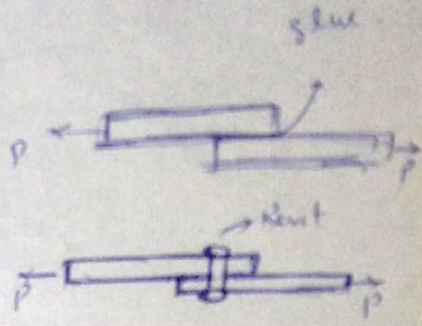
Shear stress:-
The stress produce when force acting parallel to cross sectional area.

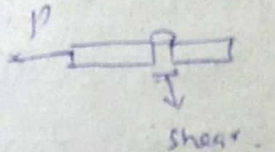
Explaination:-
Consider the fig in which two forces are applied on opposite directions on two plates as shown.



Now let consider the two plates may be connected by pin or other glue like metarial in order to cause resistance to applied loads.

→ The above figure with revit is called single shear as one surface provide resistance or crack will produce in one place.

(*) When we applied two force than the both parts will seperate & rivit will break as show above parts.



Shear stress also called tengential stress.

Mathematically:-
Shear Stress = Tau = $\tau = \frac{V}{A}$.

Double shear:-
When two surface provide resistances as break at two places.
→ These shears (double) are mostly produce in steel and wood.

(Double shear)

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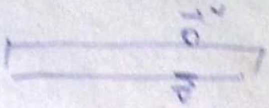
If more force applied than it will break and are shown as,



As; $\tau = \frac{P}{2A} = \frac{V}{A}$

$P = 2V$.

$V = \frac{P}{2}$

$$\boxed{\tau = \frac{P}{2A}}$$

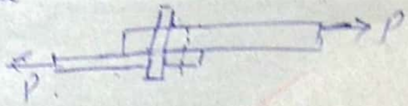
Generally it can be written as;

$$\boxed{\tau = \frac{P}{nA}}$$

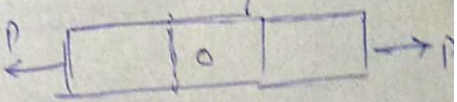
Bearing Stress:-

(*) Shear stress & bearing stress both arises at an object.

Consider the fig in which two surfaces are in contact through nut. Now if we applied two forces in opposite direction if the surface tears then we call it Bearing stress.



(*) Here tearing occur at 45°.



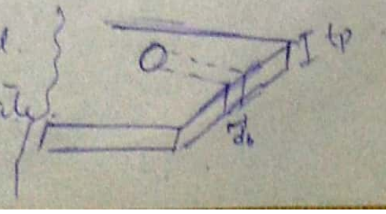
Mathematically:-

$$\delta = \frac{P}{A}$$

$A = d_b * d_t$

$$\delta = \frac{P}{d_b \times d_t}$$

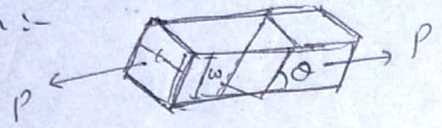
$\because d_b$ = diameter of board
$\because d_t$ = thickness of plate



Pb - 122

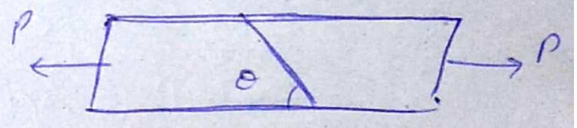
Two blocks of wood, width w and Thick t, are glued together along the joint inclined at angle θ as shown, Using free body diagram Computed illustrated in fig. Show That shearing str on glued joint is $\tau = \frac{2P\sin 2\theta}{2A}$, where A is Cross sectional area.

Diagram:-

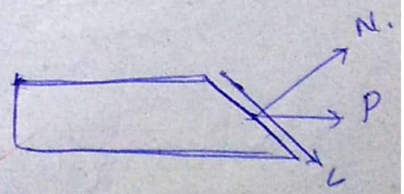


Solution:- Consider The Two blocks which are joint through glue together along The joint which is inclined.

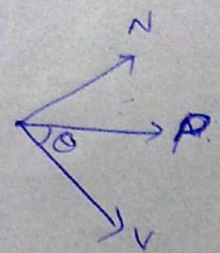
Consider block after breaking.



Show The applied load, Shear load & normal force.

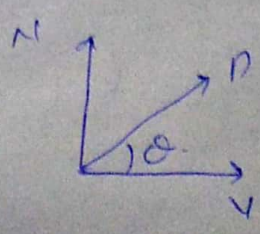


These forces can be show as;



As we know;

$$\sigma = \frac{N}{A'}$$

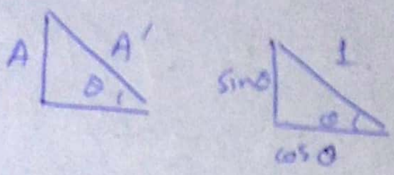


$\because A'$ = Is inclined area.

From fig.

$N = P\sin\theta$.

$$\delta = \frac{P\sin\theta}{A'} \longrightarrow (2)$$



From similar triangle

$$\frac{A'}{A} = \frac{1}{\sin\theta}$$

$$A' = \frac{A}{\sin\theta} \longrightarrow (3)$$

Only Shear Stress is Required in Question!

$$\delta = \frac{P\sin\theta}{A/\sin\theta}$$

$$\boxed{\delta = \frac{P}{A}\sin^2\theta} \rightarrow \text{Formula for normal stress.}$$

<u>Shear stress (required).</u>

$$\tau = \frac{V}{A'} \qquad \because A' = \text{Inclined area}$$

$$\tau = \frac{P\cos\theta}{A'} \qquad \because V = P\cos\theta.$$

$$= \frac{P\cos\theta}{A/\sin\theta} \qquad \because \text{Since } A' = \frac{A}{\sin\theta}$$

$$= \frac{P}{A}\sin\theta\cos\theta.$$

$\because 2\sin\theta\cos\theta = 2\sin^{2}\theta$ $\cancel{\cos\theta}$

$$= \frac{P}{2A}2\sin\theta\cos\theta$$

Multiply & divide "2"

$$\boxed{\tau = \frac{P\sin 2\theta}{2A}} \quad \text{Ans}$$

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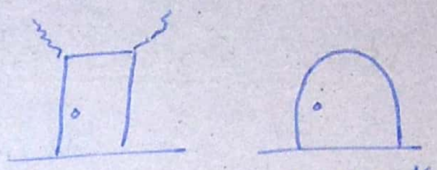
Stress Concentration:-

We have mostly seen that crack produce at sharp edge of doors as shown in fig. But If we give some radius (fillet) to its edges Than no crack will produce

It can be explain as when load of bricks upon door act down word. The stress flow similarly like water. So If there is some sharp edge in its ~~place~~ path. It will disturb its flow and therefore crack will produce.

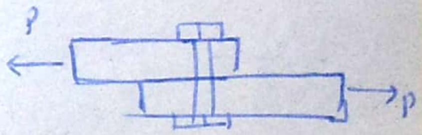


Crack at edge

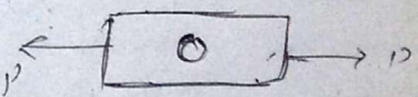
No crack

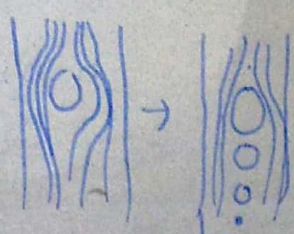
Diagram.

