Exp 4 (3)

EXP # 4

Q $\displaystyle \lim_{\substack{x \to 0 \\ y \to 0}} \frac{y^2 - x^2}{y^2 + x^2}$

Sol :-

$$l_1 \Rightarrow \lim_{x \to 0} \left(\lim_{y \to 0} \frac{y^2 - x^2}{y^2 + x^2} \right) = -1$$

$$l_2 \Rightarrow \lim_{y \to 0} \left(\lim_{x \to 0} \frac{y^2 - x^2}{y^2 + x^2} \right) \Rightarrow 1$$

$l_1 \neq l_2$ hence limit does not exist

Ex 8

$$\lim_{\substack{x \to \infty \\ y \to 2}} \frac{xy + 4}{x^2 + 2y^2}$$

$$\Rightarrow \lim_{x \to \infty} \left(\frac{2x + 4}{x^2 + 8} \right) \Rightarrow$$

Taking x common from the numerator & x^2 from denominator

$$\lim_{x \to \infty} \left(\frac{1}{x} \right) \frac{\left(2 + \frac{4}{x}\right)}{\left(1 + \frac{8}{x^2}\right)}$$

$$\Rightarrow 0 \left(\frac{2 + 0}{1 + 0} \right) = 0$$

$l_2 \Rightarrow$

$$\lim_{y \to 2} \left(\lim_{x \to \infty} \frac{\frac{y}{x} + \frac{4}{x}}{1 + 2\left(\frac{y}{x}\right)^2} \right)$$

$l_2 \Rightarrow 0$

Ex: 5: $\lim_{x\to 0} \frac{x^3-y^3}{x^2+y^2}$

Sol: $\lim_{x\to 0}\left(\lim_{y\to 0}\frac{x^3-y^3}{x^2+y^2}\right)$

$\lim_{x\to 0}\left(\frac{x^3-(0)^2}{x^2+(0)^2}\right)$

$\lim_{x\to 0}\left(\frac{x^3}{x^2}\right) = \lim_{x\to 0} x = \boxed{l_1 = 0}$

Now $\lim_{y\to 0}\left(\lim_{x\to 0}\frac{x^3-y^3}{x^2+y^2}\right)$

$\lim_{y\to 0}\left(\frac{0-y^3}{0+y^2}\right)$

$\lim_{y\to 0} -y \Rightarrow \boxed{l_2 = 0}$

Now: $y = mx$

$\lim_{x\to 0}\left(\lim_{y\to mx}\frac{x^3-y^3}{x^2+y^2}\right)$

$\lim_{x\to 0}\left(\left(\frac{x^3-m^3x^3}{x^2+m^2x^2}\right)\right) = \frac{x^3}{x^2}\left(\frac{1-m^3}{1+m^2}\right) = x\left(\frac{1-m^3}{1+m^2}\right)$

$\lim_{x\to 0} 0\left(\frac{x-m^3}{1+m^2}\right) \Rightarrow 0\left(\frac{\cancel{0}-m^3}{1+m^2}\right) = \boxed{l_3 = 0}$

$l_3 = \frac{-m^3}{1+m^2}$ now $m \in \mathbb{R}$ so l_3 can attained any val

(10) Now put $y = mx^2$

$\lim_{x\to 0}\left(\frac{x^3-m^3x^6}{x^2+m^2x^4}\right) = \frac{x^2(x+m^3x^4)}{x^2(1+m^2x^2)} = \boxed{0 = l_4}$

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Exp #6

Evaluate $\lim\limits_{\substack{x\to 1\\ y\to 2}} \dfrac{x^2+2y}{x+y^2}$

Sol

$$l_1 = \lim_{x\to 1}\left(\lim_{y\to 2} \frac{x^2+2y}{x+y^2}\right) = \lim_{x\to 1}\left(\frac{x^2+4}{x+4}\right)$$

$$l_1 = \lim_{x\to 1}\left(\frac{x^2+4}{x+4}\right) = \frac{5}{5} = 1$$

$$l_2 = \lim_{y\to 2}\left(\lim_{x\to 1} \frac{x^2+2y}{x+y^2}\right) = \lim_{y\to 2}\left(\frac{1+2y}{1+y^2}\right)$$

$$= \lim_{y\to 2}\left(\frac{1+2y}{1+y^2}\right) = \frac{5}{5} = 1$$

As $l_1 = l_2$ so limit exist.

