Schermack Perfins Some Questions, Philosophy, and a Proposal

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Schermacks are known to the specialized US stamp collecting community, but are also very desirable as a specialty area for the perfin collector. While I collect all holey stamps I am far from an expert on Schermacks, so I will start by quoting from Keneth A. Wood's excellent *This is Philately: Volume 2 G-P.* On p. 554 Ken begins:

"Perforation, Schermack: The Schermack Mailing Machine Company (later the Mailometer Co., Mailom-eter Co., and the Mail-O-Meter Co.) was one of the best known producers of privately perforated coil stamps.

The company created a variety of type of perforations including one version comprising two rectangular holes instead of the more conventional round holes in various sizes and groupings.

The rectangular holes were not so much separation perforations but were intended to engage sprockets within the vending machine that dispensed the stamps.

The company, under its various names, produced its coil stamps from 1906 to 1927."

(Ironically, and totally irrelevant to this discussion, Perforations in their many forms follow immediately after 'perfins', 'The Perfins Club', and other 'perforated' definitions in Ken's book.)

My Scott *Specialized Catalog of US Stamps and Covers* (2001 ed.) has the following to say about these issues:

"Vending & Affixing Machine Perforations: Imperforate sheets of 400 were first issued in 1906 on the request of several makers of vending and affixing machines. The machine manufacturers made coils from the imperforate sheets and applied various perforations to suit the particular needs of their machines. These privately applied perforations were used for many years and form a chapter in postal history.....

Perfins listed here are punched into the stamp at the same time as the perforation. The most common pattern consisted of a 7 mm square made of nine holes. Pins would be removed to create the unique perfins for each company using the machine....."

The rectangular perforations referred to by Ken Wood are Scott's Type III perforations. Scott lists these perforations as present in coil stamps from the regular issues of 1906-08, 1908-09, 1910, 1912, 1916-17, 1918-20, and 1923-26, plus the Lincoln, Alaska-Yukon, and Hudson-Fulton issues of 1909, and the Harding issue of 1923. However, it indicates perfin presence in stamps only from the regular series issues of:

1908-09 – on unwatermarked paper (Sc#343 [1¢ green Franklin, Sc type A138], 344 [2¢ carmine Washington, Sc type A139], 345 [3¢ deep violet Washington, Sc type A140], and 346 [4¢ orange brown Washington, Sc type A140])

1910 – on single-lined letter watermarked paper, wm #190 (Sc#383 [1¢ green Franklin Sc type A138] and 384 [2¢ carmine Washington Sc type A139])

1912 - on single-lined letter watermarked paper, wm #190 (Sc#408[1¢ green Franklin, Sc type A140], and 409 [2¢ carmine Washington Sc type A140]).

The complete 9-hole Schermack perfin pattern was formed of three rows of three holes each. And this, of course, is where the perfin collector comes in.

On page 2 of *Perfins* #20 (Vol.4 # 2) of July 1948 Warren Travell wrote the first article to appear in the journal destined to become *The Perfins Bulletin*. His article begins with the following:

"There is one kind of postally-used perfins which is in a class by itself- as it is quite different from all others both in its origin and in the style of its patterns. These are the perfins which were perforated by the Schermack Company at the same time they cut the side slots which distinguish their output from that of other companies.

The Schermack patterns are derived from a square of three holes on a side... By omission of one or more holes, a large number of different patterns may be made. And this gives rise to an interesting mathematical problem in 'combinations' – how many of these Schermack patterns are possible with the three hole square, counting only those which have 4, 5, 6, 7, 8 or 9 holes as none have been reported having only 1, 2, or 3 holes. Then when the total number has been determined, there should be deducted all those which duplicate others in appearance. For instance, numbering the holes downward beginning at the upper left corner, the patterns having holes 1, 2, 3, 4, 5, 6 is identical in appearance with pattern 4, 5, 6, 7, 8, 9. As the holes are perforated at the same time as the slots are cut, they are all properly centered and face up – there are never any inverts offverts, diagonals, sideways or double punching."

Travell goes on to list the 18 patterns which he has or have been reported to him, and requests more information from those who have patterns in this issue.

Then, on page 1 of *Perfins* #22 (Vol. 4 #4) he states very simply:

"Considerable interest has been shown by members in the recent article on Schermack perfins and the number of different types reported has risen from 18 to 65... it is desirable that every member having one or more of these Schermacks in his collection should send in the description of what he has using the code:

1	4	7	
2	5	8	
3	6	9	

And be sure to include identifications if you have any."

Those of you who collect these 9-hole perfins in Schermacks will immediately note that Travell's numbering is not that which is in current use – and some of his information about the machinery also met with quick contradiction.

In *The Perfins Bulletin* #27 (Vol 5 #4) of April 1927 on page 1, information, as follows, from George P. Howard was published:

"The Schermack Company and Mailometer Company are actually one and the same company. J. J. Schermack invented, developed and produced an electrically operated stamp affixing and envelope sealing machine. By 1909, his company, then called the Schermack Co., was 'perforating' coils with two slots and providing them in rolls of 3,000 at 50 cents over face to the owners of their affixing machines... The gadget that punched the control marks was an 'extra' on the affixing machines. That is, there were no 'perfins' on the coils before they were placed in the machines. When the knife blade severed the stamp being affixed, the same operation punched the control marks on [the] adjacent stamp. Since the stamps were locked in the machine and registers counted every stamp applied, there was hardly any need for the control markings to prevent pilfering of stamps. However perforated identification marks were popular in that era and the Company had to meet the demand."

This is a very different picture of the perfinning process than that proposed by Mr. Travell. What it means, and contradicts in Travell's note, is that all patterns read correctly may be distinguishable from all other patterns. The reason is that if the holes are punched at the same time as the cut of the previous stamp is made, the distance of the basic 9-hole pattern is constant from that cut. Thus, in Travell's illustration the two blocks of 6 holes each would be at distinctly different distances from the cut. The difference in separation from the cut would make the patterns totally distinguishable from each other if a good ruler (or a micrometer) were used. (Magnification might be necessary - but the distances involved could be used to separate the two patterns.)

On page 2 of the same issue L. N. Littlefield notes that the patent for the 9-hole pattern was granted to the Mailometer Co. of Detroit, Michigan on Feb. 9, 1915 (#1127543). He then goes on to the following:

"To 'read' the designs correctly, the stamp must be held face up and head up, for this is the position in which the stamps were punched as the coils were fed through the machines....

To form a simple way of numbering these designs for classification in the perfin catalog, the holes are numbered thus: (not to scale)

• • •	1 2 3
The holes • • • are numbered	456
• • •	789
with the stamps face and head up.	

The number assigned to each design is determined by the positions of the eliminated holes; the unpunched positions, that is. Thus a design having all nine holes is given the description '0'. Other examples follow:

is 24568

The lowest number is always used first."

Littlefield's, and not Travell's, system was ultimately adopted by the Perfins Club in numbering the non-holes used in describing the potential, but less than, 9-hole patterns.

In the April 1952 *Perfins Bulletin* 55 patterns (fewer than the 65 previously announced) are listed (by description) from the work of George P. Howard, with 8 identified to user and an additional one identified only to city of use. This article includes the note that:

"Blind holes are common, examine your stamps carefully with a glass before deciding you have a new variety."

On the second page of this article, a composite list of patterns is presented in which sixty-seven patterns are identified, 30 to user.

A similar listing of sixty-seven patterns is presented in July of 1957.

This information has persisted with limited elaboration till the present.

So now we move to speculation and my questions.

First to speculation – Travell in his writings posed the question:

"...this gives rise to an interesting mathematical problem in 'combinations' – how many of these Schermack patterns are possible with the three hole square, counting only those which have 4, 5, 6, 7, 8 or 9 holes as none have been reported having only 1, 2, or 3 holes?"

I have never seen that question answered in the *Bulletin*. So, here is the answer.

The formula to compute the number of combinations is relatively straightforward: $n (n-1) (n-2) \dots (n[R-1]) /R!$ where n is the number of factors, R is the number

of factors considered at a given time and ! = the math 'factorial' function (high school algebra anyone??) Thus for 9 items taken * at a time:

1: (9) / (1) = 9 2: (9 x 8) / (2 x 1) = 36 3: (9 x 8 x 7) / (3 x 2 x 1) = 84 4: (9 x 8 x 7 x 6) / (4 x 3 x 2 x 1) = 126 5: (9 x 8 x 7 x 6 x 5) = (5 x 4 x 3 x 2 x 1) = 126 6: 9 x...4) / (6 x1) = 84 7: 9 x...3) / (7 x1) = 36 8: 9 x...2) / (8 x1) = 9 9: 9 x...1) / (9 x1) = 1

So, all possible combinations - if we assume that all stamps were perforated face up and head up - is 9 + 36 + ... = 511. And, if we impose Travell's condition (no 3, 2 or 1 hole patterns: which incidentally we know to be incorrect since at least one 3 hole pattern is now recognized) then we subtract 129 from 511 and we have the possibility of 382 patterns in Schermack coils. If we impose Travell's second condition that similar patterns be eliminated (based on a speculated inability of distinguishing patterns with the same apparent pattern of holes and punched with fewer than either three rows or three columns in the pattern; for example 1, 2 = 2, 3 = 4, 5 = 5, 6 = 7, 8 = 8, 9 and six patterns become only one). This assumes collectors can't effectively use rulers (what the heck!) or that while the patterns were created equal (7 mm squares) their positioning in perforator heads was not consistent, we still have 339 possible *patterns*. (More on this question later.)

Randall's 1998 *Catalog of United States Perfins* lists only 67 of these possible patterns as actually being known. And he adds an additional uncertainty to that listing when on page Design 15 he states quite simply:

"One great problem with the Schermack patterns is that if a single pin breaks on the punch the result is immediately a different pattern. There are insufficient covers of this scarce issue to make a good study, so there [is] a serious question concerning which are real patterns and which are broken pin varieties."