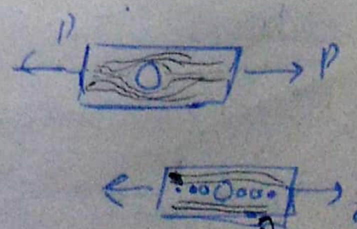
(2) → Consider two plates That are fixed through rivit. If load is applied it will break at hole side as at hole (area) is less so stress will maximum There and crack will produce there. In order to not happen we make small holes in lines with that hole so That to reduce (stress) as in water flow we provide more obstructs in order to smoothen flow of water as shown in fig.



Top view







Stress $\propto 1/\text{Area}$

Smooth Flow.

*) Stress does not like sharp edges.

→ In our daily life we can reduce stress concentration by avoiding sharp edges and giving fillet to its edges.

Stress Concentration factor:-

It is define as;

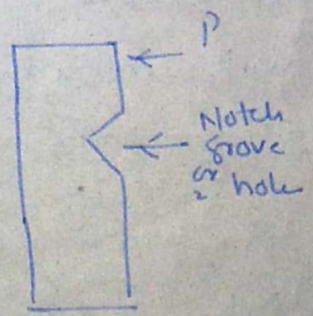
It is the ratio of maximum stress which is present at notch (groove or whole) to nominal stress (real stress) present at same section.

Formula:-

Stress concentration factor = $k = \dfrac{\text{Maximum Stress}}{\text{Nominal stress}}$.

Diagram:-

In order to understand stress concentration factor consider the diagram.



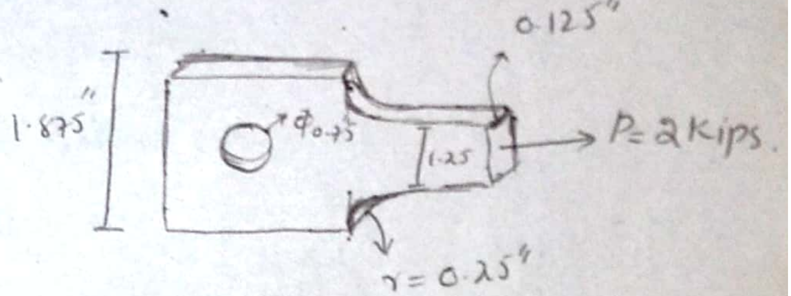
At notch section or portion the stress will maximum as area decrease since stress $\propto \frac{1}{A}$.

So the ratio of this maximum stress to average or nominal stress (which is produce due normal force at any section without grove)

Here K is called stress concentration factor whose value can be determined from the graph between w/h and r/h.

Example on Stress Concentration factor:-

Find the maximum normal stress?



Solution:-

Given data

force applied = P = 2 Kips.

Fillet radius = r = 0.25"

Height of small portion = h = 1.25"

Height of larger portion = H = 1.875"

Diameter of hole = Φ = 0.75"

Wedth of small portion = w = 0.125"

Required data:-

Maximum Stress = ?

At small portion:-

Since we know that;

$$K = \frac{\sigma_{max}}{\sigma}$$

$$\sigma_{max} = K\sigma \longrightarrow (1)$$

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$$\sigma = \frac{F}{A} = \frac{P}{A}.$$

$\because P = 2\,kips$

$A = l \times w.$

$$= \frac{2\,kips}{l \times w} = \frac{2\,kips}{(0.125'')(1.25'')}$$

$$\sigma = 12.8\ ksi \longrightarrow (2).$$

To find $k = ?$

Since; k value determine from graph of w/h and r/h. So we have first find w/h and r/h.

$$\frac{r}{h} = \frac{0.25''}{1.25''} = 0.20$$

$\because r =$ Fillet radius
$h =$ height of small portion.

$$\frac{W}{h} = \frac{1.875''}{1.25''} = 1.5.$$

$\therefore W =$ hight to large portion

By locating such points on graph we get $k = 1.75$.

So of (1) $\Rightarrow$ $\sigma_{max} = \sigma \times k.$

$$= 12.8\ ksi \times (1.75).$$

$$\boxed{\sigma_{max} = 22.4\ ksi}$$

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Now for Circle (hole):-

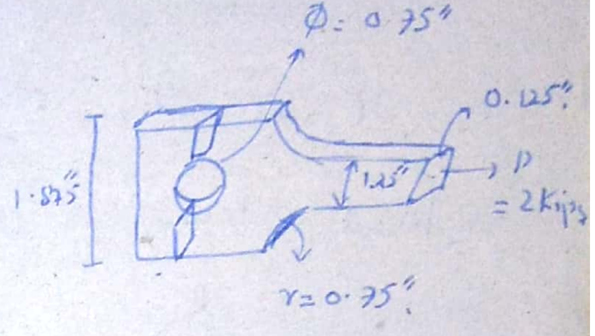
Now calculate the stress on hole portion.

Since;

$$\frac{\delta_{max}}{\delta} = k.$$

$$\delta_{max} = \delta k \longrightarrow (1)$$

As $\delta = \frac{P}{A} = \frac{2}{(1.875-0.75)(1.25)}$



$$\boxed{\delta = 14.22 \text{ kips}}$$

To find k first we have to find w/h & r/h.

∴ For hole.

$$\frac{w}{h} = \frac{\phi}{h}$$

$$\frac{w}{h} = \frac{\phi}{h} = \frac{0.75}{1.875} = 0.4.$$

∴ As stress is to find in hole i.e why we subtract the above & below area to get area of hole.

$$\frac{r}{h} = \frac{0.75''}{1.25} = 1.5$$

For these value $k = 2.2$.

eq (1) ⇒ $\delta_{max} = \delta k.$

$$= 14.22 \text{ kips } (2.2).$$

$$\boxed{\delta_{max} = 31.3 \text{ kips.}}$$

So the max. stress will be at hole section which will be controlling factor.

Factor of Safety:-

factor of safety can be defined as;

" It is the ratio of maximum stress to the permissible or working stress"

OR

It is the ratio of failure stress to actual stress"

OR

It is the ratio of strength of meterial to maximum computed stress."

Formula:- (Generally)

$$\text{Factor of Safety} = \frac{\text{Actual failure stress}}{\text{Actual stress}}.$$

For ductile Meterial:-

$$F.O.S = \frac{\text{Yield Stress}}{\text{Working Stress}}.$$

For Brittle meterial:-

$$F.O.S = \frac{\text{Ultimate stress}}{\text{Working stress}}.$$

(*) Factor of safety is just a number which should always greater than 1.

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For Example:-

In case of building

F.O.S $\geq$ 2.

