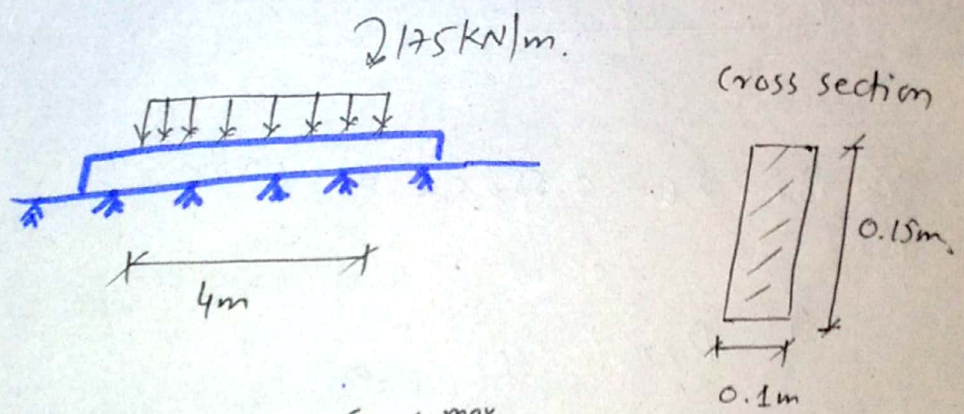
(*). Problems on last lecture:-

Pb#1 A very long beam of rectangular cross section 0.1m width & 0.15m depth is subjected to uniform loading over 4m of its length. Magnitude of load is 175 KN/m. The beam is supported on elastic foundation having modulus $K = 14\,MPa$ & $E = 200\,GPa$.

Diagram:-

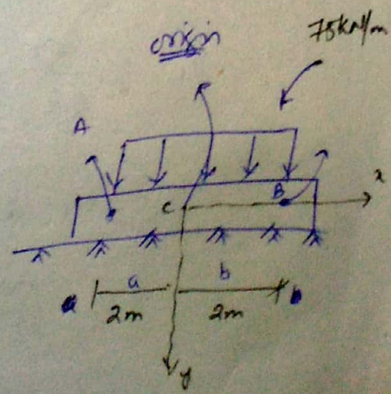


a) Calculate y_{max}, M_{max} & δ_f^{max}.

b) Calculate the max. rxn forces as well.

Solution:-

As we know that the max. deflection will occur under the load. The force as resultant for UDL will act as centre C as shown.



As for max deflection due to UDL

$$y_c = \frac{q}{2k}\left[2 - D_{\lambda a} - D_{\lambda b}\right] \rightarrow ①.$$

$$\& \; M_c = \frac{q}{4\lambda^2}\left[B\lambda a + B\lambda B\right].$$

As max. deflection will occur at middle where $x = 0$.

So;

$$\lambda^2 = \sqrt{\frac{K}{4EI}} \Rightarrow \lambda = \sqrt[4]{\frac{14\times10^6 \times 12}{4 * 200\times10^9 (0.1)(0.15)^3}}$$

$$\boxed{\lambda = 0.888\,m^{-1}}$$

$$\lambda a = \lambda b = 0.888 \times 2 = 1.776\,m^{-1}.$$

As $D\lambda x = e^{-\lambda x}[\cos\lambda x]$.

$x = a = 2$.

By putting values.

$$D a\lambda = D b\lambda = e^{-1.776}\cos[1.776)$$

$$\boxed{D\lambda a = D\lambda b = \cancel{0.169}\; -0.0345}$$

$\because q$ is uniformly distributed loa

$\&$ (1)=)

$$y_c = \frac{q}{2k}\left[2 - D\lambda a - D\lambda b\right].$$

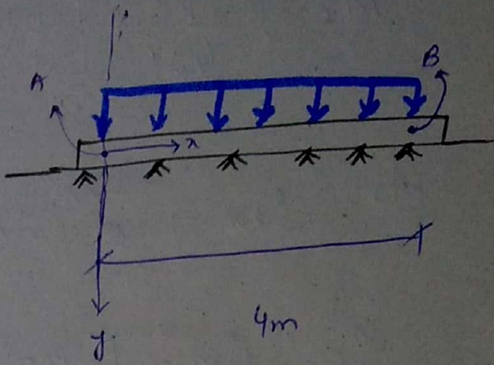
$$y_c = \frac{175\times10^3}{2\times140\times10^6}\left[2 - (-0.0345) - (-0.0345)\right]$$

$$\boxed{y_c = 0.01\,mm}$$

to find deflection at point A.

As point C is still in load so we can used same equation for UDL.

