The Equation For Excellence

How to Make Your Child Excel at Math

Arvin Vohra

1: WHY STUDY MATH?

When children ask why they need to study math, the answer usually has something to do with either daily life or applications to science and technology. The problem with the first motivation is that it is an obvious and transparent lie. The second type of "motivation" tends to have the opposite of the intended effect.

The "daily life" explanation tells students that they will need math for their daily activities. For example, they will need to calculate the tip in a restaurant, or determine how much they should pay for their groceries. Most students are quick to point out that this problem can be solved by carrying around a calculator. And anyone who is worried about running out of batteries can carry around a spare set of batteries, or even two calculators. Even cell phones have built in calculators.

The arguments against the "daily life" explanation continue. In daily life, you never need to do more than add, subtract, multiply, or divide. Why learn trigonometry? Why study calculus? Why do anything beyond arithmetic? Even math-oriented jobs rarely require any really advanced math. When I worked as an actuary, the only math I used on the job was multiplication and the occasional exponent (the actuarial profession is one of the most math-oriented professions in the world.)

The other rationale for studying math focuses on science and technology. We need math to design space shuttles and satellites, to work in laboratories, and to build the newest computers. In one way this argument makes sense. Much of that work requires intensive use of advanced math. But very few people work in those areas. Those that work in those areas usually do so because of an internal passion, not because of any external motivation.

In fact, from the perspective of most students, there is very little external motivation to be a scientist. The strongest external motivators for most teenagers are money, fame, power, popularity, and attraction to the opposite sex. None of these powerfully motivate students to pursue careers in science. For every million dollars a scientist makes, the businessmen for whom he works make a billion. For every famous scientist, there are a thousand famous musicians and actors. The scientists who made the nuclear bomb were not the ones to use it; that power belonged to politicians. And in American culture, scientists have no more popularity or sex appeal than anyone else.

Thus, this argument not only fails to motivate students; it actually does the reverse. A student with no interest in being a scientist who hears the technology argument now thinks that advanced math is useful only for scientists. Thus, he does not need to learn it. If his goal is personal gain, his time is better spent doing almost anything else – studying politics, learning to play the guitar, working out, or thinking of ways to make himself rich. Math becomes just an annoying requirement.

So then why should a student learn math at all?

Kings used to play chess to learn military strategy. When I first heard this at age ten, the idea struck me as unbelievably stupid. In chess the bishop can move only diagonally. The knight can move in an L shape. A real soldier, on the other hand, can move in any direction. How would studying chess help in any real war?

I had, of course, completely missed the point. Strategy has nothing to do with L shapes or diagonals. A chess player learns to anticipate his opponent. He learns to look for strong positions, rather than short term gains. He learns to make intelligent sacrifices, and be wary of the strategic artifices of his opponent. He learns to predict his opponent's future responses to his actions, rather than focusing on the immediate gains. This mental discipline makes his mind sharper, and he becomes a much more capable strategist.

Similarly, math is important not because it teaches a student how to use trigonometry to measure the height of a building, but because it develops a student's ability to analyze and solve unfamiliar problems. Math develops concrete reasoning, spatial reasoning, and logical reasoning. Math does not just develop skills that can be applied to science and technology; when math is taught right, it develops the student's fundamental cognitive architecture, increasing his intelligence. The student will develop the logical reasoning skills that allow a lawyer to analyze a legal situation and to present a coherent and convincing argument. He will develop the ability, essential for any businessperson, to isolate the key components of a system. He will develop mental skills that can be used in any problem-solving situation. His mind will become faster, sharper, and more precise.

What lifting weights does for muscles, math does for the mind. In no sport will an athlete suddenly lie down on his back and lift a weight ten times. However, the vast majority of athletes do the bench press. Why? It makes them stronger, and thus prepares them for athletic endeavors in general.

When you teach a child math in the right way, you are giving him the gift of a sharper and more powerful intelligence. You are helping him actually develop his mind. You are making him smarter. You are giving him the ultimate ability to succeed in the world, and to build a happier life for himself. You are not just making him better at math; you are making him better at thinking.

This book will show you how to make any student excel at math, even a student who is extremely lazy or innately bad at math. You will learn how to motivate any student and what to teach. Whether you are great at math or barely able to do algebra, there are techniques in this book that you can use.

There are a few things you need to know before continuing. The first is that the methods in this book are designed to be effective. They are not designed to be easy, nor are they designed to be fun.