In case of automobile

f.o.s $\geq$ 33

In case of air/space craft:-

f.o.s $\geq$ 1.2 – 2.5

In case of boiler:-

f.o.s $\geq$ 8.5

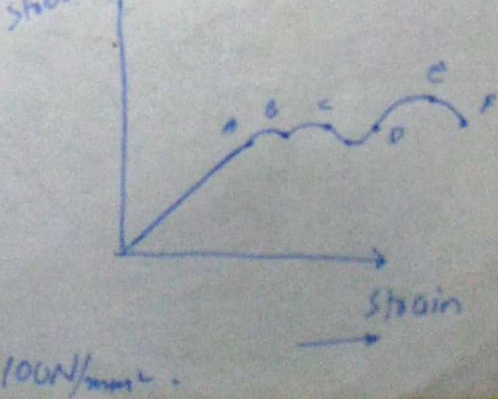
In case of bolts:-

f.o.s $\geq$ 8.5

Importance of factor of safety:-

Factor of safety help us in designing of metelial.

Consider the stress-strain curve for ductile meterial. If the yield stress is 400 N/mm² & factor of safety is 4. So the allowable stress that meterial can carry will be; $\frac{400}{4}$ = 100N/mm².



Since the maximum stress after that meterial break is $400 N/mm^2$ but because of F.O.S we reduce the stress on meterial and now we can safely operate meterial, as stress now will not increase from $100 N/mm^2$.

Factors on which F.O.S depend:-

i. Meterial selection:-
We will see whether it is ductile or brittle.

ii. Type of loading:-
Whether the load is static, variable or shock.

iii. Cost:-
If we want to increase the F.O.S we have to make more dimension which will cost more.

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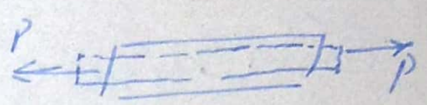
Poisson's Ratio:-

It is defined as;

"When the meterial is loaded with in elastic limit, the ratio of literal strain to longitudinal strain remain constant and is called poisson's ratio."

Explaination:-

Consider The fig. In which load is applied on Two parts of object. So here elongation is produce, at a same time The width of bar will also reduce.



Simeon D. Poisson showed in 1811, that the ratio of unit deformation or strain in these directions is constant for stresses with in proportional limit.

Denotion:-

It is denoted by mew i.e ν.

Mathematically:-

$$\text{Poisson's ratio} = -\frac{\text{Literal Strain}}{\text{Longitudinal Strain}}$$

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We take negative sign to get positive value for Poisson ratio. As Poisson ratio is always positive except in case of honey comb where it is negative.

Poisson's ratio can be written as;

$$\nu = -\frac{\varepsilon'}{\varepsilon} \longrightarrow ①$$

ε' = Literal strain.

$$\varepsilon' = \frac{\delta\ lateral}{W_o}$$

ε = Longitudinal strain.

$$\varepsilon = \frac{\delta_{long.}}{L_o}$$

Unit:-

Poison ratio has no unit.

Value range:-

Typical value of Poisson's ratio ranges from 0.1 to 0.5.

Some values of Pisson's ratio:-

For Cork = ν = 0.5

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For Rubber = 0.48 – 0.5

For Copper = 0.37.

For Aluminum = 0.35.

For Stainless Steel = 0.3.

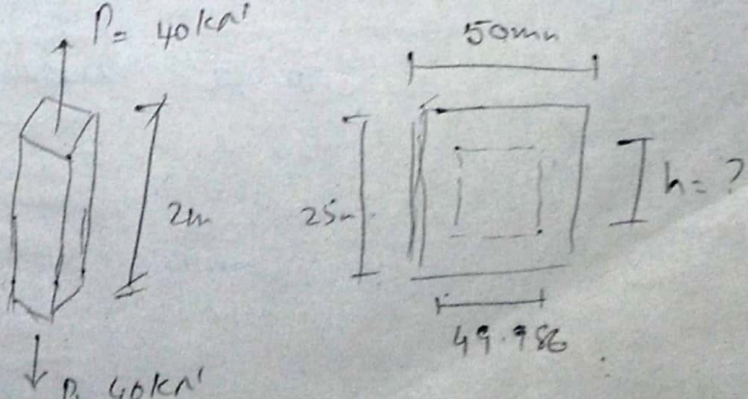
Importance of Poisson's Ratio:-

(i) Poisson's ratio is used for analysis of structural components of different members of building.

Pb on Pisson Ratio:-

Find the Piosson's ratio in the following fig. when P = 40kN applied at both sides. Its original longitudinal length is 2m and linear deformation is 2mm. The breadth & wedth is 50mm x 25mm. The deformation in breadth is 49.986mm. Find the wedth i.e height also.

Diagram:-



The metrial is isotropic.

Sol:- As Piosson's ratio is given as;

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$$\nu' = \frac{\text{Literal strain}}{\text{longitudinal Strain}} = \frac{\varepsilon'}{\varepsilon}.$$

$$\nu = -\frac{\varepsilon'}{\varepsilon} \longrightarrow ①.$$

First we have to find literal & longitudinal strain.

→ Literal strain:-

$$\varepsilon' = \frac{-\delta \text{ (change In length)}}{\text{original length } (\omega)}$$

$$= \frac{49.986 - 50}{50}.$$

∴ -ve sign show it reduces.

$$\varepsilon' = -0.00028$$

(ii) Longitudinal stress:-

$$\varepsilon = \frac{\delta \text{ (change In length linearly)}}{\text{Original length}}$$

$$= \frac{2\,mm}{2 \times 1000\,mm}.$$

$$\varepsilon = 0.001$$

& ① ⇒ $\nu = -\frac{(-0.00028)}{0.001}$

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$\boxed{\nu = 0.28}$

As for Isotropic metrial, the poisson's ratio for width is equal to Poisson's ratio for height.

i.e $\nu_w = \nu_h$.

$\nu_h = 0.28$ $\therefore \nu_w = 0.28$.

$\frac{\varepsilon'}{\varepsilon} = 0.28$. $\therefore \nu_h = \frac{-\varepsilon'}{\varepsilon}$.

$\varepsilon' = \varepsilon(0.28)$. $\therefore \varepsilon = 0.001$.

$= 0.001(0.28)$.

$\varepsilon' = 0.00028 \longrightarrow (2)$.

Since

$\varepsilon' = \frac{\delta h}{ho}$. $\varepsilon' = \frac{\delta h}{h_o}$.

$\frac{\delta h}{25} = 0.00028$.

$\Rightarrow \delta h = 25(0.00028)$.

$\boxed{\delta h = 0.007_{mm}}$

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Pb – 2:–

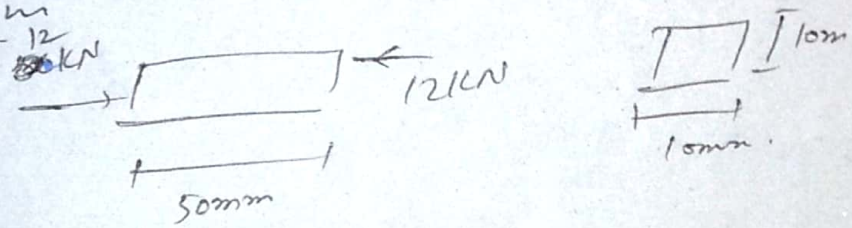
Consider Magnesium alloy on which 12kN forces apply at both sides. The original length is 50mm. The wedth & height is 10mm respectively. Longitudinal strain is 0.0015 and Poisson's ratio is 0.35 Find.