Example #7

$$\lim_{\substack{x\to\infty \\ y\to 3}} \frac{2x-3}{x^3+4y^3}$$

Sol:-

$$\lim_{\substack{x\to\infty \\ y\to 3}} \frac{2x-3}{x^3+4y^3} \Rightarrow \lim_{\substack{x\to\infty \\ y\to 3}} \frac{x^3(2/x^2 - 3/x^3)}{x^3(1+4y^3/x^3)}$$

$$\lim_{y\to 3}\left(\lim_{x\to\infty} \frac{(2/x^2 - 3/x^3)}{1+4y^3/x^3}\right)$$

$$l_1 = \lim_{y\to 3}\left(\frac{0-0}{1+0}\right) \Rightarrow 0.$$

Now;

$$\cancel{\lim_{\substack{x\to\infty \\ y\to 3}} x^3(2/x^2 -}$$

$$\lim_{\substack{x\to\infty \\ y\to 3}} \frac{2x-3}{x^3+4y^3}$$

$$\lim_{x\to\infty}\left(\lim_{y\to 3} \frac{2(x)-3}{x^3+4(\cancel{27})^3}\right)$$

$$= \lim_{x\to\infty}\left(\frac{2x-3}{x^3+4(3)^3}\right) \Rightarrow \lim_{x\to\infty}\left(\frac{2x-3}{x^3+(27)4}\right)$$

$$l_2 = \lim_{x\to\infty} \frac{x^3(2/x^2 - 3/x^3)}{x^3(1+108)} \Rightarrow \frac{0-0}{1+108} = 0$$

As $l_1 = l_2$ so limit exist at ∞ & 3.

Exp # 08

$$\lim_{\substack{x \to \alpha \\ y \to 2}} \frac{xy+4}{x^2+2y^2}$$

$$l_1 \Rightarrow \lim_{x \to \alpha} \left(\lim_{y \to 2} \frac{xy+4}{x^2+2y^2} \right) = \lim_{x \to \alpha} \left(\frac{2x+4}{x^2+8} \right)$$

$$l_1 = \lim_{x \to \alpha} \left(\frac{x(2+4/x)}{x^2(1+8/x^2)} \right) = \lim_{x \to \alpha} \left(\frac{1}{x} \frac{(2+4/x)}{(1+8/x^2)} \right)$$

$$l_1 = 0$$

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$$l_2 = \lim_{y\to 2}\left(\lim_{x\to \alpha}\frac{xy+4}{x^2+2y^2}\right) = \lim_{y\to 2}\left(\lim_{x\to \alpha}\frac{x(y+4/x)}{x^2\left(1+2\frac{y^2}{x^2}\right)}\right)$$

$$l_2 = \lim_{y\to 2}\left(\lim_{x\to \alpha}\frac{1}{x}\,\frac{(y+4/x)}{1+2y^2/x^2}\right) = \lim_{y\to 2}(0) = 0$$

As

$l_1 = l_2$ So limit exist

(1, 7)

Ex 1.1 :- Q_1 :- $\lim_{\substack{x \to 1 \\ y \to 2}} \frac{2x^2 + y^2}{2xy}$

$\rightarrow l_1$

$\Rightarrow \lim_{x \to 1} \left(\lim_{y \to 2} \frac{2x^2 + y}{2xy} \right) \Rightarrow \lim_{x \to 1} \left(\frac{2x^2 + \cancel{2}}{4x} \right)$

$\Rightarrow \lim_{x \to 1} \frac{2(x^2 + 1)}{2 \cancel{4} x} \rightarrow 1$

$l_2 \Rightarrow \lim_{y \to 2} \left(\lim_{x \to 1} \frac{2x^2 + y^2}{2xy} \right) \Rightarrow \lim_{y \to 2} \left(\frac{2 + y^2}{2y} \right)$

