> *restart*

> $x[b, 2] := (4a+2)\cdot z^2 + z + \frac{(163(4a+2)-2)}{4}$

$$x_{b,2} := (4a+2)z^2 + z + 163a + 81 \quad (1)$$

> *# a = 1,2,3, ...*

> $h := n^2 + n + 41$

$$h := n^2 + n + 41 \quad (2)$$

> $y[b, 2] := factor(subs(n = x[b, 2], h))$

$$y_{b,2} := (4z^2 + 163)(4a^2z^2 + 4az^2 + 163a^2 + 2az + z^2 + 163a + z + 41) \quad (3)$$

> *# $4z^2 + 163$ is the form of the smallest factor of h when n is of the form x[b, 2].*

> $x[b, 3, 1] := 3(3a+1)z^2 + (3a+2)\cdot z + \frac{(367(3a+1)-1)}{3}$

$$x_{b,3,1} := 3(3a+1)z^2 + (3a+2)z + 367a + 122 \quad (4)$$

> $y[b, 3, 1] := factor(subs(n = x[b, 3, 1], h))$

$$y_{b,3,1} := (9z^2 + 3z + 367)(9a^2z^2 + 3a^2z + 6az^2 + 367a^2 + 4az + z^2 + 245a + z + 41) \quad (5)$$

> $x[b, 3, 2] := 3(3a+2)\cdot z^2 + (3a+1)\cdot z + \frac{(367(3a+2)-2)}{3}$

$$x_{b,3,2} := 3(3a+2)z^2 + (3a+1)z + 367a + 244 \quad (6)$$

> *# a = 1, 2, 3, ...*

> $y[b, 3, 2] := factor(subs(n = x[b, 3, 2], h))$

$$y_{b,3,2} := (9z^2 + 3z + 367)(9a^2z^2 + 3a^2z + 12az^2 + 367a^2 + 2az + 4z^2 + 489a + 163) \quad (7)$$

> $x[b, 4, 1] := 4(4a+1)\cdot z^2 + (2(4a+1)+1)\cdot z + \frac{(653(4a+1)-1)}{4}$

$$x_{b,4,1} := 4(4a+1)z^2 + (8a+3)z + 653a + 163 \quad (8)$$

> $y[b, 4, 1] := factor(subs(n = x[b, 4, 1], h))$

$$y_{b,4,1} := (16z^2 + 8z + 653)(16a^2z^2 + 8a^2z + 8az^2 + 653a^2 + 6az + z^2 + 327a + z + 41) \quad (9)$$

> *#a = 1, 2, 3, ...*

> $x[b, 4, 3] := 4(4a+3)\cdot z^2 + (2(4a+3)-1)\cdot z + \frac{(653(4a+3)-3)}{4}$

$$x_{b,4,3} := 4(4a+3)z^2 + (8a+5)z + 653a + 489 \quad (10)$$

> $y[b, 4, 3] := factor(subs(n = x[b, 4, 3], h))$

$$y_{b,4,3} := (16z^2 + 8z + 653)(16a^2z^2 + 8a^2z + 24az^2 + 653a^2 + 10az + 9z^2 + 979a + 3z + 367) \quad (11)$$

> $x[b, 5, 1] := 5\cdot(5a+1)\cdot z^2 + (3\cdot(5a+1)+1)\cdot z + \frac{(1021(5\cdot a+1)-1)}{5}$

$$x_{b,5,1} := 5(5a+1)z^2 + (15a+4)z + 1021a + 204 \quad (12)$$

> $y[b, 5, 1] := factor(subs(n = x[b, 5, 1], h))$

$$y_{b,5,1} := (25z^2+15z+1021)(25a^2z^2+15a^2z+10az^2+1021a^2+8az+z^2+409a+z+41) \quad (13)$$

> $x[b,5,2] := 5(5a+2)\cdot z^2+(5a+1)\cdot z+\frac{(1019\cdot(5a+2)-3)}{5}$

$$x_{b,5,2} := 5(5a+2)z^2+(5a+1)z+1019a+407 \quad (14)$$

> $y[b,5,2] := factor(subs(n=x[b,5,2],h))$

$$y_{b,5,2} := (25z^2+5z+1019)(25a^2z^2+5a^2z+20az^2+1019a^2+2az+4z^2+815a+163) \quad (15)$$