(a) Stress.

(b) Change in longitudinal length $\delta_{long} = ?$

(c). $h', w' = ?$

Diagram



Solution:–

(a) Since

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

$$= \frac{12\,KN}{100\,mm}$$

$$\boxed{\sigma = 120MPa}$$

(b) As we know That;

$$\varepsilon = -\frac{\delta_{long}}{\text{Original length}}$$

$$\delta_{long} = -\varepsilon\,(L_o)$$

$$= -0.0015(50\,mm)$$

$$\boxed{\delta_{long} = -0.075mm}$$

(c) $\delta h = ?$

As; $\varepsilon' = \frac{\delta h}{h_o} \rightarrow (1)$.

to find $\varepsilon' = ?$

Since; $-\frac{\varepsilon'}{\varepsilon} = \nu$.

$\because \nu = 0.35$.
$\varepsilon = -0.0015$.

$\varepsilon' = -\varepsilon \nu$.

$= -0.35(-0.0015)$.

$\varepsilon' = 0.000525$

Put in eq (1).

$\delta h = \varepsilon' h_o$.

$= 0.000525 (10)$.

$\boxed{\delta h = 0.00525 \text{ mm}}$.

$\delta w = ?$ Since; $\varepsilon' = \frac{\delta w}{w_o}$.

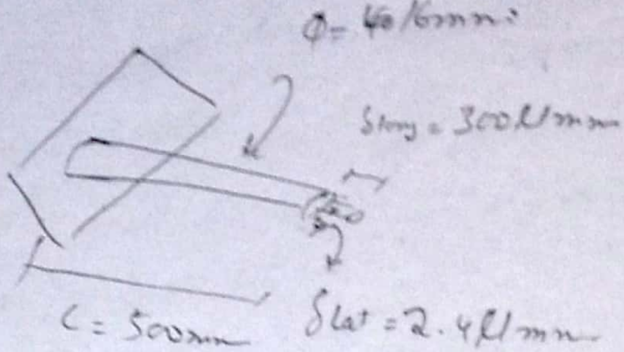
$\delta w = \varepsilon' (w_o)$

$= 0.000525 (10)$.

$\boxed{\delta w = 0.00525 \text{ mm}}$

Q(2) Find the Poisson ratio in the following Problem.

Figure:-



Solution:- As we know;

$$\nu = -\frac{\varepsilon'}{\varepsilon} \rightarrow ①.$$

longitudinal ~~literal~~ strain $= \varepsilon = \frac{\delta L}{L_0} = \frac{300\ \mu mm}{500 mm}$.

$$\varepsilon = \frac{3}{5}\mu \Rightarrow \frac{3\times 10^{-6}}{5}$$

Literal strain $= \varepsilon' = -\frac{\delta d}{d_0} = -\frac{2.4\times10^{-6} mm}{40 mm}$.

$$\varepsilon' = -0.15\times10^{-6}.$$

Put in eq ①.

$$\nu = \frac{-(-0.15\times10^{-6})}{3/5\times10^{-6}}.$$

$$\boxed{\nu = 0.25}$$ Ans

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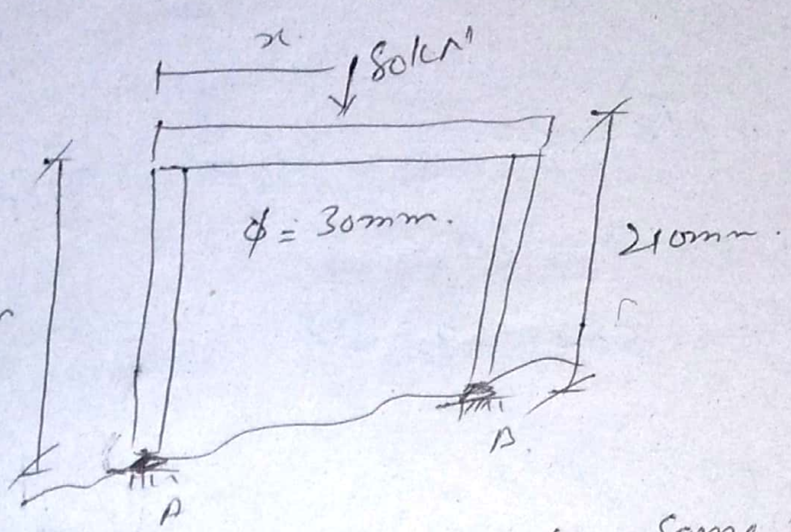
Pb-4 The Poisson ratio is 0.35, modulus of elasticity is 73.10 GPa. Find the x so that load P equally distribute.

Figure:-



Both columns are made of same materials.

Solution:-

As the columns are made of same material so; deformation will be same in column A & B.

$$\Delta_A = \Delta_B.$$

$$\frac{PL}{AE} = \frac{PL}{AE}$$

∴ As Area & E is same for both column.

So;

$$P_A L_A = P_B L_B.$$

$$P_A = \frac{P_B L_B}{L_A} \longrightarrow ①.$$

By putting values.

$$P_A = \frac{210}{220} P_B.$$

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$$\boxed{P_A = 0.95 P_B} \rightarrow (2).$$

Apply condition of equillibrium.

$(+\uparrow)\ \Sigma F_y = 0$

$P_A + P_B = 80.$

$\because P_A = 0.95 P_B.$

$P_B = 80 - P_A.$

$= 80 - (0.95 P_B).$

$P_B + 0.95 P_B = 80.$

$1.95 P_B = 80.$

$$\boxed{P_B = 41.02\,kN}$$

eq (2) $\Rightarrow$ $P_A = 0.95 P_B.$

$= 0.95(41.02)$

$$\boxed{P_A = 38.97\,kN.}$$

Now; $\overset{+}{\curvearrowleft}$ $\Sigma M_A = 0$

$80 \times x = 41.02 \times 3.$

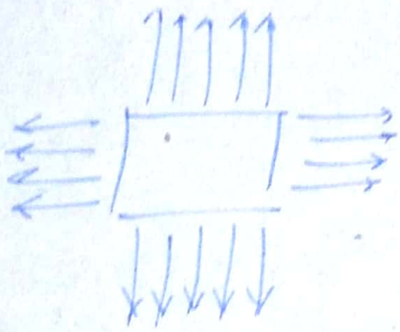
$x = \dfrac{41.02 \times 3}{80}.$

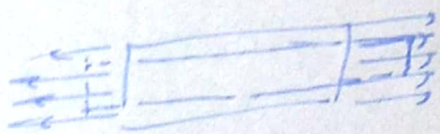
$$\boxed{x = 1.53\,mm}$$

At this value the load will equally distribute in both column.

Biaxial stresses:-
(Generalized Hook's law)

Upto now we consider the stresses produce due to one side force applied. Now in order to study the stresses produce when force is applied from both sides as shown in fig. We will divide such problem into two parts. First to solve it in x-direction and than solve for y-direction.





along x-axis:-

The strain is given as;

$$\varepsilon_x = \frac{\delta x}{E} \longrightarrow ①$$

From Hook's law (stress = E strain.

