9.87

Resistance, Current & Voltage.

A resistor of an unknown resistance is placed in an insulated container filled w/ 0.75 kg of water. A voltage source is connected in series w/ the resistor & a current of 1.2 amps flows through the resistor for 10 minutes. During this time the temperature of the water is measured & the temperature change during this time is $\Delta T = 10.00\ °C$.

ⓐ What is the resistance of the resistor?

$V = IR$ $0.75\,kg - H_2O\ (m)$, $I = 1.2A$, $t = 10\,min$, $\Delta T = 10.0\,C$

(C) Specific heat = $4186\ J/kg \cdot C$

Here $P = I^2 R$

$Q = mc\Delta T$, $P = Q/t$

$Q = 0.75\,(kg)\,(4186\ J/kg \cdot C)\,10.0\,C = 31395\,J = Q$

$P = \dfrac{31395\,J}{10\,min\,(60\,s/min)} = \boxed{52.325\ J/s = P}$ $J/s = 1\ watt$

$R = \dfrac{P}{I^2} = \dfrac{52.325\,W}{(1.2A)^2} = 36.33\ W/A$ $W/A = \Omega$

$\boxed{R = 36.33\,\Omega}$

(b) What is the voltage supplied by the power supply?

$V = IR$

$= (1.2A)(36.33\Omega)$

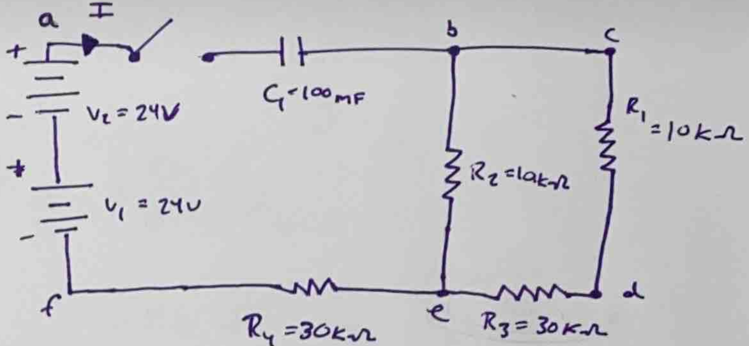
$V = 43.596 \ A\Omega$

$V = 43.596 \ V$

10.93

Circuits & Resistance

Consider the circuit below (a) What is the RC time constant of the circuit? (b) What is the initial current in the circuit once the switch is closed? (c) How much time passes between the instant the switch is closed & the time the current has reached half the initial current?



RC time => $\tau = RC$

the time constant (s) of an RC circuit. It is the time required to charge the voltage of 0 to approximately 63.2% of the value of an applied DC voltage, or discharging the same resistor to approximately 36.8% of its initial charge voltage.

$63.2\% = 1 - e^{-1}$

$36.8\% = e^{-1}$

$\Rightarrow V_0: V(t) = V_0(1 - e^{-t/\tau})$

1 step which are in series or parallel? R_1 & R_3

The The loop enclosed from bc = $R_{be} \Rightarrow R_1 + R_3 = 10k\Omega + 30k\Omega = 40k\Omega$

Thus now having the components in parallel

$$\frac{1}{R_{bc}} = \frac{1}{R_2} + \frac{1}{R_{13}} = \frac{1}{10k\Omega} + \frac{1}{40k\Omega} = \frac{40}{10(40)} + \frac{10}{10(40)} = \frac{40+10}{10(40)} = \frac{500}{400} = \frac{1}{R_{bc}}$$

$$R_{bc} = \frac{400k\Omega}{50k\Omega} = \boxed{8k\Omega = R_{bc}}$$

Thus now in series with R_4 (f)

$$R_{be} + R_4(f) = 8 + 30 = 38k\Omega$$

Thus $\tau = RC = (100 \times 10^{-3} F)(38k\Omega)$, $1F \approx s/\Omega$

$F\Omega \approx s$

$= 3800\ F\Omega$

$\boxed{\tau = 3800\ s}$

(b) initial current in the circuit once it is closed.

$V_T = V_1 + V_2 + V_3 + \ldots$, $V_1 = 24V$, $V_2 = 24V$

$= 24V + 24V$

$= 48V$

$V = IR$, $I = \frac{V}{R} = \frac{48V}{38 \times 10^{+3} \Omega}$

$= 1.26 \times 10^{-3} V/\Omega$, $V/\Omega = A$

$\boxed{I = 1.26 \times 10^{-3} A}$

(c) Time passed instant switch is closed & the time current has to reach the surface.

$V(t) = V_0 (1 - e^{-t/\tau}) \Rightarrow I(t) = I_0 (1 - e^{-t/\tau})$

I_0 = initial current , I = final current

here stated $I = \frac{I_0}{2}$, half the initial current.

$\frac{I_0}{2} = I_0 (1 - e^{-t/\tau}) \Rightarrow \frac{1}{2} = 1 - e^{-t/\tau}$

$= \frac{1}{2} - 1 = -e^{-t/3800s} \rightarrow -\frac{1}{2} = -e^{-t/3800}$

$\ln(\frac{1}{2}) = \ln(e^{-t/3800s})$

$\ln(\frac{1}{2}) = -t/3800s$

$3800s \ln(\frac{1}{2}) = -t$

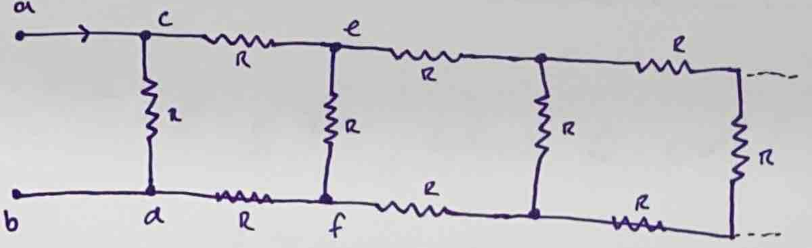
$(-0.693)(3800s) = -t$

$\boxed{t = 2633.97s}$

Current & Resistance

10.95

Consider the infinitely long chain of resistors shown below. What is the resistance between terminals a & b?



Series

$R_T = R_1 + R_2$

Parallel

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$

After $R_{ef} \to R_\infty$

Thus @ $R_{cd} = R_1 + R_2 + R_\infty$

Searching for equivalent resistance.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

$$= \frac{1}{R_1} + \frac{1}{R_{cd}} \quad , \text{ where } \frac{1}{R_{cd}} = \frac{1}{R + R + R_\infty} = \frac{1}{2R + R_\infty}$$

$$= \frac{1}{R_1} + \frac{1}{2R + R_\infty} \quad , \quad \frac{1}{R_\infty} = \frac{1}{R_4} + \frac{1}{R_5} + \cdots$$

$$= \frac{1}{R_1} + \frac{1}{2R + \left(\frac{1}{R_4} + \frac{1}{R_5} + \cdots\right)} \qquad \frac{(R_5 + R_4)(R_\infty)}{R_4 R_5 (R_\infty)}$$

$$= \frac{1}{R_1} + \frac{1}{2R + \left(\frac{(R_5 + R_4) R_\infty}{R_4 R_5 R_\infty}\right)}$$

$$R_{eq} = R_1 + 2R + \frac{R}{\left(\frac{R_4 R_5 R_\infty}{((R_5 + R_4) R_\infty}\right)} \quad , \quad R_\infty = \frac{1}{R_6} + \frac{1}{R_7} + \frac{1}{R_8}$$