An exploration of a special polynomial

Namely, $x^2 + x + 41$

By Matthew Charles Anderson

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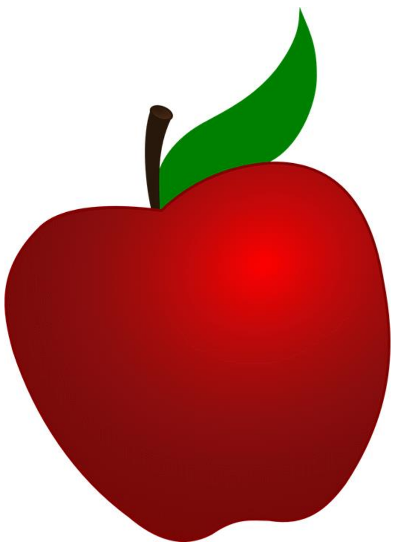
2025



Any set of prime numbers, with count ‘k’, is a k-tuple. There are no restrictions.

A prime producing polynomial.

Observations on the trinomial $n^2 + n + 41$.

by Matt C. Anderson

August 2016

The story so far

We assume that n is an integer. We focus our attention on the polynomial n^2 + n + 41.

Further, we analyze the behavior of the factorization of integers of the form

h(n) = n^2 + n + 41 (expression 1)

where n is a non-negative integer. It was shown by Legendre, in 1798 that if $0 \le n < 40$ then h(n) is a prime number.

Certain patterns become evident when considering points (a,n) where

$h(n) \equiv 0 \bmod a.$ (expression 2)

The collection of all such point produces what we are calling a "graph of discrete divisors" due to certain self-similar features. From experimental data we find that the integer points in this bifurcation graph lie on a collection of parabolic curves indexed by pairs of relatively prime integers. The expression for the middle parabolas is –

$p(r,c) = (c*x - r*y)^2 - r*(c*x - r*y) - x + 41*r^2.$ (expression 3)

The restrictions are that 0<r<c and gcd(r,c) = 1 and all four of r,c,x, and y are integers.

Each such pair (r,c) yields (again determined experimentally and by observation of calculations) an integer polynomial a*z2 + b*z + c, and the quartic h(a*z2 + b*z + c) then factors non-trivially over the integers into two quadratic expressions. We call this our "parabola conjecture". Certain symmetries in the bifurcation graph are due to elementary relationships between pairs of co-prime integers. For instance if m<n are co-prime integers, then there is an observable relationship between the parabola it determines that that formed from (n-m, n).

We conjecture that all composite values of h(n) arise by substituting integer values of z into $h(a*z^2 + b*z + c)$, where this quartic factors algebraically over **Z** for $a*z^2 + b*z + c$ a quadratic polynomial determined by a pair of relatively prime integers. We name this our "no stray points conjecture" because all the points in the bifurcation graph appear to lie on a parabola.

We further conjecture that the minimum x-values for parabolas corresponding to (r, c) with gcd(r, c) = 1 are equal for fixed n. Further, these minimum x-values line up at $163*c^2/4$ where c = 2, 3, 4, ... The numerical evidence seems to support this. This is called our "parabolas line up" conjecture.

The notation gcd(r, c) used above is defined here. The greatest common devisor of two integers is the smallest whole number that divides both of those integers.

Theorem 1 - Consider h(n) with n a non negative integer.
h(n) never has a factor less than 41.

We prove Theorem 1 with a modular construction. We make a residue table with all the prime factors less than 41. The fundamental theorem of arithmetic states that any integer greater than one is either a prime number, or can be written as a unique product of prime numbers (ignoring the order). So if h(n) never has a prime factor less than 41, then by extension it never has an integer factor less than 41.

For example, to determine that h(n) is never divisible by 2, note the first column of the residue table. If n is even, then h(n) is odd. Similarly, if n is odd then h(n) is also odd. In either case, h(n) does not have factorization by 2.

Also, for divisibility by 3, there are 3 cases to check. They are n = 0, 1, and 2 mod 3. h(0) mod 3 is 2. h(1) mod 3 is 1. and h(2) mod 3 is 2. Due to these three cases, h(n) is never divisible by 3. This is the second column of the residue table.

The number 0 is first found in the residue table for the cases h(0) mod 41 and h(40) mod 41. This means that if n is congruent to 0 mod 41 then h(n) will be divisible by 41. Similarly, if n is congruent to 40 mod 41 then h(n) is also divisible by 41.
After the residue table, we observe a bifurcation graph which has points when h(y) mod x is divisible by x. The points (x,y) can be seen on the bifurcation graph.

< insert residue table here >

Thus we have shown that h(n) never has a factor less than 41.

Theorem 2

Since $h(a) = a^2 + a + 41$, we want to show that h(a) = h(-a -1).

Proof of Theorem 2
Because h(a) = a*(a+1) + 41,
Now h(-a -1) = (-a -1)*(-a -1 +1) + 41.
So h(-a -1) = (-a -1)*(-a) +41,
And h(-a -1) = h(a).
Which was what we wanted.
End of proof of theorem 2.