Compounding the problem is the possibility (and according to Murphy's law – the probability) that someone, or more than just one someone,

played with the stamp supply and generated Schermack perfins face up/head down, or back up with the head up or down relative to the feed mechanism. While the stamp supply was locked into the machines, this would make the challenge even greater rather than precluding the possibility of the manipulation.

This would make for four times the possibilities for 'patterns' – if direction of feed were considered. But what it would really do is 'create' false patterns when read face up/head up since all four will appear different in the face up/head up position (excluding only symmetrical patterns which will always look the same regardless of the orientation of the stamp at the time it is perfinned). As an example, below is presented the same pattern punched into a stamp as noted and then turned and flipped as necessary to get it into the face up/head up orientation.

Punched Face U/Head U	Rotated to Face U/Head U	Punched Face D/head U	Rotated to J Face U/Head U
•	•	•	•
•• ==>	• •	••	==> ••
• • •	• • •	• • •	• • •
Face U/head D		Face D/head	D
•	• • •	•	• • •
•• ==>	• •	••	==> ••
• • •	•	• • •	•

Despite the final appearance of patterns in this example all were punched with the same perforator, but - with the face up/head up requirement - would be classified as four different patterns. It is important to note that this manipulation would not affect the possible 339 (face up/head up) pattern total – it could simply wind up with three of the four patterns being falsely listed as having been legitimately used or user misidentification for 'known' users of a pattern. Note also that I have not chosen to include the feeding of stamps sideways (face and head / up or down; again four possibilities) through the perforator since the feeding of stamps in that manner might have defeated even the most diehard prankster or philatelist.

I would further **speculate** here that these manipulated patterns would be more likely to survive than the 'true' face up/head up pattern because the need to manipulate the perforator to produce them suggests to me only two possible reasons for so doing - '...because the operator could...', or as favors for a stamp collector friend. In the first case chances are it was done for the thrill of the doing and the stamps would have simply been used and have no greater chance of survival than any of the 'true'

pattern stamps. In the latter case there is a very good chance they would survive in a collection or in a collector's stock as likely as not in mint condition.

Question 1 (after a very long introduction...): How many of our identified patterns are real; rephrased are some of the patterns currently listed and unidentified possibly an artifact of production tricks by the machine operator? The answer to this one will be as speculative as the question – but I think it really comes down to a question of whether any of the user-unidentified patterns in the list is known only in mint stamps. Were this to be the case, I think it would be time to start rotating and flipping the stamp to see if the pattern is possibly a rotation or flip punch of another reported pattern.

Question 2 (one of cataloging consistency): Since I first became aware of the Schermack perfins I have wondered why these patterns are described **backwards** from all other U.S. perfin patterns.

In all cases of multiple offices or users (except for Schermacks) where added holes indicate the branch office or a secondary user pattern, description assumes the addition of pins to a base pattern (see the International Harvester patterns [I43.7], or those of the New York Life Insurance Company [N113] to pick just two of many examples). Differing from this mode of describing patterns, the removal of pins from some idealized 'complete' pattern is done for the Schermacks. This definition for the purposes of cataloging assumes that 9-pin dies were sitting on a shelf somewhere at Mail-0-Meter and when a new pattern was ordered selected pine were removed from these complete dies. This description of the process is rather hard to take in the face of the rather more obvious – only the pins needed to generate the pattern were ever placed in the perforator head. Nowhere in the Bulletin have I see mention of this discrepancy between the cataloging systems or the underlying fallacy that the description of Schermacks seems to propagate.

In creating the 1998 US perfin catalog John Randall took on several serious inconsistencies. Straightening out the bulk of the confusion which had crept in over time concerning the appending of a decimal descriptor versus the assignment of a capital letter suffix to pattern numbers, is a prime example of the sticky changes accomplished by Randall.

On the next four pages I present all 511 possible Schermack patterns for a stamp read face up/head up. They are sequenced additively rather than in the current subtractive manner, and many are yet to be found or non-existent patterns. Also included in the listing are patterns that would be indistinguishable from each other based purely on pattern.

Question 3: Should we impose Travell's second condition that similar patterns be eliminated from consideration as they would be indistinguishable from each other when punched in stamps?

To try to get a handle on the answer to this question I began to approach it with simple statistics and two, possibly very flawed, assumptions.

My first assumption was that machine punching is consistent and all patterns punched by a single perforator will be in the exact same position relative to the rectangular perforations used to advance the stamps for affixing. Thus stamps punched using the same perforator should have the same distance between the fight side of the left side rectangles and the right side of holes 1, 4, & 7. In the same manner the right side of holes in positions 3, 6, & 9 should always be at the same distance from the left side of the rectangular holes on the right side of the stamp.

And my second assumption was that different perforators were created equal. All of the 9-hole patterns are 7 mm on a side within very serious tolerance. So why not punch a bunch of heads and base plates at the same time such that when they were fit into different machines (each with a different pattern) the position of the pattern remained constant in the stamps punched when cross-compared between machines.

The second assumption is relatively easy to check – get out the old micrometer and the collection and measure the suggested distances. I have 20 (19 + a Randall deleted pattern) mounted in my collection so there are enough observations here to check the second assumption. And this is where you hear hte buzzer on 'Jeopardy' – wrong answer. With the stamps face up / head up measurements from the left rectangle to the first hole encountered to its right ranged from 4.1 mm to 7.2 mm with a mean +/- 95% standard deviation of 5,72 +/- 0,93 mm. for the same measure on the right sides of the pattern the measurements ranged from 5.7 mm – 9.1 mm with a mean of 7.15 +/- 0.97 mm. If these observations come from the same 'population 95% of the observations should fall within the range of the interval given for each measurement. Again, the Jeopardy buzzer sounds; 7 of the 20 right side and 12 of the 20 left side measurements did not fall within the confidence interval based on the applicable average & standard deviation. Similar measurements for top to bottom position in the stamps gave similar results.

As to the first assumption, my statistics don't really carry nay weight since I only had two duplicate items and stats based on n-1 observations where n - 1 = 1 are meaningless. However, my observations on these two pairs are instructive. The two -44s which I have show distinctly different positions on the stamps in which they are punched – one being at left = 5.9mm and right at 7.8 mm while the other is virtually in the exact opposite location relative to measurements - left at 7.9 mm and right at 5.9 mm. The two copies I have of -59 show a similar, if not as dramatic, discrepancy in position relative to the rectangular perforations; one is found at left = 5 mm, right = 8 mm while the other is centered relative to the perforations with both left and right measurements being 6.5 mm.

So, after checking the material at hand, I would say unequivocally that there is no usable consistency of position to the various Schermack patterns that can be used to distinguish between similar patterns which lack at least one hole in each of the rows or columns. In fact, given the ranges of position a 16 hole (4 x 4 holes) square would be possible within the space described. What this means in practical terms is that given the range of hole positions his 4 5 6 7 8 9 pattern could be shifted in position such that it could legitimately be read as 1 2 3 4 5 6.

Thus Travell's second premise must be accepted.

I am less happy with his first condition that 1-, 2- and 3-hole patterns do not exist, and have

retained these patterns in the illustrations below, and in my proposed sequential numbering.

Question 4: Should Schermack descriptions be redone to reflect holes (as is done for all other patterns) rather than lacl of holes? Clearly, since I raise the question. I feel the answer is yes. Maybe, thankfully, it is not my call since the system has been in place a long time any manipulation will result in confusion, for a time – bu I tend to be compulsive about consistency and see the long term benefit to changing the descriptions to reflect presence rather than absence of holes.

Question 5: Should the Schermacks be renumbered to recognize the more standard additive pattern of holes. Smae argument here. Any renumbering will cause at least short-trerm confusion, but in the long term I feel it will serve us well to be consistent.

Question 6: If the answer to Q. 5 is yes, should the sequence of the catalog numbers take advantage of the unique reality of the Scermacks, a maximum of 511 possible face up/head up patterns, and assign the numbers reflecting that sequence. Thus current pattern numbering is a linear sequence (-1 => -75) recognizing known patterns and having gaps in the numbering only where patterns have been deleted for cause.

In my proposed system numbers are assigned sequentially in the overall scheme of possible added pin patterns, beginning at Des 91-1 (not yet known and ending with Des 91-511 (current Des 91-1). Numbering would recognize the use of only the lowest number in a similar 'position set' (Travell's condition #2), and lacking absolute confirmation of user for two different positions in the set, the other numbers in the set would never be used, but would be retained in the overall scheme, as they are redundant to the cataloged position. Using this logic Pattern Des 91-1 would be assigned and then all of the other single pin patterns would be cataloged Des 91-1.

On the next six pages (Tablel 1) are presented all of the 511 patterns (framed to distinguish one from another). Patterns are numbered below the frame with a sequential number assigned when the pattern was located in the logical additive (row "A") position in the listing. Below that number is numbering in reverse sequence, the old subtractive pattern (row "S"). And, below that in row "P" is a proposed new number for the pattern.

The proposed numbers are in the reverse order of the current numbering with two significant modifications. I chose this pattern when I recognized that the reality of these patterns is that they apparently were (as Travell apparently correctly reflects in his numbering) assigned using the fullest patterns first 9 hole, then 8 hole, then 7 hole, etc.) and working back to the single 3 hole pattern currently recognized. The two differences introduced are: some numbers appear in the 'P' rows more than once (with the highest proposed catalog number being 400), and the numbering for each "hole count section is done in the additive rather than in the subtractive manner currently used in the catalog. The total of 400 possible distinct patterns is based on a statement made earlier -- position of a pattern with no pin in either a row or column (or both) can not be distinguished from a similar pattern created using the same pin orientation but not the same position (for example - pattern 1-2-4-5 can not be distinguished from 2-3-5-6, 4-5-7-8 or 5-6-8-9 when punched in a stamp). One hundred eleven patterns are from indistinguishable another numbered. sequenced pattern, and share a proposed catalog number (the lowest number proposed for the complex). This is easily seen in the very first line of Table 1.1 where the first nine patterns. the single pin patterns, share proposed catalog number 400.