$\rightarrow \frac{6}{4} = \frac{3}{2}$

$l_1 \neq l_2$ limit does not exist

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EXE 1.1 2,5

Q2 $\lim_{\substack{x\to 2\\ y\to 3}} \frac{x^3+y^2}{x^2-y}$

Sol

$$l_1 = \lim_{x\to 2}\left(\lim_{y\to 3} \frac{x^3+y^2}{x^2-y}\right) = \lim_{x\to 2}\left(\frac{x^3+9}{x^2-3}\right)$$

$$l_1 = \lim_{x\to 2}\left(\frac{x^3+9}{x^2-3}\right) = \frac{17}{1} = 17$$

$$l_2 = \lim_{y\to 3}\left(\lim_{x\to 2} \frac{x^3+y^2}{x^2-y}\right) = \lim_{y\to 3}\left(\frac{8+y^2}{4-y}\right)$$

$$l_2 = \lim_{y\to 3}\left(\frac{8+y^2}{4-y}\right) = \left(\frac{8+9}{4-3}\right) = 17$$

Thus

$l_1 = l_2 = 17$

& hence limit exist

3.6 P: 8.

Ex: 1.1.

Q3: $\lim_{\substack{x\to\infty \\ y\to 3}} \frac{2xy-3}{x^3+4y^3}$

27
$\times 4$
108 2

Sol: $\lim_{x\to\infty}\left(\lim_{y\to 3} \frac{2xy-3}{x^3+4y^3}\right)$

$\lim_{x\to\infty}\left(\frac{2x(3)-3}{x^3+4(3)^3}\right) = \lim_{x\to\infty}\left(\frac{6x-3}{x^3+108}\right).$

$l_1 = \lim_{x\to\infty}\left(\frac{6x-3}{x^3+108}\right) \Rightarrow l_1 = \frac{\cancel{x}\left(6-\frac{3}{x}\right)}{\cancel{x}\left(x^2+\frac{108}{x}\right)} = \left(\frac{6-\frac{3}{\infty}}{\infty+\frac{108}{\infty}}\right)$

$\boxed{l_1 = 0}$

Now $l_2 = \lim_{y\to 3}\left(\lim_{x\to\infty} \frac{2xy-3}{x^3+4y^3}\right)$

$l_2 = \lim_{y\to 3}\left(\lim_{x\to\infty} \frac{\cancel{x}\left(2y-\frac{3}{x}\right)}{\cancel{x}\left(x^2+\frac{4y^3}{x}\right)}\right) = l_2 = \lim_{y\to 3}\left(\frac{2y-\frac{3}{\infty}}{(\infty)^2+\frac{4y^3}{\infty}}\right)$

$l_2 = \lim_{y\to 3}\left(\frac{2y}{\infty}\right) \Rightarrow \boxed{l_2 = 0}$

Now $l_1 = l_2$ - so limit exist

Q4:- $\lim\limits_{\substack{x \to 0 \\ y \to 0}} \frac{xy}{y - x^2}$; $x \neq 0$, $y \neq 0$.

Sol:- $l_1 = \lim\limits_{y \to 0} \left(\lim\limits_{x \to 0} \left(\frac{xy}{y - x^2} \right) \right) = \lim\limits_{y \to 0} (0) = 0.$

$l_2 = \lim\limits_{x \to 0} \left(\lim\limits_{y \to 0} \left(\frac{xy}{y - x^2} \right) \right) = \lim\limits_{x \to 0} (0) = 0.$

l_3: $y = mx$.

$$l_3 = \lim_{x \to 0} \left(\frac{x(mx)}{mx - x^2} \right) = \lim_{x \to 0} \left(\frac{mx^2}{mx - x^2} \right).$$

$$= \lim_{x \to 0} \frac{\cancel{x^2}}{\cancel{x^2}} \left(\frac{m}{m/x - 1} \right).$$

$$= \lim_{x \to 0} \left(\frac{m}{-1} \right) = -m.$$

l_4: $y = mx^2$. Put value of y.

$$l_4 = \lim_{x \to 0} \left(\frac{x(mx^2)}{mx^2 - x^2} \right) \Rightarrow \lim_{x \to 0} \frac{mx^3}{mx^2 - x^2}.$$

$$= \lim_{x \to 0} \frac{\cancel{x^2}(mx)}{\cancel{x^2}(m-1)} \Rightarrow \lim_{x \to 0} \frac{mx}{m-1}$$

$$= 0.$$

As $l_1 = l_2 \neq l_3 = l_3$. So limit does not exist.

