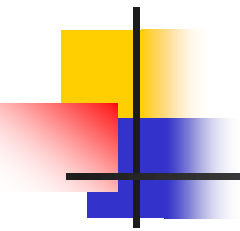


Geodetic Astronomy

Introduction & Basic definitions



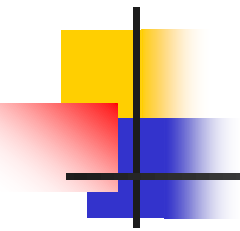
Introduction

Astronomy is defined as

“ The scientific study of the universe beyond the earth, especially the observation, calculation, and theoretical interpretation of the positions, dimensions, motion, and evolution of the celestial bodies and phenomena” (Morris, 1975)

The oldest of the natural sciences dating back to ancient Chinese and Babylonian civilizations

Before inventing the telescope (1609), the naked eye was used for measurements

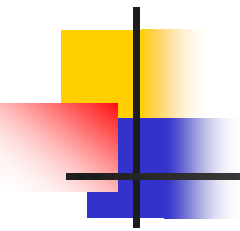


Introduction

Geodetic Astronomy is described as (Mueller, 1969)

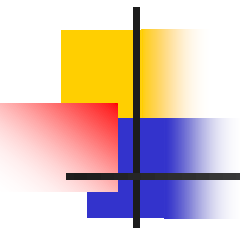
The art and science for determining, by astronomical observations, the positions of points on the earth and the azimuths of the geodetic lines connecting such points.

The fundamental concepts and basic principles of **Spherical Astronomy**, which is the basis for geodetic astronomy, were developed principally by the Greeks



The Use of Geodetic Astronomy

- ⑩ A knowledge of celestial coordinate systems, transformations amongst them, and variations in each of them
- ⑩ Celestial coordinate systems define the “link” between satellite and terrestrial coordinate systems
- ⑩ The concepts of time for geodetic purposes
- ⑩ Tidal studies require a knowledge of geodetic astronomy
- ⑩ To deal with new technologies (e.g. inertial survey systems), an understanding of the local astronomic coordinate system is essential

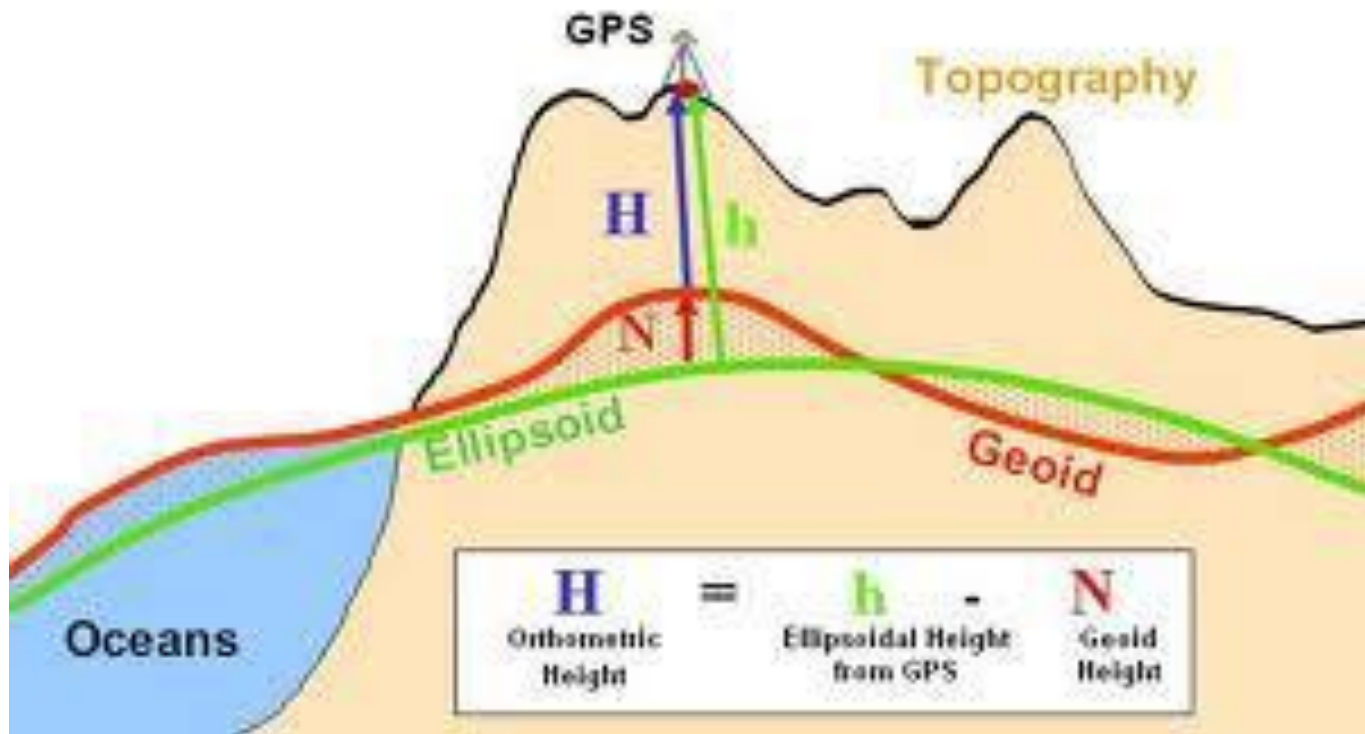


The Use of Geodetic Astronomy

- ⑩ Astronomic coordinates of terrain points, which are expressed in a “natural” coordinate system, are important when studying 3-D terrestrial networks
- ⑩ Astronomically determined azimuths provide orientation for terrestrial networks
- ⑩ The determination of astrogeodetic deflections of the vertical are useful for geoid determination
- ⑩ The determination of the origin and orientation of independent surveys in remote regions
- ⑩ Demarcation of astronomically defined boundaries (constellations).

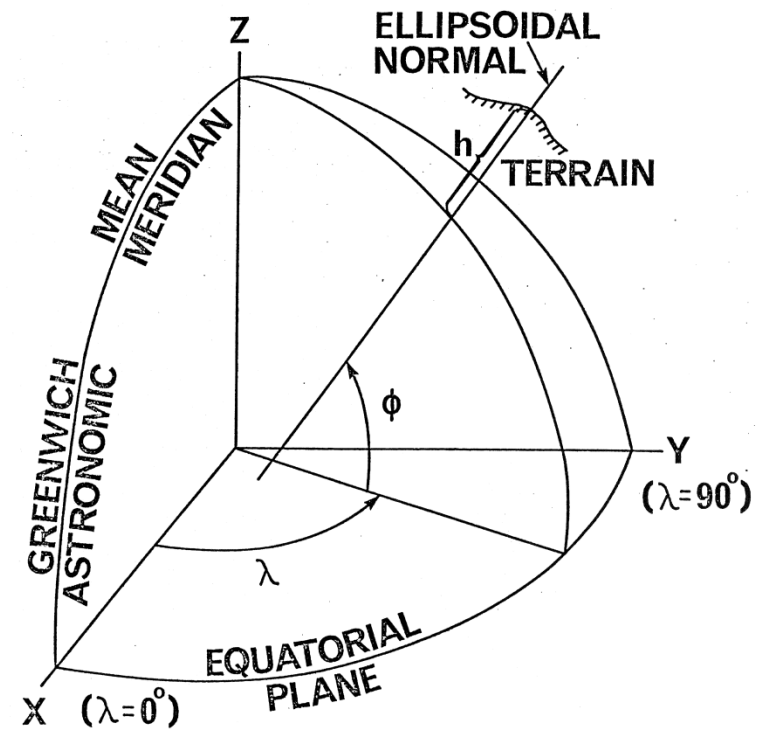
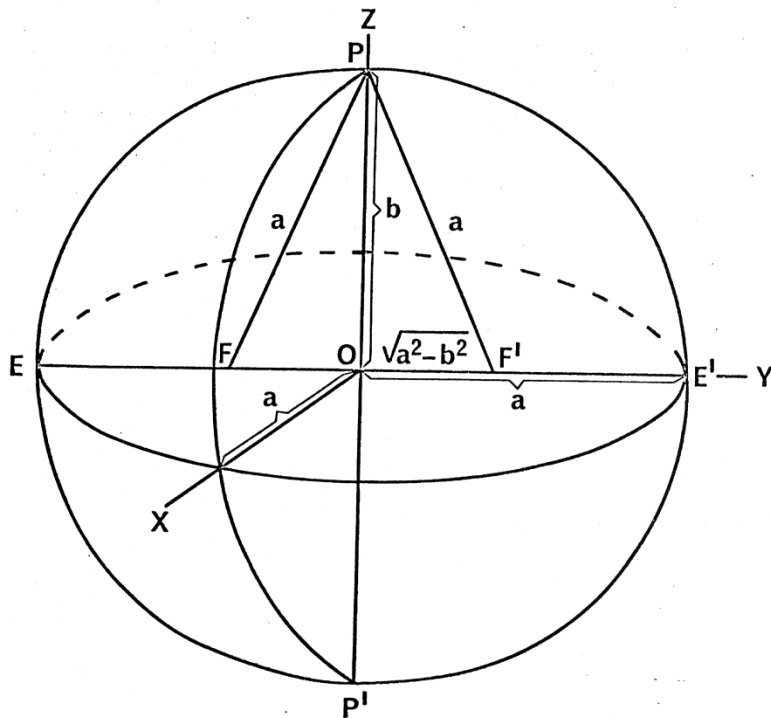
Basic Definitions

Three different surfaces: the terrain, an ellipsoid, and the geoid



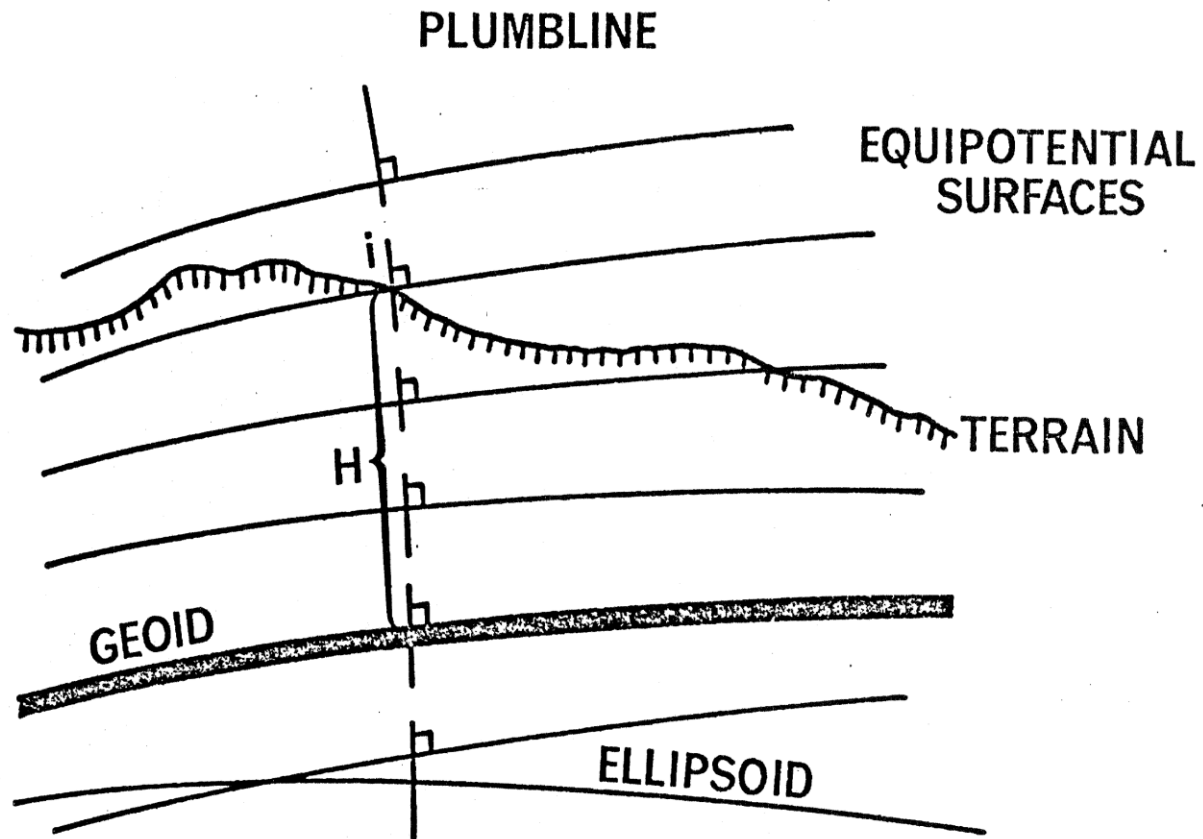
Basic Definitions

The mathematical figure of the earth, biaxial ellipsoid (reference ellipsoid)



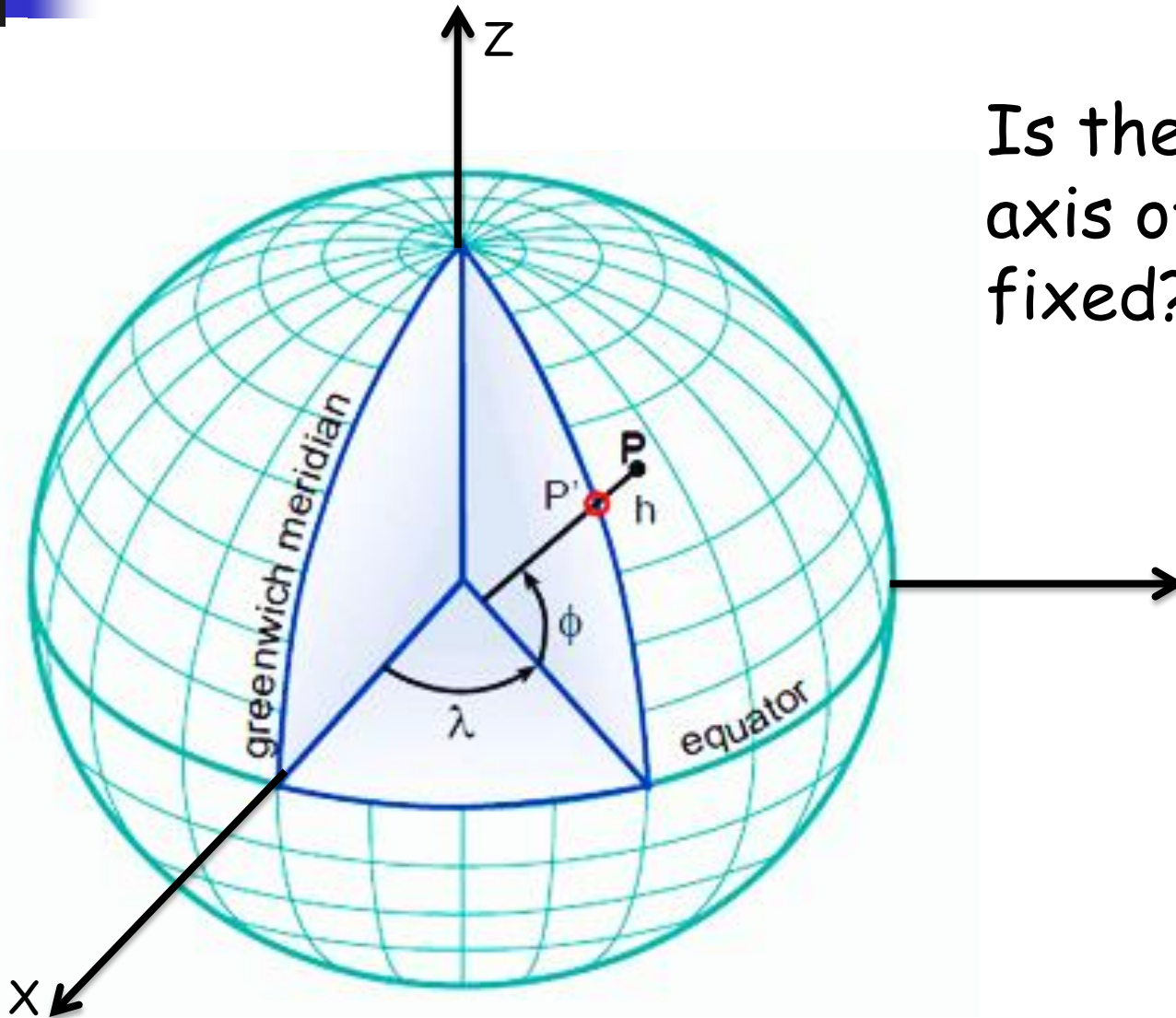
Basic Definitions

Equipotential surface – real or physical surfaces



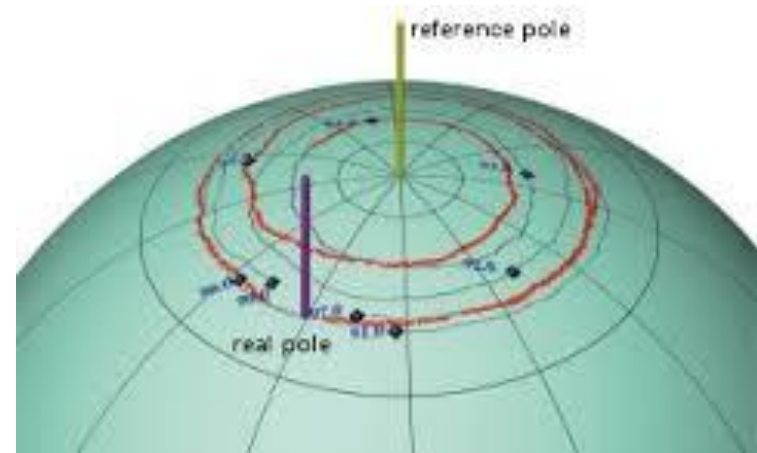
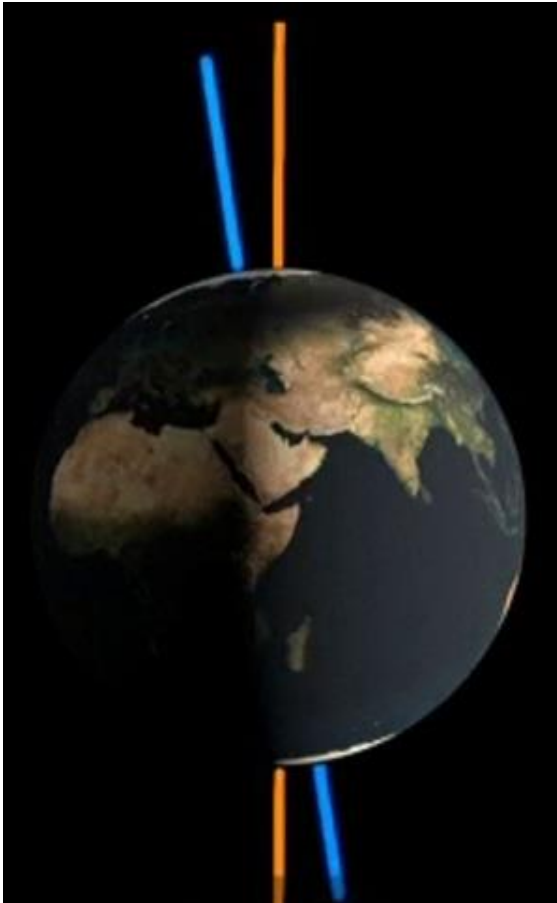
Geodetic Coordinates

Is the rotational axis of the Earth fixed?



Geodetic Coordinates

Polar Motion

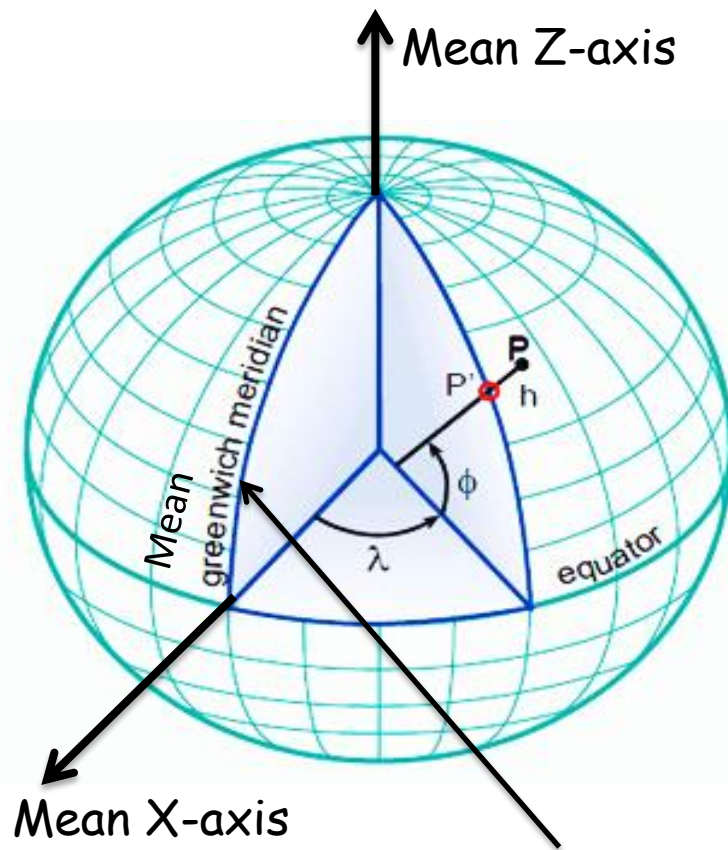


Mean Z-axis is defined by ITRS (or CTS) maintained by [IERS](#)

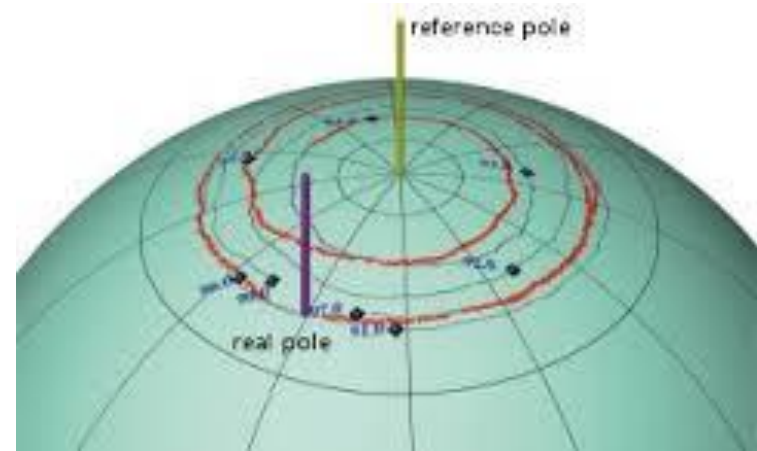
The mean pole - In the past, Conventional International Origin (CIO) & Today, International reference pole (IRP)

Geodetic Coordinates

Polar Motion



IERS (International) Reference Meridian Plane

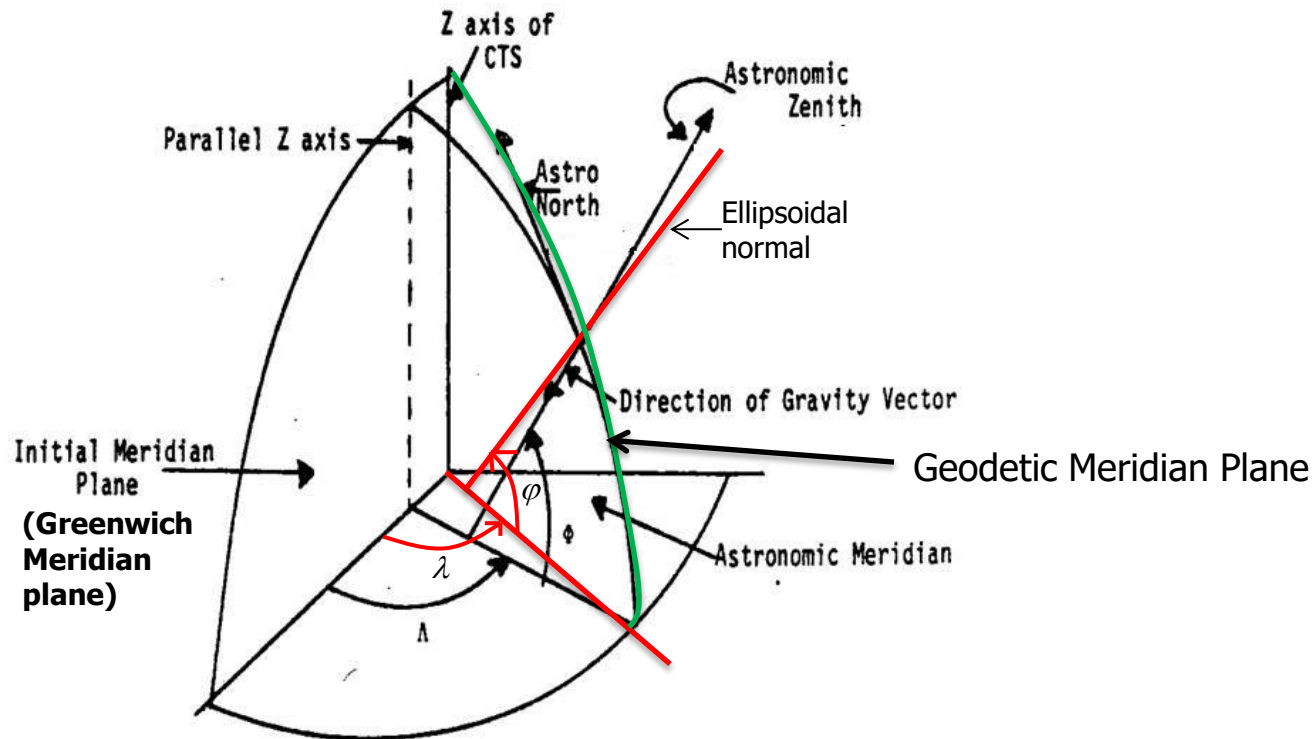


Mean Z-axis is defined by ITRS (or CTS) maintained by IERS

The mean pole - In the past, Conventional International Origin (CIO) & Today, International reference pole (IRP)

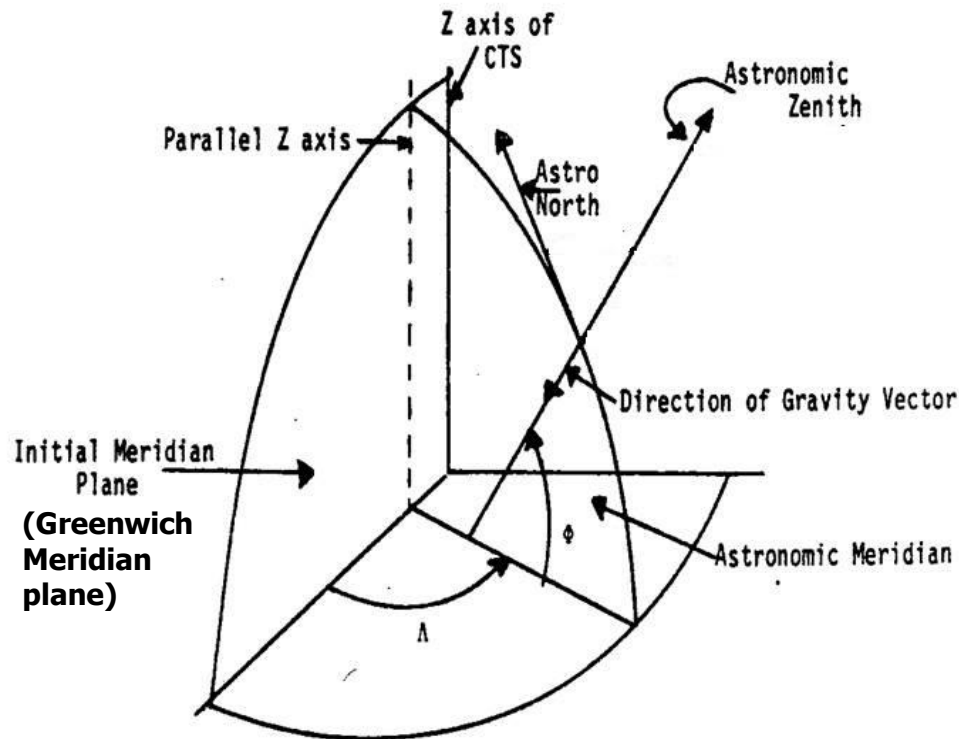
Astronomic Coordinates

Astronomic coordinates are based on the direction of the **gravity vector**, in contrast on the direction of the ellipsoidal normal on Geodetic coordinates.



