Hello, eager reader.

This note book contains several Micro soft Paint Application images,
And they describe the mathematical concept of continued fractions.

Composition and typing and net searching , and ground work, needs collaboration

by

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2025

Comments and questions and feedback, email
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Other books by Matt

1 An introduction to Prime Constelations, with original calcuations
2 Prime producing polynomial, only natural numbers, and positive trinomial
3 Photo book
4 Online Encyclopedia of Integer Sequences,
https://OEIS.org/ contributions

I assume basic knowledge of arithmetic, and some familiarity with algebra.
Background knowledge can be gained from Kahn Academy
https://www.khanacademy.org/

Section one
For starters, we have this Paint application image,

a three level, fully generalized finite
continued fraction with constant coefficients
Let

$$c = b0 + \cfrac{a1}{b1 + \cfrac{a2}{b2 + \cfrac{a3 +}{b3 + \ \ldots}}}$$

This paint image shows a finite continued fraction.

Here, there is a fun coefficient c, which,
if all {a1, a2, a3}, and {b0; b1, b2, b3}
and all coefficients are rational, then
c will also be rational.

Also, there are variables in the numerator of a generalize continued fraction.

Finite continued fractions, with rational coefficients,
Evaluate to rational numbers.

Infinite continued fractions, and irrational numbers are a completely different matter.
And are more complicated.

Section two,
Also, there is this fun part about a general, continued fraction expression for tan(x).

This infinite continued fraction was, used to prove, for the first time, the irrationality of the constant pi.

$$\tan(x) = \cfrac{x}{1 + \cfrac{-x^2}{3 + \cfrac{-x^2}{5 + \cfrac{-x^2}{\cdots}}}}$$

(in 1761) by Johann Heinrich Lambert

It was found in Lambert’s notebook,
Deep in his notebook,
He told nobody

This was in 1761, so things were different then,
They did not have telephones, or the net.

But Lambert was a pioneer, a great mathematician.

This continued fraction is infinite, thus the ellipsis ...
Also, this fraction is not simple or regular.

Question
What do you call a mathematician who is afraid?
Answer
A mathema chicken

End joke

Next page

Section three
There is this, fun, continued fraction for the tangent of pi over two, has the following form see this image, with an example for the tangent to pi over two

By the way, pi/2 radians is the same as 90 degrees, a right angle.

example where x = pi/2

$$\tan(pi/2) = \cfrac{pi/2}{1 + \cfrac{-(pi/2)^2}{3 + \cfrac{-(pi/2)^2}{5 + \text{more}}}}$$

pi/2 radians is
a right angle

mca

See that the “numerators” of the continued fraction are the positive odd numbers, {1,3,5, …}
The pattern of positive odd numbers is easy to follow.
And, it is an infinite set.

Blank page for notes

Outside the scope of continued fractions, there is this fun approximation and
Exact expression for the constant e,

What’s more, regarding the mathematical constant e, there is an approximation

see

$$e \approx 2.718$$

$$e^{\frac{2*pi}{5}} \approx 1.257$$

MCA

I bet you didn't know that before you read it here.
Good fun for the good reader

Also, technically, e is defined with the approximate value of 2.718

What’s more, in this image, see the fun facts that the infinite continued fractions,
Have given forms
See

Now, there are, some continued fraction results
They use Euclid's Algorithm,
so, simply

$\sqrt{2} = [1;2,2,\overline{2}, \ldots]$ **line over number to indicate repeat**