> $x[b,5,3] := 5(5a+3)\cdot z^2+(5a+4)\cdot z+\frac{(1019\cdot(5a+3)-2)}{5}$

$$x_{b,5,3} := 5(5a+3)z^2+(5a+4)z+1019a+611 \quad (16)$$

> $y[b,5,3] := factor(subs(n=x[b,5,3],h))$

$$y_{b,5,3} := (25z^2+5z+1019)(25a^2z^2+5a^2z+30az^2+1019a^2+8az+9z^2+1223a+3z+367) \quad (17)$$

> $x[b,5,4] := 5(5a+4)\cdot z^2+(3\cdot(5a+4)-1)\cdot z+\frac{(1021\cdot(5a+4)-4)}{5}$

$$x_{b,5,4} := 5(5a+4)z^2+(15a+11)z+1021a+816 \quad (18)$$

> $y[b,5,4] := factor(subs(n=x[b,5,4],h))$

$$y_{b,5,4} := (25z^2+15z+1021)(25a^2z^2+15a^2z+40az^2+1021a^2+22az+16z^2+1633a+8z+653) \quad (19)$$

> $x[b,6,1] := 6(6a+1)\cdot z^2+(4(6a+1)+1)\cdot z+\frac{(1471(6a+1)-1)}{6}$

$$x_{b,6,1} := 6(6a+1)z^2+(24a+5)z+1471a+245 \quad (20)$$

> $y[b,6,1] := factor(subs(n=x[b,6,1],h))$

$$y_{b,6,1} := (36z^2+24z+1471)(36a^2z^2+24a^2z+12az^2+1471a^2+10az+z^2+491a+z+41) \quad (21)$$

> *with*(*CurveFitting*)

[ArrayInterpolation, BSpline, BSplineCurve, Interactive, LeastSquares, PolynomialInterpolation, RationalInterpolation, Spline, ThieleInterpolation] **(22)**

>

> $x[b,d,1] := c\cdot(c\cdot a+1)\cdot z^2+(c\cdot(c-2)\cdot a+(c-1))\cdot z+(41\cdot c^2-c+1)\cdot a+41\cdot c-1$

$$x_{b,d,1} := c(ac+1)z^2+(c(c-2)a+c-1)z+(41c^2-c+1)a+41c-1 \quad (23)$$

> $y[b,d,1] := factor(subs(n=x[b,d,1],h))$

$$y_{b,d,1} := (c^2z^2+c^2z+41c^2-2cz-c+1)(a^2c^2z^2+a^2c^2z+41a^2c^2-2a^2cz+2acz^2-a^2c+2acz+a^2+82ac-2az+z^2-a+z+41) \quad (24)$$

> *# 3 parameter equation best so far.*

> *#Matt C. Anderson 7 30 2015*

> # I usesd x[b,3,1] and x[b,4,1] and x[b,5,1] for a curve fit.

> $x[b, 6, 5] := 6(6\,a+5)\cdot z^2 + (4(6\,a+5)-1)\cdot z + \frac{(1471(6\,a+5)-5)}{6}$

$$x_{b,\,6,\,5} := 6\,(6\,a+5)\,z^2 + (24\,a+19)\,z + 1471\,a + 1225 \qquad \textbf{(25)}$$

```
> for c from 3 to 20 do
    x[b, d, 1];
    y[b, d, 1];
  end do;
```