Astronomic Coordinates

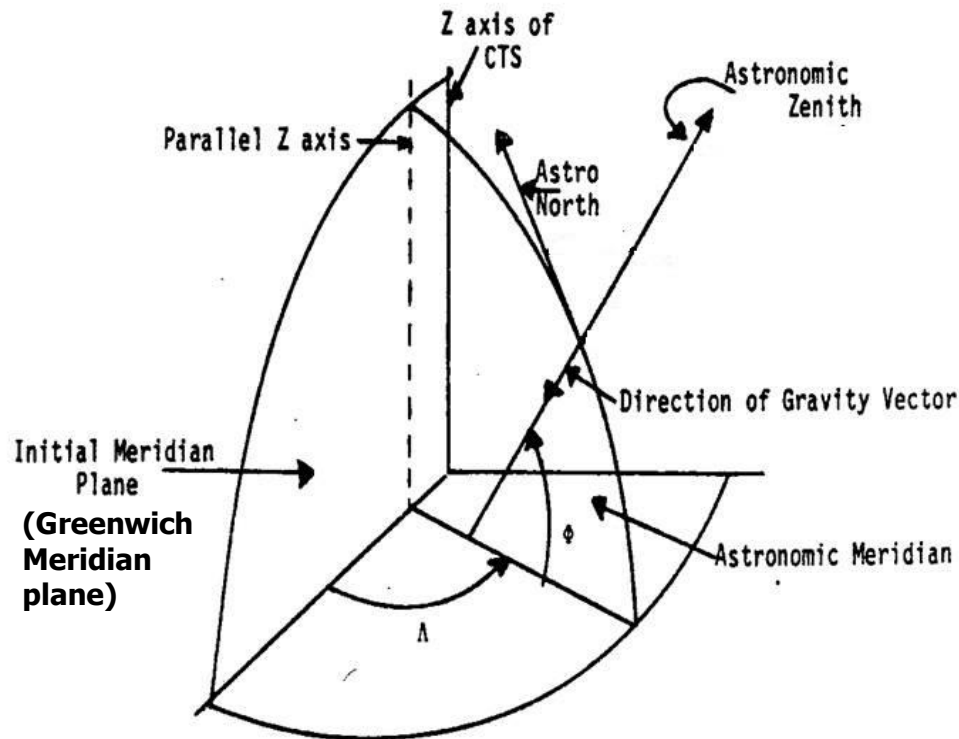
Astronomic coordinates are based on the direction of the **gravity vector**, in contrast on the direction of the ellipsoidal normal on Geodetic coordinates.



- Direction of the gravity vector is determined naturally by the arbitrary mass distribution (**plumb line does not intersect Earth center of mass**)
- Determination of latitudes, longitudes and azimuths from stars with known coordinates based on the gravity direction called **Astronomic or Natural coordinates**

Astronomic Coordinates

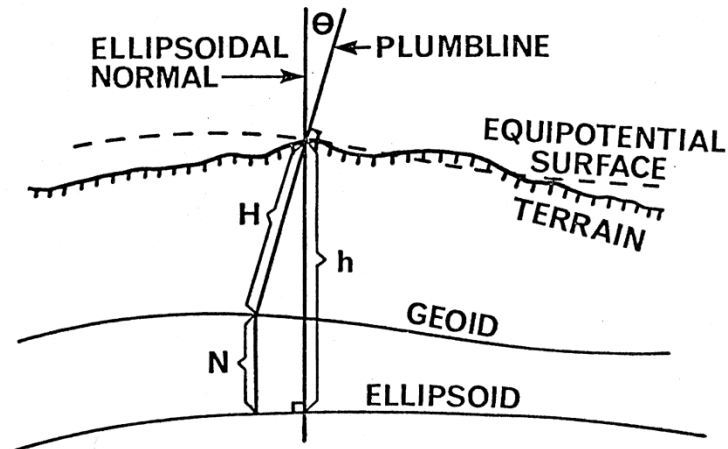
Astronomic coordinates are based on the direction of the gravity vector, in contrast on the direction of the ellipsoidal normal on Geodetic coordinates.



- **Astronomic meridian** is the plane containing the astronomic normal at a point and the parallel to the Z axis of CTS
- The **initial meridian** is the meridian which passes through the center of the transit instrument at the observatory of Greenwich
- **Astronomical latitude** of a point is the angle between Equator and direction of the gravity at that point
- **Astronomical longitude** is the angle between an initial meridian and the astronomical meridian of the point

Astro-Geodetic Deflection of the Vertical

The angle between the astronomic normal (normal to the geoid) and geodetic normal (normal to the ellipsoid) at a point and



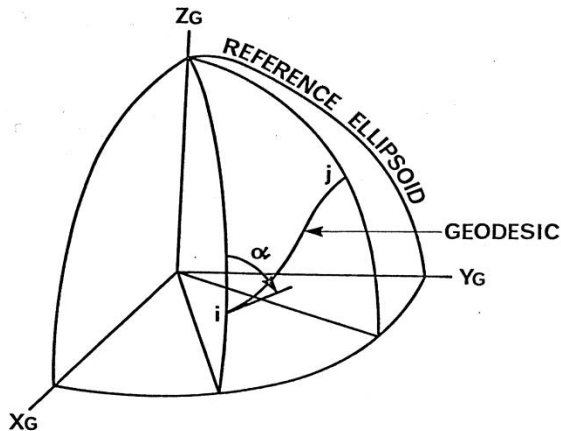
Two components: ξ - meridian and η - prime vertical

$$\xi = \Phi - \varphi$$

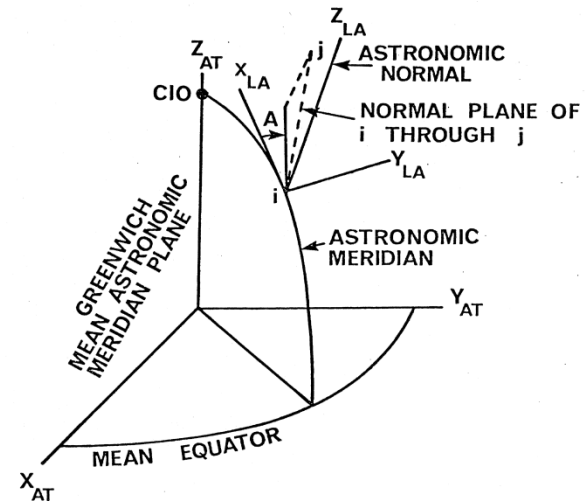
$$\eta = (\Lambda - \lambda) \cos \varphi$$

Geodetic & Astronomic Azimuths

A geodetic azimuth (α), on the surface of the reference ellipsoid, is the clockwise angle from north between the geodetic meridian of **i** and the tangent to the geodesic between **i** and **j**

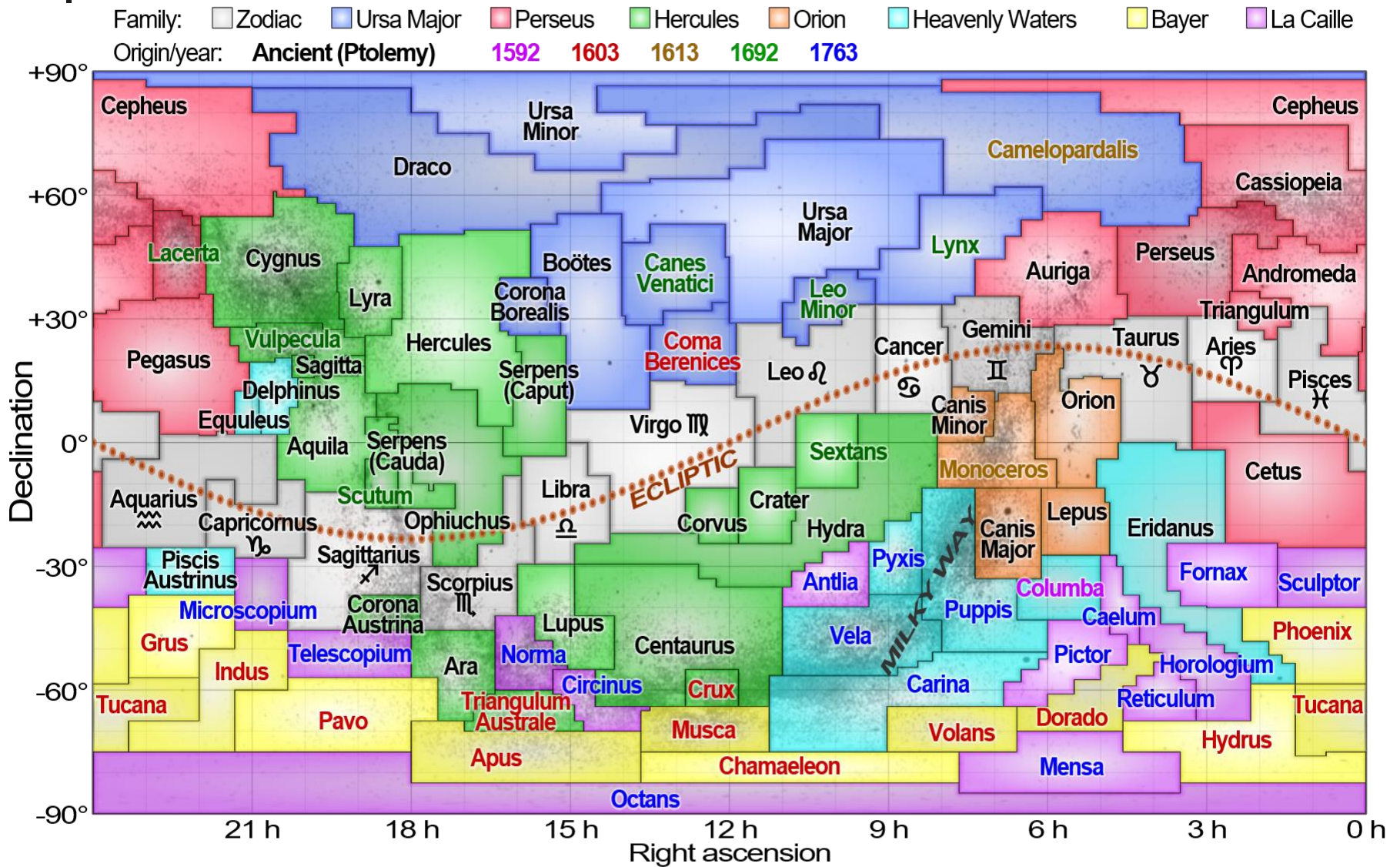


The astronomic azimuth (A) is the angle between the astronomic meridian plane of **i** and the astronomic normal plane of **i** through **j**, measured clockwise from north



$$A - \alpha = \eta \tan \varphi + (\xi \sin \alpha - \eta \cos \alpha) \cot z$$

Constellations



Geodetic Astronomy – L02

Dr. H.M.I.Prasanna

The use of Field Astronomy

- Finding a position (latitude & longitude) of points on the Earth surface
- Finding the azimuth (Orientation)

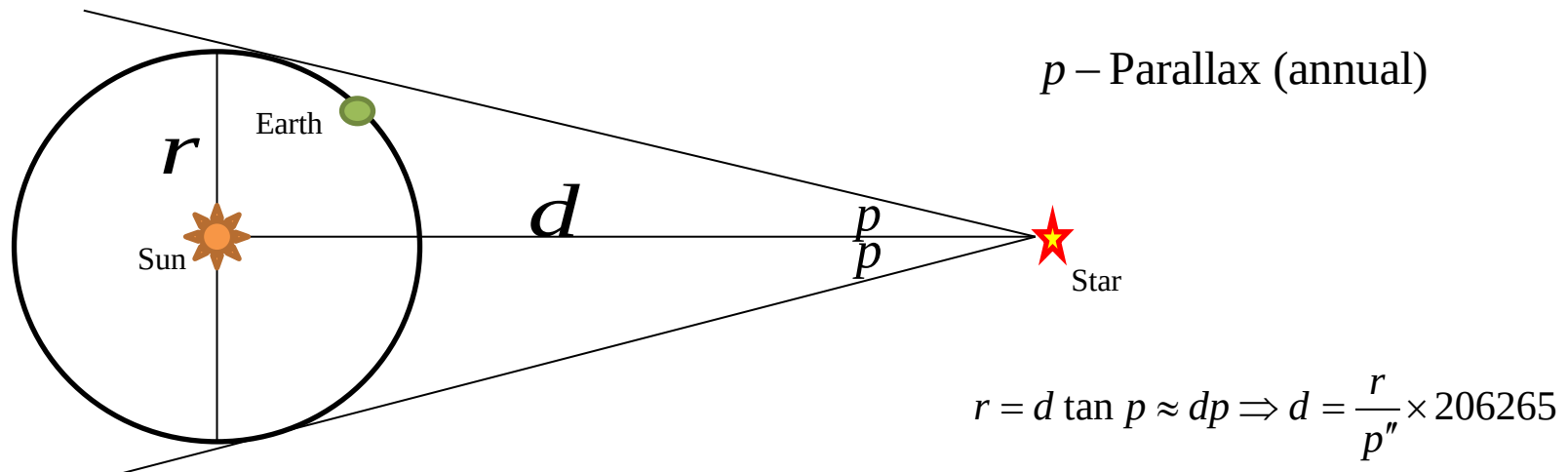
Astronomical Unit (AU)

Mean value of the distance from the sun to the Earth (1 AU, 93 million miles, 499 light seconds)

Planet	Distance (AU)
Mercury	0.39
Venus	0.72
Earth	1
Mars	1.52
Jupiter	5.2
Saturn	9.54
Uranus	19.18
Neptune	30.06
Pluto	39.5

Astronomical Unit (AU)

Stars are located very long distances from the Earth, and Earth's orbit around the Sun is used to compute these distances



For the nearest star to the Earth, $p = 1''$

$$\begin{aligned} \text{So, } d &= \frac{93 \times 10^6 \times 206265}{1} \\ &= 19.18 \times 10^{12} \text{ miles} \end{aligned}$$

Astronomical Unit (AU)

The velocity of light is constant in a vacuum (186000 miles/second), and can be used for measuring astronomical distances

The time taken by light to reach us from stars

Sun - 8.25 minutes

α - Centauri - 4 years

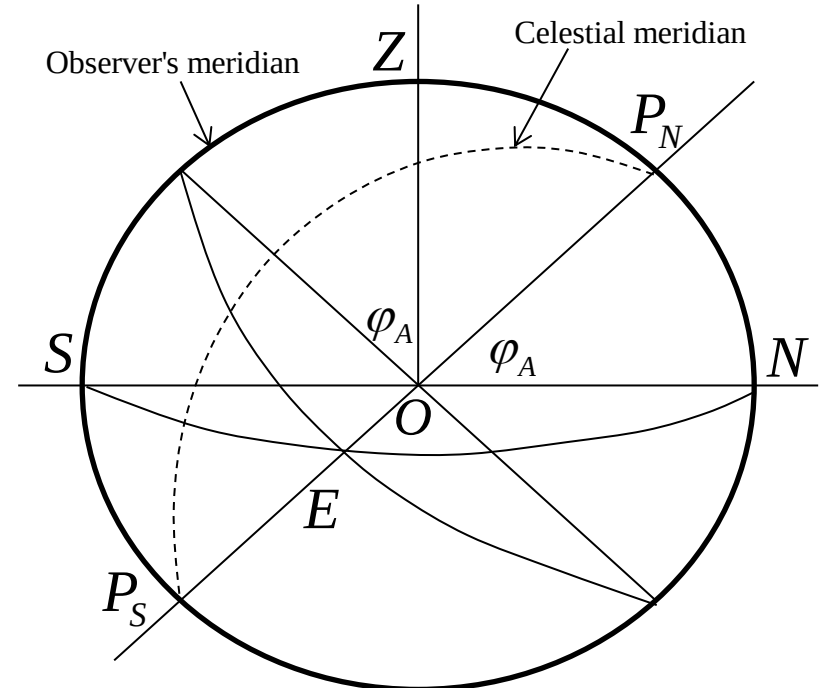
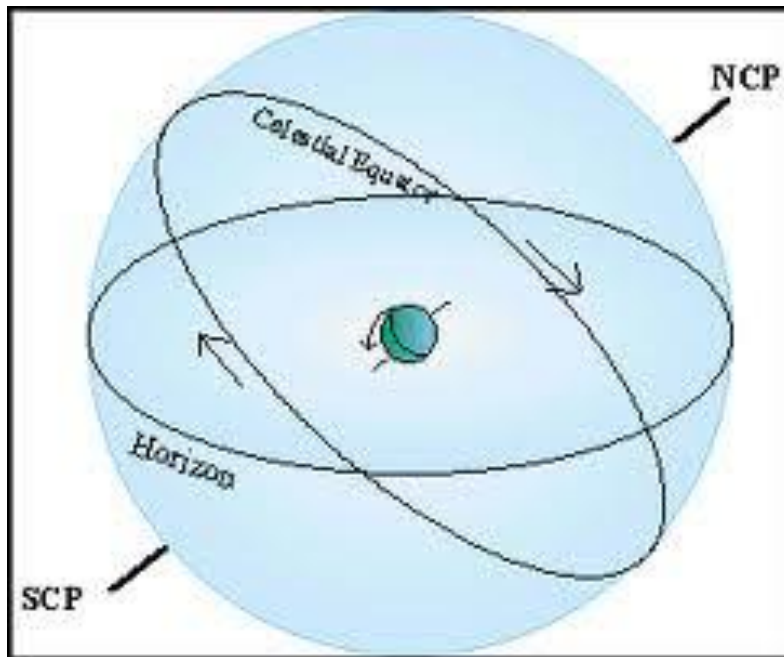
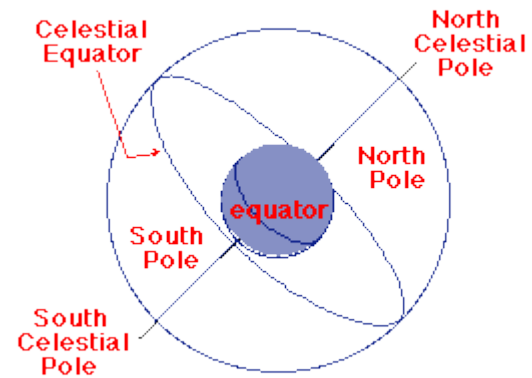
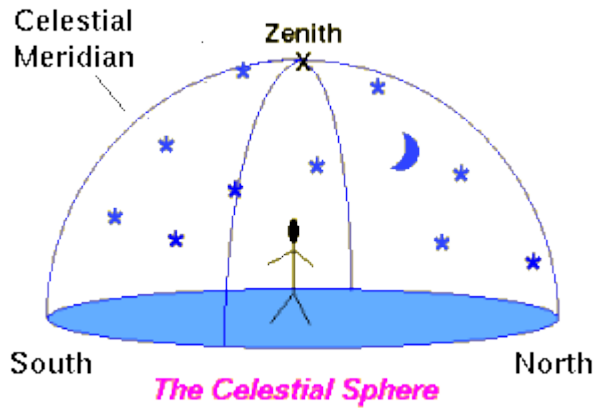
Vega – 30 years

Polaris (Polar star) – 50 years

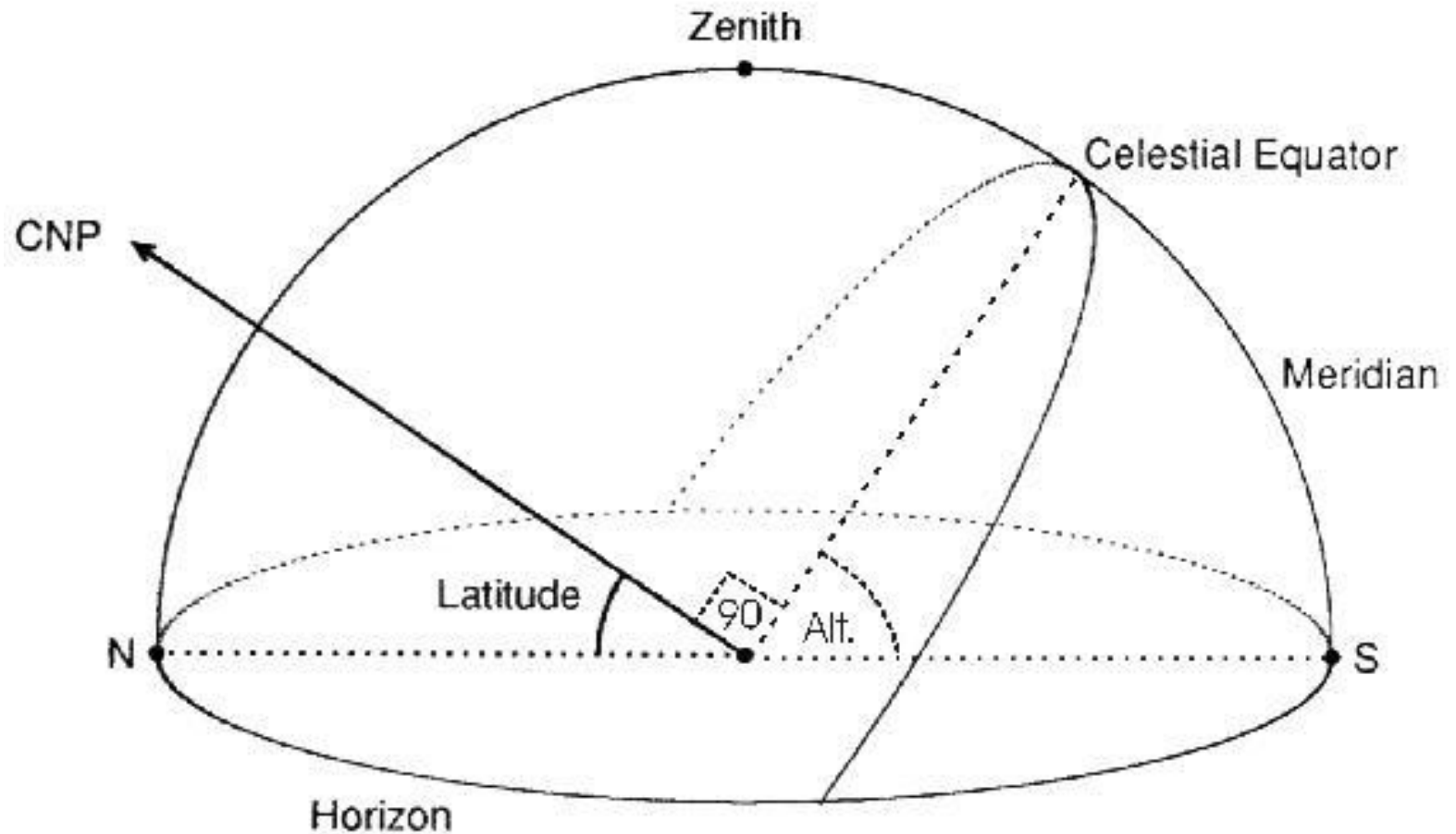
Motion of Planets

- Kepler's laws of planetary motion (17th century)
 - 1st law: The orbit of a planet is an ellipse with the sun at one focus
 - 2nd law: As the planet moves around the sun, it sweeps out equal areas in equal time
 - 3rd law: The square of the time of revolution of its orbit is proportional to the cube of its mean distance from the sun