On the flip side, this book does not advocate a "beat your kids to make them strong" type of approach. I never yell at any student, and I obviously do not use any physical punishment. If you do your part right, you will never need to yell at a student to teach him math.

Similarly, the techniques here are not ones designed to cause antagonism. Many of my students spend a good portion of their tutoring sessions frustrated with a math problem, begging for an answer, or literally groaning. And yet the ones who complain the most are the ones who seem to appreciate my training the most. In fact, many of those students pay for part of their tutoring fees with money received from allowances, jobs, and internships, rather than switch to a more moderately priced tutoring service. Instead of spending that money on entertainment, they voluntarily spend it to learn math.

Why would a teenager actually spend his own money to learn math? Because at some level every person desires ability more than entertainment. Although we often believe the opposite, most teenagers would rather gain intelligence than momentary enjoyment. I may make my students struggle more than another educator would. But my methods bring out their very best, and they can see it.

About half of this book focuses on motivation. Right now, some of the finest minds in the country are using every advertising trick they know of to persuade your child to act in certain ways. Alcohol and tobacco companies spend millions of dollars per year on advertising, as do hundreds of junk food, clothing, and entertainment companies. Thus, in today's world, weak motivational methods simply cannot compete. Parents and educators who want to be effective must use motivational methods as powerful as those used by today's professional persuasion artists.

In fact, you will have to be even more persuasive. Unlike an advertiser promoting entertainment or recreation, to effectively teach your child math, you will have to persuade him to take the more difficult, yet ultimately more rewarding, path.

For example, many schools allow students to use calculators. As this book explains, chronic calculator use can dramatically weaken a child's math abilities. Thus, you may be the one persuading your child to not use his calculator, even though his teacher encourages calculator use.

While that task might seem impossible, the methods in this book will show you what to do. Once students understand the damage that calculator use causes, most of them voluntarily stop using calculators altogether. Several of my students have even taken the SAT, the most important test of their lives, without calculators. Almost all of them returned with perfect SAT math scores.

The fact that you will be working to fundamentally improve your child's life will make motivation a bit easier. Even when kids complain, they know what benefits them. And over time, as they see themselves becoming more intelligent and more successful, motivation will become easier.

The rest of this book explains what to teach, and how to teach it. It explores the primary effective math teaching methods, including the legendary Asian system and the methods that underlie the success of my company, Arvin Vohra Education.

2: THE ASIAN SYSTEM

The belief that Asians are good at math is held with good reason: even in America, students with Asian parents tend to significantly outperform every other ethnic group. For example, 2005 math proficiency testing showed that Asian students had higher math proficiency scores than White, Black, and Hispanic students at all age levels *(Source: Child Trends Databank)*.

This section examines the techniques used by Asian parents and educators. Of course, there are variations depending on the country of origin and the individual, but there are techniques and principles that are almost universal among Asian parents and educators.

The Asian system is built on memorization. At an early age, children are taught to memorize multiplication tables and the like. As they get older, they memorize formulas, and even memorize step by step ways to solve specific problem types.

The Asian system is radically different from current American methods, which emphasize understanding over memorization. Where American parents and math teachers focus on explaining why a technique works, the Asian educators simply require that the student memorize the technique, and be ready to use it.

One might expect that such a technique would create students who simply have formulas memorized and are unable to understand what they are doing. But the reality is just the opposite. Once students have the information memorized, the understanding seems to come naturally. On the other hand, systems that drop memorization and focus on only understanding seem to have the reverse effect. Students often end up confused – unable to understand the problem, or to solve it.

This is one of the strangest paradoxes in math education, one that I wrestled with extensively at the beginning of my career as an educator. Why does memorization work in math? Why does focusing exclusively on understanding fail? Isn't math about understanding? Shouldn't memorization be saved for history?

To unravel this mystery, we will undertake a journey that will help us understand some of the most important cognitive principles involved in math education.

COGNITIVE OVERLOAD

Memorize the following list of words:
Cow, dog, horse, tree, sea, frog

Not too hard, right? Now memorize this list:
Frog, moss, grass, house, cow, mouse, deer, phone, well, spoon, table.

The fact that the list is longer makes it much harder. There are various memorization techniques a person can use to memorize the list, but it is not nearly as easy to memorize as the first list.

Most people can hold about 7 pieces of information in their working memory at any given time (usually between 5 and 9 items, depending on their complexity. Working memory is used to remember information for a short period of time. Long-term memory is used to remember information for years.) The first list only had 6 items. The second list had 11 items. But it was more than twice as hard to memorize. Why? It had gone over the limit.

