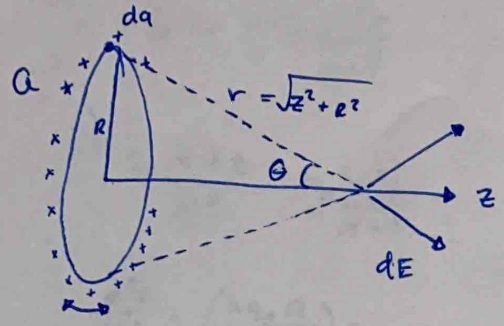
SOHCAHTOA

What frequency will the electron oscillate with.

Uniform charge distribution

①



$dE_x = 0$

$dE_y = 0$

All q points on the ring produce an $\vec{E}$

Components in the opposites will cancel giving us only contributions in the z direction

$$dE_z = \frac{k\,dq}{r^2}\cos\theta \quad , \quad \cos\theta = \frac{A}{H} = \frac{z}{\sqrt{z^2+R^2}}$$

$$dE_z = \frac{k\,dq}{\sqrt{z^2+R^2}}\,\frac{z}{(z^2+R^2)}$$

$$dE_z = \frac{k\,dq\,z}{(z^2+R^2)(z^2+R^2)^{1/2}}$$

, $dq = \lambda ds$, s = length along the ring.

$$\int dE_z = \int_0^{2\pi R} \frac{k\,z(\lambda ds)}{(z^2+R^2)^{3/2}}$$

, $s \neq z$, not dependent on z.

$$= \frac{2\pi R\, k\lambda z}{(z^2+R^2)^{3/2}}$$

, where $2\pi R\lambda = Q$

$$E_z = \frac{kzQ}{(z^2+R^2)^{3/2}}$$

, by displacing the electron slightly $z \ll R$

Using Taylor expansion.

$$\Rightarrow E_z = \frac{kQz}{(R^2)^{3/2}}$$

$$\boxed{E_z = \frac{kQz}{R^3}}$$

Finding the oscillation frequency

$$\Sigma F_{e^-} = m_e a = m_e \frac{d^2 z}{dt^2}$$

, where $F = qE$

Thus $q_e E_z = m_e \frac{d^2 z}{dt^2}$

$$\Rightarrow q_e \frac{kQz}{R^3} = m_e \frac{d^2 z}{dt^2}$$

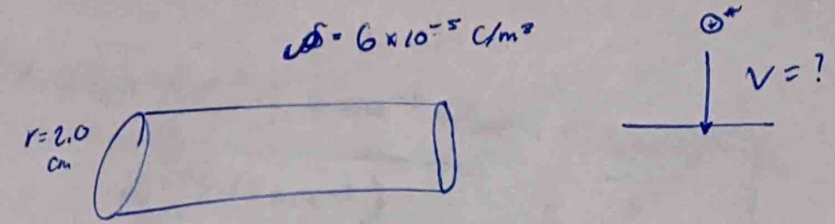
$$\frac{d^2 z}{dt^2} - \underbrace{\left(\frac{kq_e Q}{m_e R^3}\right)}_{\omega^2} z = 0$$

* This is a differential equation *

$$\boxed{\omega = \sqrt{\frac{kq_e Q}{m_e R^2}}}$$

Solution to differential equations is either $\sin\theta$ or $\cos\theta$.

②



What speed should the ion have as it enters the plasma cylinder so it's velocity is 0 as it reaches the axis of the cylinder?

$q = 1.6\times10^{-19}\,C$, $m = 5.0\times10^{-27}\,kg$

$r = 0.02\,m$, $\rho = 6\times10^{-5}\,C/m^3$

Here the charge enclosed by a cylinder is given by

$q_{enc} = \rho\pi r^2 L$

Gauss' Law $\int E\varepsilon_0\, dA = \rho\pi r^2 L$ ← Charge enclosed

$E\varepsilon_0 \int dA = \rho\pi r^2 L$, where $dA = 2\pi r L$ = Surface area of cylinder