Celestial Sphere

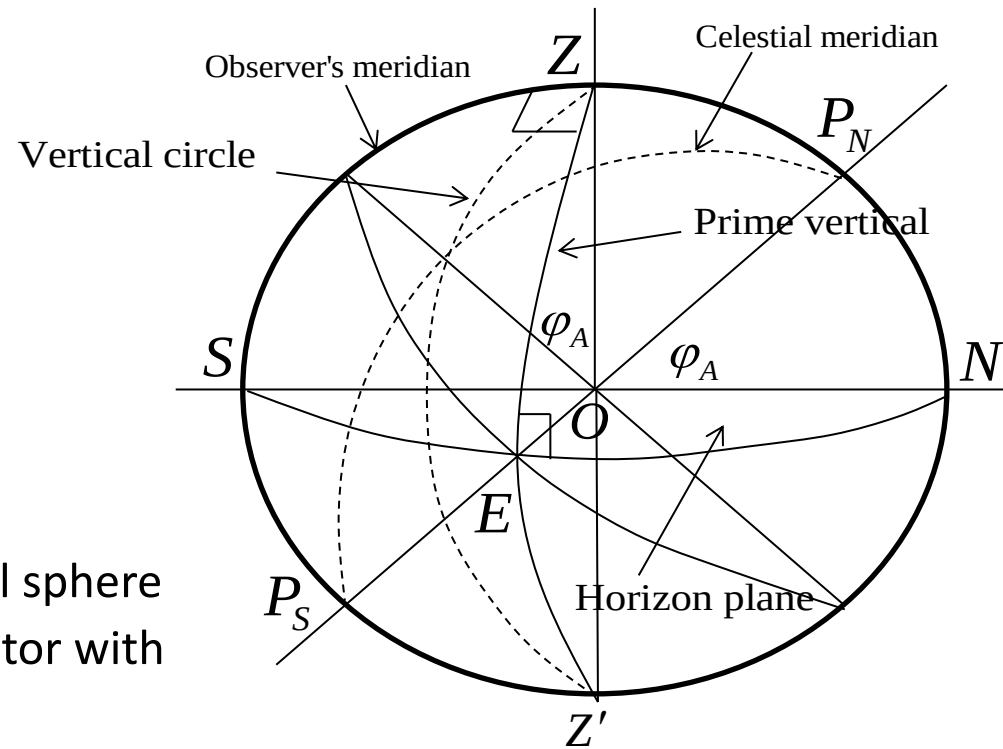


Celestial Sphere

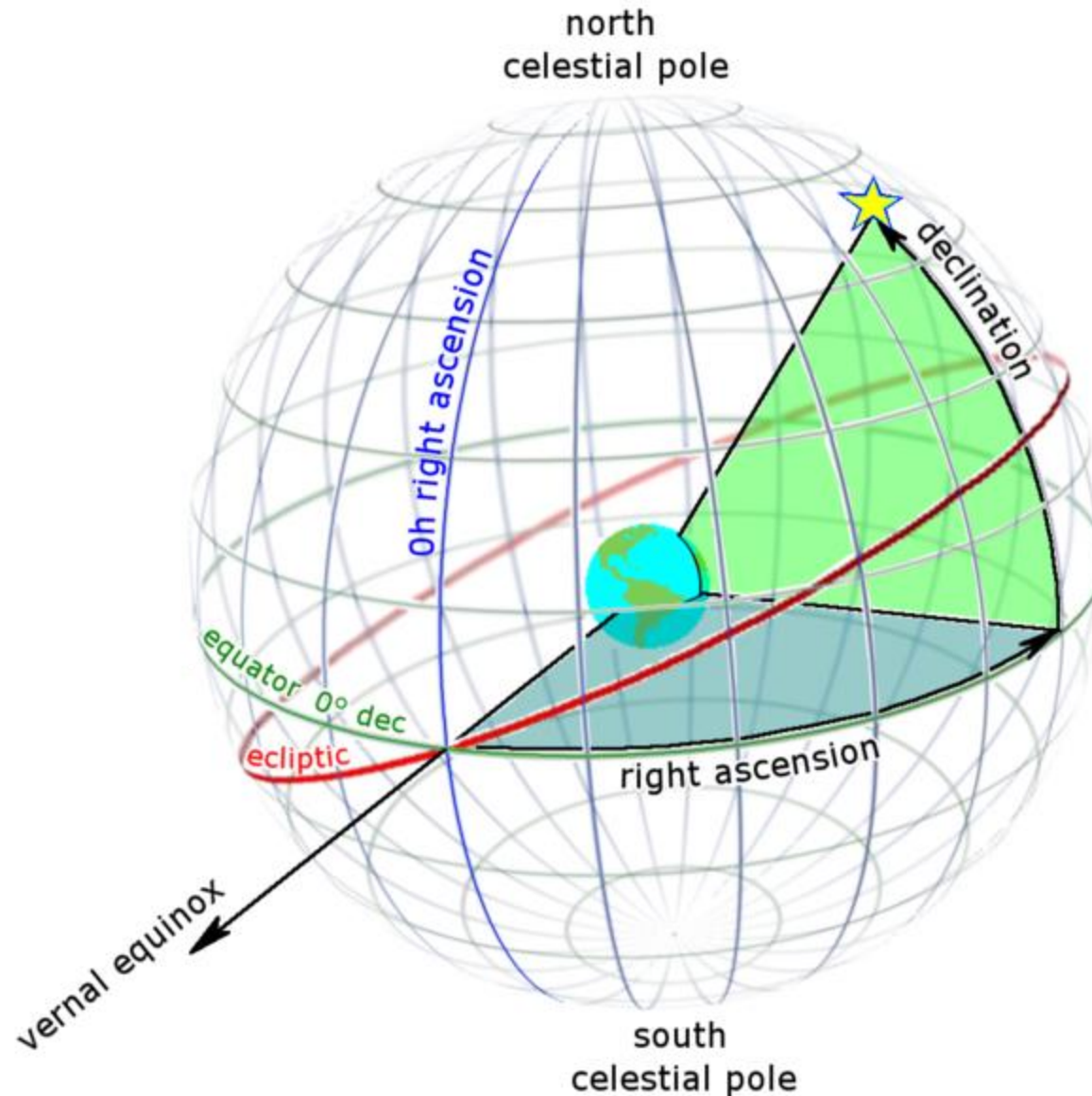


Celestial Sphere

- O is Observer's position. Z is zenith point
- Celestial sphere rotates opposite to the direction of Earth's rotation
- P_N, P_S - Celestial poles
- Celestial equator – great circle of celestial sphere formed by intersection of terrestrial equator with celestial sphere
- Zenith line – point on the celestial sphere, vertically above the observer (opposite of direction of gravity)
- Horizon plane – plane through the observer's position perpendicular to gravity direction (right angle to zenith line)
- Celestial meridian – great circles on celestial sphere through celestial poles (declination circles)
- Observer's meridian – the celestial meridian that passes through zenith
- Vertical circles and Prime vertical – great circles passing through zenith which are perpendicular to horizon plane, and one passes through East and West points called prime vertical



Right ascension & Declination



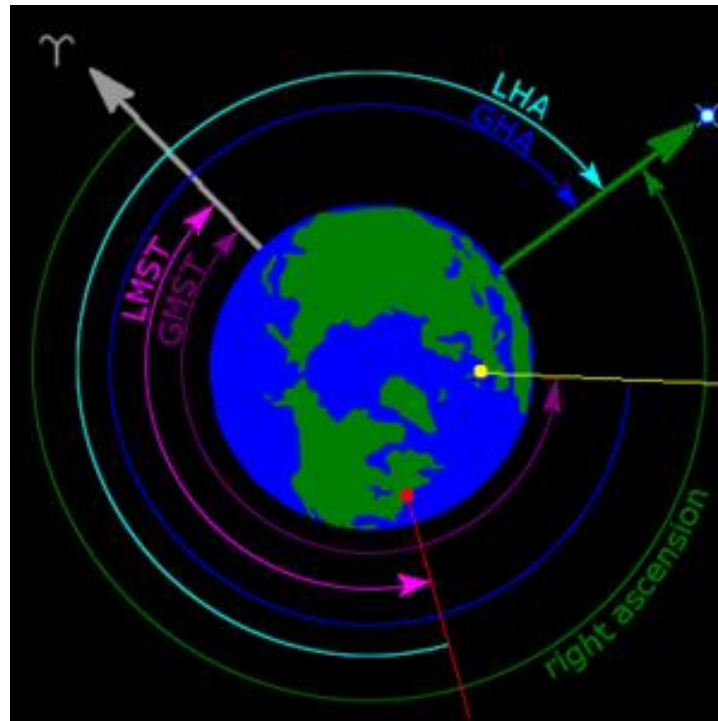
Right ascension (symbol α , abbreviated RA) is the angular distance between meridian through the star along the celestial equator from the **vernal equinox**. The vernal equinox point is one of the two where the ecliptic intersects the celestial equator. **Measured against the motion of celestial sphere**

Declination (symbol δ , abbreviated dec) measures the angular distance of an object **perpendicular to the celestial equator**, positive to the north, negative to the south

Hour Angle

Angle of the pole between the declination circle and the observer's meridian.

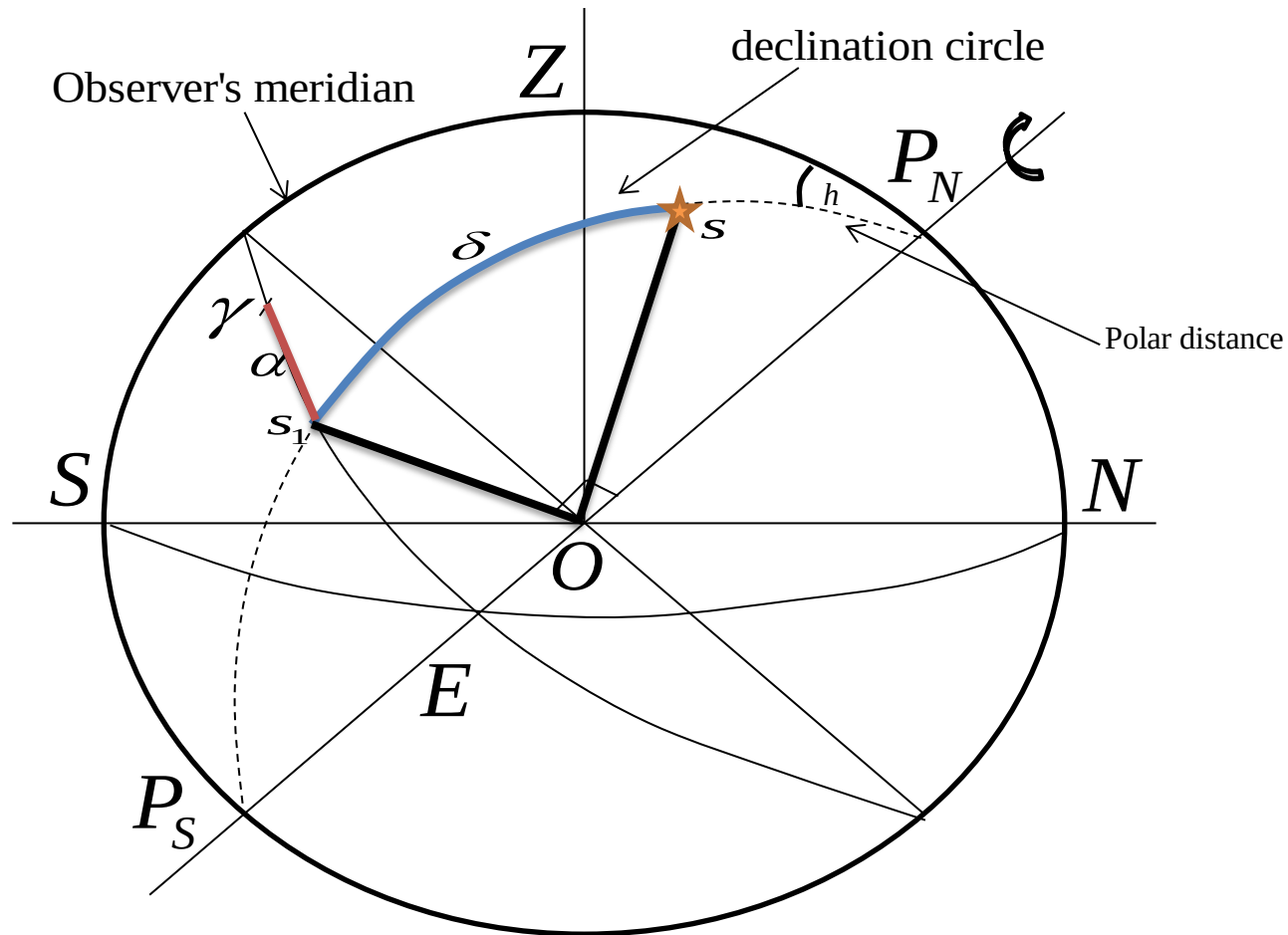
The angle may be expressed as negative east of the meridian plane and positive west of the meridian plane, or as positive westward from 0° to 360° . The angle may be measured in degrees or in time, with $24^h = 360^\circ$ exactly.



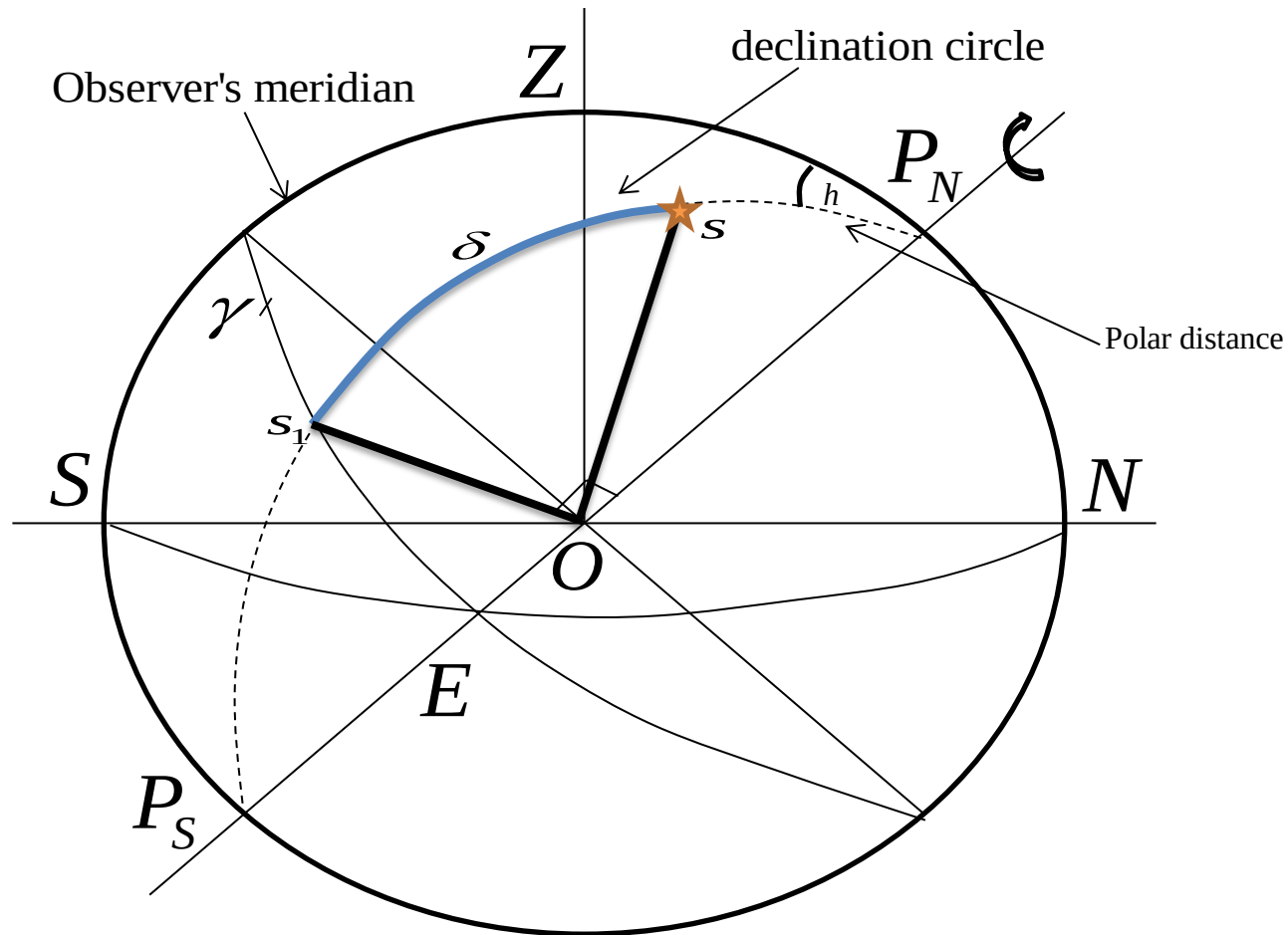
Coordinate Systems

- Right ascension & declination
- Hour angle & declination
- Altitude & Azimuth

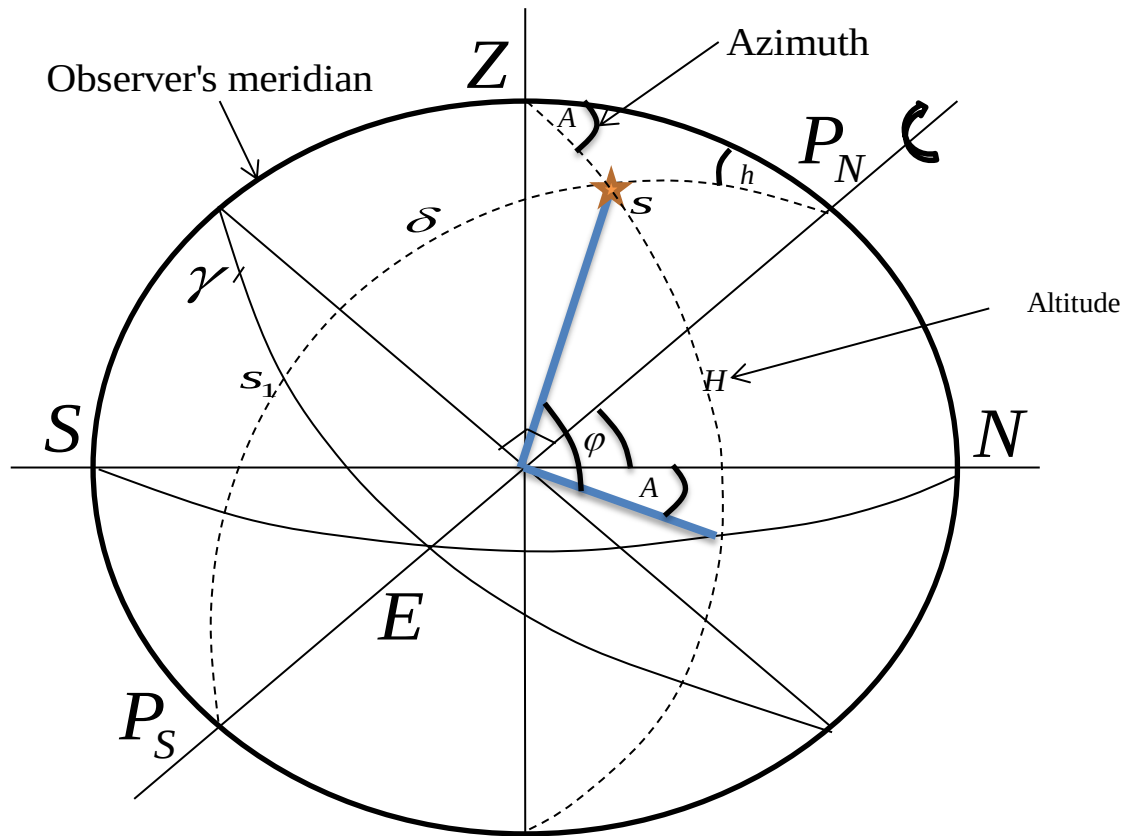
Right ascension & Declination System(α, δ)



Hour Angle C & Declination System (h, δ)



Altitude & Azimuth System (H, A)

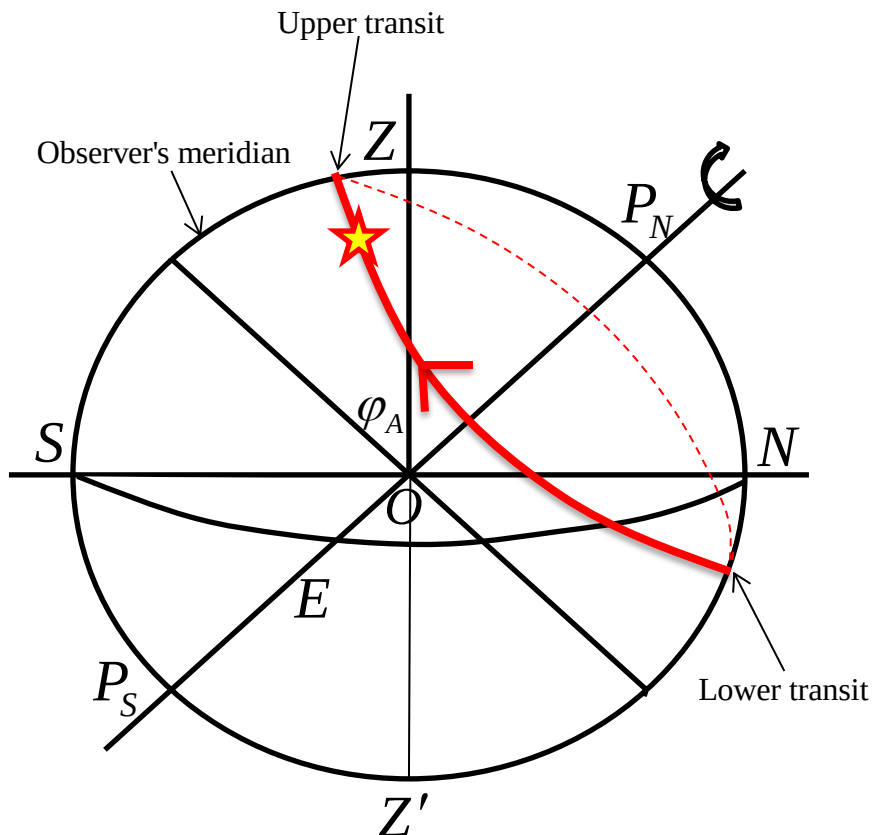


Altitude: angular distance to the star from the horizon. Measured along the vertical circle through the star

Azimuth: Angle between observer's meridian and vertical circle. Measured clockwise from North.

Transit & Elongation

When a star (celestial body) crosses the observer's meridian, it is said to transit. Happened twice a day: upper and lower transit.