Corrolary 1
Further, if h(b) mod c ≡ = then h(c –b -1) mod c ≡ 0.

We can observe interesting patterns in the “graph of discrete divisors” on a following page.

Residue Table

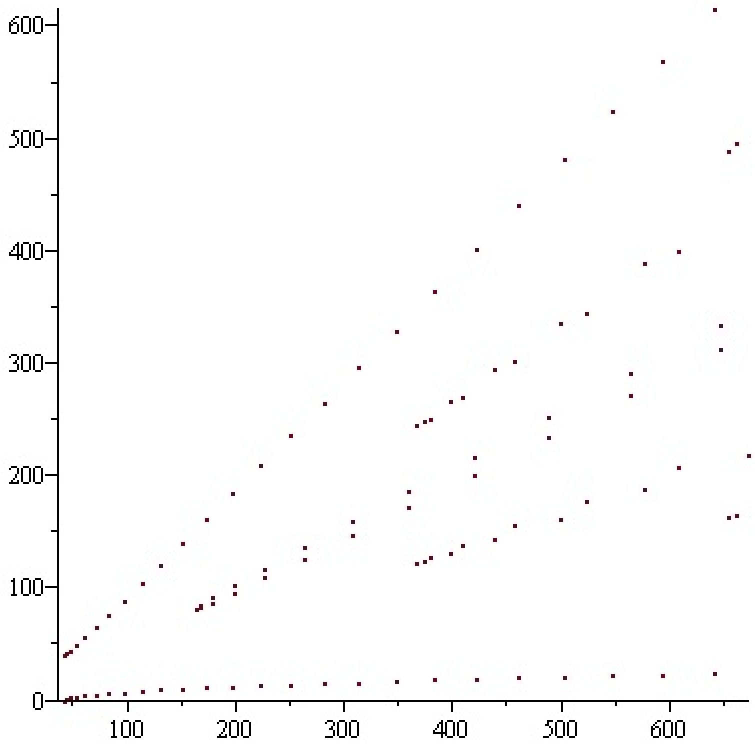
	2	3	5	7	11	13	17	19	23	29	31	37	41	43
0	1	2	1	6	8	2	7	3	18	12	10	4	0	41
1	1	1	3	1	10	4	9	5	20	14	12	6	2	0
2		2	2	5	3	8	13	9	1	18	16	10	6	4
3			3	4	9	1	2	15	7	24	22	16	12	10
4			1	5	6	9	10	4	15	3	30	24	20	18
5				1	5	6	3	14	2	13	9	34	30	28
6				6	6	5	15	7	14	25	21	9	1	40
7					9	6	12	2	5	10	4	23	15	11
8					3	9	11	18	21	26	20	2	31	27
9					10	1	12	17	16	15	7	20	8	2
10					8	8	15	18	13	6	27	3	28	22
11						4	3	2	12	28	18	25	9	1
12						2	10	7	13	23	11	12	33	25
13							2	14	16	20	6	1	18	8
14							13	4	21	19	3	29	5	36
15							9	15	5	20	2	22	35	23
16							7	9	14	23	3	17	26	12
17								5	2	28	6	14	19	3
18								3	15	6	11	13	14	39
19									7	15	18	14	11	34
20									1	26	27	17	10	31
21									20	10	7	22	11	30
22									18	25	20	29	14	31
23										13	4	1	19	34
24										3	21	12	26	39
25										24	9	25	35	3
26										18	30	3	5	12
27										14	22	20	18	23
28										12	16	2	33	36
29											12	23	9	8
30											10	9	28	25
31												34	8	1
32												24	31	22
33												16	15	2
34												10	1	27
35												6	30	11
36												4	20	40
37													12	28
38													6	18
39													2	10
40													0	4
41														0
42														41

Explaination of Residue Table
column index, C are across the top
row index, R are found along the side

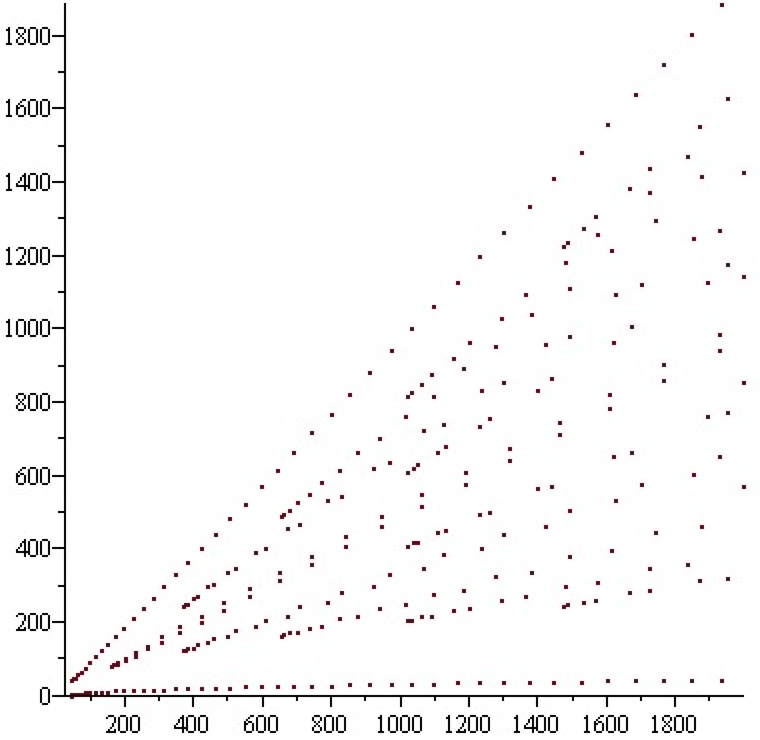
table values are calculated by
R^2 + R + 41 mod C