Boldfaced patterns, with a number to the left of the grid, are the currently identified patterns with the catalog number to the left ('Des 90 -' is assumed to preface each of these numbers).

Following this listing is Table 2, a listing of the patterns that are indistinguishable from oneanother. In the left column is the sequence number of the pattern in question (additive pattern list; Rows A in Table 1) and on the right (after the '=') the number of the allowed pattern to which it is identical in the additive list. Numbers in Table 1, Row A that are struck through are found in this table (2) with equivalent patterns noted.

In Table 3 I cross index the various numbers assigned within the four sequencing patterns; the Continued p. 16

	$\begin{array}{c c} \bullet \\ \hline 2 \\ \hline 4 \\ \hline 9 \\ \hline 5 \\ \hline 9 \\ \hline \end{array}$	•	• 5 0 7	$ \begin{array}{c c} $	• 5 0 4 1 0 0	2 hole • • • • • • • • • • • • • • •
A 1 1 S 5 0 1 5	0 0 4 0 0 • • 1 2 1 3 0 0 4 9 9 9 0 3 9 1	4 0 0 • • 1 4 4 9 8 3 9 2	4 0 0 • 1 5 4 9 7 3 9 3	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c} 4 & 0 & 0 \\ & \bullet & \bullet \\ & & \bullet \\ $	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
A 21 S 491 P 391 3	• 2 2 9 0 9 7 3 9 3	• 2 4 4 8 8 3 9 4	• 2 5 4 8 7 3 9 8	2 6 4 8 3 9	2 8 4 8 4 3 9 9	29 30 483 397 393
A 31 S 4 81 P 3 8 8 3	• 3 2 8 0 8 9 3 9 0	• 34 478 391	•	3 6 3 7 4 7 6 4 7 5 3 8 3 9 6 3 hole patterns: 1 1 1	• • 4 7 4 3 9 0	• • 39 40 473 472 391 398
A 41 S 471 P 396 3	• 4 2 4 3 7 0 4 6 9 9 0 3 8 8	4 4 4 6 8 3 8 9	45 467 388	4 6 4 7 4 6 6 4 6 5 3 4 0 3 4 1	• • • 4 8 4 6 4 3 4 2	• • 4 9 5 0 4 6 3 4 6 2 3 4 3 3 4 4
	• • 5 2 6 0 4 6 3 4 7	5 4 4 5 8 3 4 8	55 457 349	• • • # 7 5 6 5 7 5 5 7 4 5 6 5 7 4 5 5 7 4 5 5 7 4 5 5 7 4 5 5 7 3 5 6 3 5 1 1 1 1 1 1 1 1	5 8 4 5 4 3 5 2	• • • 5 9 6 0 4 5 3 4 5 2 3 5 3 3 5 4
• • • A 6 1 S 4 5 1 P 3 5 5 3	• 6 2 5 0 5 6 3 5 7	• • • 6 4 4 4 8 3 5 8	• • • • • • • • • • • • • • • • • • •	• • 6 6 6 7 4 4 6 4 4 5 3 6 0 3 6 1	• • • • • • • • • • • • • • • • • • • •	• • 6 9 7 0 4 4 3 4 4 2 3 6 3 3 6 4
• • • • • • • • • • • • • • • • • • •	7 2 4 0 6 6 7 3 4 3 9 3 6 7	• • • • 7 4 4 3 8 3 6 8	••• • 4 3 7 3 4 1	7 6 7 7 4 3 6 4 3 5 3 4 2 3 6 9	7 8 4 3 4 3 4 4	79 80 433 432 345 370
A 81 S 431 P 371 3	8 2 8 3 3 0 4 2 9 7 2 3 7 3	• • 4 2 8 3 7 4	• • • 8 5 4 2 7 3 5 3	• • • •	• • • • • • • • • • • • • • • • • • •	• • 8 9 9 0 4 2 3 4 2 2 3 7 6 3 5 9

Table 1.1: Possible Schermack pats sequenced additively. Bold cells = pats currently listed in Randall's US Catalog (his catalog numbers at left); struck out # = pat duplicates shape of another (see Tab. 2).

3 hole pat (cont.)				
A 91 92 93	9495 418417	9697 416415	9899 414413	$\begin{array}{c} 1 \\ 0 \\ 4 \\ 1 \\ 2 \end{array}$
S 4 2 1 4 1 9 P 3 6 0 3 7 7 8	4 1 8 4 1 7 3 6 5 3 7 9	380 381	382 383	370
			•	• • •
$ \begin{array}{c c} \bullet \\ \bullet \\ A & 1 & 0 & 1 \end{array} \begin{array}{c} \bullet \\ \hline \bullet & 0 & 2 \end{array} \begin{array}{c} \bullet \\ \hline \bullet & 0 & 2 \end{array} \begin{array}{c} \bullet \\ \hline \bullet & 0 & 3 \end{array} $	$ \begin{array}{c c} \bullet \\ 1 & 0 & 4 \end{array} \begin{array}{c} \bullet \\ 1 & 0 & 4 \end{array} $	$\begin{array}{c c} \bullet \\ \hline \hline \bullet \\ \hline \hline \hline \bullet \\ \hline \hline \hline \bullet \\ \hline \hline \hline \hline$	$ \begin{array}{ c c c } \bullet & \bullet \\ \hline 1 & 0 & 8 \end{array} \begin{array}{ c } \bullet & \bullet \\ \hline 1 & 0 & 8 \end{array} \begin{array}{ c } \bullet & \bullet \\ \hline 1 & 0 & 9 \end{array} $	<u>+</u> + 0
A 1 0 1 ± 0 2 ± 0 3 S 4 1 1 4 ± 0 4 0 9 P 3 8 4 3 7 2 3 7 3	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	406 405 355 386	4 0 4 4 0 3 3 8 7 3 7 7	4 0 2 3 4 0
		••• ••		•
A ± <td>1 1 4 1 1 5 3 9 8 3 9 7 3 4 7 3 4 8</td> <td>++6 ++7</td> <td>++8 ++9</td> <td>120 392</td>	1 1 4 1 1 5 3 9 8 3 9 7 3 4 7 3 4 8	++6 ++7	++8 ++9	120 392
S 4 0 1 4 0 0 3 9 9 P 3 4 1 3 4 2 3 4 3	3 9 8 3 9 7 3 4 7 3 4 8	3 9 6 3 9 5 3 4 9 3 5 3	394 393 354 358	3 9 2 3 6 8 4 hole:
				• • •
A 1 2 1 1 2 2 1 2 3	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{ c c c c c } \bullet \bullet & \bullet & \bullet \\ \hline 1 & 2 & 6 & 1 & 2 & 7 \end{array}$	• • • • • ± 2 8 ± 2 9	1 3 0
A ± 2 ± ± 2 2 ± 2 3 S 3 9 1 3 9 0 3 8 9 P 3 4 1 3 4 2 3 7 0	1 2 4 1 2 5 3 8 8 3 8 7 3 7 1 3 5 3	1 2 6 1 2 7 3 8 6 3 8 5 3 7 9 3 8 0	1 2 8 1 2 9 3 8 4 3 8 3 3 7 0 3 4 0	3 8 2 2 4 3
	•••	••	••	• •
• • 7 4 •		••		•
A 1 3 1 1 3 2 1 3 3 S 3 8 1 3 8 0 3 7 9 P 2 4 4 2 4 5 2 4 6	1 3 4 3 7 8 3 7 7	1 3 6 1 3 7 3 7 6 3 7 5	1 3 8 1 3 9 3 7 4 3 7 3	1 4 0 3 7 2
<u>P 2 4 4 2 4 5 2 4 6</u>	247 248	249 250	251 252	253
				••
A 1 4 1 S 3 7 1 A 1 4 1 S 3 7 1 A 7 0 A 1 4 3 C 0 C	1 4 4 1 4 5 3 6 8 3 6 7	• • 1 4 6 1 4 7 3 6 6 3 6 5		1 5 0 3 6 2
S 3 7 1 3 7 0 3 6 9 P 2 5 4 2 5 5 2 5 6	3 6 8 3 6 7 2 5 7 2 5 8	1 4 6 1 4 7 3 6 6 3 6 5 2 5 9 2 6 0	3 6 4 3 6 3 2 6 1 2 6 2	263
••• ••• 7••			* • • • • • • • • • • • • • • • • • • •	•••
A 1 5 1 1 5 2 1 5 3	• •		0 • •	• 1 6 0
S 3 6 1 3 6 0 3 5 9 P 2 6 4 2 6 5 2 6 6	1 5 1 5 5 3 5 8 3 5 7 2 6 7 2 6 8	1 5 6 1 5 7 3 5 6 3 5 5 2 6 9 2 7 0	158 159 354 353 271 272	1 6 0 3 5 2 2 7 3
				•
	6			••
A 1 6 1 1 6 2 1 6 3	164 165	1 6 1 6 7 3 4 6 3 4 5 2 7 9 2 8 0	1 6 8 1 6 9 3 4 4 3 4 3 2 8 1 2 8 2	1 7 0 3 4 2 2 8 3
S 3 5 1 3 5 0 3 4 9 P 2 7 4 2 7 5 2 7 6	3 4 8 3 4 7 2 7 7 2 7 8	279 280	281 282	283
				•
A 1 7 1 1 7 2 1 7 3	• •		• • • 1 7 8 1 7 9	• • 1 8 0
A 1 7 1 7 2 1 7 3 S 3 4 1 3 4 0 3 3 9 P 2 8 4 2 8 5 3 8 6	1 7 4 1 7 5 3 3 8 3 3 7 3 8 7 3 8 8	1 7 6 1 7 3 3 6 3 3 5 3 8 9 3 9 0	1 7 8 1 7 9 3 3 4 3 3 3 3 9 1 3 9 2	1 8 0 3 3 2 3 9 3

Table 1.2: Possible Schermack pats sequenced additively. Bold cells = pats currently listed in Randall's US Catalog (his catalog numbers at left); struck out # = pat duplicates shape of another (see Tab. 2).

