

Problems:- 258.

Given data;

$$\text{length} = l = 2.5 \text{ m}$$

$$t_1 = 20^\circ \text{C}$$

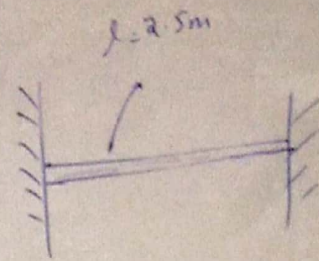
$$t_2 = -20^\circ \text{C}$$

$$\delta = ?$$

$$A = 1200 \text{ mm}^2$$

$$\alpha = 11.7 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$$

$$E = 200 \text{ GPa}$$



t_1

$$\frac{-20 - 20}{2} = -20$$

$$\delta = ?$$

$$\text{Since, } \epsilon_T = \alpha \Delta T$$

$$= (11.7 \times 10^{-6})(-40)$$

$$\epsilon_T = -4.68 \times 10^{-4}$$

Since,

$$\delta = E(\epsilon_T)$$

$$= 200 \times 10^9 (+4.68 \times 10^{-4})$$

$$\delta = 936 \text{ MPa}$$

Part (b)

Since we know that

if the other end is considered free.

$$\delta_{\text{total}} = \delta_T - \delta_P$$

$$\delta \delta_T = \delta_{\text{tot}} + \delta_P$$

$$(\alpha \Delta T) L = \frac{\delta L}{E} + 0.5$$

$$(11.7 \times 10^{-6})(-40)(2.5 \times 1000) = \frac{\delta(2.5 \times 1000)}{200 \times 10^9} + 0.5$$

$$2.34 \times 10^{-4} = \frac{\delta(2500)}{2 \times 10^{11}} + 0.5$$

$$1.74 \times 10^{-4} = \frac{\delta(2500)}{2 \times 10^{11}}$$

$$\delta = 53600000 = 53.6 \text{ MPa}$$

Pb-259

Given data:

Area of st = $\frac{3}{4} \text{ in}^2$
 Area of bz = 1.5 in^2

Toughness:-

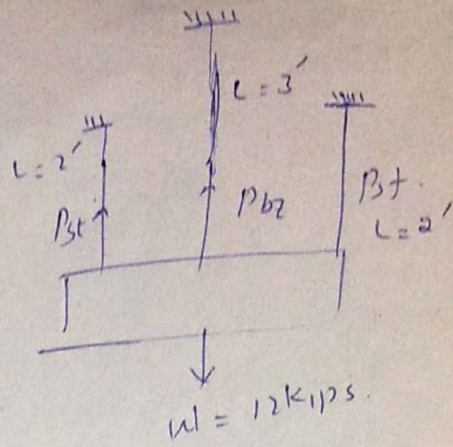
ca.

$E_{st} = 29 \times 10^6 \text{ psi}$

$E_{bz} = \frac{12 \times 10^6 \text{ psi}}{6.5 \times 10^6}$

$\alpha_{st} = 6.5 \times 10^{-6} / ^\circ\text{F}$ A

$\alpha_{bz} = 10 \times 10^{-6} / ^\circ\text{F}$



$T_1 = 0^\circ\text{F}$
 $T_2 = 100^\circ\text{F}$

$\sigma_{fy} = 0$

$2P_{st} + P_{bz} = 12000 \text{ lb} \rightarrow \text{①}$

Stress in each rod?

Pb-259

Pb-260

Geometric relation = ?

Pb-261

Given data:

$A = 0.25 \text{ in}^2$

$P = 1200 \text{ lb}$

$T_1 = 70^\circ\text{F}$

$\delta = ?$

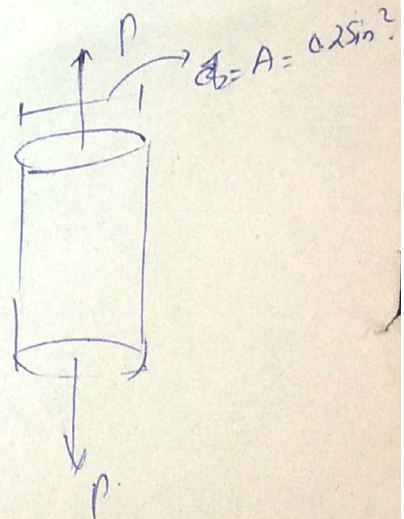
$T_2 = 0^\circ\text{F}$

$T = ? \text{ at } \delta = 0$

$\alpha = 6.5 \times 10^{-6} / ^\circ\text{F}$ $E = 29 \times 10^6 \text{ psi}$

$\delta = (\alpha \Delta T) L$

$\delta = \frac{P}{A} \Rightarrow$



Pb-261

As we know that:

$$\delta_{total} = \delta_T - \delta_P$$

$$\frac{\delta L}{E} = \alpha \Delta T L + \frac{PL}{AE}$$

$$\Delta T = 0 - 70$$

$$2 - 70$$

$$\delta = \alpha \Delta T E + \frac{P}{A}$$

$$\delta = (6.5 \times 10^{-6})(70)(29 \times 10^6) - \frac{1200}{6.5}$$

$$\delta = 15595 \text{ Psi}$$

For temp that cause zero stress?

$$\delta_T = \delta_P$$

$$\delta_T = \delta_T - \delta_P$$

$$\delta_T = 0$$

$$\alpha \Delta T L = \frac{PL}{AE}$$

$$\Delta T = \frac{1200}{(0.25 \times 29 \times 10^6)(6.5 \times 10^{-6})}$$

$$T - 70 = 25.46$$

$$T = 70 + 25.46$$

$$T = 95.46^\circ \text{F} / \text{Ans}$$

Pb-262

$$P = 5000 \text{ N}$$

$$T_1 = 20^\circ \text{C}$$

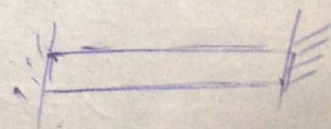
$$\sigma = 130 \text{ MPa}$$

$$T_2 = -20^\circ \text{C}$$

$$d = ?$$

$$\alpha = 11.7 \times 10^{-6} / ^\circ \text{C}$$

$$E = 200 \text{ GPa}$$



Soln:

Since

$$\delta_{net} = \delta_T + \delta_P$$

$$\frac{\delta L}{E} = \alpha \Delta T L + \frac{PL}{AE}$$

$$\frac{PK}{AE} = \frac{SK}{E} - \alpha \Delta T L$$

$$\frac{AE}{P} = \frac{E}{S} - \frac{1}{\alpha \Delta T}$$

$$\frac{(\pi/4 D^2) E}{P} = \frac{E}{S} - \frac{1}{\alpha \Delta T}$$

$$\Delta T = 40^\circ C$$

$$\pi/4 D^2 = \frac{P}{S} - \frac{P}{\alpha \Delta T E}$$

$$D = \sqrt{4 \left(\frac{P}{S} - \frac{P}{\alpha \Delta T E} \right)}$$

$$= \sqrt{4 \left(\frac{5000}{130 \times 10^6} - \frac{5000}{(11.7 \times 10^{-6})(40)(200 \times 10^9)} \right)}$$

$$= 2 \left(\sqrt{3.84 \times 10^{-5} + 5.3418 \times 10^{-5}} \right)$$

10 mm

Pb-263-

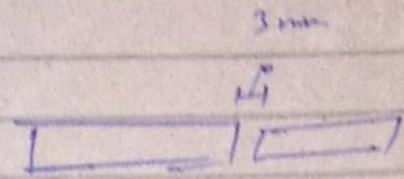
~~L = 7m~~

$$L = 10m$$

$$D = 3mm$$

$$T_i = 15^\circ C$$

at what temp
will it just touch?



Sol:- As we know that;
Free expansion due to temp change is;

$$\delta_T = \alpha \Delta T L$$

$$\Delta T = \frac{\delta_T}{\alpha L} \Rightarrow 25.64^\circ C$$

$$T_f = 25.64 + 15 \Rightarrow 40.64^\circ C$$

(b) What stress would be induced if there
no initial clearance.