Upper transit: Transit on the side of the pole to the zenith. Reach maximum altitude

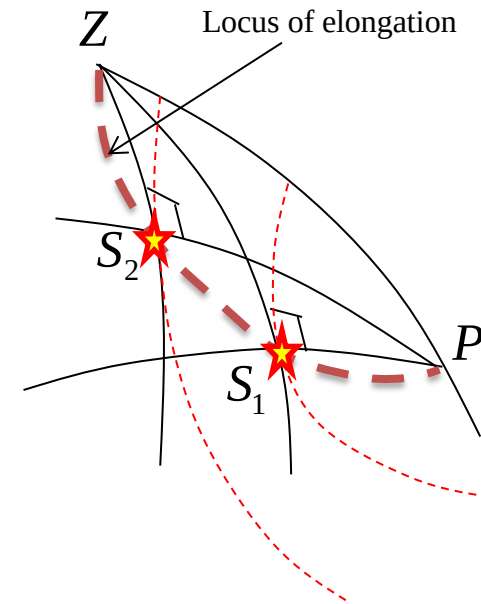
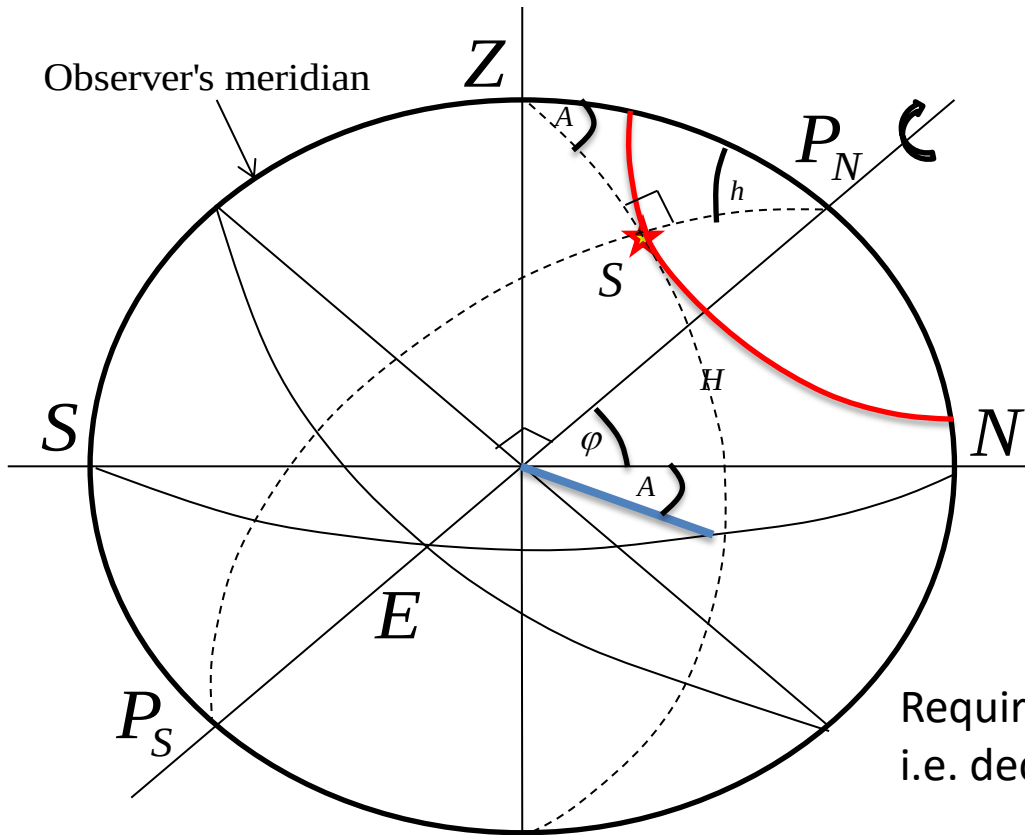
Lower transit: transit on the opposite side of the pole to the zenith.

Most of stars are below the horizon at lower transit.

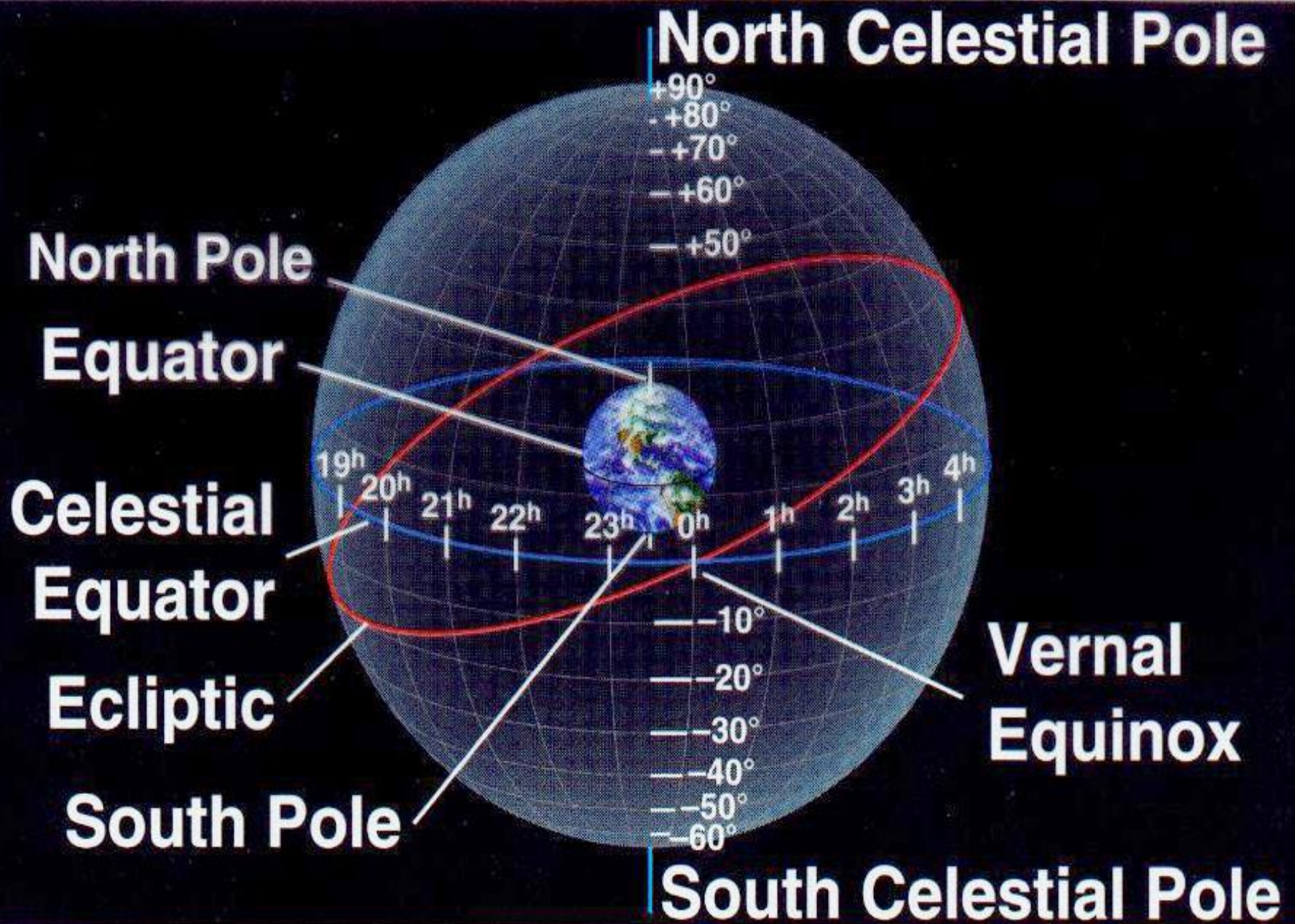
Stars always visible at lower transit called "*close polar stars*"

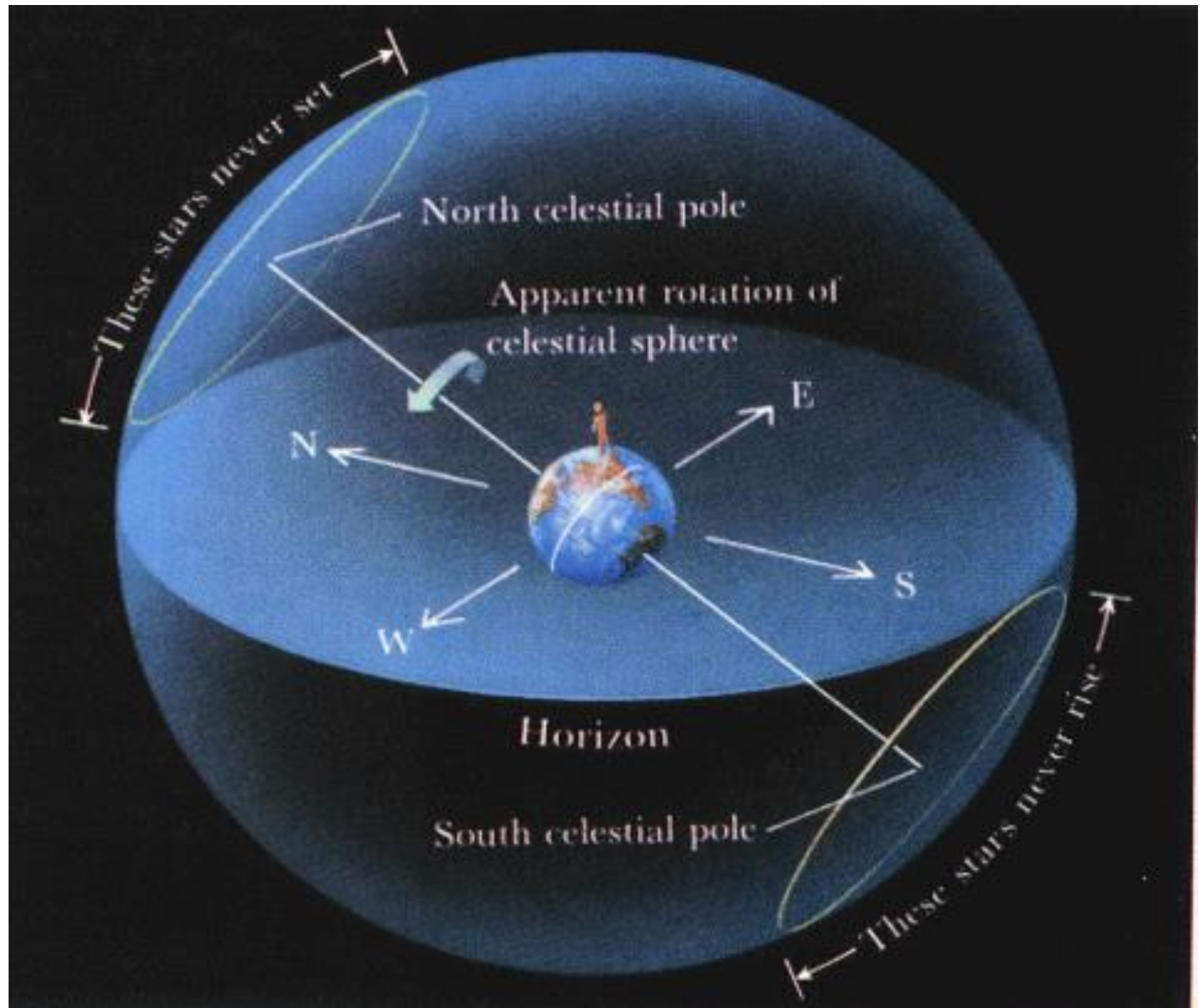
Transit & Elongation

A star is set to be elongation, when it is moving tangential to the vertical circle through the star. At this position, the star gets maximum East or West.



Requirement for elongation- $\delta \geq \varphi$
i.e. declination of star $\geq$ latitude of observer

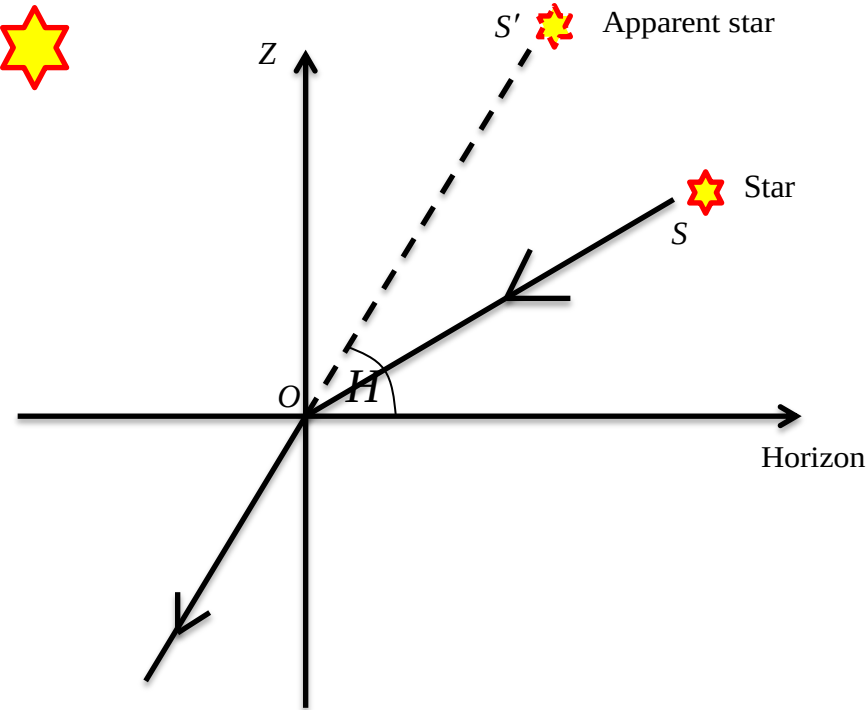
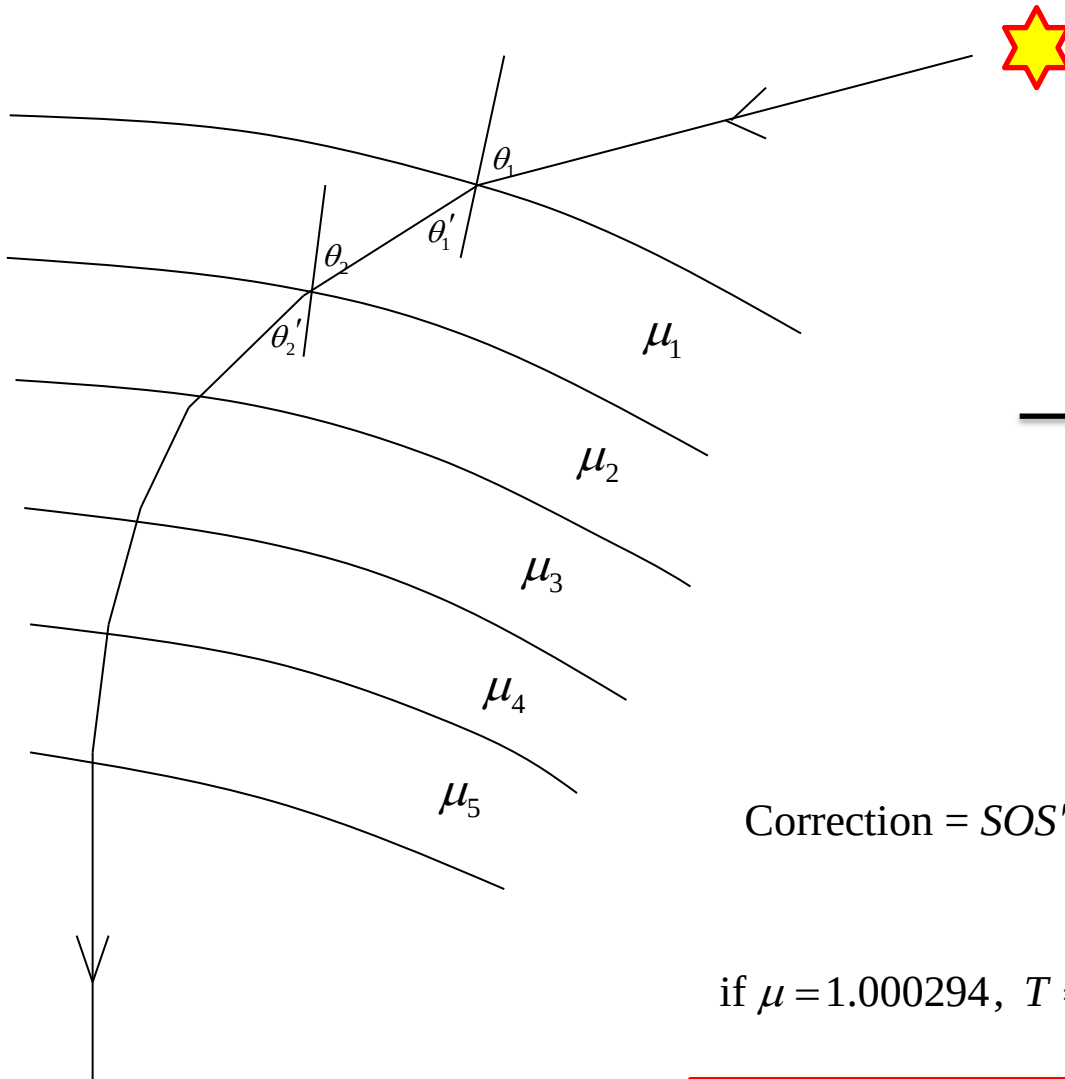




Correction of Altitude

- Refraction
- Semi diameter of Sun
- Parallax

Refraction

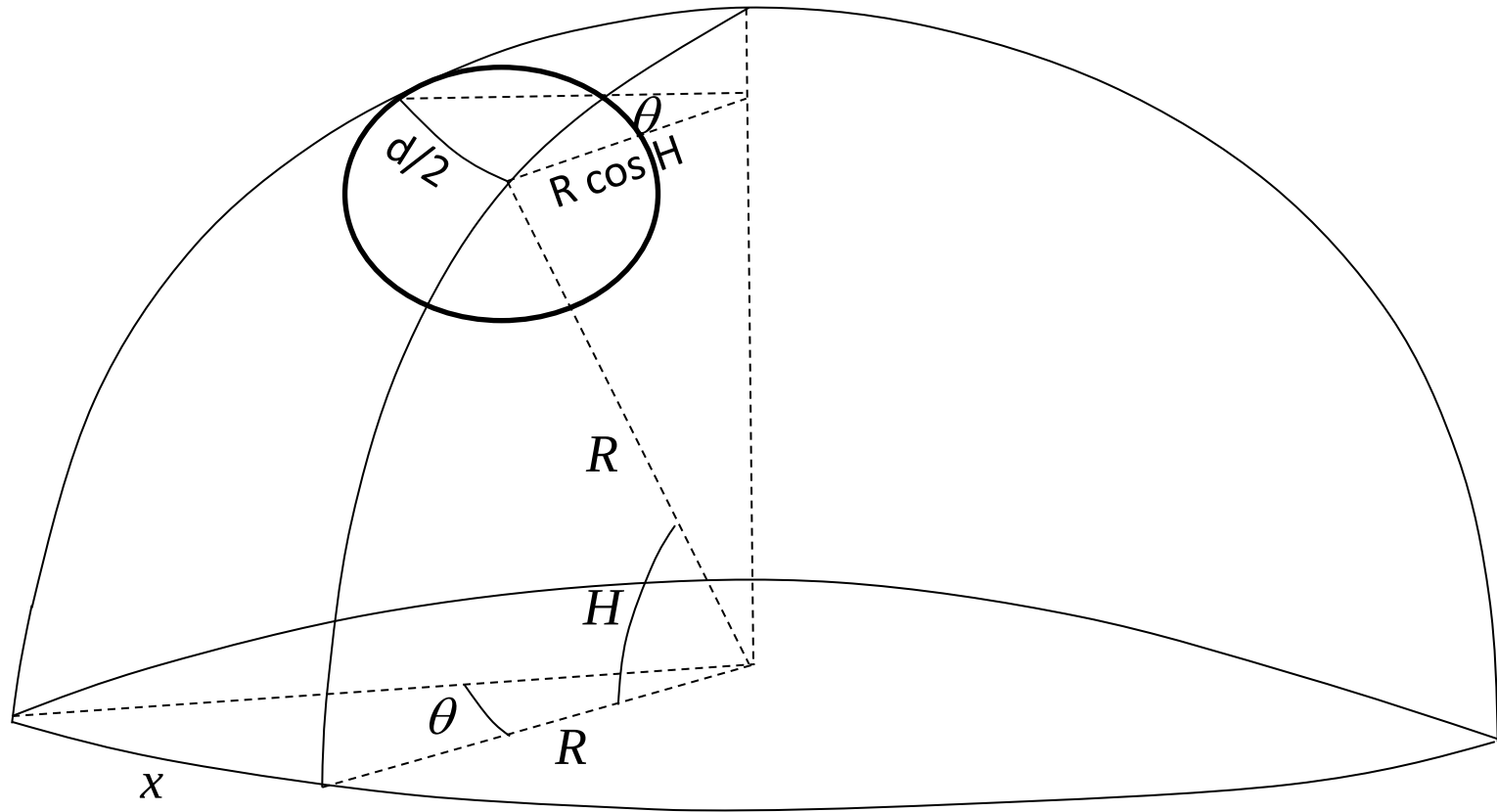


$$\text{Correction} = SOS' = \frac{(\mu - 1) \cot H}{\sin 1''}, \text{ always negative}$$

