1)$ is true and

(2) "if $P(x)$, then $P(x+1)$" is true. Then $P(n)$ is true for all positive integer.

Eg: 1. Prove that $1+2+3+..........n=\frac{n}{2}(n+1)$ where n is a natural number.

Proof

Let $P(n)=1+2+3.......+n=\frac{n}{2}(n+1)$

Let $n=1$ $L.H.S\ =P(1)=1$

$$R.H.S\ =\frac{1(1+1)}{2}=1$$

$$L.H.S\ =R.H.S$$

$P(1)$ is true.

Assume $P(x)$ is true.

Then $1+2+3+........+x=\frac{x}{2}(x+1)$

Add $(x+1)$ both side

$$1+2+3+........+x+(x+1)=\frac{x}{2}(x+1)+(x+1)$$

$$=(x+1)\left(\frac{x}{2}+1\right)$$

$$=(x+1)\left(\frac{x+2}{2}\right)$$

$$=\frac{(x+1)}{2}[(x+1)+1]$$

i.e $P(x+1)$) is true.

Therefore "if $P(x)$, then $P(x+1)$ is true.

By principle of mathematical induction. $P(n)$ is true for all natural number n

i.e $1+2+3+.........+n=\frac{n}{2}(n+1)$ for all natural number *n.*

Eg: 2. Use mathematical induction to show that

$1+2+2^2+........+2^{n-1}=2^n-1$ for all positive integer *n*.

Proof

Let $P(n)=1+2+2^2+......2^{n-1}=2^n-1$

$L.H.S\ =P(1)=1$

$R.H.S\ =2^1-1=1$

$L.H.S\ =R.H.S$

hence. $P(1)$ is true.

Assume $P(k)$ is true.

Then $1+2+2^2+..........2^{k-1}=2^k-1$

Therefore

$$1+2+2^2+..........+2^{k-1}+2^k=2^k-1+2^k$$
$$=2^k2-1$$
$$=2^{k+1}-1$$

hence, $1+2+2^2+..........+2^{k-1}+2^k=2^{k+1}-1$

i.e $P(k+1)$ is true.

Therefore "if $P(k)$, them $P(k+1)$" is true

By principle of mathematical induction $P(n)$ true for all natural number *n*.

i.e $1+2+2^2+..........2^{n-1}=2^n-1$ *f*or all positive integer *n*

3. Guide the students to solve the different involving different types of proof.

Eg : (1) The product of two odd integers is an odd integer.

(2) The sum of two even integers is an even integer

(3) If n is an odd integer, then 5n+11 is an even integer

(4) $1+3+5+.......+(2n-1)=n^2$ for all natural numbers *n*

(5) $1+4+7+...........+3n-2=\dfrac{n(3n-1)^2}{2}$ for all natural numbers *n*

Mathematics - II

Competency 3 : **Interprets the behavior of a frequency distribution**

Competency level 3.1 : Analyses mean as a measure of central tendency

Number of periods : 10

Learning out comes : 1. Finds the central tendency measurement

Guidelines to learning - teaching process :

1 • Introduce mean as a measures of central tendency.

- When a variables is measured from a homogeneous group most of the measurments will be gathered aroud the middle of the data set. This tendency is known as central tendency.

  Eg. Height of the students in grade 12 varing around 160 cm. Most of the students hight are around 160 cm less students height are around 150 cm and 170 cm. Therefore centre of height is here is 160cm.

- Introduce the following formula to find mean of ungrouped data set.

- Let $x_1, x_2, x_3, \ldots, x_n$ are n observation then arithmetic mean is $\frac{\sum_{i=1}^{n} x_i}{n}$ where

  $\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \ldots\ldots + x_n$

- Introduce the coding method to find mean of a data set

  - Introduce code as $y_i = x_i - A$ where A is any number

  - Obtain the formula as $\overline{x} = A + \overline{y}$

- Introduce the following formula to find the mean of ungrouped frequency distribution.

  Let $x_1, x_2, x_3, \ldots, x_n$ are n observation with corresponding frequencies $f_1, f_2, f_3, \ldots, f_n$

  Then $\overline{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

  where $\sum_{i-1}^{n} f_i x_i = f_1 x_1 + f_2 x_2 + f_3 x_3 + \ldots + f_n x_n$

  $\sum_{i=1}^{n} f_i = f_1 + f_2 + f_3 + \ldots + f_n$

- Introduce the method of coding to find the mean Cording is $y_i = x_i - A$

  Then $\overline{y} = \overline{x} - A$

  Where $\overline{y} = \frac{\sum_{i=1}^{n} f_i y_i}{\sum_{i=1}^{n} f_i}$

  Give examples in different type and guide students to solve different problems.

- Introduce the following formulas to find the mean of grouped frequency distribution. Let $x_1, x_2, x_3, ...x_n$ are the mid prints of the also intervals with corresponding frequency $f_1, f_2, f_3, ..., fn$

  Then $\overline{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

  Introduce the method of cording to find the mean

  Coding is $y_i = \frac{x_i - a}{b}$

  Where x_i is the mid point *a, b* are comfortably chosen numbers
  Then $\overline{x} = b\overline{y} + A$

  - Prove the above result.
  - Uses the above result to find mean of grouped frequency distribution

Guide students to solve problems involving mean.