$$\text{As, } \delta = \delta_T - \delta_P$$

$$\delta_P = \delta_T$$

Since $\delta = 0$.
No clearance

$$\frac{\delta L}{E} = \alpha \Delta T L$$

$$\delta = E \alpha \Delta T$$

$$\delta = \alpha E (T_f - T_i)$$

$$\delta = (11.7 \times 10^{-6}) (200 \times 10^3) (40.64 - 15)$$

$$\boxed{\delta = 60 \text{ MPa}} \text{ Ans}$$

Pb-264:-

$$Q = 3'$$

$$A = 0.25 \text{ in}^2$$

$$P = 1200 \text{ lb}$$

$$T_i = 40^\circ F.$$

$$E = 29 \times 10^6 \text{ psi}$$

$$\alpha = 6.5 \times 10^{-6} / ^\circ \text{F}$$

(2) Temp at which stress in bar will

$b_c = 10 \text{ ksi}$

As;

we know That;

$$S_{\text{Total}} = S_T + S_P$$

At $\delta_{total} = 0$ Since rigid support

$$\delta p = \delta T \Rightarrow \frac{\delta L}{L} = 2 \delta T$$

$$\Delta T = \frac{10 \times 10^3}{29 \times 10^6 (6.5 \times 10^6)}$$

$$\Delta T = 53^\circ\text{C} \Rightarrow \frac{T_{f2} - 53^\circ\text{C} - 40^\circ\text{C}}{T_{f2} - 13^\circ\text{C}}$$

⑥ Temp at which stress will be zero.
to; simply;

Simply i

$$\delta T = \delta p.$$

$$\frac{PV}{AE} = 2 \Delta T \sqrt{\rho} \Rightarrow \Delta T = \frac{P}{2 \sqrt{\rho} AE}$$

$$ST = 25.64 \quad \bar{f} =$$

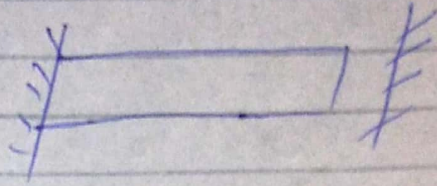
Pb-265:-

$$l = 3\text{m}$$

$$A = 320\text{mm}^2$$

$$T_i = 20^\circ\text{C}$$

$$D = 25\text{mm}$$



$$D = 25\text{mm}$$

If at which
compressive stress
in bar will be 35MPa.

$$\sigma = 35\text{MPa}$$

$$E = 80\text{GPa} \quad \alpha = 18 \times 10^{-6}/^\circ\text{C}$$

Sol:- As we know

$$\delta = \delta_T + \delta_P \quad (\text{As Temp. decrease})$$

$$25 = \alpha \Delta T L + \frac{PL}{AE} \Rightarrow \alpha \Delta T L = \frac{\sigma L}{E}$$

$$25 + \frac{35 \times 10^6 (3 \times 10^3)}{80 \times 10^9} = 18 \times 10^{-6} (3 \times 10^3) \Delta T$$

$$25 + 1.3125 =$$

$$26.3125 = 0.054 \Delta T$$

$$70.6^\circ\text{C}$$

$$\Delta T = 487.22$$

$$T_f = 487.22 + 20$$

$$T_f = 507.22^\circ\text{C}$$
$$50.6^\circ\text{C}$$

Note:-

In case of expansion. $\rightarrow (T \uparrow)$

$$\delta = \delta_T + \delta_P$$

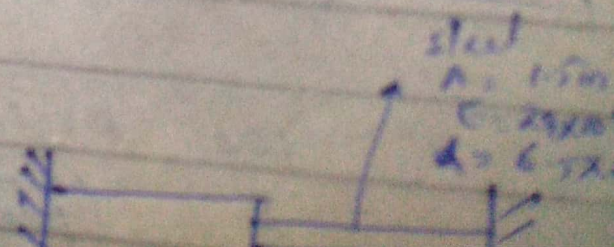
In case of contraction ($T \downarrow$)

$$\delta = \delta_T - \delta_P$$

Pb-266:-

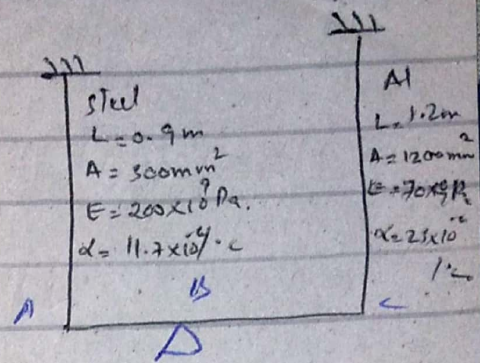
$\delta = ?$ (Each segment)