if $\mu = 1.000294$, $T = 0^\circ \text{C}$, $P = 76 \text{ Hgcm}$

$$\text{Refraction correction} = -58'' \cot H$$

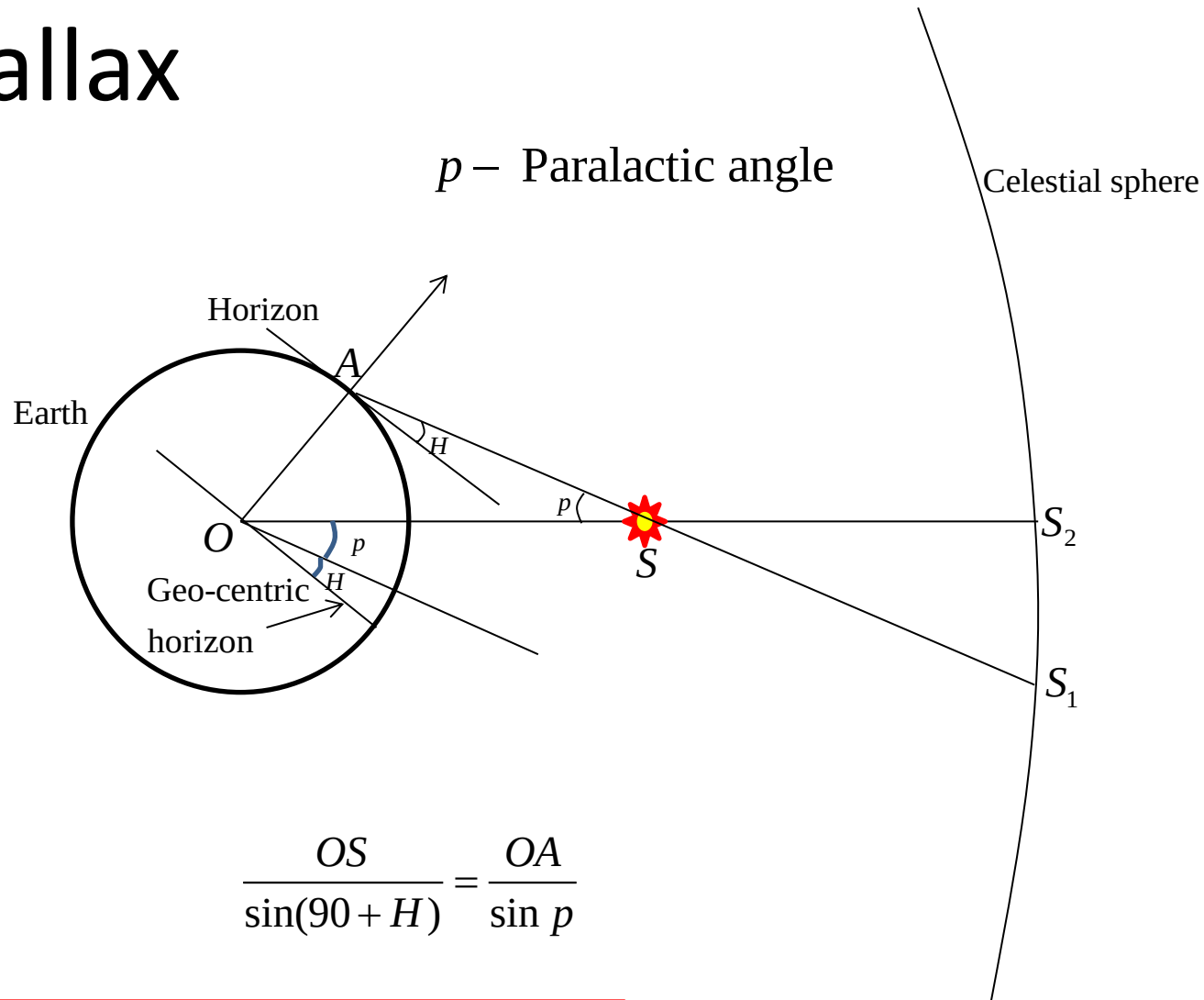
Semi Diameter of Sun



$$\theta = \frac{x}{R} = \frac{d/2}{R \cos H}$$

$$x' = \frac{d'}{2} \sec H = 16' \sec H$$

Parallax



$$\frac{OS}{\sin(90 + H)} = \frac{OA}{\sin p}$$

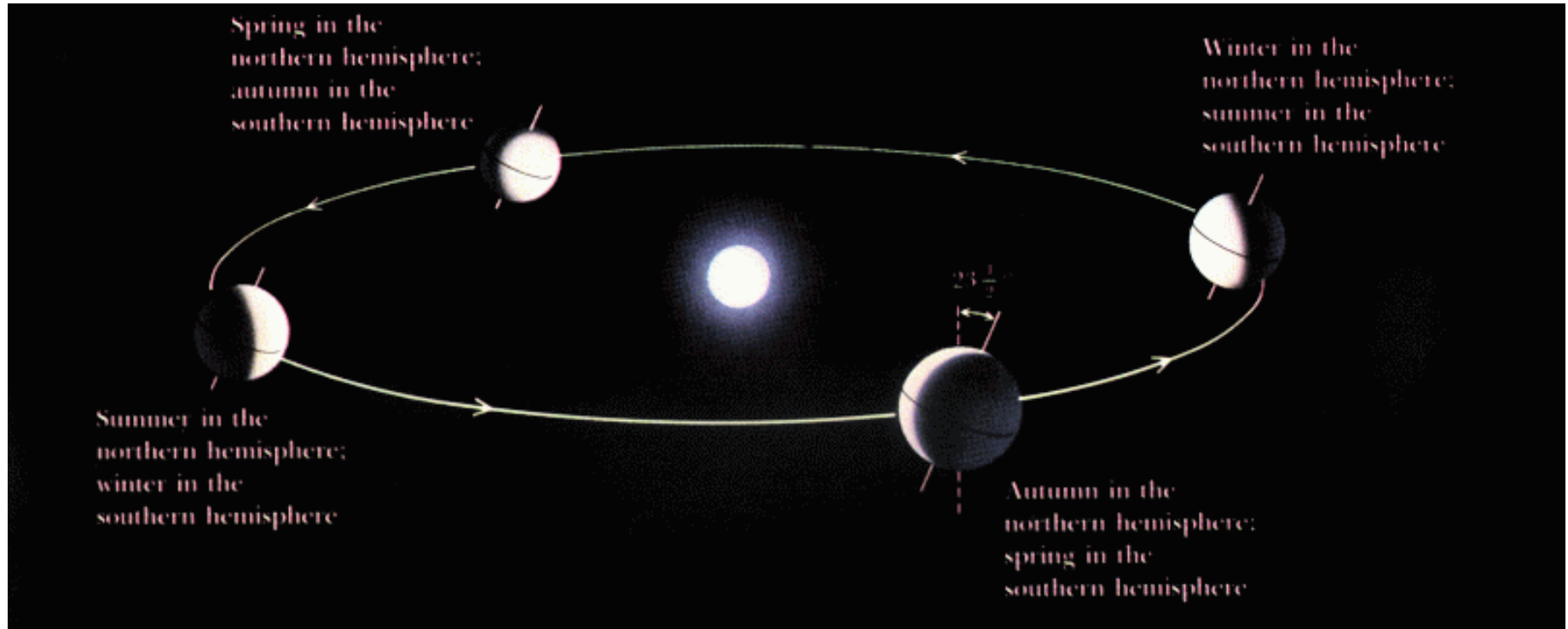
Corrected altitude = $H + p$

$$p = +9'' \cos H$$

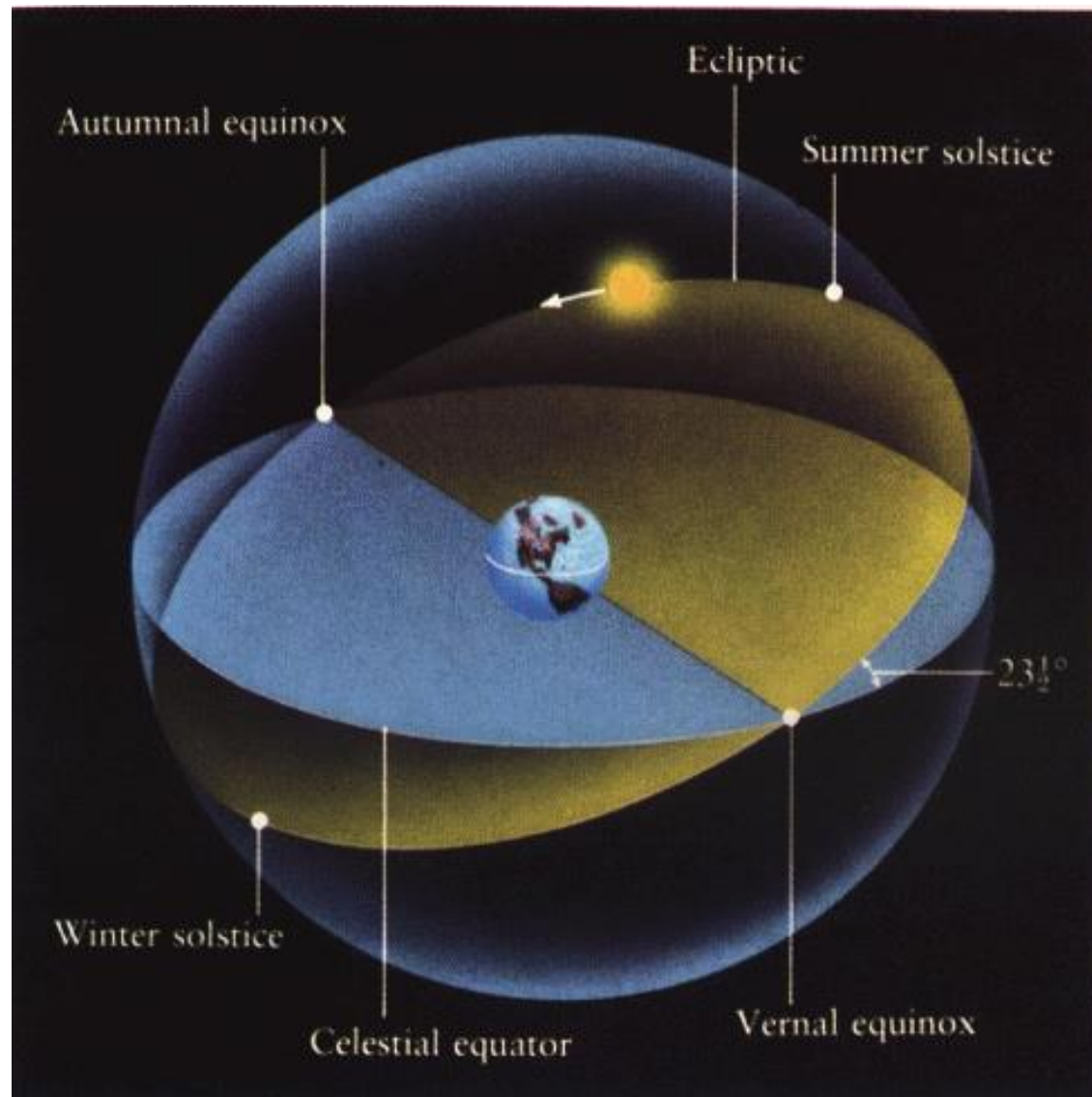
Geodetic Astronomy – L03

Dr. H.M.I.Prasanna

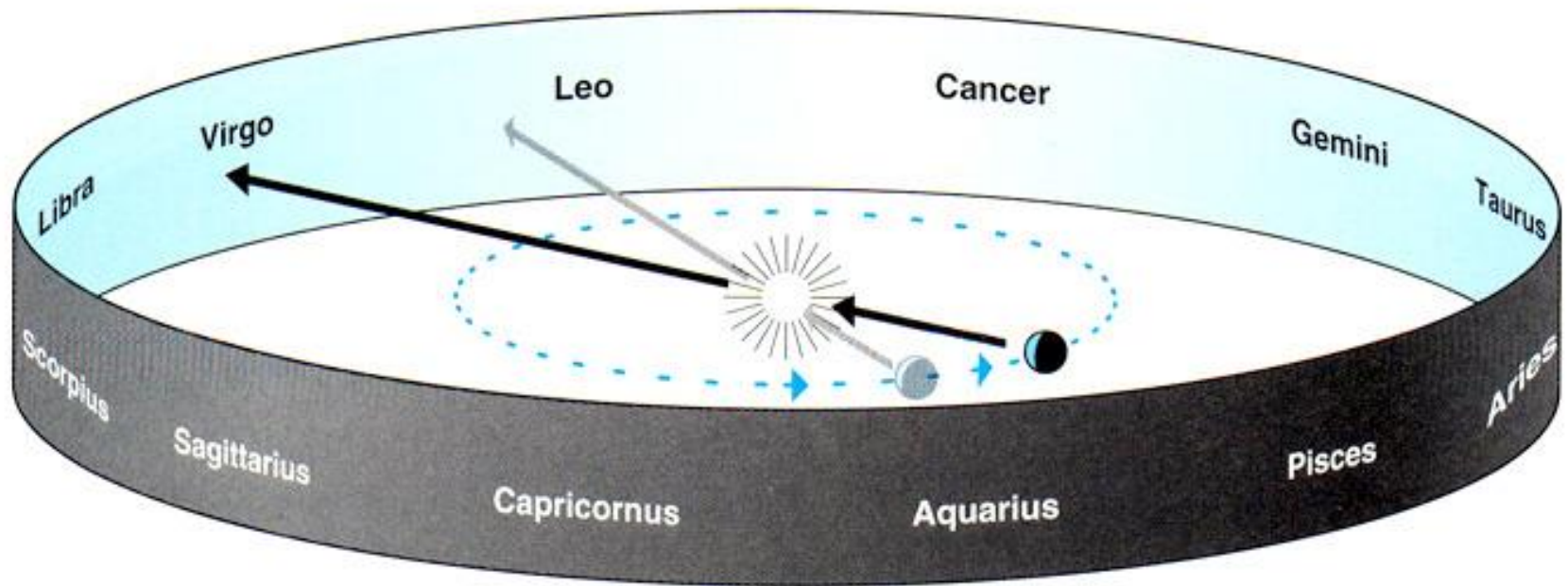
Movement of the Sun



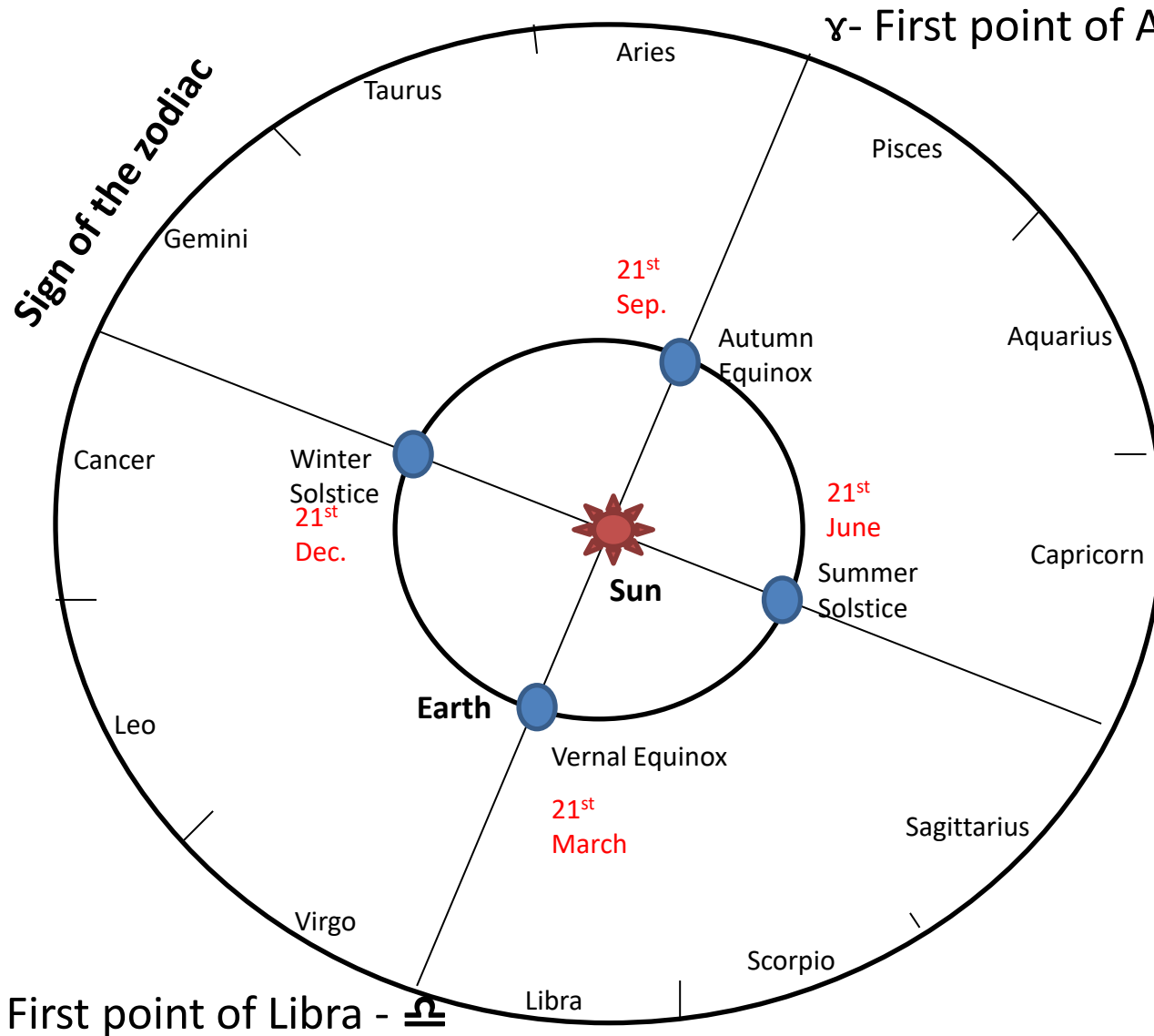
Movement of the Sun



Movement of the Sun

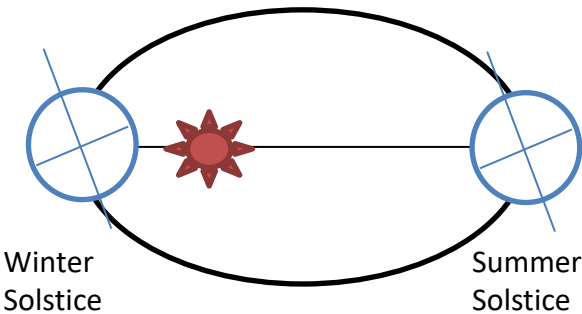
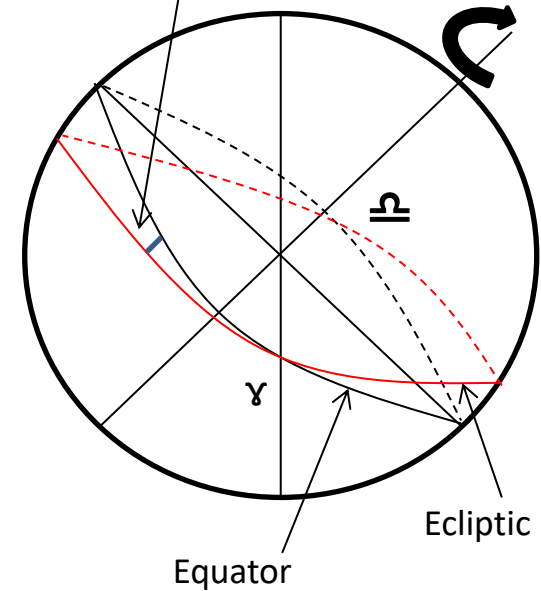


Movement of the Sun

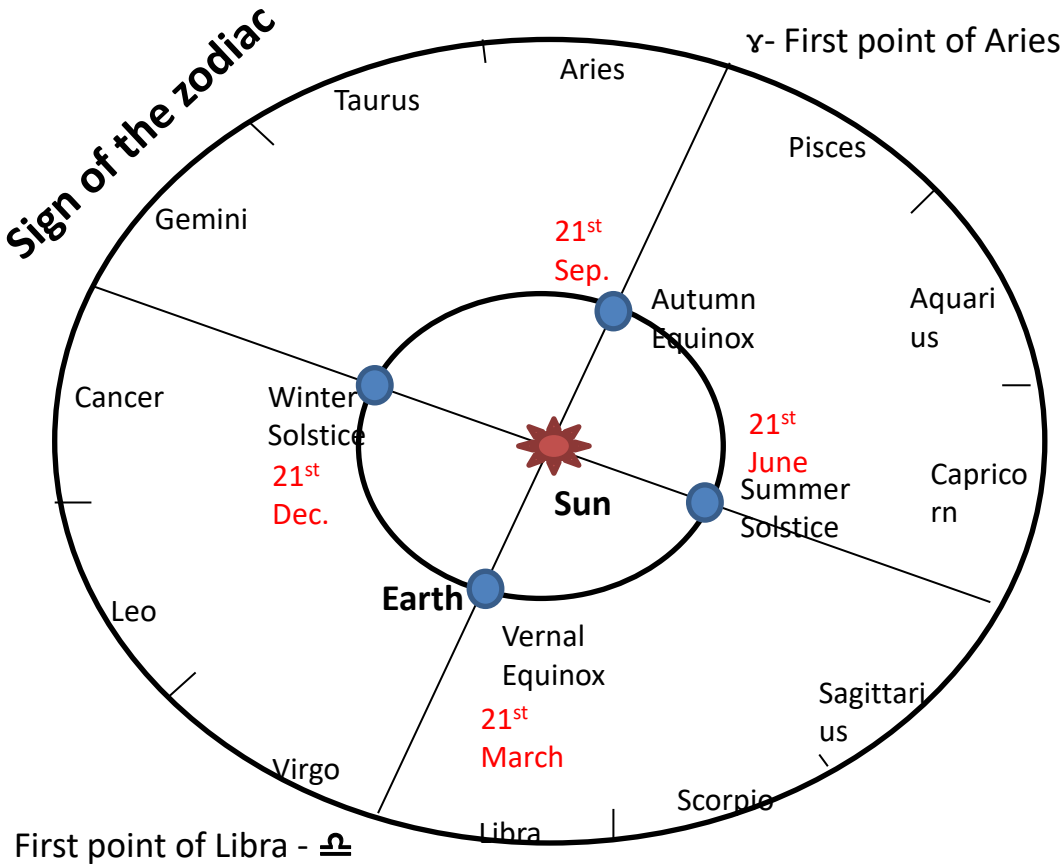


γ - First point of Aries

Obliquity of equator



Movement of the Sun



Date	Earth/Sun position	Declination	Right ascension
21 st March	Vernal Equinox/ ♈	0	0
21 st June	Summer Solstice/ 1 st point of Cancer	Max. North 23°27'	6h
21 st September	Autumn Equinox/ ♎	0	12h
21 st December	Winter Solstice/ 1 st point of Capricorn	Max. South -23°27'	18h

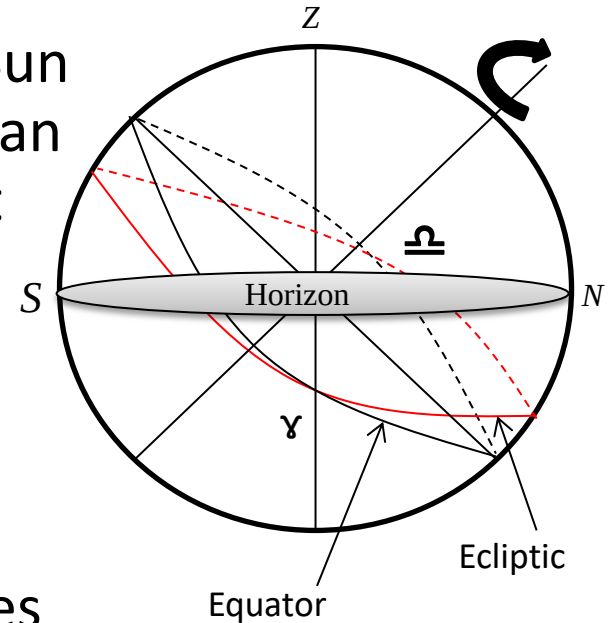
Movement of the Sun

- On 21st March (vernal equinox), sun at γ . Sun rises exactly East, crosses observers meridian south of zenith and sets exactly West point

day period = night period

- On the next day, sun moves slight along ecliptic, and now North of equator. Sun rises slightly North of East and sets slightly North of West

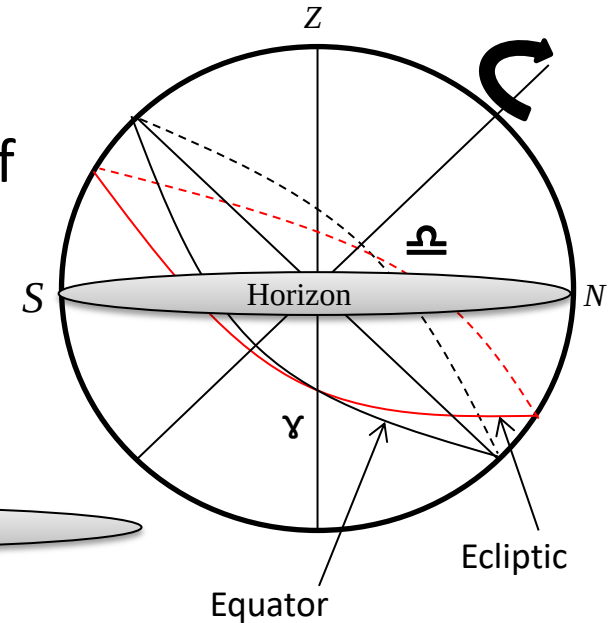
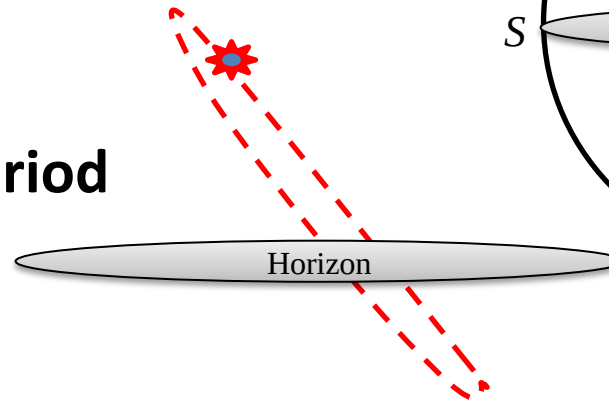
day period > night period



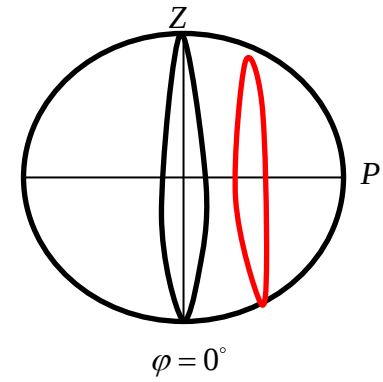
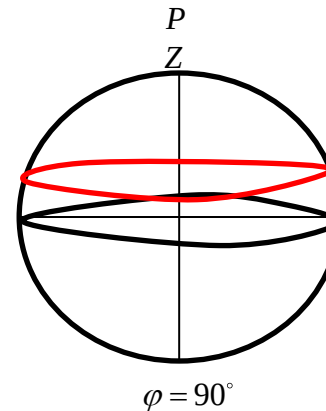
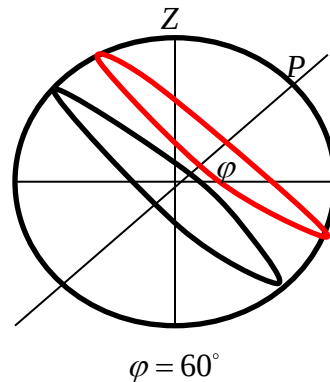
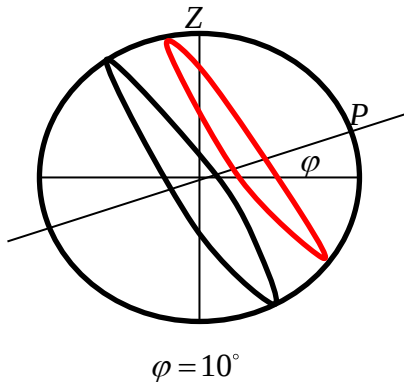
Movement of the Sun

- On 21st June (summer solstice), sun rises 23°27' North of East and sets 23°27' North of West. So,

day period >>> night period



depending on observer's latitude:

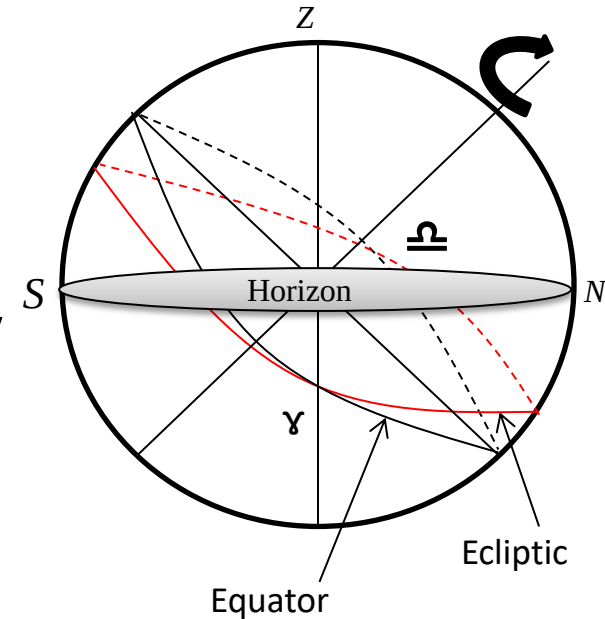


Movement of the Sun

- Now sun starts moving southwards.
- On 21st September (Autumn equinox), sun again back to equator (α). So, it rises exactly East and sets exactly West.