Notice that columns
with 41 and 43 contain 0 twice.

These 0 values become points in the
graph of discrete divisors.



600
500
400
300
200
100
0
100
200
300
400
500
600


1800
1600
1400
1200
1000
800
600
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200
0
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800
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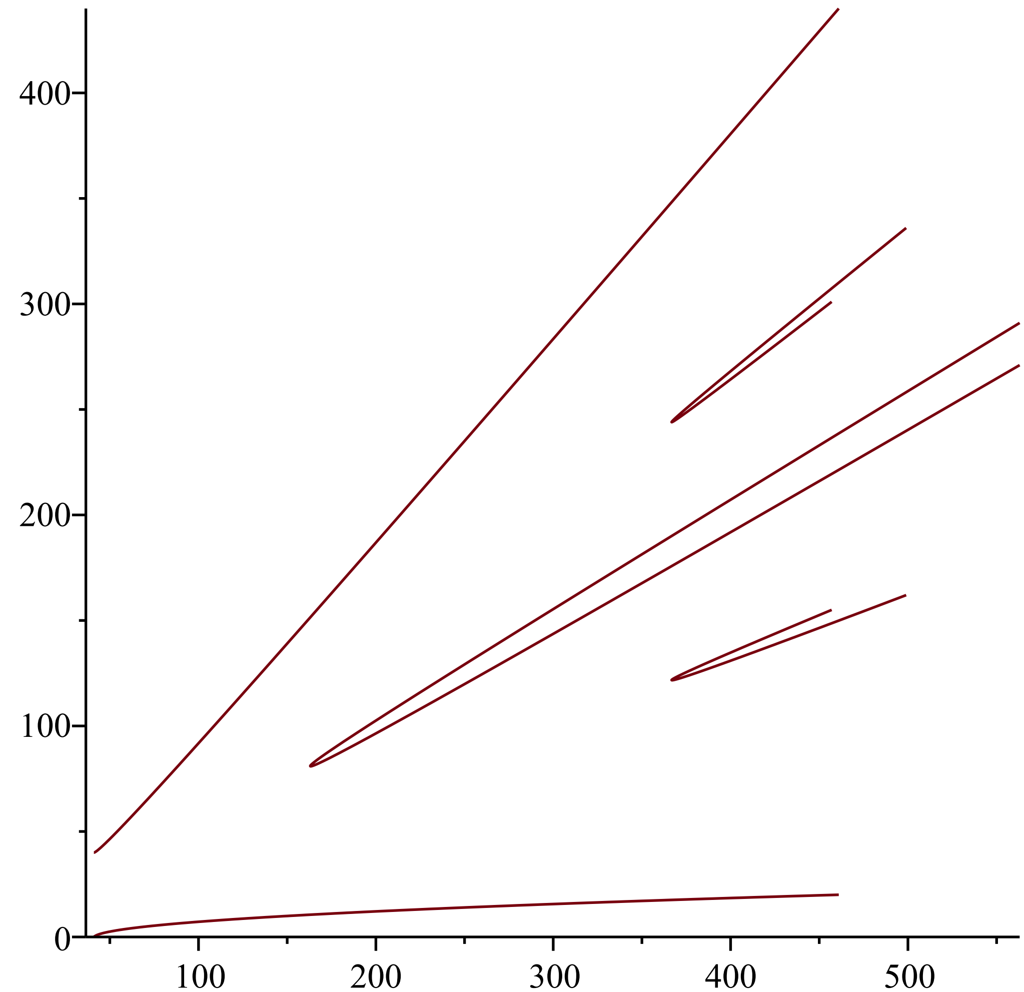
1600
1800