Thus

$E\varepsilon_0(2\pi r L) = \rho\pi r^2 L$

$E = \frac{\rho r}{\varepsilon_0 2}$

The electric field potential

$V = -\int E \cdot dr$

& Potential difference relation $U = qV$

$U = q\left(-\int E\cdot dr\right)$

$= -q\int_{0.02}^{0}\frac{\rho r}{2\varepsilon_0}dr = +\frac{q\rho}{2\varepsilon_0}\left[\frac{r^2}{2}\right]_0^{0.02} = \frac{q\rho}{2\varepsilon_0}\left[\frac{(0.02)^2}{2} - \frac{(0)^2}{2}\right]$

②
$$\frac{(1.6\times10^{-19}\,C)(6\times10^{-5}\,C/cm^3)}{2(8.85\times10^{-12})}\left[\frac{(0.02)^2}{2}\right]$$

$$\Rightarrow \boxed{U = 1.1\times10^{-16}\,J}$$

In order to find the velocity

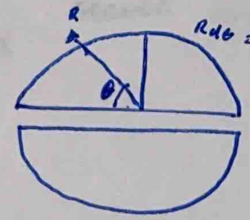
$$U = \frac{1}{2}mv^2$$

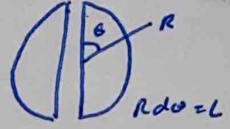
$$V = \sqrt{\frac{2U}{m}}$$

$$V = \sqrt{\frac{2(1.1\times10^{-16}\,J)}{5.0\times10^{-27}\,kg}}$$

$$\boxed{V = 2.1\times10^{5}\ m/sec}$$

③ Calculate the electric field in the center of the circle as a function of the amount of positive charge on the half circle, the amount of negative charge on the ½ circle & radius of the circle.





The length of the semicircle ~~[illegible]~~ $L = R\theta \Rightarrow dL = R\,d\theta$

Having a charge of Q $\quad \lambda = \frac{Q}{(\pi R)}$, $dQ = dL\,\lambda = \lambda R\,d\theta$

The electric field of a point charge

$$\vec{E} = \left|\frac{KQ}{r^2}\right| \Rightarrow d\vec{E} = \frac{K\,dQ}{R^2} = \frac{K\lambda R\,d\theta}{R^{\cancel{2}}}$$

$$d\vec{E} = \left|\frac{K\lambda\,d\theta}{R}\right|$$

Based off of symmetry the $\vec{E}_x$ with be 0, since $d\vec{E}$ is a vector, look @ each component at the center.

$$\int_0^{\pi} d\vec{E}_x = \int_0^{\pi} d\vec{E}\cos\theta$$

$$E_x = \int_0^{\pi}\left|\frac{K\lambda}{R}\,d\theta\right|\cos\theta$$

$$= +\frac{K\lambda}{R}\int_0^{\pi}\cos\theta\,d\theta$$

$$= \frac{K\lambda}{R}\left[-\sin\theta\right]_0^{\pi} = 0$$

$$\int_0^{\pi} d\vec{E}_y = \int_0^{\pi} -d\vec{E}\sin\theta$$

$$E_y = \int_0^{\pi}\left[\frac{K\lambda}{R}\,d\theta\right]$$

$$= -\frac{K\lambda}{R}\cos\theta\Big|_0^{\pi}$$

$$= -\frac{K\lambda}{R}\left[\cos\pi - \cos 0\right], \quad \lambda = \frac{Q}{\pi R}$$

$$\boxed{E_y = -\frac{2KQ}{\pi R^2}}$$

③

Thus for the opposite $\frac{1}{2}$ circle pointing the other direction

$$E_y = \frac{+2kQ}{\pi R^2}$$

Thus $E_{y_T} = -\frac{2kQ_1}{\pi R^2} + \frac{2kQ_2}{\pi R^2}$. recall each ~~sh~~ sphere has different charge Q_1, Q_2

