Prime Producing Polynomial

By Matt

Irreducible polynomial

- An irreducible polynomial does not factor under the rational numbers.
- W^2 + 1 is irreducible
- Let h(n) = n^2 + n + 41
- Then h(n) is irreducible.

- Assume n is an integer
- Again, $h(n) = n^2 + n + 41$.

We can describe when h(n) is a composite number by

1)a data table / list,

2) by parametric expressions, and

3) by a single expression in two variables.

Data table

h(y) = y2+y+41

h(y) mod x ≡ 0

x	y
41	0
41	40
43	1
43	41
47	2
47	44
53	3
53	49
61	4
61	56

Parametric expressions

$n(2,1) = z^2 + 40$

$n(3,1) = 3z^2 + 2z + 122$

$n(3,2) = 6z^2 + z + 244$

Restrictions on $n(r,c)$ – both r and c are integers, $1<c$, $0<r<c$, and $\gcd(r,c) = 1$

Single expression p(r,c)

$p(r,c) = (c*x - r*y)2 - r*(c*x - r*y) - x + 41*r2.$

Probrably, if p(r,c) = 0 then there are points on a graph.

The graph is not shown here.

End.

A prime producing polynomial

Observations on the Trinomial $n^2 + n + 41$

By Matt C. Anderson

March 9 2021

In number theory,

We assume that n is an integer. We focus our attention on the polynomial $n^2 + n + 41$. Further, we analyze the behavior of the factorization of integers of the form

$q(n) = n^2 + n + 41.$ (expression 1)

where n is a non-negative integer. It was shown by Legendre, in 1798 that if $0 \le n \le 40$ then r(n) is a prime number. Certain patterns become evident when considering points (a,n) where

$q(n) \equiv 0 \bmod a.$ (expression 2)

The collection of all such points produces what we are calling a "graph of discrete divisors". It has certain repeated features. From experimental data, we find that the integer points in this dataset are contained by parabolas. And more, the parabolas are described by a closed form expression. We see that the parabolas are indexed (r,c) by pairs of relatively prime integers. The expression for the middle parabolas is

$p(r,c) = (c*x - r*y)^2 - x*(c*x - r*y) - x + 41*r^2$ (expression 3)

The restrictions on p(r,c) are that $0<r<c$ and gcd(r,c) = 1. Where gcd() means greatest common divisor of two arguments. And all four of r,c,x, and y are integers.

When we take the derivative of p(r,c) with respect to x and set this expression equal to zero, we obtain

$x = (163*r^2)/4$ (expression 4)

Each such pair (r,c) yields (again determined experimentally and by observation of calculation in a computer algebra system) an integer polynomial $a*z^2 + b*z + c$. The first few (r,c) pairs are (2,1); (3,2); (3,1); (4,3); (4,1) and (5,4). Again, r and c must be relatively prime numbers. Further, the quartic $r(a*z^2 + b*z + c)$ will factor algebraically over the integers into two quadratic expressions. We call this our "parabola conjecture" (or conjecture 'a'). Certain structure in the 'graph of discrete divisors' are do to elementary relationships between pairs of co-prime integers.

We conjecture that all composite values of r(n) arise by substituting integer values of z into $q(a*z^2 + b*z + c)$, where this quartic divisors algebraically over **Z** for $a*z^2 + b*z + c$ a quadratic polynomial determined by a pair of relatively prime integers (r, c). We are confident of this conjecture because of the structure of the graph of discrete divisors produced by some computer code in our computer algebra system (Maple). We call this our "no stray points conjecture" (or conjecture 'b') because all the points in the graph appear to lie on a parabola.

We further conjecture that the minimum x-values for parabolas corresponding to (r, c) are given by expression 4. The vertical lines $x = 163*c^2/4$ where c = 2, 3, 4, … The numerical evidence seems to support this. This is called our "parabolas line up conjecture.

Theorem 1 – Consider r(n) with n a non negative integer. Then,
r(n) never has a factor less than 41.

We prove Theorem 1 with a modular construction. We make a residue table of r(y) mod x, with all the prime divisors less than 41. A form of the fundamental theorem of arithmetic states that any integer greater than one is either a prime number, or can be written as a unique product of prime numbers (ignoring the order). So if r(n) never has a prime factor less than 41, then by extension it never has a prime factor less than 41.

For example, to determine that r(n) is never divisible by 2, note the first column of the residue table. If n is even then r(n) is odd. Similarly, if n is odd then r(n) is also odd. In either case, r(n) does not have factorization by 2. Since all integers are either even or odd, r(n) is never divisible by 2 when n is a positive integer.

Also, for divisibility by 3, there are 3 cases to check. They are n ≡ 0, 1, and 2 mod 3. r(0) mod 3 is 2. r(1) mod 3 is 1 and r(2) mod 3 is 2. Since none of these results is 0, we have that r(n) is never divisible by 3. This is the second column of the residue table.

The number 0 is first found in the residue table for the cases r(0) mod 41 and r(40) mod 41. We can see that $40^2 + 40 + 41 = 41^2$. This means that if n is congruent to 0 mod 41 then r(n) will be divisible by 41. What's more is that these are the only two cases for divisibility by 41. Similarly, if n is congruent to 40 mod 41 the r(n) will also be divisible by 41.

After the residue table, we observe a curve fit to our 'graph of discrete divisors' which has points when q(y) mod x is divisible by x. This is an exact curve fit. The points (x,y) can be seen in a data table, and on a bifurcation graph.

< see residue table >

Thus we have shown that q(n) never has a factor less than 41.

Theorem 2

Since $q(a) = a^2 + a + 41$, we want to show that q(a) = q(-a-1).

Proof of theorem 2
Because q(a) = a*(a+1) + 41,
Now q(-a -1) = (-a -1)*(-a -1 +1) + 41.
So q(-a -1) = (-a –1)*(-a) + 41,
And q(-a -1) = q(a).
End of proof of theorem 2.

Corrolary 1
Further, if r(b) mod c ≡ 0 then q(c –b -1) mod c ≡ 0.

We see that it is amazing that the data points all fall within an exact curve fit. All the parabolas have integer coefficients.

End

5/4/2016

A composite number producing polynomial project

Observations on the trinomial $n^2 + n + 41$

By Matthew C. Anderson

We assume that n is an integer. We consider the composite values of $n^2 + n + 41$. We only consider positive integer values for n in this paper.

The story so far

We consider the behavior of the factorization of integers of the form $h(n) = n^2 + n + 41$ where n is a non-negative integer. It was shown by Legendre, in 1798 that if $0 \le n < 40$ then h(n) is a prime number.

Certain patterns become evident when considering points (x,y) where $h(y) \equiv 0 \bmod x$. These points can be enumerated using a computer tool such as a Computer Algebra System or spreadsheet program. The collection of all such point produces what we are calling a “graph of discrete divisors” for h(n) due to certain self-similar features. From experimental computer data we find that the integer points in this graph lie on a collection of parabolic curves indexed by pairs of relatively prime integers. Each such pair yields (again determined experimentally and by observation of calculations) an integer polynomial $a*z^2 + b*z + c$, and the quartic $h(a*z^2 + b*z + c)$ then factors non-trivially over the integers into two quadratic expressions. A quadratic expression, when graphed forms a parabola.

We call this above statement our "parabola conjecture".

Conjecture is a mathematical term that means possibly true statement.

Certain symmetries in the graph of divisors are due to elementary relationships between pairs of co-prime integers. For instance if m<n are co-prime integers, then there is an observable relationship between the parabola it determines that that formed from (n-m, n).

We conjecture that all composite values of h(n) arise by substituting integer values of z into $h(a*z^2 + b*z + c)$, where this quartic factors algebraically over $\mathbf{Z}$ for $a*z^2 + b*z + c$ a quadratic polynomial determined by a pair of relatively prime integers.

We name this above statement our "no stray points conjecture" because all the points in the graph of discrete divisors appear to lie on parabolas.

We further conjecture that the minimum x-values for parabolas corresponding to (m, n) with gcd(m, n) = 1 are equal for fixed n. Further, these minimum x-values of parabolas line up at 163*d^2/4 where d = 1, 2, 3, ... The numerical evidence seems to support this.

This statement above is called our "parabolas line up" conjecture.

The notation gcd(m, n) used above is defined here. The greatest common devisor of two integers is the smallest whole number that divides both of those integers.

Theorem 1 - Consider h(n) with n a non negative integer.
h(n) never has a factor less than 41.

We prove Theorem 1 with a modular construction. We make a residue table with all the prime factors less than 41. The fundamental theorem of arithmetic states that any integer greater than one is either a prime number, or can be written as a unique product of prime numbers (ignoring the order). So if h(n) never has a prime factor less than 41, then by extension it never has an integer factor less than 41.

For example, to determine that h(n) is never divisible by 2, note the first column of the residue table. If n is even, then h(n) is odd. Similarly, if n is odd then h(n) is also odd. In either case, h(n) does not have factorization by 2.

Also, for divisibility by 3, there are 3 cases to check. They are n = 0, 1, and 2 mod 3. h(0) mod 3 is 2. h(1) mod 3 is 1. and h(2) mod 3 is 2. Due to these three cases, h(n) is never divisible by 3. This is the second column of the residue table.

The number 0 is first found in the residue table for the cases h(0) mod 41 and h(40) mod 41. This means that if n is congruent to 0 mod 41 then h(n) will be divisible by 41. Similarly, if n is congruent to 40 mod 41 then h(n) is also divisible by 41. After the residue table, we observe a bifurcation graph which has points when h(y) mod x is divisible by x. The points (x,y) can be seen on the bifurcation graph.

see residue table page

Thus we have shown with a proof that h(n) never has a factor less than 41.

Theorem 2

Since h(a) = a^2 + a + 41, we want to show that h(a) = h(-a -1).

Proof of Theorem 2
Because h(a) = a*(a+1) + 41,
Now h(-a -1) = (-a -1)(-a - 1 + 1) + 41.
So h(-a -1) = (-a -1)*(-a) + 41,
And h(-a -1) = h(a)

Which was what we wanted
End Proof of Theorem 2

Corollary 1

Further if h(b) mod c ≡ 0 the h(c -b -1) mod c ≡ 0.

We can observe interesting patterns in the graph of discrete divisors on a following page.

Residue Table

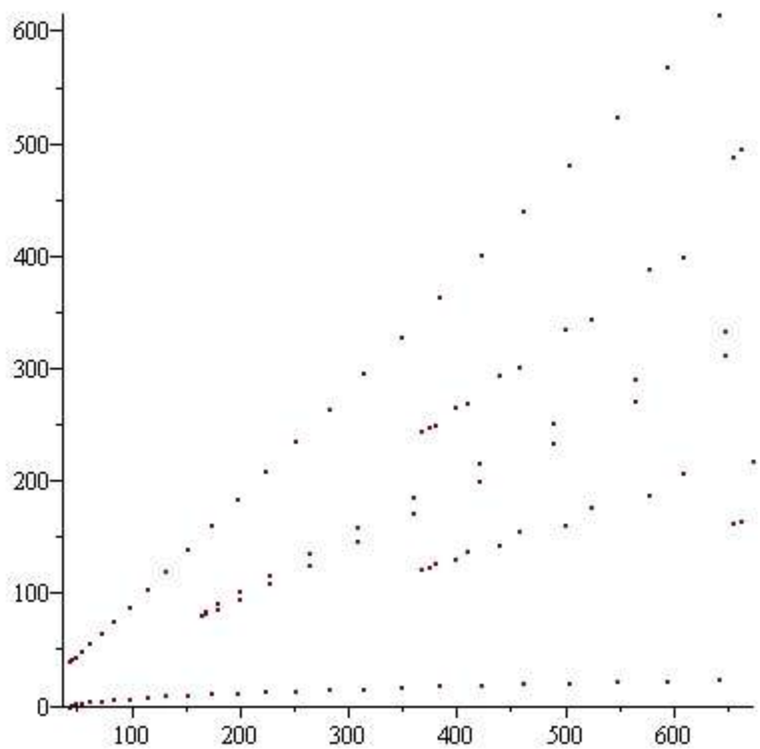
	2	3	5	7	11	13	17	19	23	29	31	37	41	43
0	1	2	1	6	8	2	7	3	18	12	10	4	0	41
1	1	1	3	1	10	4	9	5	20	14	12	6	2	0
2		2	2	5	3	8	13	9	1	18	16	10	6	4
3			3	4	9	1	2	15	7	24	22	16	12	10
4			1	5	6	9	10	4	15	3	30	24	20	18
5				1	5	6	3	14	2	13	9	34	30	28
6				6	6	5	15	7	14	25	21	9	1	40
7					9	6	12	2	5	10	4	23	15	11
8					3	9	11	18	21	26	20	2	31	27
9					10	1	12	17	16	15	7	20	8	2
10					8	8	15	18	13	6	27	3	28	22
11						4	3	2	12	28	18	25	9	1
12						2	10	7	13	23	11	12	33	25
13							2	14	16	20	6	1	18	8
14							13	4	21	19	3	29	5	36
15							9	15	5	20	2	22	35	23
16							7	9	14	23	3	17	26	12
17								5	2	28	6	14	19	3
18								3	15	6	11	13	14	39
19									7	15	18	14	11	34
20									1	26	27	17	10	31
21									20	10	7	22	11	30
22									18	25	20	29	14	31
23										13	4	1	19	34
24										3	21	12	26	39
25										24	9	25	35	3
26										18	30	3	5	12
27										14	22	20	18	23
28										12	16	2	33	36
29											12	23	9	8
30											10	9	28	25
31												34	8	1
32												24	31	22
33												16	15	2
34												10	1	27
35												6	30	11
36												4	20	40
37													12	28
38													6	18
39													2	10
40													0	4
41														0
42														41

The function h(n) which was defined as $n^2 + n + 41$ has interesting properties. Especially when n is restricted to the integers. As we know h(n) is a prime number for as n goes from 0 to39.

h(40) = 41^2, which is a composite number.