Since from Piosson ratio;

$$\nu = -\frac{\varepsilon_{y_1}}{\varepsilon_x}$$

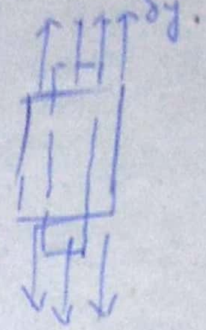
$$\varepsilon_{y_1} = -\nu \varepsilon_x \longrightarrow ②$$

Put values in eq ①.

$$\boxed{\varepsilon_{y_1} = -\nu \frac{\delta y}{E}} \longrightarrow ③$$

Along y-axis :-

From Hook's law;



$$\varepsilon_{y_2} = \frac{\delta y}{E}$$

$$\varepsilon_{x_2} = -\upsilon\, \varepsilon_{y_2}.$$

$$\varepsilon_{x_2} = -\frac{\upsilon\, \delta y}{E} \longrightarrow ④.$$

Now add those who along x-axis with corresponding ε along y-axis

$$\varepsilon_x = \varepsilon_{x_1} + \varepsilon_{x_2}.$$

$$\varepsilon_x = \frac{\delta x}{E} - \frac{\upsilon\, \delta y}{E} \longrightarrow Ⓐ$$

$$\varepsilon_y = \varepsilon_{y_1} + \varepsilon_{y_2}.$$

$$\varepsilon_y = \frac{\delta y}{E} - \frac{\upsilon\, \delta x}{E} \longrightarrow Ⓑ.$$

Multiply eq ① by E & Eυ with eq Ⓑ we get

$$E\varepsilon_x = \delta x - \upsilon\, \delta y \longrightarrow Ⓒ.$$

$$\varepsilon_y E\upsilon = \upsilon\, \delta y - \upsilon^2 \delta x \longrightarrow Ⓓ$$

Add eq Ⓒ & D.

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$$\varepsilon_x E = \delta x - \nu \cancel{\delta y}$$

$$\varepsilon_y \times \nu E = \nu \cancel{\delta y} - \nu^2 \delta x$$

$$\varepsilon_x E + \varepsilon_y \nu E = \delta x - \nu^2 \delta x .$$

$$E(\varepsilon x + \varepsilon y \nu) = \delta x (1-\nu^2).$$

$$\boxed{\delta x = \frac{E(\varepsilon x + \nu \varepsilon y)}{(1-\nu^2)}}$$

Similarly;

$$\boxed{\delta y = \frac{E(\varepsilon y + \nu \varepsilon x)}{1-\nu^2}}$$

Relation between G, E & ν.

The relation between modulus of rigidity (G) Young's modulus (E) and Poisson's ratio is given as;

$$G = \frac{E}{2(1+\nu)} \longrightarrow (1)$$

For steel this ratio is;

$$\boxed{G = 0.38 E}$$

For concrete;

$$\boxed{G = 0.4E}$$

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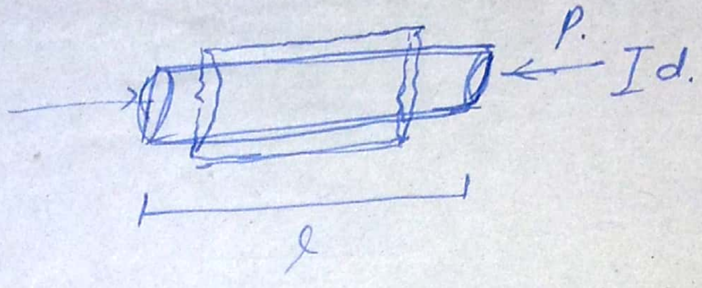
Pb—222:— A solid cylinder of diameter d carries an axial load P, show that its change in diameter is $4Pv/\pi Ed$.

Solution:—

Consider the figure.



As compressive force is applied so length will get decrease and diameter increases.

As; We know that;

$$v = -\frac{\varepsilon_y}{\varepsilon_x} \qquad \because \text{Pisson's ratio.}$$

$$\varepsilon_y = -v\varepsilon_x \rightarrow ①$$

From Hookes law.
$\because$ stress = (E)strain.

$$\varepsilon_x = \frac{\delta x}{E}$$

$$\varepsilon_y = -v\frac{\delta x}{E}$$

$$\because \varepsilon_y = \frac{\delta y}{d}$$

$$\frac{\delta y}{d} = -v\frac{\delta x}{E}$$

$$\because \delta x = \frac{P}{A}$$

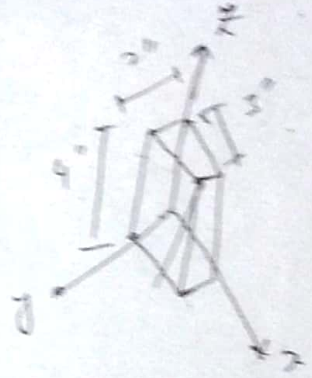
$$\delta y = -\frac{vPd}{AE}$$

$$= -\frac{vPd}{\frac{1}{4}\pi d^2 E} \Rightarrow \boxed{\delta y = \frac{-v(4P)}{\pi d E}}$$

Proved

Problem 223

A rectangular steel block is 3" long in x-direction, 2 inches in y-direction and 4 inches in z-direction. The block is subjected to triaxial loading of 3 uniformly forces (distributed) as follows; 48 kips tension in x-direction, 60 kips compression in y-direction and 54 kips in z-direction. If $\nu = .30$ & $E = 29 \times 10^6$ psi determine the single uniformly distributed load in the x direction that would produce the same deformation in y-direction as original loading.



Solution:-

Given data;

Dimension of block;

x-direction = 3 inches.

y direction = 2 inches.

z-direction = 4 inches.

Triaxial loads;

X-direction = 48 kips.

Y-direction = 60 kips.

Z- direction 54 kips.

Pisson ratio = 0.30.

Modulus of elasticity = E = 29×10^6 Psi.

Required data;

$P_x = ?$ that produce same deformation in y direction.

As we know that;

$$\delta x = \frac{P}{A} \longrightarrow ①.$$

or

$$P = \delta x \, A \longrightarrow ①.$$

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As for x-direction.

Area = 4(2) = 8 in^2. ∵ From fig.

To find $\delta x = ?$

Since,

$$\delta x = E \varepsilon_{x} \longrightarrow (A).$$

From Poisson's ratio;

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$\varepsilon_y = -\varepsilon_x \nu \longrightarrow (2).$$

As; $\varepsilon_x = \frac{\delta x}{E}$ ∵ From eq (A).

$$\varepsilon_y = -\frac{\delta x}{E} \nu \longrightarrow (3).$$

Now find ε_y :-

Since;

$$\varepsilon_y = \frac{1}{E}\left(\delta y - \nu(\delta x + \delta z)\right) \longrightarrow (4)$$

$$\delta x = \frac{P_x}{A} = \frac{48}{2 \times 4} \Rightarrow 6 \text{ ksi. (tension).}$$

$$\delta y = \frac{P_y}{A} = \frac{60}{4 \times 3} \Rightarrow 5 \text{ ksi (compression).}$$

$$\delta z = \frac{P_z}{A} = \frac{54}{2 \times 3} \Rightarrow 9 \text{ ksi (Tension).}$$

eq (4) ⇒

$$\varepsilon_y = \frac{1}{2900 \times 10^4}\left|-5000 - 0.30(6000 + 9000)\right|.$$

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$\varepsilon_y = -3.276 \times 10^{-4} \rightarrow 5$

Put values in eq 3

$-3.276 \times 10^{-4} = \frac{-\delta x}{29 \times 10^6}(0.30)$

$\delta x = 31666.67 \ Psi \rightarrow 6$

Put values in eq 1.