Now let's look at an example that is a bit closer to math.

Here is a rule: When you see a cow, hit it with a frog.

Easy to memorize. Easy to understand. You might even find yourself remembering this "formula" several weeks from now.

Here is another formula:

When you see a ztyq, hit it with a tfgh. (Note: A ztyq is just a cow missing a leg. A tfgh is a frog with more that seven spots on his back.)

If you focus, you will be able to memorize this formula and this explanation for a few minutes. But you will probably forget it by tomorrow. There are two reasons for this. First, there are more items of information. Secondly, picturing this requires a bit more work.

If you have ever struggled with math, the feeling you get from the above "formula" may be familiar. Now try this:

When you see a mtyq that is lacking a tfgh, hit it with a mrtg. (A mtyq is a ftzz or a qoiu. A qoiu is a frog without feet. A mtyq is half a dandelion. The definition of a tfgh is given above. An fter is a half of *a rofr, which is a cow's left hoof.)*

This formula is extremely difficult to follow. Few people even bother to read the formula the whole way through, and those that do forget it quickly.

What does this have to do with math? Look at the following math problem:

Paint costs $3 for enough paint for one square foot. Fred wants to paint a rectangular wall that is 4 yards wide by 5 yards long. How much will it cost to buy enough paint for 3 coats?

If you know that the area of a rectangle is length times width, and that a yard is 3 feet, this problem should not cause cognitive overload. But if you do not have the facts and formulas memorized, you end up with:

Paint costs $3 for enough paint for one square foot. Fred wants to paint a rectangular wall that is 4 yards wide by 5 yards long. How much will it cost to buy enough paint for 3 coats? (The area of a rectangle is length times width. A yard is three feet.).

It looks familiar, right? We have not even come close to cognitive overload, but there is more to juggle now. Because the student must juggle the information in the problem and unfamiliar formulas, he is not able to focus exclusively on solving the problem.

Now look at this problem:

Fred wants to paint a can red. The can is a cylinder with height 20

> *inches and radius 10 inches. He wants to cover the sides of the can with three coats of paint and the top with four coats of paint. Paint costs 10 cents for enough to cover a square inch. How much will it cost, in dollars, for enough paint to paint the can?*

This problem is a bit more complicated, but it is still only a prealgebra problem. Now look at what a student who does not know the formulas must juggle:

> *Fred wants to paint a can red. The can is a cylinder with height 20 inches and diameter 10 inches. He wants to cover the sides of the can with three coats of paint and the top with four coats of paint. Paint costs 10 cents for enough to cover a square inch. How much will it cost, in dollars, for enough paint to paint the can. (The top and bottom are circles; the area of a circle is* π**radius*^{2}*. The radius is half the diameter. The lateral surface area is the circumference times the height. The circumference is* π**diameter.)*

Of course, in a real problem, the relevant information would not be neatly written in parentheses after the problem. The student would have to look it up, or ask a parent or teacher. He would not only have to juggle the information, but also keep it all together while he got it from different sources. He would have very few cognitive resources available to analyze and solve the problem, because his mind would be too occupied keeping the formulas straight. He would have little chance of getting the problem right; on a test, he would just hope for partial credit.

This problem would be given in a prealgebra class, usually to seventh or eighth graders. And yet many high school seniors would struggle with this problem. In fact, many adults would struggle with this problem. But the problem is not actually difficult; it just tends to create cognitive overload.

When a student hits cognitive overload, the signs are usually easy to see. He becomes visibly frustrated. He may act out emotionally, by yelling, crying, or swearing. This is often viewed as a deep-seated behavioral problem, but it often is not. The student is faced with an impossible situation that may seem pointless. How would you react if you woke up tomorrow morning locked in a cage, for no apparent reason?

Other students may withdraw, seeming as if they are somewhere else. Their faces may stop showing any expression, and they may show little reaction to instructions or questions. Teachers and parents often mistakenly conclude that such students are stupid. In reality, they are withdrawing from an incomprehensible situation.

Some students may write something completely random on the page, or blurt out a formula that has nothing to do with the problem. For example, they might just say "quadratic formula?" or "Pythagorean theorem?" They might even just guess a random number. A student who does this does not believe that his random utterance is the correct answer. In desperation, he just says something, knowing full well it is wrong.

If a student has been struggling for a long time, you are dealing with an even bigger problem. Remember this?

When you see a cow, hit it with a frog.

You know what a cow is, and you know what a frog is. So it is easy for you to understand the above rule.

