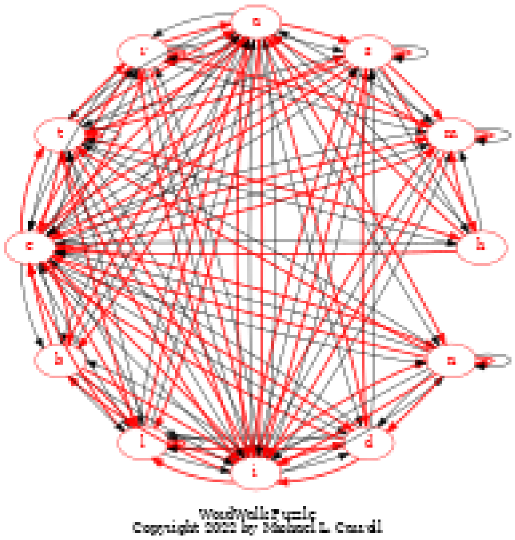
WordWalk^{TM}
Introduction and Examples



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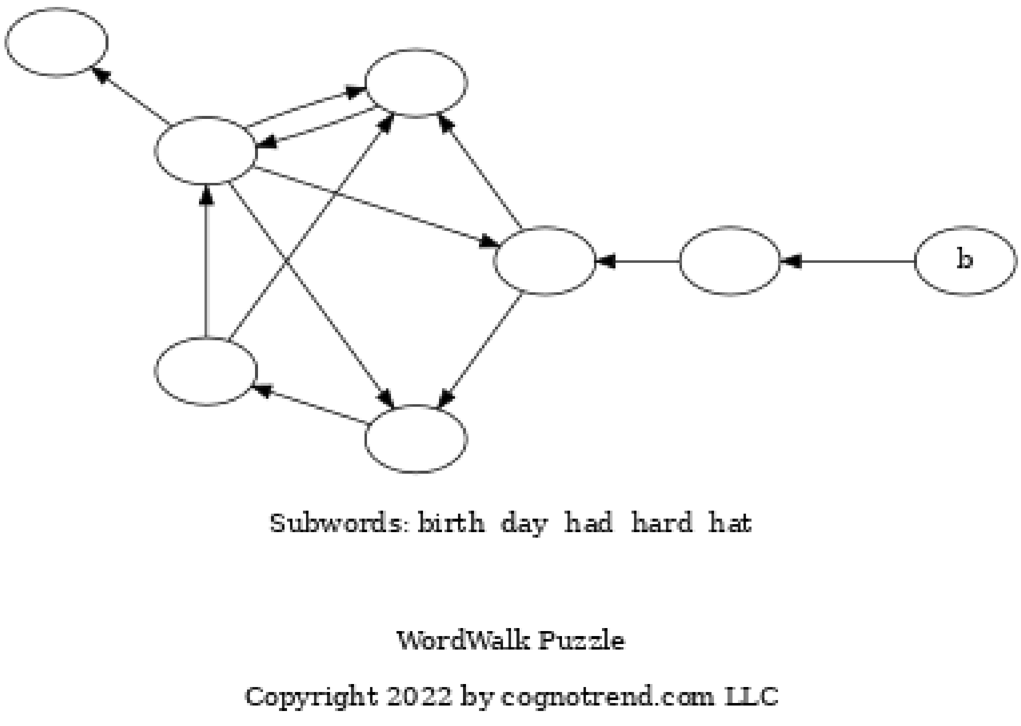
Chapter 1

Introduction

A WordWalk puzzle hides several words in a diagram consisting of nodes and arrows.[1] An example is shown in Figure 1.1 with the letter 'b' assigned to one of the nodes. The solution to this puzzle is shown in Figure 1.2. The red arrows shown in the solution trace out all the letters of a hidden word called the **root word**; in this case the root word is 'birthday'.

Figure 1.1: Unsolved WordWalk Puzzle for root word 'birthday'



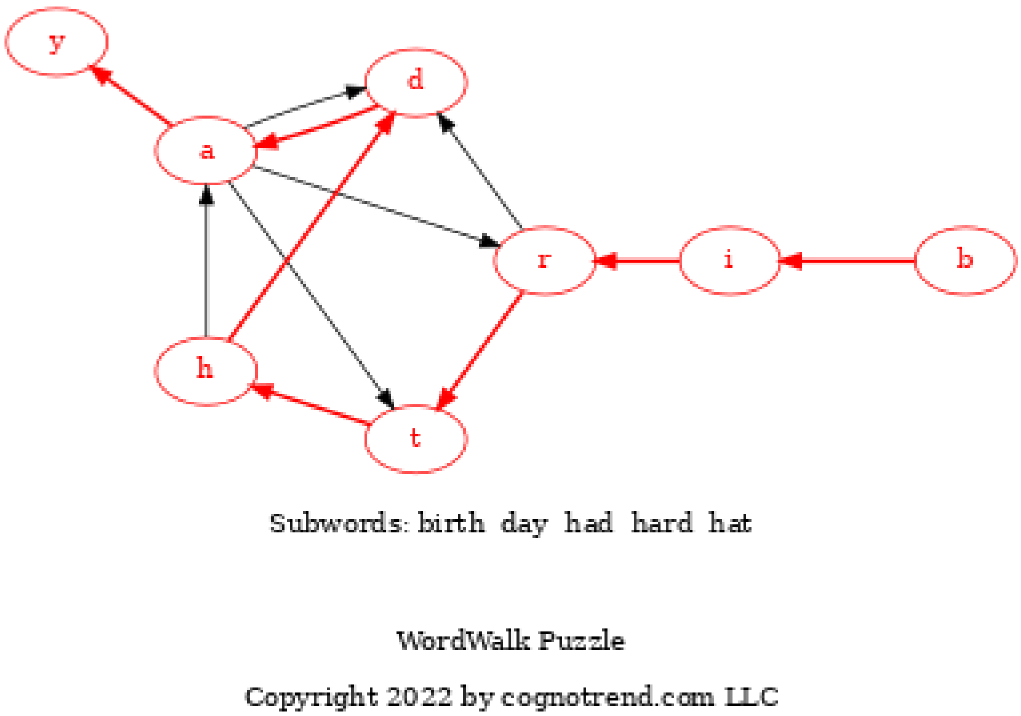
But there are also other words embedded in the diagram. In Figure 1.1 the words 'birth', 'day', 'had', 'hard' and 'hat' can also be found. These words are given as clues to the puzzle solver and are called **subwords**.

A word is considered to be a subword of another word if each letter of the first word is also used somewhere in the second word.[2] In a WordWalk

[1]In mathematics, this type of structure is called a directed graph and its nodes and arrows are also known as vertices and edges.

[2]In the language of mathematics, if we look at the set of letters occurring in a word, then the set of letters of a subword

Figure 1.2: Solved WordWalk Puzzle for root word 'birthday'



puzzle, all of the clue words are subwords of the root word and each of the letters of the root word occurs at least once in one or more of the subwords.

To solve a WordWalk puzzle, one must fill in all the blank nodes with the correct letters and then correctly guess the root word of the puzzle.

Although there is no need to guess the subwords (since they are provided already as clues), they do need to be found in the puzzle in order to verify the correctness of the proposed root word solution. If the paths which trace out each subword cannot be found after the assignment of letters, then that assignment of letters is incorrect – even though there may be an incidental path tracing out the whole root word.[3]

Sometimes the puzzles are made easier by using different node shapes for vowels. E.g., we sometimes use a diamond-shaped node to indicate that a vowel is to be assigned to that node.

must be a subset of the set of letters of the root word. Also note that a subword is not necessarily shorter than the root word. 'wherever' is a subword of 'however', but 'wherever' is the longer of the two words.