- Guide students to find a cording if the mean of the new set of data is given.

  Eg: 1. Find the mean at the following data set

  4, 6, 8, 12, 14, 16, 17

  Hence find the mean of the following data sets using suits coding

  (i) 104, 106, 108, 112, 113, 117

  (ii) 10.4, 10.6, 11.2, 11.3, 11.7

  Eg: 2. The mean marks obtained by 10 students for an examination is 40. This marks to be scaled such as the mean will be 50. Using the Codi $y = x_i - A$. where x_i is the original marks and y_i is the marks after coding.

  (i) Find the value of A

  (ii) It one student original marks 50 then find the new marks.

  (iii) It one student new marks is to find the students original marks

2. **Weighted mean :** In a data set, according to their importance we can assign some weight for them, before finding their mean. Then we can find their mean according to their weight this mean is called weighted mean.

Let $x_1, x_2, x_3, \ldots, x_n$ are numbers with weight $w_1, w_2 \ldots w_n$ then weighted mean of then above numbers are

$$\left(\frac{w_1x_1 + w_2x_2 + \ldots + w_nx_n}{w_1 + w_2 + \ldots + w_n}\right)$$

$$w_1x_1 + w_2x_2 + \ldots + w_nx_n$$

since $w_1 + w_2 + \ldots + w_n = 1$

Give examples for weighted mean

- It $x_1, x_2, x_3 \ldots x_n$ are n positive numbers then Geometric mean of there numbers are denoted by

$$G.M. = (x_1 \times x_2 \times x_3 \times \ldots \times x_n)^{\frac{1}{n}}$$

$$G.M. = \sqrt[n]{x_1 \times x_2 \times x_3 \times \ldots \times x_n}$$

Note that Geometric mean defined only for positive numbers and the answer also positive.

Competency level 3.2 : Interprets the frequency distribution in terms of values of relative positions

Number of periods : 14

Learning out come : 1. Finds the relative position of frequency distribution

Guidelines to learning / teaching process :

1. Median
   - Introduce median for ungrouped data set.
   - Introduce median for ungrouped frequency distribution
- Introduce median for grouped frequency distribution
  - Median class
  - Median using cumulative frequency distrubuion.
  - Median using liner interpolation

Quartiles

- Introduce quartiles for ungrouped data set.
- Introduce quartiles for ungrouped frequency distribution.
- Introduce quartiles for ungrouped frequency distribution using
    - Cumulative frequency curve
    - Linear interpolation
    - Formula

Deciles and percentiles

- Introduce deciles and percentiles to ungrouped frequency distribution.
- Introduce deciles and percentiles to ungrouped frequency distribution using
    - Cumulative frequency curve
    - Linear interpolation

Third Term

Mathematics - I

Competency 8 : **Manipulates inequalities.**

Competency level 8.1 : Solves problems involving linear and quadratic inequalities

Number of periods : 10

Learning out comes :
1. Solves linear and quadratic inequalities
2. Solves Simultaneous linar inequalities

Guidelines to learning / teaching process :

- When "a" and "b" are real numbers
  (i) $a > b$ if and only if *(a-b)* is positive
  $a > b$, $(a - b) > 0$
  (ii) $a < b$ if and only if *(a-b)* is negative
  $a < b$, $(a - b) < 0$

- Explain the above in these definitions using the number line
  When x and y are any two real-numbers either one of the following is true.
  $x > y, x < y, x = y$ this is said be trichotomy law

- Explain inequalities (trichotomy law) by using number line.
  Introduce the following interval notations for a set of numbers.
  When $a, b, a < b,$

Interval	Notation
$\{x \in \mathbb{R} \quad a \le x \le b\}$	$[a, b]$
$\{x \in \mathbb{R} \quad a \le x < b\}$	$[a, b)$
$\{x \in \mathbb{R} \quad a < x \le b\}$	$(a, b]$
$\{x \in \mathbb{R} \quad a < x < b\}$	(a, b)

- Explain following intervals as well.

$\{x \in \mathbb{R} \quad x \ge a\}$	$[a, +\infty]$
$\{x \in \mathbb{R} \quad x > a\}$	$[a, +\infty]$
$\{x \in \mathbb{R} \quad x \le a\}$	$[-\infty, \ a]$
$\{x \in \mathbb{R} \quad x < a\}$	$[-\infty, \ a]$

- States and proves fundamentals result in inequalities

  Results

  When $a, b, c \in \mathbb{R}$

  - $a > b$ and $b > c \Rightarrow a > c$
  - $a > b \Rightarrow a + c > b + c$
  - $a > b$ and $c > 0 \Rightarrow ac > bc$
  - $a > b > 0$ and $c < 0 \Rightarrow ac < bc$
  - $a > b$ and $c = 0 \Rightarrow ac = bc = 0$
  - $a > b$ and $c < d \Rightarrow a + c < b + d$
  - $a > b > 0$ and $c > d > 0 \Rightarrow ac > bd$
  - $a > b > 0 \Rightarrow \frac{1}{a} < \frac{1}{b}$
  - $a < b < 0 \Rightarrow \frac{1}{a} > \frac{1}{b}$