<u>4 hole p</u> a	ts <u>(cont)</u>			·		
••	· · · ·	• •	#• 6		• • •	
A 1 8 1		184	8 • • • 1 8 5	186 18		• • 189 190
S 3 3 1 P 2 9 4	3 3 0 3 2 9 2 9 5 2 9 6	328 297	327 298	3 2 6 3 2 2 9 9 3 0	2 5 3 2 4) 0 3 0 1	3 2 3 3 2 2 3 0 2 3 0 3
# • •	# • • •	••	••	••	• • •	••
7	6	•				
A 191 S 321 P 249	192 320 349 304 251	$\frac{1}{3}$ $\frac{9}{4}$ $\frac{4}{3}$ $\frac{1}{2}$ $\frac{8}{2}$ $\frac{2}{5}$ $\frac{2}{2}$	$ \begin{array}{r} 1 & 9 & 5 \\ 3 & 1 & 7 \\ 3 & 0 & 5 \end{array} $	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c cccc} \hline 7 & 1 & 9 & 8 \\ \hline 5 & 5 & 3 & 1 & 4 \\ \hline 5 & 6 & 3 & 0 & 6 \\ \hline \end{array}$	$\begin{array}{c c} 1 9 9 & 2 \theta \theta \\ 3 1 3 & 3 \frac{1}{2} 2 \end{array}$
<u>P 2 4 9</u>	304 251	252			56 306	307 261
•••			• # • • 6 • 5	•••		
A 2 0 1 S 3 1 1	2 0 2 2 0 3 3 1 0 3 0 9 3 0 9 3 1 0	2 0 4 3 0 8	2 0 5 3 0 7 3 1 2	206 20	0 2 0 0 7 2 0 0 5 3 0 4 3 1 5	• • • • 2 0 9 2 1 0 3 0 3 0 2 1 0 3 1 6 3 1 7
S 3 1 1 P 3 0 8	309 310	3 0 8 3 1 1	3 1 2	306 30 313 31	0 5 3 0 4 1 4 3 1 5	3 1 6 3 1 7
			• #			
A 2 1 1	• • •	• • 2 1 4	••• 3	2 1 6 2 1		
A 2 1 1 S 3 0 1 P 3 1 8	2 1 2 1 3 3 0 0 2 9 9 2 8 0 2 8 1	2 1 4 2 9 8 3 1 9	2 9 7 3 2 0	296 29 286 32	1 7 2 1 8 9 5 2 9 4 2 1 3 2 2	2 1 9 2 2 0 2 9 3 2 9 2 9 2 9 2 9 2 3 2 3 2 3 2 3 2 3
•	•		# •	•	• •	• •
• • •			6•• 2•	•••	• •	
A 2 2 1 S 2 9 1 P 3 2 4	2 2 2 3 2 9 0 2 8 9 3 2 5 3 2 6	2 2 4 2 8 8 3 2 7	2 2 5 2 8 7 3 2 8	2 2 6 2 2 2 8 6 2 8 3 2 9 3 3	2 7 2 2 8 3 5 2 8 4 3 0 3 3 1	2 2 9 2 3 0 2 8 3 2 8 2 3 3 2 3 3 3
<u>P 3 2 4</u>	325 326	327	328	329 33	30 331	3 3 2 3 3 3
•		•	•	•	• •	• # • • 6
• A 2 3 1 S 2 8 1	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	•• 234 278	• • 2 3 5	$\begin{array}{c c} \bullet \bullet \\ \hline 2 & 3 & 6 \end{array} \begin{array}{c} \bullet \bullet \\ \hline 2 & 3 \\ \hline \end{array}$	• •	<u>••</u> 1 <u>•••</u> 2 3 9 2 4 0
S 2 8 1 <u>P 3 3 4</u>	2 8 0 2 7 9 3 0 9 3 1 0	278 335	277 336	276 27 31433	75274 37338	2 7 3 2 7 2 3 1 9 3 3 9
	$\begin{array}{c c} \bullet \bullet \bullet \\ \bullet \\ \hline 2 4 2 \end{array} \begin{array}{c} \bullet \bullet \bullet \\ \hline 2 4 3 \end{array}$	••• ••		$\begin{array}{c c} \bullet \bullet \\ \hline \bullet \bullet \\ \hline 2 4 6 \end{array}$	$\begin{array}{c c} \bullet \\ \bullet $	$\begin{array}{ c c c c c } \hline \bullet & \bullet \\ \hline \bullet & \bullet \\ \hline 2 4 9 & 2 5 0 \end{array}$
A 2 4 1 S 2 7 1 P 2 4 3	2 4 2 4 3 2 7 0 2 6 9 2 4 4 2 4 5	$\frac{2}{2} + 4$ $\frac{2}{6} + 8$ $\frac{2}{2} + 4$	2 4 5 2 6 7 2 5 0	2 4 6 2 4 2 6 2 6 2 5 4 2	55 264	2 4 9 2 5 0 2 6 3 2 6 2 2 6 9 2 7 8
				5 hole patterns		
					• # • • • • 5 • 9 •	
A 2 5 1 S 2 6 1 P 2 9 9	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\frac{2}{2}$ $\frac{5}{4}$	2 5 5 2 5 7 3 2 4	25625 25625 12913	5 7 2 5 8 5 5 2 5 4 3 0 1 3 1	2 5 9 2 6 0 2 5 3 2 5 2 1 3 2 1 3 3
P 2 9 9	300 249	258 308	324	129 13	<u>131</u>	<u>1 3 2 1 3 3</u>
•••	•••• •5 • 8	# • • • 5 • 7 •	•••	••• •5 •6	•••	•••
	• 8 • • • • • • • • • • • • • • • • • •			266 26	• • • 5 7 2 6 8	
A 2 6 1 S 2 5 1 P 1 3 4	2 6 2 2 6 3 2 5 0 2 4 9 1 3 5 1 3 6	2 6 4 2 4 8 1 3 7	2 6 5 2 4 7 1 3 8	2 6 6 2 6 2 4 6 2 4 1 3 9 1 4	2 6 8 5 2 4 4 0 1 4 1	2 6 9 2 7 0 2 4 3 2 4 2 1 4 2 1 4 3

Table 1.3: Possible Schermack pats sequenced additively. Bold cells = pats currently listed in Randall's US Catalog (his catalog numbers at left); struck out # = pat duplicates shape of another (see Tab. 2).