$\delta x = \frac{Px}{A} \Rightarrow Px = \delta x A$

$= 31666.67(8)$

$= 253333.33 \ lb$ (tension)

$\boxed{Px = 235.33 \text{ Kips}}$ Ans

Q224:- For The ~~load~~ block loaded triaxially described in Prob 223, Find The uniformly distributed load That must be added in x-direction to produce no deformation in The Z-direction.

Solution:-

Since we know That;

$\varepsilon_z = \frac{1}{E}(\delta_z - v(\delta x + \delta y)] \rightarrow 1$

As from problem 223.

$\delta z = 9 \ ksi$

$\delta x = 6 \ ksi$

$\delta y = 5 \ ksi$ & $E = 25 \times 10^6 \ ksi$

By putting values.

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$$E_z = \frac{1}{29\times10^6}\left[9000 - 0.3(6000-5000)\right].$$

$$= \frac{1}{29\times10^6}(9000 - 300).$$

$$\varepsilon_z = 3\times10^{-3}.$$

As;

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x+\sigma_y)\right].$$

As in z-direction no deformation produced

so; $\varepsilon_z = 0$.

$$0 = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x+\sigma_y)\right).$$

$$\sigma_z = \nu(\sigma_x+\sigma_y).$$

$$9000 = 0.3(\sigma_x - 5000).$$

$$\sigma_x - 5000 = \frac{\cancel{9000}^{3000}}{0.3}\times10$$

$$= 30000$$

$$\sigma_x = 35000\ psi.$$

$$\sigma_{added} + 6000 = 35000.$$

$$\sigma_{added} = 29000\ psi$$

$$\sigma_{add} = P/A \Rightarrow P_{add} = \sigma_{add}\times A.$$

$$\because 2\times4 = A$$

$$= 29000(8).$$

$$= 232000\ lb.$$

$$\boxed{P_{add} = 232\,Kips}$$ Ans

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Pb-225:- A welded steel cylindrical drum made of 10mm plate has internal diameter of 1.20m. Compute the change in diameter that would caused by an internal pressure of 1.5MPa. Assume that Pisson's ratio 0.30 & E = 200GPa

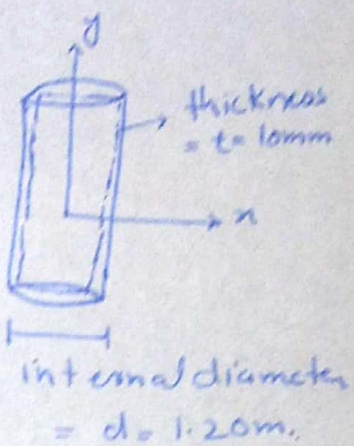
Ans:- Given data;

Internal diameter
$= d = 1.2m.$

Internal Pressure;
$P = 1.5 MPa$

$\nu = 0.30.$

$E = 200 GPa.$



Required data;

Change in diameter $= \Delta d = ?$

Solution:-

As we know that;

$$\varepsilon_x = \frac{\Delta D}{D} \rightarrow (1).$$

To find $\varepsilon_x = ?$

Since;

$$\varepsilon_x = \frac{\delta_x}{E} - \frac{\nu \delta_y}{E} \rightarrow (2).$$

finding δ_x & $\delta_y = ?$

As; $\delta_x = \frac{P}{A}$ $\quad \because \pi/4 D^2 = A.$

$\therefore$

Chapter # 02

Strain

Definition:-

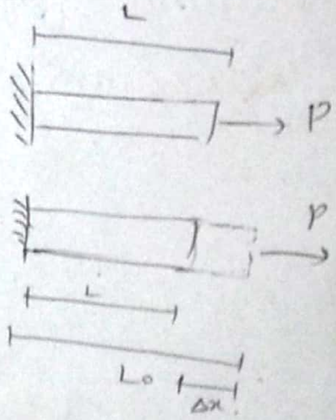
strain can be define as;

"Change in length per unit original length is called strain."

Explaination:-

When a load is applied on a body, there deformation (Change in length occur).

Consider the figure in which a cylinder of length is fixed on one side and on other side a load P is shown.



Its original length is L, but as load is applied so it extends as shown.

Now this change in length over original length is called strain.

Denotion:-

It is denoted by ϵ. (Epsilon)

Formula:-

$$\epsilon = \frac{\delta}{L} \qquad \because \delta = L_{Final} - L_{original}$$

(*) Stress is dimensionless as it is ratio of two same units (lengths)

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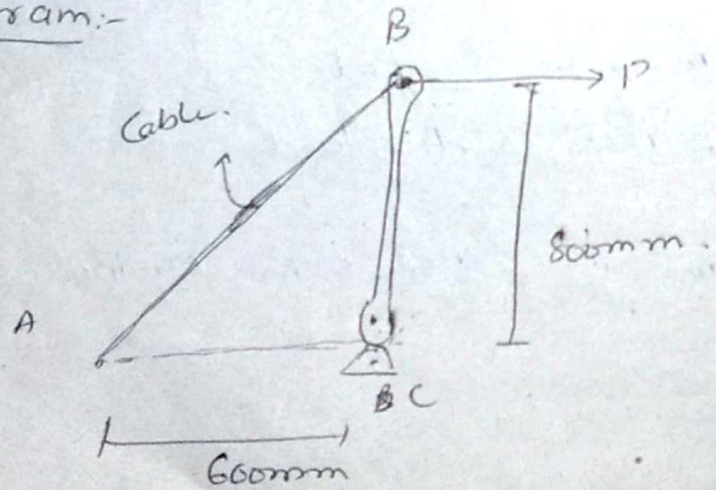
In case of elongation (tension) strain will be positive.

→ Where in case of compression i.e (Compression) Strain will be negative.

Pb:- Consider the fig. Find the strain in cable AB if force is applied P which inclined the rod at 5°. The length (height) of rod is 800mm and connected pin at C. The distance from A where cable fixed and C is 600mm.

Diagram:-



Ans:- Since we know that;

$$\epsilon_{AB} = \frac{L_{Final} - L_{initial}}{\text{Original length}} \longrightarrow (1)$$

As

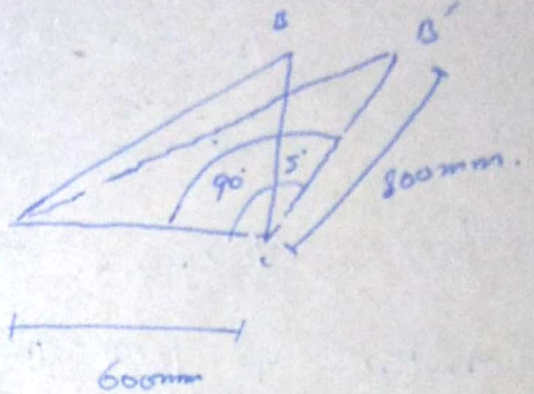
$L_{Final} = ?$

And $L_{initial} = 1000mm$. = original length.

to find $L_{Final} = ?$

Consider the diagram.