Q5 $$\lim_{\substack{x\to 0\\ y\to 0}} \frac{x-y}{x^2+y^2}$$

$$l_1 = \lim_{x\to 0}\left(\lim_{y\to 0} \frac{x-y}{x^2+y^2}\right) = \lim_{x\to 0}\left(\frac{x}{x^2}\right) = \lim_{x\to 0}\left(\frac{1}{x}\right)$$

$$l_1 = \lim_{x\to 0}\left(\frac{1}{0}\right) = \infty$$

$$l_2 = \lim_{y\to 0}\left(\lim_{x\to 0} \frac{x-y}{x^2+y^2}\right) = \lim_{y\to 0}\left(\frac{-y}{y^2}\right)$$

$$\lim_{y\to 0}\left(\frac{-1}{y}\right) = -\infty$$

As $l_1 \neq l_2$ so limit does not exist

Q:3, Ex 1.2 can't solved

P: 11. Ex: 1.2

Q:6.

Show that funct.

$$f(x,y) = \begin{cases} 2x^2+y, & (x,y) \neq (1,2) \\ 0 & (x,y) = (1,2) \end{cases}$$ is

discountinous at (1,2)

Sol: $\lim_{\substack{x\to 1\\ y\to 2}} f(x,y) = 2x^2+y$.

$l_1 = \lim_{x\to 1} (\lim_{y\to 2} 2x^2+y)$

$l_1 = \lim_{x\to 1} (2x^2+2)$

$l_1 = 2(1)+2 \Rightarrow \boxed{l_1 = 4}$

Now $\lim_{\substack{y\to 2\\ x\to 1}} = \lim_{y\to 2} (\lim_{x\to 1} 2x^2+y)$

$l_2 = \lim_{y\to 2} (2(1)+y)$

$l_2 = \lim_{y\to 2} (2+y)$

$\boxed{l_2 = 4.}$

Now ~~fun~~ $f(0,0) = 0$

so $l_1 = l_2 \neq f(0,0)$.

so funtion $f(x,y)$ is discontinou at (1,2)

Put $y = mx$

$\lim_{\substack{x\to 2\\ y\to 2}} 2x^2+mx$.

$l_3 = \lim_{x\to 2} (2(2)^2+2m)$

$l_3 = \cancel{\lim} \; 8+2m$

$l_3 = 8+2m$.

Put $y = mx^2$

$l_4 = \lim_{\substack{x\to 2\\ y\to mx^2}} (2x^2+y)$

$l_4 = \lim_{x\to 2} (2x^2+mx^2)$

$l_4 = 2(2)^2+m2^2$

$l_4 = 8+4m$

Here 'm' can attain diff ~~erent~~ valu

so $l_1 = l_2 \neq l_3 \neq l_4$

so does't exist

Q7: $\lim\limits_{\substack{x \to 0 \\ y \to 0}} \dfrac{x^3 + 2y^3}{x^2 + 4y^2}$, $x \neq 0$, $y \neq 0$

$l_1 \Rightarrow$

$$\lim_{x \to 0} \left(\lim_{y \to 0} \frac{x^3 + 2y^3}{x^2 + 4y^2} \right) \Rightarrow \lim_{x \to 0} x \Rightarrow 0$$

$l_2 \Rightarrow$

$$\lim_{y \to 0} \left(\lim_{x \to 0} \frac{x^3 + 2y^3}{x^2 + 4y^2} \right) \Rightarrow \lim_{y \to 0} \frac{y}{2} \Rightarrow 0$$