<u>5 hole pat</u>	ts <u>(cont)</u>	- p	·					ī	
A 271	272	273	274	275	276	2 7 7 2 3 5	278	279	280
S 2 4 1 P 1 4 4	2 4 0 1 4 5	2 3 9 1 4 6	238 147	237 148	236 149	$\begin{array}{cccc} 2 & 3 & 5 \\ 1 & 5 & 0 \end{array}$	234 151	233 152	2 3 2 1 5 3
	••				# • •	••	••	••	• •
••	••		•	•	5•	•			
A 2 8 1	282	283	284	285	286	2 8 7 2 2 5	288	289	• • • 2 9 0 2 2 2
A 2 8 1 S 2 3 1 P 1 5 4	$\begin{array}{ccc} 2 & 3 & 0 \\ 1 & 5 & 5 \end{array}$	2 8 3 2 2 9 1 5 6	2 2 8 1 5 7	227 158	226 159	2 2 5 1 6 0	224 161	223 162	222 163
••	••	••	••	••	••	••	••	••	••
•••	••	••	•••	•••	•••	•••	•	•	•
A 291 S 221 P 164	292 220	293 219	294 218	295 217	2 9 6 2 1 6 1 6 9	297 215	298 214	299 213	300 212
<u>P 1 6 4</u>	1 6 5	2 1 9 1 6 6	1 6 7	1 6 8	1 6 9	1 7 0	1 7 1	2 1 3 1 7 2	1 7 3
•••	•••	•••	••#		•••	•••	•••	•	•••
A 3 0 1			•• 4		306	307	••		•••
S 2 1 1	3 0 2 2 1 0	3 0 3 2 0 9	208	305207	206	205	3 0 8 2 0 4	3 0 9 2 0 3	3 1 0 2 0 2
<u>P 1 7 4</u>	175	176	177	178	1 7 9	180	181	182	183
•••	••••	•			•••	•••	•••	• • #	5•
•	•	•							
A 3 1 1	312	3 1 3	314	• • 3 1 5	316	•• 3 1 7	• • 3 1 8	3 1 9	3 <u>• • •</u> 3 2 0
A 3 1 1 S 2 0 1 P 1 8 4	$ \begin{array}{r} 3 1 2 \\ 2 0 0 \\ 1 8 5 \end{array} $	313 199	314 198	3 1 5 1 9 7	316 196	317 195	318 194	319 193	320 192
A 3 1 1 S 2 0 1 P 1 8 4	3 1 2 2 0 0 1 8 5	3 1 3	3 1 4	3 1 5	316	3 1 7 1 9 5 1 9 0	3 1 8 1 9 4 1 9 1	3 1 9 1 9 3 1 9 2	320
$ \begin{array}{c} A & 3 & 1 & 1 \\ S & 2 & 0 & 1 \\ P & 1 & 8 & 4 \\ \end{array} $	200	3 1 3 1 9 9 1 8 6	3 1 4 1 9 8 1 8 7	3 1 5 1 9 7	316 196	3 1 7 1 9 5 1 9 0	3 1 8 1 9 4 1 9 1	319 193	320 192
S 2 0 1 P 1 8 4	$\begin{array}{c} 2 & 0 & 0 \\ 1 & 8 & 5 \\ \hline \bullet & \bullet \\ 3 & 2 & 2 \end{array}$	$ \begin{array}{c} 3 & 1 & 3 \\ 1 & 9 & 9 \\ 1 & 8 & 6 \\ \end{array} $	$ \begin{array}{r} 3 1 4 \\ 1 9 8 \\ 1 8 7 \end{array} $	3 1 5 1 9 7 1 8 8	$ \begin{array}{c} 3 & 1 & 6 \\ 1 & 9 & 6 \\ 1 & 8 & 9 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 2 & 6 \end{array} $	$ \begin{array}{c} 3 & 1 & 7 \\ 1 & 9 & 5 \\ 1 & 9 & 0 \\ \end{array} $	3 1 8 1 9 4 1 9 1 \$ 5 2 3 2 8	$ \begin{array}{c} 3 & 1 & 9 \\ 1 & 9 & 3 \\ 1 & 9 & 2 \end{array} $	3 2 0 1 9 2 1 9 3
S 2 0 1 P 1 8 4	2 0 0 1 8 5	3 1 3 1 9 9 1 8 6	3 1 4 1 9 8 1 8 7	3 1 5 1 9 7 1 8 8	3 1 6 1 9 6 1 8 9	3 1 7 1 9 5 1 9 0	3 1 8 1 9 4 1 9 1 \$ • • 2 •	3 1 9 1 9 3 1 9 2	3 2 0 1 9 2 1 9 3
S 2 0 1 P 1 8 4	$ \begin{array}{c} 2 & 0 & 0 \\ 1 & 8 & 5 \\ \end{array} $	$ \begin{array}{c} 3 & 1 & 3 \\ 1 & 9 & 9 \\ 1 & 8 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 3 \\ 1 & 8 & 9 \\ \end{array} $	$ \begin{array}{c} 3 & 1 & 4 \\ 1 & 9 & 8 \\ 1 & 8 & 7 \end{array} $ $ \begin{array}{c} \bullet \\ \bullet \\ 3 & 2 & 4 \\ 1 & 8 & 8 \end{array} $	3 1 5 1 9 7 1 8 8 • • • 3 2 5 1 8 7	$ \begin{array}{c} 3 & 1 & 6 \\ 1 & 9 & 6 \\ 1 & 8 & 9 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 2 & 6 \\ 1 & 8 & 6 \end{array} $	$ \begin{array}{c} 3 & 1 & 7 \\ 1 & 9 & 5 \\ 1 & 9 & 0 \\ \end{array} $	3 1 8 1 9 4 1 9 1 \$ • • 5 • • 5 • • 5 • • 3 2 8 1 8 4 2 0 1 • • • #	3 1 9 1 9 3 1 9 2 • • • 3 2 9 1 8 3 2 0 2	3 2 0 1 9 2 1 9 3
$ \begin{array}{c} S & 2 & 0 & 1 \\ P & 1 & 8 & 4 \\ \hline $	$ \begin{array}{c} 2 & 0 & 0 \\ 1 & 8 & 5 \\ \hline $	3 1 3 1 9 9 1 8 6 • • • 3 2 3 1 8 9 1 9 6	$ \begin{array}{c} 3 & 1 & 4 \\ 1 & 9 & 8 \\ 1 & 8 & 7 \\ \hline $	$ \begin{array}{c} 3 & 1 & 5 \\ 1 & 9 & 7 \\ 1 & 8 & 8 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 5 \\ 1 & 8 & 7 \\ 1 & 9 & 8 \end{array} $	$ \begin{array}{c} 3 & 1 & 6 \\ 1 & 9 & 6 \\ 1 & 8 & 9 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 6 \\ 1 & 8 & 6 \\ 1 & 9 & 9 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & $	$ \begin{array}{c} 3 & 1 & 7 \\ 1 & 9 & 5 \\ 1 & 9 & 0 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet \\ 3 & 2 & 7 \\ 1 & 8 & 5 \\ 2 & 0 & 0 \\ \end{array} $	3 1 8 1 9 4 1 9 1 \$ 5 • • 2 3 2 8 1 8 4 2 0 1 • • • 5 • • 5 • • 2 • • 3 2 8 1 8 4 2 0 1	$ \begin{array}{c} 3 & 1 & 9 \\ 1 & 9 & 3 \\ 1 & 9 & 2 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 2 & 9 \\ 1 & 8 & 3 \\ 2 & 0 & 2 \end{array} $	3 2 0 1 9 2 1 9 3 • • • • • • 3 3 0 1 8 2 2 0 3 • • • • • • • • • • • •
$ \begin{array}{c} S & 2 & 0 & 1 \\ P & 1 & 8 & 4 \\ \hline $	$ \begin{array}{c} 2 & 0 & 0 \\ 1 & 8 & 5 \\ \hline $	$ \begin{array}{c} 3 & 1 & 3 \\ 1 & 9 & 9 \\ 1 & 8 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 3 \\ 1 & 8 & 9 \\ 1 & 9 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \\ 3 & 3 & 3 \\ \end{array} $	$ \begin{array}{c} 3 & 1 & 4 \\ 1 & 9 & 8 \\ 1 & 8 & 7 \\ \end{array} $ • • • • • • • • • • • • • • • • • •	$ \begin{array}{c} 3 & 1 & 5 \\ 1 & 9 & 7 \\ 1 & 8 & 8 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 5 \\ 1 & 8 & 7 \\ 1 & 9 & 8 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \\ 3 & 3 & 5 \end{array} $	$ \begin{array}{c} 3 & 1 & 6 \\ 1 & 9 & 6 \\ 1 & 8 & 9 \\ \hline $	$ \begin{array}{c} 3 & 1 & 7 \\ 1 & 9 & 5 \\ 1 & 9 & 0 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet \\ 3 & 2 & 7 \\ 1 & 8 & 5 \\ 2 & 0 & 0 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet \\$	3 1 8 1 9 4 1 9 1 \$ • • • 3 2 8 1 8 4 2 0 1 • • • • • • • • • • • • • • • • • • •	$ \begin{array}{c} 3 & 1 & 9 \\ 1 & 9 & 3 \\ 1 & 9 & 2 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 9 \\ 1 & 8 & 3 \\ 2 & 0 & 2 \end{array} $	3 2 0 1 9 2 1 9 3 • • • • • • 3 3 0 1 8 2 2 0 3 • • • • • • • • • • • • • • • • •
$ \begin{array}{c} S & 2 & 0 & 1 \\ P & 1 & 8 & 4 \\ \hline $	$ \begin{array}{c} 2 & 0 & 0 \\ 1 & 8 & 5 \\ \hline $	$ \begin{array}{c} 3 & 1 & 3 \\ 1 & 9 & 9 \\ 1 & 8 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 3 \\ 1 & 8 & 9 \\ 1 & 9 & 6 \\ \end{array} $	$ \begin{array}{c} 3 & 1 & 4 \\ 1 & 9 & 8 \\ 1 & 8 & 7 \\ \hline $	$ \begin{array}{c} 3 & 1 & 5 \\ 1 & 9 & 7 \\ 1 & 8 & 8 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 5 \\ 1 & 8 & 7 \\ 1 & 9 & 8 \end{array} $	$ \begin{array}{c} 3 & 1 & 6 \\ 1 & 9 & 6 \\ 1 & 8 & 9 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 6 \\ 1 & 8 & 6 \\ 1 & 9 & 9 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & $	$ \begin{array}{c} 3 & 1 & 7 \\ 1 & 9 & 5 \\ 1 & 9 & 0 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet \\ 3 & 2 & 7 \\ 1 & 8 & 5 \\ 2 & 0 & 0 \\ \end{array} $	3 1 8 1 9 4 1 9 1 \$ 5 • • 2 3 2 8 1 8 4 2 0 1 • • • 5 • • 5 • • 2 • • 3 2 8 1 8 4 2 0 1	$ \begin{array}{c} 3 & 1 & 9 \\ 1 & 9 & 3 \\ 1 & 9 & 2 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 2 & 9 \\ 1 & 8 & 3 \\ 2 & 0 & 2 \end{array} $	3 2 0 1 9 2 1 9 3 • • • • • • 3 3 0 1 8 2 2 0 3 • • • • • • • • • • • •
$ \begin{array}{c} S & 2 & 0 & 1 \\ P & 1 & 8 & 4 \\ \hline $	$ \begin{array}{c} 2 & 0 & 0 \\ 1 & 8 & 5 \\ \hline $	$ \begin{array}{c} 3 & 1 & 3 \\ 1 & 9 & 9 \\ 1 & 8 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 3 \\ 1 & 8 & 9 \\ 1 & 9 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 3 & 3 \\ 1 & 7 & 9 \\ \end{array} $	$ \begin{array}{c} 3 & 1 & 4 \\ 1 & 9 & 8 \\ 1 & 8 & 7 \\ \end{array} $ • • • • • • • • • • • • • • • • • • •	$ \begin{array}{c} 3 & 1 & 5 \\ 1 & 9 & 7 \\ 1 & 8 & 8 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 5 \\ 1 & 8 & 7 \\ 1 & 9 & 8 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \\ 3 & 3 & 5 \\ 1 & 7 & 7 \end{array} $	$ \begin{array}{c} 3 & 1 & 6 \\ 1 & 9 & 6 \\ 1 & 8 & 9 \\ \hline $	$ \begin{array}{c} 3 & 1 & 7 \\ 1 & 9 & 5 \\ 1 & 9 & 0 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 2 & 7 \\ 1 & 8 & 5 \\ 2 & 0 & 0 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \\ \hline 3 & 3 & 7 \\ + & 7 & 5 \\ 1 & 4 & 5 \\ \end{array} $	3 1 8 1 9 4 1 9 1 5 • • • 3 2 8 1 8 4 2 0 1 • • • 3 3 8 1 7 4 1 4 6 # •	3 1 9 1 9 3 1 9 2 ••• 3 2 9 1 8 3 2 0 2 ••• • • • • • • • • • • • • • • • • •	3 2 0 1 9 2 1 9 3 • • • • • • • • • • • • • • • • • • • •