$$3\,(3\,a+1)\,z^2 + (3\,a+2)\,z + 367\,a + 122$$
$$(9\,z^2+3\,z+367)\,(9\,a^2 z^2 + 3\,a^2 z + 6\,a\,z^2 + 367\,a^2 + 4\,a\,z + z^2 + 245\,a + z + 41)$$
$$4\,(4\,a+1)\,z^2 + (8\,a+3)\,z + 653\,a + 163$$
$$(16\,z^2+8\,z+653)\,(16\,a^2 z^2 + 8\,a^2 z + 8\,a\,z^2 + 653\,a^2 + 6\,a\,z + z^2 + 327\,a + z + 41)$$
$$5\,(5\,a+1)\,z^2 + (15\,a+4)\,z + 1021\,a + 204$$
$$(25\,z^2+15\,z+1021)\,(25\,a^2 z^2 + 15\,a^2 z + 10\,a\,z^2 + 1021\,a^2 + 8\,a\,z + z^2 + 409\,a + z + 41)$$
$$6\,(6\,a+1)\,z^2 + (24\,a+5)\,z + 1471\,a + 245$$
$$(36\,z^2+24\,z+1471)\,(36\,a^2 z^2 + 24\,a^2 z + 12\,a\,z^2 + 1471\,a^2 + 10\,a\,z + z^2 + 491\,a + z + 41)$$
$$7\,(7\,a+1)\,z^2 + (35\,a+6)\,z + 2003\,a + 286$$
$$(49\,z^2+35\,z+2003)\,(49\,a^2 z^2 + 35\,a^2 z + 14\,a\,z^2 + 2003\,a^2 + 12\,a\,z + z^2 + 573\,a + z + 41)$$
$$8\,(8\,a+1)\,z^2 + (48\,a+7)\,z + 2617\,a + 327$$
$$(64\,z^2+48\,z+2617)\,(64\,a^2 z^2 + 48\,a^2 z + 16\,a\,z^2 + 2617\,a^2 + 14\,a\,z + z^2 + 655\,a + z + 41)$$
$$9\,(9\,a+1)\,z^2 + (63\,a+8)\,z + 3313\,a + 368$$
$$(81\,z^2+63\,z+3313)\,(81\,a^2 z^2 + 63\,a^2 z + 18\,a\,z^2 + 3313\,a^2 + 16\,a\,z + z^2 + 737\,a + z + 41)$$
$$10\,(10\,a+1)\,z^2 + (80\,a+9)\,z + 4091\,a + 409$$
$$(100\,z^2+80\,z+4091)\,(100\,a^2 z^2 + 80\,a^2 z + 20\,a\,z^2 + 4091\,a^2 + 18\,a\,z + z^2 + 819\,a + z + 41)$$
$$11\,(11\,a+1)\,z^2 + (99\,a+10)\,z + 4951\,a + 450$$
$$(121\,z^2+99\,z+4951)\,(121\,a^2 z^2 + 99\,a^2 z + 22\,a\,z^2 + 4951\,a^2 + 20\,a\,z + z^2 + 901\,a + z + 41)$$
$$12\,(12\,a+1)\,z^2 + (120\,a+11)\,z + 5893\,a + 491$$
$$(144\,z^2+120\,z+5893)\,(144\,a^2 z^2 + 120\,a^2 z + 24\,a\,z^2 + 5893\,a^2 + 22\,a\,z + z^2 + 983\,a + z + 41)$$
$$13\,(13\,a+1)\,z^2 + (143\,a+12)\,z + 6917\,a + 532$$
$$(169\,z^2+143\,z+6917)\,(169\,a^2 z^2 + 143\,a^2 z + 26\,a\,z^2 + 6917\,a^2 + 24\,a\,z + z^2 + 1065\,a + z + 41)$$
$$14\,(14\,a+1)\,z^2 + (168\,a+13)\,z + 8023\,a + 573$$
$$(196\,z^2+168\,z+8023)\,(196\,a^2 z^2 + 168\,a^2 z + 28\,a\,z^2 + 8023\,a^2 + 26\,a\,z + z^2 + 1147\,a + z + 41)$$

$$15\ (15\ a+1)\ z^2+(195\ a+14)\ z+9211\ a+614$$

$$(225\ z^2+195\ z+9211)\ (225\ a^2\ z^2+195\ a^2\ z+30\ a\ z^2+9211\ a^2+28\ a\ z+z^2+1229\ a+z+41)$$

$$16\ (16\ a+1)\ z^2+(224\ a+15)\ z+10481\ a+655$$

$$(256\ z^2+224\ z+10481)\ (256\ a^2\ z^2+224\ a^2\ z+32\ a\ z^2+10481\ a^2+30\ a\ z+z^2+1311\ a+z+41)$$

$$17\ (17\ a+1)\ z^2+(255\ a+16)\ z+11833\ a+696$$

$$(289\ z^2+255\ z+11833)\ (289\ a^2\ z^2+255\ a^2\ z+34\ a\ z^2+11833\ a^2+32\ a\ z+z^2+1393\ a+z+41)$$

$$18\ (18\ a+1)\ z^2+(288\ a+17)\ z+13267\ a+737$$

$$(324\ z^2+288\ z+13267)\ (324\ a^2\ z^2+288\ a^2\ z+36\ a\ z^2+13267\ a^2+34\ a\ z+z^2+1475\ a+z+41)$$