Also h(n) can be generated recursively as h(0) = 0 and h(n) = h(n-1) + 2*n.

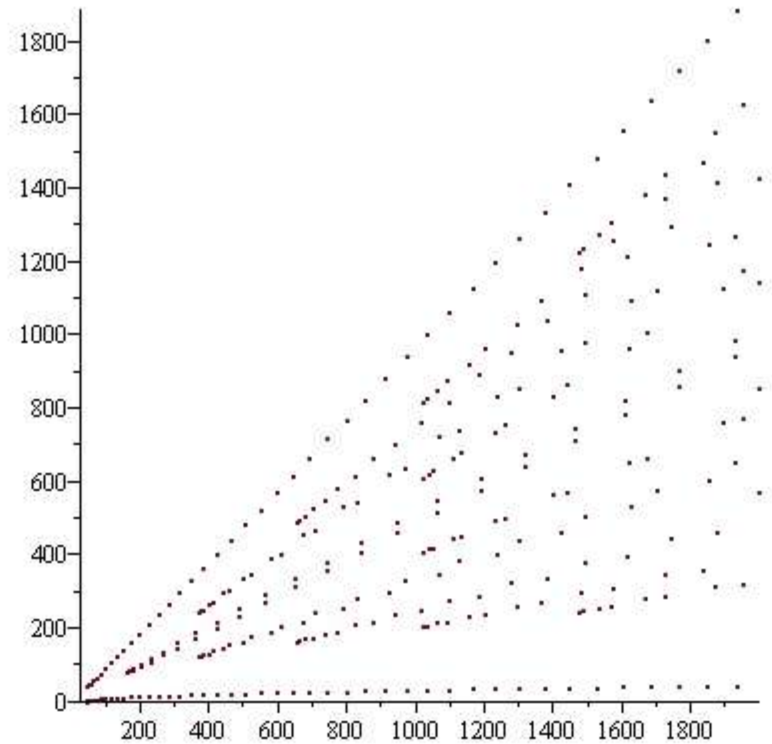
This is a linear recurrence with constant coefficients.



Bifurcation Graph

These are pairs of numbers (x,y) such that h(y) mod x ≡ 0.

And $h(y) = y^2 + y + 41$.



Here is a zoomed out iteration of the same graph as the previous page.

There seems to b an apparent regular structure in this graph of divisibility.

The points give themselves to an exact curve fit of parabolas.

The general form of these parabolas is –

$p(r,c) = c^2x^2 - 2c*r*x*y + r^2y^2 - (c*r + 1)*x + r^2y + 41r^2$. (Equation 1).

p is for parabola, r is for row index, c is for column index, x is the horizontal axis and y is the vertical axis.

This does not include the top and bottom parabolas.

There are also 3 restrictions.

$1<r$

$0<c<r$

$Gcd(r,c) = 1$.

All the parabolas can be described exactly and algebraically.

The x minimum of p(r,c) is

Pmin = (163*r^2)/4.   (expression 2)

This can be found with the Mape Commmand extrema.

To wit –

```
> # p is for parabola
> p[r, c] := c^2·x^2 − r·c·2·x·y + r^2·y^2 − (r·c + 1)·x + r^2y + 41·r^2;
```

$$p_{r,c} := c^2 x^2 - 2\,r\,c\,x\,y + r^2 y^2 - (c\,r + 1)\,x + r^2 y + 41\,r^2 \qquad \textbf{(1)}$$

```
> e2 := extrema(x, p[r, c] = 0, {x, y});
```

$$e2 := \left\{ \frac{163}{4}\,r^2 \right\} \qquad \textbf{(2)}$$

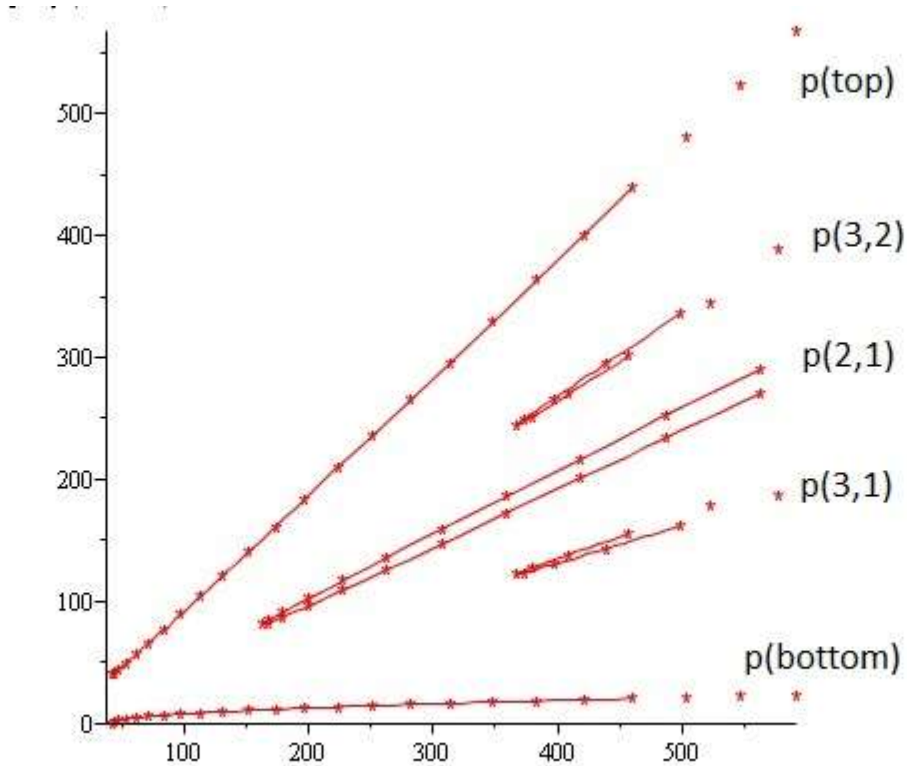
This project is not finished.

Here is some Maple code to show the exact curve fit for the graph of divisors.

```
> # Maple code
> x[bottom] := z^2+z+41; y[bottom] := z;
> p2 := plot([x[bottom], y[bottom], z = 0 .. 20]);
> with(plots);
> x[1, 1, top] := z^2+z+41; y[top] := z^2+40;
> p3 := plot([x[top], y[top], z = 0 .. 20]);
>
> y[2, 1] := 2*z^2+z+81; x[2, 1] := 4*z^2+163;
> p4 := plot([x[2, 1], y[2, 1], z = -10 .. 10]);
>
> y[3, 1] := 3*z^2+2*z+122; x[3, 1] := 9*z^2+3*z+367;
> p5 := plot([x[3, 1], y[3, 1], z = -4 .. 3]);
>
> y[3, 2] := 6*z^2+z+244; x[3, 2] := 9*z^2+3*z+367;
> p6 := plot([x[3, 2], y[3, 2], z = -4 .. 3]);
>
> d1 := display([p2, p3, p4, p5, p6])
> # code for graph of divisors
> xv := Vector[row](89); yv := Vector[row](89); counter := 1;
> for a from 2 to 600 do
    for b from 0 to a-1 do
      if `mod`(b^2+b+41, a) = 0 then
        xv[counter] := a; yv[counter] := b; counter := counter+1
      end if
    end do
  end do;
> counter;
> d2 := plot(xv, yv, style = point, symbol = asterisk);
> display(d1, d2)
> # This produces a graph.
```

The graph of divisors with 5 parabolas appears on the next page.

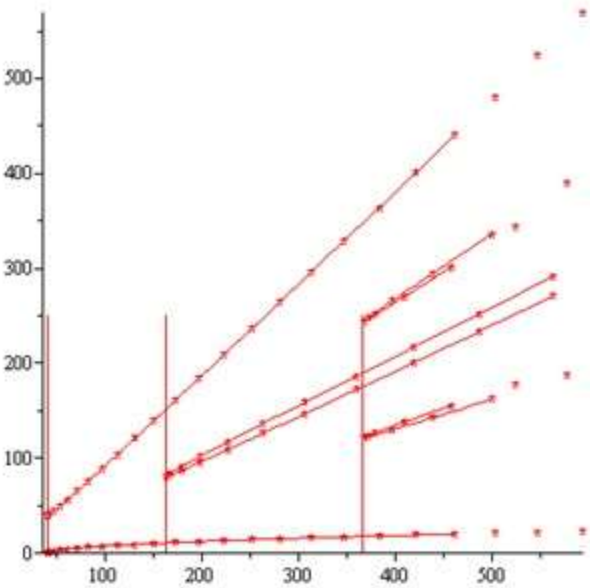
Graph of Divisors with parabolas that exactly fit the points



> "Notice the exact curve fit of parabolas to divisibility points"

Graph of divisibility

Vertical lines at
163*c^2/4



Notice the vertical lines are tangent to the parabolas.

There is still more to be done with this project.

A prime producing polynomial.

Observations on the trinomial $n^2 + n + 41$.

by Matt C. Anderson

May 2021

In number theory,

We analyze the behavior of the factorization of integers of the form

$$h(n) = n^2 + n + 41 \qquad \text{(expression 1)}$$

where n is a non-negative integer. It was shown by Legendre, in 1798 that if $0 \le n < 40$ then h(n) is a prime number.

Given that n is restricted to positive integers, it is an unsolved problem whether or not h(n) is a prime number an infinite number of times. I suspect that h(n) is prime infinitely often. Numerical evidence supports this.

Certain patterns become evident when considering points (a,n) where

$$h(n) \equiv 0 \bmod a. \qquad \text{(expression 2)}$$

The collection of all such point produces what we are calling a "graph of discrete divisors" due to certain self-similar features. From experimental data we find that the integer points in this bifurcation graph lie on a collection of parabolic curves indexed by pairs of relatively prime integers. The expression for the middle parabolas is –

$$p(r,c) = (c*x - r*y)^2 - r*(c*x - r*y) - x + 41*r^2. \qquad \text{(expression 3)}$$

The restrictions are that $0<r<c$ and $\gcd(r,c) = 1$ and all four of r,c,x, and y are integers.

Each such pair (r,c) yields (again determined experimentally and by observation of calculations) an integer polynomial a*z2 + b*z + c, and the quartic h(a*z2 + b*z + c) then factors non-trivially over the integers into two quadratic expressions. We call this our "parabola conjecture". Certain symmetries in the bifurcation graph are due to elementary relationships between pairs of co-prime integers. For instance if $m<n$ are co-prime integers, then there is an observable relationship between the parabola it determines that that formed from (n-m, n).

We conjecture that all composite values of h(n) arise by substituting integer values of z into $h(a*z^2 + b*z + c)$, where this quartic factors algebraically over **Z** for $a*z^2 + b*z + c$ a quadratic polynomial determined by a pair of relatively prime integers. We name this our "no stray points conjecture" because all the points in the bifurcation graph appear to lie on a parabola.

We further conjecture that the minimum x-values for parabolas corresponding to (r, c) with gcd(r, c) = 1 are equal for fixed n. Further, these minimum x-values line up at $163*c^2/4$ where c = 2, 3, 4, ... The numerical evidence seems to support this. This is called our "parabolas line up" conjecture.

The notation gcd(r, c) used above is defined here. The greatest common devisor of two integers is the smallest whole number that divides both of those integers.

Theorem 1 – The only small factors theorem - Consider h(n) with n a non negative integer.
h(n) never has a factor less than 41.

We prove Theorem 1 with a modular construction. We make a residue table with all the prime factors less than 41. Also, we test all possible residues for each prime.