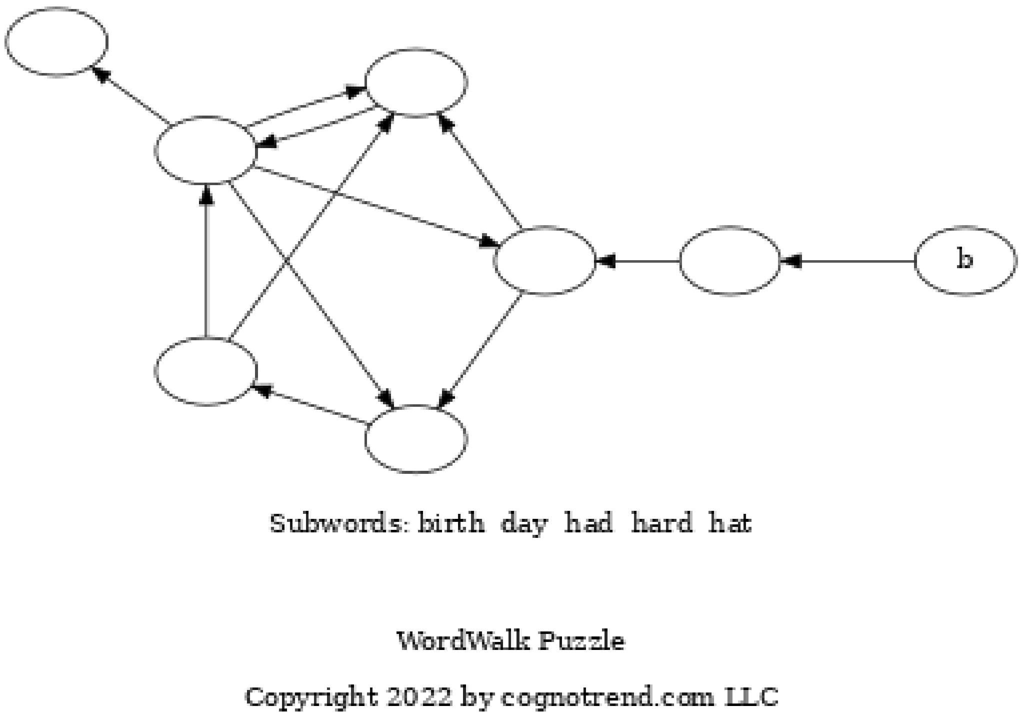
[3]In future online/interactive versions of WordWalk, the app itself will verify that each subword has been found.

Chapter 2

Examples

As an example, consider the unsolved WordWalk puzzle in Figure 1.1, repeated here for convenience as Figure 2.1. This puzzle was built from the root word 'birthday' with the subwords 'birth', 'day', 'had', 'hard', and 'hat'.

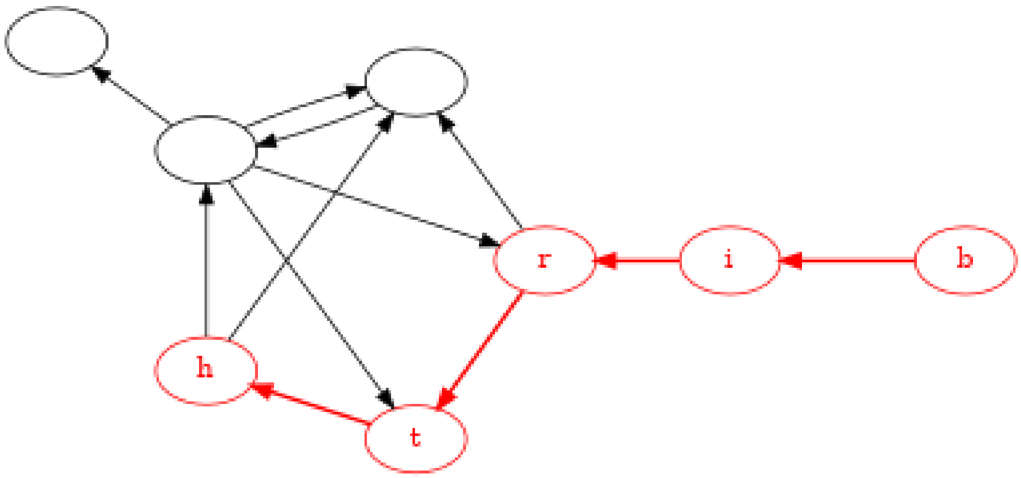
Figure 2.1: Unsolved WordWalk Puzzle for root word 'birthday' and subwords 'birth', 'day', 'had', 'hard', 'hat'



Although the solution can be seen in Figure 2.7, let's consider how we might go about solving for each of the given subwords.

Let's begin with the subword 'birth'. It should be pretty obvious how to solve for this. The 'b', 'i', and 'r' are obviously the first three nodes starting at the right. Then we have a choice of where to place the next letter 't'. This is where some knowledge of letter frequency (in English words) might come in handy. You probably already know that the vowels 'a' and 'e' both occur frequently in English words, and this is also true of our subwords. Look at how often ‘a’ occurs in the given subwords. It both follows and is followed by several different letters. On the other hand, the letter ‘t’, which is the next letter to be assigned, occurs only a couple of times in in the subwords. So, we should assign probably assign the ‘t’ to the bottom most node, since it only has two incoming arrows while the upper node has three. Doing this immediately implies that the ‘h’ should go to the only node pointed to by the ‘t’ node. Now we’ve filled out the graph for the subword ‘birth’ as shown in Figure 2.2.

Figure 2.2: WordWalk Puzzle for root word ‘birthday’ showing only the hidden subword ‘birth’.



Subwords: birth day had hard hat

WordWalk Puzzle

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Now let’s consider the next subword ‘day’. Since the ‘y’ only occurs at the end of this subword, and since there are no letters in any of the subwords that follow this letter, it must be that the upper left hand corner node is the right one for ‘y’ (because it doesn’t have any arrows emanating from it). But from this it follows that the only node that points at this last node must in fact be the node for ‘a’. And this leaves only one last node to be filled out for ‘d’. See Figure 2.3

At this point all the nodes have been filled out and we should be pretty confident that this is the solution. However, to fully verify this, we must find the remaining subwords, ‘had’, ‘hard’, and ‘hat’. But these can be easily found as seen in the Figs. 2.4, 2.5, and 2.6, respectively.

Figure 2.3: WordWalk Puzzle for root word ‘birthday’ showing only the hidden subword ‘day’.

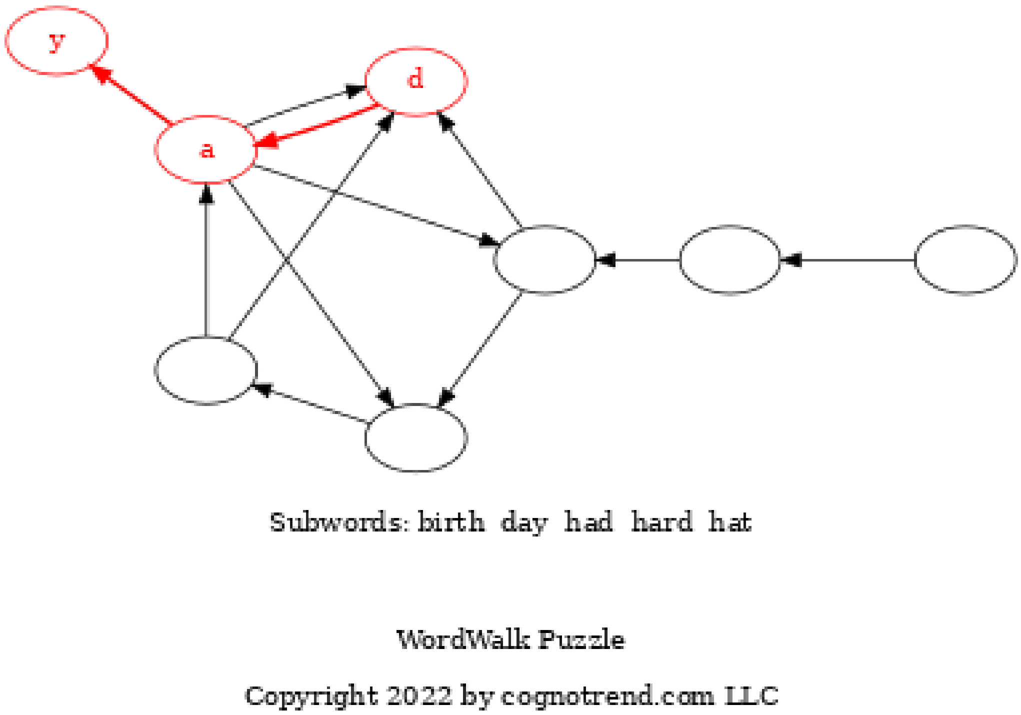


Figure 2.4: WordWalk Puzzle for root word ‘birthday’ showing only the hidden subword ‘had’.