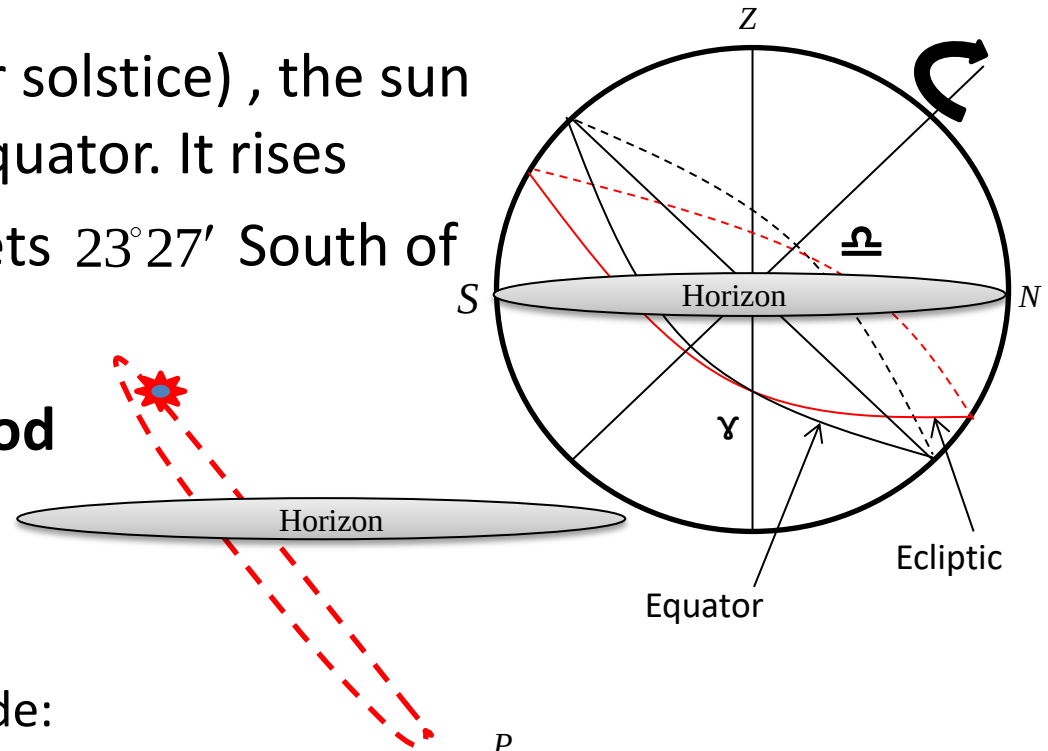
day period = night period

- This southward movement goes on and gradually, **day period < night period**

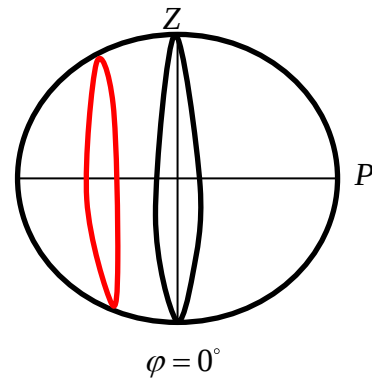
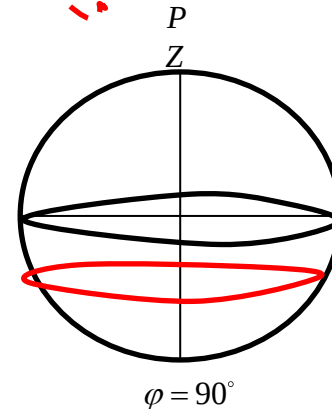
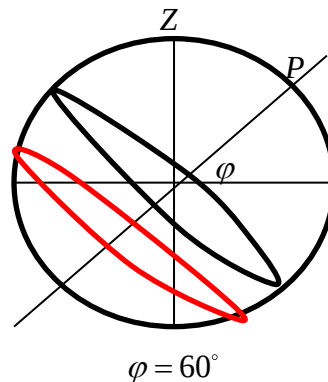
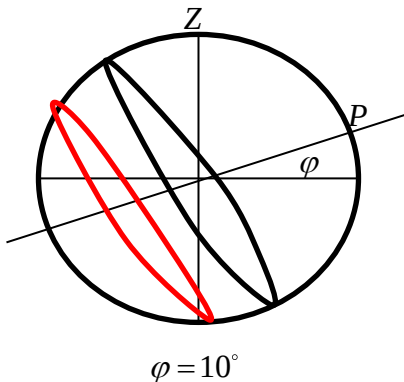


Movement of the Sun

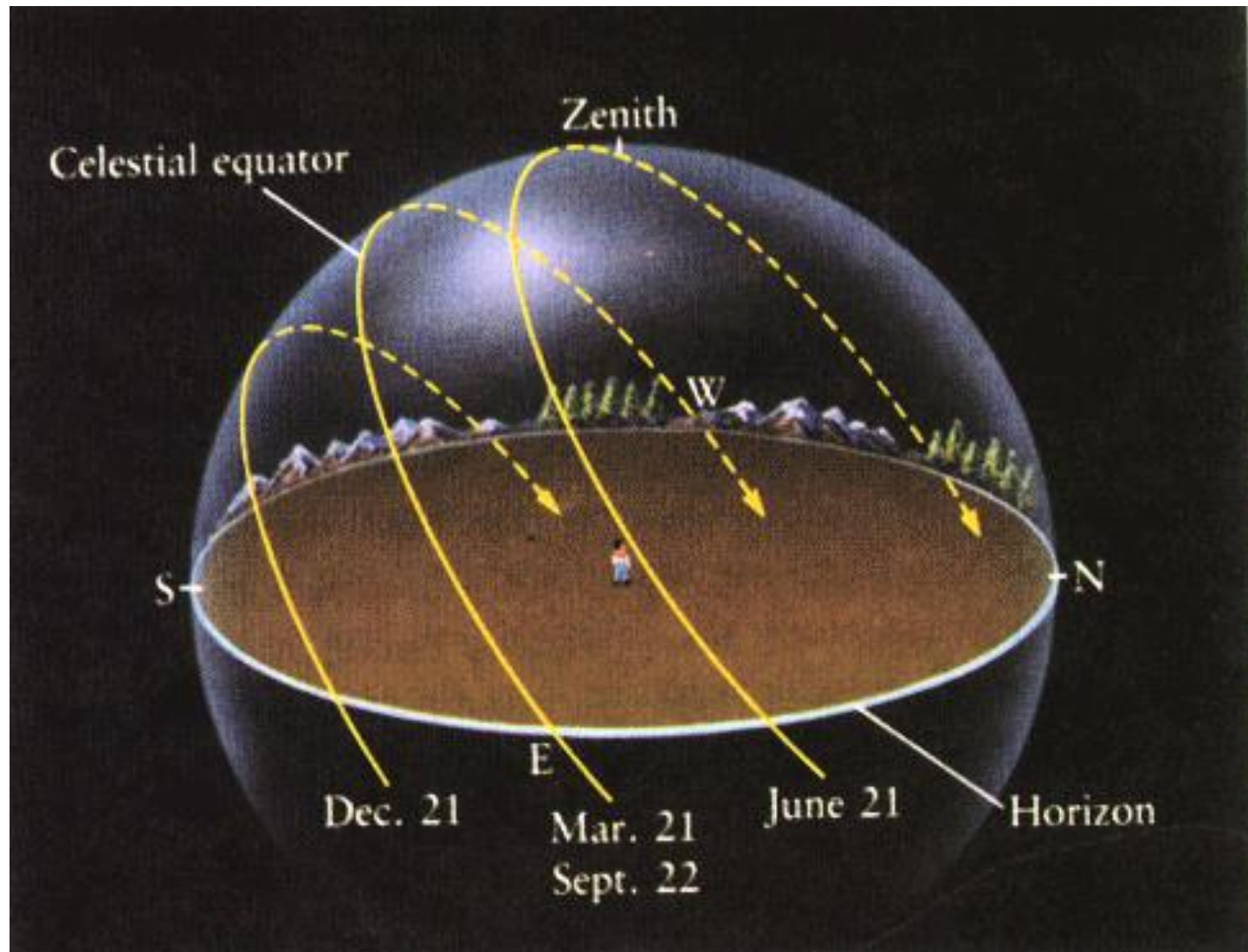
- On 21st December (Winter solstice), the sun is at maximum south of equator. It rises $23^{\circ}27'$ South of East and sets $23^{\circ}27'$ South of West. So,
day period <<< night period



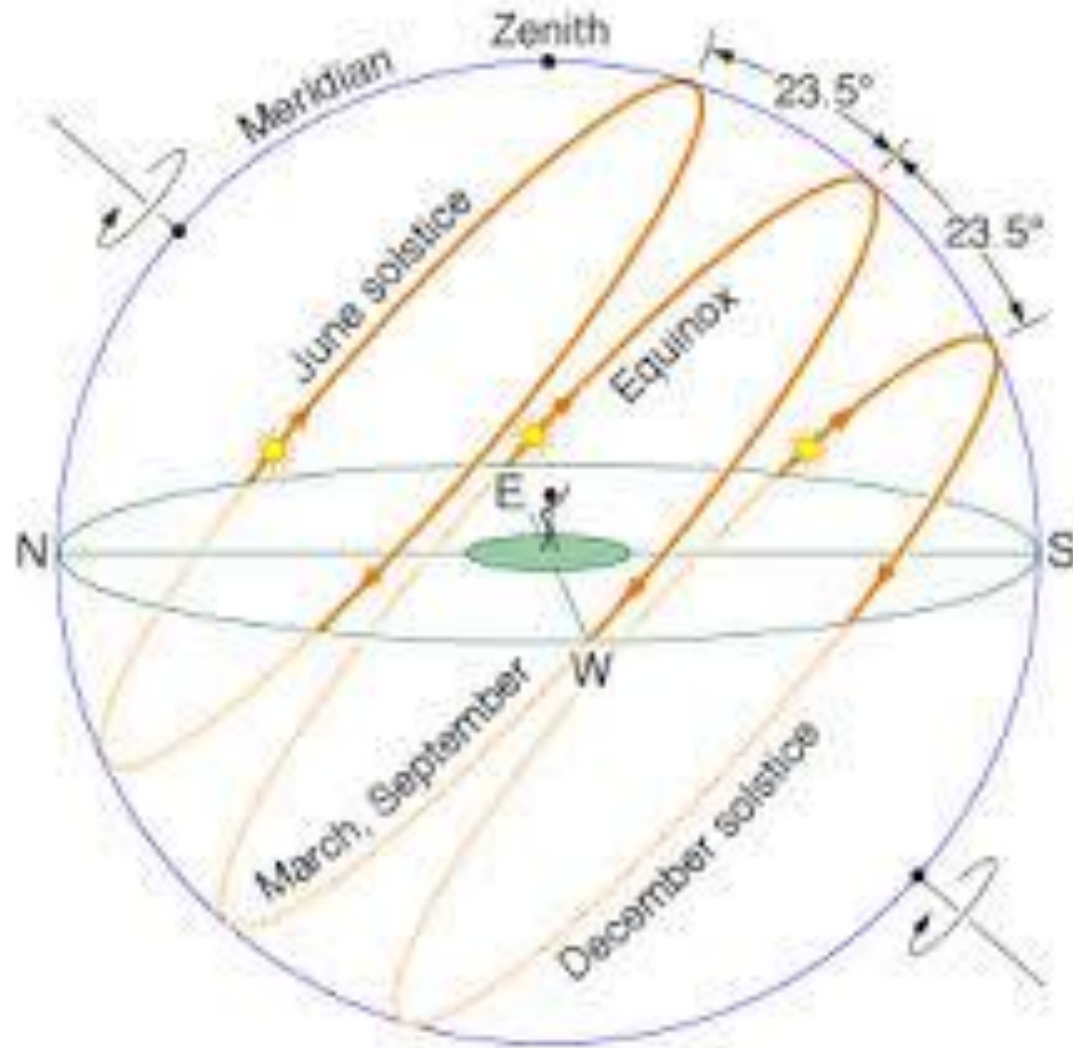
depending on observer's latitude:



Movement of the Sun



Movement of the Sun



Geodetic Astronomy – L04

Dr. H.M.I.Prasanna

Time

- Year and Day
 - Calendar year and solar year
 - Day
 - Sidereal day and Solar day
 - Sidereal time and Solar time
 - GMT

Calendar year and Solar year

- Calendar year/ Civil year – 365 or 366 days
- Solar/ Tropical year – Interval between successive vernal equinox

Day is the interval between successive upper/lower transits of some celestial body

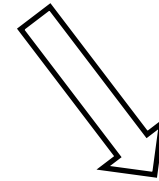
Celestial bodies have unique movements which create different type of days

Sidereal day/time

- **Sidereal day** – Time interval between successive upper transit of the first point of Aries. Day is of uniform length
- **Sidereal time** – Hour angle of the first point of Aries. Time is local because it is related to the local meridian of the observer.

If the first point of Aries is at the upper transit of the observer's meridian, sidereal time is 0h

View Direction
for time diagram



The diagram illustrates the celestial sphere with the following components:

- Observer's Position:** Indicated by a point P at the center of the sphere.
- Local Sidereal Time (LST):** Represented by the angle h (hour angle) between the meridian (vertical line) and the hour circle passing through the star S .
- Right Ascension (RA):** Represented by the angle γ (gamma) between the meridian and the great circle passing through the star S and the celestial equator.
- Star Position:** The star is labeled S and is located on the celestial sphere.
- Coordinate System:** The vertical line represents the meridian, with L at the top (North) and L' at the bottom (South). The horizontal line represents the celestial equator, with E on the left (East) and W on the right (West).
- Green Circle:** A green circle is drawn around the star S , representing a circle of constant Right Ascension (RA).
- Red Dashed Line:** A red dashed line connects the observer's position P to the star S .
- Angle h :** The angle between the meridian and the hour circle passing through the star S .
- Angle γ :** The angle between the meridian and the great circle passing through the star S and the celestial equator.

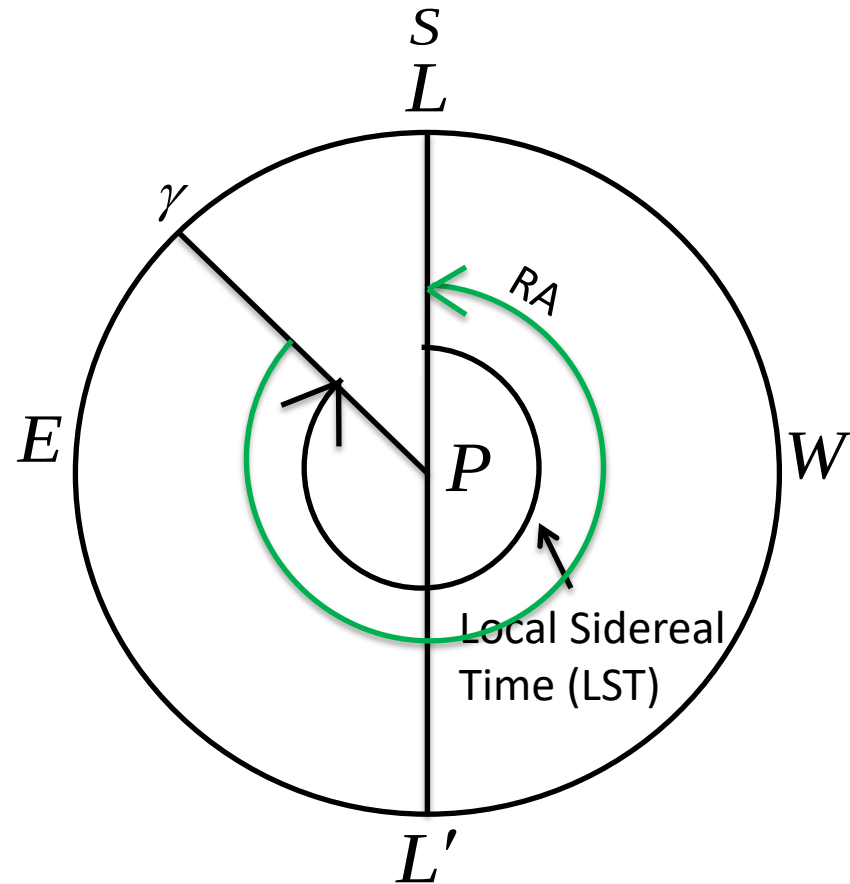
$$LST = RA_{star} - h_{star}$$

Sidereal day/time

When the star is on the observer's meridian, i.e. upper transit of the star, $h=0$

$$LST = RA_{star}$$

Time Diagram

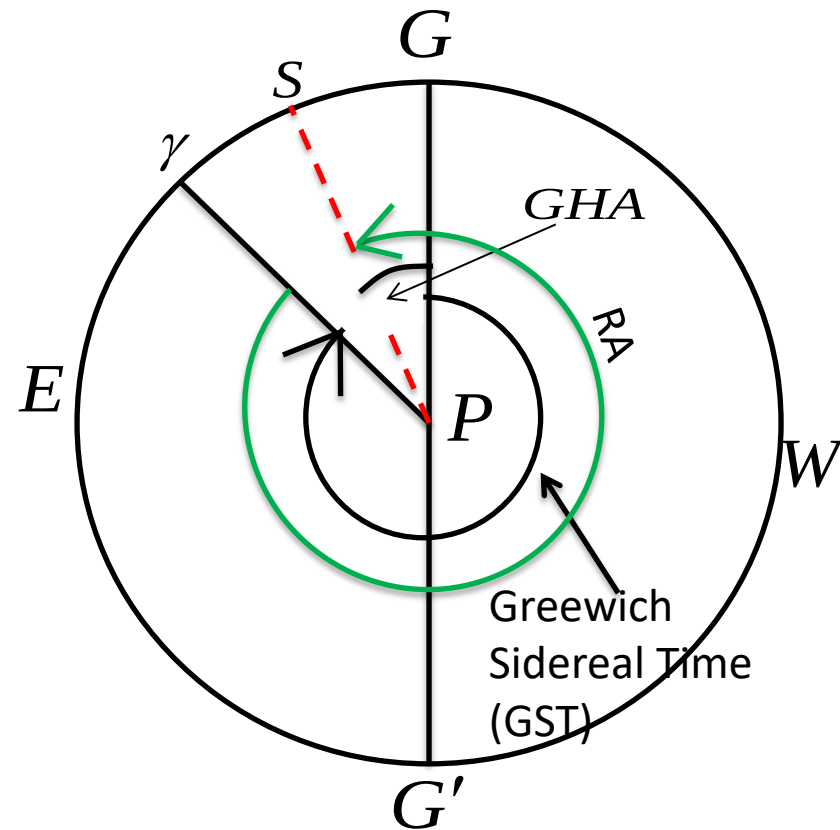


Sidereal day/time

When the observer is on the Greenwich meridian

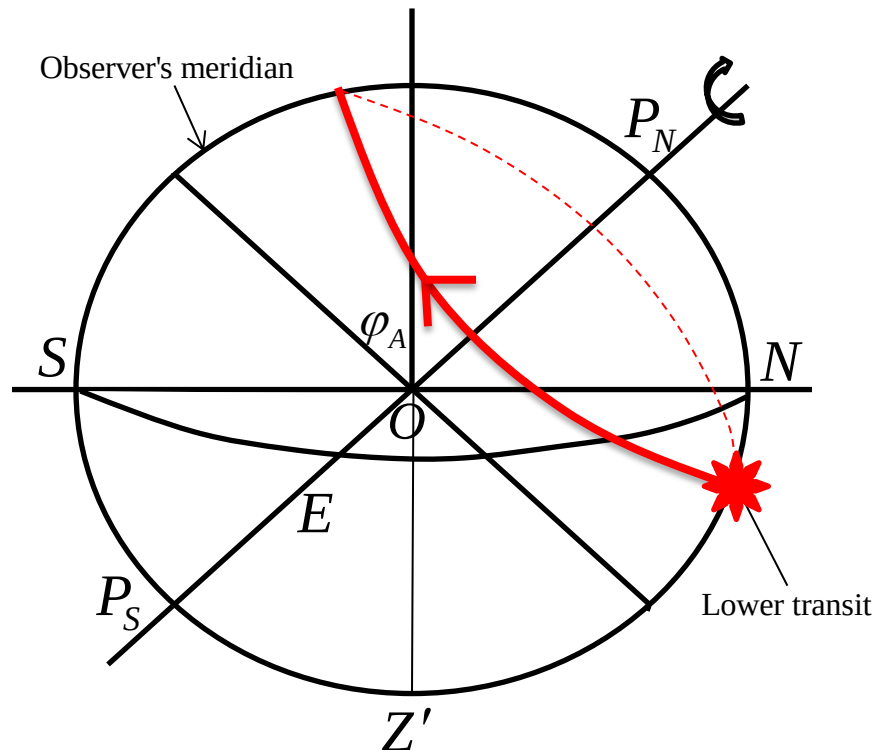
$$GST = RA - GHA_{Star}$$

Time Diagram



Solar day/time

- The apparent solar day is the time interval between successive lower transit of the Sun
- Solar day begins at mid night



Solar day/time

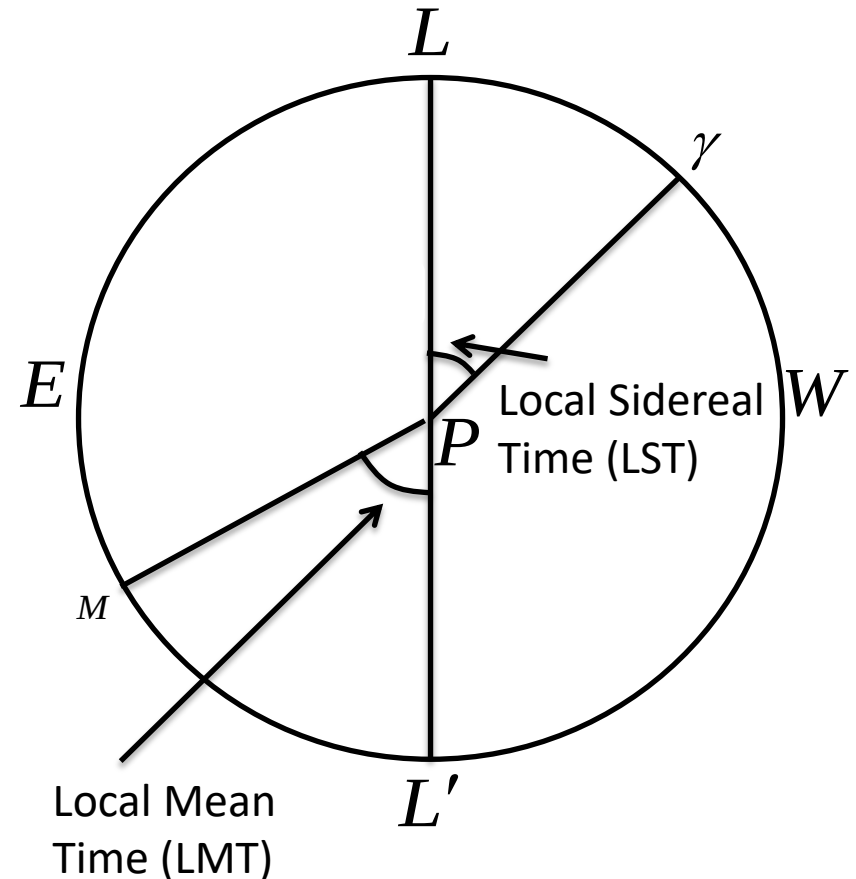
- Solar day varies due to:
 - Kepler's law – the movement of the Sun is not uniform
 - The Sun moves along the ecliptic but not equator

To overcome this problem, a **Mean Sun** is introduced, which is an imagined to move along the equator with constant angular velocity

- **Apparent Sun is moving along the ecliptic with non-uniform velocity**
- **Mean Sun is moving along the equator with uniform angular velocity**

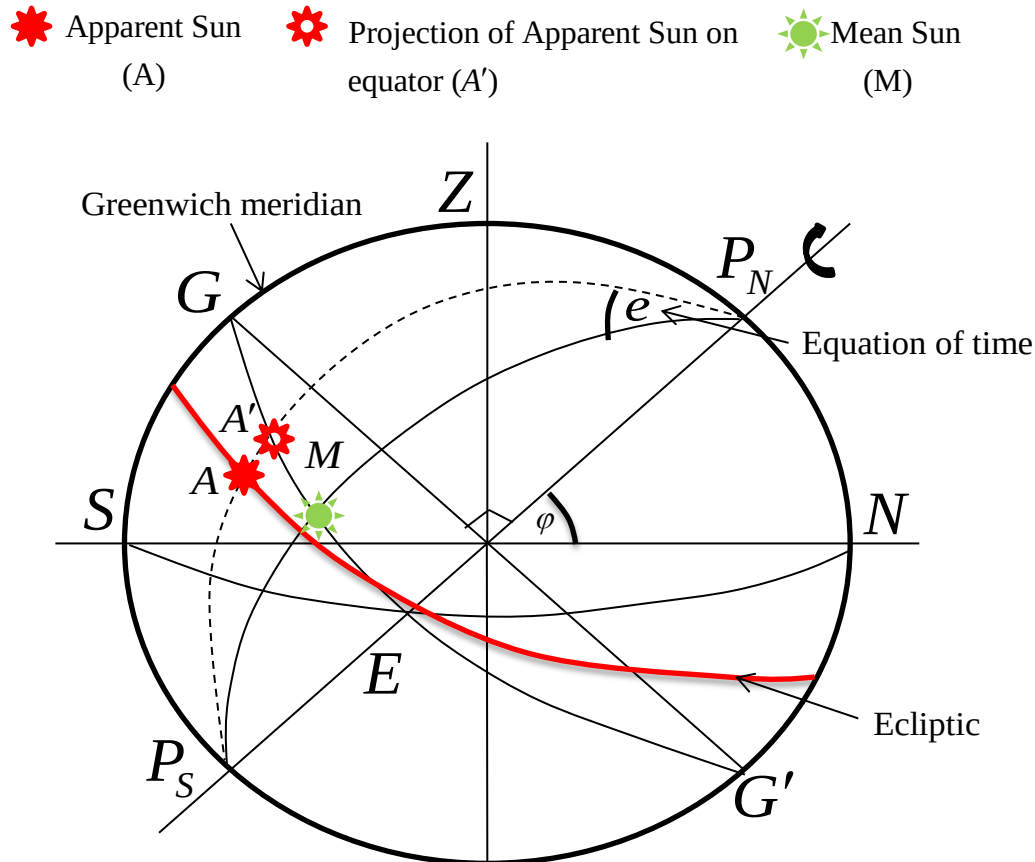
Solar day/time

- Mean Solar Day
 - The average length of all solar days taken from one Vernal equinox to the next, and is taken on lower transit.
- Mean Solar Time
 - The hour angle of the mean Sun, taken from the lower transit