    For $a > b > 0$ and n is a positive rational number, a”> b” and a”< b”

1. Guide students to solves linear and quadratic inequalities
2. Guide students to solves simultaneous linar inequalities

Competency level 8.2 : Solves quadratic inequality using graphical method

Number of periods : 06

Learning out come : 1. Solves quadratic and simultaneous inequalities using graphs

Guidelines to learning / teaching process :

1. - Guide students to solve quadratic inequalities by using graphs

   - Guide students to solves problems involving simple taneous quadratic inequalities graphically

Competency level 8.3 : Solves inequalities involving rational functions.

Number of periods : 08

Learning out come : 1. Solves in inequalities of the form
where $f(x)$, $g(x)$ are polynomials of x (degree 3) and g (x)

Guidelines to learning / teaching process :

- Guide students to solve rational functions in the from $\dfrac{P(x)}{Q(x)}$ where $P(x)$, $Q(x)$ are polynomials in x.
- Steers that Highest order of $P(x)$, $Q(x)$ is less then or equal to 3. (Graphical method are not expected)

Competency 11 : Determines the limit of the function

Competency level 11.1 : Interprets the limit of a function and solves problems using the heorems on limits.

Number of periods : 08

Learning out comes : 1 .States the intuitive idea of a limit and theorem on limits

2. Proves $\lim\limits_{x\to a}\left(\dfrac{x^n - a^n}{x-a}\right) = na^{n-1}$ where n is a rational number.

3 .Applies the above theorem

Guidelines to learning-teaching process :

1. When $x \in \mathbb{R}$, discuss how the value x can approach a rational number *"a"* without being equal to it and discussed the behavior of $f(x)$ when x approach to number *"a"* and discussed x can approach to a in two ways which minus infinity to *"a"* called left hand limit $x \to a$ also plus infinity to number *"a"* called right had limit

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^2} f(x) = \lim_{x\to a} f(x) = 1$$

Also discussed the cases where $lim\ f(x)$ does not exist and distinguish between the $x \to a$ Limit of a function at a point and the value of a function at that point.

Assume *f* and *g* to be functions for which a limit exist as $x \to a$ where *a* is a real number then following theorems are can be express.

- Let $f(x) = K$ there $f(x) = K$

  $\lim_{x \to a} f(x) = K$

- Where *K* is constant $l \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

- $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

- $\lim_{x \to a} [f(x).g(x)] = \lim_{x \to a} f(x) \pm K \lim_{x \to a} g(x)$

- If $\lim_{x \to a} g(x) \neq 0,\ \lim_{x \to a} \left[\frac{f(x)}{g(x)}\right]$

- $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n,\ n \in \mathbb{N}$

- $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)},\quad n \in \mathbb{N}$

- When *f* is a polynomial function for all $x \in \mathbb{R}$

  $x \to a\ \ f(x) = f(a)$

- If $f(x) = g(x)$ for all values of x except at $x = a$ in an interval including number a than

  $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$

  the proofs of the above theorems are not excepted explain their use in solving problem with examples.

2. Prove the theorem $\lim_{x \to a} \left(\frac{x^n - a^n}{x - a}\right) = n\ a^{n-1}$ For positive integral values of *n*.

   Deduce it for negative integral values of *n* then prove that the theorem is true for any rational number *n*.

3. Students attention focus to solve the problem using there above result involving limits.

Mathematics - II

Competency 3 : **Interprets the behavior of a frequency distribution**

Competency level 3.3 : Analyses mode as a measure of central tendency

Number of periods : 04

Learning out come : 1. Finds the mode as a measure of central tendency

Guidelines to learning -teaching process :

- Introduce mode as a central tendency measure.
- Guide the students to find the mode for a
  - Ungrouped data set
  - Ungrouped frequency distribution
  - Grouped frequency distribution introduce the formula.

- Guide students to find mode for different sets of data.

mode $Mo = L + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2} \right) C$ where

L - lower boundary of the mode class

C - size of the mode clas

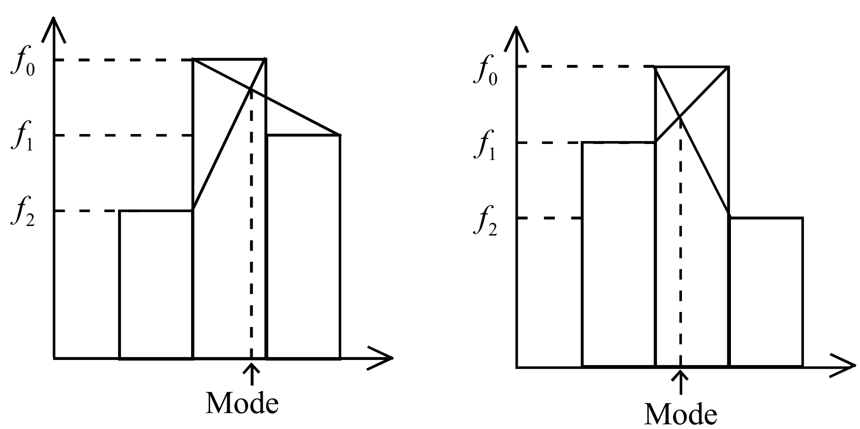
$\Delta_1 = f_0 - f_1, \quad \Delta_2 = f_0 - f_2$