For example, to determine that h(n) is never divisible by 2, note the first column of the residue table. If n is even, then h(n) is odd. Similarly, if n is odd then h(n) is also odd. In either case, h(n) does not have factorization by 2.

Also, for divisibility by 3, there are 3 cases to check. They are n = 0, 1, and 2 mod 3. h(0) mod 3 is 2. h(1) mod 3 is 1. and h(2) mod 3 is 2. Due to these three cases, h(n) is never divisible by 3. This is the second column of the residue table.

The number 0 is first found in the residue table for the cases h(0) mod 41 and h(40) mod 41. This means that if n is congruent to 0 mod 41 then h(n) will be divisible by 41. Similarly, if n is congruent to 40 mod 41 then h(n) is also divisible by 41.
After the residue table, we observe a bifurcation graph which has points when h(y) mod x is divisible by x. The points (x,y) can be seen on the bifurcation graph.

< see residue table in appendix 4 >

Thus we have shown that h(n) never has a factor less than 41. This ends our proof.

The fundamental theorem of arithmetic states that any integer greater than one is either a prime number, or can be written as a unique product of prime numbers (ignoring the order). So if h(n) never has a prime factor less than 41, then by extension it never has an integer factor less than 41.

Theorem 2 – the near mirror symmetry theorem

Since $h(a) = a^2 + a + 41$, we want to show that h(a) = h(-a -1).

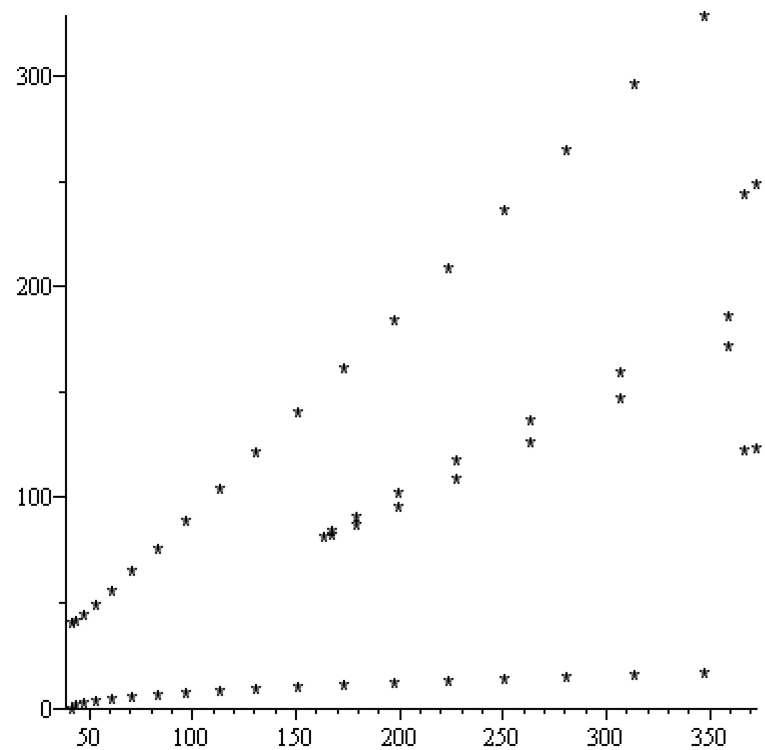
Proof of Theorem 2
Because h(a) = a*(a+1) + 41,

Now h(-a -1) = (-a -1)*(-a -1 +1) + 41.
So h(-a -1) = (-a -1)*(-a) +41,
And h(-a -1) = h(a).
Which was what we wanted.
End of proof of theorem 2.

Corollary 1
Further, if h(b) mod c ≡ = then h(c −b -1) mod c ≡ 0.

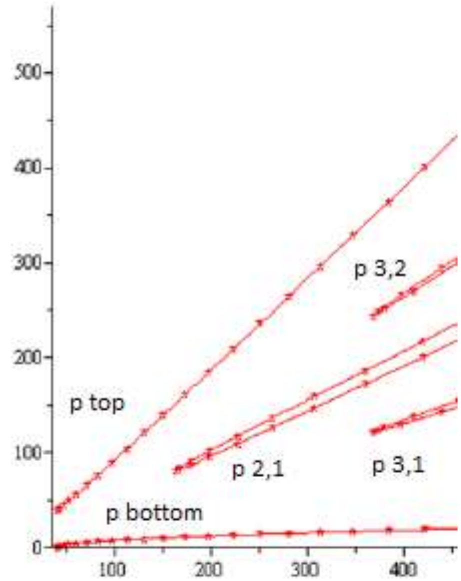
We can observe interesting patterns in the graph of discrete divisors on a following page.

plot(x, y, style = point, symbol = asterisk, color = black)



 # this is a graph of 55 data points of $y^2 + y + 41$ **mod** $x = 0$.
 # It can be curve fit with parabolas.
 # This graph shows 5 parabolas
 # The names of the parabolas are p_{top}; p_{bottom}; $p_{2,1}$; $p_{3,2}$; **and** $p_{3,1}$.

The curve fit data is shown below.



Graph of discrete divisors.

Undiscovered Expressions

So far, we want to determine when $h(n) = n^2 + n + 41$ is a prime number. We produce a dataset that satisfies the congruency $h(y) \equiv 0 \mod x$. In other words, we find ordered pairs (x,y) such that x divides h(y). The graph of all pairs (x,y) seems to have obvious regularity and patterns. We are able to tabulate coefficients of parabolas that exactly fit the data. Here are the first few parabolas :

P bottom x (z) = $z^2 + z + 41$
P bottom y (z) = z

P top x (z) = $z^2 – z + 41$
P top y (z) = $z^2 + 40$

P 2,1 x (z) = $4*z^2 + 163$
P 2,1 y (z) = $2*z^2 + z + 81$

P 3,2 x (z) = $4*z^2 + 163$
P 3,2 y(z) = $6*z^2 + z + 244$

P 3,1 x (z) = $z^2 + z + 41$
P 3,1 y (z) = $3*z^2 + 2*z + 122$

A computer tool can show that h(P 2,1 x(z)) = P 2,1 y(z) * ($z^2 + z + 41$). (equation *)

The Maple command subs() can substitute one expression into another. Also the Maple command factor() can factor quartic polynomials.

The important part of equation * is that the right hand side is the product of two integers, both greater than one. This proves that h(P 2,1(z)) is a composite number. In other words, if you put a positive integer of the form $4*z^2 +163$ as input to h(n), then you will get a composite number as output.

We have the general parabola

P c,r x(z) and P c,r y(z).

I was unable to determine these expressions. It may be impossible and it is related to the distribution of prime numbers.

My naming scheme for the parabolas requires c and r to be integers and

0>r>c and gcd(r,c) = 1

Where gcd is the Greatest Common Divisor of two integers.

So the first few parabolas are, besides top and bottom,

P 2,1

P 3,1   P 3,2

P 4,1   P 4,3

P 5,1   P 5,2   P 5,3   P 5,4

Hopefully the naming convention for P c,r is now clear.

I was able to determine an expression for P c,r that eliminates z.

This is expression 3 from before

$P_{r,c} = (c*x – r*y)^2 – r*(c*x – r*y) – x + 41*r^2$

We assume r and c are integers.

Appendix 1 - Maple Code for graph of discrete divisors

```
x := Vector(55) :
y := Vector(55) :
counter := 1 :
for a from 2 to 378 do
for b from 0 to a - 1 do
if mod(b^2 + b + 41, a) = 0
  then x[counter] := a : y[counter] := b : counter := counter + 1;
end if;
end do:
end do:
```

The number 378 was chosen by trial and error to completely fill the vector of length 55. The number 55 was chosen so that we can easily identify 5 parabolas from the data points.

This code creates a data set and stores it in two vectors.

Appendix 2 – Mape Code for exact curve fit parabolas

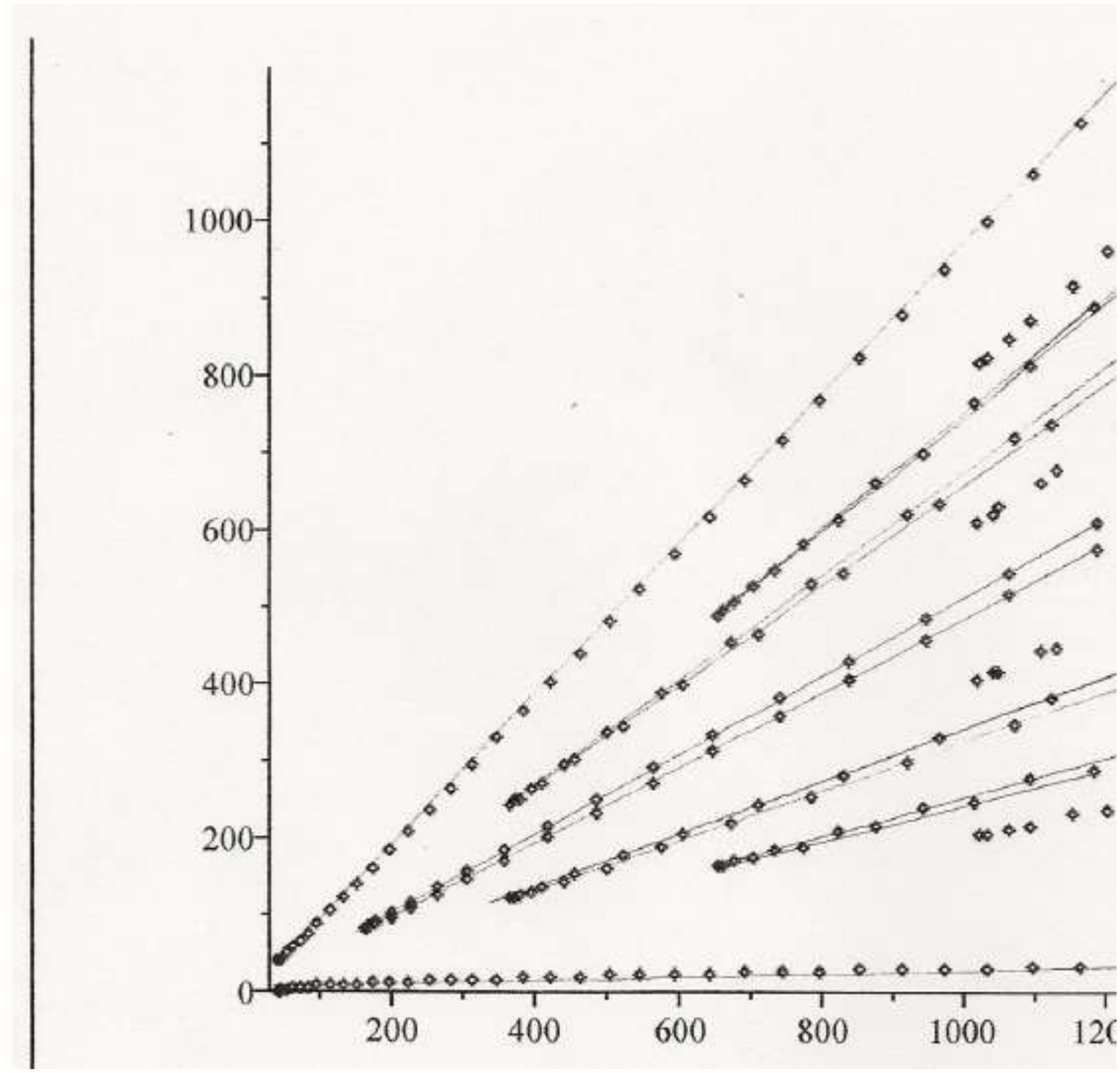
```
> x[1, 1, bottom] := z^2+z+41; y[1, 1] := z;
> p2 := plot([x[1, 1, bottom], y[1, 1], z = 0 .. 20]);
> with(plots);
> display(p2);
>
> x[1, 1, top] := z^2+z+41; y[1, 1, top] := z^2+40;
> p3 := plot([x[1, 1, top], y[1, 1, top], z = 0 .. 20]);
> display(p3);
>
> y[2, 1] := 2*z^2+z+81; x[2, 1] := 4*z^2+163;
> p4 := plot([x[2, 1], y[2, 1], z = -10 .. 10]);
> display(p4);
>
> y[3, 1] := 3*z^2+2*z+122; x[3, 1] := 9*z^2+3*z+367;
> p5 := plot([x[3, 1], y[3, 1], z = -4 .. 3]);
>
> y[3, 2] := 6*z^2+z+244; x[3, 2] := 9*z^2+3*z+367;
> p6 := plot([x[3, 2], y[3, 2], z = -4 .. 3]);
```

This code shows that parabolas exactly fit the data produced by (expression 2).