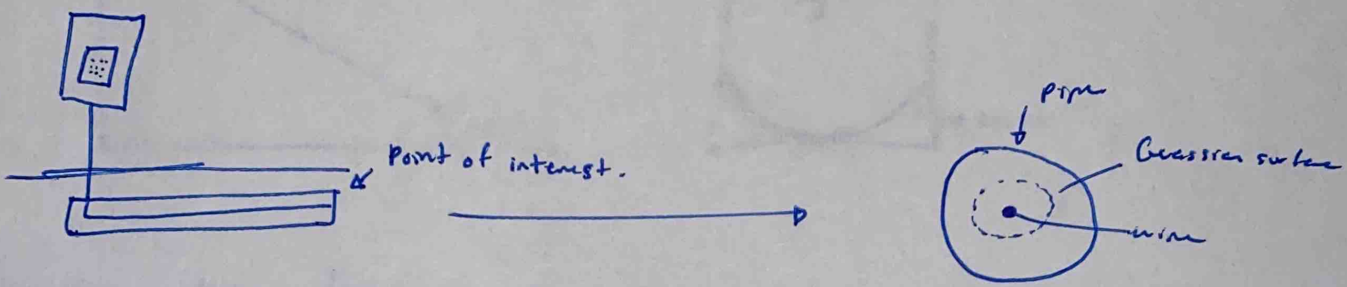
$$= \frac{2k}{\pi R^2}\left[Q_2 - Q_1\right]$$

Thus if the two charges are equal the electric field is 0 at the center.

If $Q_1 > 0$ & $Q_2 < 0$

$$E_y = \frac{-2k}{\pi R^2}\left(|Q_2| + |Q_1|\right)$$, meaning both halves contribute to the sphere with negative charge on the bottom.

(4) Calculate the electric field at the end of the pipe.



From Gauss's Law we understand that there is no charge enclosed within the guassian surface.

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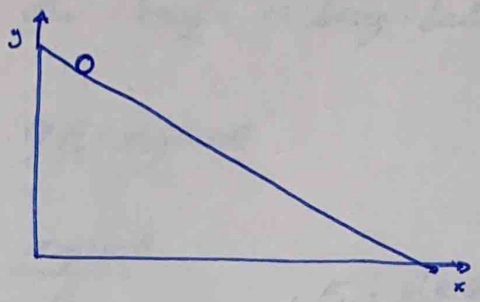

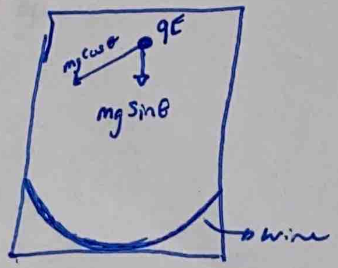

Find the electric field at the center of semicircle wire

Charge on the ring = Q

$L = \pi R$

Charge per unit length $L = \frac{Q}{\pi R}$

Electric field same from question 3

$E_x = \int dE_x = dE \sin\theta$

$, dE = \frac{KQ\, d\theta}{R^2 \pi}$

$\Rightarrow \frac{KQ}{R^2 \pi} \int_{-\pi/2}^{\pi/2} \sin\theta \, d\theta$

$= \frac{KQ}{R^2 \pi} \left[-\cos\theta \right]_{-\pi/2}^{\pi/2} = 0 \quad \because \cos\frac{\pi}{2} = 0$

$E_y = \int_{-\pi/2}^{\pi/2} dE \cos\theta = \frac{KQ}{R^2 \pi} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$

$= \frac{KQ}{R^2 \pi} \left[\sin\theta \right]_{-\pi/2}^{\pi/2} = \frac{KQ}{R^2 \pi} \left[\sin(\frac{\pi}{2}) - \sin(-\frac{\pi}{2}) \right] \Rightarrow \frac{KQ}{\pi R^2} \left[1 - (-1) \right]$

$\boxed{E_y = \frac{2KQ}{R^2 \pi}}$

(5)

Thus the charge is being balanced by

$$qE_y = mg\sin\theta$$

$$q = \frac{mg\sin\theta}{E_y} \quad , \; E_y = \frac{2kQ}{\pi R^2}$$

$$q = \frac{mg\sin\theta}{1}\left(\frac{\pi R^2}{2kQ}\right)$$

$\theta = 15$, $Q = 800 \times 10^{-6}\,C$, $m = 0.025\,kg$

$R = 0.10\,m$, $k = 9 \times 10^9$

~~Thus q = 0.025 ×0.10~~

$$q = \frac{(0.025)(9.81)\sin(15)(\pi)(0.10)^2}{2 \times (9 \times 10^9)(800 \times 10^{-6}\,C)}$$

$$\boxed{q = 1.38 \times 10^{-10}\,C}$$