$ \begin{array}{c} S & 2 & 0 & 1 \\ P & 1 & 8 & 4 \\ \hline $	$ \begin{array}{c} 2 & 0 & 0 \\ 1 & 8 & 5 \\ \hline $	$ \begin{array}{c} 3 & 1 & 3 \\ 1 & 9 & 9 \\ 1 & 8 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 3 \\ 1 & 8 & 9 \\ 1 & 9 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 3 & 3 \\ 1 & 7 & 9 \\ 2 & 0 & 6 \\ \end{array} $	$ \begin{array}{c} 3 1 4 \\ 1 9 8 \\ 1 8 7 \end{array} $ • • • • • • • • • • • • • • • • • •	$ \begin{array}{c} 3 & 1 & 5 \\ 1 & 9 & 7 \\ 1 & 8 & 8 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 5 \\ 1 & 8 & 7 \\ 1 & 9 & 8 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 3 & 5 \\ 1 & 7 & 7 \\ 2 & 0 & 8 \end{array} $	$ \begin{array}{c} 3 & 1 & 6 \\ 1 & 9 & 6 \\ 1 & 8 & 9 \\ \hline $	3 1 7 1 9 5 1 9 0 ••• •• 3 2 7 1 8 5 2 0 0 ••• •• •• •• •• 3 3 7 1 7 5 1 4 5 •• •• •• •• •• •• •• •• •• •• •• •• ••	3 1 8 1 9 4 1 9 1 5 • • • 5 • • 3 2 8 1 8 4 2 0 1 • • • 5 • • 3 3 8 1 7 4 1 4 6 # 4 9 • • • 9 • • •	3 1 9 1 9 3 1 9 2 • • • 3 2 9 1 8 3 2 0 2 • • • • 3 3 9 1 7 3 2 1 0 • •	3 2 0 1 9 2 1 9 3
$ \begin{array}{c} S & 2 & 0 & 1 \\ P & 1 & 8 & 4 \\ \hline $	$ \begin{array}{c} 2 & 0 & 0 \\ 1 & 8 & 5 \\ \hline $	$ \begin{array}{c} 3 & 1 & 3 \\ 1 & 9 & 9 \\ 1 & 8 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 3 \\ 1 & 8 & 9 \\ 1 & 9 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 3 & 3 \\ 1 & 7 & 9 \\ 2 & 0 & 6 \\ \end{array} $	$ \begin{array}{c} 3 1 4 \\ 1 9 8 \\ 1 8 7 \end{array} $ • • • • • • • • • • • • • • • • • •	$ \begin{array}{c} 3 & 1 & 5 \\ 1 & 9 & 7 \\ 1 & 8 & 8 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 5 \\ 1 & 8 & 7 \\ 1 & 9 & 8 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 3 & 5 \\ 1 & 7 & 7 \\ 2 & 0 & 8 \end{array} $	$ \begin{array}{c} 3 & 1 & 6 \\ 1 & 9 & 6 \\ 1 & 8 & 9 \\ \hline $	3 1 7 1 9 5 1 9 0 ••• •• 3 2 7 1 8 5 2 0 0 ••• •• •• •• •• 3 3 7 1 7 5 1 4 5 •• •• •• •• •• •• •• •• •• •• •• •• ••	3 1 8 1 9 4 1 9 1 5 • • • 5 • • 3 2 8 1 8 4 2 0 1 • • • 5 • • 3 3 8 1 7 4 1 4 6 # 4 9 • • • 9 • • •	3 1 9 1 9 3 1 9 2 • • • 3 2 9 1 8 3 2 0 2 • • • • 3 3 9 1 7 3 2 1 0 • •	3 2 0 1 9 2 1 9 3
$ \begin{array}{c} S & 2 & 0 & 1 \\ P & 1 & 8 & 4 \\ \hline $	$ \begin{array}{c} 2 & 0 & 0 \\ 1 & 8 & 5 \\ \hline $	$ \begin{array}{c} 3 & 1 & 3 \\ 1 & 9 & 9 \\ 1 & 8 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 3 \\ 1 & 8 & 9 \\ 1 & 9 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 3 & 3 \\ 1 & 7 & 9 \\ \end{array} $	$ \begin{array}{c} 3 & 1 & 4 \\ 1 & 9 & 8 \\ 1 & 8 & 7 \\ \end{array} $ • • • • • • • • • • • • • • • • • • •	3 1 5 1 9 7 1 8 8 • • • 3 2 5 1 8 7 1 9 8 • • • • 3 3 5 1 7 7 2 0 8	$ \begin{array}{c} 3 & 1 & 6 \\ 1 & 9 & 6 \\ 1 & 8 & 9 \\ \hline $	3 1 7 1 9 5 1 9 0 • • • • 3 2 7 1 8 5 2 0 0 • • • • • • 3 3 7 1 7 5 1 4 5 1 4 5 1 4 5	3 1 8 1 9 4 1 9 1 5 • • • 3 2 8 1 8 4 2 0 1 • • • 3 3 8 ± 7 4 1 4 6 # • • •	3 1 9 1 9 3 1 9 2 • • • 3 2 9 1 8 3 2 0 2 • • • • 3 3 9 1 7 3 2 1 0 • •	3 2 0 1 9 2 1 9 3 3 3 0 1 8 2 2 0 3 3 4 0 1 7 2 2 1 1
$ \begin{array}{c} S & 2 & 0 & 1 \\ P & 1 & 8 & 4 \\ \hline $	$ \begin{array}{c} 2 & 0 & 0 \\ 1 & 8 & 5 \\ \hline $	$ \begin{array}{c} 3 & 1 & 3 \\ 1 & 9 & 9 \\ 1 & 8 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 3 \\ 1 & 8 & 9 \\ 1 & 9 & 6 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 3 & 3 \\ 1 & 7 & 9 \\ 2 & 0 & 6 \\ \end{array} $	$ \begin{array}{c} 3 1 4 \\ 1 9 8 \\ 1 8 7 \end{array} $ • • • • • • • • • • • • • • • • • •	$ \begin{array}{c} 3 & 1 & 5 \\ 1 & 9 & 7 \\ 1 & 8 & 8 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 5 \\ 1 & 8 & 7 \\ 1 & 9 & 8 \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 3 & 5 \\ 1 & 7 & 7 \\ 2 & 0 & 8 \end{array} $	$ \begin{array}{c} 3 & 1 & 6 \\ 1 & 9 & 6 \\ 1 & 8 & 9 \\ \hline $	3 1 7 1 9 5 1 9 0 ••• •• 3 2 7 1 8 5 2 0 0 ••• •• •• •• •• 3 3 7 1 7 5 1 4 5 •• •• •• •• •• •• •• •• •• •• •• •• ••	3 1 8 1 9 4 1 9 1 5 • • • 5 • • 3 2 8 1 8 4 2 0 1 • • • 5 • • 3 3 8 1 7 4 1 4 6 # 4 9 • • • 9 • • •	3 1 9 1 9 3 1 9 2 • • • 3 2 9 1 8 3 2 0 2 • • • • 3 3 9 1 7 3 2 1 0 • • • • • • • • • • • • • • •	3 2 0 1 9 2 1 9 3
$\begin{array}{c} S & 2 & 0 & 1 \\ P & 1 & 8 & 4 \\ \hline & & & & \\ \bullet & & \bullet & \\ A & 3 & 2 & 1 \\ S & 1 & 9 & 1 \\ P & 1 & 9 & 4 \\ \hline & & \bullet & \bullet \\ A & 3 & 3 & 1 \\ S & 1 & 8 & 1 \\ P & 2 & 0 & 4 \\ \hline & & \bullet & \bullet \\ A & 3 & 4 & 1 \\ S & \frac{1}{2} & 7 & \frac{1}{2} \\ P & 1 & 5 & 1 \\ \hline & & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \hline & & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \hline \end{array}$	$ \begin{array}{c} 2 & 0 & 0 \\ 1 & 8 & 5 \\ \hline $	$ \begin{array}{c} 3 1 3 \\ 1 9 9 \\ 1 8 6 \end{array} $ $ \begin{array}{c} \bullet \\ \bullet \\$	$ \begin{array}{c} 3 & 1 & 4 \\ 1 & 9 & 8 \\ 1 & 8 & 7 \\ \hline $	$ \begin{array}{c} 3 & 1 & 5 \\ 1 & 9 & 7 \\ 1 & 8 & 8 \\ \hline & & & & \\ & & & & \\ & & & & \\ & & & &$	$ \begin{array}{c} 3 & 1 & 6 \\ 1 & 9 & 6 \\ 1 & 8 & 9 \\ \hline & \bullet & \bullet \\ 3 & 2 & 6 \\ 1 & 8 & 6 \\ 1 & 9 & 9 \\ \hline & \bullet & \bullet \\ 3 & 3 & 6 \\ 1 & 7 & 6 \\ 2 & 0 & 9 \\ \hline & \bullet & \bullet \\ 3 & 4 & 6 \\ 1 & 6 & 6 \\ 2 & 1 & 5 \\ \hline & \bullet & \bullet \\ \hline & \bullet & $	$ \begin{array}{c} 3 1 7 \\ 1 9 5 \\ 1 9 0 \\ \end{array} $ $ \begin{array}{c} \bullet \bullet \bullet \\ \bullet \bullet \bullet \\ \hline \end{array} $ $ \begin{array}{c} \bullet \bullet \bullet \\ \hline \end{array} $ $ \begin{array}{c} \bullet \bullet \bullet \\ \hline \end{array} $ $ \begin{array}{c} \bullet \bullet \bullet \\ \hline \end{array} $ $ \begin{array}{c} \bullet \bullet \bullet \\ \end{array} $ $ \begin{array}{c} \end{array} $ $ \end{array} $ $ \begin{array}{c} \end{array} $ $ \begin{array}{c} \end{array} $ $ \end{array} $ $ \end{array} $ $ \end{array} $ $ \begin{array}{c} \end{array} $ $ \end{array} $ $ \end{array} $ $ \end{array} $ $ \begin{array}{c} \end{array} $ $ \end{array} $	$3 1 8 1 9 4 1 9 1 5 \cdot \cdot$	3 1 9 1 9 3 1 9 2 • • • 3 2 9 1 8 3 2 0 2 • • • • 3 3 9 1 7 3 2 1 0 • • • • • • • • • • • • • • •	3 2 0 1 9 2 1 9 3
$\begin{array}{c} S & 2 & 0 & 1 \\ P & 1 & 8 & 4 \\ \hline \\ P & 1 & 8 & 4 \\ \hline \\ A & 3 & 2 & 1 \\ S & 1 & 9 & 1 \\ P & 1 & 9 & 4 \\ \hline \\ A & 3 & 2 & 1 \\ \hline \\ P & 1 & 9 & 4 \\ \hline \\ A & 3 & 2 & 1 \\ \hline \\ A & 3 & 2 & 1 \\ \hline \\ A & 3 & 2 & 1 \\ \hline \\ A & 3 & 2 & 1 \\ \hline \\ A & 3 & 2 & 1 \\ \hline \\ A & 3 & 3 & 1 \\ \hline \\ P & 2 & 0 & 4 \\ \hline \\ A & 3 & 4 & 1 \\ \hline \\ S & 1 & 7 & 1 \\ P & 1 & 5 & 1 \\ \hline \\$	$ \begin{array}{c} 2 & 0 & 0 \\ 1 & 8 & 5 \\ \hline $	$ \begin{array}{c} 3 & 1 & 3 \\ 1 & 9 & 9 \\ 1 & 8 & 6 \\ \end{array} $ $ \begin{array}{c} $	$ \begin{array}{c} 3 1 4 \\ 1 9 8 \\ 1 8 7 \\ \hline $	$ \begin{array}{c} 3 & 1 & 5 \\ 1 & 9 & 7 \\ 1 & 8 & 8 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 2 & 5 \\ 1 & 8 & 7 \\ 1 & 9 & 8 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ 3 & 3 & 5 \\ 1 & 7 & 7 \\ 2 & 0 & 8 \\ \end{array} $ $ \begin{array}{c} \bullet & \bullet \\ \bullet & \bullet \\ 3 & 4 & 5 \\ 1 & 6 & 7 \\ 2 & 1 & 4 \\ \end{array} $	$ \begin{array}{c} 3 & 1 & 6 \\ 1 & 9 & 6 \\ 1 & 8 & 9 \\ \hline & \bullet & \bullet \\ 3 & 2 & 6 \\ 1 & 8 & 6 \\ 1 & 9 & 9 \\ \hline & \bullet & \bullet \\ 3 & 3 & 6 \\ 1 & 7 & 6 \\ 2 & 0 & 9 \\ \hline & \bullet & \bullet \\ 3 & 4 & 6 \\ 1 & 6 & 6 \\ 2 & 1 & 5 \\ \hline & \bullet & \bullet \\ \hline & \bullet & \bullet \\ \end{array} $	3 1 7 1 9 5 1 9 0 3 2 7 1 8 5 2 0 0 3 3 7 1 7 3 3 7 1 7 3 3 7 1 4 5 3 4 7 1 6 5 2 1 6 	3 1 8 1 9 4 1 9 1 \$ • • • 3 2 8 1 8 4 2 0 1 • • 3 3 8 ± 7 4 1 4 6 # 4 9 • • • 3 4 8 1 6 4 2 1 7 • •	3 1 9 1 9 3 1 9 2 • • • 3 2 9 1 8 3 2 0 2 • • • • 3 3 9 1 7 3 2 1 0 • • • • • • • • • • • • • • •	3 2 0 1 9 2 1 9 3