$l_3 \Rightarrow y = mx$

$$\Rightarrow \lim_{x \to 0} \frac{x^3 + 2m^3x^3}{x^2 + 4m^2x^2} \Rightarrow \mathop{lt}_{x \to 0} \frac{x^3(1 + 2m^3)}{x^2(1 + 4m^2)} = 0$$

$l_4 \Rightarrow y = mx^2 \Rightarrow \lim\limits_{x \to 0} \dfrac{x^3 + 2m^3x^6}{x^2 + m^2x^4} \Rightarrow \mathop{lt}\limits_{x \to 0} \dfrac{x^3(1 + 2m^3x^3)}{x^2(1 + m^2x^2)} \Rightarrow 0$

$l_1 = l_2 = l_3 = l_4$

hence the limit exists.

Q_8:- $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^2y^3}{x^2+y^2}$, $x\neq 0, y\neq 0$.

Sol:-

$l_1 = \lim\limits_{y\to 0}\left(\lim\limits_{x\to 0} \dfrac{x^2y^3}{x^2+y^2}\right) = \lim\limits_{y\to 0}(0) = 0.$

$l_2 = \lim\limits_{x\to 0}\left(\lim\limits_{y\to 0} \dfrac{x^2y^3}{x^2+y^2}\right) = \lim\limits_{x\to 0}(0) = 0.$

l_3 : Put $y = mx$.

$l_3 = \lim\limits_{x\to 0}\left(\dfrac{x^2(mx)^3}{x^2+(mx)^2}\right).$

$= \lim\limits_{x\to 0} \dfrac{mx^5}{x^2+mx^2} \Rightarrow \lim\limits_{x\to 0} \dfrac{x^2(mx^3)}{x^2(1+m)}.$

$= 0.$

l_4 = Put $y = mx^2$.

$l_4 : \lim\limits_{x\to 0} \dfrac{\left(x^2(mx^2)^3\right)}{x^2+(mx^2)^3} \Rightarrow \lim\limits_{x\to 0} \dfrac{m x^8}{x^2+m^2x^6}.$

$= \lim\limits_{x\to 0} \dfrac{x^2 \cdot mx^6}{x^2(1+m^2x^4)}.$

$= \dfrac{0}{1+0} \Rightarrow 0.$

As $l_1 = l_2 = l_3 = l_4$ So limit exist at $x=0$.

Exp 10

Discuss the continuity of $f(x,y) = \begin{cases} \dfrac{x}{\sqrt{x^2+y^2}} & x \neq 0, y \neq 0 \\ 2 & x = 0, y = 0 \end{cases}$

$l_1 = \lim_{x \to 0} \left(\lim_{y \to 0} \dfrac{x}{\sqrt{x^2+y^2}} \right) = \lim_{x \to 0} (1) = 1$

$l_2 = \lim_{y \to 0} \left(\lim_{x \to 0} \dfrac{x}{\sqrt{x^2+y^2}} \right) = \lim_{y \to 0} \left(0/y \right) = 0$

Since $l_1 \neq l_2$ so it is not continuous at $x=0, y=0$

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Example # 11

$$f(x,y) = \begin{cases} \dfrac{x^2+xy+x+y}{x+y} & (x,y) \neq (2,-2) \\ 4 & (x,y) = (2,-2) \end{cases}$$

Sol:

$$f(x,y) = \begin{cases} \dfrac{x^2+xy+x+y}{x+y}, & (x,y) \neq (2,-2) \\ 4 & (x,y) = (2,-2). \end{cases}$$

$$l_1 = \lim_{(x,y)\to(2,-2)} \frac{x^2+xy+x+y}{x+y}.$$

$$= \lim_{(x,y)\to(2,-2)} \frac{x(x+y)+1(x+y)}{(x+y)}.$$

$$= \lim_{(x,y)\to(2,-2)} \frac{\cancel{(x+y)}(x+1)}{\cancel{(x+y)}}.$$

$$l_1 = \lim_{(x,y)\to(2,-2)} (x+1) = 2+1 = 3.$$

$$l_2 = 4.$$

As $l_1 \neq l_2$.

so limit does not exist.