$$19\ (19\ a+1)\ z^2+(323\ a+18)\ z+14783\ a+778$$

$$(361\ z^2+323\ z+14783)\ (361\ a^2\ z^2+323\ a^2\ z+38\ a\ z^2+14783\ a^2+36\ a\ z+z^2+1557\ a+z+41)$$

$$20\ (20\ a+1)\ z^2+(360\ a+19)\ z+16381\ a+819$$

$$(400\ z^2+360\ z+16381)\ (400\ a^2\ z^2+360\ a^2\ z+40\ a\ z^2+16381\ a^2+38\ a\ z+z^2+1639\ a+z+41) \quad \textbf{(26)}$$

> *# for x[b,7,1] see* x[b, d, 1]

> $x[b,7,1] := 7\ (7\ a+1)\ z^2+(35\ a+6)\ z+2003\ a+286$

$$x_{b,7,1} := 7\ (7\ a+1)\ z^2+(35\ a+6)\ z+2003\ a+286 \quad \textbf{(27)}$$

> $y[b,7,1] := factor(subs(n=x[b,7,1], h))$

$$y_{b,7,1} := (49\ z^2+35\ z+2003)\ (49\ a^2\ z^2+35\ a^2\ z+14\ a\ z^2+2003\ a^2+12\ a\ z+z^2+573\ a+z+41) \quad \textbf{(28)}$$

>

> $x[b,7,2] := 7\cdot(7\ a+2)\cdot z^2+(7\ a+1)\cdot z+\dfrac{(1997\cdot(7\ a+2)-4)}{7}$

$$x_{b,7,2} := 7\ (7\ a+2)\ z^2+(7\ a+1)\ z+1997\ a+570 \quad \textbf{(29)}$$

> $y[b,7,2] := factor(subs(n=x[b,7,2], h))$

$$y_{b,7,2} := (49\ z^2+7\ z+1997)\ (49\ a^2\ z^2+7\ a^2\ z+28\ a\ z^2+1997\ a^2+2\ a\ z+4\ z^2+1141\ a+163) \quad \textbf{(30)}$$

> *# now a curve fit with x[b,3,2] ; x[b,5,2] ; and x[b,7,2] doesn't work*

> $x[b,7,3] := 7\cdot(7\ a+3)\cdot z^2+(3\cdot(7\ a+3)-1)\cdot z+\dfrac{(1999\cdot(7\ a+3)-5)}{7}$

$$x_{b,7,3} := 7\ (7\ a+3)\ z^2+(21\ a+8)\ z+1999\ a+856 \quad \textbf{(31)}$$

> $y[b,7,3] := factor(subs(n=x[b,7,3], h))$

$$y_{b,7,3} := (49\ z^2+21\ z+1999)\ (49\ a^2\ z^2+21\ a^2\ z+42\ a\ z^2+1999\ a^2+16\ a\ z+9\ z^2 \quad \textbf{(32)}$$

$$+ 1713\,a + 3\,z + 367\big)$$

> $x[b, 7, 4] := 7\cdot(7\,a+4)\cdot z^2 + (3\cdot(7\,a+4)+1)\cdot z + \frac{(1999\cdot(7\,a+4)-2)}{7}$

$$x_{b,7,4} := 7\,(7\,a+4)\,z^2 + (21\,a+13)\,z + 1999\,a + 1142 \quad \textbf{(33)}$$

> $y[b, 7, 4] := factor(subs(n = x[b, 7, 4], h))$

$$y_{b,7,4} := \big(49\,z^2 + 21\,z + 1999\big)\,\big(49\,a^2\,z^2 + 21\,a^2\,z + 56\,a\,z^2 + 1999\,a^2 + 26\,a\,z + 16\,z^2 + 2285\,a + 8\,z + 653\big) \quad \textbf{(34)}$$

> $x[b, 7, 5] := 7\cdot(7\,a+5)\cdot z^2 + (7\,a+6)\cdot z + \frac{(1997\cdot(7\,a+5)-3)}{7}$

$$x_{b,7,5} := 7\,(7\,a+5)\,z^2 + (7\,a+6)\,z + 1997\,a + 1426 \quad \textbf{(35)}$$