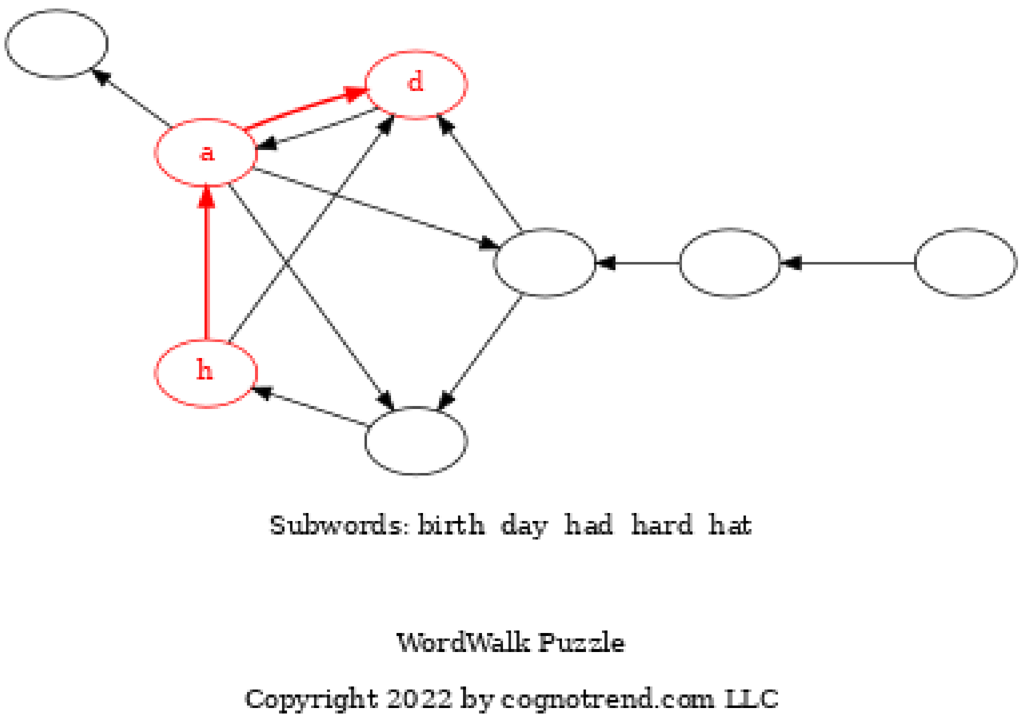


Figure 2.5: WordWalk Puzzle for root word 'birthday' showing only the hidden subword 'hard'.

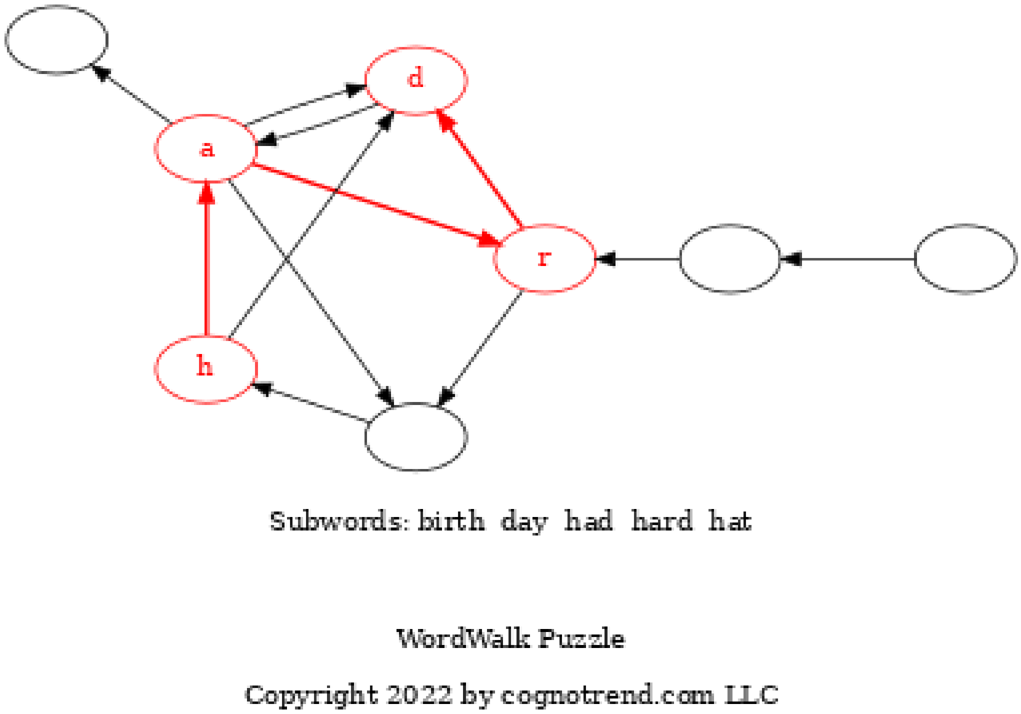


Figure 2.6: WordWalk Puzzle for root word 'birthday' showing only the hidden subword 'hat'.