Using law of Cosine to find L_{Final}. i.e length of AB after applying of load P.



$$(AB)^2 = (600)^2 + (800)^2 - 2(600)(800)\cos(95^\circ).$$

$$AB = L_{Final} = \sqrt{360000 + 640000 - 700966.618}$$

$$L_{Final} = 1040.99\ mm.$$

Eq ④ =)

$$\epsilon_{AB} = \frac{1040.99 - 1000}{1000}.$$

$$\boxed{\epsilon_{AB} = 0.041\ \frac{mm}{mm}.}$$

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Hooke's Law:-

This law states that;

Strain is directly proportional to strain with in elastic limit.

Mathematically:-

Stress $\propto$ Strain.

$\sigma \propto \epsilon$.

But Thomas Young in 1807 introduce a constant of proportionality that came to known as Young's modulus (Modulus of elasticity)

$\sigma \propto \epsilon$.

$\sigma = E\epsilon$.

$$\boxed{E = \frac{\sigma}{\epsilon}} \longrightarrow (1).$$

Since we know that;

$$\sigma = \frac{P}{A} \longrightarrow (2).$$

&

$$\epsilon = \frac{\delta}{L} \longrightarrow (3).$$

$\because$ Put eq (2) & (3) in (1).

eq (1) $\Rightarrow$

$$E = \frac{P/A}{\delta/l}.$$

$$\boxed{E = \frac{Pl}{A\delta}}$$

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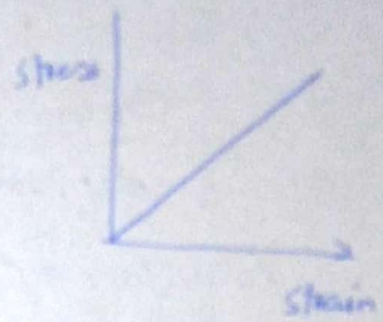
Modulus of elasticity has same unit as stress i.e N/m^2 or Pascal.

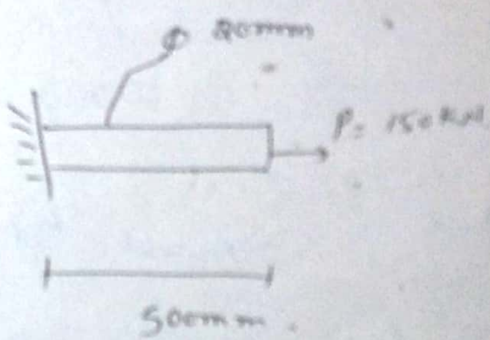
(*) Hooke's law can be explain from the graph as shown. Which is graph of stress & strain.



Withⁿ proportional limit stress is increasing with increase in strain. This graph follow hook's law.

Problem # 1.

Find the change in length when a load P of 150KN is acted as shown. If diameter of rod is 20mm & length (initial) is 500mm Modulus of elasticity is 200GPa.



Solution:-

Since we know that;

$$\delta = \frac{PL}{AE}$$ By putting values.

$$= \frac{(150\,kN)(500\,mm)}{\frac{\pi}{4}(20\,mm)^2(200\,kN/mm^2)}$$

$$\boxed{\delta = 1.19\,mm}$$

$\because 1\,GPa = kN/mm^2$

Deformation per unit length is;

$$\epsilon = \frac{\delta}{L} = \frac{1.19\,mm}{500\,mm}.$$

$$\boxed{\epsilon = 2.3 \times 10^{-3} \frac{mm}{mm}}$$

Shearing Strain:-

It is defined as;

"Angular change between two perpendicular faces of differential elements".

OR

"It is change in right angle."

Denotion:-

→ Denoted by Gamma "γ"

Formula:-

$$\gamma = \frac{\pi}{2} - \theta.$$

Unit:- It is measure in radian.

Explanation:-

Consider the figure In which a force P act on it in such a way which is parallel to its cross sectional area. Such force does not compress or elongate but change the shape of object as shown. Such change in shape or deformation is called shearing strain measure in radian.

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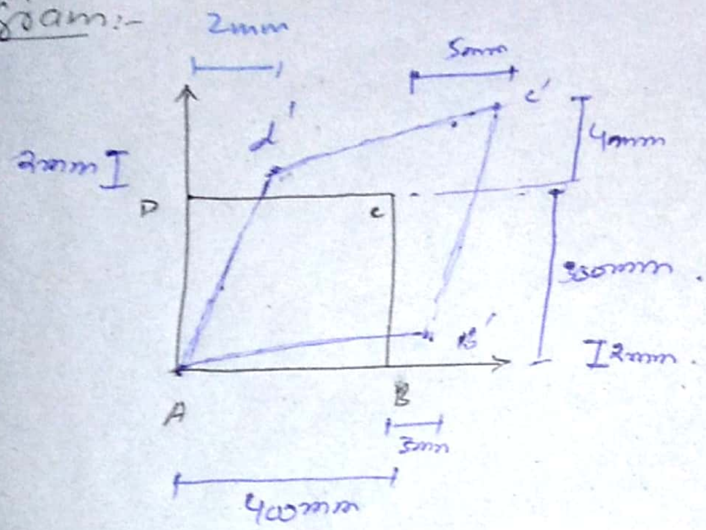
Problem on Shearing Strain:-

Consider The fig in which some force is applied due to which its shape change. The deformations & its all lengths are given. Find The shear strain at A & D.

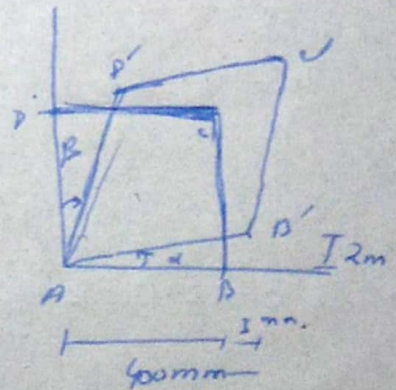
Diagram:-



Solution:-

We have to find shearing strain at A & D.