f_0 - frequency of the mode clas

f_1 - frequency of the class before the mode clas

f_2 - frequency of the class after the mode clas

- guide students to obtain the mode in geometrical ways



Competency level 3.4 : Uses suitable measures of central tendency to reach decisions on frequency distributions.

Number of periods : 04

Learning out come :
1. States the relative importance of measure of central tendency
2. Selects a suitable measure of central tendency for given situations.

Guidelines to learning- teaching process :

1. Discusses the importance of measures of central tendency.

2. Selects a suitable measure of central tendency for given situations.

- When most of the values are around the centre or takes a certain value, then mode will be the good central measure.
- In the process of mean calcultion all values should be taken therefore mean is the important valuable measure among the other central measures
- To extend calculations, mean is the suitable measure
- Explains by using suitable examples that in a symmetric distibution change in a value of a data will affect the mean of the distribution.
- Explain that in a symmetric distibution mean or median or mode will be the suitable measure
- Explains that in a situation when the are open class intervals mean is not suitable and mode or median will be the suitable measure.

Competency level 3.5 : Interprets the dispersion of a distribution using measures of deviation

Number of periods : 10

Learning out comes :

1. Describes the measures of dispersions and their importances.

2. Uses suitable measure of dispersion to make decision on frequency distribution.

3 .Describes pool mean and pool variance.

4. Calculates pool mean and pool variance.

5. Uses coding to calculate variance.

6 .Solves problems involving liner transformation.

Guidelines to learning - teaching process :

1.
- Introduce the range s the difference between the value of highest observation any the value of lowest observation.
- Introduce the inter quartile range as Q_3- Q_1
- Introduce the inter semi quartile range (Quartile deviation) as $\frac{Q_2 - Q_1}{2}$
- Provide suitable example to find range interquartile range semi inter quartile range

Give examples in
- Ungrouped distribution
- Ungrouped frequency distribution
- Grouped frequency distribution
- For Grouped frequency distribution

- Introduce the formula to calculate the quartiles

Mean deviation:

- Introduce the mean deviation for ungrouped data set

$x_1, x_2, x_3, ..., x_n$ are n observer then mean deviation of the data set is defined as

$$MD = \frac{\sum_{I=1}^{N} |x_i - \bar{x}|}{n} \quad \text{where} \quad \bar{x} = \frac{\sum_{I=1}^{N} x_i}{n}$$

- Introduce the mean deviation for ungrouped frequency distribution

  Let $x_1, x_2, x_3, ..., x_n$ are n observations and their corresponding frequencies are

- then the nean deviation is

- Introduce the mean deviation for grouped frequency distribution

  Let $x_1, x_2, x_3, ..., x_n$ are the midvalues of the class intervals and their corresponding

  frequencies are $f_1, f_2, f_3, ..., f_n$ then $MD = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{\sum_{i=1}^{n} f_i}$

Variance and standard deviations

- Introduce the formula for variance to ungrouped data set

  Variance $\sigma^2 = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|^2}{n}$ Standard deviation $\sigma = \sqrt{\frac{\sum_{i=1}^{n} |x_i - \bar{x}|^2}{n}}$

Guide students to obtain the formula $\sigma^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} - (\overline{x})^2$

- Introduce the formula for variance for ungrouped frequency distribution

$$\sigma^2 = \frac{\sum_{i=1}^{n} f_i |x_i - \overline{x}|^2}{\sum f_i}$$

Where f_i is the corresponded frequency for the value x_i

Guide students to obtain the formula $\sigma^2 = \frac{\sum_{i=1}^{n} f_i (x_i)^2}{\sum_{i=1}^{n} f} - (\overline{x})^2$

- Introduce the formula for variance for grouped frequency distribution

$\sigma^2 = \frac{\sum_{i=1}^{n} f_i (x_i - \overline{x})^2}{n}$ where Let $x_1, x_2, x_3, ..., x_n$ are the midvalues of the class intervals and their corresponding frequencies are $f_1, f_2, f_3, ..., f_n$

Guide students to obtain the formula $\sigma^2 = \frac{\sum_{i=1}^{n} f_i (x_i)^2}{\sum_{i=1}^{n} f_i} - (\overline{x})^2$

2.• Guide students to use suitable measure of dispersion to make decision on frequency distribution.