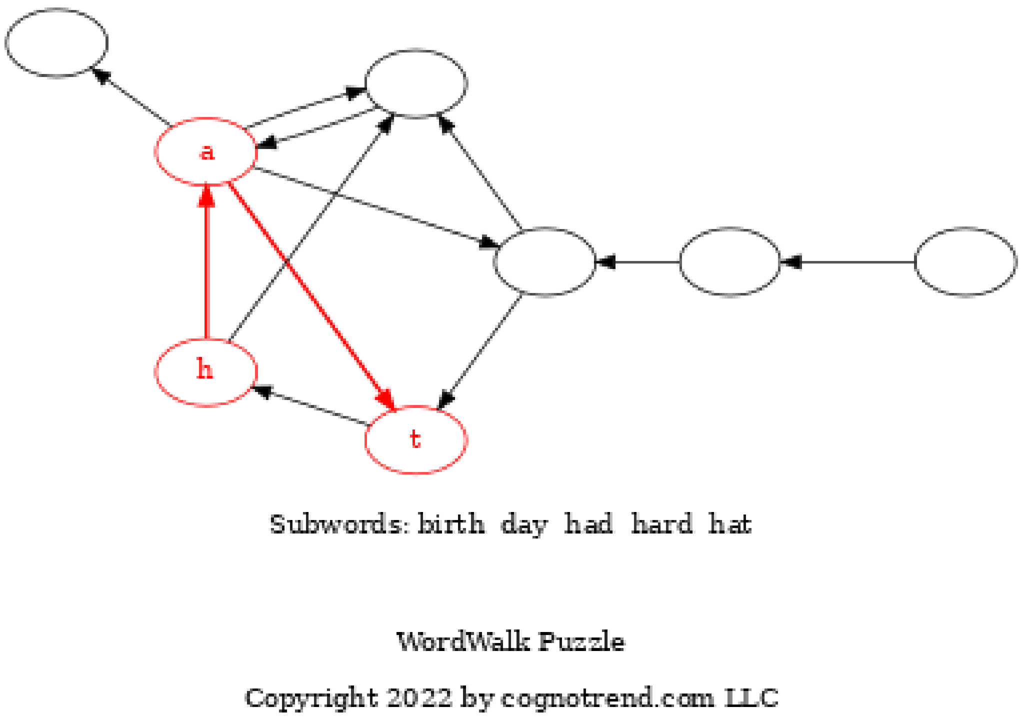
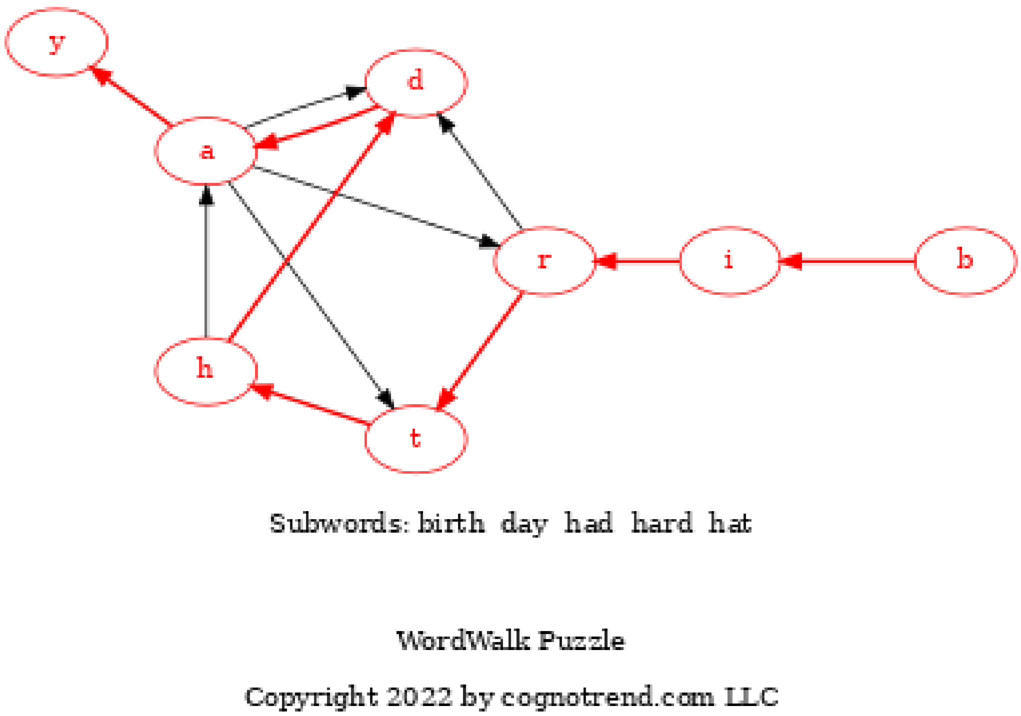


Figure 2.7: Solved WordWalk Puzzle for root word ‘birthday’ and subwords ‘birth’, ‘day’, ‘had’, ‘hard’, ‘hat’



Appendix A

The Theory Behind WordWalk Graphs

This appendix discusses some of the mathematical aspects of WordWalk graphs. It also explains the procedures used to generate them.

A.1 Graph Theory

A **directed graph** in mathematics is a pair of sets $< V, E >$, where V is non-empty and $E \subset V \times V$, i.e., E is a set of ordered pairs of elements drawn from V. The elements of V are called **vertices** and the those of E are called edges. In WordWalk we change this nomenclature slightly and refer to the elements of V as **nodes** and to the ordered pairs in E as **arrows**.

Mathematical graph theory distinguishes between directed graphs (as defined above) and more general graphs in which the edge set E is a set of unordered pairs. Since we are dealing in WordWalk solely with directed graphs we will usurp the term 'graph' to always mean a directed graph. However, the reader should always keep in mind the more precise terminology used in graph theory.

Directed graphs are often depicted as diagrams of nodes (often represented by circles, dots, ellipses, etc.)

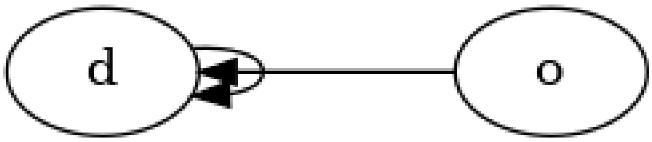
Consider a set of nodes comprised of the letters $\{w, o, r, l, d\}$ and arrows $\{(w, o), (o, r), (r, l), (l, d)\}$. The pictorial representation might be as shown in Figure 1.1.

Figure 1.1: Directed Graph $< V, E >$, where $V = \{w, o, r, l, d\}$ and $E = \{(w, o), (o, r), (r, l), (l, d)\}$

Obviously, this graph is a WordWalk graph for the word 'world'. The word 'world' has the subwords 'word', 'old', and 'odd' (among others); but they are not yet indicated in Figure 1.1.

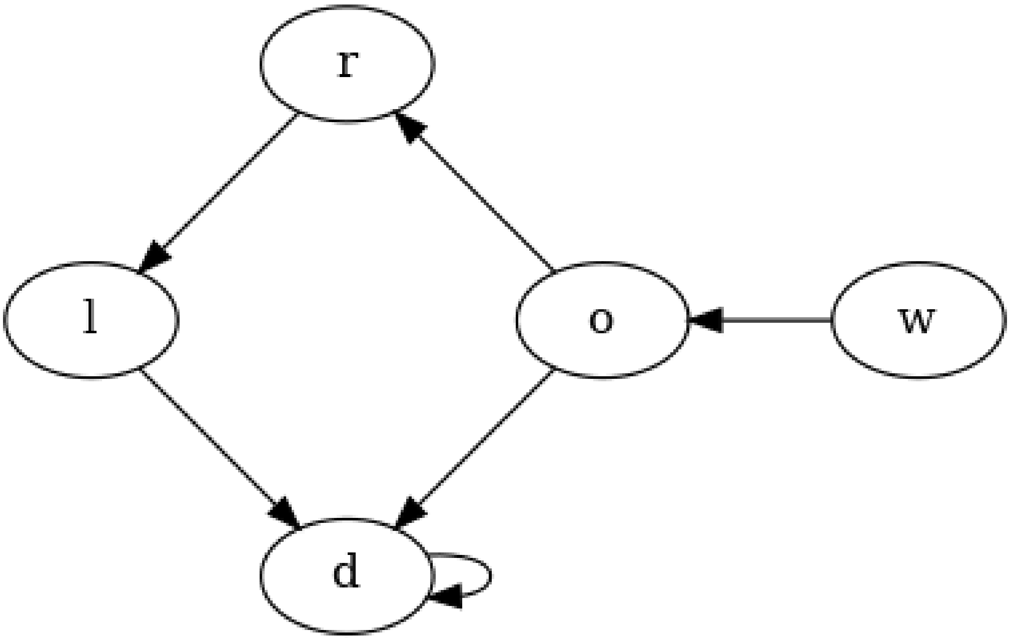
Let's now consider a graph for the word 'odd': $< \{o, d\}, \{(o, d), (d, d)\} >$. This is depicted in Figure 1.2

Figure 1.2: Directed Graph $< V, E >$, where $V = \{o, d\}$ and $E = \{(o, d), (d, d)\}$



Now it gets interesting! Let's combine these two graphs into a single graph so that the graph for 'odd' becomes a subgraph of the graph for 'world'. The result is in Figure 1.3

Figure 1.3: Directed Graph $< V, E >$, where $V = \{w, o, r, l, d\}$ and $E = \{(w, o), (o, r), (r, l), (l, d), (o, d), (d, d)\}$



You'll notice that the E set is now the union of the two previous E sets.