EXE 1.2

① $f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & x \neq 0\ y \neq 0 \\ 0 & x=0, y=0 \end{cases}$

Now for continuity $l_1 = l_2 = l_3 =$
So

$$l_1 = \lim_{x\to 0}\left(\lim_{y\to 0} \frac{xy(x^2-y^2)}{x^2+y^2}\right) = \lim_{x\to 0}(0) = 0$$

$$l_2 = \lim_{y\to 0}\left(\lim_{x\to 0} \frac{xy(x^2-y^2)}{x^2+y^2}\right) = \lim_{y\to 0}(0) = 0$$

~~put $y = mx$~~ $\boxed{l_3 = 0}$ given

$$l_3 = \lim_{x\to 0}\left(\frac{x\,mx(x^2-m^2x^2)}{x^2+m^2x^2}\right) = \lim_{x\to 0}\left(\frac{x^2m(x^2-m^2x^2)}{x^2+m^2x^2}\right)$$

$l_3 = 0$

~~put $y = mx^2$~~

$$l_4 = \lim_{x\to 0}\left(\frac{x\,mx^2(x^2-m^2x^4)}{x^2+m^2x^4}\right) = \lim_{x\to 0}(0) = 0$$

As $l_1 = l_2 = l_3 =$

So the function is continuous at the origin

Ex 1.2

(3) $\dfrac{x^3y^3}{x^3+y^3}$ when $x\neq 0$, $y\neq 0$

$0,$ when $x=0$, $y=0$

$$l_1 \Rightarrow \lim_{x\to 0}\left(\lim_{y\to 0}\frac{x^3y^3}{x^3+y^3}\right)=0$$

$$l_2 = \lim_{y\to 0}\left(\lim_{x\to 0}\frac{x^3y^3}{x^3+y^3}\right)=0$$

$$\cancel{l_3 = 0 = 0}$$

$$\cancel{l_4 = \frac{m^3x^6}{x^3(1+m^3)} \Rightarrow \frac{x^3m^3}{(1+m^3)} = 0}$$

$$\Rightarrow l_3 = \lim_{\substack{x\to 0\\ y\to 0}} \lim_{(x,y)\to(0,0)} \Rightarrow \frac{0}{0} = \infty$$

$l_4 = y = mx^2$

$$\frac{x^3m^3x^6}{x^3+m^3x^6} = \frac{x^9m^3}{x^3(1+m^3x^6)}$$

$$\frac{x^6m^3}{1+m^3x^6} = 0$$

Q.4:- $f(x,y) = \begin{cases} x^3+y^3 & \text{when } x \neq 0, y \neq 0 \\ 0 & \text{when } x=0, y=0 \end{cases}$ at origin

Sol:-

$l_1 = \lim_{y\to 0}\left(\lim_{x\to 0}(x^3+y^3)\right) = \lim_{y\to 0}(0) = 0$

$l_2 = \lim_{x\to 0}\left(\lim_{y\to 0}(x^3+y^3)\right) = \lim_{x\to 0}(0) = 0.$

~~$l_3 = \lim_{(x,y)\to(0,0)}(x^3+y^3) = 0.$~~

~~As $l_1 = l_2 = l_3$ so $f(x,y)$ is continous~~

at ~~$x=0$.~~

l_3: $y = mx$.

$\lim_{x\to 0}(x^3+(mx)^3) \Rightarrow \lim_{x\to 0}(x^3+mx^3) = 0.$

l_4: $y = mx^2$.

$\lim_{x\to 0}(x^3+(mx^2)^3) = \lim_{x\to 0}(x^3+mx^6)$.

$= 0.$

$l_1 = l_2 = l_3 = l_4$ so limit exist at origin

Q·3: Ex: 1.2 can't solved

P: 11. Ex: 1-2-

Q:6.

Show that funct.

$$f(x,y) = \begin{cases} 2x^2+y, & (x,y) \neq (1,2) \\ 0 & (x,y) = (1,2) \end{cases}$$ is

discountinous at $(1,2)$

Sol. $\lim_{\substack{x\to1\\y\to2}} f(x,y) = 2x^2+y$.