Shearing strain at A:-



$$\gamma_A = \alpha + \beta \rightarrow (1)$$

$$\tan\alpha = \frac{2\ mm}{400\ mm}$$

$$\alpha = \tan^{-1}(0.0049)$$

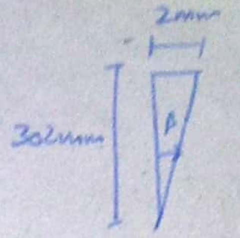
$$\alpha = 6.284^{\circ}$$

In radian

$$\alpha = 0.284 \times \pi/180$$

$$\boxed{\alpha = 4.96 \times 10^{-3}\ rad}$$

Now For β:-



$$\tan\beta = \frac{2\ mm}{302\ mm}$$

$$= 0.3794^\circ$$

In radian:-

$$= 0.3794 \times \frac{\pi}{180}$$

$$= 6.62 \times 10^{-3} \text{ radian}$$

Now eq (1) ⇒

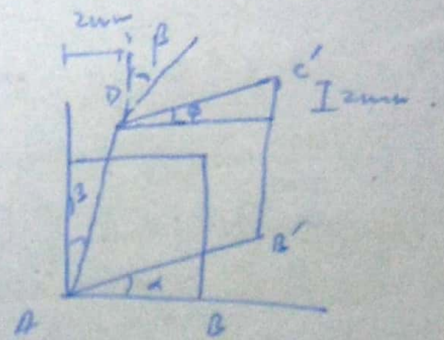
Shearing strain at $A = \gamma_A = \alpha + \beta$.

$$= 4.96 \times 10^{-3} \text{ rad} + 6.62 \times 10^{-3} \text{ rad}$$

$$= 11.58 \times 10^{-3} \text{ rad.}$$

Shearing strain at D:-

$$\gamma_D = \phi + \beta \quad \longrightarrow (2)$$



Since β is same

as corresponding angles.

$$\tan\phi = \frac{2mm}{402mm}$$

$$\tan\phi = \cancel{0.255}\ (0.0049)$$

$$\phi = \tan^{-1}(0.0049)$$

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$\phi = 0.2807$.

In radian

$\phi = 0.2807 \times \frac{\pi}{180}$.

$\phi = 4.9 \times 10^{-3}$ rad.

∴ Here we take its negative value because here compression of angle occur. i.e angle reduced.

So; $\gamma_D = \phi + \beta$.

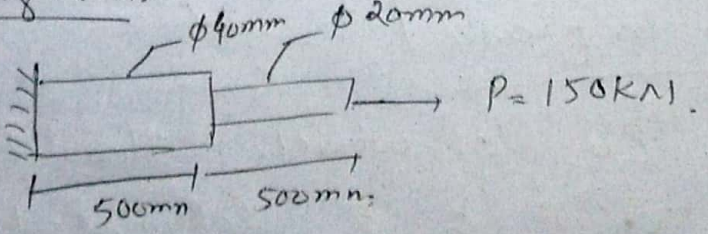
$= 4.9 \times 10^{-3} + 6.62 \times 10^{-3}$ rad.

$$\boxed{\gamma_D = -11.58 \times 10^{-3} \text{ rad}}$$

Problem → Composite

Consider the fig which consist of different diameter i.e 40mm and 20mm. A force of 150KN force is applied. The length of first section is 500mm & other length is also 500mm. Calculate change in length. (deformation).

Diagram:-



Solution:- Given lengths;

$l_1 = 500$mm

$l_2 = 500$mm

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Diameter I = 40mm.

Diameter II = 20mm.

Force = P = 150KN.

Modulus of Elasticity = E = 200GPa

Formula:-

$$\text{Deformation} = \sum_{i=1}^{n} \frac{P_i L_i}{A_i E_i}.$$

Putting values to get total deformation.

$$\delta_{Total} = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2}.$$

$$\delta_{Total} = \frac{150 \times (KN) \times (500mm)}{\frac{\pi}{4}(40)^2 * 200(KN/mm^2)} + \frac{150KN \times 500mm}{\pi/4\,(20)^2 * (200KN/mm^2)}$$

$$\delta = 0.298\,mm + 1.194mm.$$

$$\boxed{\delta_{Total} = 1.492\,mm}$$

Pb— Deflection In frame

Consider the fig. In which two columns are provided at A & B. Column at A has area of 300mm² & length 400mm while length of column at B is 200mm. where as area is 400mm. At C load of 20KN act. The

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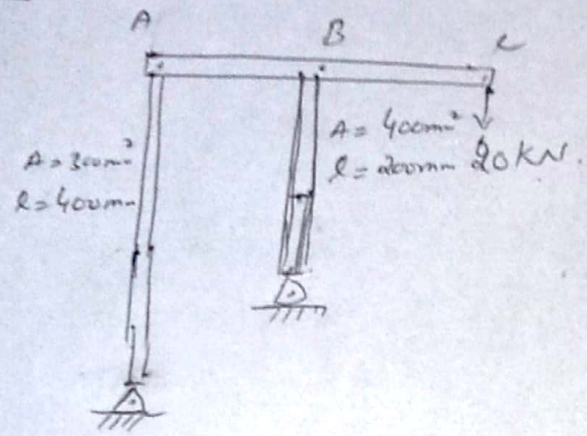
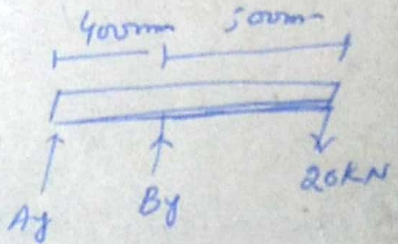
between A & B = 400mm & B and C is 500mm. Calculate deformation at C. Modulus of elasticity is 500GPa.

Diagram:-



Solution:-

First we have to find load at A & B. So draw FBD of beam.



$(+\circlearrowleft)$

$\Sigma M_A = 0$

$-B_y(400) + 20kN \times 900 = 0$

$400 B_y = (200 \times 900) kN.$

$B_y = \frac{1}{20} \times 900 \text{ kN}$

$\boxed{B_y = 45kN}$

$(\uparrow +) \Sigma F_y = 0 \; (\downarrow -)$

$A_y + B_y - 20 = 0.$

$A_y + 45 - 20 = 0$

$\boxed{A_y = -25kN}$

∴ -ve sign show our assume direction is wrong.

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Deformation at A.

$$\delta_A = \frac{PL}{AE}$$

$$= \frac{75 \times 40}{300 \times 200}.$$

$$\boxed{\delta_A = 0.167\,mm}$$

Deformation at B:-

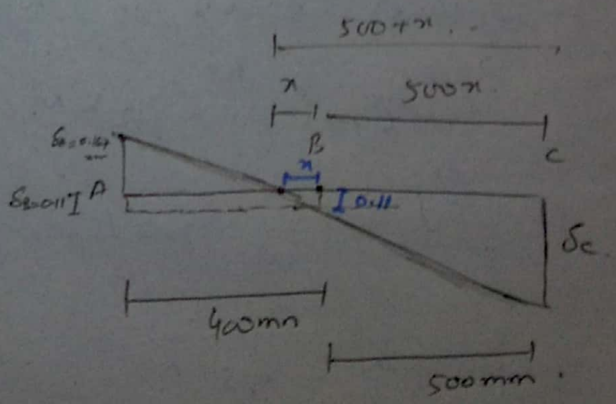
$$\delta_B = \frac{PL}{AE}.$$

$$= \frac{45 \times 200}{400 \times 200}$$

$$\boxed{\delta_B = -0.11}.$$

As length of column B will decrease due to compression.

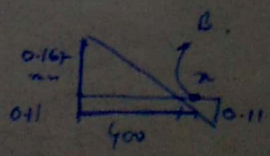
Deformation at C



From similarity of triangles.

$$\frac{\delta_c}{500+x} = \frac{0.11}{x}. \longrightarrow (1).$$

Consider the two triangles.



Now Consider triangles at Left side

$$\frac{0.167\,mm}{400\,mm} = \frac{0.11\,mm}{x}.$$

$$\boxed{x = 263.5\,mm}$$

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