Table 1.4: Possible Schermack pats sequenced additively. Bold cells = pats currently listed in Randall's US Catalog (his catalog numbers at left); struck out # = pat duplicates shape of another (see Tab. 2).

5 hole pats (cont) 4 7 7				•
A 3 6 1 3 6 2 S 1 5 1 1 5 0 P 2 2 9 2 3 0	3 6 3 3 6 4 1 4 9 1 4 8 2 3 1 2 3 2	3 6 5 3 6 6 3 6 1 4 7 1 4 6 1 4 2 3 3 2 3 4 2 3	15 144 143 1	70 42 38
# • 6 • A 3 7 1 S 1 4 1 P 2 3 9 6 hole p	••• ••• 373 374 ±39 138 218 241	# •	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	• • • • • • • • • • • • • • • • • • •
A 3 8 1 S 1 3 1 P 1 9 9 4 7	# •	• • • • • • • • # • • • • • • • • • # • • 3 8 5 3 8 6 3 8 1 2 7 1 2 6 1 2 5 0 5 1 5	3 7 3 8 8 3 8 9 3 2 5 1 2 4 1 2 3 1	• • 9 0 2 2 5 5
A 3 9 1 S 1 2 1 P 5 6 5 7	• • • • # • • • • • • # • • • • • • # • • • • • • # • • • 3 9 3 3 9 4 1 1 9 1 1 8 5 8 5 9	• • • # • • • • • 4 • • • • 4 • • 3 9 5 3 9 6 3 9 1 1 7 1 1 6 1 1 6 0 6 1 6		• • • • 0 0 1 2 6 5
• •	• •	••• ••• # ••• ••• ••• # ••• ••• ••• 8 ••• ••• 405 406 106 100 107 106 100 100 100 70 71 70 71 70	• •	• • 1 0 0 2 7 5
• • # • • • • • • • • • <td>• • • • • • • •<!--</td--><td>• • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • •</td><td> [_•_] [_•_] [• 17 418 419 4</td><td>• • 2 0 9 2 8 5</td></td>	• • • • • • • • </td <td>• • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • •</td> <td> [_•_] [_•_] [• 17 418 419 4</td> <td>• • 2 0 9 2 8 5</td>	• • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • •	[_•_] [_•_] [• 17 418 419 4	• • 2 0 9 2 8 5
• •	• •	• •	35 84 83	• • 3 0 8 2 9 5
• •	• •	3 • •	• •	• • • 4 0 7 2 0 5
• • • • • •	••• ••• ••• •	• •	# •	• • • • • • • • • • • • • • • • • • • •

Table 1.5: Possible Schermack pats sequenced additively. Bold cells = pats currently listed in Randall's US Catalog (his catalog numbers at left); struck out # = pat duplicates shape of another (see Tab. 2).



Table 1.6: Possible Schermack pats sequenced additively. Bold cells = pats currently listed in Randall's US Catalog (his catalog numbers at left); struck out # = pat duplicates shape of another (see Tab. 2).

current system (Randall catalog), the additive sequence, the subtractive sequence and a proposed sequencing based on listing all possible distinguishable patterns.

Finally, in Table 4 I present the current patterns listed with proposed additive catalog numbers ad descriptions based on the presence of a hole (rather than the current absence of one). In Table 4 you will easily see the reduction in the number of pins in the patterns from 9 to 3, and the patter of pin placement from pin 1 through pin 9 is clearly followed from pattern to pattern.

Of real interest to me is the absence of many probable patterns (the new -3, -5, -9, etc.) My guess is that many of the missing patterns at least those closest to the top of the list existed and were used but are lost

$\begin{array}{c} 3 \\ 4 \\ 5 \\ 8 \\ 7 \\ 8 \\ 9 \\ 8 \\ 2 \\ 1 \\ 3 \\ 4 \\ 8 \\ 9 \\ 8 \\ 8 \\ 8 \\ 8 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1$	2 = 3 =
$\begin{array}{c}1\\1\\1\\1\\1\\1\\0\\1\\3\\1\\5\\6\\9\\2\\2\\5\\1\\1\\1\\2\\3\\1\\0\\1\\1\\2\\3\\1\\0\\1\\1\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\1\\0\\1\\0\\1\\0\\1\\0\\1\\0\\1\\0\\1\\0\\1\\0\\1\\0\\1\\0\\0\\1\\0\\0\\1\\0$	1 1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	37 = 38 =
$\begin{array}{c}1&2\\1&3\\2&9\\1&1&0\\1&1&0\\1&4&8&0\\1&9&1&2\\5&5&9&1\\6&6&6&6&1\\8&8&8\\8&8\\\end{array}$	19 12
$\begin{array}{c} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 2 & 0 \\ 1 & 1 & 1 & 2 & 0 \\ 1 & 1 & 1 & 0 &$	$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$
$\begin{array}{c} 9 \\ 9 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 5 \\ 5 \\ 5 \\ 6 \\ 6 \\ 7 \\ 4 \\ 8 \\ 1 \\ 9 \\ 5 \\ 5 \\ 6 \\ 0 \\ 1 \\ 9 \\ 5 \\ 6 \\ 0 \\ 1 \\ 9 \\ 5 \\ 6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$	86 61
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$

Table 2: Patterns indistinguishable from others when punched; lowest available number selected for pattern (based on the additive model, row A in the previous table.)