A.2 WordWalk: An Application of Graph Theory to Word Puzzles

WordWalk puzzles are directed graphs generated from several words, one (or more) of which is unknown– called the root word – while the other words are known – called the subwords.

WordWalk puzzle graphs are formed from root words and its subwords as follows: Each unique letter in the root word is assigned to a unique node in the graph. Thus, there are often fewer nodes in the graph than there are letters in the word, because letters in the graph are not counted more than once. A letter that occurs multiple times in a word is only counted once and is assigned to a single node in the graph.

If a certain letter in the word is followed by another letter, then an arrow is drawn in the graph from the node of the first letter to the node of the second letter. If a letter is immediately repeated in a word – as the letter 'o' is in the word 'book' – then the arrow is drawn as a loop pointing right back at the same originating node.

In Figure 1.2 we saw such a loop for the letter 'd'. At any rate, for each edge in the edge set E, there will be one (and only one) arrow pointing from the node of the first member of the edge's ordered pair to the node of the second member. Thus, the arrow from 'o' to 'r' in Figure 1.3 corresponds to the pair $(o, r) \in E$ while the arrow from 'o' to the 'd' in the graph corresponds to the pair $(o, d) \in E$.

Given an initial graph generated in this way from a hidden word called the root word, additional subwords of that root word are chosen and also contribute new arrows to the graph.[1] A subword is a word whose letters also occur in the root word. For example, the word 'test' is a subword of the root word 'street'.

Note that the letters of a subword do not need to occur in the same contiguous order that they exhibit in the root word, as would otherwise be the case for the subword 'birth' in the root word 'birthday'. It is only required that the **set** of unique letters occurring in a word be a **subset** of the **set** of letters of the root word. Thus, the word 'hard' is considered to be a subword of 'birthday', because $\{h, a, r, d\} \subset \{b, i, r, t, h, d, a, y\}$.

Subwords do not contribute new nodes to a graph, since all of their nodes are already present in the graph. What they do contribute, usually, are new nodes.[2]

Even very short root words can have many subwords. E.g. the word 'would' has at least the following subwords: 'do', 'old', 'low', 'wood', 'odd',

[1] Some subwords do not contribute new arrows to the graph. E.g., the words 'birth' and ''day' do not contribute any new arrows to the graph for 'birthday'.

[2] As long as they are not contiguous substrings of the root word.

'loud', 'wow', 'doll'.[3]

By contributing new arrows, the subword graphs become embedded as subgraphs in the composite graph based on the root word and the composite WordWalk graph becomes all the more complex.

In general graph theory, a **path** from a vertex x to a vertex y is a sequence of edges and nodes of the form $x_1, e_1, x_2, \ldots, x_{n-1}, e_{n-1}, x_n$, where $x_1 = x$, $x_n = y$, and each edge e_i is in fact an edge of the form (x_i, x_{i+1}). The subgraphs corresponding to subwords are in WordWalk in fact path subgraphs of the composite graph, and they offer multiple ways to walk through the graph and trace out the subwords.

As mentioned previously, each node in a WordWalk puzzle represents one letter of the root word. But only the first letter of the root word is generally given in the initial (mostly blank) graph to be solved. For simpler versions of this puzzle designed for kids, more than one letter of the root word can also be given. Sometimes the graph can be made easier by using a different shape for certain letters like vowels.

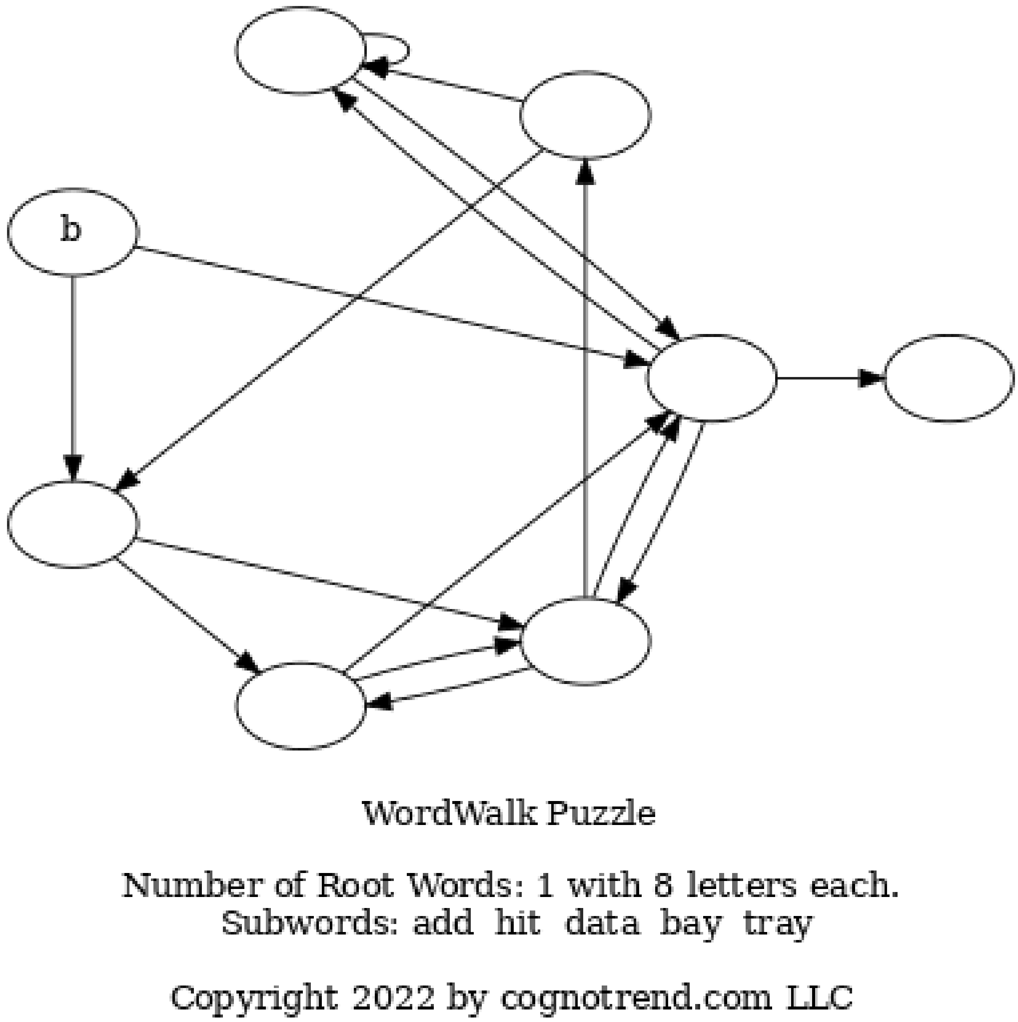
The arrows from node to node show which letters follow which. Letters that repeat somewhere in a word represent repeated crossings of the associated node(s). Only if the repeated letters are adjacent does its associated node exhibit a loop.

The WordWalk graph in Figure 1.4 shows how different a graph can be for the same root word ('birthday') yet involving different subwords and also a different *number* of subwords. The solution for the word 'birthday' with this different subwords is shown in Figure 1.5. The original WordWalk puzzle for 'birthday' in Figure 2.1 did not exhibit any loops. That's because neither the root word nor the subwords contained consecutive double letters. Indeed, none of these words that make up the WordWalk graph require that any of the letter nodes be re-visited at all. However, the second graph for 'birthday' given in Figure 1.5 did in fact exhibit a loop due the double 'dd' in the subword 'add'.

The subgraph and path tracing out the subword 'data' can be seen in Figure 1.6 Another puzzle illustrating the presence of double letters is shown for the root word 'street' in Figure 1.7.