With the cylinder problem above, the really struggling student sees something more akin to this:

When you see a faquat, hit it with a potwu.

Why? He might not know what a cylinder is. The phrase "lateral surface area" might as well be written in Babylonian. Diameter? Radius? Because he cannot picture the problem properly, the steps he must take are a meaningless series of commands. If by some miracle he remembers them for a quiz, he is sure to forget them by the exam. He is dealing with foreign concepts that he can not picture.

To picture the problem, he must store the following information in working memory:

1. What a cylinder is
2. What a circumference is
3. The formula for the circumference
4. The formula for area of a circle
5. What a radius is
6. What a diameter is
    6a. The relationship between radius and diameter
7. The formula for the lateral surface area (which is really just the area of a rectangle)

The student has hit seven before even starting the problem. His working memory is full, and the problem has not even begun! He has nowhere to store the information for the problem (what the height is, what the cost is, etc.)..

Additionally, when a student's cognitive resources are being fully used, the student is less able to check for random errors. The rate at which he makes careless mistakes goes up dramatically. In fact, a high number of careless mistakes is one of the signs that tell me that a student's cognitive resources are being overstretched during the problem-solving process.

This is a prealgebra problem. The difficulty increases as the student enters algebra, or moves up to calculus.

HOW THE ASIAN SYSTEM ADDRESSES COGNITIVE OVERLOAD

We discussed how working memory can hold about 7 pieces of information at one time. But you know more than seven facts. You know more than 7000 facts, for that matter.

That information is stored in long-term memory. The Asian system helps students store information in their long-term memory in such a way that it is readily accessible. To be more precise, the Asian system forces students to store information in their long-term memory, and to have it ready for use.

The Asian system is fantastically effective, and extremely simple. As soon as the child is able to talk, math training begins. The child is constantly taught to memorize math facts and drilled daily on the facts. He is quizzed constantly on his multiplication tables. He is quizzed constantly on formulas (e.g. area of a triangle, circumference of a circle, quadratic formula, etc.).

He is repeatedly given specific problem types until he can do them in a few seconds. For example, he might be asked to find the surface area of a cylinder every day. After a few weeks, he can find the surface area of a cylinder with incredible speed. With the constant drilling, the information is always readily accessible and is stored in long-term memory.

The student does not need to have any amazing innate intelligence. His intelligence can be just average. For that matter, it can be below average.

The results speak for themselves, but let's look at how this student analyzes the above problem. Remember, the formulas are so ingrained into his mind that he barely needs to think about them to use them. He has done problems like this one so many times that the process has become virtually automatic.

Here is the problem mentioned in the last section:

Fred wants to paint a can red. The can is a cylinder with height 20 inches and radius 10 inches. He wants to cover the sides of the can with three coats of paint and the top with four coats of paint. Paint costs 10 cents for enough to cover a square inch. How much will it cost, in dollars, for enough paint to paint the can.

Here is the Asian student's way of thinking about it:

Find the top area and multiply by 4. Find the lateral surface area and multiply by 3. Add the two areas, and then multiply the sum by 10 to get number of cents. Divide by 100 to get the number of dollars.

The correct mathematical steps are:

$\pi*(10)^2 = 100\pi$ is the top area. Multiply this by 4 to get 400π. The lateral surface area is $2\pi(10)(20) = 400\pi$.
Multiply this by 3 to get 1200π. Add those two numbers together to get $400\pi + 1200\pi = 1600\pi$. Multiply this number by 10 to get

the number of cents, which is 16000π cents. Divide this by 100 to get 160π dollars. Note that π equals approximately 3.14.

COGNITIVE OVERLOAD IN ARITHMETIC AND ALGEBRA

We have seen how cognitive overload can be an issue when solving word problems. What about regular arithmetic and algebra problems?

Look at this problem:

$$\begin{array}{r} 45 \\ \underline{\times\ 37} \end{array}$$

Most adults would find this problem fairly straightforward. But what if you had not memorized your multiplication tables? Then rather than starting out by doing 7x5 = 35, your first step would be to add 5+5+5+5+5+5+5, to get 35. You would then carry the 3, and do 7+7+7+7 (instead of 4*7) to get 28, and the process would continue like that. You might even forget what you were doing before you finished, and have to restart. In other words, you would reach cognitive overload.

The chance of making a careless mistake would be pretty high. You might even be tempted to do 45+45+45+45… (37 times) rather than the step by step multiplication.