Exact Curve Fit

```
> # trinomials that curve fit the bifurcation graph.

> x[1, 1, bottom] := z^2 + z + 41 :
  y[1, 1] := z :
> p2 := plot([x[1, 1, bottom], y[1, 1], z = 0 ..20]);
                                   p2 := PLOT(...)                          (1)
> with(plots) :
> display(p2) :

> x[1, 1, top] := z^2 + z + 41 :
  y[1, 1, top] := z^2 + 40 :
> p3 := plot([x[1, 1, top], y[1, 1, top], z = 0 ..20]);
                                   p3 := PLOT(...)                          (2)
> display(p3) :
> # this is  correct

> y[2, 1] := 2 z^2 + z + 81 :
  x[2, 1] := 4 z^2 + 163 :
  p4 := plot([x[2, 1], y[2, 1], z =-10 ..10]);
  display(p4) :
                                   p4 := PLOT(...)                          (3)
> display([p2, p3, p4]) :
> # this multiple plot is correct.

> y[3, 1] := 3 z^2 + 2 z + 122 :
  x[3, 1] := 9 z^2 + 3 z + 367 :
> p5 := plot([x[3, 1], y[3, 1], z =-4 ..3])
                                   p5 := PLOT(...)                          (4)
> display(p5) :

> y[3, 2] := 6 z^2 + z + 244 :
  x[3, 2] := 9 z^2 + 3 z + 367 :
> p6 := plot([x[3, 2], y[3, 2], z =-4 ..3])
                                   p6 := PLOT(...)                          (5)
> display(p6) :
```

> *display*([*p2*, *p3*, *p4*, *p5*, *p6*])



> # I like this plot

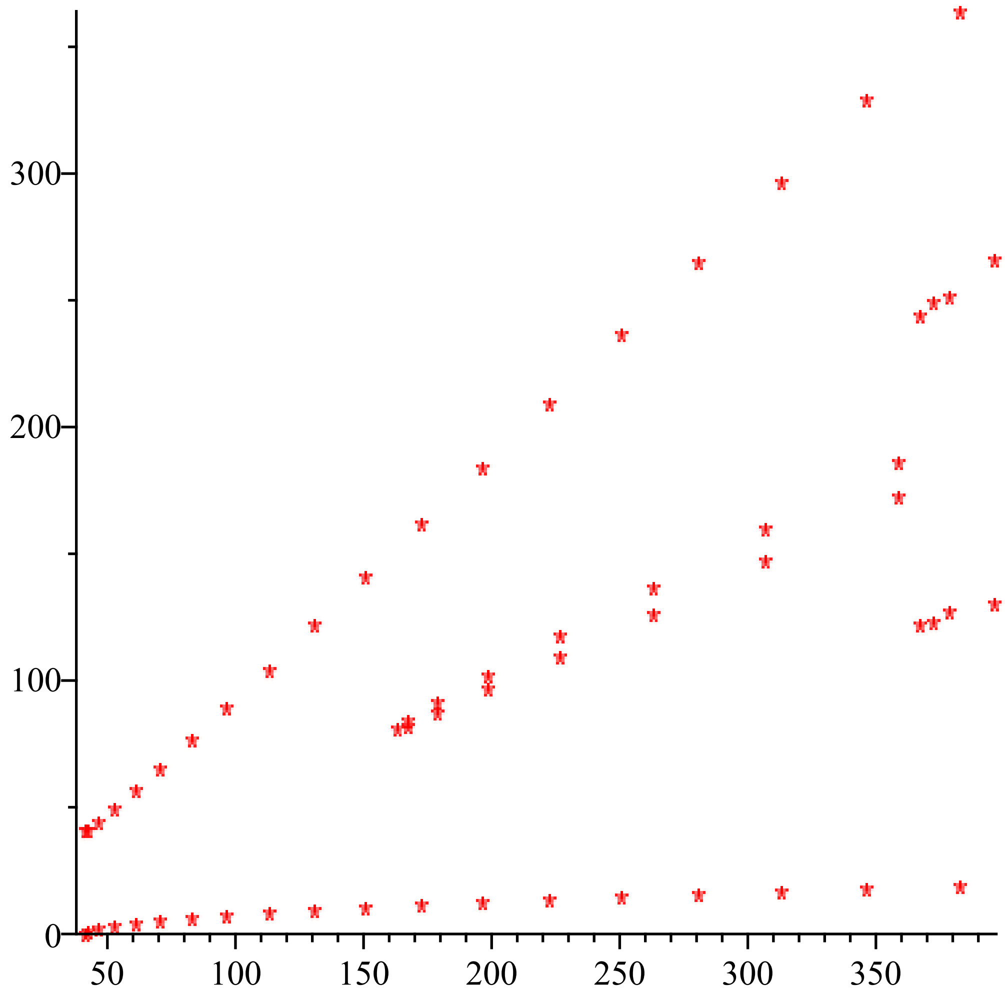
> #Matt C. Anderson

> #1 − 19 − 2016

>

```
> x := Vector(61) :
  y := Vector(61) :
  counter := 1 :
  for a from 2 to 400 do
   for b from 0 to a − 1 do
   if mod(b^2 + b + 41, a) = 0 then x[counter] := a : y[counter] := b : counter := counter + 1;
       end if;
   end do:
   end do:
```

```
> plot(x , y, style = point, symbol = asterisk)
```



> *# this is a graph of pairs (x,y) such that* $y^2 + y + 41$ **mod** $x = 0$.