See graph above.

Appendix 3

Graph of discrete divisors with 7 parabolas.



The data in this graph seems to appear with a (mostly) regular pattern.

Appendix 4 – residue table

Residue Table

	2	3	5	7	11	13	17	19	23	29	31	37	41	43
0	1	2	1	6	8	2	7	3	18	12	10	4	0	41
1	1	1	3	1	10	4	9	5	20	14	12	6	2	0
2		2	2	5	3	8	13	9	1	18	16	10	6	4
3			3	4	9	1	2	15	7	24	22	16	12	10
4			1	5	6	9	10	4	15	3	30	24	20	18
5				1	5	6	3	14	2	13	9	34	30	28
6				6	6	5	15	7	14	25	21	9	1	40
7					9	6	12	2	5	10	4	23	15	11
8					3	9	11	18	21	26	20	2	31	27
9					10	1	12	17	16	15	7	20	8	2
10					8	8	15	18	13	6	27	3	28	22
11						4	3	2	12	28	18	25	9	1
12						2	10	7	13	23	11	12	33	25
13							2	14	16	20	6	1	18	8
14							13	4	21	19	3	29	5	36
15							9	15	5	20	2	22	35	23
16							7	9	14	23	3	17	26	12
17								5	2	28	6	14	19	3
18								3	15	6	11	13	14	39
19									7	15	18	14	11	34
20									1	26	27	17	10	31
21									20	10	7	22	11	30
22									18	25	20	29	14	31
23										13	4	1	19	34
24										3	21	12	26	39
25										24	9	25	35	3
26										18	30	3	5	12
27										14	22	20	18	23
28										12	16	2	33	36
29											12	23	9	8
30											10	9	28	25
31												34	8	1
32												24	31	22
33												16	15	2
34												10	1	27
35												6	30	11
36												4	20	40
37													12	28
38													6	18
39													2	10
40													0	4
41														0
42														41

Explaination of Residue Table
column index, C are across the top
row index, R are found along the side

table values are calculated by
R^2 + R + 41 mod C

Notice that columns
with 41 and 43 contain 0 twice.

These 0 values become points in the
graph of discrete divisors.

Thus we have tried all prime divisors from 2 to 37 inclusive. None of them give a zero residue. The four residues in the residue table involve divisibility by 41 and 43.

A prime producing quadratic expression.

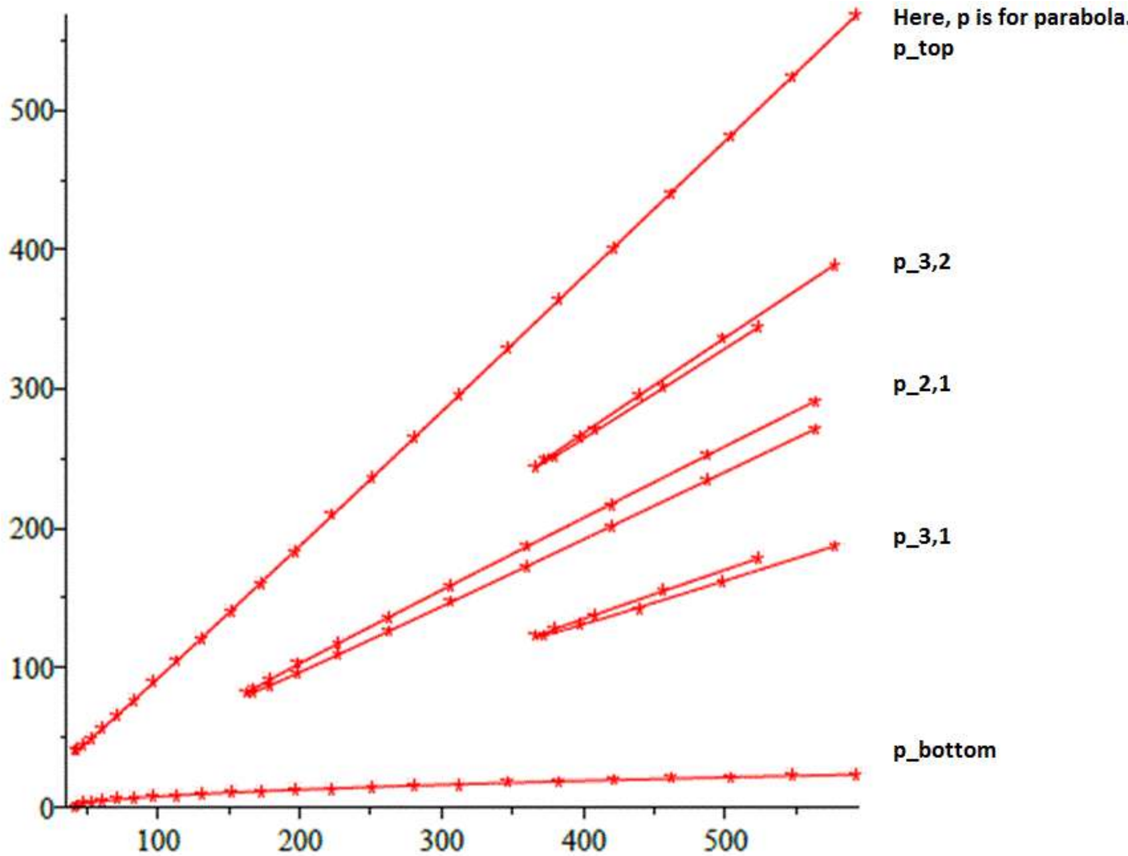
An exploration on the trinomial f(n) = n^2 + n + 41. Where n is a non-negative integer.

Apparently, all cases where f(n) is a composite number can be listed systematically.

Maple Code for exact curve fit parabolas. Parabolas are described parametrically.

```
> x[1, 1, bottom] := z^2+z+41; y[1, 1] := z;

> p2 := plot([x[1, 1, bottom], y[1, 1], z = 0 .. 20]);

> with(plots);

> display(p2);

>

> x[1, 1, top] := z^2+z+41; y[1, 1, top] := z^2+40;

> p3 := plot([x[1, 1, top], y[1, 1, top], z = 0 .. 20]);

> display(p3);

>

> y[2, 1] := 2*z^2+z+81; x[2, 1] := 4*z^2+163;

> p4 := plot([x[2, 1], y[2, 1], z = -10 .. 10]);

> display(p4);

>

> y[3, 1] := 3*z^2+2*z+122; x[3, 1] := 9*z^2+3*z+367;

> p5 := plot([x[3, 1], y[3, 1], z = -4 .. 3]);

>

> y[3, 2] := 6*z^2+z+244; x[3, 2] := 9*z^2+3*z+367;

> p6 := plot([x[3, 2], y[3, 2], z = -4 .. 3]);
```

Now see plot on next page

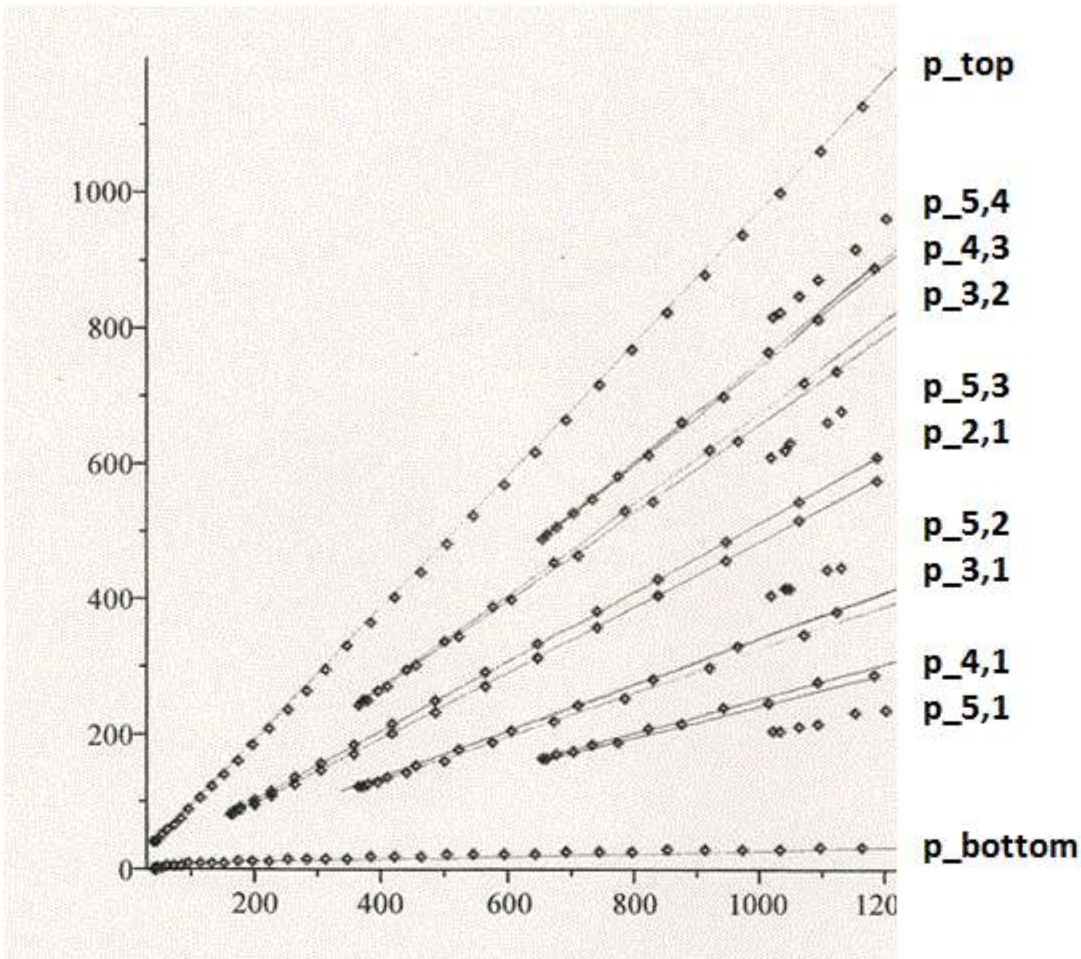


Data points of $y^2+y+41 \equiv 0 \; mod \; x$. Also, parabolic exact curve fit of this data.

Rules for naming parabolas

p_r,c with p for parabola, r for row and c for column. Require that r and c are positive integers. Also, 0<r<c and gcd(r,c) = 1. Where gcd stands for greatest common divisor. Also, the count of the number of c parabolas for a given r is Euler's phi function phi(r). This enumerates as phi(r) = 1,2,2,4,2, … see oeis.org/A10.

Here is a zoomed out view of the same graph.



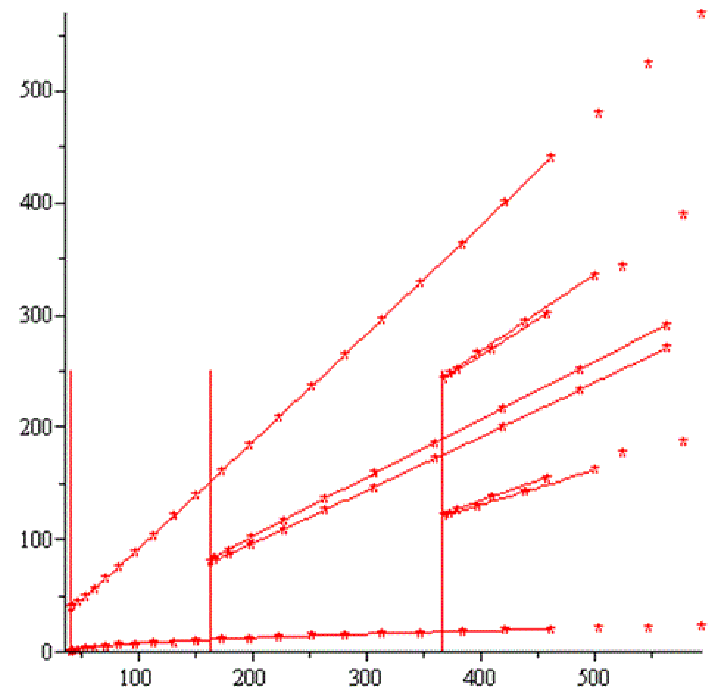
y^2+y+41 mod x is congruent to 0.

Horizontal minimum of parabolas (not including p_top and p_bottom) is 163*(x^2)/4. For some reason, the parabolas line up. Such is the nature of the integers.

A prime producing polynomial graph again with more analysis.

Graph of divisibility

Vertical lines at 163*n^2/4



Notice the vertical lines are tangent to the parabolas.

See that 163*1/4 = 40.75. And, $163*(2^2)/4 = 163$. And $163*(3^2)/4 = 366.75$. So we have 3 vertical lines. The x minimum of the curve fit graphs line up exactly with the vertical lines. The parabolas are tangent there.

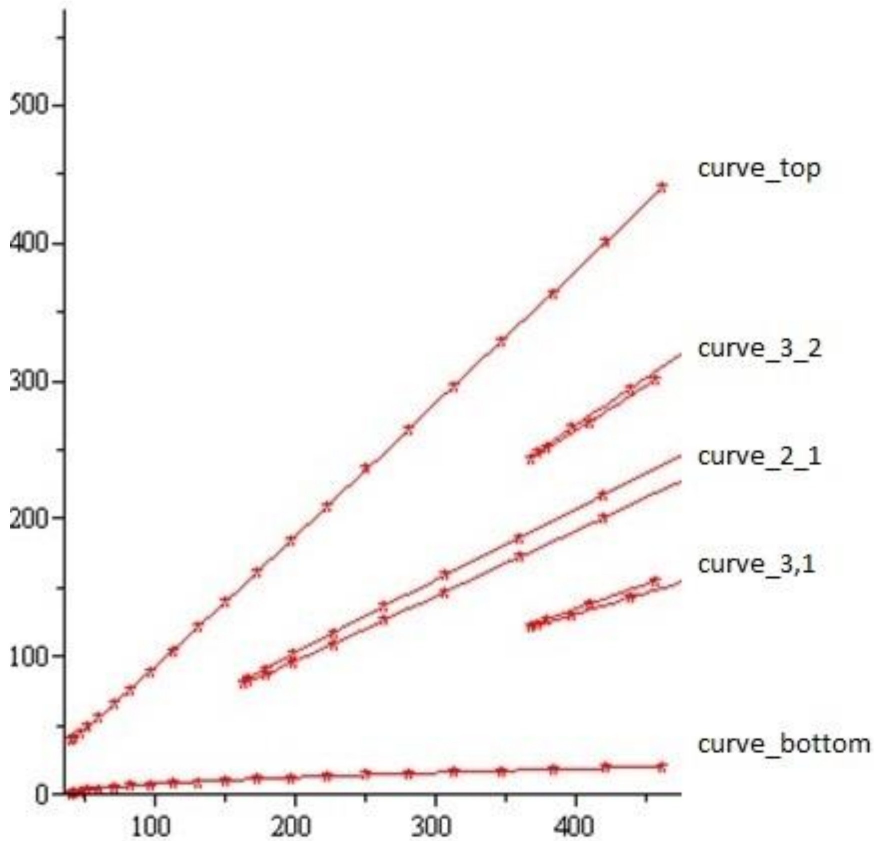
Prime Producing Polynomail project rehash

By Matt C. Anderson

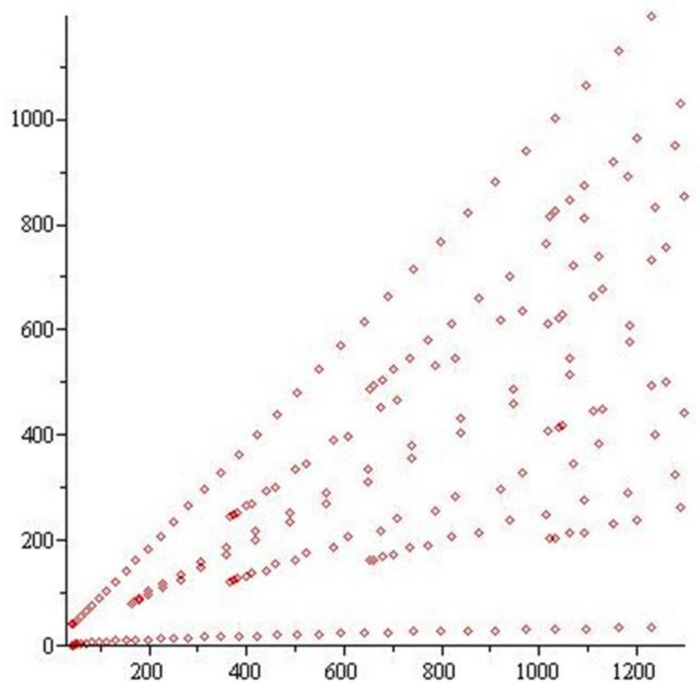
9/11/2016

We assume n is an integer. From before, h(n) = n^2 + n + 41. Our “graph of discrete divisors” shows values of y such that 0<y<x and h(y) mod x is congruent to 0. See graph.

The points on the graph can be connected by exact curve fit. The connecting curves are parabolas. We have defined a numbering system for each of the parabolas. All the parabolas are defined parametrically.



Curve_R_C is defined where R and C are integers and 0<C<R. Also gcd(R,C) = 1. That is to say, the row index and column index must be relatively prime.

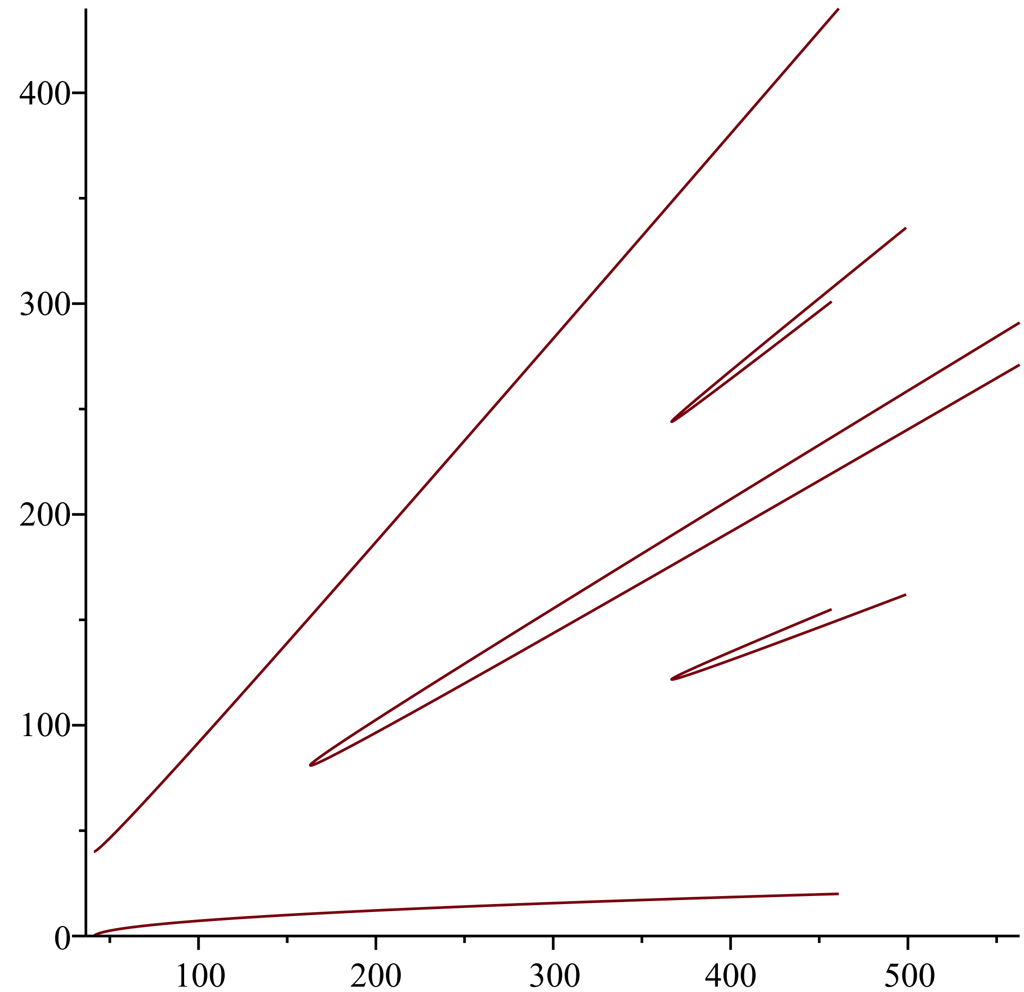


Take this for what it’s worth.

Matt

```
> # trinomials that curve fit the bifurcation graph.

> x[1, 1, bottom] := z^2 + z + 41 :
  y[1, 1] := z :
> p2 := plot([x[1, 1, bottom], y[1, 1], z = 0 ..20]);
                              p2 := PLOT(...)                              (1)
> with(plots) :
> display(p2) :

> x[1, 1, top] := z^2 + z + 41 :
  y[1, 1, top] := z^2 + 40 :
> p3 := plot([x[1, 1, top], y[1, 1, top], z = 0 ..20]);
                              p3 := PLOT(...)                              (2)
> display(p3) :
> # this is  correct

> y[2, 1] := 2 z^2 + z + 81 :
  x[2, 1] := 4 z^2 + 163 :
  p4 := plot([x[2, 1], y[2, 1], z =-10 ..10]);
  display(p4) :
                              p4 := PLOT(...)                              (3)
> display([p2, p3, p4]) :
> # this multiple plot is correct.

> y[3, 1] := 3 z^2 + 2 z + 122 :
  x[3, 1] := 9 z^2 + 3 z + 367 :
> p5 := plot([x[3, 1], y[3, 1], z =-4 ..3])
                              p5 := PLOT(...)                              (4)
> display(p5) :

> y[3, 2] := 6 z^2 + z + 244 :
  x[3, 2] := 9 z^2 + 3 z + 367 :
> p6 := plot([x[3, 2], y[3, 2], z =-4 ..3])
                              p6 := PLOT(...)                              (5)
> display(p6) :
```

> *display*([*p2*, *p3*, *p4*, *p5*, *p6*])



> # I like this plot

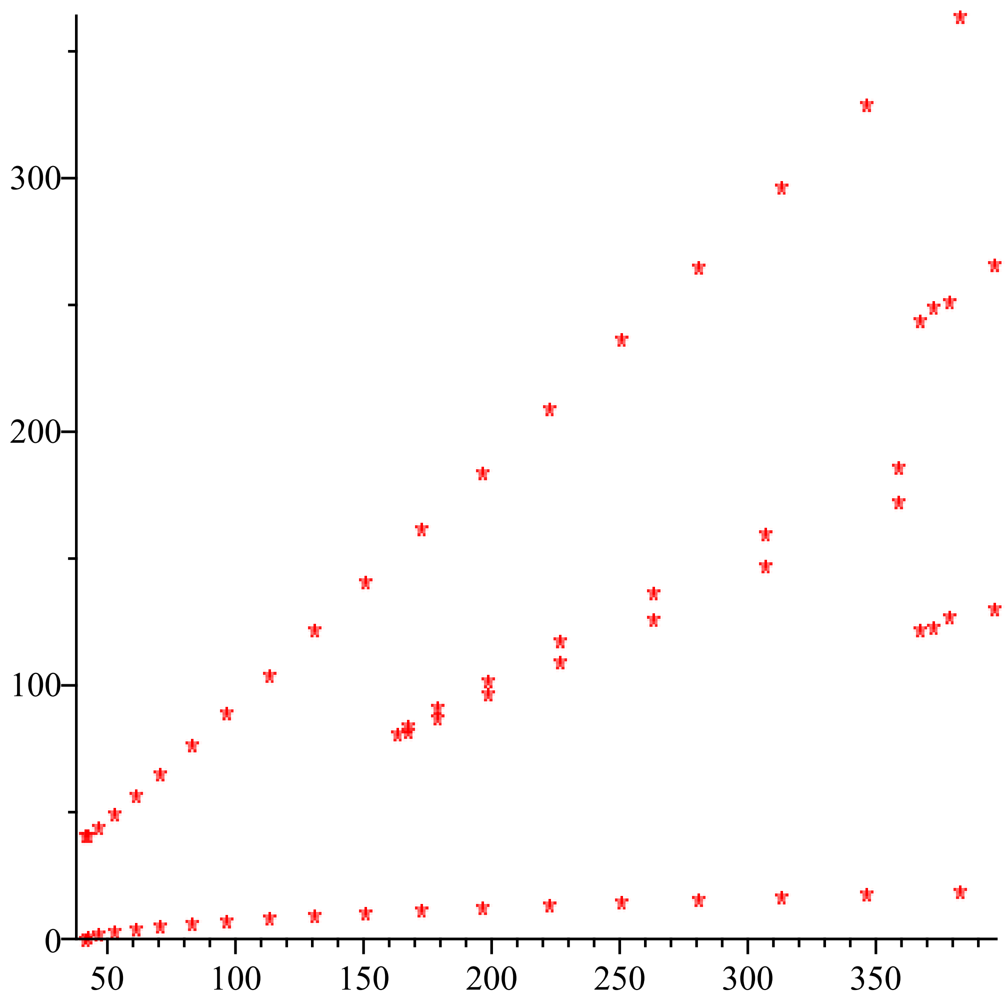
> #Matt C. Anderson

> #1 −19 −2016

>

```
> x := Vector(61) :
  y := Vector(61) :
  counter := 1 :
  for a from 2 to 400 do
   for b from 0 to a - 1 do
   if mod(b^2 + b + 41, a) = 0 then x[counter] := a : y[counter] := b : counter := counter + 1;
      end if;
   end do:
   end do:
```

```
> plot(x , y, style = point, symbol = asterisk)
```



> # *this is a graph of pairs (x,y) such that* $y^2 + y + 41$ **mod** $x = 0$.

> # *there is a point if* $y^2 + y + 41$ *is divisible* **by** x **and** *thus composite*

> # *this graph is if and only if. If h(n) is composite then there is a point on the graph. Also, if there is a point on the graph then h(n) is composite.*

> # *This page was coded in Maple.*

> #*Matt C. Anderson 12−14−2015*

>

> *# list of pairs (x,y) such that* $y^2 + y + 41 \ \mathbf{mod}\ x \equiv 0.$

```
> for a from 1 to 40 do
  x[a], y[a]
  end do;
```

41, 0
41, 40
43, 1
43, 41
47, 2
47, 44
53, 3
53, 49
61, 4
61, 56
71, 5
71, 65
83, 6
83, 76
97, 7
97, 89
113, 8
113, 104
131, 9
131, 121
151, 10
151, 140
163, 81
167, 82
167, 84
173, 11
173, 161
179, 87
179, 91
197, 12
197, 184
199, 96
199, 102
223, 13
223, 209
227, 109
227, 117
251, 14
251, 236

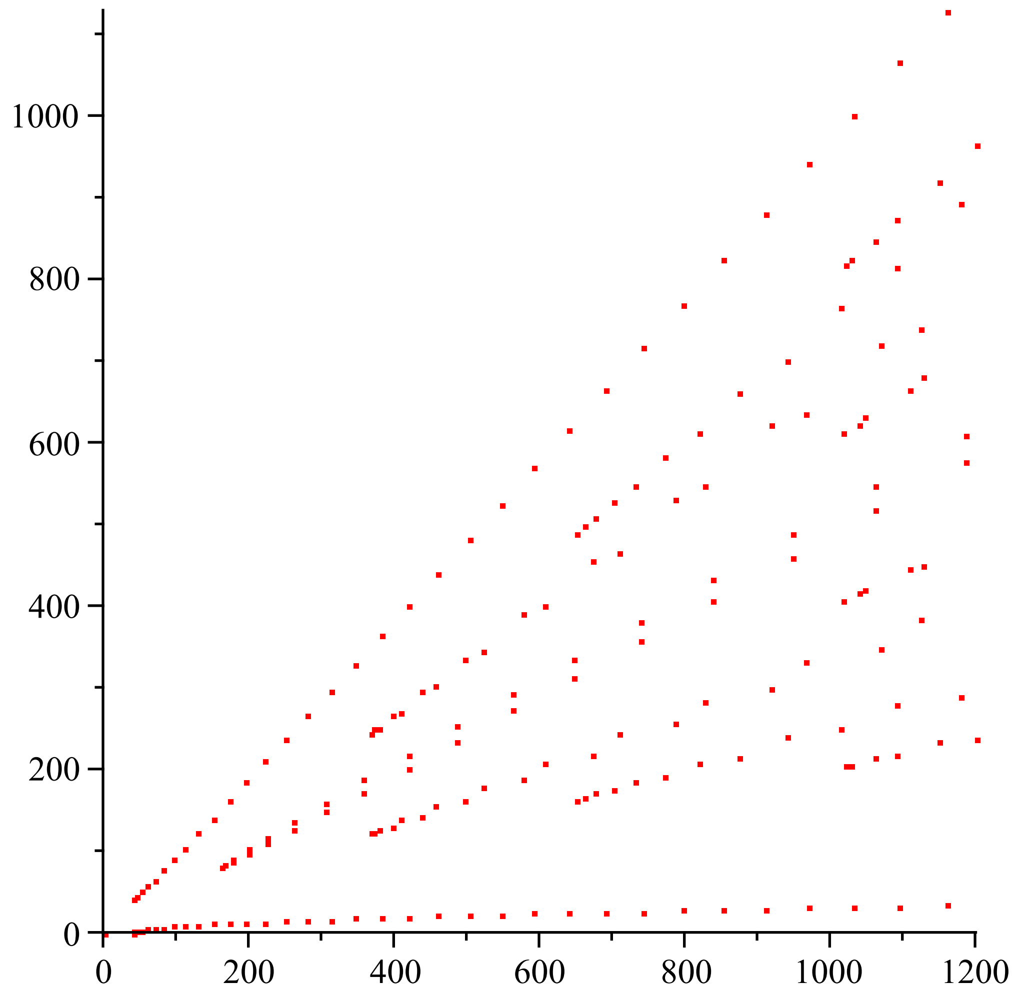
263, 126 (1)

>

```
> x := Vector(200) :
  y := Vector(200) :
  counter := 1 :
  for a from 1 to 200 do
  b := ithprime(a) :
  for c from 0 to b do
  if mod(c^2 + c + 41, b) = 0 then x[counter] := b : y[counter] := c : counter := counter + 1;
     end if;
  end do:
  end do:
> counter
```

187 **(1)**

```
> xs := x[1 ..186] :
> ys := y[1 ..186] :
> plot(x , y, style=point, symbol=point)
```




```
> # this is a graph of pairs (x,y) such that y^2 + y + 41 mod x = 0.
>
> # Matt Anderson 11-10-2013
>
```

```
> h := n^2 + n + 41 :
> f :=proc(y)
  description "factors the substitution of the eypression into n^2+n + 41";
  factor(y^2 + y + 41);
  end proc;
```

$$f := \mathbf{proc}(y)$$

description "factors the substitution of the eypression into n^2+n + 41";

factor(y^2 + y + 41)

end proc (1)

```
>
> # Small equation coeffieients doublecheck
>
> #The question I am attempting to answer in this project is — what integer values of n cause
    h(n) to be a composite number, and by extention, when is h(n) prime.
> # r is for row and c is for column.  So y[r,c] is a composition of functions h( y[r,c]).
> # when y[r,c] is carefully chosen, it makes y[r,c]  algebraically.  This means that y[r,c] is the
    product of two integers, neither of which is 1 or −1, and thus y[r, c] is composite
> # I am pretty sure that any n below a threshold lies on one of the lines described by the
    expressions below.
>
>
> y[1, 1] := z:
  x[1, 1] := f(%);
```

$$x_{1,\,1} := z^2 + z + 41 \tag{2}$$

```
> y[1, 2] := z^2 + 40 :
  x[1, 2] := f(%);
```

$$x_{1,\,2} := (z^2 + z + 41)\,(z^2 - z + 41) \tag{3}$$

```
> y[2, 1] := 2 z^2 + z + 81 :
  x[2, 1] := f(%);
```

$$x_{2,\,1} := (4\,z^2 + 163)\,(z^2 + z + 41) \tag{4}$$

```
> y[3, 1] := 3 z^2 + 2 z + 122 :
  x[3, 1] := f(%);
```

$$x_{3,\,1} := (z^2 + z + 41)\,(9\,z^2 + 3\,z + 367) \tag{5}$$

```
> y[3, 2] := 6 z^2 + z + 244 :
  x[3, 2] := f(%);
```

$$x_{3,\,2} := (4\,z^2 + 163)\,(9\,z^2 + 3\,z + 367) \tag{6}$$

```
> y[4, 1] := 4 z^2 + 3 z + 163 :
  x[4, 1] := f(%);
```

$$x_{4,\,1} := (16\,z^2 + 8\,z + 653)\,(z^2 + z + 41) \tag{7}$$

```
> y[4, 3] := 12 z^2 + 5 z + 489 :
  x[4, 3] := f(%);
```

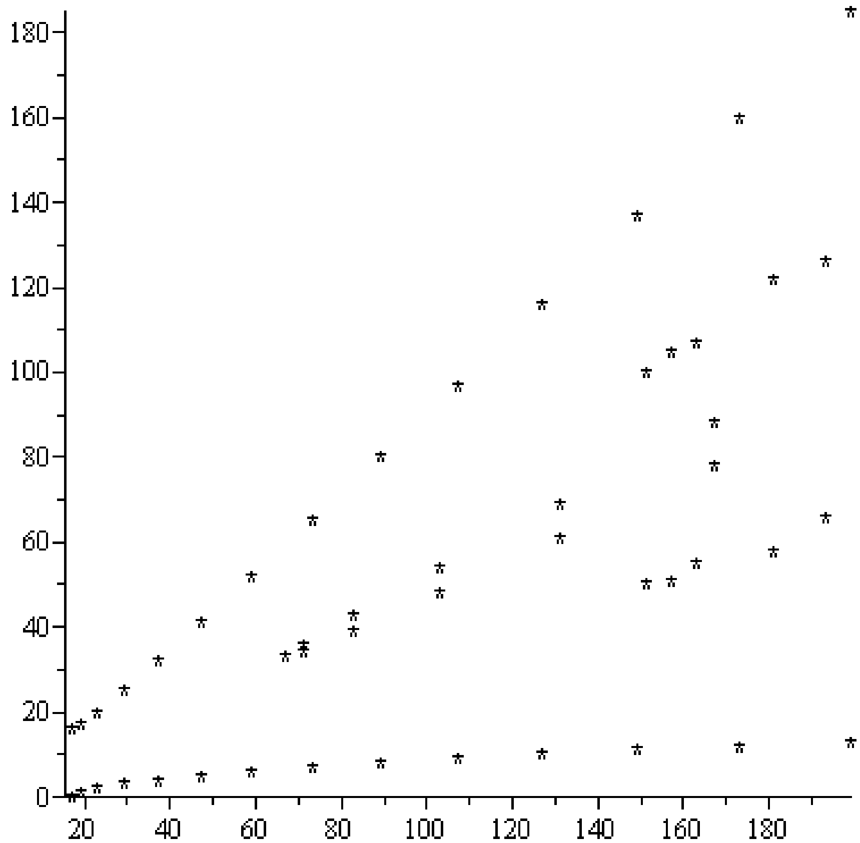
(8)

Analysis of the trinomial f(n) = $n^2 + n + 17$.

Abstract – Assuming that n is an non-negative integer, we find a pattern of when f(n) = $n^2 + n + 17$ is a composite number. We assign n as n = $A*x^2+B*x+C$. Where A, B, and C are determined by numerical evidence. The f(n) factors algebraically, and f(n) is a composite number.

We use the Maple program to calculate the values of ‘n’ where f(n) is a composite number. Then we graph these results. The graph shows some structure for the composite cases. See Maple code.

```
> # 6 −29 −2023
>
  x := Vector[row](49) :
  y := Vector[row](49) :
  counter := 1 :
  for a from 2 to 200 do
  for b from 0 to a − 1 do
  if mod(b^2 + b + 17, a) = 0
    then x[counter] := a : y[counter] := b : counter := counter + 1;
  end if;
  end do:
  end do:

> counter

> plot(x, y, style = point, symbol = asterisk, color = black )
```



> # *this is a graph of 49 data points of* $y^2 + y + 17 \ \mathbf{mod}\ x = 0$.

> # *It can be curve fit with parabolas.*

> # *This graph shows 5 parabolas*

> # *The names of the parabolas are* p_{top}; p_{bottom}; $p_{2,1}$; $p_{3,2}$; **and** $p_{3,1}$

.

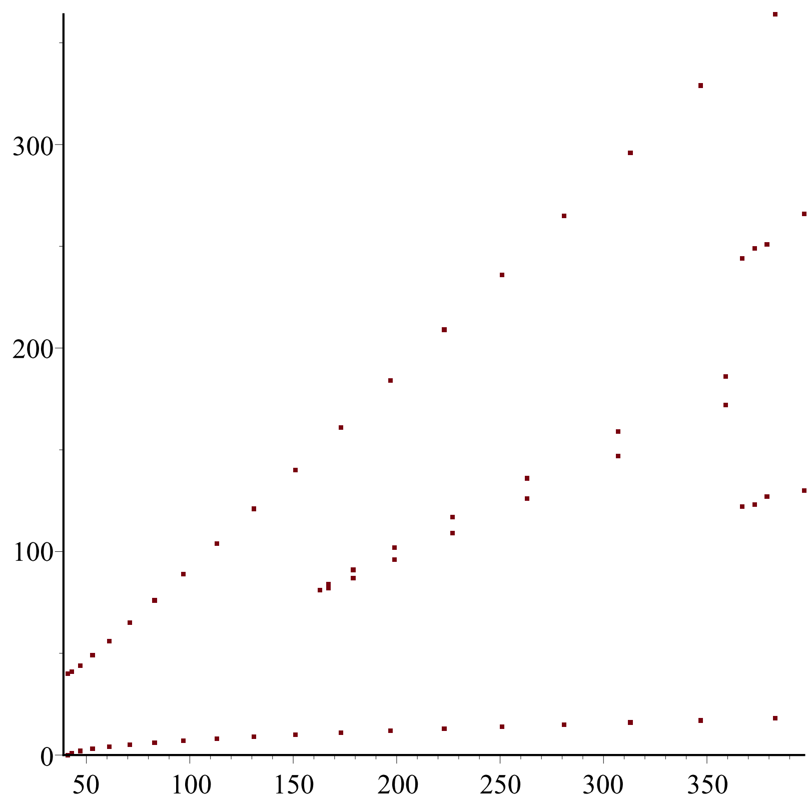
>

Hope you find this page interesting.

```
> # 3-2-2019
> # prime producing polynomial
> # h:=n^2 + n + 41 :
> x := Vector[row](100) :
> y := Vector[row](100) :
> counter := 1 :
> for a from 2 to 500 do
   for b from 0 to a − 1 do
   if mod(b^2 + b + 41, a) = 0 then
   x[counter] := a :
   y[counter] := b :
   counter := counter + 1 :
   end if;
   end do:
   end do:
Error, Vector index out of range
> counter
```

101 **(1)**

```
> plot(x[1 ..92], y[1 ..92], style=point, symbol=point)
```



```
> save x, "xdat41.txt"
> save y, "ydat41.txt"
>
```

```
> h := n^2 + n + 41 :
> f :=proc(y)
  description "factors the substitution of the eypression into n^2+n + 41";
  factor(y^2 + y + 41);
  end proc;
```

$f :=$ **proc**(y) **(1)**

description "factors the substitution of the eypression into n^2+n + 41";

factor$(y\hat{}2 + y + 41)$

end proc

```
>
> # Small equation coeffieients doublecheck
>
> #The question I am attempting to answer in this project is - what integer values of n cause
      h(n) to be a composite number, and by extention, when is h(n) prime.
> # r is for row and c is for column.  So y[r,c] is a composition of functions h( y[r,c]).
> # when y[r,c] is carefully chosen, it makes y[r,c]  algebraically.  This means that y[r,c] is the
      product of two integers, neither of which is 1 or -1, and thus y[r, c] is composite
> # I am pretty sure that any n below a threshold lies on one of the lines described by the
      expressions below.
>
>
> y[1, 1] := z:
  x[1, 1] := f(%);
```

$$x_{1,1} := z^2 + z + 41 \quad \textbf{(2)}$$

```
> y[1, 2] := z^2 + 40 :
  x[1, 2] := f(%);
```

$$x_{1,2} := (z^2 + z + 41)\,(z^2 - z + 41) \quad \textbf{(3)}$$

```
> y[2, 1] := 2 z^2 + z + 81 :
  x[2, 1] := f(%);
```

$$x_{2,1} := (4z^2 + 163)\,(z^2 + z + 41) \quad \textbf{(4)}$$

```
> y[3, 1] := 3 z^2 + 2 z + 122 :
  x[3, 1] := f(%);
```

$$x_{3,1} := (z^2 + z + 41)\,(9z^2 + 3z + 367) \quad \textbf{(5)}$$

```
> y[3, 2] := 6 z^2 + z + 244 :
  x[3, 2] := f(%);
```

$$x_{3,2} := (4z^2 + 163)\,(9z^2 + 3z + 367) \quad \textbf{(6)}$$

```
> y[4, 1] := 4 z^2 + 3 z + 163 :
  x[4, 1] := f(%);
```

$$x_{4,1} := (16z^2 + 8z + 653)\,(z^2 + z + 41) \quad \textbf{(7)}$$

```
> y[4, 3] := 12 z^2 + 5 z + 489 :
  x[4, 3] := f(%);
```

(8)

$$x_{4,\,3} := (16\,z^2 + 8\,z + 653)\,(9\,z^2 + 3\,z + 367) \quad \textbf{(8)}$$

> $y[5, 1] := 5\,z^2 + 4\,z + 204:$
> $x[5, 1] := f(\%);$

$$x_{5,\,1} := (z^2 + z + 41)\,(25\,z^2 + 15\,z + 1021) \quad \textbf{(9)}$$

> $y[5, 2] := 10\,z^2 + z + 407:$
> $x[5, 2] := f(\%);$

$$x_{5,\,2} := (4\,z^2 + 163)\,(25\,z^2 + 5\,z + 1019) \quad \textbf{(10)}$$

> $y[5, 3] := 15\,z^2 + 4\,z + 611:$
> $x[5, 3] := f(\%);$

$$x_{5,\,3} := (25\,z^2 + 5\,z + 1019)\,(9\,z^2 + 3\,z + 367) \quad \textbf{(11)}$$

> $y[5, 4] := 20\,z^2 + 11\,z + 816:$
> $x[5, 4] := f(\%);$

$$x_{5,\,4} := (16\,z^2 + 8\,z + 653)\,(25\,z^2 + 15\,z + 1021) \quad \textbf{(12)}$$

> $y[6, 1] := 6\,z^2 + 5\,z + 245:$
> $x[6, 1] := f(\%);$

$$x_{6,\,1} := (z^2 + z + 41)\,(36\,z^2 + 24\,z + 1471) \quad \textbf{(13)}$$

> $y[6, 5] := 30\,z^2 + 19\,z + 1225:$
> $x[6, 5] := f(\%);$

$$x_{6,\,5} := (36\,z^2 + 24\,z + 1471)\,(25\,z^2 + 15\,z + 1021) \quad \textbf{(14)}$$

> $y[7, 1] := 7\,z^2 + 6\,z + 286:$
> $x[7, 1] := f(\%);$

$$x_{7,\,1} := (z^2 + z + 41)\,(49\,z^2 + 35\,z + 2003) \quad \textbf{(15)}$$

> $y[7, 2] := 14\,z^2 + z + 570:$
> $x[7, 2] := f(\%);$

$$x_{7,\,2} := (4\,z^2 + 163)\,(49\,z^2 + 7\,z + 1997) \quad \textbf{(16)}$$

> $y[7, 3] := 21\,z^2 + 8\,z + 856:$
> $x[7, 3] := f(\%);$

$$x_{7,\,3} := (9\,z^2 + 3\,z + 367)\,(49\,z^2 + 21\,z + 1999) \quad \textbf{(17)}$$

> $y[7, 4] := 28\,z^2 + 13\,z + 1142:$
> $x[7, 4] := f(\%);$

$$x_{7,\,4} := (49\,z^2 + 21\,z + 1999)\,(16\,z^2 + 8\,z + 653) \quad \textbf{(18)}$$

> $y[7, 5] := 35\,z^2 + 6\,z + 1426:$
> $x[7, 5] := f(\%);$

$$x_{7,\,5} := (25\,z^2 + 5\,z + 1019)\,(49\,z^2 + 7\,z + 1997) \quad \textbf{(19)}$$

> $y[7, 6] := 42\,z^2 + 29\,z + 1716:$
> $x[7, 6] := f(\%);$

$$x_{7,\,6} := (49\,z^2 + 35\,z + 2003)\,(36\,z^2 + 24\,z + 1471) \quad \textbf{(20)}$$

>

> $y[8,1] := 8z^2 + 7z + 327:$
$x[8,1] := f(\%);$

$$x_{8,1} := (64z^2 + 48z + 2617)(z^2 + z + 41) \quad \textbf{(21)}$$

> $y[8,3] := 24z^2 + 7z + 978:$
$x[8,3] := f(\%);$

$$x_{8,3} := (64z^2 + 16z + 2609)(9z^2 + 3z + 367) \quad \textbf{(22)}$$

> $y[8,5] := 40z^2 + 9z + 1630:$
$x[8,5] := f(\%);$

$$x_{8,5} := (64z^2 + 16z + 2609)(25z^2 + 5z + 1019) \quad \textbf{(23)}$$

> $y[8,7] := 56z^2 + 41z + 2289:$
$x[8,7] := f(\%);$

$$x_{8,7} := (49z^2 + 35z + 2003)(64z^2 + 48z + 2617) \quad \textbf{(24)}$$

>

> $y[9,1] := 9z^2 + 8z + 368:$
$x[9,1] := f(\%);$

$$x_{9,1} := (z^2 + z + 41)(81z^2 + 63z + 3313) \quad \textbf{(25)}$$

> $y[9,2] := 18z^2 + z + 733:$
$x[9,2] := f(\%);$

$$x_{9,2} := (81z^2 + 9z + 3301)(4z^2 + 163) \quad \textbf{(26)}$$

> $y[9,4] := 36z^2 + 19z + 1469:$
$x[9,4] := f(\%);$

$$x_{9,4} := (16z^2 + 8z + 653)(81z^2 + 45z + 3307) \quad \textbf{(27)}$$

> $y[9,5] := 45z^2 + 26z + 1837:$
$x[9,5] := f(\%);$

$$x_{9,5} := (81z^2 + 45z + 3307)(25z^2 + 15z + 1021) \quad \textbf{(28)}$$

> $y[9,7] := 63z^2 + 8z + 2567:$
$x[9,7] := f(\%);$

$$x_{9,7} := (81z^2 + 9z + 3301)(49z^2 + 7z + 1997) \quad \textbf{(29)}$$

> $y[9,8] := 72z^2 + 55z + 2944:$
$x[9,8] := f(\%);$

$$x_{9,8} := (81z^2 + 63z + 3313)(64z^2 + 48z + 2617) \quad \textbf{(30)}$$

>

> $y[10,1] := 10z^2 + 9z + 409:$
$x[10,1] := f(\%);$

$$x_{10,1} := (100z^2 + 80z + 4091)(z^2 + z + 41) \quad \textbf{(31)}$$

> $y[10,3] := 30z^2 + 11z + 1223:$
$x[10,3] := f(\%);$

(32)

$$x_{10,\,3} := (9z^2+3z+367)\,(100z^2+40z+4079) \tag{32}$$

> y[10, 7] := 70 z^2 + 29 z + 2855 :
x[10, 7] := f(%);

$$x_{10,\,7} := (100z^2+40z+4079)\,(49z^2+21z+1999) \tag{33}$$

> y[10, 9] := 90 z^2 + 71 z + 3681 :
x[10, 9] := f(%);

$$x_{10,\,9} := (100z^2+80z+4091)\,(81z^2+63z+3313) \tag{34}$$

>

> y[11, 1] := 11 z^2 + 10 z + 450 :
x[11, 1] := f(%);

$$x_{11,\,1} := (z^2+z+41)\,(121z^2+99z+4951) \tag{35}$$

> y[11, 2] := 22 z^2 + z + 896 :
y[11, 2] := f(%);

$$y_{11,\,2} := (121z^2+11z+4931)\,(4z^2+163) \tag{36}$$

> y[11, 3] := 33 z^2 + 10 z + 1345 :
x[11, 3] := f(%);

$$x_{11,\,3} := (9z^2+3z+367)\,(121z^2+33z+4933) \tag{37}$$

> y[11, 4] := 44 z^2 + 21 z + 1795 :
x[11, 4] := f(%);

$$x_{11,\,4} := (16z^2+8z+653)\,(121z^2+55z+4937) \tag{38}$$

> y[11, 5] := 55 z^2 + 34 z + 2246 :
x[11, 5] := f(%);

$$x_{11,\,5} := (25z^2+15z+1021)\,(121z^2+77z+4943) \tag{39}$$

> y[11, 6] := 66 z^2 + 43 z + 2696 :
x[11, 6] := f(%);

$$x_{11,\,6} := (36z^2+24z+1471)\,(121z^2+77z+4943) \tag{40}$$

> y[11, 7] := 77 z^2 + 34 z + 3141 :
x[11, 7] := f(%);

$$x_{11,\,7} := (121z^2+55z+4937)\,(49z^2+21z+1999) \tag{41}$$

> y[11, 8] := 88 z^2 + 23 z + 3587 :
x[11, 8] := f(%);

$$x_{11,\,8} := (64z^2+16z+2609)\,(121z^2+33z+4933) \tag{42}$$

> y[11, 9] := 99 z^2 + 10 z + 4034 :
x[11, 9] := f(%);

$$x_{11,\,9} := (121z^2+11z+4931)\,(81z^2+9z+3301) \tag{43}$$

> y[11, 10] := 110 z^2 + 89 z + 4500 :
x[11, 10] := f(%);

$$x_{11,\,10} := (121z^2+99z+4951)\,(100z^2+80z+4091) \tag{44}$$

>

> $y[12, 1] := 12\,z^2 + 11\,z + 491:$
$x[12, 1] := f(\%);$

$$x_{12,\,1} := \left(z^2 + z + 41\right)\left(144\,z^2 + 120\,z + 5893\right) \tag{45}$$

> $y[12, 5] := 60\,z^2 + 11\,z + 2445:$
$x[12, 5] := f(\%);$

$$x_{12,\,5} := \left(25\,z^2 + 5\,z + 1019\right)\left(144\,z^2 + 24\,z + 5869\right) \tag{46}$$

> $y[12, 7] := 84\,z^2 + 13\,z + 3423:$
$x[12, 7] := f(\%);$

$$x_{12,\,7} := \left(144\,z^2 + 24\,z + 5869\right)\left(49\,z^2 + 7\,z + 1997\right) \tag{47}$$

> $y[12, 11] := 132\,z^2 + 109\,z + 5401:$
$x[12, 11] := f(\%);$

$$x_{12,\,11} := \left(121\,z^2 + 99\,z + 4951\right)\left(144\,z^2 + 120\,z + 5893\right) \tag{48}$$

>

> $y[13, 1] := 13\,z^2 + 12\,z + 532:$
$x[13, 1] := f(\%);$

$$x_{13,\,1} := \left(169\,z^2 + 143\,z + 6917\right)\left(z^2 + z + 41\right) \tag{49}$$

> $y[13, 2] := 26\,z^2 + z + 1059:$
$x[13, 2] := f(\%);$

$$x_{13,\,2} := \left(169\,z^2 + 13\,z + 6887\right)\left(4\,z^2 + 163\right) \tag{50}$$

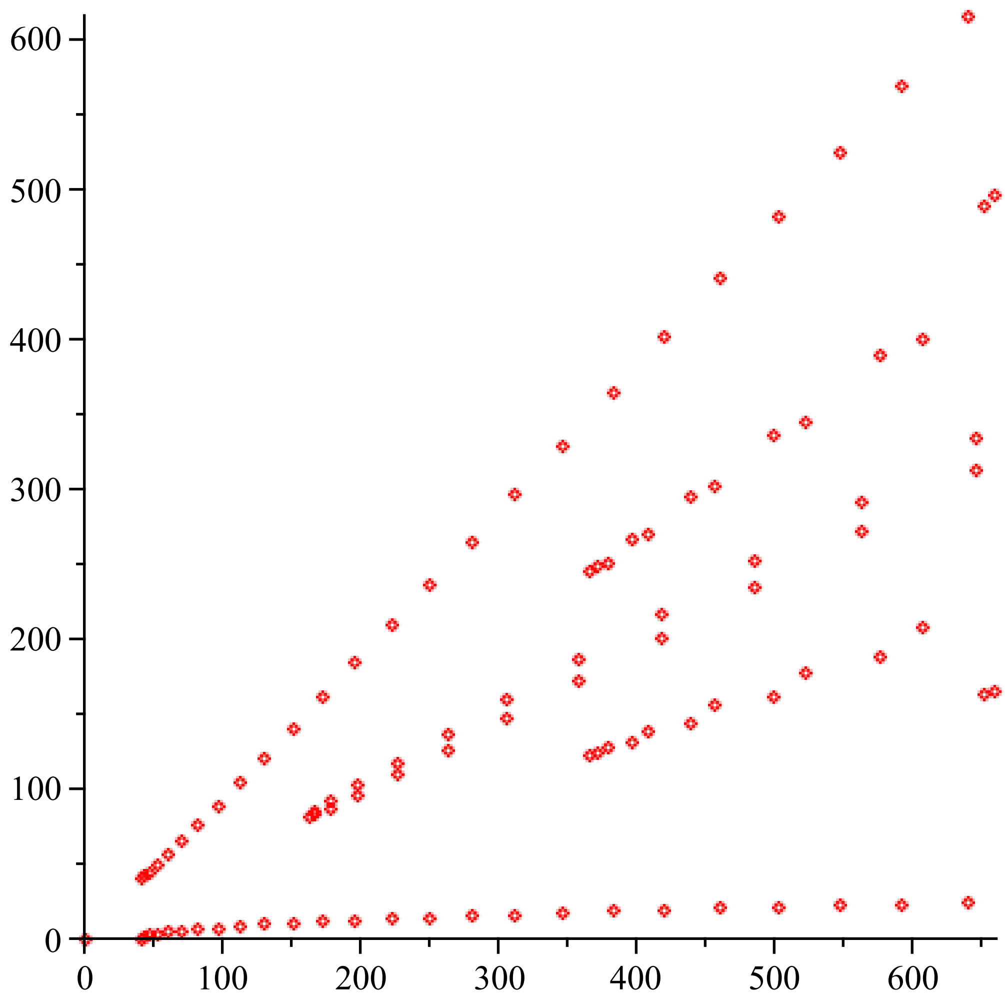
>

> *# 11 −1 −2016 M. A.*

```
> restart
> x := Vector(100) :
> y := Vector(100) :
> counter := 1 :
> for a from 2 to 672 do
   for b from 0 to a − 1 do
   h := b^2 + b + 41 :
   if mod(h, a) = 0 then x[counter] := a : y[counter] := b : counter := counter + 1;
   end if;
   end do:
   end do:
> counter
```

$$100 \quad (1)$$

```
> # this example was cooked.  The number 672 was chosen so that xy would exactly fill.
> # there are 100 rows and 2 columns in the xy matrix.
> plot(x, y, style=point);
```




```
> # This plot is 100 points of cases where (y^2 + y + 41) mod x ≡ 0.
> # In other words h(y) is divisible by x.
> # now the dataset.
```

```
> ('y_vector before x_vector');
for c from 1 to 30 do
print(y[c], x[c]);
end do;
```

y_{vector} *before* x_{vector}

0, 41
40, 41
1, 43
41, 43
2, 47
44, 47
3, 53
49, 53
4, 61
56, 61
5, 71
65, 71
6, 83
76, 83
7, 97
89, 97
8, 113
104, 113
9, 131
121, 131
10, 151
140, 151
81, 163
82, 167
84, 167
11, 173
161, 173
87, 179
91, 179
12, 197 **(2)**

```
> # for example h(0) is divisible by 41.  Also h(40) is divisble by 41.  etc...
>
```

Residue Table

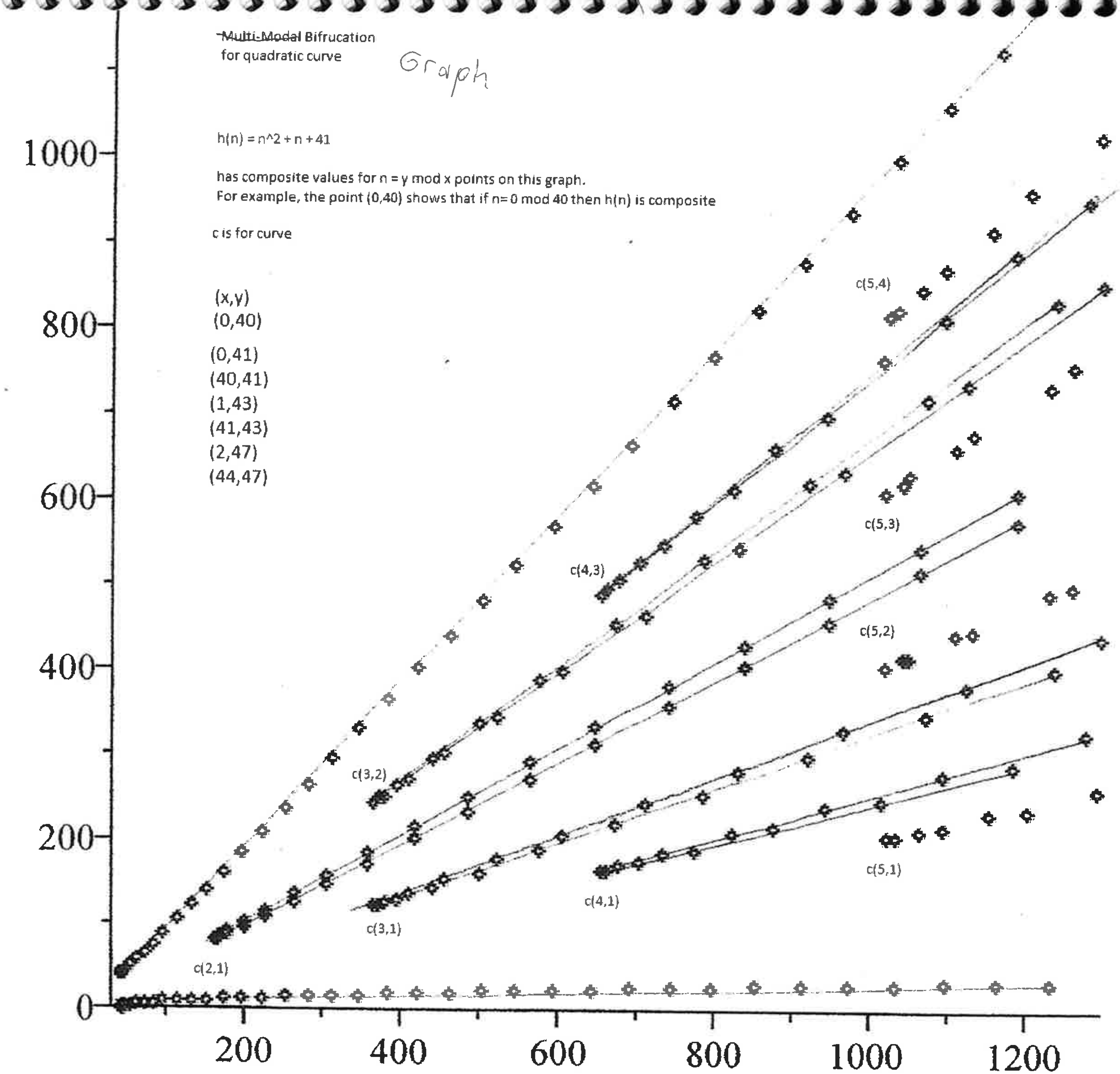
	2	3	5	7	11	13	17	19	23	29	31	37	41	43
0	1	2	1	6	8	2	7	3	18	12	10	4	0	41
1	1	1	3	1	10	4	9	5	20	14	12	6	2	0
2		2	2	5	3	8	13	9	1	18	16	10	6	4
3			3	4	9	1	2	15	7	24	22	16	12	10
4			1	5	6	9	10	4	15	3	30	24	20	18
5				1	5	6	3	14	2	13	9	34	30	28
6				6	6	5	15	7	14	25	21	9	1	40
7					9	6	12	2	5	10	4	23	15	11
8					3	9	11	18	21	26	20	2	31	27
9					10	1	12	17	16	15	7	20	8	2
10					8	8	15	18	13	6	27	3	28	22
11						4	3	2	12	28	18	25	9	1
12						2	10	7	13	23	11	12	33	25
13							2	14	16	20	6	1	18	8
14							13	4	21	19	3	29	5	36
15							9	15	5	20	2	22	35	23
16							7	9	14	23	3	17	26	12
17								5	2	28	6	14	19	3
18								3	15	6	11	13	14	39
19									7	15	18	14	11	34
20									1	26	27	17	10	31
21									20	10	7	22	11	30
22									18	25	20	29	14	31
23										13	4	1	19	34
24										3	21	12	26	39
25										24	9	25	35	3
26										18	30	3	5	12
27										14	22	20	18	23
28										12	16	2	33	36
29											12	23	9	8
30											10	9	28	25
31												34	8	1
32												24	31	22
33												16	15	2
34												10	1	27
35												6	30	11
36												4	20	40
37													12	28
38													6	18
39													2	10
40													0	4
41														0
42														41

Explaination of Residue Table
column index, C are across the top
row index, R are found along the side

table values are calculated by
R^2 + R + 41 mod C

Notice that columns
with 41 and 43 contain 0 twice.

These 0 values become points in the
graph of discrete divisors.


~~Multi-Modal~~ Bifrucation
for quadratic curve
Graph
h(n) = n^2 + n + 41
has composite values for n = y mod x points on this graph.
For example, the point (0,40) shows that if n= 0 mod 40 then h(n) is composite
c is for curve
(x,y)
(0,40)
(0,41)
(40,41)
(1,43)
(41,43)
(2,47)
(44,47)
c(5,4)
c(5,3)
c(4,3)
c(5,2)
c(3,2)
c(5,1)
c(4,1)
c(3,1)
c(2,1)
1000
800
600
400
200
0
200
400
600
800
1000
1200

Citations

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10) Weisstein, Eric W. "Landau's Problems." From Mathworld - A Wolfram Web Resource. http://mathworld.wolfram.com/LandausProblems.html see problem #4