$l_1 = \lim_{x\to1} \left(\lim_{y\to2} 2x^2+y \right)$

$l_1 = \lim_{x\to1} (2x^2+2)$

$l_1 = 2(1)+2 \Rightarrow \boxed{l_1 = 4}$

Now $\lim_{\substack{y\to2\\x\to1}} = \lim_{y\to2} \left(\lim_{x\to1} 2x^2+y \right)$

$l_2 = \lim_{y\to2} (2(1)+y)$

$l_2 = \lim_{y\to2} (2+y)$

$\boxed{l_2 = 4.}$

Now ~~f~~ $f(0,0) = 0$

so $l_1 = l_2 \neq f(0,0)$.

so funtion $f(x,y)$ is discontinou at $(1,2)$

put $y = mx$

$\lim_{\substack{x\to2\\y\to2}} 2x^2+mx.$

$l_3 = \lim_{x\to2} (2(2)^2+2m)$

$l_3 = \cancel{\lim} \; 8+2m$

$l_3 = 8+2m.$

Put $y = mx^2$

$l_4 = \lim_{\substack{x\to2\\y\to mx^2}} (2x^2+y)$

$l_4 = \lim_{x\to2} (2x^2+mx^2)$

$l_4 \; 2(2)^2+m2^2$

$l_4 = 8+4m$

Here 'm' can attain different val-
so $l_1 = l_2 \neq l_3 \neq l_4$
so doesn't exist

Exp 15

$U = (1-2xy+y^2)^{-1/2}$ prove That

$$x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2 U^3$$

Now

$$\frac{\partial u}{\partial x} = -\frac{1}{2}(1-2xy+y^2)^{-3/2}(-2y$$

$$= \frac{y}{(1-2xy+y^2)^{3/2}}$$

$$x\frac{\partial u}{\partial x} = \frac{xy}{(1-2xy+y^2)^{3/2}} \quad \text{--- } 1$$

Also

$$\frac{\partial u}{\partial y} = -\frac{1}{2}(1-2xy+y^2)^{-3/2}(-2x+2y)$$

$$= \frac{-\cancel{2}(x-y)}{(1-2xy+y^2)^{3/2}}\left(-\frac{1}{\cancel{2}}\right)$$

$$\frac{\partial u}{\partial y} = \frac{x-y}{(1-2xy+y^2)^{3/2}}$$

$$y\frac{\partial u}{\partial y} = \frac{xy-y^2}{(1-2xy+y^2)^{3/2}} \quad \text{--- } 2$$

$$(1) - (2) \Rightarrow x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} = \frac{xy}{(1-2xy+y^2)^{3/2}} - \frac{(xy-y^2)}{(1-2xy+y^2)^{3/2}}$$

$$= \frac{xy}{(1-2xy+y^2)^{3/2}} - \frac{xy+y^2}{(1-2xy+y^2)^{3/2}}$$

$$= \frac{1}{(1-2xy+y^2)^{3/2}}\left[\cancel{xy} - \cancel{xy} + y^2\right]$$

$$= \cancel{\frac{1}{v^{3/2}}(y^2)} = \cancel{\frac{y^2}{v^{3/2}}}$$

$$= \frac{y^2}{\left((1-2xy+y^2)^{-\frac{1}{2}}\right)^{-3}} = \frac{y^2}{v^{-3}} = y^2v^3$$

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Example # 16

$z = e^{ax+by} f(ax-by)$. prove That,

$$b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$$

Sol:-

$$\frac{\partial z}{\partial x} = e^{ax+by}(a).f(ax-by) + e^{ax+by}.f'(ax-by)(a)$$

$$= a\left(e^{ax+by} f(ax-by) + e^{ax+by} f'(ax-by)\right).$$

$$b\frac{\partial z}{\partial x} = abe^{ax+by}\left(f(ax-by) + f'(ax-by)\right).$$

$\hookrightarrow$ (1).