> *# there is a point if* $y^2 + y + 41$ *is divisible* **by** x **and** *thus composite*

> *# this graph is if and only if. If h(n) is composite then there is a point on the graph. Also, if there is a point on the graph then h(n) is composite.*

> *# This page was coded in Maple.*

> *#Matt C. Anderson 12−14−2015*

>

> # *list of pairs (x,y) such that* $y^2 + y + 41$ **mod** $x \equiv 0$.

```
> for a from 1 to 40 do
  x[a], y[a]
  end do;
```

41, 0
41, 40
43, 1
43, 41
47, 2
47, 44
53, 3
53, 49
61, 4
61, 56
71, 5
71, 65
83, 6
83, 76
97, 7
97, 89
113, 8
113, 104
131, 9
131, 121
151, 10
151, 140
163, 81
167, 82
167, 84
173, 11
173, 161
179, 87
179, 91
197, 12
197, 184
199, 96
199, 102
223, 13
223, 209
227, 109
227, 117
251, 14
251, 236

$$263,\ 126 \tag{1}$$

>

```
> # the divisibility or bifurcation graph shows values (x,y) such that h(y) mod x is congurent to 0.
> h := n^2 + n + 41 :
> x := Vector[row](291) :
  y := Vector[row](291) :
> counter := 1 :
> for a from 2 to 2000 do
   for b from 0 to a - 1 do
   if mod(b^2 + b + 41, a) = 0 then
     x[counter] := a :
     y[counter] := b :
     counter := counter + 1 :
   end if;
   end do:
   end do:
> counter
```

292 **(1)**

```
> # note we read ... a from 2 to ... because we ignore divisibility by one.
>
> plot(x, y, style = point, symbol = point)
  plot(x[1 ..100], y[1 ..100], style = point, symbol = point)
```

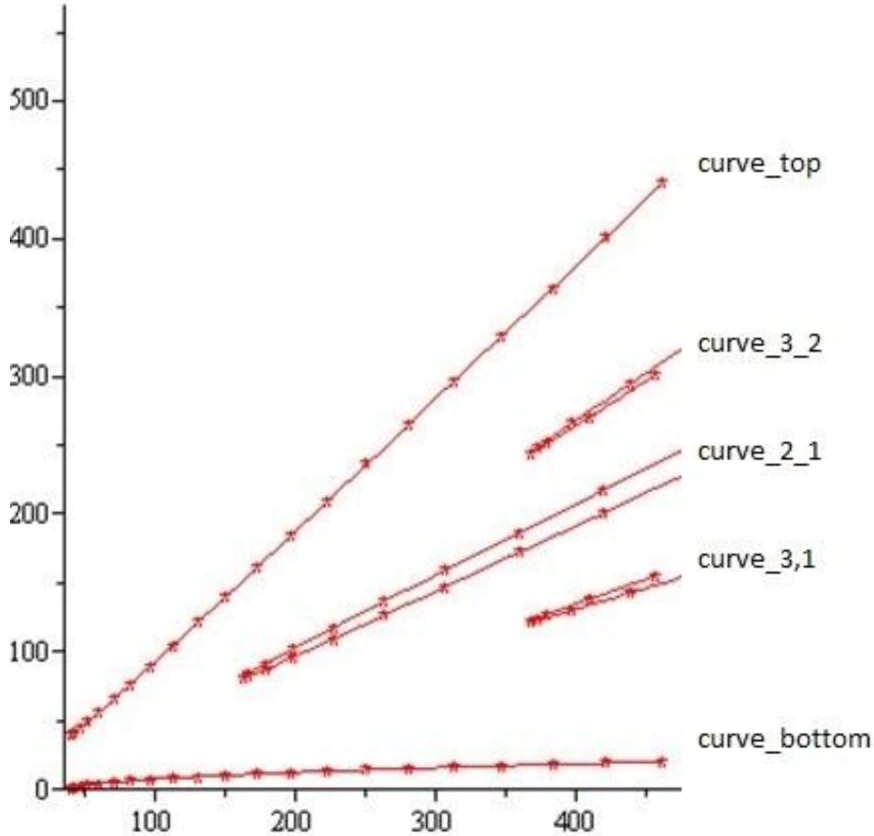
Prime Producing Polynomail project rehash