Universal Time (UT/GMT)

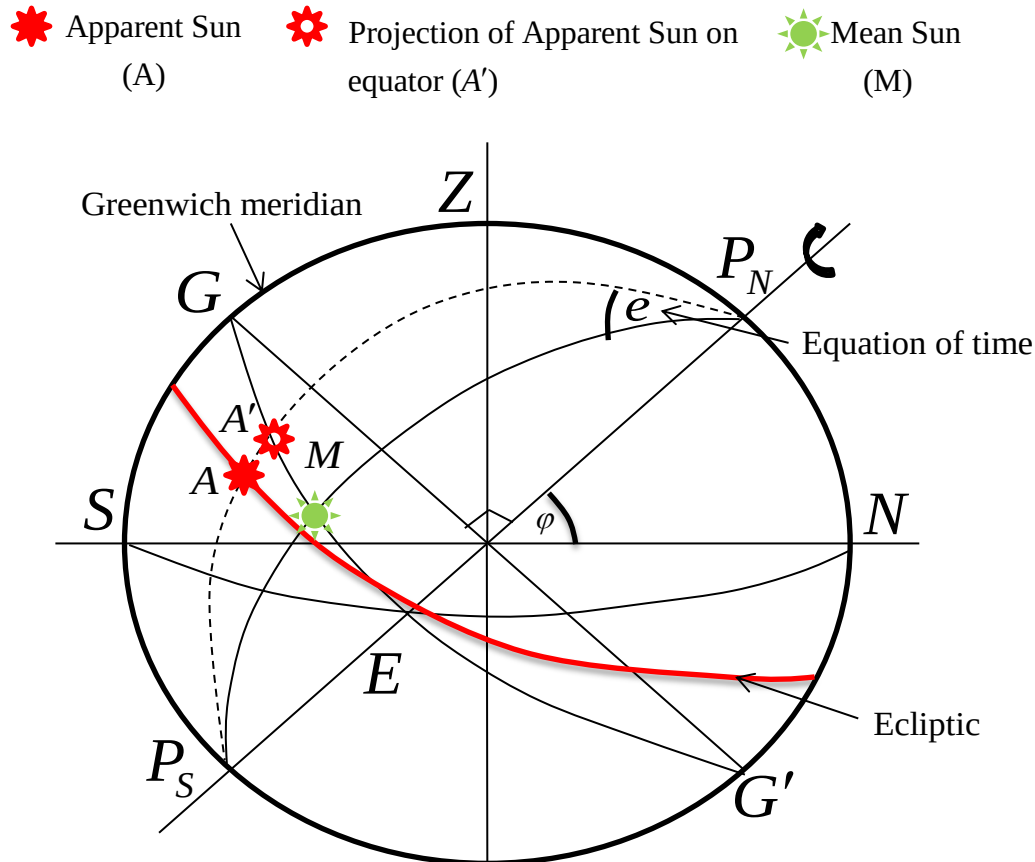
- UT/GMT is hour angle measured from the lower transit of the Mean Sun from Greenwich meridian



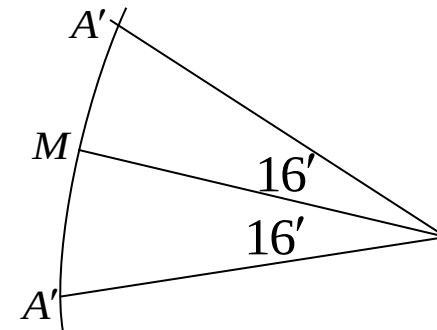
- Apparent Sun (or its projection) and the mean Sun are two different objects
- They are together 4 times a year
 - Two Equinoxes
 - Two Solstices

Universal Time (UT/GMT)

- UT/GMT is hour angle measured from the lower transit of the Mean Sun from Greenwich meridian

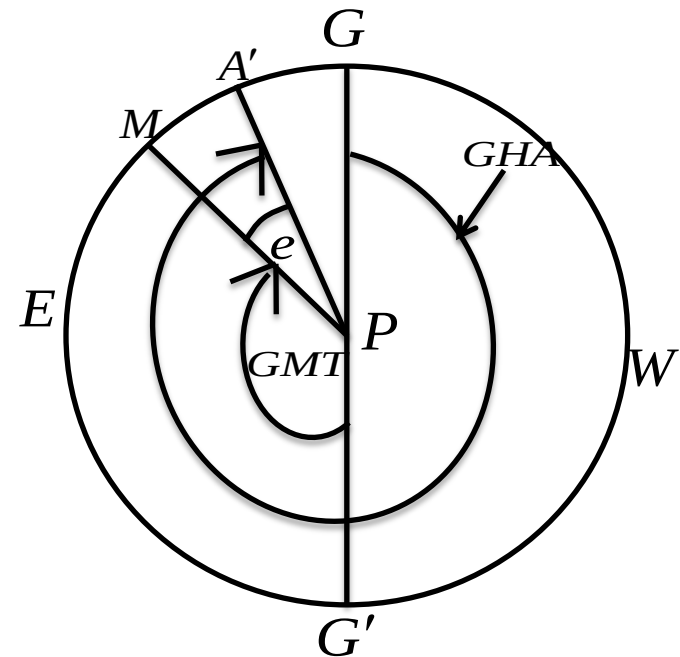
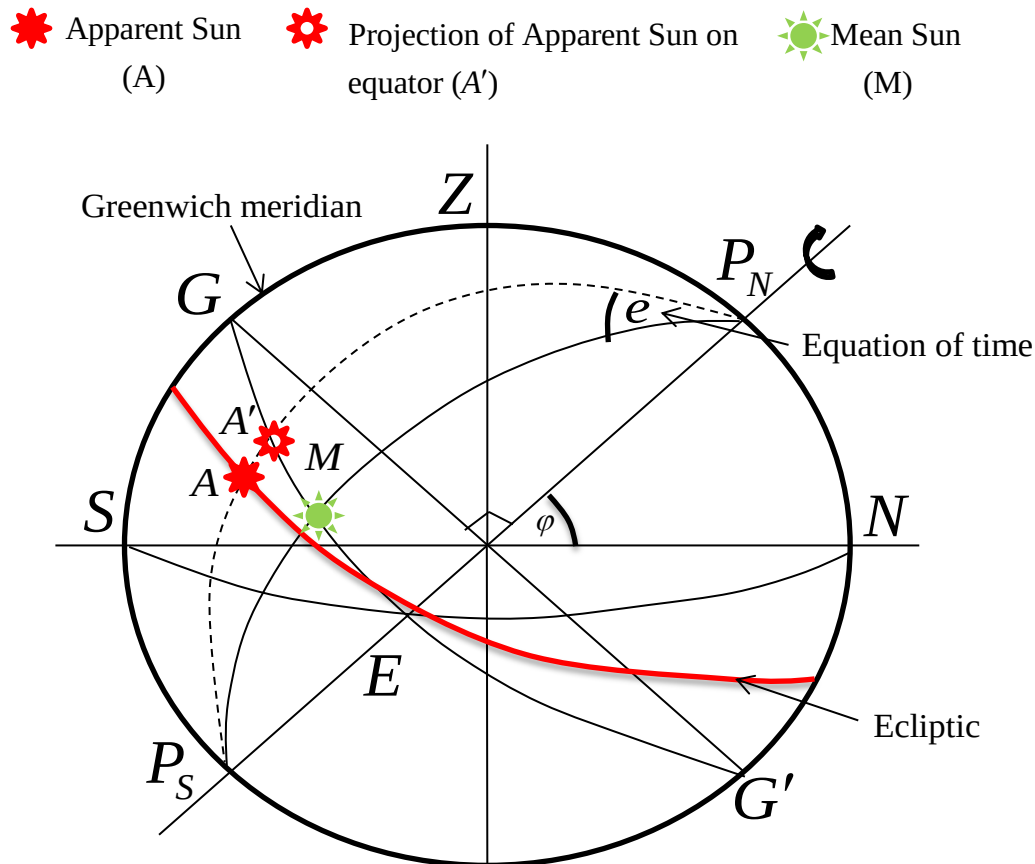


- **Equation of time** – The angle between apparent and mean Sun
- **Max** – 16' of arc



Universal Time (UT/GMT)

- UT/GMT is hour angle measured from the lower transit of the Mean Sun from Greenwich meridian



$$GHA = GMT + (12^h \pm e)$$

$$GHA = GMT + E$$

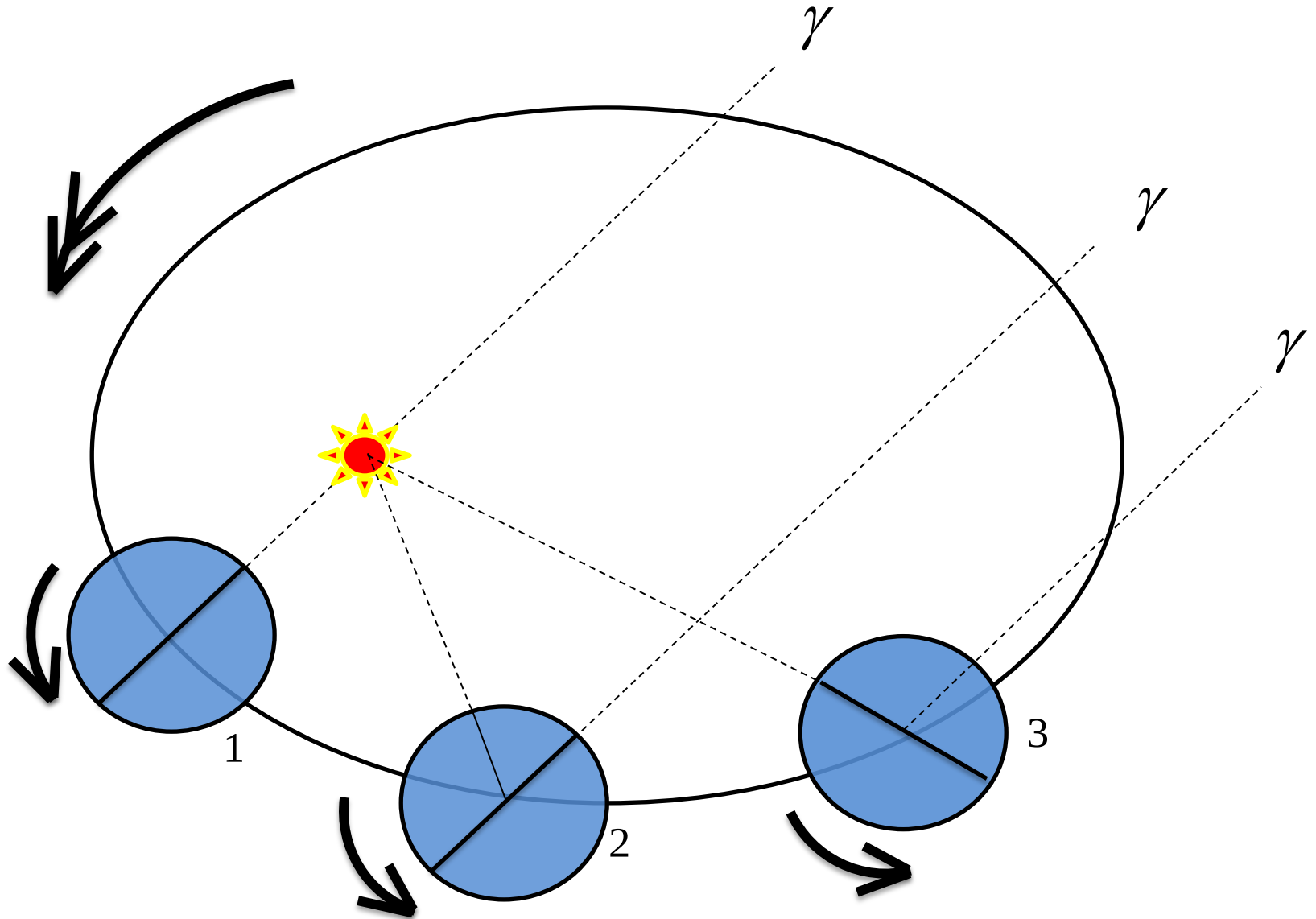
Sidereal Time Vs Solar Time

- First point of Aries, Mean Sun & Apparent Sun are used to measure the time
- If relative positions of celestial bodies on the celestial sphere were constant, their hour angle is equally changed during the same time interval

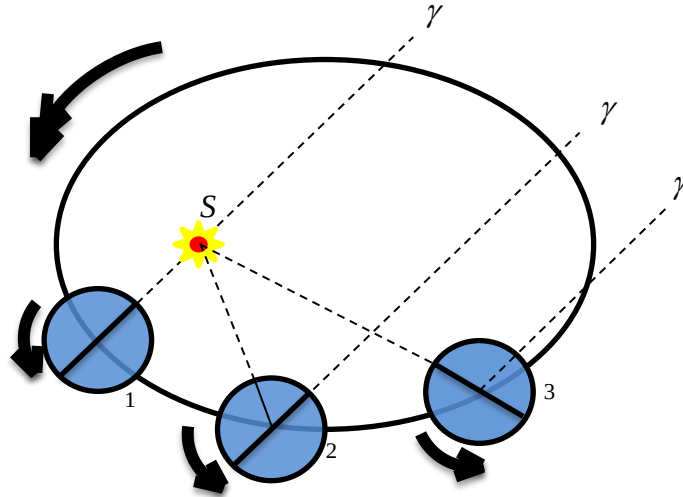
i.e. 1 hour of mean time = 1 hour of Sidereal time

???

Sidereal Time Vs Solar Time

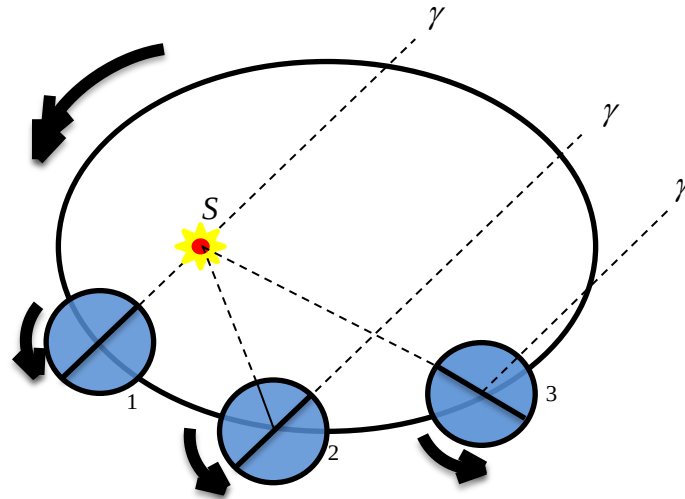


Sidereal Time Vs Solar Time

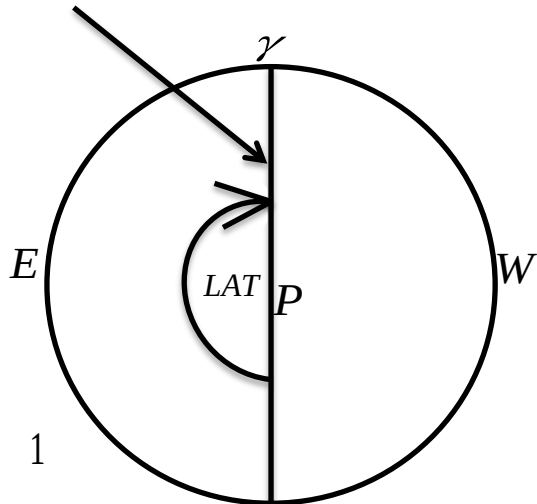


- At position 1, **S** and **γ** are together on the observer's meridian
- At position 2, Earth has rotated around its axis until **γ** is again on the observer's meridian, in the meantime, Earth has travelled a short distance around the Sun.
 - **S** and **γ** are not together
 - One sidereal day has lapsed
- At position 3, Earth has rotated a little further, so that **S** now on the observer's meridian
 - **γ** has moved passing transit position
 - One solar day has lapsed

Sidereal Time Vs Solar Time

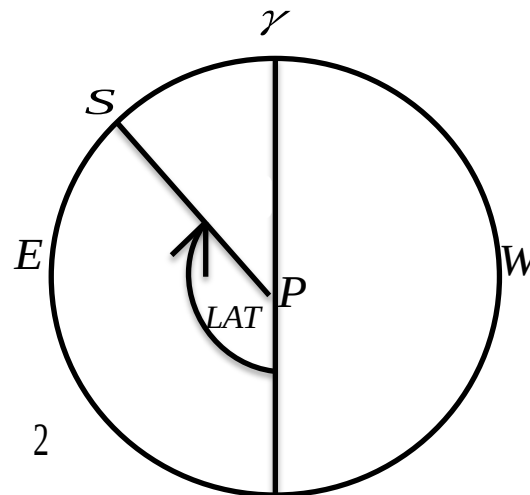


Observer's meridian S



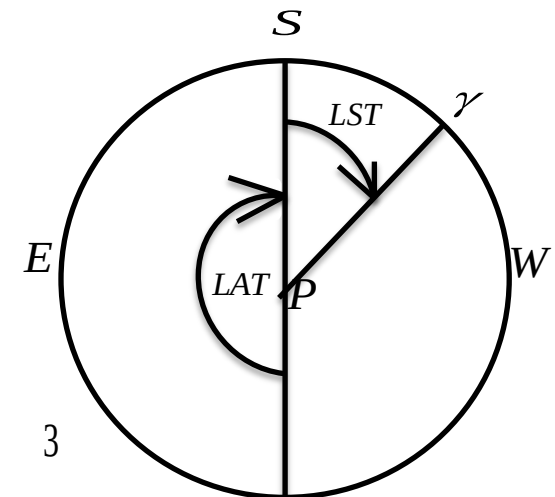
$LST = 0$

$LAT = 12^h$



$LST = 0$

$LAT = \text{less than } 12^h$

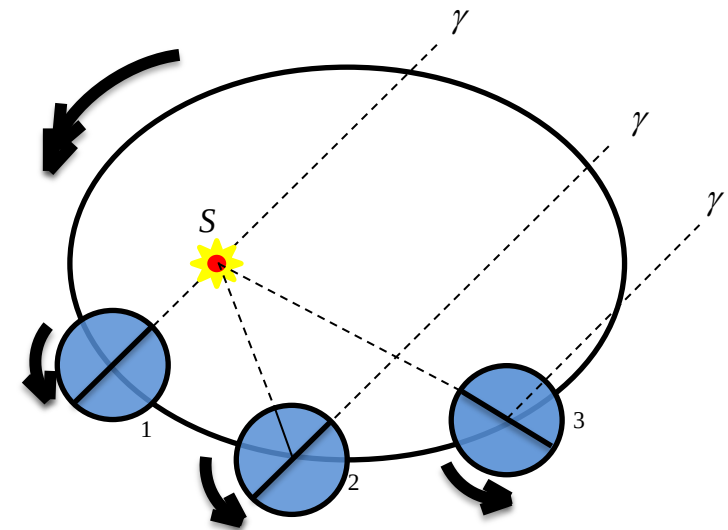


$LST = \text{more than } 0^h$

$LAT = 12^h$

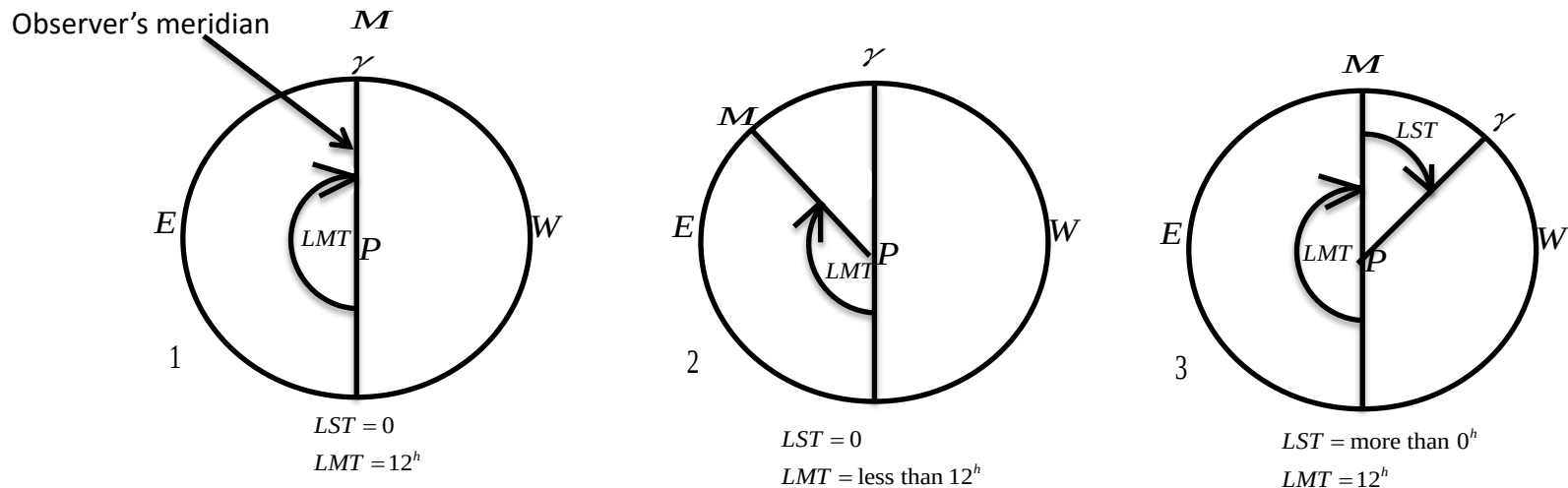
Sidereal Time Vs Solar Time

- Solar day is slightly longer than Sidereal day
- After passing position 1, when the Earth is again at position 1, i.e. after one solar year, S and γ are again together. But Sun has lost one round, i.e. one solar day has lost.
- There are 366.2422 sidereal days in a Solar year. i.e. 365.2422 solar days in a Solar year



Sidereal Time Vs Solar Time

Lets now replace the apparent Sun with Mean Sun

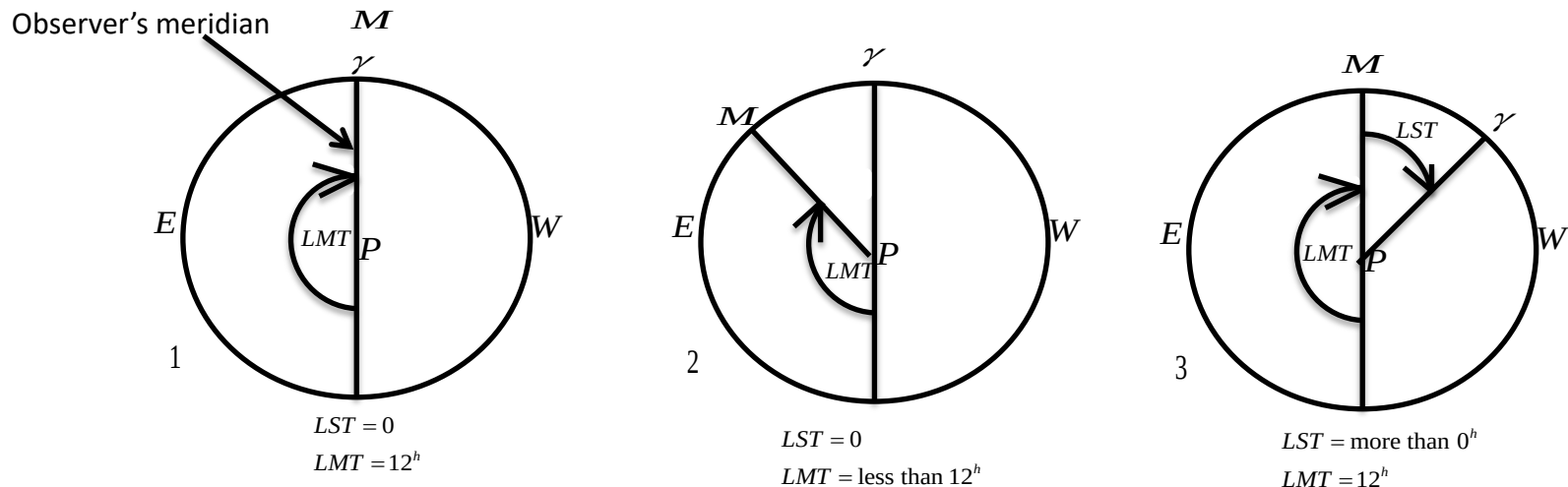


- M has lost 24 hours against 366.2422 sidereal days, then one sidereal day:

- $$\frac{24^h}{366.2422} = 3^m 55.81^s - \text{Retardation of Mean time on Sidereal time}$$