3. **Pooled mean (Combined mean)**

Let $\overline{x}_1$ and $\overline{x}_2$ be the means of two sets of data with sizes n_1 and n_2 respectively.
Show that the pooled mean $\overline{x} = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2}$

Pooled varience

Let σ_1^2 and σ_2^2 be the variances of two sets of data with sizes n_1 and n_2 respectively. Show that the pooled variance.

$$\sigma^2 = \frac{1}{n_1 + n_2}\left\{n_1\sigma_1^2 + n_2\sigma_2^2\right\} + \frac{n_1 n_2}{(n_1 + n_2)^2}(\overline{x}_1 - \overline{x}_2)^2$$

- Guide students to prove and use the above formula.

4. Guide students to calculate pooled mean and pooled variance .

5 • Guide students to obtain the formula for coded data set

Let $x_1, x_2....x_n$ are the data set with frequencies $f_1, f_2... f_n$.

Consider the cording $y_i = \dfrac{x_i - A}{b}$

Obtain the formula $\sigma_x^2 = b^2\sigma_y^2$ and $\sigma_x = |b|\sigma_y$

Solves problems involving above coding

6 • Discuss about the linear transformation $y = ax+b$ and prove that,

- $\bar{y} = a\bar{x} + b$ • $\sigma_x = |b|\sigma_y$
- Guide students to solve problems involving linear transformation.

Eg: An exam consist two papers paper A and paper B. Marks obtains by the students having mean and variance such as

	Mean	Standard deviation
Paper A	62	13
Paper B	27	6

This marks are transformed such that both mean and standard deviation are 50 and 20 respectively.

(i) Find the transformation equation for paper A and paper B

(ii) A student got 80 marks paper A and 46 in paper B find his new marks according to the transformation given above States and prove the formula

Competency level 3.6 : Interprets coefficient of variation as a measures of dispersion

Number of periods : 03

Learning outcomes :
1. Explains coefficient of variation and solves problems
2. Solves problems involving coefficient variation

Guidelines to learning -teaching process :

1. Define the coefficient of variations (C.V)

2 Solves problems involving coefficient variance

Competency level 3.7 : Decides on the shape of a distribution using measures of skewness

Number of periods : 02

Learning out comes :
1. Defines the measure of skewness
2. States relationship between mean, median and mode
3. Finds measures of skewness
4. Describes the shape of distribution using measures of skewness

Guidelines to learning -teaching process :

1. Discuss the distribution of data and relates with skewness.

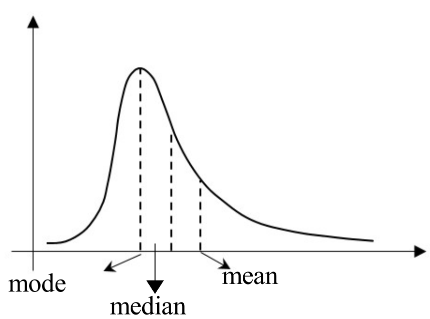
2. States the relations between mean, median mode
   mean - mode = 3 (mean - median)

3. Defines the Karl -Pearsons coefficient of skewness as (SK)

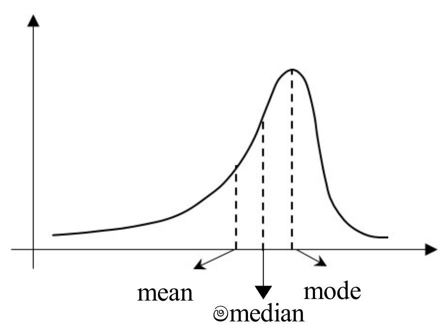
$$S_k = \frac{\text{Mean}}{\text{Standard deviation}}$$

4. Explains the shapes of distribution using coefficient of skewness skewed, positively skewed, negatively skewed.

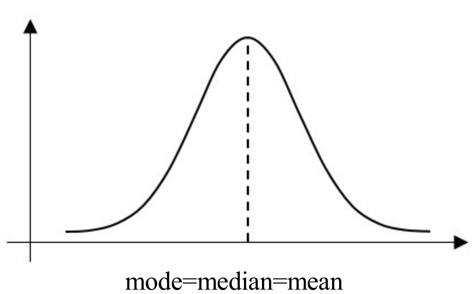
Positively skewed



Negatively skewed



Symmetric



Competency 4 : **Analyses random phenomena mathematically**

Competency level 4.1 : Determines the events of a random experiment

Number of periods : 08

Learning out comes :

1. Explains deterministic experiments.
2. Explains deterministic experiments.
3.Explains random experiments
4. Defines sample space and sample points
5. Defines an event
6. Explains types of events
7. Classifies the event
8. Defines union and intersection of two events
9. Explains mutually exclusive events Exhaustive events
10. Explains equally probable events
11. Explains event space
12.Solves Problems involving above concepts.

Guidelines to learning - teaching process :

1. Guide students to describe deterministic experiments.
2. Guide students to describe non- deterministic experiments.

3. A random experiment is an experiment, trial or observation that can be repeated numerous times under same. Conditions the out come of an individual random experiment must be independent and identically distributed it.
Must in no way be affected by any previous out come and cannot be predicated with certainly.
Eg: (1) The tossing of a coin.
(2) The roll of a die.
(3) The selection of a numbered ball 1 to 50 in an urn.
(4) Percentage of calls dropped due to errors over a particular time period.
(5) The time difference between two massages arriving at a massage center.
(6) The time difference between two different voice calls over a particular network.