By Matt C. Anderson

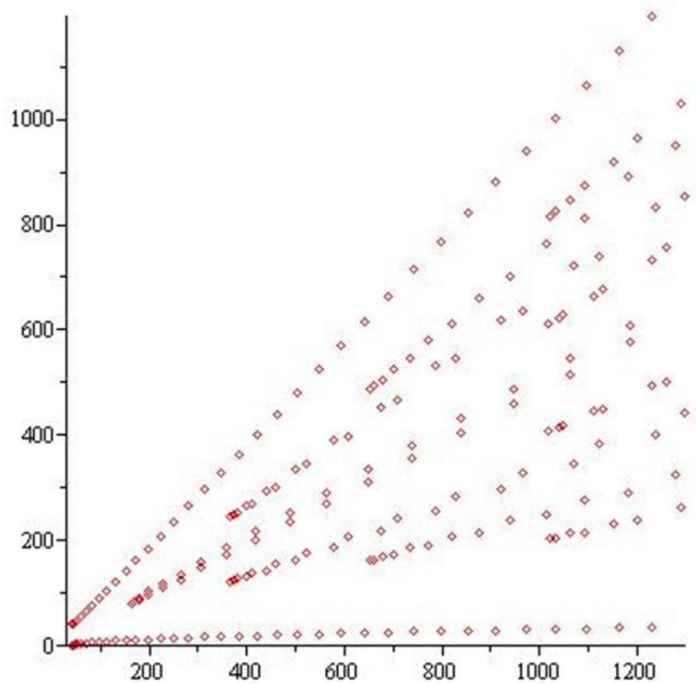
9/11/2016

We assume n is an integer. From before, $h(n) = n^2 + n + 41$. Our “graph of discrete divisors” shows values of y such that $0<y<x$ and h(y) mod x is congruent to 0. See graph.

The points on the graph can be connected by exact curve fit. The connecting curves are parabolas. We have defined a numbering system for each of the parabolas. All the parabolas are defined parametrically.



Curve_R_C is defined where R and C are integers and 0<C<R. Also gcd(R,C) = 1. That is to say, the row index and column index must be relatively prime.



Take this for what it’s worth.

Matt

> $h := n^2 + n + 41$:

> f :=**proc**(y)
description "factors the substitution of the eypression into n^2+n + 41";
$factor(y^2 + y + 41)$;
end proc;

f := **proc**(y) **(1)**
description "factors the substitution of the eypression into n^2+n + 41";
$factor(y\text{^}2 + y + 41)$
end proc

>

> *# Small equation coeffieients doublecheck*

>

> *#The question I am attempting to answer in this project is − what integer values of n cause h(n)* **to** *be a composite number,* **and by** *extention, when is h(n) prime.*

> *# r is for row and c is for column. So y[r,c] is a composition of functions h(y[r,c]).*

> *# when y[r,c] is carefully chosen, it makes y[r,c] algebraically. This means that y[r,c] is the product of two integers, neither of which is 1 or −1, and thus y[r, c] is composite*

> *# I am pretty sure that any n below a threshold lies on one of the lines described by the expressions below.*

>

>

> $y[1, 1] := z$:
$x[1, 1] := f(\%)$;

$$x_{1,\,1} := z^2 + z + 41 \quad \textbf{(2)}$$

> $y[1, 2] := z^2 + 40$:
$x[1, 2] := f(\%)$;

$$x_{1,\,2} := (z^2 + z + 41)\,(z^2 - z + 41) \quad \textbf{(3)}$$

> $y[2, 1] := 2\,z^2 + z + 81$:
$x[2, 1] := f(\%)$;

$$x_{2,\,1} := (4\,z^2 + 163)\,(z^2 + z + 41) \quad \textbf{(4)}$$

> $y[3, 1] := 3\,z^2 + 2\,z + 122$:
$x[3, 1] := f(\%)$;

$$x_{3,\,1} := (z^2 + z + 41)\,(9\,z^2 + 3\,z + 367) \quad \textbf{(5)}$$

> $y[3, 2] := 6\,z^2 + z + 244$:
$x[3, 2] := f(\%)$;

$$x_{3,\,2} := (4\,z^2 + 163)\,(9\,z^2 + 3\,z + 367) \quad \textbf{(6)}$$

> $y[4, 1] := 4\,z^2 + 3\,z + 163$:
$x[4, 1] := f(\%)$;

$$x_{4,\,1} := (16\,z^2 + 8\,z + 653)\,(z^2 + z + 41) \quad \textbf{(7)}$$

> $y[4, 3] := 12\,z^2 + 5\,z + 489$:
$x[4, 3] := f(\%)$;

(8)

$$x_{4,\,3} := (16\,z^2 + 8\,z + 653)\,(9\,z^2 + 3\,z + 367) \quad \textbf{(8)}$$

> $y[5,1] := 5\,z^2 + 4\,z + 204$:
> $x[5,1] := f(\%)$;

$$x_{5,\,1} := (z^2 + z + 41)\,(25\,z^2 + 15\,z + 1021) \quad \textbf{(9)}$$

> $y[5,2] := 10\,z^2 + z + 407$:
> $x[5,2] := f(\%)$;

$$x_{5,\,2} := (4\,z^2 + 163)\,(25\,z^2 + 5\,z + 1019) \quad \textbf{(10)}$$

> $y[5,3] := 15\,z^2 + 4\,z + 611$:
> $x[5,3] := f(\%)$;

$$x_{5,\,3} := (25\,z^2 + 5\,z + 1019)\,(9\,z^2 + 3\,z + 367) \quad \textbf{(11)}$$