$$\Delta T = 100^\circ\text{C}$$

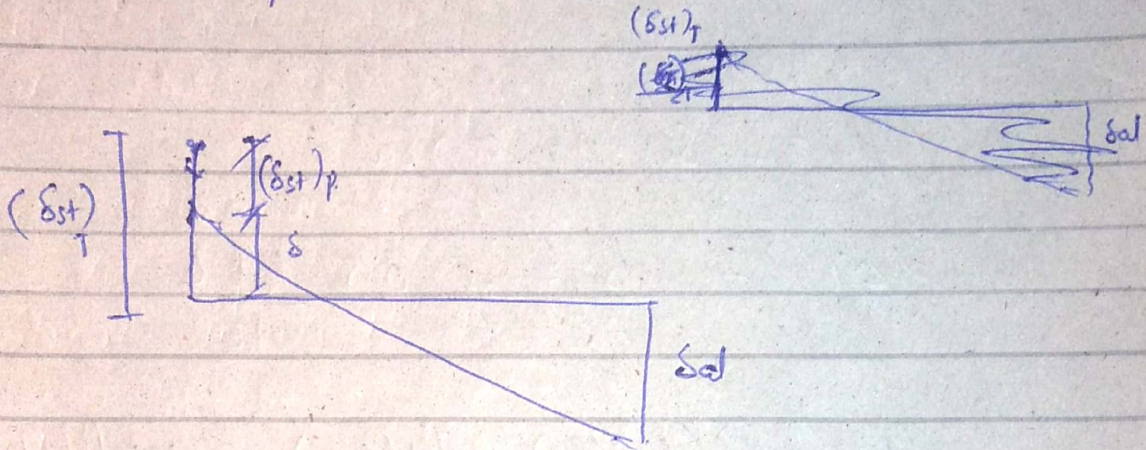


1/b - 268:-

Temp decrease $\Rightarrow$
 steel to 40°C .
 $\delta_{al} = ?$



Sol:- As temp decrease.

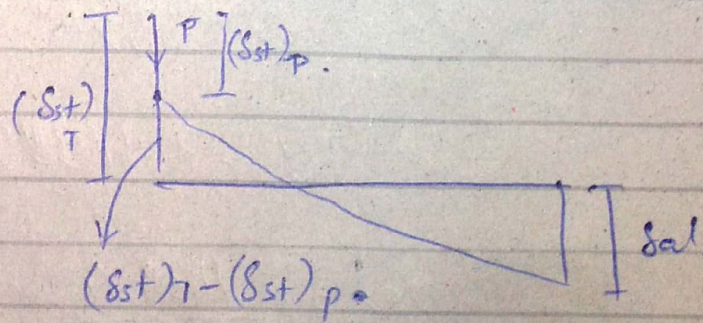


$$(\sigma_{st})_T = (\sigma_{st})_P = \delta_{al}$$

$$(\sigma_{st})_P + \delta_{al} = (\sigma_{st})_T$$

From similarity
 $\Rightarrow \Delta s$.

$$\frac{(\sigma_{st})_T - \sigma_{st}}{0.6} = \frac{\delta_{al}}{1.2}$$



$$(\sigma_{st})_T - (\sigma_{st})_P = 0.5 \delta_{al} \rightarrow \text{①}$$

Pb-269:-

Given data:

$$t = 10^\circ\text{C}$$

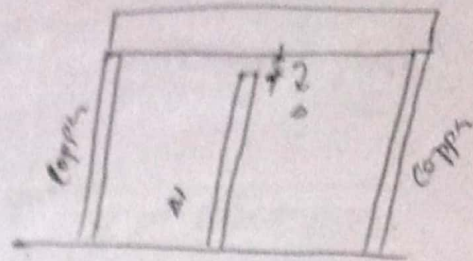
$$a = \text{gap} = 0.18 \text{ mm}$$

Required data:

Calculate stress in each

rod if temperature increased

$$\text{to } 95^\circ\text{C}$$



For copper bar

$$A = 500 \text{ mm}^2$$

$$E = 120 \text{ GPa}$$

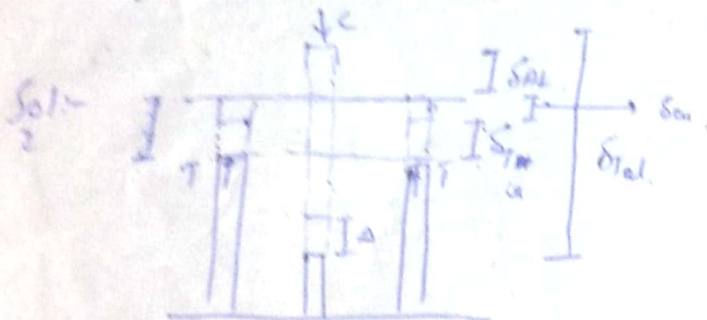
$$\alpha = 16.8 \times 10^{-6} / ^\circ\text{C}$$

For aluminum bar

$$A = 400 \text{ mm}^2$$

$$E = 70 \text{ GPa}$$

$$\alpha = 23.1 \times 10^{-6} / ^\circ\text{C}$$



From above fig.

By compatibility;

$$\delta_{al} = \Delta + \delta_{cu} + \delta_{cu} + \delta_{al} \rightarrow \text{①}$$

$$\begin{aligned} \delta_{al} &= \frac{P}{E_{al}} \times L \\ &= \frac{(23.1 \times 10^{-6}) (95 - 10) (750)}{70 \times 10^9} \\ \delta_{al} &= \end{aligned}$$

Since;

$$\delta T_{al} = \alpha_{al} \Delta T_{al} L_{al}$$

$$= (23.1 \times 10^{-6}) (95 - 10) (750 - 0.18)$$

$$\boxed{\delta T_{al} = 1.472 \text{ mm}}$$
 change in length in aluminium due to increase of Temp.

$$\delta T_{cu} = \alpha_{cu} \Delta T_{cu} L_{cu}$$

$$= (16.8 \times 10^{-6}) (95 - 10) (750)$$

$$\boxed{\delta T_{cu} = 1.071 \text{ mm}}$$

eq (1)

$$\delta T_{al} = \Delta + \delta T_{cu} + \delta c_{al} + \delta a_{al}$$

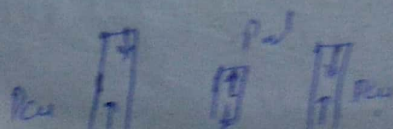
$$1.472 = 0.18 + 1.071 + \left(\frac{PL}{AE} \right)_{cu} + \left(\frac{PL}{AE} \right)_{al}$$

$$0.221 = \frac{P_{cu} (750)}{(400)(70 \times 10^9)} + \frac{P_{al} (750 - 0.18)}{(500)(120 \times 10^9)}$$

$$0.221 = 2.67 \times 10^{-5} P_{cu} + P_{al} (1.249 \times 10^{-3})$$

$$P_{al} = 17,684.2 - 2.13 \times 10^4 P_{cu} \rightarrow (1)$$

Draw FBD



$$\sum F_y = 0 (1+)$$

$$P_{cu} + P_{cu} - P_{al} = 0$$

$$2P_{cu} - P_{al} = 0$$

$$2P_{cu} - (17684.2 - 2.1377P_{cu}) = 0$$

$$2P_{cu} - 17684.2 + 2.1377P_{cu} = 0$$

$$4.1377P_{cu} = 17684.2$$

$$P_{cu} = 4273.91$$

$$P_{al} = 8547.819$$

$$\sigma_{al} = \frac{8547.819}{400} = \boxed{21.36 \text{ MPa}}$$

$$\sigma_{cu} = \frac{4273.91}{500} = \boxed{\sigma_{cu} = 8.5 \text{ MPa}}$$

$$\frac{P}{E} = \frac{271}{2}$$

Given data:

Weight $w = 80 \text{ kN}$

Stress in steel $= \sigma_{st} = 55 \text{ MPa}$

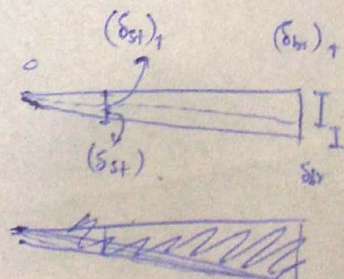
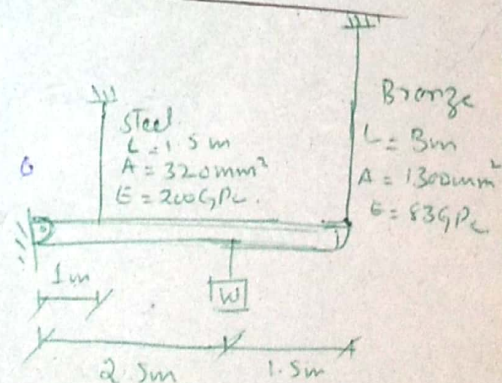
$$\alpha_{st} = 11.7 \times 10^{-6} / ^\circ\text{C}$$

$$\alpha_{br} = 18.8 \times 10^{-6} / ^\circ\text{C}$$

Required $\Delta T = ?$

Sol:-

2 Draw diagram when temp change
Let suppose increases. Then
draw when load is applied



Using similarity of triangle

$$\frac{(\delta_{st})_T + \delta_{st}}{1} = \frac{(\delta_{br})_T + \delta_{br}}{4}$$

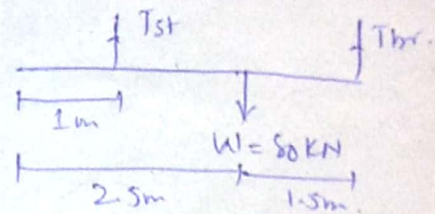
$$(\delta_{st})_T + \delta_{st} = \frac{(\delta_{br})_T}{4} + \frac{\delta_{br}}{4} \rightarrow (1)$$

Since,

$$\frac{(\delta_{st})_T}{1} = \sigma_{st} = 55 \text{ MPa} \rightarrow (2)$$

For $\sigma_{br} = ?$

$$\sum M_0 = 0$$



$$T_{st}(1) + 4T_{br} - W \times 2.5 = 0$$

$$T_{st} + 4T_{br} = (80 \times 10^3)(2.5)$$

$$T_{st} + 4T_{br} = 2 \times 10^5 \rightarrow (3)$$

$$\sigma_{st} A_{st} + 4(\sigma_{br} A_{br}) = 2 \times 10^5$$

$$T_{st} = \sigma_{st} A_{st}$$

$$T_{br} = \sigma_{br} A_{br}$$

$$(55)(320) + 4(\sigma_{br})(1300) = 2 \times 10^5$$

$$\sigma_{br} = 35.6769 \text{ MPa}$$

2. (1) ~~st~~

$$(\delta_{st})_T + \delta_{st} = \frac{(\delta_{br})_T}{4} + \frac{\delta_{br}}{4}$$

$$\left(\Delta T L \right)_{st} + \left(\frac{PL}{AE} \right)_{st} = 0.25 \left(\left(\Delta T L \right)_{br} + \left(\frac{PL}{AE} \right)_{br} \right)$$

$$(\alpha \Delta T L)_{st} - 0.25(\alpha \Delta T L)_{br} = 0.25 \left(\frac{\delta L}{E} \right)_{br} - \left(\frac{\delta_{st} L}{E} \right)$$

$$(11.7 \times 10^{-6}) \Delta T (1500) - 0.25 (18.9 \times 10^{-6}) (\Delta T) (3000) = \frac{0.25 (35.08 \times 10^6) (3000)}{83 \times 10^9} - \frac{55 (1500)}{200 \times 10^9}$$

$$\Delta T (0.01755 - 0.014175) = 0.31698 - 0.4125$$

$$= \frac{-0.09552}{3.36 \times 10^{-3}}$$

$$\boxed{\Delta T = -28.3^\circ \text{C}} \quad \text{Ans}$$

1/b - 272:-

Given data:-

$W = 120 \text{ kN}$

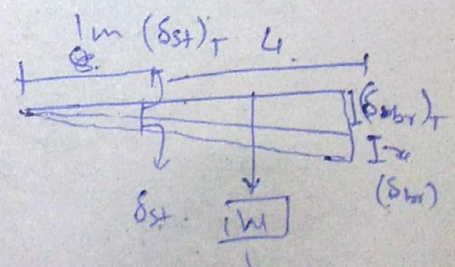
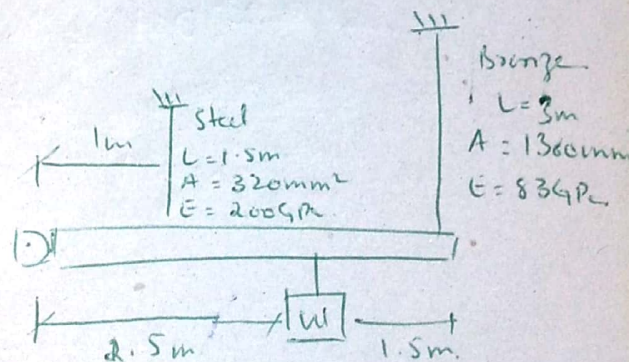
$\Delta T = 30^\circ \text{C}$

Find stresses in each rod = ?