4. The set of all the possible outcomes in considered random experiment is called the sample space of the experiment and it is usually dented by
Eg: 1. Tossing a coin.
Sample space $S = \{H, T\}$
Eg: 2. Tossing a die.
Sample space $S = \{1, 2, 3, 4, 5, 6\}$
Eg: 3. Tossing a die twice.
Sample space $S = \{(i, j) \,;\, i, j = 1, 2, 3, 4, 5, 6\}$

Eg: 4. Measuring the life time of a light bulb.

Sample space S = {(t : t≥0}

Eg: 5. Keeping on tossing a coin until one get a heds.

Sample space S = {H, TH, TTH, TTTH, ...}

5. An event is a subset of the sample space.

Eg: Tossing a two coins.

Sample space S = {HT, HH, TH, TT}

Event E_1 = {HT, TH}

E_2 = {HH}

6. Events distinguish between two types of events. Called simple events and compound event.

(a) Simple or elementary event.

If there be only are element of the sample space in the set representing an event then this event is called simple elementary event.

Eg: "throwing a die"

Sample space S= {1, 2, 3, 4, 5, 6}

Simple event E_1= {2}

E_2= {6}

(b) Compound event

If there are more than element of the sample space in the set representing an event and this event is called s Compound event

Eg: "throwing a die"

Sample space S= {1, 2, 3, 4, 5, 6}

Simple event E_1= {2, 4, 6}

E_2= {2, 3, 5}

7. (a) Certain events.

An event which is sure to occur at every performance of an experiment is called a certain events.

(b) Impossible events.

An event which cannot occur at any performance of the experiment is called an impossible event.

Eg: "Seven in case of throwing a die"

(c) Complementary events.

An event which consists in the negation of another event is called complementary event of the event.

Eg: In case of throwing a die, even face and odd face are complementary to each other

8. (a) The event A or B

When events A and B are two events associated with a sample space then "$A \cup B$" is the event either A or B or both. This event '$A \cup B$' is also called 'A or B' therefor event (A or B) = $A \cup B$

$$= \{x : x \in A \text{ } or \text{ } x \in B\}$$

(b) The event A and B

If A and B are the events then the set $A \cap B$ denotes the event A and B

Thus $A \cap B = \{x : x \in A \text{ } and \text{ } x \in B\}$

9. Mutually exclusive events

If there is no element incommonm for two or more events. i. e between two or more sub sets of the sample space, then these events are called mutually exclusive events

If E_1 and E_2 are two mutually exclusive events.then $E_1 \cap E_2 = \phi$

Eg: "Throwing a die"

Sample space S = { 1, 2, 3, 4, 5, 6}

Events E_1= {2, 4, 6} E_2= {1, 3, 6}

$E_1 \cap E_2 = \phi$

Hence E_1 and E_2 are Mutually exclusive

If E_4 = {2, 3, 5} and E_5 = {3, 6}

Then $E_4 \cap E_5 = \{3\}$

Then E_4 and E_5 are not mutually exclusive.

10. Exhaustive events

All the possible outcomes of the experiment are known as exhaustive events

Eg: "Throwing a die"

Sample space S = { 1, 2, 3, 4, 5, 6}

E_1= {1, 2, 3} E_2= {3, 4} E_3= {5, 6}

$E_1 \cup E_2 \cup E_3 = \{1, 2, 3\} \cup \{3, 4\} \cup \{5, 6\} = S$

Such events E_1, E_2, E_3 are called exhaustive

11. Equally likely events

When there is no reason to expect the happening of on event in preference to the other then the events are known equally likely events.

Eg: "Throwing a die"

Sample space S = { 1, 2, 3, 4, 5, 6}

E_1= {1,} E_2= {2} E_3= {3}

E_4= {2, 3}

$E_{1,}$ E_2 *and* E_3 are equally likely events.

$E_{1,}$ *and* E_4 not equally likely events.

12. Let an experiment be dented by *A* and the simple events connected with *A* will be called event points and the set of all possible event points is called event space of *A*.

Competency level 4.2 : Interprets probability

Number of periods : 10

Learning out comes :

1. States classical definition of probability and its limitation.
2. States the axiomatic definition of probability.
3. State the frequency approximation to probability.
4. Proves the theorems in probability using axiomatic definition
5. Solves problems using the axiometic definition of probability

Guidelines to learning - teaching process :

1. If all the outcomes of a sample space are equally likely, then the probability that an event will occur is equal to the ratio.

$$\frac{\text{The number of outcomes favorable to event}}{\text{The total number of outcome of the sample space.}}$$

Suppose that event E can be happen in h way of a total of n possible equally likely *n* ways then,

$$P(E)=\frac{h}{n}$$

$$P(not\ E)=\frac{n-h}{n}=\frac{n}{n}-\frac{h}{n}=1-P(E)$$

$$P(E)+P(not\ E)=1$$

2. Let S be the sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S. ie P (s) and range is the interval [0, 1] i. e. $P:P(s)\rightarrow[0,1]$ statisfies the following axioms.