[3] However, only a few words are generally chosen (perhaps at random) to be included when forming the WordWalk graph.

Figure 1.4: Unsolved WordWalk Puzzle for root word ‘birthday’ and subwords ‘add’, ‘hit’, ‘data’, ‘bay’, ‘tray’



The subword ‘rest’ can be seen in Figure 1.8

Figure 1.5: Solved WordWalk Puzzle for root word 'birthday' and subwords 'add', 'hit', 'data', 'bay', 'tray'

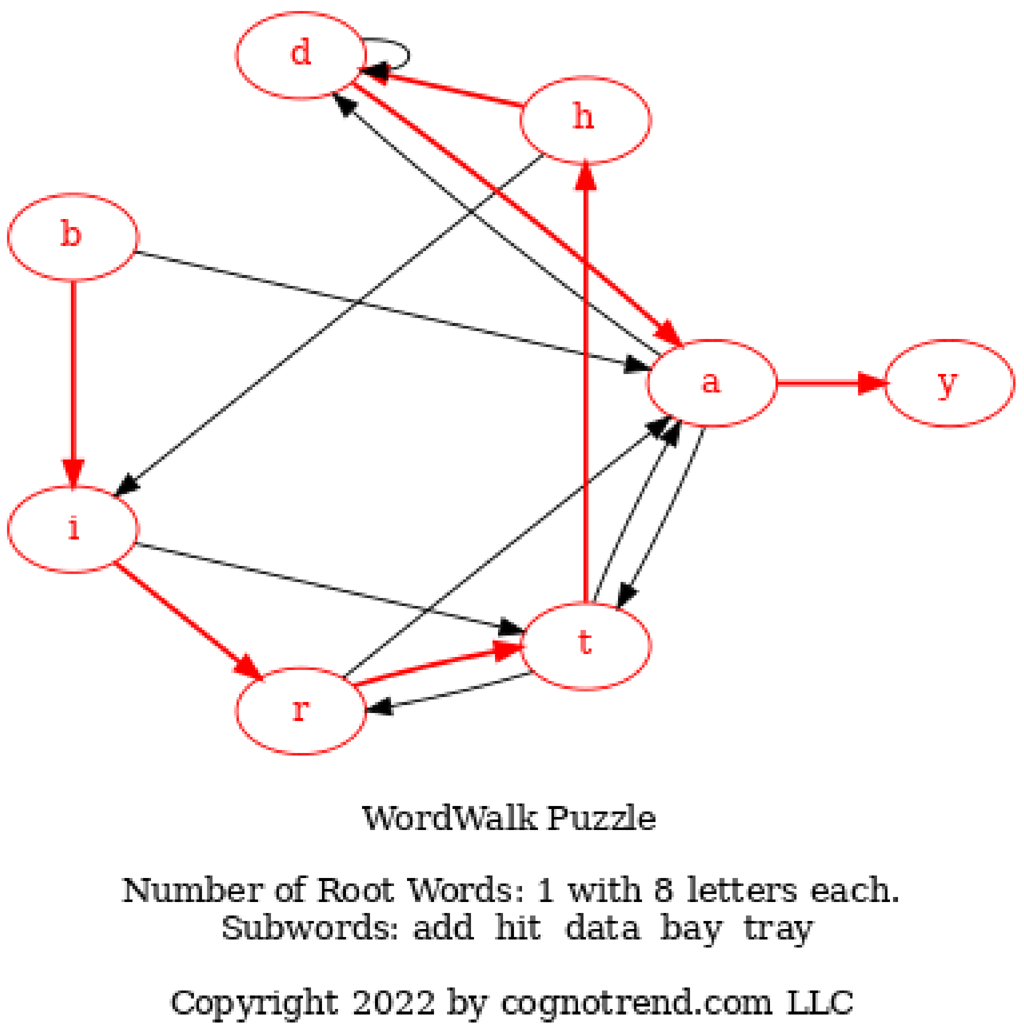
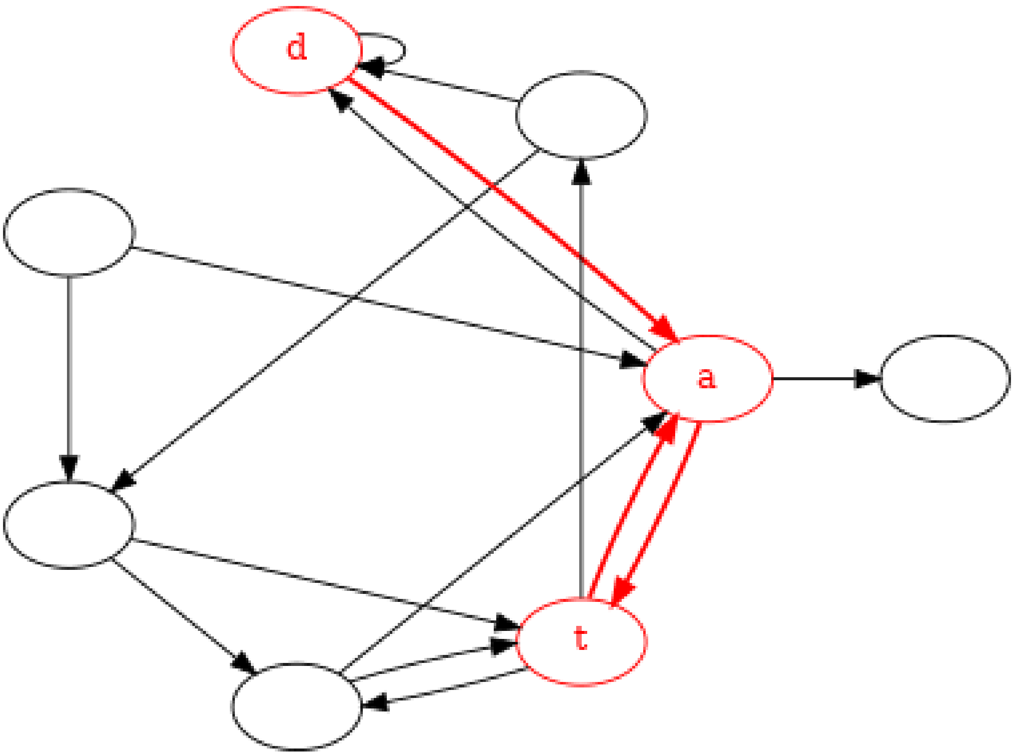


Figure 1.6: Subword path for 'data' in WordWalk graph for 'birthday'.



WordWalk Puzzle

Number of Root Words: 1 with 8 letters each.
Subwords: add hit data bay tray

Figure 1.7: Solved WordWalk Puzzle for root word 'street' and subwords 'see', 'set', 'test' and 'rest'.