> $y[5,4] := 20\,z^2 + 11\,z + 816$:
> $x[5,4] := f(\%)$;

$$x_{5,\,4} := (16\,z^2 + 8\,z + 653)\,(25\,z^2 + 15\,z + 1021) \quad \textbf{(12)}$$

> $y[6,1] := 6\,z^2 + 5\,z + 245$:
> $x[6,1] := f(\%)$;

$$x_{6,\,1} := (z^2 + z + 41)\,(36\,z^2 + 24\,z + 1471) \quad \textbf{(13)}$$

> $y[6,5] := 30\,z^2 + 19\,z + 1225$:
> $x[6,5] := f(\%)$;

$$x_{6,\,5} := (36\,z^2 + 24\,z + 1471)\,(25\,z^2 + 15\,z + 1021) \quad \textbf{(14)}$$

> $y[7,1] := 7\,z^2 + 6\,z + 286$:
> $x[7,1] := f(\%)$;

$$x_{7,\,1} := (z^2 + z + 41)\,(49\,z^2 + 35\,z + 2003) \quad \textbf{(15)}$$

> $y[7,2] := 14\,z^2 + z + 570$:
> $x[7,2] := f(\%)$;

$$x_{7,\,2} := (4\,z^2 + 163)\,(49\,z^2 + 7\,z + 1997) \quad \textbf{(16)}$$

> $y[7,3] := 21\,z^2 + 8\,z + 856$:
> $x[7,3] := f(\%)$;

$$x_{7,\,3} := (9\,z^2 + 3\,z + 367)\,(49\,z^2 + 21\,z + 1999) \quad \textbf{(17)}$$

> $y[7,4] := 28\,z^2 + 13\,z + 1142$:
> $x[7,4] := f(\%)$;

$$x_{7,\,4} := (49\,z^2 + 21\,z + 1999)\,(16\,z^2 + 8\,z + 653) \quad \textbf{(18)}$$

> $y[7,5] := 35\,z^2 + 6\,z + 1426$:
> $x[7,5] := f(\%)$;

$$x_{7,\,5} := (25\,z^2 + 5\,z + 1019)\,(49\,z^2 + 7\,z + 1997) \quad \textbf{(19)}$$

> $y[7,6] := 42\,z^2 + 29\,z + 1716$:
> $x[7,6] := f(\%)$;

$$x_{7,\,6} := (49\,z^2 + 35\,z + 2003)\,(36\,z^2 + 24\,z + 1471) \quad \textbf{(20)}$$

>

> $y[8,1] := 8z^2 + 7z + 327:$
$x[8,1] := f(\%);$

$$x_{8,1} := (64z^2 + 48z + 2617)(z^2 + z + 41) \quad \textbf{(21)}$$

> $y[8,3] := 24z^2 + 7z + 978:$
$x[8,3] := f(\%);$

$$x_{8,3} := (64z^2 + 16z + 2609)(9z^2 + 3z + 367) \quad \textbf{(22)}$$

> $y[8,5] := 40z^2 + 9z + 1630:$
$x[8,5] := f(\%);$

$$x_{8,5} := (64z^2 + 16z + 2609)(25z^2 + 5z + 1019) \quad \textbf{(23)}$$

> $y[8,7] := 56z^2 + 41z + 2289:$
$x[8,7] := f(\%);$

$$x_{8,7} := (49z^2 + 35z + 2003)(64z^2 + 48z + 2617) \quad \textbf{(24)}$$

>

> $y[9,1] := 9z^2 + 8z + 368:$
$x[9,1] := f(\%);$

$$x_{9,1} := (z^2 + z + 41)(81z^2 + 63z + 3313) \quad \textbf{(25)}$$

> $y[9,2] := 18z^2 + z + 733:$
$x[9,2] := f(\%);$

$$x_{9,2} := (81z^2 + 9z + 3301)(4z^2 + 163) \quad \textbf{(26)}$$

> $y[9,4] := 36z^2 + 19z + 1469:$
$x[9,4] := f(\%);$

$$x_{9,4} := (16z^2 + 8z + 653)(81z^2 + 45z + 3307) \quad \textbf{(27)}$$

> $y[9,5] := 45z^2 + 26z + 1837:$
$x[9,5] := f(\%);$

$$x_{9,5} := (81z^2 + 45z + 3307)(25z^2 + 15z + 1021) \quad \textbf{(28)}$$

> $y[9,7] := 63z^2 + 8z + 2567:$
$x[9,7] := f(\%);$

$$x_{9,7} := (81z^2 + 9z + 3301)(49z^2 + 7z + 1997) \quad \textbf{(29)}$$

> $y[9,8] := 72z^2 + 55z + 2944:$
$x[9,8] := f(\%);$

$$x_{9,8} := (81z^2 + 63z + 3313)(64z^2 + 48z + 2617) \quad \textbf{(30)}$$

>

> $y[10,1] := 10z^2 + 9z + 409:$
$x[10,1] := f(\%);$

$$x_{10,1} := (100z^2 + 80z + 4091)(z^2 + z + 41) \quad \textbf{(31)}$$

> $y[10,3] := 30z^2 + 11z + 1223:$
$x[10,3] := f(\%);$

(32)