- For any event E, $P(E) \geq 0$
- P(S) = 1
- If E and F are mutually exclusive events. Then $P(E \cup F) = P(E) + P(F)$

3. If random experiment had been done *N* times and the event *A* occurred *n(A)* times then $\frac{n(A)}{N}$ is called as the relative frequency of event *A*. If *N* is infinite then the ratio comes to a limited value which is called the probability of that event *A*

   If $N \geq n(A) \geq 0 \ then \ 0 \leq P(A) \leq 1$

4. Theorem 1: $\mathrm{P}(\phi) = 0$

   $\mathrm{P}(\mathrm{S} \cup \phi) = \mathrm{P}(\mathrm{S})$

   Since S , ϕ are Mutually exclusive events

   Hence

   $\mathrm{P}(\mathrm{S} \cup \phi) = \mathrm{P}(\mathrm{S}) + \mathrm{P}(\phi) = \mathrm{P}(\mathrm{S})$

   $\therefore \ \mathrm{P}(\phi) = 0$

   Theorem 2: If *E* is any events

   $\mathrm{P}(\mathrm{E}^1) = 1 - \mathrm{P}(\mathrm{E})$

   Hence *E* and E^1 are mutually exclusive.

   $\mathrm{P}(\mathrm{E} \cup \mathrm{E}^1) = \mathrm{P}(\mathrm{S})$

   $\mathrm{P}(\mathrm{E}) + \mathrm{P}(\mathrm{E}^1) = \mathrm{P}(\mathrm{S})$

   $\mathrm{P}(\mathrm{E}) + \mathrm{P}(\mathrm{E}^1) = 1$

   $\mathrm{P}(\mathrm{E}^1) = 1 - \mathrm{P}(\mathrm{E})$

   Theorem 3: *A, B* two events

   $\mathrm{P}(\mathrm{A}) = \mathrm{P}(\mathrm{A} \cap \mathrm{B}) + \mathrm{P}(\mathrm{A} \cap \mathrm{B}^1)$

   $(\mathrm{A} \cap \mathrm{B})$ and $(\mathrm{A} \cap \mathrm{B}^1)$ are Mutually exclusive.

   $(\mathrm{A} \cap \mathrm{B}) \cup (\mathrm{A} \cap \mathrm{B}^1) = \mathrm{A}$

   $\mathrm{P}\left\{(\mathrm{A} \cap \mathrm{B}) \cup (\mathrm{A} \cap \mathrm{B}^1)\right\} = \mathrm{P}(\mathrm{A})$

   $\mathrm{P}(\mathrm{A} \cap \mathrm{B}) + \mathrm{P}(\mathrm{A} \cap \mathrm{B}^1) = \mathrm{P}(\mathrm{A})$

   Theorem 4: A and B are two events

$$P(A \cap B) = P(A) + P(B) - P(A \cap B)$$

$(A \cap B^1)$ and B are Mutually exclusive events.

$$(A \cap B^1) \cup B = A \cup B$$

$$P[A \cap B^1] + P(B) = P(A \cap B) \longrightarrow ①$$

$$P(A) = P(A \cap B) + P(A \cap B^1) \longrightarrow ②$$

From ① and ② $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

5. Guide students to solve the problem using axiometricdent and laws of probability.

Competency 6 : **Uses Permutation and combination to solve Mathematical problems**

Competency level 6.1: Uses Permutations as a technique of solving mathematical problems

Number of periods : 10

Learning out comes :
1. Explains the fundamental principle's of counting.
2. Defines the factorial.
3. States the recursive relation for factorial.
4. Defines ^nP_r and obtains the formulae for ^nP_r.
5. Finds the permutation of r objects taken from n objects all differents.
6. Finds the permutation of r objects taken from n objects not all differenct
7. Solves problems involving permutations.

Guidelines to learning / teaching process :

1. Fundamental principle of counting:

   If one operation can be performed in m different ways and a second operation can be per. formed in n different ways, then will be $m.n$ different ways performing the two operation in succession.

2. Definition of factorial n

   Normal form : $n! = 1, 2, 3....n$ and $0! = 1$

3. Recursive form : F(n) = n F (n-1) and F(0) = 1
Where n is a positive integer.

4. Defines that the number of permutations of n different objects taken r $(0 \le r \le n)$ at a time is ${}^{n}p_{r}$ and show that ${}^{n}\text{P}_{r} = \dfrac{n!}{(n-r)!}$

5. Guide students to find the number of permutations of n different objects taken r $(0 \le r \le n)$ at a time

6.
- Finds the permutation of r objects aken from n objects not all differenct
- Define that the number of permutation of n different objects taken all at a time is ${}^{n}p_{n}$ and ${}^{n}p_{n} = n!$
- Show that the number of permutation of n different objects taken r at a time when each object may occur any number of times is n^{r}
- Show that the number of permutation of n objects r of which are one kind and the remaining all are different is $\dfrac{n!}{r!}$