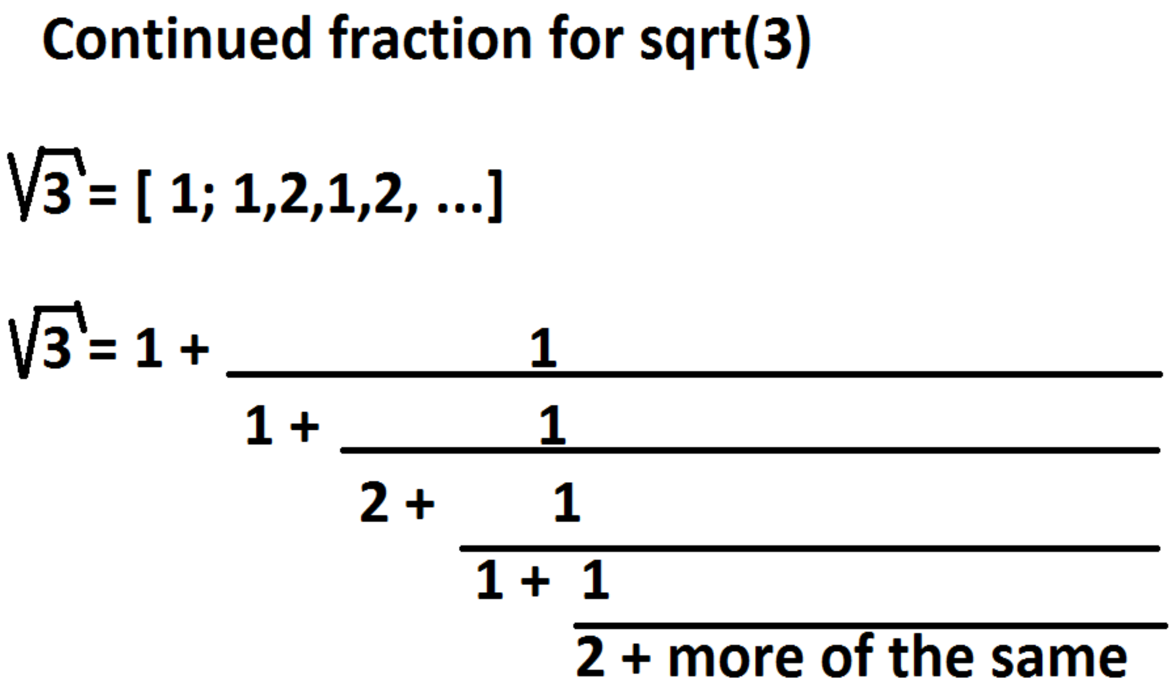
$\sqrt{3} = [1;1,2,\overline{1,2}, \ldots]$

$\sqrt{46} = [6;1,3,1,1,2,6,2,1,1,3, \ldots]$
no aparent pattern for square root of 46

We are dealing with irrational numbers here.

Most infinite continued fractions are irrational.

Similarly, there is this fun image for the square root of three



Here we have the simple pattern that,
Sqrt(3) = [1;1,2,1,2,1,2,1,2, …]

That looks like real fun for the interested reader.
It does to the writer.

Also, there is this fun image for the square root of 198,
As a continued fraction
Here, we assume that the coefficients are all positive.

aslo, note that

$$\sqrt{198} = 14 + \cfrac{1}{14 + \cfrac{1}{28 + \cfrac{1}{14 + \cfrac{1}{28 + \ldots}}}}$$