Sidereal Time Vs Solar Time

Lets now replace the apparent Sun with Mean Sun

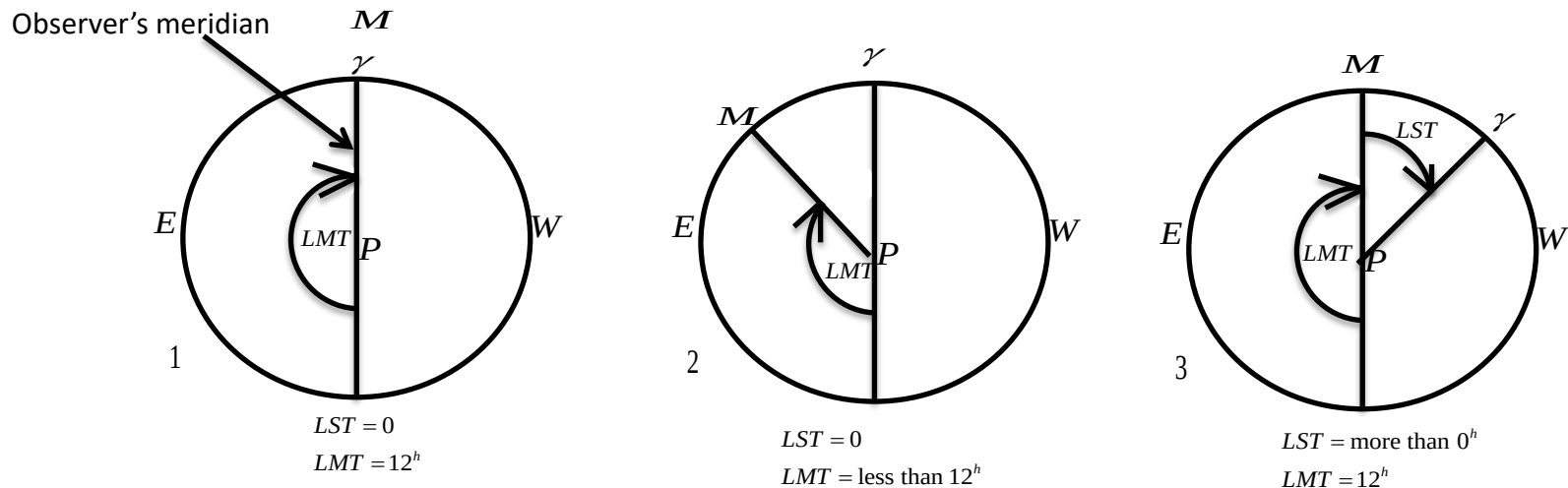


- **γ** is moving faster than M. In 365.2422 solar days **γ** has one round (24 hours) ahead :

- $\frac{24^h}{365.2422} = 3^m 56.56^s$ - **Acceleration of Sidereal time on mean time**

Sidereal Time Vs Solar Time

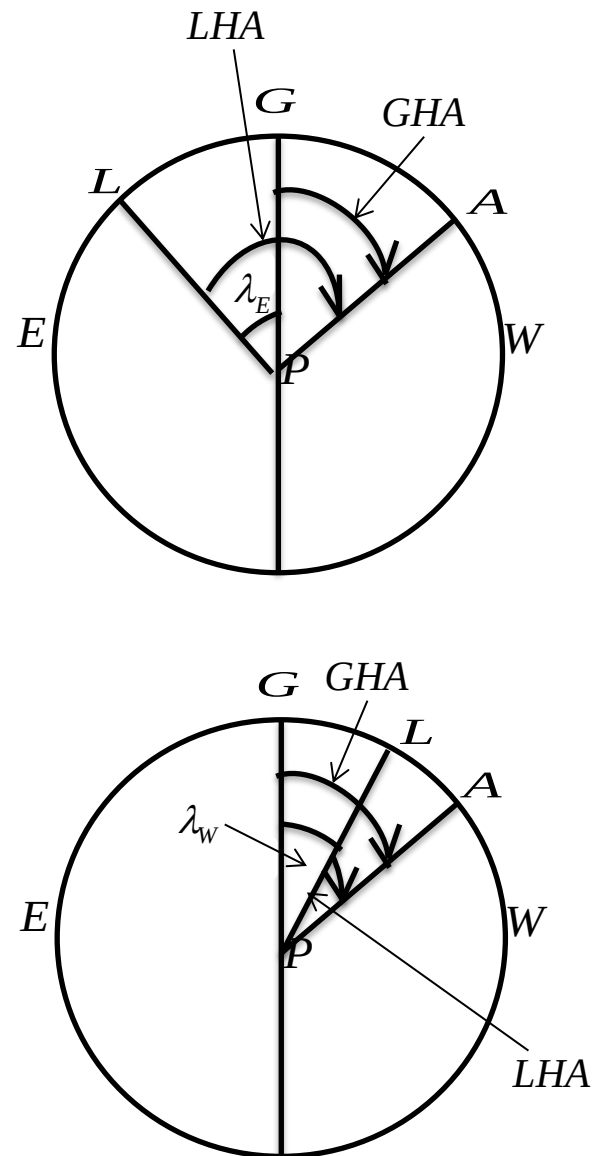
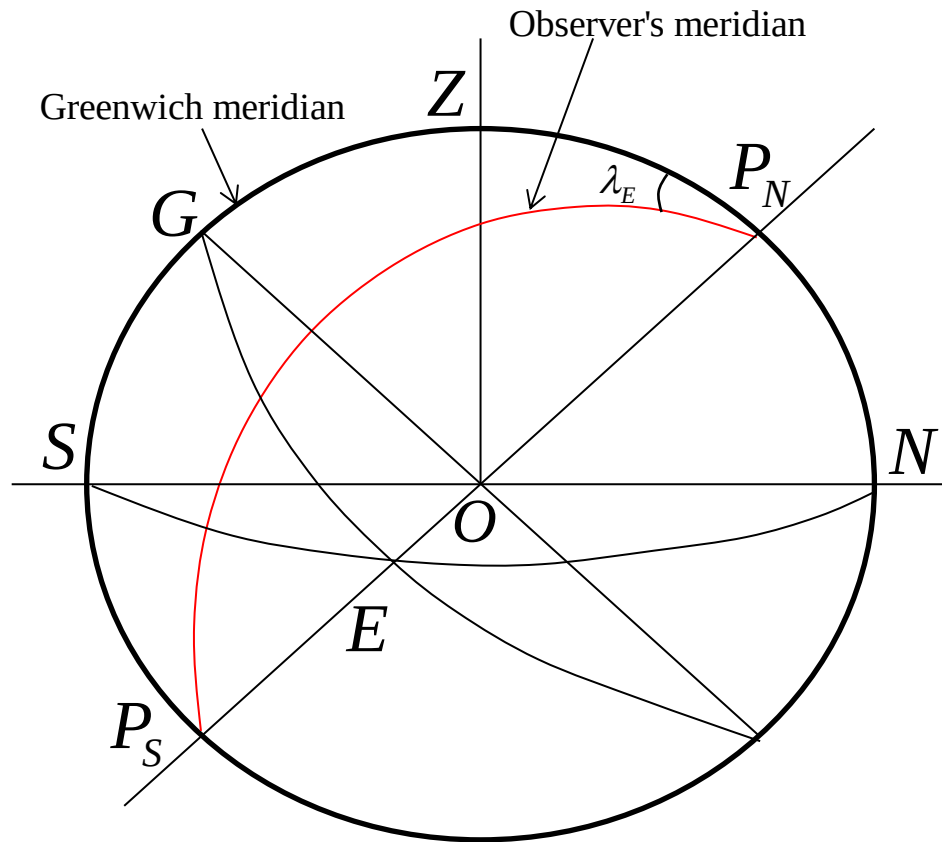
Lets now replace the apparent Sun with Mean Sun



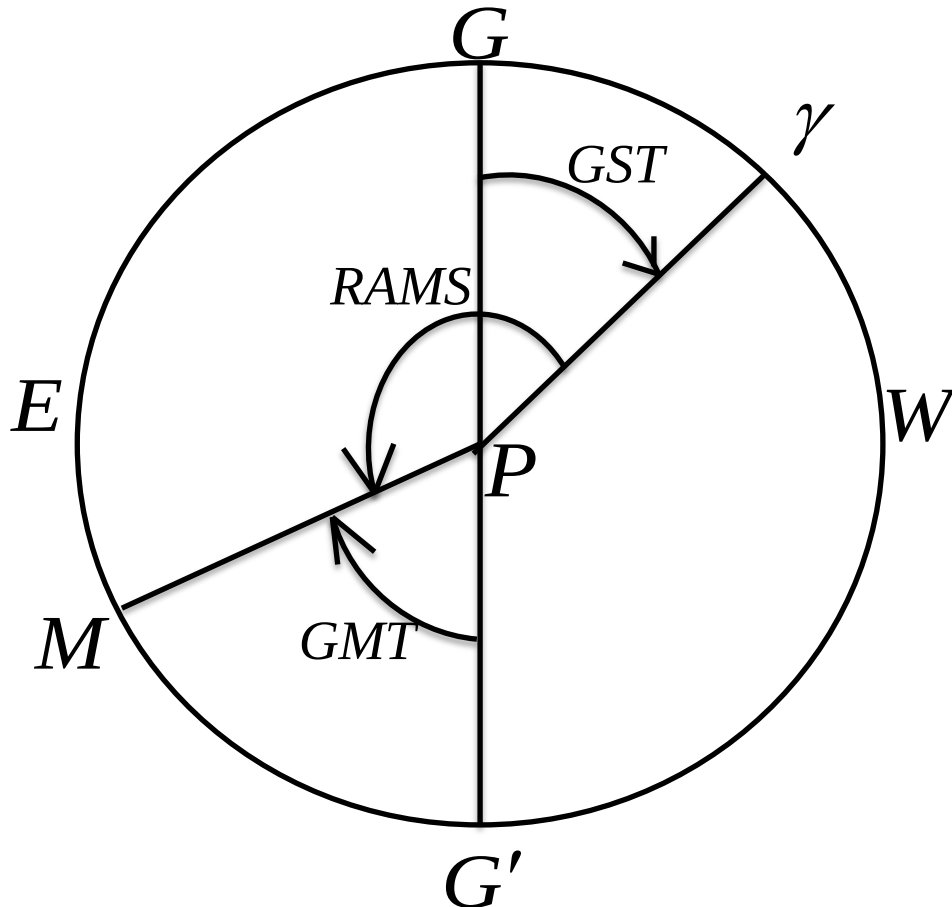
$$1 \text{ Solar Day} = 24^h 3^m 56.56^s \text{ Sidereal hours} \\ = MT + \text{Acceleration} = ST$$

$$GST = GMT + R$$

Time Vs Longitude



Sidereal Time Vs Mean Time



We know

$$GST = GMT + R$$

From the diagram

$$GST = GMT + RAMS \pm 12^h$$

Therefore,

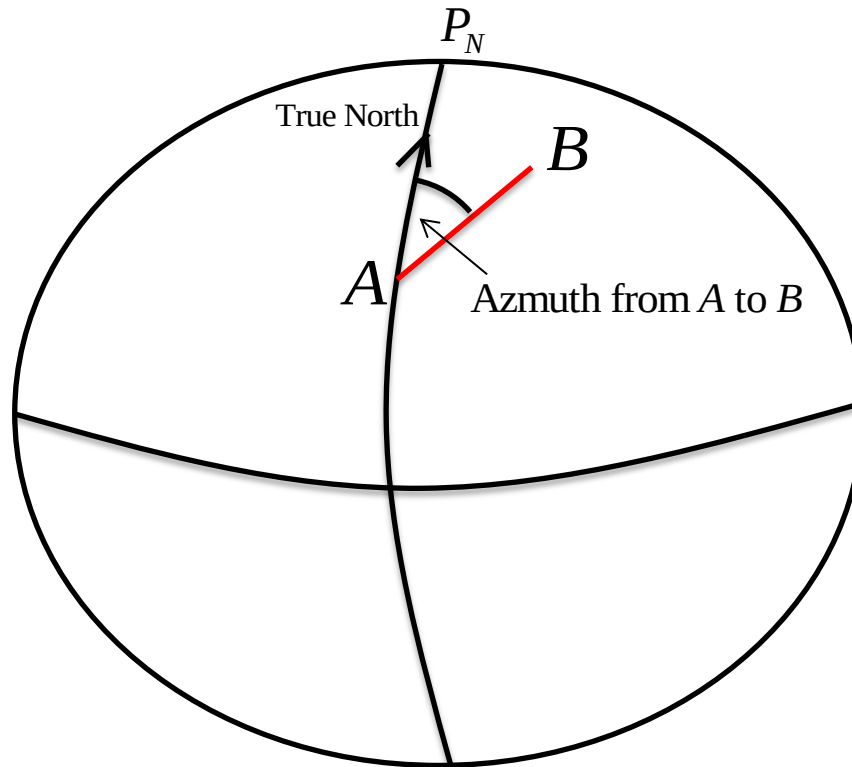
$$R = RAMS \pm 12^h$$

$RAMS$ – Right Ascension of Mean Sun

Computation of Sun or Star Hour Angles

Computation	Given	To find	Reverse Computation
A	GMT	GST	A1
B	GST	LST	B1
C	LST	LHA of a star	C1
D	GMT	GHA of a star	D1
E	GHA of the Sun	LHA of the Sun	E1
F	GMT	LMT	F1
G	GMT	Standard Time (ST)	G1

Determination of Azimuth



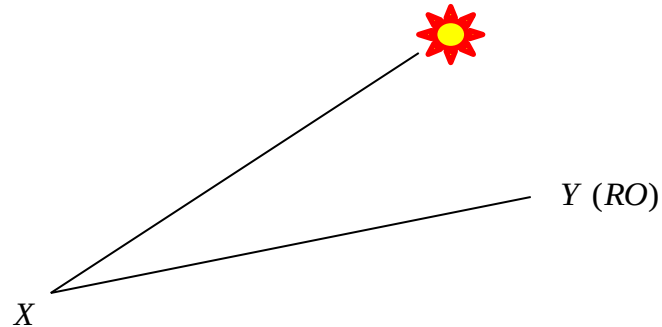
Azimuth of a line at a point on the Earth Surface is **the angle between that line and the meridian at that point**

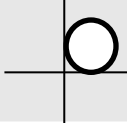
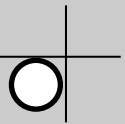
Determination of Azimuth

4 methods:

1. Observation to the sun, recording horizontal and vertical angles
2. Observation to the sun, recording horizontal angle and precise time
3. Observation to stars, recording horizontal and vertical angles
4. Observation to close-polar stars (or star at elongation), recording horizontal and precise time

Observation to the sun, recording horizontal and vertical angles



Time	Object	Face	HA	VA
	RO	FL	-----	-----
-----		FL	-----	-----
-----		FR	-----	-----
	RO	FR	-----	-----

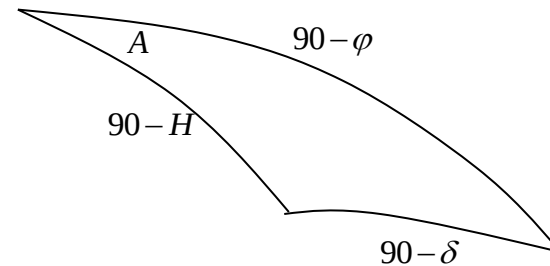
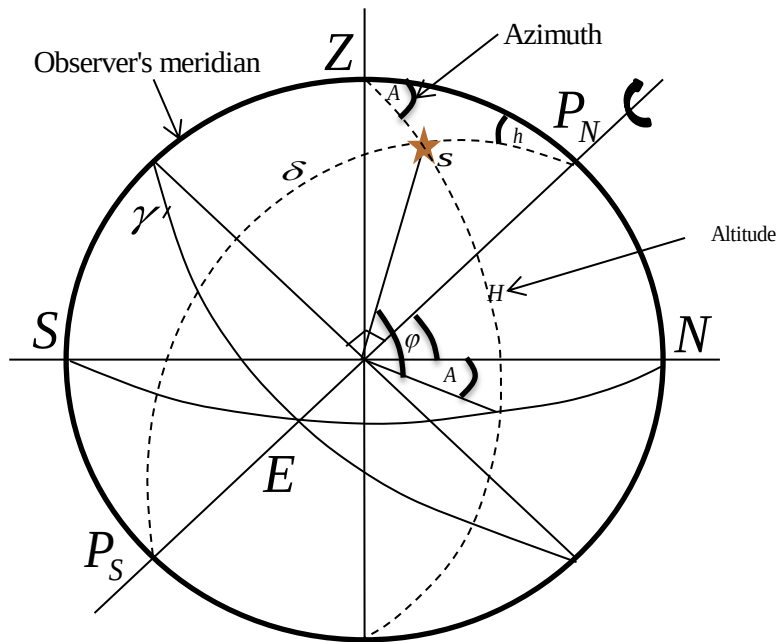
Set 1

Observation to the sun, recording horizontal and vertical angles

- Usually, 4 sets in the morning and 4 sets in the afternoon
- VA should be 20° - 40°
- Computation:
 - Mean HA to Ro, Mean HA to Sun, Mean VA to Sun, Mean time
 - Refraction & Parallax correction for mean VA
 - Declination of the Sun
 - Latitude of the observer
 - Azimuth of the sun from celestial triangle
 - Azimuth of RO

Observation to the sun, recording horizontal and vertical angles

- Azimuth of the sun from celestial triangle



$$\cos(90 - \delta) = \cos(90 - H) \cos(90 - \varphi) + \sin(90 - H) \sin(90 - \varphi) \cos A$$

$$\cos A = \frac{\sin \delta - \sin H \sin \varphi}{\cos H \cos \varphi}$$

φ – From topo map

δ – From star almanac

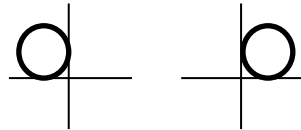
H – Observed

Observation to the sun, recording horizontal and vertical angles

Example:

Observation to the sun, recording horizontal angle and precise time

- From this method, it is possible to get more accurate bisection of the Sun. Only vertical cross-hair is of concerned.

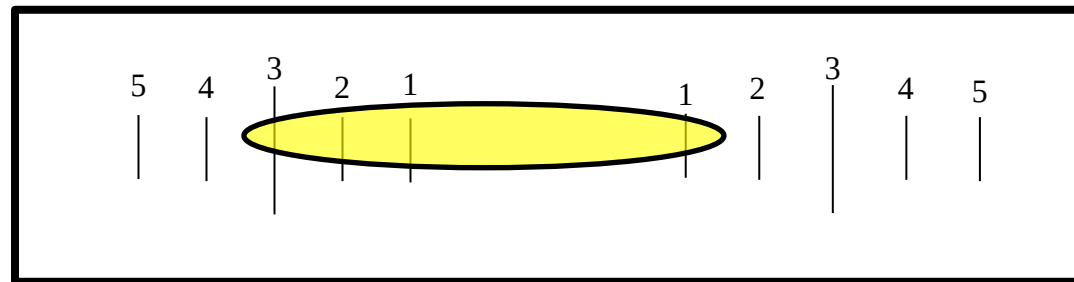


- Since VA is not recorded, no refraction or parallax correction

Observation to the sun, recording horizontal angle and precise time

- **Plate Bubble**

To indicate any dislevelment of the instrument which effect HA reading



$$L = 3.5$$

$$R = 1.5$$

ν – Sensitivity of the bubble

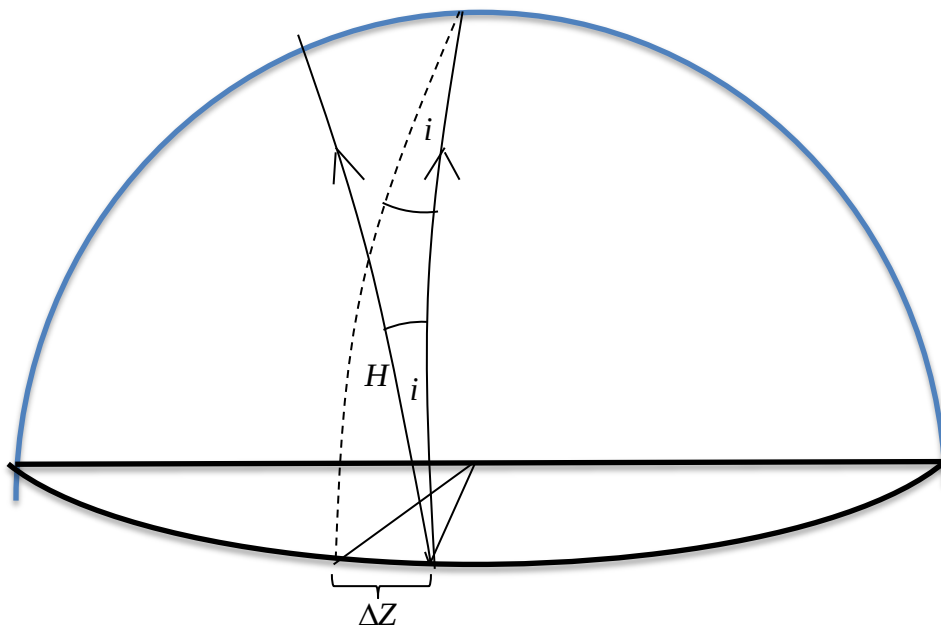
The dislevelment is $i = \left(\frac{L - R}{2} \right) \nu$

For n number of sets, taking mean

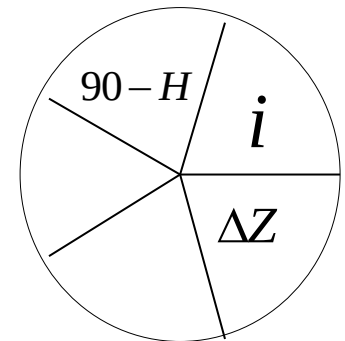
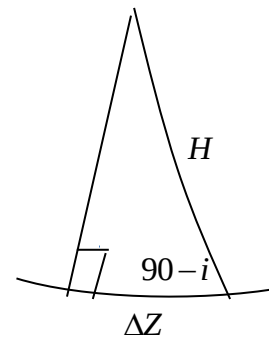
$$i = \frac{(\sum L - \sum R) \nu}{2 + n}$$

Observation to the sun, recording horizontal angle and precise time

- Plate Bubble correction for HA reading**



H – Altitude



$$\sin i = \tan(90 - H) \tan \Delta Z$$

$$\Delta Z'' = i'' \tan H$$

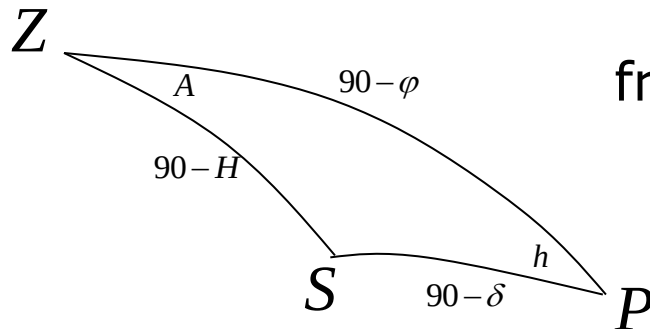
Observation to the sun, recording horizontal angle and precise time

Computation Procedure

- Correct HA to Sun from dislevelment
- Declination of the Sun
- LHA of the Sun ($h = 360^\circ - LHA$)

$$ST \rightarrow GMT \rightarrow GHA_{Sun} \rightarrow LHA_{Sun}$$

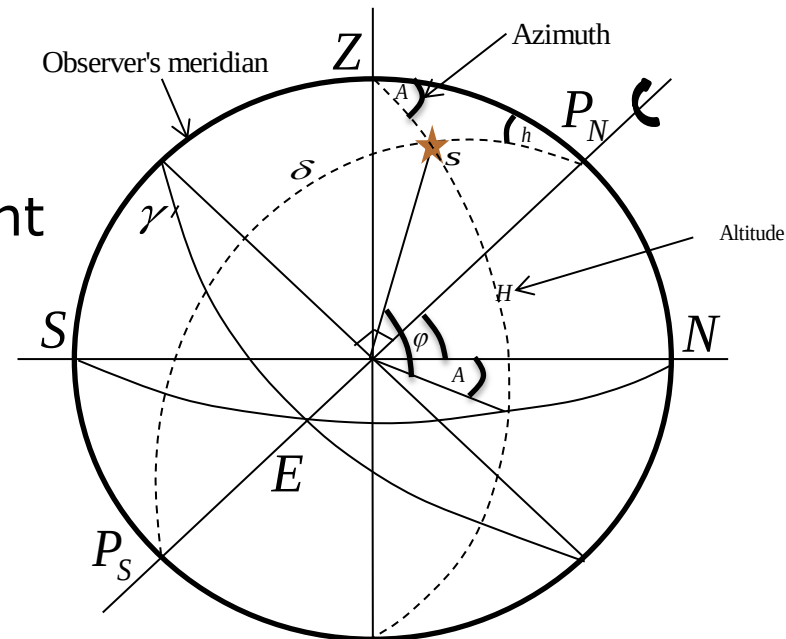
- Latitude from topo map



from cot formula,

$$\tan A = \frac{\sin h}{\cos \phi \tan \delta - \sin \phi \cos h}$$

- Azimuth of RO



Observation to the sun, recording horizontal angle
and precise time

Example: