We have a Fibonacci Derivation

We are aware that

1) $F(n) = F(n-1) + F(n-2)$ where $F(1) = 1$ and $F(2) = 1$.

This implies

2) $F(n-1) = F(n-2) + F(n-3)$.

Substituting 2) into 1) gives

3) $F(n) = 2*F(n-2) + F(n-3)$

Similarly,

4) $F(n-2) = F(n-3) + F(n-4)$. So
5) $F(n) = 3*F(n-3) + 2*F(n-4)$. So
6) $F(n-3) = F(n-4) + F(n-5)$. So
7) $F(n) = 5*F(n-4) + 3*F(n-5)$.

A pattern can be noticed here. For example

8) $F(n) = F(5)*F(n-4) + F(4)*F(n-5)$

Change n to m

9) $F(m) = F(a)*F(m-a+1) + F(a-1)*F(m-a)$.

Now m=2m

We have

10) $F(2*n) = F(a)*F(2*n-a+1)+F(a-1)*F(2*n-1)$.

Where a=n. Finally

11) $F(2n)=F(n)*F(n+1) + F(n-1)*F(n)$

Extra 12) $F(2n) = F(n)*[F(n+1) + F(n-1)]$

End Derivation.