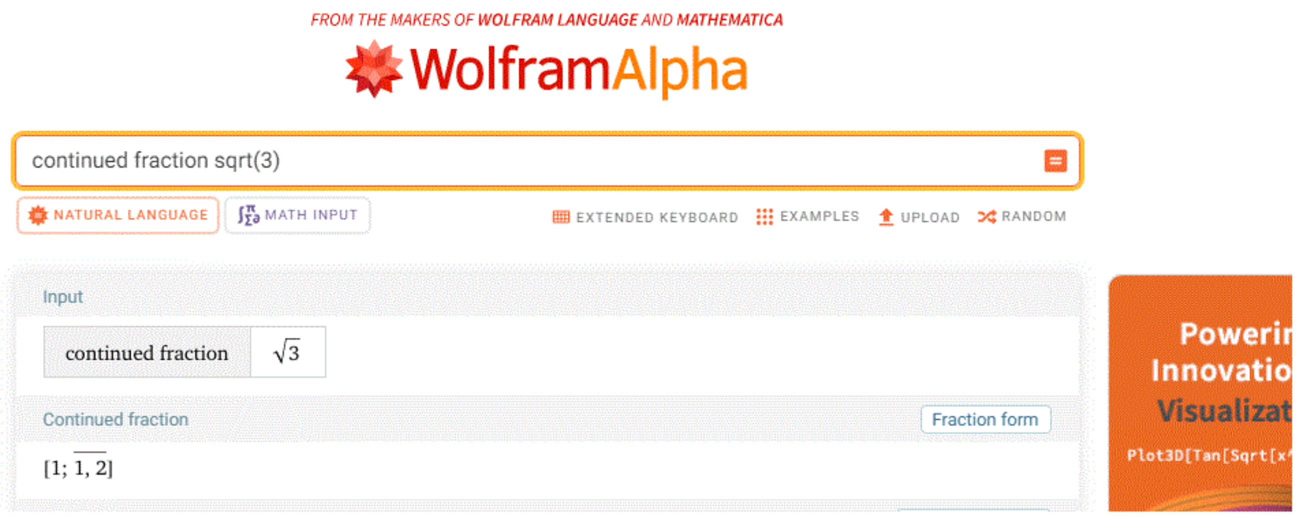
in notation,

$\sqrt{198}$ = [14; 14, 28, 14, 28, 14 ...]

the 14 and 28 repeat infinitely

Also, there is no apparent pattern to the “denominator” coefficients.
This continued fraction is both simple and regular.
All the “numerator” coefficients are equal to one.

Here comes something new, we recalculate the continued fraction for the
Square root of three., for your viewing pleasure,
The square root of three, a second time, this time with
A screen shot from the 'net



that is, sqrt(3) = a and

$$a = 1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cdots}}}}$$

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So let "a" equal exactly the square root of three,
Then there is the fun result that this infinite, repeating continued fraction equals exactly the value of the continued fraction shown.

In My Humble Opinion (IMHO)
Wolfram Alpha, by Waterloo Maple, Canada,
Is the best free online calculator available today.
See
https://www.wolframalpha.com/input?i=partial+fractions

and simply www.wolframalpha.com/

What’s more, there is this table

table of continued fractions of $\sqrt{n}$ for $1 < n < 102$

The simple continued fractions for the square roots of positive integers (which aren't perfect powers) are non-terminating but they are periodic. In the following table, the square roots of the integers from 2 to 101 (excluding perfect powers) are listed in compact form: first the integer part followed by semicolon, then the periodic part stated once, its individual terms separated by commas. For example, the notation "14; 14, 28" for 198 means

$$\sqrt{198} = 14 + \cfrac{1}{14 + \cfrac{1}{28 + \cfrac{1}{14 + \cfrac{1}{28 + \cdots}}}},$$

where the dots mean a periodic repetition of 14 and 28 in the denominators.

n	Continued fraction of $\sqrt{n}$
2	1; 2
3	1; 1, 2
5	2; 4
6	2; 2, 4
7	2; 1, 1, 1, 4

from planetmath.org

This is interesting, because someone else worked out all the continued fractions for these
Positive counting numbers.
It is unclear to me which parts repeat.
The interested reader can figure that out and tell me.
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What's more,

Outside the scope of continued fractions, we have that

Considering the Reimann Zeta function,
Or just the Zeta(s) function for short,
There is the expression that

EDIT

Just to make it clear, note that

$$\begin{aligned}\frac{1}{\zeta(s)} &= \left(1-\frac{1}{2^s}\right)\left[\left(1-\frac{1}{3^s}\right)-\left(1-\frac{1}{3^s}\right)\frac{1}{5^s}-\left(1-\frac{1}{3^s}\right)\left(1-\frac{1}{5^s}\right)\frac{1}{7^s}-\cdots\right]\\ &= \left(1-\frac{1}{2^s}\right)\left(1-\frac{1}{3^s}\right)\left[\left(1-\frac{1}{5^s}\right)-\left(1-\frac{1}{5^s}\right)\frac{1}{7^s}-\cdots\right]\\ &\vdots\\ &= \prod_{p\in\mathbb{P}}\left(1-\frac{1}{p^s}\right)\end{aligned}$$

where $\mathbb{P}$ is the set of the prime numbers.

from math.stackexchange.com/
Google search words
continued fraction riemann zeta 2

So there is an interesting reference

References

1 https://en.wikipedia.org/wiki/Continued_fraction

2 https://math.stackexchange.com/questions/2698904/how-to-find-a-continued-fraction

3 https://mathworld.wolfram.com/ContinuedFraction.html

Thank you for reading and looking

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