Sol:- As we know that after apply load the beam will deform mean inclined.

Using similarity of triangle.

$$\delta_{st} + (\delta_{st})_T = (\delta_m) \quad \dots \delta_m$$



$$(\delta_{st})_T + \delta_{st} = 0.25((\delta_{br})_T + \delta_{br})$$

$$(\delta_{st})_T = \alpha \Delta T$$

$$(\alpha \Delta T L)_{T_{st}} + \left(\frac{PL}{AE}\right)_{st} = 0.25 \left((\alpha \Delta T L)_{br} + \left(\frac{PL}{AE}\right)_{br} \right)$$

$$(11.7 \times 10^{-6})(30)(1300) + \frac{\delta_{st}(1500)}{200 \times 10^3} = 0.25(18.9 \times 10^{-6})(30)(3000)$$

$$0.5268 + 7.5 \times 10^{-3} \delta_{st} = 0.4252 + \frac{0.25(\delta_{br}(3000))}{83 \times 10^3}$$

$$0.1013 + 7.5 \times 10^{-3} \delta_{st} = 9 \times 10^{-3} \delta_{br}$$

$$7.5 \times 10^{-3} \delta_{st} = 9 \times 10^{-3} \delta_{br} - 0.1013$$

$$\delta_{st} = 1.2 \delta_{br} - 13.5066 \rightarrow \textcircled{1}$$

Draw FBD of fig:-

$$\sum M_O = 0$$

$$P_{st}(1) + P_{br}(4) - 120(2.5) = 0$$

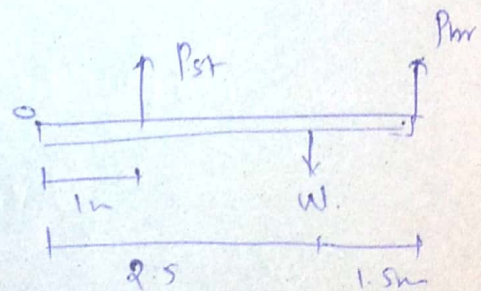
$$P_{st} + 4P_{br} - 300000 = 0$$

$$\delta_{st}(A_{st}) + 4(\delta_{br} A_{br}) - 300000 = 0$$

$$\delta_{st}(320) + 4(\delta_{br}(1300)) - 300000 = 0$$

$$320 \delta_{st} + 5200 \delta_{br} - 300000 = 0$$

$$\delta_{st} = 937.5 - 16.25 \delta_{br} \rightarrow \textcircled{2}$$



Comparing σ_1 & σ_2 (1)

$$1.2\sigma_{br} - 13.5066 = 937.5 - 16.25\sigma_{br}$$

~~1.25~~

$$1.2\sigma_{br} - 13.5 - 937.5 + 16.25\sigma_{br} = 0$$

$$17.45\sigma_{br} - 951 = 0$$

$$\sigma_{br} = \frac{951}{17.45}$$

$$\boxed{\sigma_{br} = 54.5 \text{ MPa}}$$

$$\sigma_{st} = 1.2\sigma_{br} - 13.5066$$

$$= 1.2(54.5) - 13.5066$$

$$\sigma_{st} = 65.398 - 13.5066$$

$$\boxed{\sigma_{st} = 51.89 \text{ MPa}}$$

B sol. answer is wrong

Pb:- 273

The composite bar.