Table 3: Potential renumbering of the Schermacks accounting for all possible positions. Note that the sequence of additive patterns is the reverse numerically of the current catalog (subtractive) numbering; 'proposed' is a combination of both in which "position patterns" share the same number.

Old	Additive	Subtractive	Proposed	1	Old	Additive	Subtractive Proposed
	- 5 1 1				- 37	= - 4 1 2	= -100 = -77
- 2 =	- 5 1 0	= -2	= - 1 0		- 38	= - 4 0 7	= -105 = -72
- 3 =	- 5 0 8	= -4	= -8		- 4 0	= -404	= -108 = -69
- 4 =	- 5 0 7	= -5	= - 7		- 4 1	= - 3 9 6	= -116 = -61
		= - 5					
- 5 =	- 506	= - 6	= - 6		- 4 2	= - 3 9 4	1 1 0 0 0 0
- 6 =	- 504	= - 8	= -4		- 4 3	= - 3 8 7	= - 1 2 5 = - 5 2
- 7 =	- 502	= -10	= - 2		- 4 4	= - 3 8 3	= - 1 2 9 = - 4 8
- 8 =	- 500	= -12	- 45		- 4 5	= - 3 7 5	= - 1 3 7 = - 2 4 2
- 9 =	- 499	= -13	- 4 4		- 4 6	= - 3 7 1	= - 1 4 1 = - 2 3 9
- 1 0 =	- 496	= -16	- 4 1		- 47	= - 3 6 3	= -150 = -230
- 1 1 =	- 4 9 4	- 18	- 39		- 4 9	= - 3 4 8	= - 1 6 4 = - 2 1 7
- 1 2 =	- 4 9 1	- 21	- 36		- 50	= - 3 4 7	= - 1 6 5 = - 2 1 6
- 1 3 =	- 488	- 24	- 33		- 51	= - 3 3 9	= - 1 7 3 = - 2 1 0
- 1 4 =	- 484	= -28	- 29		- 5 2	= - 3 2 8	= - 1 8 4 = - 2 0 1
- 1 5 =	- 4 8 3	= - 2 9	= - 2 8		- 5 3	= - 3 2 0	= -192 = -193
- 1 6 =	- 4 8 0	= - 3 2	= - 2 5		- 54	= - 3 0 5	= -207 = -178
	- 4 7 9	= - 3 3	= -24		- 5 5	= - 2 8 6	= -226 = -159
	- 4 7 8	= - 3 4	= - 2 3		- 56	= - 2 6 7	= -245 = -140
- 1 9 =	- 4 7 7	= - 3 5	= - 2 2		- 57	= - 2 6 4	= -248 = -137
-							
	- 4 7 6	= - 3 6				= - 2 6 3	
- 2 1 =	- 4 7 5	= - 3 7	= - 2 0		- 5 9	= - 2 5 8	= -254 = -131
- 2 2 =	- 4 7 3	= - 3 9	= -18		- 61	= - 2 4 0	= - 2 7 2 = - 3 3 9
- 2 3 =	- 4 7 0	= - 4 2	= -15		- 62	= - 2 2 5	= - 2 8 7 = - 3 2 8
- 2 4 =	- 4 6 9	= -43	= -14		- 63	= - 2 1 6	= - 2 9 6 = - 2 8 6
- 2 5 =	-467	= -45	= -12		- 6 5	= - 2 0 6	= - 3 0 6 = - 3 1 3
- 2 6 =	- 464	= -48	= -128		- 66	= - 1 9 2	= - 3 2 0 = - 2 0 4
- 2 7 =	- 461	- 51	= -125		- 6 7	= - 1 9 1	= - 3 2 1 = - 2 4 9
- 2 9 =	- 4 5 6	- 56	= -120		- 68	= - 1 8 5	= - 3 2 7 = - 2 9 8
- 3 1 =	- 4 5 3	- 5 9	= -117		- 6 9	= - 1 6 4	= - 3 4 8 = - 2 7 7
- 3 2 =	- 4 4 8	= - 6 4	= -113		- 70	= - 1 5 8	= - 3 5 4 = - 2 7 1
- 3 3 =	- 4 3 9	= -73	= -101		- 71	= - 1 5 2	= - 3 6 0 = - 2 6 5
- 3 4 =	- 4 3 5	= -77	= -100		- 74	= -133	= -379 = -246
- 3 5 =	- 4 3 4	= - 7 8	= -99		- 7 5	= - 57	= -455 = -351
- 36 =	- 4 3 0	= - 8 2	= -95	'	. , , ,	/	<u> </u>
	- J U	- 02	- 95	1			

Proposed #	Old #	Description]	Proposed #	Old #	Description
Des 90 - 1	-1	1-2-3-4-5-6-7-8-9	1	Des 90 - 100	-34	1-4-5-7-8-9
Des 90 - 2	-7	1-2-3-4-5-6-7-8		Des 90 - 104	-33	2-3-4-5-6-8
Des 90 - 4	-6	1-2-3-4-5-6-8-9		Des 90 - 113	-32	2-3-5-6-7-8
Des 90 - 6	-5	1-2-3-4-6-7-8-9		Des 90 - 117	-31	2-4-5-6-7-8
Des 90 - 7	-4	1-2-3-5-6-7-8-9		Des 90 - 120	-29	2-4-5-7-8-9
Des 90 - 8	-3	1-2-4-5-6-7-8-9		Des 90 - 1256	-27	3-4-5-6-8-9
Des 90 - 10	-2	2-3-4-5-6-7-8-9		Des 90 - 128	-26	3-4-5-7-8-9
Des 90 - 12	-25	1-2-3-4-5-6-8		Des 90 - 131	-59	1-2-3-4-7
Des 90 - 14	-24	1-2-3-4-5-7-8		Des 90 - 136	-58	1-2-3-5-8
Des 90 - 15	-23	1-2-3-4-5-7-9		Des 90 - 137	-57	1-2-3-5-9
Des 90 - 18	-22	1-2-3-4-6-7-9		Des 90 - 140	-56	1-2-3-6-9
Des 90 - 20	-21	1-2-3-4-7-8-9		Des 90 - 159	-55	1-2-5-8-9
Des 90 - 21	-20	1-2-3-5-6-7-8		Des 90 - 178	-54	1-3-5-7-9
Des 90 - 22	-19	1-2-3-5-6-7-9		Des 90 - 193	-53	1-4-7-8-9
Des 90 - 23	-18	1-2-3-5-6-8-9		Des 90 - 201	-52	2-3-4-5-8
Des 90 - 24	-17	1-2-3-5-7-8-9		Des 90 - 210	-51	2-3-5-7-8
Des 90 - 25	-16	1-2-3-6-7-8-9		Des 90 - 216	-50	2-4-5-6-8
Des 90 - 28	-15	1-2-4-5-6-8-9		Des 90 - 217	-49	2-4-5-6-9
Des 90 - 29	-14	1-2-4-5-7-8-9		Des 90 - 230	-47	3-4-5-6-8
Des 90 - 33	-13	1-3-4-5-6-7-9		Des 90 - 239	-46	3-5-6-7-8
Des 90 - 36	-12	1-3-4-6-7-8-9		Des 90 - 242	-45	3-6-7-8-9
Des 90 - 39	-11	2-3-4-5-6-7-8		Des 90 - 246	-74	1-2-3-7
Des 90 - 41	-10	2-3-4-5-6-8-9		Des 90 - 249	-67	1-2-4-5
Des 90 - 44	-9	2-3-5-6-7-8-9		Des 90 - 265	-71	1-3-4-6
Des 90 - 45	-8	2-4-5-6-7-8-9		Des 90 - 271	-70	1-3-5-8
Des 90 - 48	-44	1-2-3-4-5-7		Des 90 - 277	-69	1-3-7-9
Des 90 - 52	-43	1-2-3-4-6-8		Des 90 - 286	-63	1-4-7-8
Des 90 - 59	-42	1-2-3-5-6-9		Des 90 - 398	-68	1-7-8-9
Des 90 - 61	-41	1-2-3-5-7-9		Des 90 - 304	-66	2-3-5-7
Des 90 - 69	-40	1-2-4-5-6-9		Des 90 - 313	-65	2-4-6-8
Des 90 - 72	-38	1-2-4-5-8-9		Des 90 - 328	-62	3-4-6-7
Des 90 - 77	-37	1-2-5-6-7-8		Des 90 - 367	-61	3-7-8-9
Des 90 - 95	-36	1-3-5-7-8-9		Des 90 - 394	-75	1-3-8
Des 90 - 99	-35	1-4-5-6-8-9				

Table 4: Proposed numbers and descriptions based on the additive description of pin use in pattern, beginning with 'full' 9 hole pattern, followed sequentially by 8 hole, 7 hole, 6 hole etc. patterns.

And, here's a last bit of speculation. Six-, seven-, eight-,-, and nine-hole patterns account for 128 of the 400 distinguishable patterns in the 'P' listing. This count is big enough to include all recognized patterns if the 5, 4, and 3 hole patterns were found to be broken pin varieties of these 128. Most of the 5, 4, and the 3 hole patterns can be generated by breaking one or more vulnerable corner pin(s) (# 1,3,7,or 9) from a 6 -> 9-hole pattern. (Proposed -271, -277, -313, -131, -140, -178, -192, -216 and -242 are the possible exceptions [all cool patterns --just plain fun]).

Having reviewed the basics of the 9-hole Schermacks, I have shown that cataloging of these patterns is inconsistent with the remainder of perfin patters. I have proposed a catalog sequence consistent with the other multi-office patterns; one based on the presence (not the absence) of holes. Opinions - ideas?? Write them...