So;



$$y_c = \frac{q}{2k}\left(2 - D_{\lambda A} - D_{\lambda B}\right) \rightarrow (A')$$

$x_A = 0.$

a = distance of point A from origin.

$a = 0 \quad , b = 4$

b = Distance of point B from origin.

$\lambda a = a * \lambda = 0.$

$\lambda b = b * \lambda = 0.888 \times 4$

$\lambda b = 3.552 \, m^{-1}$

$D_{\lambda a} = e^{-\lambda a}\left[\cos \lambda a\right] = 1.$

$D_{\lambda b} = e^{-\lambda b}\left[\cos \lambda b\right] = e^{-3.552}\left[\cos(3.552)\right].$

$\boxed{D_{\lambda b} = -0.02628}$

of (A') $\Rightarrow$ $y_c = \frac{175 \times 10^3}{2 \times 140 \times 10^6}\left[2 - 1 - (-0.02628)\right]$

$\boxed{y_c = 0.00064}$

To find moment:-

As;

$$M_c = \frac{q}{4\lambda^2}\left[B\lambda a + B\lambda b\right] \rightarrow (B)$$

As $B\lambda x = e^{-\lambda x} \sin \lambda x.$

So; $B\lambda a = B\lambda b = e^{-1.776} \sin(1.776)$

$$\boxed{B\lambda a = B\lambda b = 0.165}$$

$\{ (B) \Rightarrow$

$$M_c = \frac{175\times10^3}{4(0.888)^2}\left[0.165+0.165\right]$$

$$\boxed{M_c = 18393.58 \text{ N mm.}}$$

(b) To find σ_f^{max}

$$\sigma_f^{max} = \frac{M_{max} c}{I}$$

$c = \frac{0.15}{2}$

$c = 0.075.$

$M_{max} = 18393.58$ Nm

$$= \frac{18393.58 * 0.075 * 12}{(0.1)(0.15)^3}.$$

$$\boxed{\sigma_f^{max} = 49.04 MPa}$$ Ans

(b) Max. rxn force b/w soil & beam.

As $P = ky$. Max. force will be when y is max.

$P_{max} = k\, y_{max}$

$P_{max} = 14 \times 10^6 \times 0.0129$

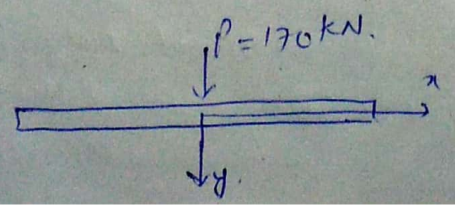
$\boxed{P_{max} = 180600\ N}$

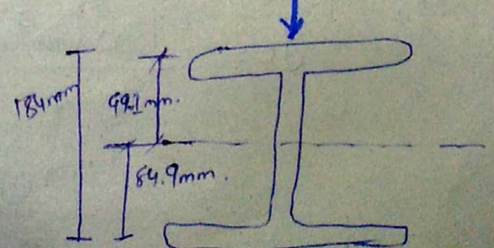
or $\boxed{P_{max} = 180.6\ kN}$

Pb# A rail road uses steel rails ($E = 200\ GPa$) with a depth of 184mm. The distance from top of rail to centroid is 99.1mm & $I = 36 \times 10^6\ mm^4$. Rail is supported by ties, ballast & a road bed that together are assumed to act as elastic foundation with modulus $k = 14\ N/mm^2$.

(a) Determine y_{max}, M_{max} & σ_{max}. Assume only one wheel $P = 170kN$.

Sol: As cross section of rail is I section





As Concentrated load P_o

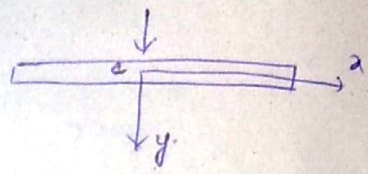
So max deflection will take place under load

The deflection for point load is given by;

$$y = \frac{P\lambda \; A_{\lambda x}}{2k} \longrightarrow ①$$

Let take deflection under load. where $x = 0$.

To find $\lambda = ?$



As $\lambda = \sqrt[4]{\frac{K}{4EI}}$

$$\lambda = \sqrt[4]{\frac{14}{4(200\times10^3)(36\times10^6)}} = \left(\frac{14}{4(200\times10^3)(36\times10^6)}\right)^{1/4}$$

$\lambda = 0.000835\ mm^{-1}$

So; $\lambda x = 0$.

$A_{\lambda x} = e^{-\lambda x}[\cos\lambda x + \sin\lambda x] = e^{0}[\cos 0 + \sin 0] = 1$.