Given data;

$$P = 50 \text{ kips}$$

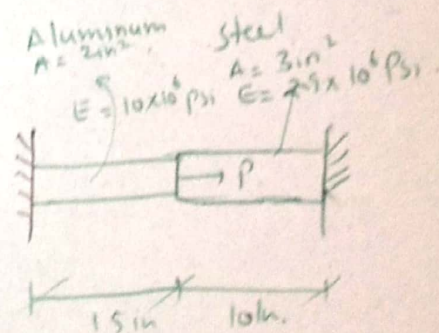
$$T_1 = 60^\circ\text{F}$$

$$T_2 = 120^\circ\text{F}$$

stress in each member = ?

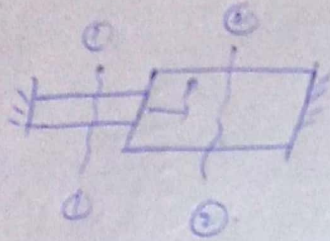
$$\sigma_{st} = 6.5 \times 10^6 / \text{in}^2$$

$$\sigma_{al} = 12.8 \times 10^6 / \text{in}^2$$

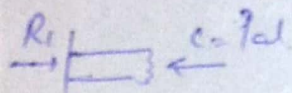


Solution:-

Divide each
Section into Two
Parts, i.e

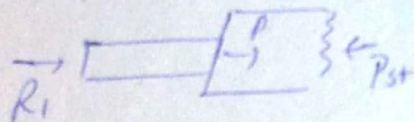


Take Section ①-①.



$$R_1 = P l$$

section - ②-②.



$$R_1 + P = 2 P_{st}$$

$$P_{st} = R_1 + P \rightarrow \text{①}$$

From rough work sketch.

$$(\delta_{st})_T + (\delta_{al})_T = \delta_{al} + \delta_{st}$$

$$(\alpha \Delta T L)_{st} + (\alpha \Delta T L)_{al} = \left(\frac{P L}{A E} \right)_{st} + \left(\frac{P L}{A E} \right)_{al}$$

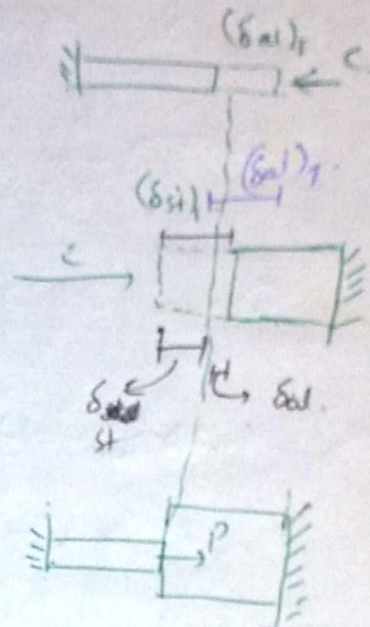
$$\Rightarrow (6.5 \times 10^{-6})(60)(10) + (12.8 \times 10^{-6})(60)(15) = \frac{P_{st}(10)}{3(2.9 \times 10^6)} + \frac{P_{al}(15)}{2(10 \times 10^6)}$$

$$3.9 \times 10^{-3} + 0.0115 = (1.15 \times 10^{-7}) P_{st} + 7.5 \times 10^{-6} P_{al} \rightarrow \text{②}$$

Rough Work

As Temp Increase in
each segment will
increase its length.

For Aluminum



Final
Position

Since:

$$P_{al} = R_1$$

$$\sum P_{st} = R_1 + P$$

Using these eqn in eq (2)

$$0.0154 = (1.15 \times 10^7)(R_1 + P) + 7.5 \times 10^6(R_1)$$
$$= (1.15 \times 10^7)(R_1 + 50000) + 7.5 \times 10^6 R_1$$

$$0.0154 = 1.15 \times 10^7 R_1 + 5.75 \times 10^3 + 7.5 \times 10^6 R_1$$

$$9.65 \times 10^3$$
$$0.01539 = 7.615 \times 10^6 R_1$$

$$\boxed{R_1 = 1267.23 \text{ lb}}$$

$$P_{al} = R_1 = 1267.23 \text{ lb}$$

$$P_{st} = R_1 + 50000 = 51267.23 \text{ lb}$$

Stress in steel:-

$$\sigma_{st} = \frac{51267.23}{3}$$

$$\boxed{\sigma_{st} = 17089.1 \text{ psi}}$$

Stress in Al:-

$$\sigma_{al} = \frac{1267.23}{2}$$

$$\boxed{\sigma_{al} = 633.617 \text{ psi}}$$