7. Guide students to solevs problems involving permutation.

Competency level 6.2 : Uses combinations as a technique of solving Mathematical problems

Number of periods : 14

Learning out comes :
1. Defines combination.
2. Explains the difference between permutations and combinations.
3. Defines ${}^{n}C_{r}$ .and finds a formula for ${}^{n}C_{r}$
4. Applies the formulae to related problems.
5. States the properties of ${}^{n}C_{r}$.
6. Finds numbers of combinations. of r objects taken from n objects all different.
7. Finds number of combinations of r objects taken trom n objects not all diffrent
8. Solves problems involving combinations

Guidelines to learning - teaching process :

1. Defines that the number of combinations of n different objects taken r at a time is ${}^{n}C_{r}$
2. Explain with examples that in permutation the order is important, but in combination order is immaterial.

3. Defines that the number of combinations of n different objects taken r at a time is ${}^{n}C_{r}$ and obtain the formula for ${}^{n}C_{r}$

Show that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$, ${}^{n}P_{r} = r\,!\,{}^{n}C_{r}$

i) ${}^{n}C_{r} = {}^{n}C_{n-r}$ ii) ${}^{n}C_{r} + {}^{n}C_{n-1} = n + {}^{n}C_{r}$

4. Guide students to apply the formula to diiferemt situations

5. Guide students to prove the following results

$(\text{i})\ {}^{n}\text{C}_{0} = {}^{n}\text{C}_{n} = 1$ $(\text{ii})\ {}^{n}\text{C}_{r} = {}^{n}\text{C}_{n-r}$ $(\text{iii})\ {}^{n}\text{C}_{r} + {}^{n}\text{C}_{r-1} = {}^{n+1}\text{C}_{r}$

6. Show that the total number of combinations of n different objects taken any number of a time is $2^{n} - 1$.

7. Guide students to find number of combination of r objects taken from n objects not all different

8. Guide students to solve problems on permutations and combination.

School Based Assessment

Introduction - School Based Assessment

Learning -Teaching and Evaluation are three major components of the process of Education. It is a fact that teachers should know that evaluation is used to assess the progress of learning-teaching process. Moreover, teachers should know that these components influence mutually and develop each other. According to formative assessment (continuous assessment) fundamentals; it should be done while teaching or it is an ongoing process. Formative assessment can be done at the beginning, in the middle, at the end and at any instance of the learning teaching process.

Teachers who expect to assess the progress of learning of the students should use an organized plan. School based assessment (SBA) process is not a mere examination method or a testing method. This programme is known as the method of intervening to develop learning in students and teaching of teachers. Furthermore, this process can be used to maximize the student's capacities by identifying their strengths and weaknesses closely.

When implementing SBA programmes, students are directed to exploratory process through Learning Teaching activities and it is expected that teachers should be with the students facilitating, directing and observing the task they are engaged in.

At this juncture students should be assessed continuously and the teacher should confirm whether the skills of the students get developed up to expected level by assessing continuously. Learning teaching process should not only provide proper experiences to the students but also check whether the students have acquired them properly. For this, to happen proper guiding should be given.

Teachers who are engaged in evaluation (assessment) would be able to supply guidance in two ways. They are commonly known as feed-back and feed- forward. Teacher's role should be providing Feedback to avoid learning difficulties when the students' weaknesses and inabilities are revealed and provide feed-forward when the abilities and the strengths are identified, to develop such strong skills of the students.

Student should be able to identify what objectives have achieved to which level, leads to Success of the Learning Teaching process. Teachers are expected to judge the competency levels students have reached through evaluation and they should communicate information about student progress to parents and other relevant sectors. The best method that can be used to assess is the SBA that provides the opportunity to assess student continuously.

Teachers who have got the above objective in mind will use effective learning, Teaching, evaluation methods to make the Teaching process and learning process effective. Following are the types of evaluation tools student and, teachers can use. These types were introduced to teachers by the Department of Examination and National Institute of Education with the new reforms. Therefore, we expect that the teachers in the system know about them well.

Types of assessment tools:

1.	Assignments	2.	Projects
3.	Survey	4.	Exploration
5.	Observation	6.	Exhibitions
7.	Field trips	8.	Short written
9.	Structured essays	10.	Open book test
11.	Creative activities	12.	Listening Tests
13.	Practical work	14.	Speech
15.	Self creation	16	Group work
17.	Concept maps	18.	Double entry journal
19.	Wall papers	20.	Quizzes
21.	Question and answer book	22.	Debates
23.	Panel discussions	24.	Seminars
25.	Impromptus speeches	26.	Role-plays

Teachers are not expected to use above mentioned activities for all the units and for all the subjects. Teachers should be able to pick and choose the suitable type for the relevant units and for the relevant subjects to assess the progress of the students appropriately. The types of assessment tools are mentioned in Teacher's Instructional Manuals.

If the teachers try to avoid administering the relevant assessment tools in their classes there will be lapses in exhibiting the growth of academic abilities, affective factors and psycho- motor skills in the students.

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