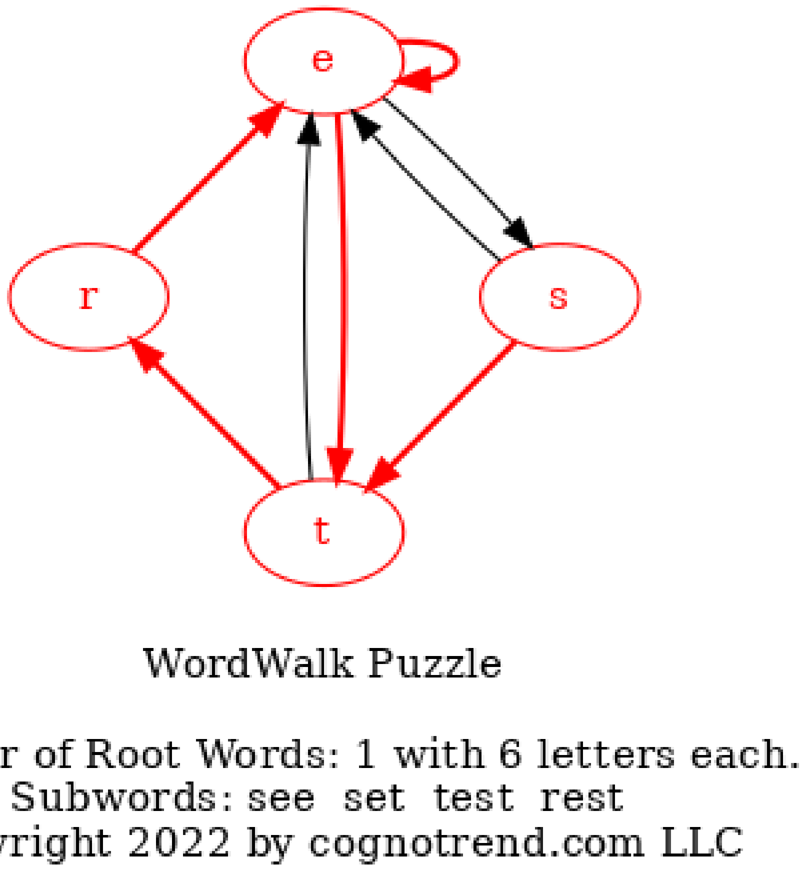
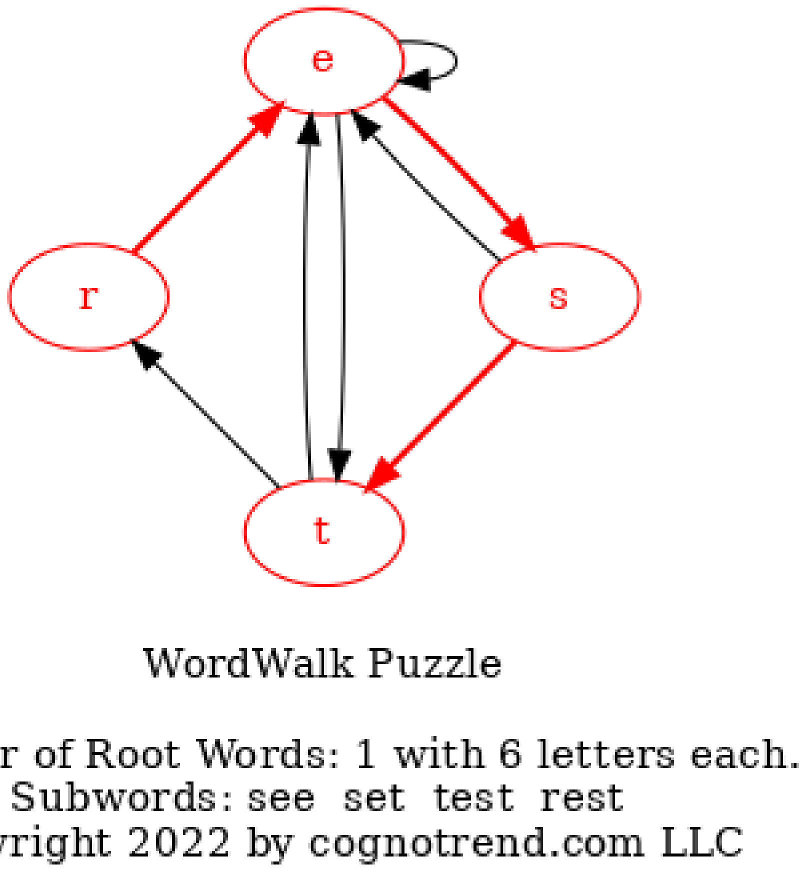


Figure 1.8: Subword 'rest' in WordWalk Puzzle for root word 'street' and subwords 'see', 'set', 'test' and 'rest'.



At the lowest level of difficulty, all the subwords from which the graph was made are given to you. At higher levels, the number of letters in the root word is generally larger, and the number of possible subwords is therefore also greater; yet not all of the subwords contained in the graph have to be shown to the player. Think of the given subwords as clues. The more clues given, the easier it is to solve the puzzle. However, there is also a downside to having more clues: the number of arrows and hence the number of possible letter assignments does increase as well. At even higher levels of difficulty WordWalk puzzles may be generated by more than one root word and their chosen subwords.

Very complex WordWalk graphs can be generated using multiple root words and their subwords. Consider Figure 1.9. Here there are three root words (whose first letters are given) and five subwords. Also, to make it slightly easier to solve, diamond shapes have been used for the vowels.

Some of the subwords can be seen in Figs 1.10, 1.11 and 1.12. The solution consists of the root word 'faith', 'hope' and 'love'. It is left to the reader to find these root words and the mandatory subwords in the WordWalk graph.

Figure 1.9: WordWalk Puzzle with three root words five subwords.

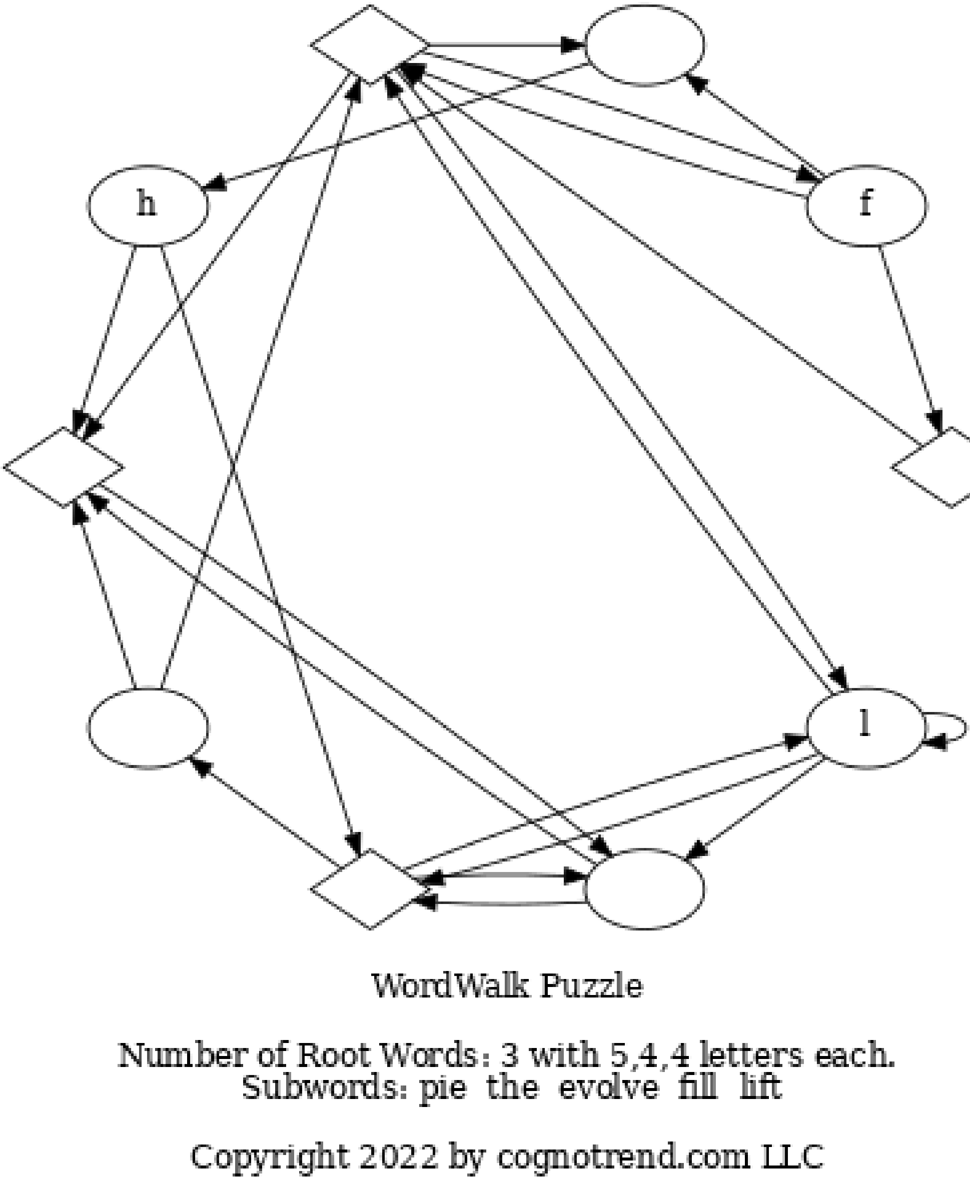


Figure 1.10: WordWalk Puzzle with three root words five subwords.

