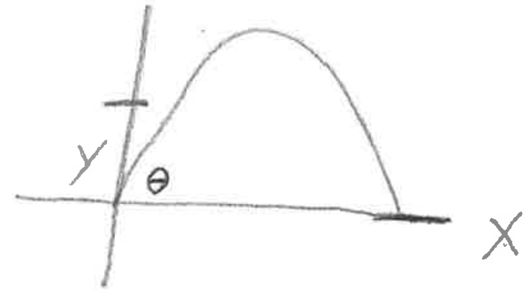
Projectile Motion

ASSUME MINIMAL AIR RESISTANCE

GIVEN RAMP MEASURE ϕ, CHOOSE ANGLE θ TO MAXIMIZE X DISPLACEMENT.

ANSWER $\theta = \frac{\pi}{4} + \frac{\phi}{2}$.

From wikipedia - Projectile motion



① $y = V_0 t \sin(\theta) - \frac{g t^2}{2}$

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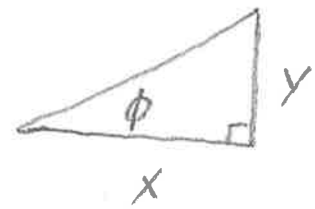
② $x = V_0 t \cos \theta$



③ $\frac{y}{x} = \tan \phi$

$\frac{①}{②}$ $\frac{y}{x} = \frac{V_0 t \sin \theta}{V_0 t \cos \theta} - \frac{g t^2}{2 V_0 t \cos \theta}$

SIMPLIFY

$$\tan \phi = \tan \theta - \frac{g t}{2 V_0 \cos \theta}$$

clear fraction

$$2 V_0 \cos \theta \tan \phi = 2 V_0 \cos \theta \tan \theta - g t$$

USE $\cos \theta \cdot \tan \theta = \sin \theta$

THEN

$$2 V_0 \cos \theta \tan \phi = 2 V_0 \sin \theta - g t$$

WANT t_D

$$2v_0 \cos\theta \tan\phi = 2v_0 \sin\theta - gt$$

$$gt = 2v_0 \sin\theta - 2v_0 \cos\theta \tan\phi$$

(4) $$t_{FLIGHT} = \frac{2v_0}{g} \sin\theta - \frac{2v_0}{g} \cos\theta \tan\phi$$

~ FROM WIKIPEDIA PAGE 5

(5) $$x = v_0 t_{FLIGHT} \cos\theta$$

$$t_{FLIGHT} = \frac{x}{v_0 \cos\theta}$$

COMBINE (4) AND (5)

$$\frac{x}{v_0 \cos\theta} = \frac{2v_0}{g} \sin\theta - \frac{2v_0}{g} \cos\theta \tan\phi$$

FROM BEFORE

$$\frac{x}{V_0 \cos\theta} = \frac{2V_0}{g}\sin\theta - \frac{2V_0}{g}\cos\theta\tan\phi$$

$$x = \frac{2V_0^2}{g}\sin\theta\cos\theta - \frac{2V_0^2}{g}\cos^2\theta\tan\phi$$

$$x(\theta) = \frac{2V_0^2}{g}\cos\theta\left(\sin\theta - \cos\theta\tan\phi\right)$$

NEXT $\frac{dx}{d\theta} = 0$

$(UV)' = U'V + UV'$

SIMPLIFIED

$$X_{SIMP} = \cos\theta\left(\sin\theta - \cos\theta\tan\phi\right)$$

FIND

$$\frac{d\,X_{SIMP}}{d\theta} = -\sin\theta\left(\sin\theta - \cos\theta\tan\phi\right) +$$

$$\cos\theta\left(\cos\theta + \sin\theta\tan\phi\right)$$

SET $\frac{dX_{SIMP}}{d\theta} = 0$

$$0 = -\sin^2\theta + \sin\theta\cos\theta\tan\phi + \cos^2\theta + \sin\theta\cos\theta\tan\phi$$

$$0 = \cos^2\theta - \sin^2\theta + \tan\phi\left(\sin\theta\cos\theta + \sin\theta\cos\theta\right)$$

(6) $$0 = \cos^2\theta - \sin^2\theta + \left(\tan\phi\right)\cdot 2\sin\theta\cos\theta$$

Trigonetric Identities

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\sin(2\theta) = 2\sin\theta \cdot \cos\theta$$

From (6)

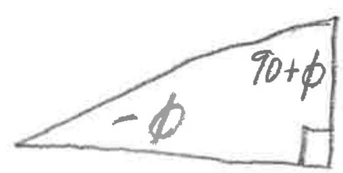
$$0 = \cos(2\theta) + (\tan\phi) \cdot \sin(2\theta)$$

$$0 = \frac{\cos(2\theta)}{\sin(2\theta)} + \tan\phi$$

$$\frac{\cos(2\theta)}{\sin(2\theta)} = -\tan\phi$$

$$\tan(2\theta) = -\cot\phi$$

$$\tan(2\theta) = \cot -\phi$$



$$2\theta = \tan^{-1}(\cot -\phi)$$

$$2\theta = 90° + \phi$$

$$\theta = 45° + \phi/2$$

$$\theta = \frac{\pi}{4} + \frac{\phi}{2}$$

WHICH WAS WHAT WE WANTED.