$$x_{10,3} := (9z^2 + 3z + 367)(100z^2 + 40z + 4079) \quad (32)$$

> $y[10, 7] := 70z^2 + 29z + 2855 :$
> $x[10, 7] := f(\%);$

$$x_{10,7} := (100z^2 + 40z + 4079)(49z^2 + 21z + 1999) \quad (33)$$

> $y[10, 9] := 90z^2 + 71z + 3681 :$
> $x[10, 9] := f(\%);$

$$x_{10,9} := (100z^2 + 80z + 4091)(81z^2 + 63z + 3313) \quad (34)$$

>

> $y[11, 1] := 11z^2 + 10z + 450 :$
> $x[11, 1] := f(\%);$

$$x_{11,1} := (z^2 + z + 41)(121z^2 + 99z + 4951) \quad (35)$$

> $y[11, 2] := 22z^2 + z + 896 :$
> $y[11, 2] := f(\%);$

$$y_{11,2} := (121z^2 + 11z + 4931)(4z^2 + 163) \quad (36)$$

> $y[11, 3] := 33z^2 + 10z + 1345 :$
> $x[11, 3] := f(\%);$

$$x_{11,3} := (9z^2 + 3z + 367)(121z^2 + 33z + 4933) \quad (37)$$

> $y[11, 4] := 44z^2 + 21z + 1795 :$
> $x[11, 4] := f(\%);$

$$x_{11,4} := (16z^2 + 8z + 653)(121z^2 + 55z + 4937) \quad (38)$$

> $y[11, 5] := 55z^2 + 34z + 2246 :$
> $x[11, 5] := f(\%);$

$$x_{11,5} := (25z^2 + 15z + 1021)(121z^2 + 77z + 4943) \quad (39)$$

> $y[11, 6] := 66z^2 + 43z + 2696 :$
> $x[11, 6] := f(\%);$

$$x_{11,6} := (36z^2 + 24z + 1471)(121z^2 + 77z + 4943) \quad (40)$$

> $y[11, 7] := 77z^2 + 34z + 3141 :$
> $x[11, 7] := f(\%);$

$$x_{11,7} := (121z^2 + 55z + 4937)(49z^2 + 21z + 1999) \quad (41)$$

> $y[11, 8] := 88z^2 + 23z + 3587 :$
> $x[11, 8] := f(\%);$

$$x_{11,8} := (64z^2 + 16z + 2609)(121z^2 + 33z + 4933) \quad (42)$$

> $y[11, 9] := 99z^2 + 10z + 4034 :$
> $x[11, 9] := f(\%);$

$$x_{11,9} := (121z^2 + 11z + 4931)(81z^2 + 9z + 3301) \quad (43)$$

> $y[11, 10] := 110z^2 + 89z + 4500 :$
> $x[11, 10] := f(\%);$

$$x_{11,10} := (121z^2 + 99z + 4951)(100z^2 + 80z + 4091) \quad (44)$$

>

> $y[12, 1] := 12\,z^2 + 11\,z + 491:$
$x[12, 1] := f(\%);$

$$x_{12,\,1} := (z^2 + z + 41)\,(144\,z^2 + 120\,z + 5893) \quad (45)$$

> $y[12, 5] := 60\,z^2 + 11\,z + 2445:$
$x[12, 5] := f(\%);$

$$x_{12,\,5} := (25\,z^2 + 5\,z + 1019)\,(144\,z^2 + 24\,z + 5869) \quad (46)$$

> $y[12, 7] := 84\,z^2 + 13\,z + 3423:$
$x[12, 7] := f(\%);$

$$x_{12,\,7} := (144\,z^2 + 24\,z + 5869)\,(49\,z^2 + 7\,z + 1997) \quad (47)$$

> $y[12, 11] := 132\,z^2 + 109\,z + 5401:$
$x[12, 11] := f(\%);$

$$x_{12,\,11} := (121\,z^2 + 99\,z + 4951)\,(144\,z^2 + 120\,z + 5893) \quad (48)$$

>

> $y[13, 1] := 13\,z^2 + 12\,z + 532:$
$x[13, 1] := f(\%);$

$$x_{13,\,1} := (169\,z^2 + 143\,z + 6917)\,(z^2 + z + 41) \quad (49)$$

> $y[13, 2] := 26\,z^2 + z + 1059:$
$x[13, 2] := f(\%);$

$$x_{13,\,2} := (169\,z^2 + 13\,z + 6887)\,(4\,z^2 + 163) \quad (50)$$

>

> *# 11−1−2016 M. A.*

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