$$\frac{\partial z}{\partial y} = e^{ax+by}(b).f(ax-by) + e^{ax+by} f'(ax-by)(-b).$$

$$= e^{ax+by} b\left(f(ax-by) - f'(ax-by)\right).$$

$$a\frac{\partial z}{\partial y} = abe^{ax+by}\left(f(ax-by) - f'(ax-by)\right)$$

Now add.

$$\Rightarrow b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = abe^{ax+by}\left(f(ax-by) + f'(ax-by)\right)$$
$$+ abe^{ax+by}\left(f(ax-by) - f'(ax-by)\right)$$

$$= abe^{ax+by}\left[f(ax-by) + \cancel{f'(ax-by)} + f(ax-by) - \cancel{f'(ax-by)}\right]$$

$$= abe^{ax+by}\left(2f(ax-by)\right).$$

$$= 2ab\,\underbrace{e^{ax+by}\left(f(ax-by)\right)}_{z}$$

$$= 2abz \text{ proved}$$

Exp 18

$$z = \tan(y+ax) + (y-ax)^{3/2} \quad \text{Show that}$$

$$\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$$

Sol

$$z = \tan(y+ax) + (y-ax)^{3/2}$$

$$\frac{\partial z}{\partial x} = \sec^2(y+ax)a + (3/2)(y-ax)^{1/2}(-a)$$

$$= a\sec^2(y+ax) - \frac{3}{2}a(y-ax)^{1/2}$$

$$\frac{\partial^2 z}{\partial x^2} = a^2 2\sec^2(y+ax)\tan(y+ax) + \frac{3}{4}a^2(y-ax)^{-1/2}$$

1

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RHS of $\frac{\partial z}{\partial y} = \sec^2(y+ax) + \frac{3}{2}(y-ax)^{1/2}$

$$\frac{\partial^2 z}{\partial y^2} = 2\sec^2(y+ax)\tan(y+ax) + \frac{3}{4}(y-ax)^{-1/2}$$

$$a^2\frac{\partial^2 z}{\partial y^2} = 2a^2\sec^2(y+ax)\tan(y+ax) + \frac{3}{4}a^2(y-ax)^{-1/2} \quad (2)$$

from (1) & (2)

Proved.

Example #6

$u = u(y-z, z-x, x-y)$, prove That;

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$$

Sol:- let $r = y-z$, $s = z-x$ & $t = x-y$.

From formula.

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial u}{\partial s}\cdot\frac{\partial s}{\partial x} + \frac{\partial u}{\partial t}\cdot\frac{\partial t}{\partial x}.$$

$$= \frac{\partial u}{\partial r}(0) + \frac{\partial u}{\partial s}(-1) + \frac{\partial u}{\partial t}\cdot(+1).$$

$$= -\frac{\partial u}{\partial s} + \frac{\partial u}{\partial t} \longrightarrow (1).$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial u}{\partial s}\cdot\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\cdot\frac{\partial t}{\partial y}$$

$$= \frac{\partial u}{\partial r}(+1) + \frac{\partial u}{\partial s}(0) + \frac{\partial u}{\partial t}(-1).$$

$$= +\frac{\partial u}{\partial r} - \frac{\partial u}{\partial t} \longrightarrow (2).$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial z} + \frac{\partial u}{\partial s}\cdot\frac{\partial s}{\partial z} + \frac{\partial u}{\partial t}\cdot\frac{\partial t}{\partial z}.$$

$$= \frac{\partial u}{\partial r}(-1) + \frac{\partial u}{\partial s}(1) + \frac{\partial u}{\partial t}(0).$$

$$= -\frac{\partial u}{\partial r} + \frac{\partial u}{\partial s} \longrightarrow (3).$$

Add eq (1), (2) & (3).

$$= \cancel{-\frac{\partial u}{\partial s}} + \cancel{\frac{\partial u}{\partial t}} - \cancel{\frac{\partial u}{\partial t}} + \cancel{\frac{\partial u}{\partial r}} - \cancel{\frac{\partial u}{\partial r}} + \cancel{\frac{\partial u}{\partial s}}$$

$$= 0 \quad \text{Ans}$$