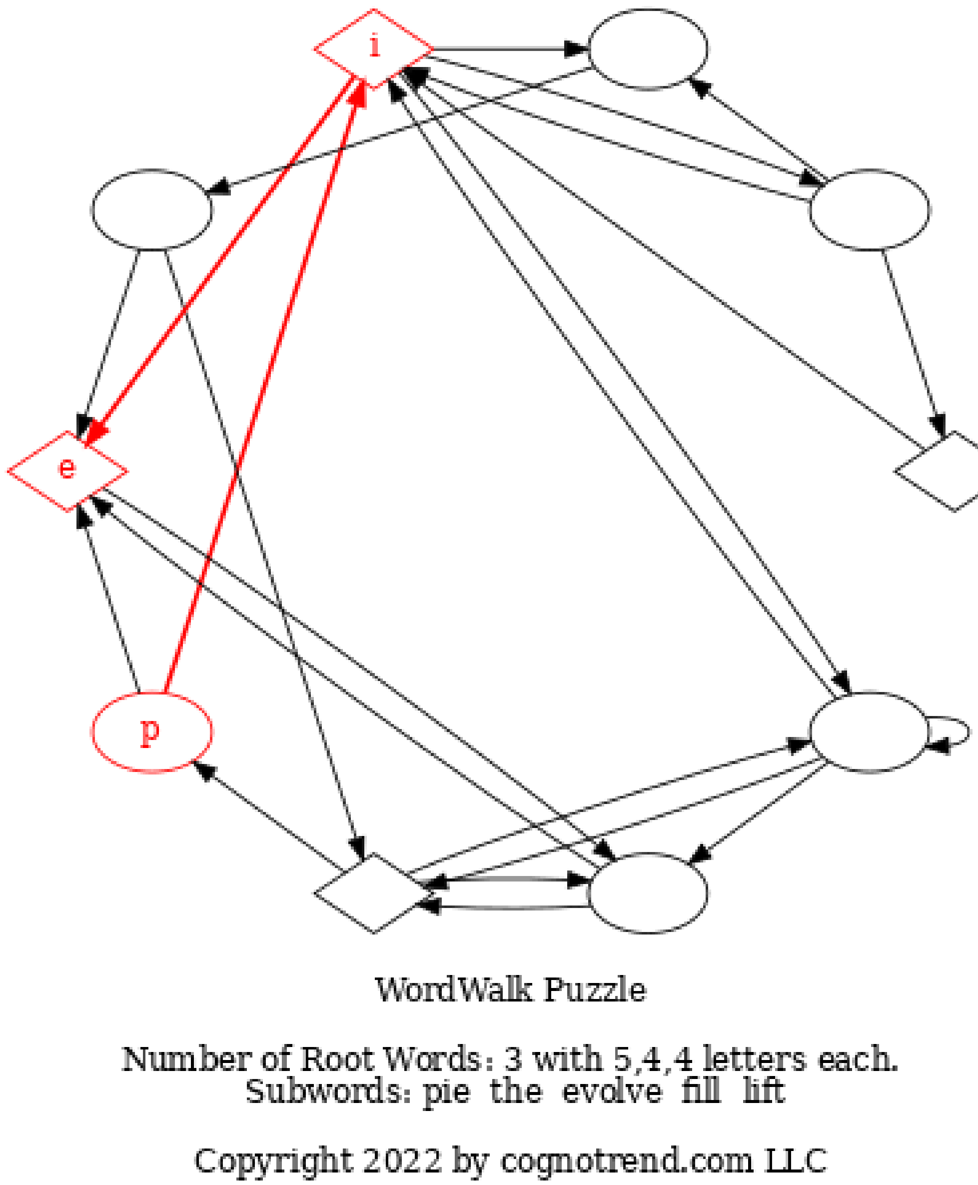
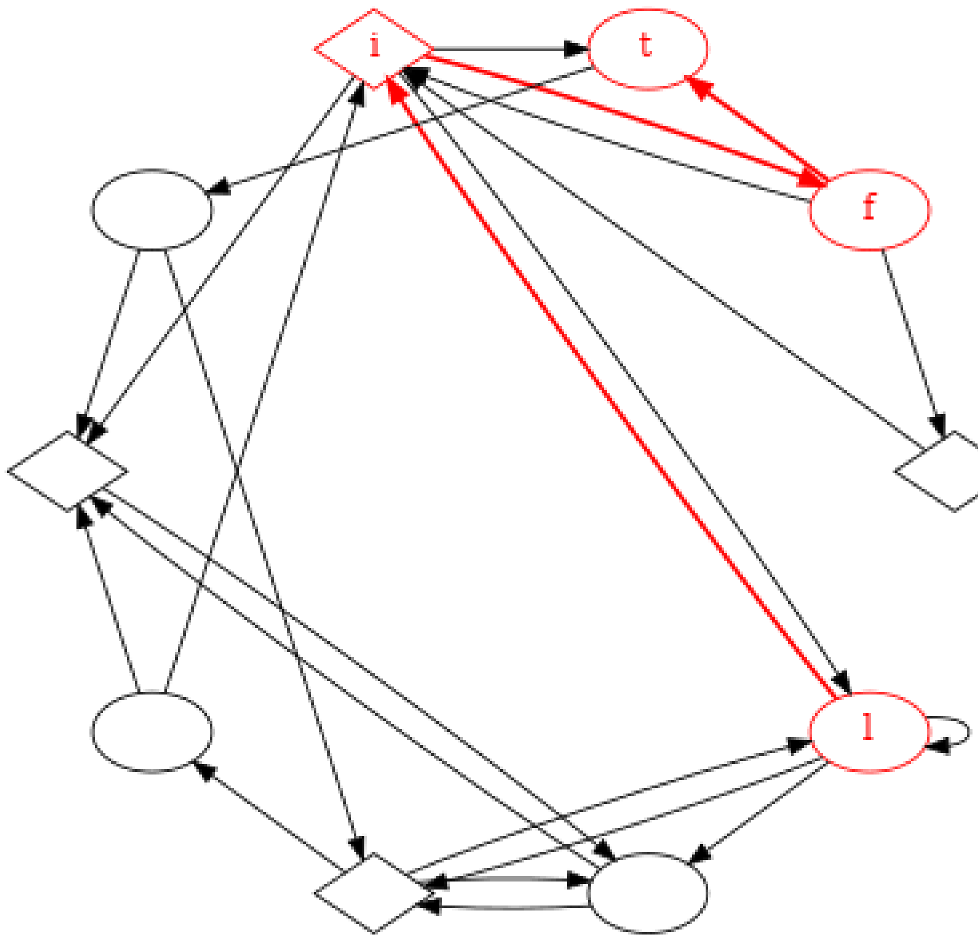


Figure 1.11: WordWalk Puzzle with three root words five subwords.



WordWalk Puzzle

Number of Root Words: 3 with 5,4,4 letters each.
Subwords: pie the evolve fill lift

Figure 1.12: WordWalk Puzzle with three root words five subwords.