How about a harder problem?

$$\begin{array}{r} 4563 \\ \underline{\times\ 7452} \end{array}$$

A bit tougher. But if you did not know the multiplication tables, it would be incredibly difficult.

Keep imagining that you did not know the multiplication tables. Do this:

$$^{1}/_{7} + ^{5}/_{42}$$

It is getting harder, right? If you do not know multiplication tables, it is hard to get started with this one.

Now let's move to algebra:

$$^{3}/_{a} + ^{4}/_{(a+3)}$$

This problem has nothing to do with multiplication tables. But a person who had not memorized multiplication tables would never have really understood how to add fractions. That person would never be able to understand how to do this problem. He would probably memorize some way of doing the problem in the short term, and forget it by the time the exam comes around.

The above problem is a beginner level algebra problem. More advanced problems would require the student to solve similar problems as just one part of a multi-step problem. As the difficulty increases, the problems become impossible for the child to attempt at all. The child has "slipped through the cracks." There is no way for the child to do the problem without cognitive overload. To understand and solve the problem, he would have to hold years' worth of material in his working memory, which is impossible. Alternatively, he would have to actually learn several years of math before attempting the problem.

Because the Asian system focuses on long-term memorization of basic math facts, those trained with this system never face cognitive overload on these types of math problems. In fact, the Asian system takes it one step further. Not only does the system force students to memorize multiplication tables, etc., it constantly drills students on basic problem types. The student would not just find it easy to figure out how to do the above algebra problem. He would not need to "figure it out" in the first place. He would have practiced similar problems so many times that the process would be virtually automatic; it would be no more difficult than walking.

DISTANCING

Remember this?
"When you see a cow, hit it with a frog."

That is a lot easier to remember than
"When you see a terqp, hit it with a srato."

Why? The two statements are equally complex. However, the first statement means something to you, while the second statement means nothing. You cannot visualize it, or make sense of it beyond committing it to rote. You can memorize it, but you do not really know what you have memorized. Your mind distances itself from the information. You might memorize the information, but it will never be fully incorporated into your understanding.

Let's see how this applies to math. Student A is a strong math student. Student F is a weak math student. Both are given the following formula:

Area of a circle is π**radius squared, where* π *=3.14159…*

They are both given the following problem:

The radius of a circle is 6. Find the area, in terms of π.

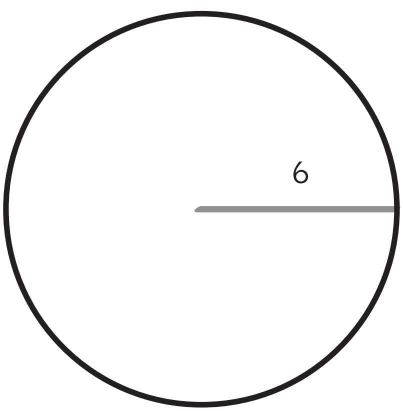
Both students do the following:

Area=radius^{2*} π

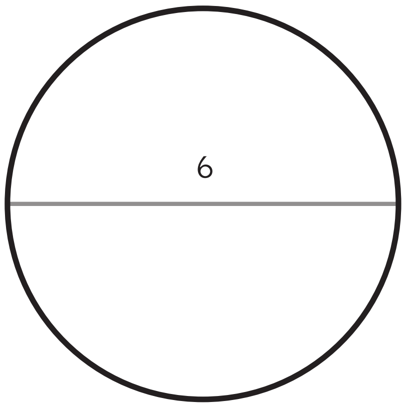
Area = 6^{2*} π

Area = 36 π

Internally, however, they did something completely different. Student A (the strong student) pictured the problem. Before he began, he visualized the following:



He knows what a radius is, and understands what he is doing as he calculates the area. If he is instead given a problem that says the diameter is 6 and asked to find the area, he will visualize this:



He will instantly see that the radius is 3, and will then solve the problem.

Student F (the weak student), did something very different. At some level, he thought "I don't know what a radius is, and I don't want to know. All I need to know is that if I am given a radius, I should multiply the radius by the radius, and then multiply by π."

I call this phenomenon "distancing", because the student distances himself from the concept. He keeps the information outside of his perception of the world. If the strong math student sees a pizza, he recognizes that it has a radius, and can picture the radius. The weak math student thinks of the word "radius" as descriptive only in his math class. He does not even consider that every circle he ever sees has a radius.

If Student F is given a problem that gives him the diameter instead of the radius, he is confused. Often, weak students end up just memorizing that the radius is the diameter divided by two, without visualizing the relationship. Not only does this further distance the student from the concept, it also gives him an extra