So; $\boxed{A_{\lambda x} = 1}$

$C_{\lambda x} = e^{-\lambda x}[\cos\lambda x - \sin\lambda x] = e^{0}[\cos 0 - \sin 0] = 1$.

So; eq ① ⇒

$$y_c = \frac{170\times10^3}{2\times14} \times 1 \times \lambda$$

$$= \frac{170\times10^3}{28} \times 0.000835$$

$$\boxed{y_{max} = 5.06 \, mm}$$

To find $M_{max} = ?$

As $M_{max} = \frac{P}{4\lambda} C_\lambda x.$

By putting values.

$$M_{max} = \frac{170 \times 10^3}{4 \times (0.000835)} \times 1$$

$$\boxed{M_{max} = 50.89 \times 10^6 \, Nmm}$$

To find $\sigma_f^{max} = ?$

As $\sigma_f^{max} = \frac{M_{max} \times C_{max}}{I}$

$$= \frac{50.8 \times 10^6 \times 99.1}{36 \times 10^6}$$

∴ Max flexural stress will occur at a point where moment is max.

$$\boxed{\sigma_f^{max} = 140.11 \, MPa}$$

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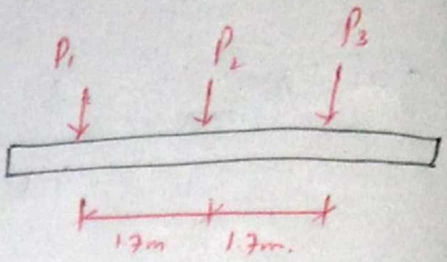
(b) Calculate y_{max}, M_{max} & σ_f^{max} when there are three wheels each of magnitude 170 kN That are equally spaced at 1.7m.

Here we used principle of superposition.

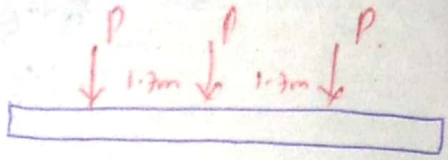


As;

$$y = \frac{P\lambda A_{\lambda x}}{2k} \longrightarrow (A)$$

$P_1 = P_2 = P_3 = 170kN.$

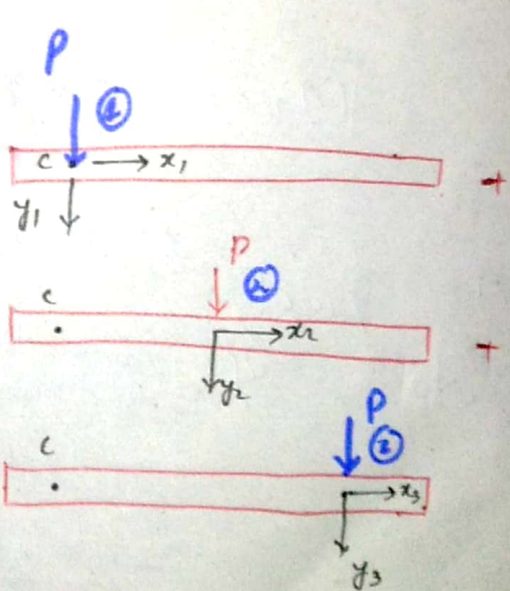
Principle of Superposition.



&

$$M = \frac{P}{4\lambda} C_{\lambda x} \longrightarrow (B).$$

Deflection & moment at point 1.



As

$$\lambda = \sqrt[4]{\frac{k}{4EI}}$$

$$\lambda = \left(\frac{14}{4\times 200\times10^3 \times 36\times10^6}\right)^{1/4}$$

$$\boxed{\lambda = 0.000535 m^{-1}}$$

Note# Do not used concept of mechanics of solid in this case to direct judge to determine deflection, max. moment etc.

For Load 1:-

$x = 0.$ $\lambda x = 0$

$A_{\lambda x} = 1$

$C_{\lambda x} = 1$ > As calculated

So;

$$y_1 = \frac{170\times10^3}{2\times14}\times 0.000835\times 1$$

$$\boxed{y_1 = 5.06\,mm}$$

∴ Distance is take from load to origin.

For load #2

$X = -1.7m = -1700mm$

$\lambda x = -1700\times0.000835 = -1.4195$.

So;

$$A_x = e^{-\lambda x}[\cos\lambda x + \sin\lambda x]$$

$$= e^{1.4195}(\cos(-1.4195)+\sin(-1.4195)].$$

$$\boxed{A\lambda x = 0.2797}$$

$$\boxed{C\lambda x = -0.2018}$$

For load 3 :-

$$\boxed{x = -3.4m}$$

After calculation we get

$$\boxed{A\lambda x = -0.0377}$$

$$\boxed{C\lambda x = -0.0752}$$