> $y[b, 7, 5] := factor(subs(n = x[b, 7, 5], h))$

$$y_{b,7,5} := \big(49\,z^2 + 7\,z + 1997\big)\,\big(49\,a^2\,z^2 + 7\,a^2\,z + 70\,a\,z^2 + 1997\,a^2 + 12\,a\,z + 25\,z^2 + 2853\,a + 5\,z + 1019\big) \quad \textbf{(36)}$$

> $x[b, 7, 6] := 7\cdot(7\,a+6)\cdot z^2 + (5(7\,a+6)-1)\cdot z + \frac{(2003\cdot(7\,a+6)-6)}{7}$

$$x_{b,7,6} := 7\,(7\,a+6)\,z^2 + (35\,a+29)\,z + 2003\,a + 1716 \quad \textbf{(37)}$$

> $y[b, 7, 5] := factor(subs(n = x[b, 7, 5], h))$

$$y_{b,7,5} := \big(49\,z^2 + 7\,z + 1997\big)\,\big(49\,a^2\,z^2 + 7\,a^2\,z + 70\,a\,z^2 + 1997\,a^2 + 12\,a\,z + 25\,z^2 + 2853\,a + 5\,z + 1019\big) \quad \textbf{(38)}$$

> $x[b, 8, 1] := 8\,(8\,a+1)\,z^2 + (48\,a+7)\,z + 2617\,a + 327$

$$x_{b,8,1} := 8\,(8\,a+1)\,z^2 + (48\,a+7)\,z + 2617\,a + 327 \quad \textbf{(39)}$$

> *# that was from the 3 parameter expression*

> $y[b, 8, 1] := factor(subs(n = x[b, 8, 1], h))$

$$y_{b,8,1} := \big(64\,z^2 + 48\,z + 2617\big)\,\big(64\,a^2\,z^2 + 48\,a^2\,z + 16\,a\,z^2 + 2617\,a^2 + 14\,a\,z + z^2 + 655\,a + z + 41\big) \quad \textbf{(40)}$$

> $x[b, 8, 3] := 8(8\,a+3)\cdot z^2 + (2(8\,a+3)+1)\cdot z + \frac{(2609(8\,a+3)-3)}{8}$

$$x_{b,8,3} := 8\,(8\,a+3)\,z^2 + (16\,a+7)\,z + 2609\,a + 978 \quad \textbf{(41)}$$

> $y[b, 8, 3] := factor(subs(n = x[b, 8, 3], h))$

$$y_{b,8,3} := \big(64\,z^2 + 16\,z + 2609\big)\,\big(64\,a^2\,z^2 + 16\,a^2\,z + 48\,a\,z^2 + 2609\,a^2 + 14\,a\,z + 9\,z^2 + 1957\,a + 3\,z + 367\big) \quad \textbf{(42)}$$

> $x[b, 8, 5] := 8(8\,a+5)\cdot z^2 + (2(8\,a+5)-1)\cdot z + \frac{(2609(8\,a+5)-5)}{8}$

$$x_{b,8,5} := 8\,(8\,a+5)\,z^2 + (16\,a+9)\,z + 2609\,a + 1630 \quad \textbf{(43)}$$

> $y[b, 8, 5] := factor(subs(n = x[b, 8, 5], h))$

$$y_{b,8,5} := \big(64\,z^2 + 16\,z + 2609\big)\,\big(64\,a^2\,z^2 + 16\,a^2\,z + 80\,a\,z^2 + 2609\,a^2 + 18\,a\,z + 25\,z^2 + 3261\,a + 5\,z + 1019\big) \quad \textbf{(44)}$$

> $x[b, 8, 7] := 8(8\,a+7)\cdot z^2 + (6(8\,a+7)-1)\cdot z + \frac{(2617(8\,a+7)-7)}{8}$

$$x_{b,\,8,\,7} := 8\,(8\,a+7)\,z^2 + (48\,a+41)\,z + 2617\,a + 2289 \qquad \textbf{(45)}$$

> $y[b, 8, 7] := factor(subs(n = x[b, 8, 7], h))$

$$y_{b,\,8,\,7} := (64\,z^2 + 48\,z + 2617)\,(64\,a^2 z^2 + 48\,a^2 z + 112\,a\,z^2 + 2617\,a^2 + 82\,a\,z + 49\,z^2 + 4579\,a + 35\,z + 2003) \qquad \textbf{(46)}$$

> *# Matt C. Anderson, 7 31 2015*

>